WBJEE - 2024

Answer Keys by

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MATHEMATICS

Q.No.	○	✚	✿	□
01	A	B	C	A
02	B	B	C	C
03	B	C	B	C
04	B	D	D	D
05	A	B	B	D
06	D	D	C	A
07	B	B	A	B
08	A	C	C	C
09	A	B	A	D
10	C	B	C	D
11	A	C	B	D
12	D	B	C	B
13	C	A	B	B
14	C	B	C	C
15	B	A	A	C
16	A	B	B	B
17	B	A	C	D
18	C	D	A	B
19	C	B	D	C
20	C	A	A	B
21	A	A	C	B
22	C	C	C	B
23	B	B	D	B
24	A	D	A	A
25	B	C	B	B
26	C	C	C	A
27	A	C	D	B
28	B	A	D	A
29	C	B	C	D
30	C	A	D	C
31	D	C	B	A
32	A	C	B	A
33	C	B	B	C
34	B	C	C	B
35	D	C	B	D
36	A	A	D	C
37	D	B	C	B
38	C	C	B	C
39	D	A	B	A
40	B	B	B	C
41	C	B	B	A
42	D	C	A	C
43	B	D	A	C
44	D	A	B	B
45	B	A	B	B
46	C	B	A	C
47	B	D	D	A
48	B	D	C	B
49	C	D	A	C
50	B	C	A	A
51	C	B	D	A
52	C	A	B	D
53	B	A	A	A
54	D	B	D	B
55	A	C	D	B
56	D	C	D	A
57	D	D	A	A
58	D	B	D	C
59	A	A	A	C
60	A	D	A	B
61	D	D	B	D
62	A	D	B	A
63	A	D	A	D
64	B	A	C	D
65	B	A	C	D
66	B,D	A,C	B,C	D
67	A,B	C,D	A,D	A,B,C
68	A,D	A,B	A,C	C,D
69	B,C	B,D	A,C	A,C
70	A,C	B,C	A,B,C	B,D
71	A,C	A,D	D	A,B
72	D	A,C	A,C	A,D
73	A,B,C	A,C	C,D	B,C
74	C,D	A,B,C	A,B	A,C
75	A,C	D	B,D	A,C

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ANSWERS & HINTS
for
WBJEE - 2024
SUB : MATHEMATICS

CATEGORY - 1 (Q:1 to Q50)

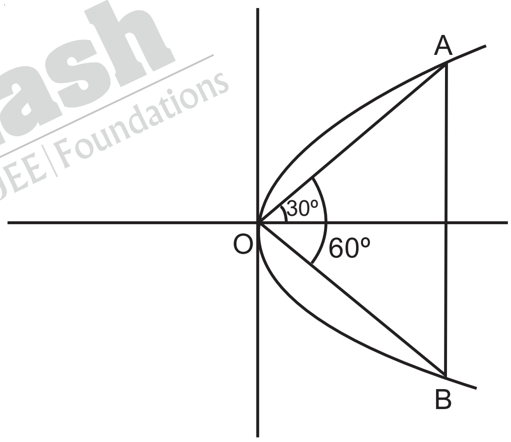
(Carry 1 mark each. Only one option is correct. Negative marks: – ¼)

1. ΔOAB is an euilateral triangle inscribed in the parabola $y^2 = 4ax$, $a > 0$ with O as the vertex, then the length of the side of ΔOAB is

(A) $8a\sqrt{3}$ unit (B) $8a$ unit (C) $4a\sqrt{3}$ unit (D) $4a$ unit

Ans : (A)

Hint : Slope of OA : $M_{OA} = \tan 30^\circ = \frac{1}{\sqrt{3}}$



Equation of OA : $y = \frac{1}{\sqrt{3}}x$ from parabola

and line OA , $\frac{x^2}{3} = 4ax$

$\Rightarrow x = 12a \Rightarrow y = 4\sqrt{3}a \quad \therefore AB = 2 \times 4\sqrt{3}a = 8a\sqrt{3}$ unit

2. The plane $2x - y + 3z + 5 = 0$ is rotated through 90º about its line of intersection with the plane $x + y + z = 1$. The equation of the plane in new position is

(A) $3x + 9y + z + 17 = 0$ (B) $3x + 9y + z = 17$ (C) $3x - 9y - z = 17$ (D) $3x + 9y - z = 17$

Ans : (B)

Hint : Equation of a plane passing through the line of intersection of the given planes is

$2x - y + 3z + 5 + \lambda(x + y + z - 1) = 0$

$\Rightarrow (2 + \lambda)x - (1 - \lambda)y + (3 + \lambda)z + 5 - \lambda = 0$ this will be perpendicular to the plane $2x - y + 3z + 5 = 0$

$\Rightarrow (2+\lambda)2 + (1-\lambda)\times 1 + 3(3+\lambda) = 0 \quad \Rightarrow \lambda = \frac{-7}{2}$

$\therefore$ Equation of plane $3x + 9y + z = 17$

3. If the relation between the direction ratios of two lines in $\mathbb{R}^3$ are given by $l + m + n = 0$, $2\,lm + 2\,mn - ln = 0$ then the angle between the lines is (l, m, n have their usual meaning)

(A) $\frac{\pi}{6}$ (B) $\frac{2\pi}{3}$ (C) $\frac{\pi}{2}$ (D) $\frac{\pi}{4}$

Ans : (B)

Hint : Quadratic in $\frac{m}{n} : 2\left(\frac{m}{n}\right)^2 - \frac{m}{n} - 1 = 0 \Rightarrow \frac{m}{n} = 1, -\frac{1}{2}$

$\Rightarrow$ n = m or n = – 2m & $l = -m - n$, $\Rightarrow l : m : n \Rightarrow -2 : 1 : 1$ or $1 : 1 - 2$

$\therefore$ Angle = $\cos^{-1}\left(\frac{-2}{\sqrt{6}}\frac{1}{\sqrt{6}} + \frac{1}{\sqrt{6}}.\frac{1}{\sqrt{6}} + \frac{1}{\sqrt{6}}\left(\frac{-1}{\sqrt{6}}\right)\right) = \cos\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$

4. If U_n (n = 1, 2) denote the n^{th} derivative (n = 1, 2) of U(x) = $\frac{Lx + M}{x^2 - 2Bx + C}$ (L, M, B, C are constants), then $PUI_2 + QU_1 + RU = 0$, holds for

(A) $P = x^2 - 2B$, $Q = 2x$, $R = 3x$
(B) $P = x^2 - 2Bx + C$, $Q = 4((x - B)$, $R = 2$
(C) $P = 2x$, $Q = 2B$, $R = 2$
(D) $P = x^2$, $Q = x$, $R = 3$

Ans : (B)

Hint : U(x) = $\frac{Lx + m}{x^2 - 2Bx + c}$, $U(x)(x^2 - 2Bx + c) = Lx + m$

$U(x)(2x - 2B) + (x^2 - 2Bx + c)\,U_1(x) = L$

$U(x) \times 2\,(2x - 2B)xU_1(x) + (2x - 2B)U_1(x) + (x^2 - 2Bx + c)\,U_2(x) = 0$

$\Rightarrow U_2(x^2 - 2B + c) + U_1\,(4x - 4B) + 2U = 0$

$\therefore P = x^2 - 2Bx + c$, $\theta = 4\,(x - B)$, $R = 2$

5. For every real number $x \neq -1$, let $f(x) = \frac{x}{x+1}$. Write $f_1(x) = f(x)$ & for $n \geq 2$, $f_n(x) = f(f_{n-1}(x))$. Then $f_1(-2).\,f_2(-2)$ .... must be

(A) $\frac{2^n}{1.3.5......(2n-1)}$ (B) 1 (C) $\frac{1}{2}\binom{2n}{n}$ (D) $\binom{2n}{n}$

Ans : (A)

Hint : $f(x) = \frac{x}{x+1}$, $f_2(x) = \frac{\frac{x}{x+1}}{\frac{x}{x+1} + 1} = \frac{x}{2x+1}$

$f_3(x) = \frac{x}{3x+1}$, $\Rightarrow f_1(-2).f_2(-2)$ .... $f_n(-2)$

$= \frac{-2}{-1} \times \frac{-2}{-3} \times \frac{-2}{-5} \times .... \times \frac{-2}{(2n-1)} = \frac{2^n}{1.3.5....\,(2n-1)}$

6. If α, β are the roots of the equation $ax^2 + bx + c = 0$ then $\lim\limits_{x \to \beta} \dfrac{1 - \cos(ax^2 + bx + c)}{(x - \beta)^2}$ is

(A) $(\alpha - \beta)^2$ (B) $\dfrac{1}{2}(\alpha - \beta)^2$ (C) $\dfrac{a^2}{4}(\alpha - \beta)^2$ (D) $\dfrac{a^2}{2}(\alpha - \beta)^2$

Ans : (D)

Hint : $ax^2 + bx + c = a(x - \alpha)(x - \beta)$

$$\lim_{x \to \beta} \frac{1 = \cos(ax^2 + bx + c)}{(x - \beta)^2}$$

$$= \lim_{x \to \beta} \frac{1 - \cos(a(x - \alpha)(x - \beta))}{(x - \beta)^2}$$

$$= \lim_{x \to \beta} \frac{2\sin^2\left(\dfrac{a(x-\alpha)(x-\beta)}{2}\right)}{(x-\beta)^2} = \lim_{x \to \beta} \frac{2\sin^2\left(\dfrac{a(x-\alpha)(x-\beta}{2}\right)}{\dfrac{a^2(x-\alpha)^2(x-\beta)^2}{4}} \times \frac{a^2(x-\alpha)^2}{4}$$

$$= 2 \times 1x \frac{a^2(\beta - \alpha)^2}{4} = \frac{a^2(\alpha - \beta)^2}{2}$$

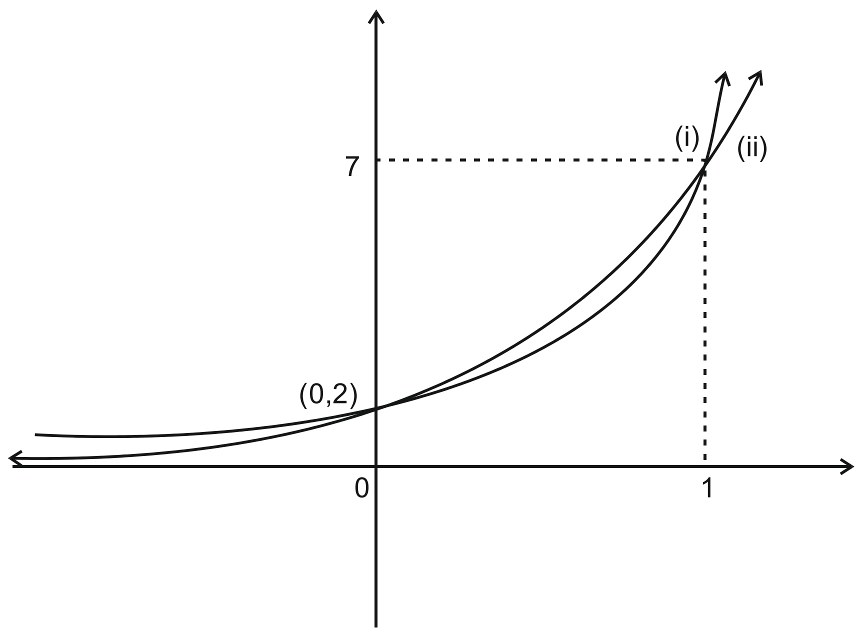
7. The equation $2^x + 5^x = 3^x + 4^x$ has

(A) no real solution (B) only one non-zero real soltuion

(C) infinitely many solutions (D) only three non-negative real soltuions

Ans : (B)*

Hint :



(i) $y = 2^x + 5^x$

(ii) $y = 3^x + 4^x$

option (B)*

* x = 0 and 1 are the roots

$\Rightarrow$ one non zero real solution

8. Consider the function $f(x) = (x-2)\log_e.x$. Then the equation $x\log_e x = 2-x$

(A) has at least one root in (1, 2)
(B) has no root in (1, 2)
(C) is not at all solvable
(D) has infinitely many roots in (–2, 1)

Ans : (A)

Hint : Let $g(x) = x\log_e x - 2 + x$

$g'(x) = 1 + \log_e x + 1 = 2 + \log_e x$

$g(1).g(2) = -1\times(2\log_e 2) = -ve$

9. If $\int \frac{\log_e(x+\sqrt{1+x^2})}{\sqrt{1+x^2}}dx = f(g(x)) + c$ then

(A) $f(x) = \frac{x^2}{2}, g(x) = \log_e(x+\sqrt{1+x^2})$
(B) $f(x) = \log_e(x+\sqrt{1+x^2}), g(x) = \frac{x^2}{2}$
(C) $f(x) = x^2, g(x) = \log_e(x+\sqrt{1+x^2})$
(D) $f(x) = \log_e(x-\sqrt{1+x^2}), g(x) = x^2$

Ans : (A)

Hint : Let $\log(x+\sqrt{1+x^2}) = t \Rightarrow \frac{\frac{\sqrt{1+x^2}+x}{\sqrt{1+x^2}}}{x+\sqrt{1+x^2}}dx = dt$

$\Rightarrow \frac{dx}{\sqrt{1+x^2}} = dt \quad \therefore I = \int \frac{\log(x+\sqrt{1+x^2})}{\sqrt{1+x^2}}dx = \int t.dt$

$I = \frac{t^2}{2} + c = \frac{\left[\log(x+\sqrt{x^2+1})\right]^2}{2} + c$

$\therefore f(x) = \frac{x^2}{2}, g(x) = \log_e(x+\sqrt{1+x^2})$

10. If $f(x) = \frac{e^x}{1+e^x}, I_1 = \int_{f(-a)}^{f(a)} x\, g(x(1-x))\, dx$ and $I_2 = \int_{f(-a)}^{f(a)} g(x(1-x))dx$, then the value of $\frac{I_2}{I_1}$ is

(A) –1
(B) –3
(C) 2
(D) 1

Ans : (C)

Hint : $f(a) + f(-a) = \frac{e^a}{1+e^a} + \frac{e^{-a}}{1+e^{-a}} = 1$

Let $f(-a) = t \therefore f(a) = 1-t$

Now, $I_1 = \int_t^{1-t} xg(x(1-x))dx = \int_t^{1-t}(1-x)g((1-x)(1-(1-x))dx$

$$=\int_t^{1-t}(1-x)g(x(1-x))dx$$

$$\therefore 2I_1=\int_t^{1-t}g(x(1-x))dx=I_2 \Rightarrow \frac{I_2}{I_1}=2$$

11. Let $f:\mathbb{R}\to\mathbb{R}$ be a differentiable function and f(1) = 4. Then the value of $\lim_{x\to 1}\int_4^{f(x)}\frac{2t}{x-1}dt,$ if $f'(1) = 2$ is

(A) 16 (B) 8 (C) 4 (D) 2

Ans : (A)

Hint : $\lim_{x\to 1}\int_4^{f(x)}\frac{2t}{x-1}dt$, $=\lim_{x\to 1}\left[\frac{t^2}{x-1}\right]_4^{f(x)}$

= $=\lim_{x\to 1}\frac{[f(x)]^2-4^2}{x-1}$ $\left(\frac{0}{0}\text{ form}\right)$ $[\because f(1)=4]$

$=\lim_{x\to 1}\frac{2f(x)f'(x)}{1}$ = 2f(1) f′(1) = 2(4) (2) = 16

12. If $xy' + y - e^x = 0$, y(a) = b, then $\lim_{x\to 1}$ y(x) is

(A) $e + 2ab - e^a$ (B) $e^2 + ab - e^{-a}$ (C) $e - ab + e^a$ (D) $e + ab - e^a$, $\left(y'=\frac{dy}{dx}\right)$

Ans : (D)

Hint : $xy' + y - e^x = 0$

$\Rightarrow$ $x\frac{dy}{dx}+y=e^x$

$\Rightarrow$ $x\,dy + y\,dx = e^x\,dx$

$\Rightarrow$ $\int d(xy)=\int e^x dx$

$\Rightarrow$ $xy = e^x + c$

$y(a) = b \Rightarrow c = ab - e^a$

$\therefore$ $y(x) = \frac{e^x+ab-e^a}{x}$

$\therefore$ $\lim_{x\to 1} y(x) = e + ab - e^a$

13. All values of a for which the inequality $\frac{1}{\sqrt{a}}\int_1^a\left(\frac{3}{2}\sqrt{x}+1-\frac{1}{\sqrt{x}}\right)dx<4$ is satisfied, lie in the interval

(A) (1, 2) (B) (0, 3) (C) (0, 4) (D) (1, 4)

Ans : (C)

Hint : $\frac{1}{\sqrt{a}}\int_1^a\left(\frac{3}{2}\sqrt{x}+1-\frac{1}{\sqrt{x}}\right)dx<4$

$\Rightarrow \quad \frac{1}{\sqrt{a}}\cdot\left[\frac{3}{2}\cdot\frac{x^{3/2}}{\frac{3}{2}}+x-\frac{x^{1/2}}{1/2}\right]_1^a<4 \quad \Rightarrow \frac{1}{\sqrt{a}}\left(a^{3/2}+a-2a^{1/2}-1-1+2\right)>4$

$\Rightarrow \quad \frac{1}{a^{1/2}}\cdot\left(a^{3/2}+a-2a^{1/2}\right)<4 \quad \Rightarrow a+a^{1/2}-2<4$

$\Rightarrow \quad \left(a^{1/2}\right)^2+a^{1/2}-6<0 \quad \Rightarrow \left(a^{1/2}+3\right)\left(a^{1/2}-2\right)<0$

but, $a^{1/2}+3\not<0 \quad \therefore\ a^{1/2}-2<0 \ \Rightarrow a < 4$ and $a > 0 \ \therefore\ a \in (0, 4)$

14. For any integer n, $\int_0^{\pi} e^{\cos^2 x}\cdot\cos^3(2n+1)x\,dx$ has the value

(A) π (B) 1 (C) 0 (D) $\frac{3\pi}{2}$

Ans : (C)

Hint : $I=\int_0^{\pi}e^{\cos^2 x}.\cos^3(2n+1)x\,dx \quad \Rightarrow \int_0^{\pi}e^{\cos^2(\pi-x)}.(\cos(2n+1)(\pi-x))^3dx$

$\Rightarrow \quad \int_0^{\pi}e^{\cos^2 x}.\left\{\cos\left((2n+1)\pi-(2n+1)x\right)\right\}^3dx \Rightarrow -\int_0^{\pi}e^{\cos^2 x}.\cos^3(2n+1)x\,dx$

$= \quad -I \qquad \Rightarrow 2I = 0 \ \Rightarrow I = 0$

15. Let f be a differential function with $\lim_{x\to\infty} f(x) = 0$. If $y' + yf'(x) - f(x)f'(x) = 0$, $\lim_{x\to\infty} y(x) = 0$ then

(A) $y + 1 = e^{f(x)} + f(x)$ (B) $y + 1 = e^{-f(x)} + f(x)$ (C) $y + 2 = e^{-f(x)} + f(x)$ (D) $y - 1 = e^{-f(x)} + f(x)$

Ans : (B)

Hint : $y'+yf'(x)-f(x)f'(x)=0 \quad \Rightarrow \frac{dy}{dx}+yf'(x)=f(x)f'(x) \ \rightarrow$ Linear DE $\frac{dy}{dx}+py=Q$ form

$\therefore \quad$ If $= \int_e f'(x)dx=e^{f(x)} \quad \therefore\ ye^{f(x)}=\int e^{f(x)}.f(x).f'(x)dx \ = \int e^{f(x)}.f(x).d(f(x))$

$= \quad f(x).e^{f(x)}-e^{f(x)}+c \ \Rightarrow y=f(x)-1+ce^{-f(x)} \ \Rightarrow y+1=f(x)+c.e^{-f(x)}$

$= \quad \lim_{x\to\infty}y(x)=\lim_{x\to\infty}\left(f(x)-1+c.e^{-f(x)}\right) = 0 \ \Rightarrow 0-1+c.\,e^0=0 \ \Rightarrow c=1$

from (1), $y + 1 = f(x) + e^{-f(x)}$

16. Let y = f(x) be any curve on the X–Y plane & P be a point on the curve. Let C be a fixed point not on the curve. The length PC is either a maximum or a minimum, then

(A) PC is perpendicular to the tangent at P
(B) PC is parallel to the tangent at P
(C) PC meets the tangent at an angle of 45°
(D) PC meets the tangent at an angle of 60°

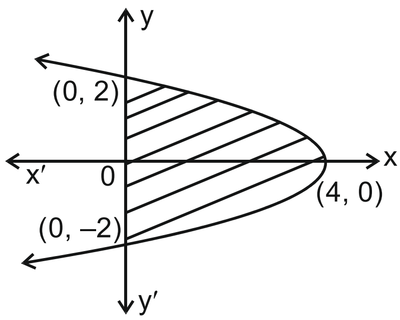
Ans : (A)

17. The are bounded by the curves $x = 4 - y^2$ and the Y-axis is

(A) 16 sq. unit
(B) $\frac{32}{3}$ sq. unit
(C) $\frac{16}{3}$ sq. unit
(D) 32 sq. unit

Ans : (B)

Hint : $x = 4 - y^2 \Rightarrow y^2 = -x + 4$



$x = 0 \Rightarrow y^2 = 4 \Rightarrow y = \pm 2$

Area bounded by the curve $x = 4 - y^2$ and y – axis is,

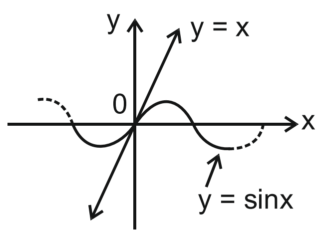
$$= 2\int_0^2 x\,dy = 2\int_0^2 \left(4 - y^2\right)dy = 2\left[4y - \frac{y^3}{3}\right]_0^2 = 2\left(8 - \frac{8}{3}\right) = \frac{32}{3} \text{ sq. unit}$$

18. $F(x) = \cos x - 1 + \frac{x^2}{2!}$, $x \in \mathbb{R}$. Then f(x) is

(A) decreasing function
(B) increasing function
(C) neither increasing nor decreasing
(D) constant $\forall$ x > 0

Ans : (C)

Hint : $f(x) = \cos x - 1 + \frac{x^2}{2!}, x \in R \Rightarrow f'(x) = -\sin x - 0 + \frac{2x}{2!} = x - \sin x$



$f'(x) > 0 \quad \forall\ x > 0 \quad \because x > \sin x$

$f'(x) < 0 \quad \forall\ x < 0 \quad \because x < \sin x$

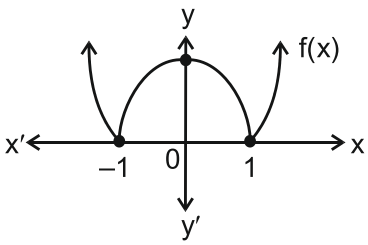
$\therefore$ f(x) is neither increasing nor decreasing .

19. Let $f : \mathbb{R} \to \mathbb{R}$ be given by $f(x) = |x^2 - 1|$, then

(A) f has a local minima at x = ±1 but no local maxima
(B) f has a local maxima at x = 0, but no local minima
(C) f has a local minima at x = ±1 and a local maxima at x = 0
(D) f has niether any local maxima nor any local minima

Ans : (C)

Hint : $f(x) = |x^2 - 1|$



By figure,

f has a local minima at x = ± 1 and a local maxima at x = 0

20. If a particle moves in a straight line according to the law $x = a\sin\left(\sqrt{\lambda}t + b\right)$, then the particle will come to rest at two points whose distance is [symbols have their usual meaning]

(A) a (B) $\frac{a}{2}$ (C) 2a (D) 4a

Ans : (C)

Hint : $\frac{dx}{dt} = a\cos\left(\sqrt{\lambda}t + b\right).\sqrt{\lambda} = 0 \begin{bmatrix} \text{at rest,} \\ \frac{dx}{dt} = 0 \end{bmatrix} \Rightarrow \cos\left(\sqrt{\lambda}t + b\right) = 0$

$\Rightarrow \sqrt{\lambda}t + b = \frac{\pi}{2}, \frac{-\pi}{2}$

$\therefore$ $x_1 = a\sin\frac{\pi}{2}$ and $x_2 = a\sin\left(\frac{-\pi}{2}\right)$

$= a$ $= -a$

$\therefore$ Distance between the points = $|x_2 - x_1|$ = 2a

21. A unit vector in XY-plane making an angle 45° with $\hat{i} + \hat{j}$ and an angle 60° with $3\hat{i} - 4\hat{j}$ is

(A) $\frac{13}{14}\hat{i} + \frac{1}{14}\hat{j}$ (B) $\frac{1}{14}\hat{i} + \frac{13}{14}\hat{j}$ (C) $\frac{13}{14}\hat{i} - \frac{1}{14}\hat{j}$ (D) $\frac{1}{14}\hat{i} - \frac{13}{14}\hat{j}$

Ans : (A)

Hint : Let the vector be $x\hat{i} + y\hat{j} = \vec{r}$ $\vec{a} = \hat{i} + \hat{j}$ and $\vec{b} = 3\hat{i} - 4\hat{j}$

$\vec{r}.\vec{a} = |\vec{r}||\vec{a}|\cos 45° = 1.\sqrt{2}.\frac{1}{\sqrt{2}} = 1$ $\Rightarrow$ x+y = 1 ------ (1)

$\vec{r}.\vec{b} = |\vec{r}||\vec{b}|\cos 60°$ $\Rightarrow 3x - 4y = 1.5.\frac{1}{2}$ $\Rightarrow \frac{2}{5}(3x - 4y) = 1$ $\Rightarrow 6x - 8y = 5$ ----(2)

Solving (1) and (2), 14x = 13 $\Rightarrow x = \frac{13}{14}$ $\therefore y = \frac{1}{14}$

$\therefore$ Unit vector = $\frac{13}{14}\hat{i} + \frac{1}{14}\hat{j}$

22. If for the series $a_1, a_2, a_3, ...$ etc, $a_r - a_{r+1}$ bears a constant ratio with $a_r.a_{r+1}$; then $a_1, a_2, a_3 ....$ are in

(A) A.P. (B) G.P. (C) H.P. (D) Any other series

Ans : (C)

Hint : $\frac{a_r - a_{r+1}}{a_r a_{r+1}} = K(\text{constant}) \Rightarrow \frac{1}{a_{r+1}} - \frac{1}{a_r} = K \quad \therefore \frac{1}{a_1}, \frac{1}{a_2}, \frac{1}{a_3}, ...$ is an A.P

$\Rightarrow a_1, a_2, a_3, ...$ is a H.P

23. Given an A.P. and a G.P. with positive terms, with the first and second terms of the progressions being equal. If a_n and b_n be the n^{th} term of A.P. and G.P. respectively then

(A) $a_n > b_n$ for all $n > 2$ (B) $a_n < b_n$ for all $n > 2$ (C) $a_n = b_n$ for some $n > 2$ (D) $a_n = b_n$ for some odd n

Ans : (B)

Hint : $a_1, a_2, a_3, ..., a_n \rightarrow AP$

$a_1, a_2, b_3, ..., b_n \rightarrow GP$

$a_n = a_1 + (n-1)(a_2 - a_1)$

$$b_n = a_1\left(\frac{a_2}{a_1}\right)^{n-1} = a_2^{n-1}.a_1^{1-n+1} = a_2^{n-1}.a_1^{2-n}$$

for n = 3, $a_3 = a_1 + 2(a_2 - a_1) \Rightarrow a_3 = 2a_2 - a_1$

$b_3 = a_2^2.\ a_1^{-1} = \frac{a_2^2}{a_1}$

$$a_3 - b_3 = 2a_2 - a_1 - \frac{a_2^2}{a_1} = \frac{2a_2a_1 - a_1^2 - a_2^2}{a_1} = \frac{2a_1a_2 - (a_1^2 + a_2^2)}{a_1} < 0 \quad \left[\because a_1^2 + a_2^2 > 2a_1a_2\right]$$

$\therefore a_3 < b_3 \ \therefore a_n < b_n$ for all $n > 2$

24. If $(x^2 \log_x 27).\log_9 x = x + 4$ then the value of x is

(A) 2 (B) $-\frac{4}{3}$ (C) –2 (D) $\frac{4}{3}$

Ans : (A)

Hint : $(x^2 \log_x 27).\log_9 x = x + 4$

$\Rightarrow x^2.\log_9 27 = x + 4 \Rightarrow x^2.\ \log_{3^2} 3^3 = x + 4 \Rightarrow \frac{3}{2}x^2 = x + 4$

$\Rightarrow 3x^2 - 2x - 8 = 0 \Rightarrow 3x^2 - 6x + 4x - 8 = 0 \Rightarrow (3x + 4)(x - 2) = 0$

$x \neq -\frac{4}{3} \ \therefore x = 2$

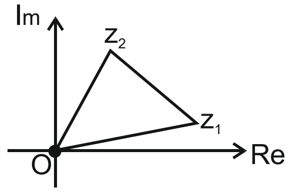
25. If z_1 and z_2 be two roots of the equation $z^2 + az + b = 0$, $a^2 < 4b$, then the origin, z_1 and z_2 form an equilateral triangle if

(A) $a^2 = 3b^2$ (B) $a^2 = 3b$ (C) $b^2 = 3a$ (D) $b^2 = 3a^2$

Ans : (B)

Hint : Here, $z_1 + z_2 = -a$ and $z_1z_2 = b$

$z_2 = z_1 e^{i\pi/3} = z_1\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right) = z_1\left(\frac{1}{2} + i.\frac{\sqrt{3}}{2}\right)$



$\Rightarrow 2z_2 - z_1 = \sqrt{3}\,.iz_1 \Rightarrow (2z_2 - z_1)^2 = -3z_1^2 \Rightarrow 4z_2^2 + 4z_1^2 = 4z_1z_2 \Rightarrow z_1^2 + z_2^2 = z_1z_2$

$\Rightarrow (z_1 + z_2)^2 - 2z_1z_2 = z_1z_2 \Rightarrow a^2 - 2b = b \Rightarrow a^2 = 3b$

26. If $\cos\theta + i\sin\theta$, $\theta \in \mathbb{R}$, is the root of the equation

$a_0x^n + a_1.x^{n-1} + ... + a_{n-1}x + a_n = 0$, $a_0, a_1, ...a_n \in \mathbb{R}, a_0 \neq 0,$

then the value of $a_1\sin\theta + a_2\sin 2\theta + ... + a_n\sin n\theta$ is

(A) 2n (B) n (C) 0 (D) n+1

Ans : (C)

Hint : $a_0x^n + a_1.x^{n-1} + ... + a_{n-1}x + a_n = 0$

divided by x^n

$a_0 + a_1\frac{1}{x} + ...+ a_{n-1}\frac{1}{x^{n-1}} + a_n\frac{1}{x^n} = 0$

put $x = \cos\theta + i\sin\theta$ and equating imaginary part

$a_1\sin\theta + a_2\sin 2\theta + ...+ a_n\sin 2\theta = 0$

27. If a, b, c are distinct odd natural numbers, then the number of rational roots of the equation $ax^2 + bx + c = 0$

(A) must be 0 (B) must be 1

(C) must be 2 (D) cannot be determined from the given data

Ans : (A)

Hint : $D = b^2 - 4ac \geq 0$

for rational roots D is perfect square

$D = b^2 - 4ac$

$\downarrow$

odd – even = odd

Let $D = (2k + 1)^2$

$b^2 - 4ac = (2k + 1)^2$

$4ac = (2n + 1)^2 - (2k + 1)^2$

$= 4(n - k)(n + k + 1)$

$ac = (n - k)(n + k + 1)$

= even integer

but given ac = odd integer

$(a = 2n + 1)$

28. If $P(x) = ax^2 + bx + c$ and $Q(x) = -ax^2 + dx + c$ where $ac \neq 0$, then $P(x) . Q(x) = 0$ has (a, b, c, d are real)

(A) 2 real roots (B) at least two real roots (C) 4 real roots (D) no real root

Ans : (B)

Hint : $D_1 = b^2 - 4ac$

$D_2 = d^2 + 4ac$

$D_1 + D_2 = b^2 + d^2 > 0$

$D_1 + D_2 > 0$

At least one of D_1 or D_2 is +ve so at least two real roots

29. Let N be the number of quadratic equations with coefficients from {0, 1, 2, ..., 9) such that 0 is a solution of each equation. Then the value of N is

(A) 2^9 (B) 3^9 (C) 90 (D) 81

Ans : (C)

Hint : $ax^2 + bx + c = 0$

one root is zero$\Rightarrow c = 0$

$a \neq 0$, a = 1, 2, 3, ... 9

b = 0, ...9

Number of value of N = 9 × 10 = 90

30. The coefficient of $a^{10}b^7c^3$ in the expansion of $(bc + ca + ab)^{10}$ is

(A) 140 (B) 150 (C) 120 (D) 160

Ans : (C)

Hint : $(bc + ca + ab)^{10} = \sum \frac{\lfloor 10}{\lfloor n_1 \lfloor n_2 \lfloor n_3}(bc)^{n_1}(ca)^{n_2}(ab)^{n_3} = \sum \frac{\lfloor 10}{\lfloor n_1 \lfloor n_2 \lfloor n_3} b^{n_1+n_3} c^{n_1+n_2} a^{n_2+n_3}$

$n_1 + n_3 = 7$ $n_1 + n_2 + n_3 = 10$

$n_1 + n_2 = 3$ $n_1 = 0, n_2 = 3$

$n_2 + n_3 = 10$ $n_3 = 7$

Coefficient of $a^{10}b^7c^3 = \frac{\lfloor 10}{\lfloor 3 \lfloor 7} = \frac{10 \times 9 \times 8}{3 \times 2} = 10 \times 3 \times 4 = 120$

31. The numbers 1, 2, 3, ..., m are arranged in random order. The number of ways this can be done, so that the numbers 1, 2, ..., r (r < m) appears as neighbours is

(A) $(m - r)!$ (B) $(m - r + 1)!$ (C) $(m - r)!r!$ (D) $(m - r + 1)!\, r!$

Ans : (D)

32. If $A = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$ and $\theta = \frac{2\pi}{7}$, then $A^{100} = A \times A \times$ .....(100 times) is equal to

(A) $\begin{pmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix}$ (B) $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$ (C) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ (D) $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

Ans : (A)

Hint : $\because A^{100} = \begin{bmatrix} \cos 100\theta & -\cos 100\theta \\ \sin 100\theta & \cos 100\theta \end{bmatrix}$

As $\theta = \frac{2\pi}{7}$

$$A^{100} = \begin{bmatrix} \cos\frac{200\pi}{7} & -\sin\frac{200\pi}{7} \\ \sin\frac{200\pi}{7} & \cos\frac{200\pi}{7} \end{bmatrix} = A^2$$

33. If $(1 + x + x^2 + x^3)^5 = \sum_{k=0}^{15} a_k x^k$ then $\sum_{k=0}^{7} (-1)^k . a_{2k}$ is equal to

(A) 2^5 (B) 4^5 (C) 0 (D) 4^4

Ans : (C)

Hint : $\sum_{k=0}^{1} (-1)^K a_{2k}$

$= a_0 - a_2 + a_4 - a_6 + a_8 .....a_{14}$

$(1+x+x^2+x^3)^5 = a_0 + a_1x + ... + a_{15}x^{15}$

put x = i

$(1+i + i^2+i^3)^5 = a_0 + a_1 i + a_2 i^2 + ...+ a_{15} i^{15}$

equating real part

$a_0 - a_2 + a_4 - a_6 + a_8 ......a_{14} = 0$

34. Let $f(x) = \begin{vmatrix} \cos x & x & 1 \\ 2\sin x & x^3 & 2x \\ \tan x & x & 1 \end{vmatrix}$, then $\lim_{x\to 0} \frac{f(x)}{x^2} =$

(A) 2 (B) –2 (C) 1 (D) –1

Ans : (B)

Hint : $f(x) = (x^3 - 2x^2)\cos x + \tan x(2x^2 - x^3)$

$$\lim_{x\to 0} \frac{f(x)}{x^2} = \lim_{x\to 0} (x-2)\cos x + \tan x(2-x)$$

$$= -2$$

35. If $\begin{vmatrix} x^k & x^{k+2} & x^{k+3} \\ y^k & y^{k+2} & y^{k+3} \\ z^k & z^{k+2} & z^{k+3} \end{vmatrix} = (x-y)(y-z)(z-x)\left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z}\right)$, then

(A) k = –3 (B) k = 3 (C) k = 1 (D) k = –1

Ans : (D)

Hint : $(xyz)^k \begin{vmatrix} 1 & x^2 & x^3 \\ 1 & y^2 & y^3 \\ 1 & z^2 & z^3 \end{vmatrix}$

$$= (xyz)^k (x-y)(z-x) - (y-z)(xy + yz + zx)$$

$$= (xyz)^{k+1} (x-y)(z-x) - (y-z)\left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z}\right)$$

k + 1 = 0

$\boxed{k = -1}$

36. If $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} \cdot A \cdot \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then A =

(A) $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ (C) $\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ (D) $\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$

Ans : (A)

Hint : $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} A \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Premultiplication by $\begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}$ both sides

$A \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}$

Post multiplication by $\begin{bmatrix} 3 & 2 \\ 5 & 3 \end{bmatrix}$ both sides

$A = \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ 5 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$

37. Let A be the set of even natural numbers that are < 8 &
B be the set of prime integers that are < 7
The number of relations from A to B are

(A) 3^2 (B) 2^{9-1} (C) 9^2 (D) 2^9

Ans : (D)

Hint : A = {2, 4, 6}
B = {2, 3, 5}
$\therefore$ Number of relations from A to B is 2^9

38. In $\mathbb{R}$, a relation p is defiened as follows :
$\forall$ a, b $\in \mathbb{R}$, a p b holds if $a^2 - 4ab + 3b^2 = 0$. Then

(A) p is equivalence relation (B) p is only symmetric
(C) p is only reflexive (D) p is only transitive

Ans : (C)

Hint : $\because$ a $\mathbb{R}$ b $\Rightarrow a^2 - 4ab + 3b^2 = 0$
$\therefore$ a $\mathbb{R}$ a because $a^2 - 4a.a + 3a^2 = 0$, $\forall a \in \mathbb{R}$
$\therefore$ it is reflective only.

39. Let $f : \mathbb{R} \to \mathbb{R}$ be a function defined by $f(x) = \dfrac{e^{|x|} - e^{-x}}{e^x + e^{-x}}$, then

(A) f is both one-one and onto (B) f is one-one but not onto
(C) f is onto but not one-one (D) f is neither one-one nor onto

Ans : (D)

Hint : $f : \mathbb{R} \to \mathbb{R}$, $f(x) = \dfrac{e^{|x|} - e^{-x}}{e^x + e^{-x}}$

Case I: If $x \geq 0$

$$f(x)=\frac{e^{x}-e^{-x}}{e^{x}+e^{-x}}=1-\frac{2e^{-x}}{e^{x}+e^{-x}}=1-\frac{2}{e^{2x}+1}>-1$$

Case II: $x < 0$

$$f(x)=\frac{e^{-x}-e^{-x}}{e^{x}+e^{-x}}=0$$

40. The expression $\cos^2\phi + \cos^2(\theta + \phi) - 2\cos\theta\cos\phi\cos(\theta + \phi)$ is

(A) independent of θ (B) independent of ϕ (C) independent of θ and ϕ (D) dependent on θ and ϕ

Ans : (B)

Hint : $\cos^2\phi + \cos^2(\theta + \phi) - 2\cos\theta.\cos\phi\cos(\theta + \phi)$

for $\theta = 0$

$\cos^2\phi + \cos^2\phi - 2\cos^2\phi = 0$

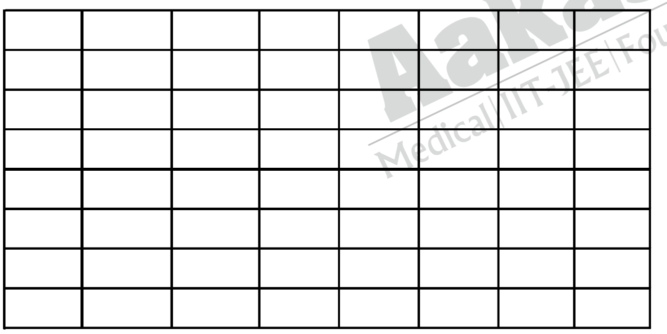
which is independent from ϕ.

41. Two smallest squares are chosen one by one on a chess board. The probability that they have a side in common is

(A) $\frac{1}{9}$ (B) $\frac{2}{7}$ (C) $\frac{1}{18}$ (D) $\frac{5}{18}$

Ans : (C)

Hint :



For each corner squarers there will be two options for each border squares there will be 3 options for each other squares there will be 4 options.

$\therefore$ favourable cases

$$\frac{4\times2+24\times3+36\times4}{2}=112$$

$\therefore$ required probability $\frac{112}{64\times63}=\frac{1}{18}$

42. Two integers r and s are drawn one at a time without replacement from the set {1, 2, ..., n}. Then P $(r \leq k/s \leq k)$ =

(A) $\frac{k}{n}$ (B) $\frac{k}{n-1}$ (C) $\frac{k-1}{n}$ (D) $\frac{k-1}{n-1}$

Ans : (D)

Hint : $p(r \le k / s \le k) = \dfrac{p(r \le k \cap s \le k)}{p(s \le k)}$

Now, $p(s \le k) = \dfrac{k}{n}$

$$p(r \le k \cap s \le k) = \frac{{}^kC_2}{{}^nC_2} = \frac{k(k-1)}{n(n-1)}$$

$$\therefore p(r \le k / s \le k) = \frac{\frac{k(k-1)}{n(n-1)}}{\frac{k}{n}} = \frac{k-1}{n-1}$$

43. A biased coin with probability p (0 < p < 1) of getting head is tossed until a head appears for the first time. If the probability that the number of tosses required is even is $\frac{2}{5}$, then p =

(A) $\frac{1}{4}$ (B) $\frac{1}{3}$ (C) $\frac{2}{3}$ (D) $\frac{3}{4}$

Ans : (B)

Hint : required probability will be

$$(1-p)p + (1-p)^3 p + \cdots \infty = \frac{2}{5}$$

$$\Rightarrow p\left[\frac{1-p}{1-(1-p)^2}\right] = \frac{2}{5}$$

$$\Rightarrow \frac{p(1-p)}{1-(1-2p+p^2)} = \frac{2}{5}$$

$$\Rightarrow \frac{p-p^2}{2p-p^2} = \frac{2}{5}$$

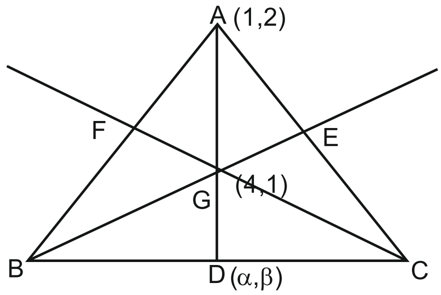
$$\Rightarrow \boxed{p = \frac{1}{3}}$$

44. In ΔABC, co-ordinates of A are (1, 2) and the equation of the medians through B and C are x + y = 5 and x = 4 respectively. Then the midpoint of BC is

(A) $\left(5, \frac{1}{2}\right)$ (B) $\left(\frac{11}{2}, 1\right)$ (C) $\left(11, \frac{1}{2}\right)$ (D) $\left(\frac{11}{2}, \frac{1}{2}\right)$

Ans : (D)

Hint :



Let D, E, F be midpoint of BC, AC and AB respectively.

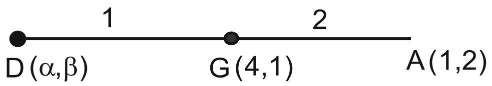
According to question equation of BE : $x + y = 5$

Equation of CF

$x = 4$

$\therefore$ Centroid (4, 1)

Let point D be (α, β)



by section formula,

$$(\alpha,\beta)=\left(\frac{11}{2},\frac{1}{2}\right)$$

45. If $0<\theta<\frac{\pi}{2}$ and $\tan 3\theta \neq 0$, then $\tan\theta + \tan 2\theta + \tan 3\theta = 0$ if $\tan\theta . \tan 2\theta = k$ where k =

(A) 1 (B) 2 (C) 3 (D) 4

Ans : (B)

Hint : $\because 3\theta=\theta+2\theta$

$\Rightarrow \quad \tan 3\theta=\tan(\theta+2\theta)=\dfrac{\tan\theta+\tan 2\theta}{1-\tan\theta\,.\,\tan 2\theta}$

$\Rightarrow \quad \tan 3\theta=\dfrac{-\tan 3\theta}{1-k}$

$\Rightarrow \quad 1=\dfrac{-1}{1-k} \qquad [\because \tan 3\theta \neq 0]$

$\Rightarrow \quad 1-k=-1 \qquad \Rightarrow \boxed{k=2}$

46. The equation $r\cos\theta = 2a\sin^2\theta$ represents the curve

(A) $x^3 = y^2(2a + x)$ (B) $x^2 = y^2(2a + x)$ (C) $x^3 = y^2(2a - x)$ (D) $x^3 = y^2(a + x)$

Ans : (C)

Hint : $r\cos\theta=2a\sin^2\theta$

$\Rightarrow \quad x\left(x^2+y^2\right)=2ay^2$

$\Rightarrow \quad x^3=y^2(2a-x)$

47. If (1, 5) be the midpoint of the segment of a line between the line $5x - y - 4 = 0$ and $3x + 4y - 4 = 0$, then the equation of the line will be

(A) $83x + 35y - 92 = 0$ (B) $83x - 35y + 92 = 0$ (C) $83x - 35y - 92 = 0$ (D) $83x + 35y + 92 = 0$

Ans : (B)

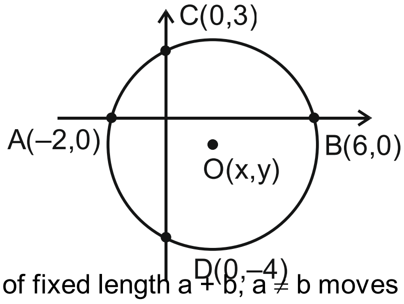
Hint :

48. Chords AB & CD of a circle intersect at right angle at the point P. If the length of AP, PB, CP, PD and 2, 6, 3, 4 units respectively, then the radius of the circle is

(A) 4 units (B) $\frac{\sqrt{65}}{2}$ units (C) $\frac{\sqrt{67}}{2}$ units (D) $\frac{\sqrt{66}}{2}$ units

Ans : (B)

Hint :



$O(x, y) = \text{centre}$

$\Rightarrow OC = OA = OB = OD$

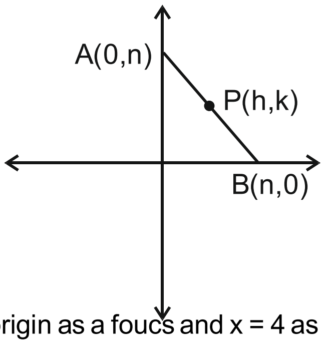
$\Rightarrow r = \frac{\sqrt{65}}{2}$

49. A line of fixed length $a + b$, $a \neq b$ moves so that its ends are always on two fixed perpendicular straight lines. The locus of a point which divides the line into two parts of legnth a and b is

(A) a parabola (B) a circle (C) an ellipse (D) a hyperbola

Ans : (C)

Hint :



$m^2 + n^2 = (a+b)^2$

$AP = a ; PB = b$

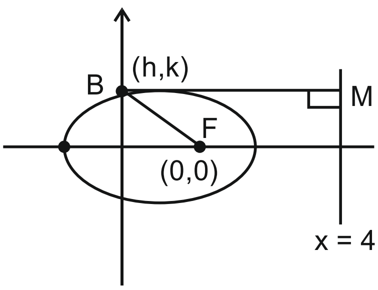
$\Rightarrow \frac{h^2}{a^2} + \frac{k^2}{b^2} = 1$

50. With origin as a foucs and x = 4 as corresponding directrix, a family of ellipse are drawn. Then the locus of an end of minor axis is

(A) a circle (B) a parabola (C) a straight line (D) a hyperbola

Ans : (B)

Hint :



$h = -ae, k = b, \ \frac{a}{e} - ae = 4 \Rightarrow a(1-e^2) = 4e \Rightarrow \frac{b^2}{a} = 4e$

$\Rightarrow b^2 = 4ae \Rightarrow k^2 = -4h \Rightarrow y^2 = -4x$

Which is a general conic and satisfying the condition of parabola.

CATEGORY - 2 (Q.51 to 65)

(Carry 2 marks each. Only one option is correct. Negative marks : –½)

51. $\lim\limits_{x\to\infty} \frac{1}{n^{k+1}}\left[2^k+4^k+6^k+....+(2n)^k\right]=$

(A) $\frac{2^k}{k}$ (B) $\frac{2^{k+1}}{k+1}$ (C) $\frac{2^k}{k+1}$ (D) $\frac{2^k}{k-1}$

Ans : (C)

Hint : $\lim\limits_{n\to\infty}\frac{1}{n}\sum\limits_{r=1}^{n}\left(\frac{2r}{n}\right)^k=\int_0^1(2x)^k dx=\frac{2^k}{k+1}$

52. If $y=\tan^{-1}\left[\frac{\log_e\left(\frac{e}{x^2}\right)}{\log_e\left(ex^2\right)}\right]+\tan^{-1}\left[\frac{3+2\log_e x}{1-6\,.\,\log_e x}\right]$, then $\frac{d^2y}{dx^2}=$

(A) 2 (B) 1 (C) 0 (D) –1

Ans : (C)

Hint : $y=\tan^{-1}(1)-\tan^{-1}(2\ln x)+\tan^{-1}(3)+\tan^{-1}(2\ln x)$

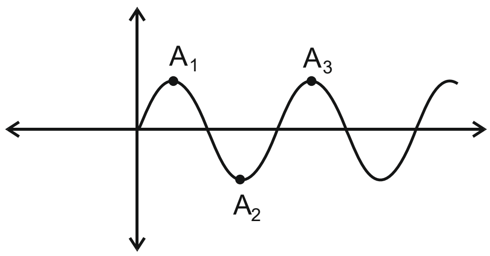
53. Consider the function f(x) = x(x – 1) (x – 2) ... (x – 100). Which one of the following is correct ?

(A) This function has 100 local maxima (B) This function has 50 local maxima

(C) This function has 51 local maxima (D) Local minima do not exist for this function

Ans : (B)

Hint : f(x) = x (x – 1) (x – 2) (x – 3) .... (x – 100)



54. Let $I(R)=\int\limits_0^R e^{-R\sin x}dx, R>0$.

then,

(A) $I(R)>\frac{\pi}{2R}\left(1-e^{-R}\right)$ (B) $I(R)<\frac{\pi}{2R}\left(1-e^{-R}\right)$

(C) $I(R)=\frac{\pi}{2R}\left(1-e^{-R}\right)$ (D) $I(R)$ and $\frac{\pi}{2R}\left(1-e^{-R}\right)$ are not comparable

Ans : (D)

Hint : Checking the different positive values of R, can't comparable.

55. In a plane $\vec{a}$ and $\vec{b}$ are the position vectors of two points A and B respectively. A point P with position vector $\vec{r}$ moves on that plane in such a way that $|\vec{r}-\vec{a}| \sim |\vec{r}-\vec{b}| = c$ (real constant). The locus of P is a conic section whose eccentricity is

(A) $\frac{|\vec{a}-\vec{b}|}{c}$ (B) $\frac{|\vec{a}+\vec{b}|}{c}$ (C) $\frac{|\vec{a}-\vec{b}|}{2c}$ (D) $\frac{|\vec{a}+\vec{b}|}{2c}$

Ans : (A)

Hint : $e = \frac{|\vec{a}-\vec{b}|}{c}$

56. If $\alpha_1, \alpha_2, \cdots, \alpha_n$ are in A.P. with common difference θ, then the sum of the series
$\sec\alpha_1 \sec\alpha_2 + \sec\alpha_2 \sec\alpha_3 + \cdots + \sec\alpha_{n-1} \sec\alpha_n = k(\tan\alpha_n - \tan\alpha_1)$
where k =

(A) $\sin\theta$ (B) $\cos\theta$ (C) $\sec\theta$ (D) $\operatorname{cosec}\theta$

Ans : (D)

Hint : $\alpha_2 - \alpha_1 = \alpha_3 - \alpha_2 = \cdots = \theta$

$$s = \frac{1}{\sin\theta}\left[\frac{\sin(\alpha_2-\alpha_1)}{\cos\alpha_2 \cos\alpha_1} + \cdots + \frac{\sin(\alpha_n - \alpha_{n-1})}{\cos\alpha_n \cos\alpha_{n-1}}\right]$$

$= \operatorname{cosec}\theta [\tan\alpha_n - \tan\alpha_1]$

57. Five balls of different colours are to be placed in three boxes of different sizes. The number of ways in which we can place the balls in the boxes so that no box remains empty is

(A) 160 (B) 140 (C) 180 (D) 150

Ans : (D)

Hint : $3^5 - {}^3C_1(3-1)^5 + {}^3C_2\, 1^5 - 0 = 150$

58. Let A = $\begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 1 & 1 & 1 \end{pmatrix}$, B = $\begin{pmatrix} 2 \\ 1 \\ 7 \end{pmatrix}$

Then for the validity of the result AX = B, X is

(A) $\begin{pmatrix} -1 \\ 1 \\ 7 \end{pmatrix}$ (B) $\begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix}$ (C) $\begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix}$ (D) $\begin{pmatrix} 4 \\ 2 \\ 1 \end{pmatrix}$

Ans : (D)

Hint : By option elimination.

59. If 1000! = $3^n \times m$ where m is an integer not divisible by 3. then n =

(A) 498 (B) 298 (C) 398 (D) 98

Ans : (A)

Hint : $E_3(1000!) = \left[\frac{1000}{3}\right] + \left[\frac{1000}{3^2}\right] + \left[\frac{1000}{3^3}\right] + \left[\frac{1000}{3^4}\right] + \left[\frac{1000}{3^5}\right] + \left[\frac{1000}{3^6}\right] = 498$

60. For the real numbers x & y, we write x p y if $x - y + \sqrt{2}$ is an irrational number. Then relation p is

(A) reflexive (B) symmetric (C) transitive (D) equivalence relation

Ans : (A)

Hint : $x\ p\ x = x - x + \sqrt{2} = \sqrt{2}$ is an irrational

61. Let $A = \begin{pmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{pmatrix}$, then

(A) A is a null matrix
(B) A is skew symmetric matrix
(C) A^{-1} does not exist
(D) $A^2 = 1$

Ans : (D)

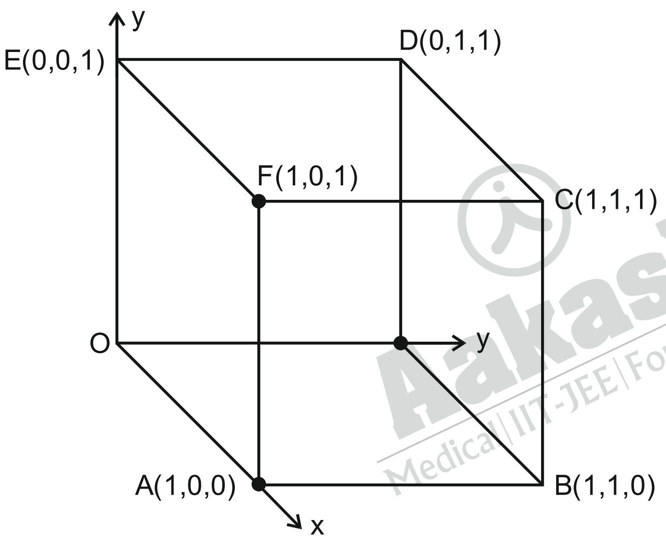
Hint : $A^2 = 1$

62. Angle between two diagonals of a cube will be

(A) $\cos^{-1}\left(\frac{1}{3}\right)$
(B) $\sin^{-1}\left(\frac{1}{3}\right)$
(C) $\frac{\pi}{2} - \cos^{-1}\left(\frac{1}{3}\right)$
(D) $\frac{\pi}{2} - \sin^{-1}\left(\frac{1}{3}\right)$

Ans : (A)

Hint :



d.c. of diagonal OC : $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$ or, $-\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}$

d.c. of diagonal AD : $-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$ or, $\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}$

$\therefore$ Angle between OC & AD = $\cos^{-1}\left(\frac{1}{3}\right)$

63. If A and B are acute angles such that $\sin A = \sin^2 B$ and $2\cos^2 A = 3\cos^2 B$, then (A, B) =

(A) $\left(\frac{\pi}{6}, \frac{\pi}{4}\right)$
(B) $\left(\frac{\pi}{6}, \frac{\pi}{6}\right)$
(C) $\left(\frac{\pi}{4}, \frac{\pi}{6}\right)$
(D) $\left(\frac{\pi}{4}, \frac{\pi}{4}\right)$

Ans : (A)

Hint : $2\cos^2 A = 3(1 - \sin^2 B)$

$\Rightarrow 2 - 2\sin^2 A = 3 - 3\sin A \quad (\because \sin^2 B = \sin A)$

$\Rightarrow \sin A = 1, \frac{1}{2}$

$\Rightarrow \sin B = \pm 1, \pm\frac{1}{\sqrt{2}}$

$\therefore (A, B) \equiv \left(\frac{\pi}{2}, \pm\frac{\pi}{2}\right)$ or $\left(\frac{\pi}{6}, \pm\frac{\pi}{4}\right)$

64. If two circles which pass through the points (0, a) and (0, –a) and touch the line y = mx + c, cut orthogonally then

(A) $c^2 = a^2(1 + m^2)$ (B) $c^2 = a^2(2 + m^2)$ (C) $c^2 = a^2(1 + 2m^2)$ (D) $2c^2 = a^2(1 + m^2)$

Ans : (B)

Hint : Let the circle be $x^2 + y^2 + 2gx + 2fy + k = 0$.

$\because$ passes through (0,a) & (0,–a)

$\Rightarrow k = -a^2, f = 0$

$\Rightarrow x^2 + y^2 + 2gx - a^2 = 0$ ($\because$ two circles are orthogonal)

$\Rightarrow 2g_1g_2 + 2f_1f_2 = c_1 + c_2$

$\Rightarrow g_1g_2 = -a^2$ —— 1

also y = mx + c is the tangent to the given circles

$\Rightarrow$ radius = distance of centre (– g, 0) from the line

$\Rightarrow \sqrt{g^2 + a^2} = \frac{|mg - c|}{\sqrt{1 + m^2}}$

$\Rightarrow g^2 - 2mcg + a^2(1 + m^2) - c^2 = 0$

$\because g_1g_2 = a^2(1 + m^2) - c^2$ —— 2

from 1 and 2

$c^2 = (m^2 + 2)a^2$

65. The locus of the midpoint of the system of parallel chords parallel to the line y = 2x to the hyperbola $9x^2 - 4y^2 = 36$ is

(A) 8x – 9y = 0 (B) 9x – 8y = 0 (C) 8x + 9y = 0 (D) 9x – 4y = 0

Ans : (B)

Hint : Locus implies diameter of the hyperbola

i.e. $y = \frac{b_2}{a^2m}x$ (m = 2)

$\Rightarrow y = \frac{9}{8}x$

$\Rightarrow 9x - 8y = 0$

CATEGORY - 3 (Q66 to Q75)

(Carry 2 marks each. One or more options are correct. No negative marks)

66. Choose the correct statement :

(A) x + sin 2x is a periodic function
(B) x + sin 2x is not a periodic function
(C) $\cos\left(\sqrt{x}+1\right)$ is a periodic function
(D) $\cos\left(\sqrt{x}+1\right)$ is not a periodic function

Ans : (BD)

Hint : f(x) = x + sin2x is not a periodic function of x (since 'x' is increasing function)

also g(x) = $\cos\left(\sqrt{x}+1\right)$ is not a periodic function of x (since g(x+T) can never be equal to g(x))

67. The points of extremum of $\int_0^{x^2} \frac{t^2-5t+4}{2+e^t}\,dt$ are

(A) ± 1
(B) ± 2
(C) ± 3
(D) $\pm\sqrt{2}$

Ans : (A, B)

Hint : $f(x) = \int_0^{x^2} \frac{t^2-5t+4}{2+e^t}dt$ ..... (i)

to find extremum points we solve f′(x) = 0

so differentiating equation (i) using Newton Leibnitz Rule,

$$f'(x) = \frac{(x^2-1)(x^2-4)}{2+e^{x^2}}\cdot(2x) \qquad \text{so, } f'(x)=0 \text{ at } x=0, \pm 1, \pm 2$$

68. Let Γ be the curve y = $be^{-x/a}$ & L be the straight line $\frac{x}{a}+\frac{y}{b}=1$ where a, b $\in \mathbb{R}$

(A) L touches the curve Γ at the point where the curve crosses the axis of y.
(B) L does not touch the curve at the point where the curve crosses the axis of y.
(C) Γ touches the axis of x at the point.
(D) Γ never touches the axis of x.

Ans : (A, D)

Hint : $\Gamma \equiv y = be^{-x/a} \xrightarrow[\text{w.r.t x}]{\text{on differentiating}} \frac{dy}{dx} = \frac{-b}{a}e^{-x/a}$ at x = 0, y = b, $\frac{dy}{dx} = \frac{-b}{a}$

so, equation of tengent at (0,b) is $y-b=-\frac{b}{a}(x-0) \Rightarrow \underbrace{\frac{x}{a}+\frac{y}{b}=1}_{L}$

so, L touches the given curve at (0, b)

$\because \frac{dy}{dx}$ can be never be 0, so, the given curve never touches the x – axis.

69. The acceleration f ft/sec^2 of a particle after a time t sec starting from rest is given by $f = 6 - \sqrt{1.2t}$. Then the maximum velocity v and time T at attend this velocity are

(A) T = 20 sec (B) v = 60 ft/sec (C) T = 30 sec (D) v = 40 tt/sec

Ans : (B, C)

Hint : Given $acc^n = f = \frac{dv}{dt} = 6 - \sqrt{1.2t}$ (t is in secs)

at t = 0, v = 0 and let the maximum velocity is 'v' which is attained at 't_0'

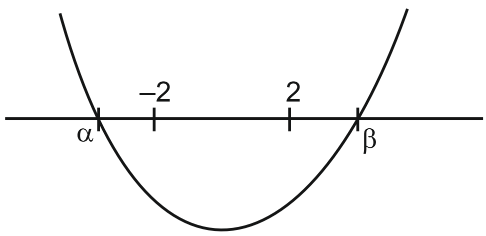
at maximum velocity, $\frac{dv}{dt} = 0 \Rightarrow f = 0 \Rightarrow t_0 = 30s$ so, $\int_0^v dv = \int_0^{30}\left(6 - \sqrt{1.2t}\right)dt \Rightarrow \boxed{v = 60 \text{ ft/s}}$

70. If the quadratic equation ax^2 + bx + c = 0 (a > 0) has two roots α and β such that $\alpha < -2$ and $\beta > 2$ then

(A) c < 0 (B) a + b + c > 0 (C) a – b + c < 0 (D) a – b + c > 0

Ans : (A, C)

Hint : f(x) = ax^2 + bx + c



so, f(–2) < 0 and f(2) > 0

also, f(0) < 0 $\Rightarrow$ c < 0

f(–1) < 0 $\Rightarrow$ a – b + c < 0

f(1) < 0 $\Rightarrow$ a + b + c < 0

71. If n is a positive integer, the value of $(2n + 1)\ {}^nC_0 + (2n-1)\ {}^nC_1 + (2n-3)\ {}^nC_2 + ... + 1\cdot {}^nC_n$ is

(A) $(n + 1)2^n$ (B) 3^n (C) f′(2) where $f(x) = x^{n+1}$ (D) $(n + 1)2^{n+1}$

Ans : (A, C)

Hint : $\sum_{r=0}^{n}(2n+1-2r)\,{}^nC_r$

$= (2n+1)\sum_{r=0}^{n}{}^nC_r - 2\sum_{r=0}^{n} r\cdot{}^nC_r$

$= (2n+1)\cdot 2^n - 2\cdot n\cdot 2^{n-1} = (n+1)\cdot 2^n$

If $f(x)=x^{n+1} \Rightarrow f'(2) = (n + 1)\cdot 2^n$

72. The function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = e^x + e^{-x}$ is

(A) one-one (B) onto (C) bijective (D) not bijective

Ans : (D)

Hint : $\because$ f(x) is even function $\therefore$ Not one - one

$\because$ $e^x + e^{-x}$ can't be negative $\therefore$ Not onto

73. If $a_i, b_i, c_i \in \mathbb{R}\,(i = 1, 2, 3)$ and $x \in \mathbb{R}$ and $\begin{vmatrix} a_1+b_1x & a_1x+b_1 & c_1 \\ a_2+b_2x & a_2x+b_2 & c_2 \\ a_3+b_3x & a_3x+b_3 & c_3 \end{vmatrix} = 0$, then

(A) $x = 1$ (B) $x = -1$ (C) $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$ (D) $x = 2$

Ans : (A, B, C)

Hint : $(1-x^2)\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$

74. If ABC is an isosceles triangle and the coordinates of the base points are B(1, 3) and C(–2, 7). The coordinates of A can be

(A) $(1, 6)$ (B) $\left(-\frac{1}{8}, 5\right)$ (C) $\left(\frac{5}{6}, 6\right)$ (D) $\left(-7, \frac{1}{8}\right)$

Ans : (C, D)

Hint : Let A (x, y) now $(AB)^2 = (AC)^2$

except mid point of BC for ΔABC to be isosceles $\therefore 8y - 6x = 43$

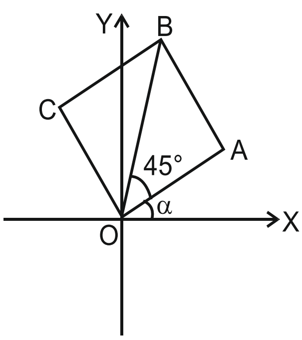
75. A square with each side equal to 'a' above the x-axis and has one vertex at the origin. One of the sides passing through the origin makes an angle $\alpha\left(0 < \alpha < \frac{\pi}{4}\right)$ with the positive direction of the axis. Equation of the diagonals of the square.

(A) $y(\cos\alpha - \sin\alpha) = x(\sin\alpha + \cos\alpha)$ (B) $y(\cos\alpha + \sin\alpha) = x(\cos\alpha - \sin\alpha)$

(C) $y(\sin\alpha + \cos\alpha) + x(\cos\alpha - \sin\alpha) = a$ (D) $y(\cos\alpha - \sin\alpha) + x(\cos\alpha + \sin\alpha) = a$

Ans : (A, C)

Hint :



OB $\rightarrow$ $y = \tan(45 + \alpha)\,x$

$y(\cos\alpha - \sin\alpha) = x(\sin\alpha + \cos\alpha)$

$\because AC \perp OB$, $A(a\cos\alpha, a\sin\alpha)$ $\therefore AC \rightarrow y - a\sin\alpha = \frac{(\sin\alpha - \cos\alpha)}{(\sin\alpha + \cos\alpha)}(x - a\cos\alpha)$

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WBJEE - 2024

Answer Keys by

Aakash Institute, Kolkata Centre

PHYSICS & CHEMISTRY

Q.No.	○	✚	✿	□
01	C	B	B	D
02	B*	B	C	A
03	A	A	D	D
04	D	D	D	C
05	B	D	B	D
06	×	D	×	A
07	C	C	D	C
08	D	B*	B	B
09	B	A	B	B
10	C	C	C	D
11	D	B	C	A
12	B	×	A	D
13	B	D	D	A
14	×	D	D	D
15	D	B	C	C
16	C	C	D	B*
17	B	B	C	×
18	C	D	A	C
19	D	B	B	B
20	A	×	B	C
21	D	C	D	D
22	D	C	A	D
23	C	B	D	B
24	A	A	B*	C
25	C	D	A	×
26	B	C	D	D
27	B	D	C	B
28	D	D	×	B
29	D	C	C	C
30	A	A	B	C
31	D	C	D	B
32	D	D	D	C
33	B	D	B	D
34	B	B	C	D
35	C	B	D	B
36	D	B,C	A,B,C	C,D
37	A,B,C	D	A,B,D	B,C
38	A,B,D	A,B,C	C,D	D
39	C,D	A,B,D	B,C	A,B,C
40	B,C	C,D	D	A,B,D
41	A	A	C	C,D #
42	C	B	B	A
43	D	A	B	D
44	B	D	C	B
45	B	C	C	B
46	C	B	C	B
47	B	A	B	C
48	C	C	C	C
49	C	D	B	A
50	C	B	C	B
51	B	B	C	A
52	B	C	B	D
53	C	C	A	D
54	A	C	A	B
55	C	C	A	A
56	C	C	C,D #	C
57	B	B	B	C
58	A	B	C	B
59	C,D #	A	D	B
60	A	A	B	C
61	B	C	B	C
62	B	C	A	C
63	C	B	D	B
64	D	A	C	C
65	B	C,D #	A	B
66	B	B	B	B
67	A	B	C	A
68	D	B	D	A
69	C	C	B	C
70	A	D	A	C
71	A	C	D	A
72	D	A	D	C
73	D	D	A	A
74	A	D	C	D
75	C	A	A	D
76	A,C	B,C	B,C	B,D
77	B,C	A,C	A,B	B,C
78	A,B	B,C	B,D	A,C
79	B,D	A,B	B,C	B,C
80	B,C	B,D	A,C	A,B

× Incorrect questions

More than one options are incorrect

* Additional information required

Aakash

Medical | IIT-JEE | Foundations

Code - ❍

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ANSWERS & HINTS
for
WBJEE - 2024
SUB : PHYSICS & CHEMISTRY

PHYSICS

CATEGORY - 1 (Q1 to Q30)

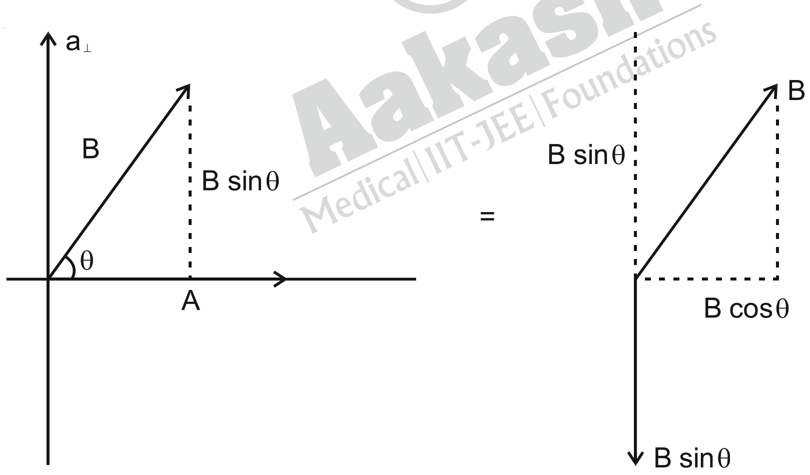
(Carry 1 mark each. Only one option is correct. Negative mark : – ¼)

1. Let θ be the angle between two vectors $\vec{A}$ and $\vec{B}$. If $\hat{a}_{\perp}$ is the unit vector perpendicular to $\vec{A}$, then the direction of $\vec{B} - B\sin\theta\,\hat{a}_{\perp}$ is

(A) along $\vec{B}$ (B) perpendicular to $\vec{B}$ (C) along $\vec{A}$ (D) perpendicular to $\vec{A}$

Ans : (C)

Hint :



$\vec{B} - B\sin\theta\,\hat{a}_{\perp}$ is along $\vec{A}$

2. The Power (P) radiated from an accelerated charged particle is given by $P \propto \frac{(q\,a)^m}{c^n}$ where q is the charge, a is the acceleration of the particle and c is speed of light in vacuum. From dimensional analysis, the value of m and n respectively, are

(A) m = 2, n = 2 (B) m = 2, n = 3 (C) m = 3, n = 2 (D) m = 0, n = 1

Ans : (B)

Hint : $\frac{q^2}{4\pi\varepsilon_0 rT} = \frac{1}{\varepsilon_0}\frac{q^m a^m}{c^n}$ {taking $\frac{1}{\varepsilon_0}$ in multiplication as constant of proportionality}

$$\frac{q^2}{rT} = \frac{q^m a^m}{c^n}$$

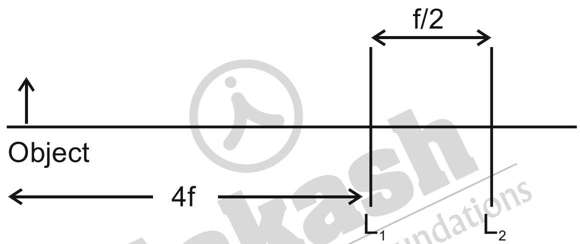
m = 2

$$\frac{a^m}{c^n} = \frac{1}{CT^2} = \frac{1\,a^2}{c\,c^2} \qquad [c = aT]$$

$$\frac{a^m}{c^n} = \frac{a^2}{c^3}$$

n = 3

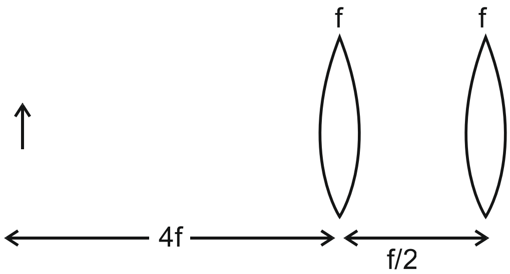
3. The convex lens (L_1 and L_2) of equal focal length f are placed at a distance $\frac{f}{2}$ apart. An object is placed at a distance 4f in the left of L_1 as shown in figure. The final image is at



(A) $\frac{5f}{11}$ right of L_2 (B) $\frac{5f}{11}$ left of L_2 (C) 5f right of L_2 (D) 5f left of L_2

Ans : (A)

Hint :



$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v} - \frac{1}{-4f} = \frac{1}{f} \Rightarrow \frac{1}{v} = \frac{1}{f} - \frac{1}{4f} \Rightarrow \frac{1}{v} = \frac{4-1}{4f} \Rightarrow v = \frac{4f}{3}$$

$$u_2 = \frac{4f}{3} - \frac{f}{2} = \frac{8f - 3f}{6} \Rightarrow u_2 = \frac{5f}{6}$$

$$\frac{1}{v_2} - \frac{1}{u_2} = \frac{1}{f} \Rightarrow \frac{1}{v_2} - \frac{6}{5f} = \frac{1}{f} \Rightarrow \frac{1}{v_2} = \frac{1}{f} + \frac{6}{5f}$$

$$\frac{1}{v_2} = \frac{11}{5f} \Rightarrow v_2 = \frac{5f}{11}$$

4. Which of the following quantity has the dimension of length?
(h is Planck's constant, m is the mass of electron and c is the velocity of light)

(A) $\frac{hc}{m}$ (B) $\frac{h}{mc^2}$ (C) $\frac{h^2}{mc^2}$ (D) $\frac{h}{mc}$

Ans : (D)

Hint : $[h] = [ML^2T^{-1}]$

$[c] = [LT^{-1}]$

$[m] = [M^1L^0T^0]$

Dimension of length

$[L] = h^x c^y m^z$

$[L^1] = [ML^2T^{-1}]^x [LT^{-1}]^y [M^1L^0T^0]^z$

Dimensional analysis on mass

$0 = x + z \Rightarrow x = -z \Rightarrow z = -1$

Dimensional analysis on length

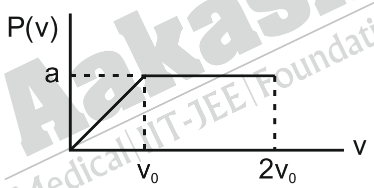
$1 = 2x + y \Rightarrow 2x - x = 1 \Rightarrow x = 1$

Dimensional analysis on time

$0 = -x - y \Rightarrow -x = +y \Rightarrow y = -1$

Therefore final formula is $\frac{h}{mc}$

5. The speed distribution for a sample of N gas particles is shown below. $P(v) = 0$ for $v > 2v_0$. How many particles have speeds between $1.2\ v_0$ and $1.8\ v_0$?



(A) 0.2 N (B) 0.4 N (C) 0.6 N (D) 0.8 N

Ans : (B)

Hint : Total area under curve = $\frac{3}{2} v_0 a = N$

Area under $1.2v_0$ to $1.8v_0$ is $0.6v_0 = 0.4N$

6. The internal energy of a thermodynamic system is given by $U = a\, s^{4/3} V^{\alpha}$ where s is entropy, V is volume and 'a' and 'α' are constants. The value of α is

(A) 1 (B) –1 (C) $\frac{1}{3}$ (D) $-\frac{1}{3}$

Ans : (No Answer)

Hint : it can't be found dimensionally.

7. A particle of mass 'm' moves in one dimension under the action of a conservative force whose potential energy has the form of $U(x) = -\frac{\alpha x}{x^2 + \beta^2}$ where α and β are dimensional parameters. The angular frequency of the oscillation is proportional to

(A) $\sqrt{\frac{\alpha^3}{m\beta^4}}$ (B) $\sqrt{\frac{\alpha}{m\beta^4}}$ (C) $\sqrt{\frac{\alpha}{m\beta^3}}$ (D) $\sqrt{\frac{\alpha}{m\beta^6}}$

Ans : (C)

Hint : $U = -\frac{\alpha x}{x^2 + \beta^2}$

here, $[\beta] = [L]$

$[\alpha] = [ML^3 T^{-2}]$

only option (C) has dimension of $\frac{1}{T}$

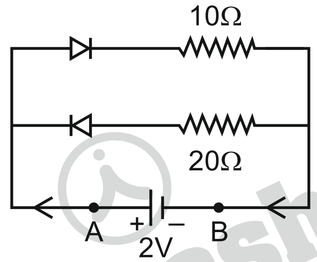
8. Longitudinal waves cannot

(A) have a unique wave length
(B) have a unique wave velocity
(C) transmit energy
(D) be polarized

Ans : (D)

Hint : Longitudinal wave can not be polarized.

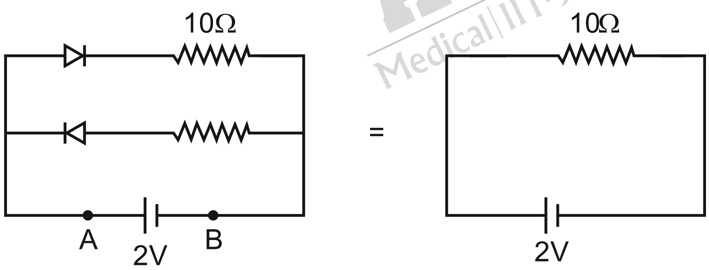
9. A 2 V cell is connected across to points A and B as shown in the figure. Assume that the resistance of each diode is zero in forward bias and infinity in reverse bias. The curring supplied by the cell is



(A) 0.5 A
(B) 0.2 A
(C) 0.1 A
(D) 0.25 A

Ans : (B)

Hint :



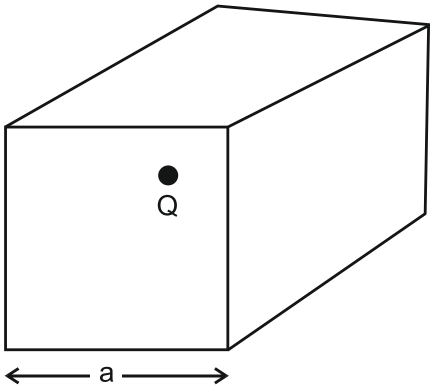
$i = \frac{2}{10} = 0.2A$

10. A charge Q is placed at the centre of a cube of sides a. The total flux of electric field through the six surfaces of the cube is

(A) $\frac{6Qa^2}{\epsilon_0}$
(B) $\frac{Qa^2}{6\epsilon_0}$
(C) Q/ϵ_0
(D) Qa^2/ϵ_0

Ans : (C)

Hint :



Flux passing through all six sides is $\frac{Q}{\varepsilon_0}$

11. The elastic potential energy of a strained body is

(A) stress × strain
(B) stress / strain
(C) stress × strain / volume
(D) $\frac{1}{2}$ × stress × strain × volume

Ans : (D)

Hint : Elastic PE = $\frac{1}{2}$ × stress × strain × V

12. Which of the following statement(s) is/are true in respect of nuclear binding energy?

(i) The mass energy of a nucleus is larger than the total mass energy of its individual protons and neutrons.
(ii) If a nucleus could be separated into its nucleons, an energy equal to the binding energy would have to be transferred to the particles during the separating process.
(iii) The binding energy is a measure of how well the nucleons in a nucleus are held together.
(iv) The nuclear fission is somehow related to acquiring higher binding energy.

(A) Statements (i), (ii) and (iii) are true
(B) Statements (ii), (iii) and (iv) are true
(C) Statements (ii) and (iii) are true
(D) All the four statements are true

Ans : (B)

Hint : [(ii), (iii) and (iv)]

13. A satellite of mass m rotates round the earth in a circular orbit of radius R. If the angular momentum of the satellite is J, then its kinetic energy (K) and the total energy (E) of the satellite are

(A) $K = \frac{J^2}{mR^2}, E = -\frac{J^2}{2mR^2}$
(B) $K = \frac{J^2}{2mR^2}, E = -\frac{J^2}{2mR^2}$
(C) $K = \frac{J^2}{2mR^2}, E = -\frac{J^2}{mR^2}$
(D) $K = \frac{J^2}{2mR^2}, E = \frac{J^2}{mR^2}$

Ans : (B)

Hint : $KE = \frac{J^2}{2I} = \frac{J^2}{2mR^2}$, $E = -KE$

14. What force F is required to start moving this 10 kg block shown in the figure if it acts at an angle of 60° as shown ? $(\mu_s = 0.6)$

(A) 22.72 N (B) 24.97 N (C) 25.56 N (D) 27.32 N

Ans : (Incorrect Question)

Hint : For block to move

$$\theta < \tan^{-1}\left[\frac{1}{\mu}\right]$$

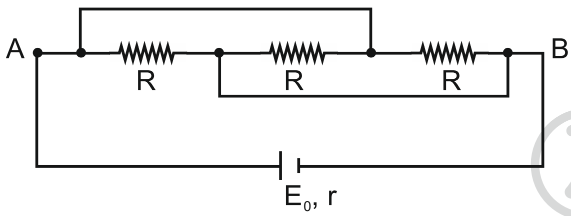
15. Light of wavelength 6000Å is incident on a thin glass plate of r.i. 1.5 such that the angle of refraction into the plate is 60°. Calculate the smallest thickness of the plate which will make dark fringe by reflected beam interference.

(A) 1.5×10^{-7}m (B) 2×10^{-7}m (C) 3.5×10^{-7}m (D) 4×10^{-7}m

Ans : (D)

Hint : $2\mu t \text{ Cos } r = n\lambda$

put n = 1

16. 

Consider a circuit where a cell of emf E_0 and internal resistance r is connected across the terminal A and B as shown in figure. The value of R for which the power generated in the circuit is maximum, is given by

(A) R = r (B) R = 2r (C) R = 3r (D) $R = \frac{r}{3}$

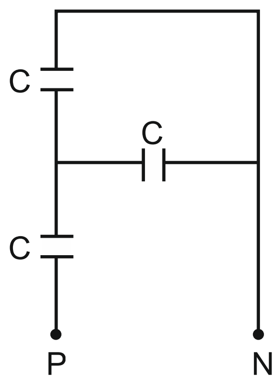
Ans : (C)

Hint : $R_{ext} = r$

$$\frac{R}{3} = r$$

R = 3r

17. The equivalent capacitance of a combination of connected capacitors shown in the figure between the points P and N is



(A) 3 C (B) $\frac{2C}{3}$ (C) $\frac{4C}{5}$ (D) $\frac{3}{2}C$

Ans : (B)

Hint : $\frac{c \times 2c}{3c} = \frac{2c}{3}$

18. In a single-slit diffraction experiment, the slit is illuminated by light of two wavelength λ_1 and λ_2. It is observed that the 2nd order diffraction minimum for λ_1 coincides with the 3rd diffraction minimum for λ_2. Then

(A) $\frac{\lambda_1}{\lambda_2} = \frac{2}{3}$ (B) $\frac{\lambda_1}{\lambda_2} = \frac{5}{7}$ (C) $\frac{\lambda_1}{\lambda_2} = \frac{3}{2}$ (D) $\frac{\lambda_1}{\lambda_2} = \frac{7}{5}$

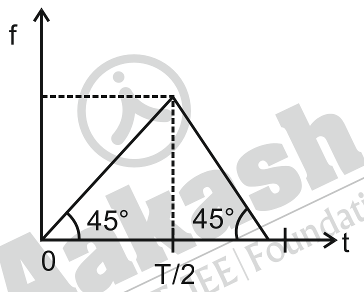
Ans : (C)

Hint : $d\sin\theta = n\lambda$

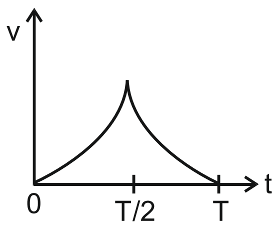
$n_1\lambda_1 = n_2\lambda_2$

$\frac{\lambda_1}{\lambda_2} = \frac{n_2}{n_1} = 3/2$

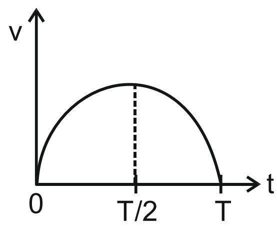
19. The acceleration-time graph of a particle moving in a straight line is shown in the figure. If the initial velocity of the particle is zero then the velocity-time graph of the particle will be



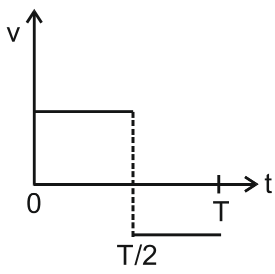
(A)



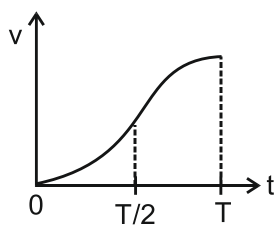
(B)



(C)



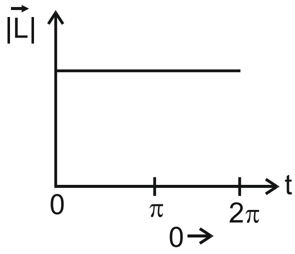
(D)

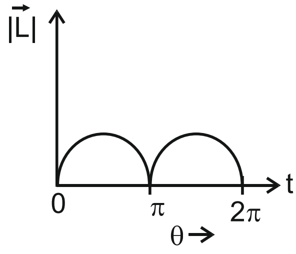


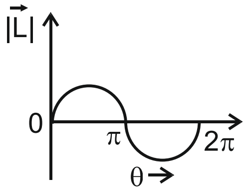
Ans : (D)

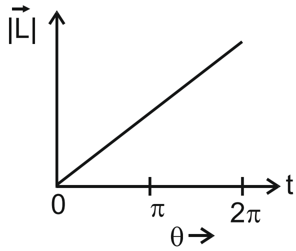
Hint : $\because$ a = slope of v – t graph

20. The position vector of a particle of mass m moving with a constant velocity $\vec{\upsilon}$ is given by $\vec{r} = x(t)\hat{i} + b\hat{j}$, where b is a constant. At an instant, $\vec{r}$ makes an angle θ with the x-axis as shown in the figure. The variation of the angular momentum of the particle about the origin with θ will be

(A) 

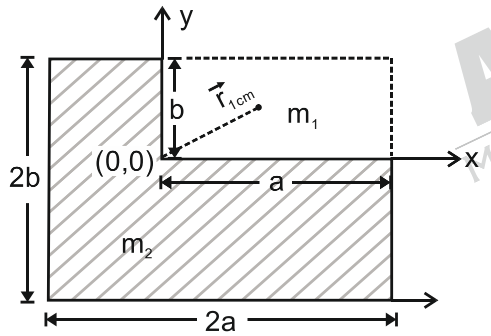
(B) 

(C) 

(D) 

Ans : (A)

Hint : L = mVx . b = constant

21. 

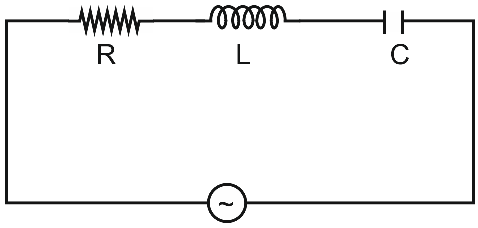
The position of the centre of mass of the uniform plate as shown in the figure is

(A) $\left(-\frac{a}{2}, -\frac{b}{2}\right)$ (B) $\left(\frac{a}{8}, \frac{b}{8}\right)$ (C) $\left(-\frac{b}{6}, -\frac{a}{6}\right)$ (D) $\left(-\frac{a}{6}, -\frac{b}{6}\right)$

Ans : (D)

Hint : Required $\vec{r} = -\frac{m_1}{m_2}(\vec{r}_{1cm})$

$m_2 = 3m_1$

22. 

$V = 50\sqrt{2}\sin\omega t$

In a series LCR circuit, the rms voltage across the resistor and the capacitor are 30 V and 90 V respectively. If the applied voltage is $50\sqrt{2}$ sin ωt, then the peak voltage across the inductor is

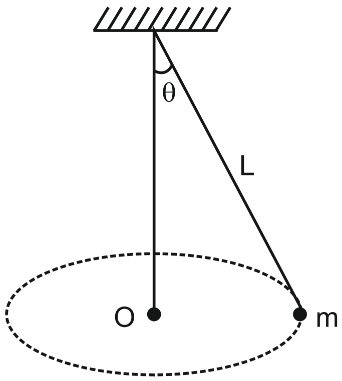
(A) 70 V (B) 50 V (C) $70\sqrt{2}$ V (D) $50\sqrt{2}$ V

Ans : (D)

Hint : $\sqrt{30^2 + (90 - V_L)^2} = 50$

$\Rightarrow V_L = 50V$

$= (V_L)_{max} = 50\sqrt{2}$ V

23. 

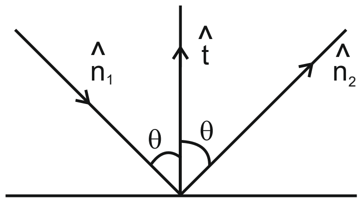
A small ball of mass m is suspended from the ceiling of a floor by a string of length L. The ball moves along a horizontal circle with constant angular velocity ω, as shown in the figure. The torque about the centre (O) of the horizontal circle is

(A) mg L sin θ (B) mg L (C) 0 (D) mg L cos θ

Ans : (C)

Hint : Angular momentum is conserved about O.

24. If $\hat{n}_1$, $\hat{n}_2$ and $\hat{t}$ represent, unit vectors along the incident ray, reflected ray and normal to the surface respectively, then



(A) $\hat{n}_2 = \hat{n}_1 - 2(\hat{n}_1.\hat{t})\hat{t}$ (B) $\hat{n}_2 = \hat{n}_1 + 2(\hat{n}_1.\hat{t})\hat{t}$ (C) $\hat{n}_2 = -\hat{n}_1$ (D) $\hat{n}_2 = 2\hat{n}_1 - (\hat{n}_1 \times \hat{t}).\hat{n}_1$

Ans : (A)

Hint : $\hat{n}_1 = -\cos\theta\,\hat{t} + \sin\theta\,\hat{i}$

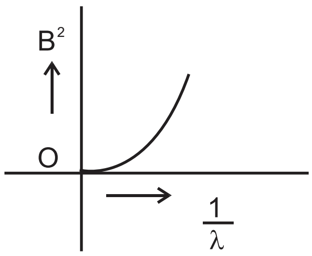
$\hat{n}_2 = \cos\theta\,\hat{t} + \sin\theta\,\hat{i}$

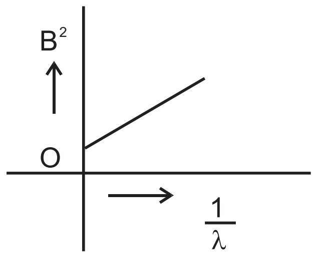
$\hat{n}_2 - \hat{n}_1 = 2\cos\theta\hat{t} = -2(\hat{n}_1.\hat{t})\hat{t}$

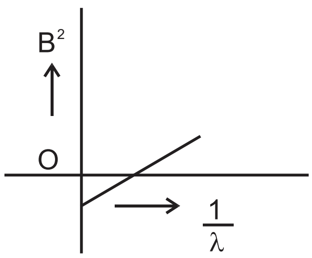
$\hat{n}_1.\hat{t} = -\cos\theta$

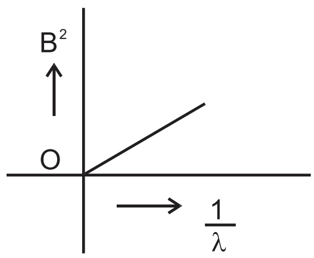
$\boxed{\hat{n}_2 = \hat{n}_1 - 2(\hat{n}_1.\hat{t})\hat{t}}$

25. A beam of light of wavelength λ falls on a metal having work function ϕ placed in a magnetic field B. The most energetic electrons, perpendicular to the field are bent in circular arcs of radius R. If the experiment is performed for different values of λ, then B^2 vs. $\frac{1}{\lambda}$ graph will look like (keeping all other quantities constant)

(A)


(B)


(C)


(D)


Ans : (C)

Hint : $\frac{hc}{\lambda} - \phi = k$, $R = \frac{\sqrt{2mk}}{qB}$ $B^2 \propto K$, $B^2 = A_1\left(\frac{1}{\lambda}\right) - A_2$

26. A charged particle moving with a volocity $\vec{v} = v_1\hat{i} + v_2\hat{j}$ in a magnetic field $\vec{B}$ experiences a force $\vec{F} = F_1\hat{i} + F_2\hat{j}$. Here v_1, v_2, F_1, F_2 all are constants. Then $\vec{B}$ can be

(A) $\vec{B} = B_1\hat{i} + B_2\hat{j}$ with $\frac{v_1}{v_2} = \frac{B_1}{B_2}$

(B) $\vec{B} = B_1\hat{i} + B_2\hat{j} + B_3\hat{k}$ with $\frac{v_1}{v_2} = \frac{B_1}{B_2}$

(C) $\vec{B} = B_3\hat{j}$ with $B_1 = B_2 = 0$

(D) $\vec{B} = B_1\hat{j} + B_2\hat{k}$ with $\frac{B_1}{B_2} = \frac{v_1}{v_2}$

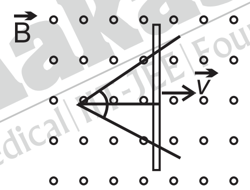
Ans : (B)

Hint : $\vec{F}.\vec{V} = 0 \Rightarrow \frac{F_1}{F_2} = -\frac{V_2}{V_1}$........(I)

$\vec{F}.\vec{B} = 0 \Rightarrow \frac{F_1}{F_2} = -\frac{B_2}{B_1}$.........(II)

Or $\boxed{\frac{B_1}{B_2} = \frac{V_1}{V_2}}$ $\vec{B} = B_1\hat{i} + B_2\hat{j} + B_3\hat{k}$

27. Two straignt conducting plates form an angle θ where their ends are joined. A conducting bar in contact with the plates and forming an isosceles triangle with them starts at the vertex at time t = 0 and moves with constant velocity $\vec{v}$ to the right as shown in figure. A magnetic field $\vec{B}$ points out of the page. The magnitude of emf induced at t = 1 second will be



(A) $Bv\tan\frac{\theta}{2}$ (B) $Bv^2\tan\frac{\theta}{2}$ (C) $2Bv^2\tan\frac{\theta}{2}$ (D) $2Bv^2\sin\frac{\theta}{2}$

Ans : (B)

Hint : $\ell = 2vt\tan\frac{\theta}{2}$

$\varepsilon = B\ell v = 2Bv^2\tan\frac{\theta}{2}$

28. Three point charges q, – 2q and q are placed along x axis at x = – a, 0 and a respectively. As $a \to 0$ and $q \to \infty$ while $q a^2 = Q$ remains finite, the clectric field at a point P, at a distance x(x >> a) from x = 0 is $E = \frac{\alpha Q}{4\pi \epsilon_0 x\beta}\hat{i}$. Then

(A) $\alpha = \beta$ (B) $\alpha = 2\beta$ (C) $\alpha = \frac{2}{3}\beta$ (D) $2\alpha = 2\beta$

Ans : (D)

Hint : $E = \frac{2kqa}{x^3}\left[\frac{1}{\left(1-\frac{a}{2x}\right)^3} - \frac{1}{\left(1+\frac{a}{2x}\right)^3}\right]$

$$=\frac{2kqa}{x^3}\left[1+\frac{3a}{2x}-1+\frac{3a}{2x}\right]=\frac{2kqa}{x^3}\left[\frac{3a}{x}\right] = \frac{6kqa^2}{x^4} = 6\frac{1}{4\pi\in}\frac{Q}{x^4}$$

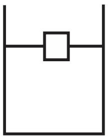
$\beta = 4 \quad \frac{\alpha}{\beta}=\frac{\cancel{6}^3}{\cancel{4}^2}$

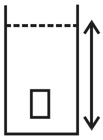
$\alpha = 6 \quad \alpha=\frac{3}{2}\beta$

29. A body floats with $\frac{1}{n}$ of its volume keeping outside of water. If the body has been taken to height h inside water and released, it will come to the surface after time t. Then

(A) $t \propto \sqrt{n}$ (B) $t \propto n$ (C) $t \propto \sqrt{n+1}$ (D) $t \propto \sqrt{n-1}$

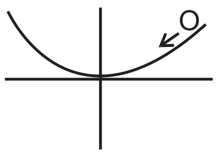
Ans : (D)

Hint :  $F_B = mg$; $V\left(1-\frac{1}{n}\right)\sigma g = V dg$, $\Rightarrow d=\left(\frac{n-1}{n}\right)\sigma$

 h $a=\frac{F_b-mg}{m}=\frac{v\sigma g}{m}-g$, $a=\frac{v\sigma g}{v\left(\frac{n-1}{n}\right)\sigma}-g$, $\Rightarrow a=\frac{ng}{n-1}-g$, $\Rightarrow a=\frac{g}{n-1}$

Now for time, $h=\frac{1}{2}at^2$, $h=\frac{1}{2}\frac{g}{(n-1)}t^2$, $\Rightarrow t=\sqrt{\frac{2h}{g}(n-1)}$ = $\Rightarrow \boxed{t\alpha\sqrt{n-1}}$

30. A small sphere of mass m and radius r slides down the smooth surface of a large hemispherical bowl of radius R. If the sphere starts sliding from rest, the total kinetic energy of the sphere at the lowest point A of the bowl will be [given, moment of inertia of sphere = $\frac{2}{5}mr^2$]



(A) mg (R – r) (B) $\frac{7}{10}mg(R-r)$ (C) $\frac{2}{7}mg(R-r)$ (D) $\frac{10}{7}mg(R-r)$

Ans : (A)

Hint : $mg(R-r) = K_f$

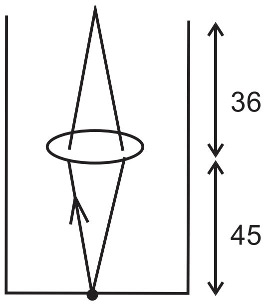
<u>Category 2 (Q. 31 to 35)</u>

(Carry 2 marks each. Only one option is correct. Negative marks – ½)

31. When a convex lens is placed above an empty tank, the image of a mark at the bottom of the tank, which is 45 cm from the lengs is formed 36 cm above the lens. When a liquid is poured in the tank to a depth of 40 cm, the distance of the image of the mark above the lens is 48 cm. The refractive index of the liquid is

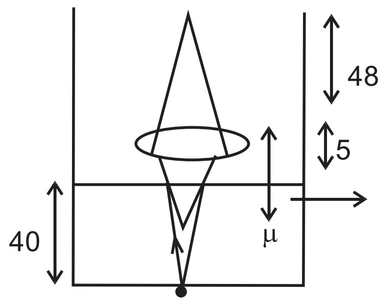
(A) 1.358 (B) 1.544 (C) 1.472 (D) 1.366

Ans : (D)

Hint : 

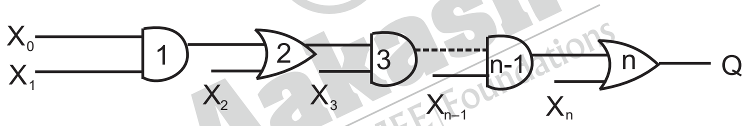
Initially without water, $\frac{1}{36}-\frac{1}{-45}=\frac{1}{f}\Rightarrow f=20$

then, after pouring water upto 40 cm



apparent object distance $=\frac{40}{\mu}+5$, $\frac{1}{48}+\frac{1}{\frac{40}{\mu}+5}=\frac{1}{20}$ $\Rightarrow \mu=1.366$

32. In the given network of AND and OR gates, output Q can be written as (assuming n even)



(A) $X_0X_1+X_2X_3+\ldots X_{n-1}X_n$

(B) $X_0X_1\ldots X_n+X_1X_2\ldots X_n+X_2X_3\ldots X_n+X_n$

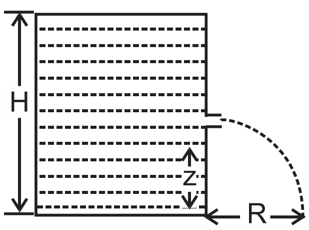
(C) $X_0X_1\ldots X_{n-1}+X_{n-2}+X_{n-2}X_{n-1}+X_n$

(D) $X_0X_1\ldots X_{n-1}+X_2X_3X_5\ldots X_{n-1}+X_{n-2}X_{n-1}+X_n$

Ans : (D)

Hint : $\left[\{(X_0X_1+X_2)X_3+X_4\}X_5+X_6\right]X_7+......$

33. Water in filled in a cylindrical vessel of height H. A hole is made at height z from the bottom, as shown in the figure. The value of of z for which the range (R) of the emerging water through the hole will be maximum for



(A) $z=\frac{H}{4}$ (B) $z=\frac{H}{2}$ (C) $z=\frac{H}{8}$ (D) $z=\frac{H}{3}$

Ans : (B)

Hint : Theory

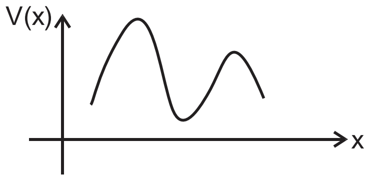
34. A metal plate of area 10^{-2} m^2 rests on a lauer of castor oil, 2×10^{-3} m thick whose coefficient of viscosity is 1.55 Ns m^{-2}. The approximate horizontal force required to move the plate with a uniform speed of 3×10^{-2} ms^{-1} is

(A) 0.6718 N (B) 0.2325 N (C) 0.2022 N (D) 0.6615 N

Ans : (B)

Hint : $F = \eta A \frac{v}{h} = 1.55 \times 10^{-2} \times \frac{3 \times 10^{-2}}{2 \times 10^{-3}}$

35. The following figure shows the variation of potential energy V(x) of a particle with distance x. The particle has



(A) Two equilibrium points, one stable another unstable (B) Two equilibrium points, both stable

(C) Three equilibrium points, one stable two unstable (D) Three equilibrium points, two stable one unstabl

Ans : (C)

Hint : Theory

Category 3 (Q36 to 40)

(Carry 2 marks each. One or more options are correct. No negative marks)

36. Monochromatic light of wavelength λ = 4770 Å is incident separately on the surface of four different metals A, B, C and D. The work functions of A, B, C and D are 4.2 eV, 3.7 eV, 3.2 eV and 2.3 eV, respectively. The metal / metals from which electrons will be emitted is /are

(A) A, B, C and D (B) B, C and D (C) C and D (D) D only

Ans : (D)

Hint : $E_{photon} = \frac{hc}{\lambda} = 2.6$ eV $>$ work function for photoelectons emission

37. Consider the integral form of the Gauss' law in electrostatics.

$$\oiint \vec{E} \cdot \vec{ds} = \frac{Q}{\epsilon_0}$$

Which of the following statements are correct?

(A) It contains law of Coulomb.

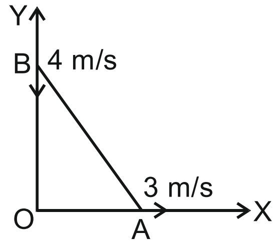
(B) It contains superposition principle.

(C) An elementary patch on the enclosing surface is a polar vector.

(D) An elementary patch on the enclosing surface is a pseudo-vector.

Ans : (A, B, C)

Hint : Theory

38. 

A uniform rod AB of length 1m and mass 4kg is sliding along two mutually perpendicular frictionless walls OX and OY. The velocity of the two ends of the rod A and B are 3 m/s and 4 m/s respectively, as shown in the figure. Then which of the following statement(s) is/are correct?

(A) The velocity of the centre of mass of the rod is 2.5 m/s.

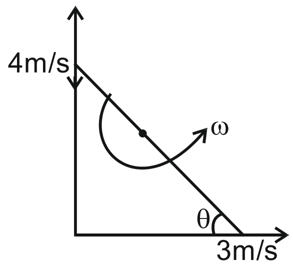
(B) Rotational kinetic energy of the rod is $\frac{25}{6}$ joule.

(C) The angular velocity of the rod is 5 rad/s clockwise.

(D) The angular velocity of the rod is 5 rad/s anticlockwise.

Ans : (A, B, D)

Hint :

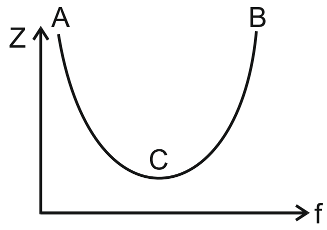


Get θ

$$3\cos\theta = 4\sin\theta$$

$$\omega = \frac{3\sin\theta + 4\sin\theta}{\ell}$$

$$(\text{KE})_{\text{rotation}} = \frac{1}{2}\,\frac{1}{12}ml^2\,\omega^2 = \frac{25}{6}$$

39. 

The variation of impedance Z of a series LCR circuit with frequency of the source is shown in the figure. Which of the following statement(s) is/are true?

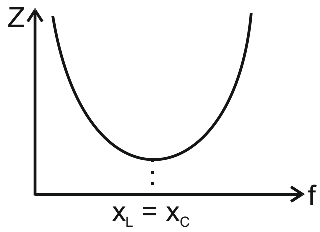
(A) The impedance Z is inductive in the portion AC

(B) The impedance Z is capacitive in the portion BC

(C) The impedance Z is inductive in the portion BC

(D) The impedance Z is capacitive in the portion AC

Ans : (C, D)

Hint : 

AC : $X_L < X_C \Rightarrow$ capacitive nature

BC : $X_L > X_C \Rightarrow$ Inductive nature

40. The electric field of a plane electromagnetic wave in a medium is given by $\vec{E}(x, y, z, t) = E_0 \hat{n} e^{ik_0[(x+y+z)-ct]}$

where c is the speed of light in free space. $\vec{E}$ field is polarized in the x – z plane. The speed of wave is ν in the medium. Then

(A) $\hat{n} = \hat{i} - \hat{k}$; $\nu = c$.

(B) $\hat{n} = \dfrac{\hat{i} - \hat{k}}{\sqrt{2}}$; $\nu = \dfrac{c}{\sqrt{3}}$

(C) refractive index of the medium is $\sqrt{3}$

(D) $\hat{n} = \dfrac{\hat{i} + \hat{k}}{\sqrt{2}}$; $\nu = \dfrac{c}{\sqrt{2}}$

Ans : (B, C)

Hint : $\vec{E} = E_0 \hat{n}\ e^{ik_0(\hat{k}\cdot\hat{r} - wt)}$

$\Rightarrow \vec{k} = \hat{i} + \hat{j} + \hat{k}$

$\Rightarrow v = \dfrac{c}{|\hat{i} + \hat{j} + \hat{k}|} = \dfrac{c}{\sqrt{3}}$

$\Rightarrow$ refractive index $= \sqrt{3}$

$\vec{E}$ is in z – x plane and $\vec{E} \cdot \vec{V} = 0$

$\Rightarrow \hat{n} = \dfrac{\hat{i} - \hat{k}}{\sqrt{2}}$

CHEMISTRY

CATEGORY - 1 (Q 41 to 70)

(Carry 1 mark each. Only one option is correct. Negative marks: –¼)

41. In the following sequence of reaction compound 'M' is

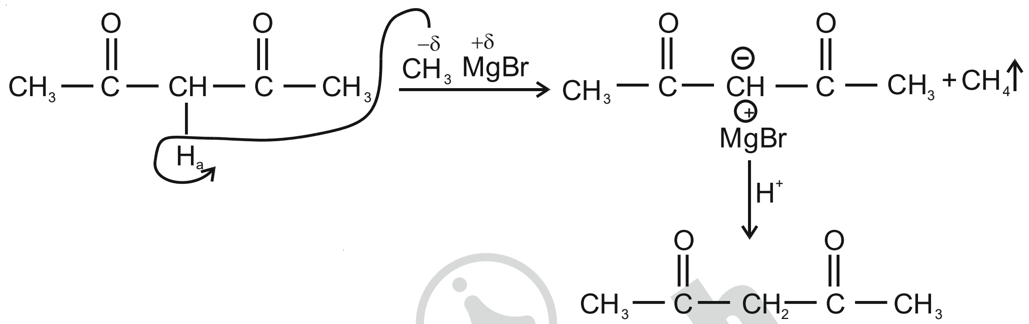
$$M \xrightarrow{CH_3MgBr} N + CH_4\uparrow \xrightarrow{H^+} CH_3COCH_2COCH_3$$

(A) $CH_3COCH_2COCH_3$ (B) $CH_3COCH_2CO_2Et$

(C) $CH_3-\overset{\overset{O}{\|}}{C}-CH_3$ (D) $CH_3-\overset{\overset{O}{\|}}{C}-OH$

Ans : (A)

Hint :



42. Identify the ion having $4f^6$ electronic configuration.

(A) Gd^{3+} (B) Sm^{3+} (C) Sm^{2+} (D) Tb^{3+}

Ans : (C)

Hint : ${}_{62}Sm = [Xe]4f^6 6s^2$

$\therefore Sm^{+2} = [xe]4f^6 6s^0$

43. Metallic conductors and semiconductors are heated separately. What are the changes with respect to conductivity?

(A) increase, increase (B) decrease, decrease

(C) increase, decrease (D) decrease, increase

Ans : (D)

Hint : For Metallic Conductors, $R \propto T$ (R is the resistance)

Hence conductivity decreases with increase of Temperature

For semi-conductors, conductivity increases with rise of temperature due increase in number of electron-hole pairs.

44. The equivalent weight of $Na_2S_2O_3$(Gram molecular weight = M) in the given reaction is

$I_2 + 2Na_2S_2O_3 = 2\,NaI + Na_2S_4O_6$

(A) M/2 (B) M (C) 2M (D) M/4

Ans : (B)

Hint : $I_2 + 2Na_2\overset{+2}{S_2}O_3 = 2NaI + Na_2\overset{+2.5}{S_4}O_6$

n·f=2 × 0.5 = 1

$\therefore$ Number of moles of electron lost per mole of $Na_2S_2O_3$ = 1

Hence equivalent weight of $Na_2S_2O_3 = \dfrac{M}{1} = M$

45. The reactivity order of the following molecules towards S_N1 reaction is

Allyl chloride	Chlorobenzene	Ethyl chloride
(I)	(II)	(III)

(A) I > II > III (B) I > III > II (C) II > I > III (D) III > I > II

Ans : (B)

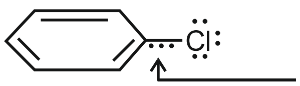
Hint : Reactivity of S_N1 α stability of carbocation

$$CH_2 = CH - CH_2 - Cl \xrightarrow{r.d.s} CH_2 = CH - \overset{\oplus}{C}H_2 + Cl^- \longleftrightarrow \overset{\oplus}{C}H_2 - CH = CH_2$$

(allyl chloride)

I

(Resonance stabilized)



Bond order > 1 due to L.P. $-\pi$ conjugation.
Hence no heterolytic cleavage.Therefore No S_N1

(chlorobenzene)

II

$$CH_3 - CH_2 - Cl \xrightarrow{r.d.s} CH_3 - \overset{\oplus}{C}H_2 + Cl^-$$

III

(1°carbocation)

$\therefore$ I > III > II

46. Toluene reacts with mixed acid at 25° C to produce

(A) nearly equal amounts of o- and m- nitrotoluene

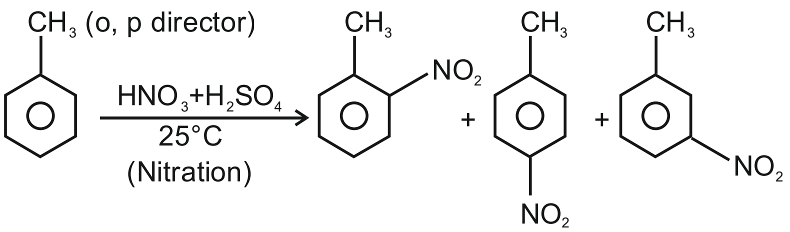
(B) p nitrotoluene (only)

(C) Predominantly o- nitrotoluene and p- nitrotoluene

(D) 2, 4, 6- trinitrotoluene (only)

Ans : (C)

Hint :



(~61.5%) (~37%) (~1.5%)

47. $PhCHO + CH_3CH_2-\overset{\overset{\large O}{\|}}{C}-O-\overset{\overset{\large O}{\|}}{C}-CH_2CH_3 \xrightarrow[\text{(anhydrous)}]{CH_3CH_2-\overset{\overset{O}{\|}}{C}-ONa} P$

The product 'P' in the above reaction is

(A) $PhCH=CHCH_2COOH$

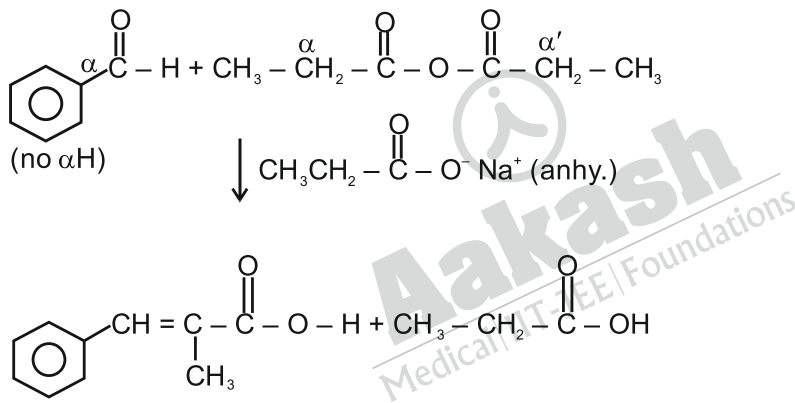
(B) $Ph-CH=\overset{\overset{\large CH_3}{|}}{C}-COOH$

(C) $Ph\,CH<^{OCOCH_3}_{OCOCH_3}$

(D) $Ph\,CH=CH-CH_2-\overset{\overset{\large O}{\|}}{C}-O-\overset{\overset{\large O}{\|}}{C}-CH_2CH_3$

Ans : (B)

Hint :



(Perkin's condensation)

48. The decreasing order of reactivity of the following alkenes towards HBr addition is

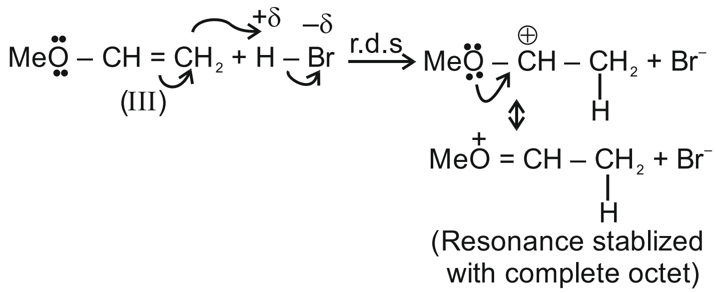
$CH_3-CH=CH_2$ (I)	$CF_3-CH=CH_2$ (II)	$MeOCH=CH_2$ (III)	$CH_3-\overset{\overset{O}{\|}}{C}-CH=CH_2$ (IV)
(A) I>II>III>IV	(B) II>IV>I>III	(C) III>IV>I>II	(D) III>I>II>IV

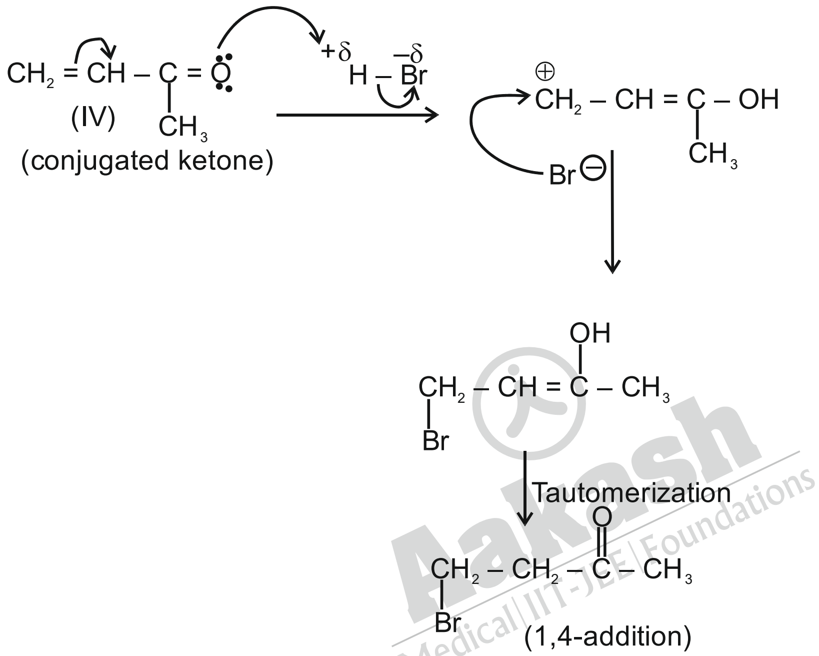
Ans : (C)

Hint : Reactivity of HBr addition to Alkene $\propto$ stability of carbocation

$$\underset{\text{(I)}}{CH_3-CH=CH_2} + \overset{+\delta}{H}-\overset{-\delta}{Br} \xrightarrow{\text{r.d.s}} \underset{(+I)}{CH_3}\rightarrow\overset{\oplus}{C}H-\underset{\underset{\large H}{|}}{CH_2} + Br^- \quad (2^\circ)$$

$$\underset{\text{(II)}}{CF_3-CH=CH_2} + \overset{+\delta}{H}-\overset{-\delta}{Br} \xrightarrow{\text{r.d.s}} \underset{\text{(strong –I)}}{F_3C}\leftarrow\underset{\underset{\large H}{|}}{CH}-\overset{\oplus}{C}H_2 + Br^- \quad (1^\circ)$$

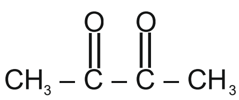




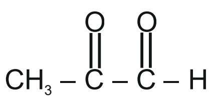
Thus stability order of carbocation

III > IV > I > II

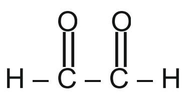
49. Ozonolysis of o-xylene produces



I



II

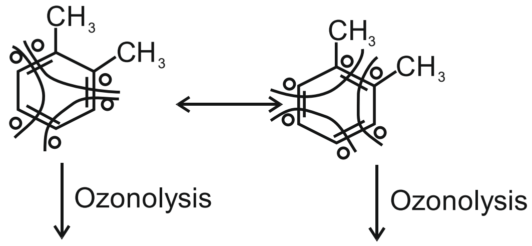


III

(A) I:III = 1:2 (B) II:III = 2:1 (C) I:II:III = 1:2:3 (D) I:II:III = 3:2:1

Ans : (C)

Hint :

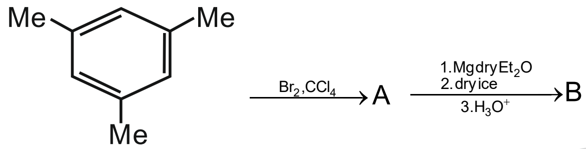


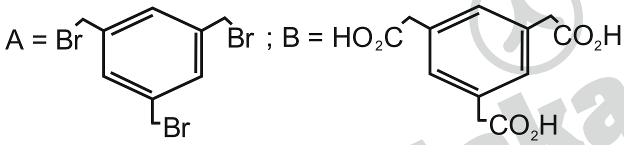
$$CH_3-\overset{\displaystyle O}{\overset{\|}{C}}-\overset{\displaystyle O}{\overset{\|}{C}}-CH_3+2\,H-\overset{\displaystyle O}{\overset{\|}{C}}-\overset{\displaystyle O}{\overset{\|}{C}}-H \qquad 2CH_3-\overset{\displaystyle O}{\overset{\|}{C}}-\overset{\displaystyle O}{\overset{\|}{C}}-H+H-\overset{\displaystyle O}{\overset{\|}{C}}-\overset{\displaystyle O}{\overset{\|}{C}}-H$$

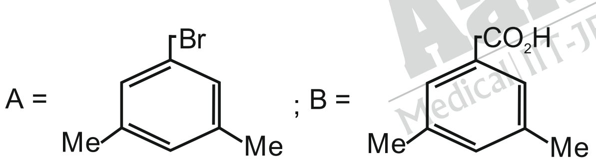
I = 1, II = 2, III = 3

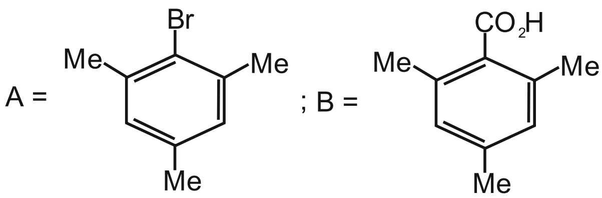
$\therefore$ I : II : III = 1 : 2 : 3.

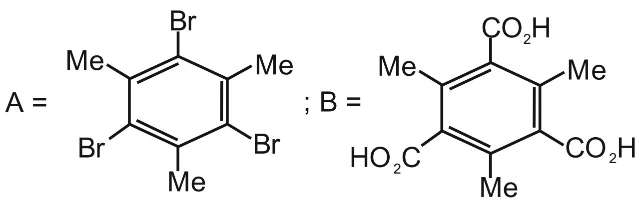
50. The compounds A and B are respectively



(A) 

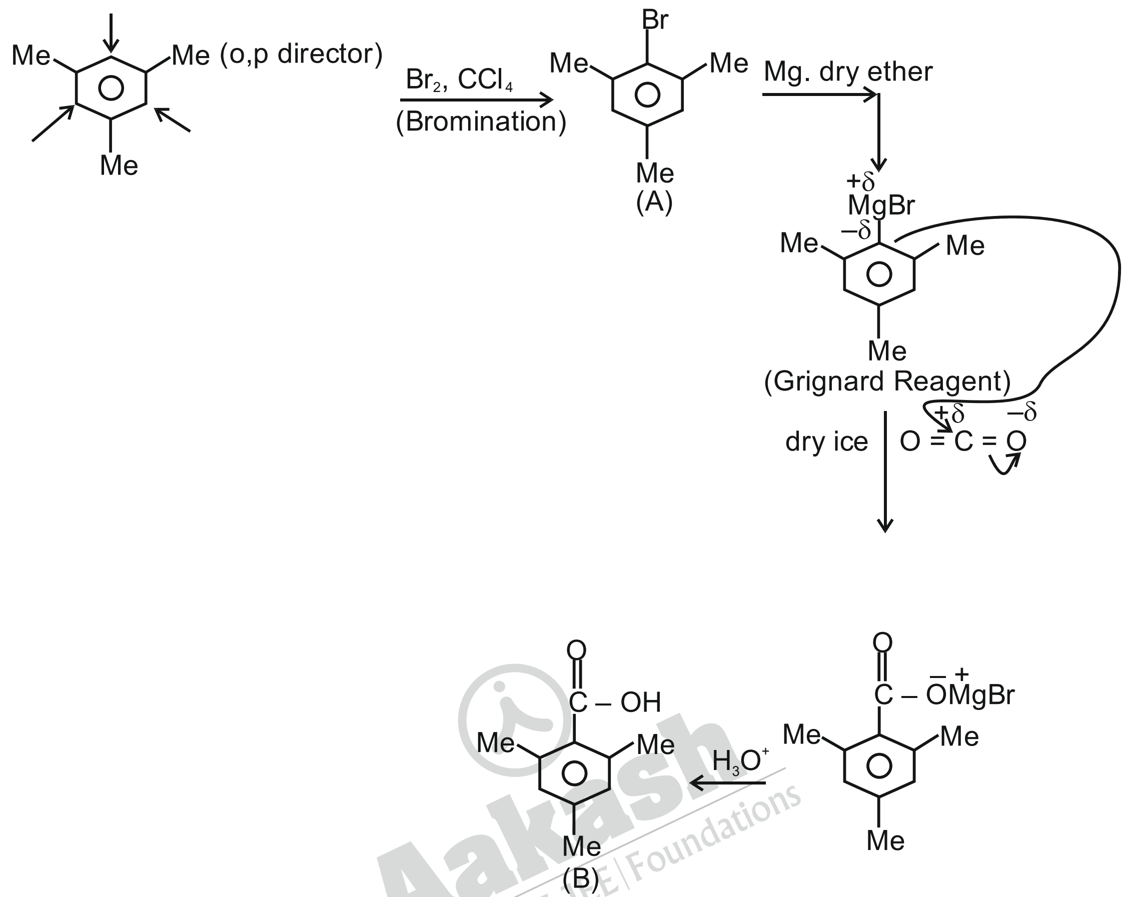
(B) 

(C) 

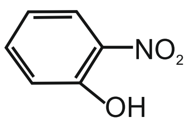
(D) 

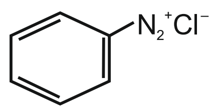
Ans : (C)

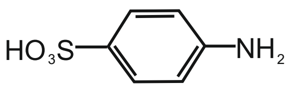
Hint :

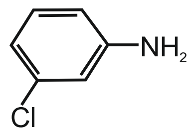


51. The compound that does not give positive test for nitrogen in Lassaigne's test is

(A) 

(B) 

(C) 

(D) 

Ans : (B)

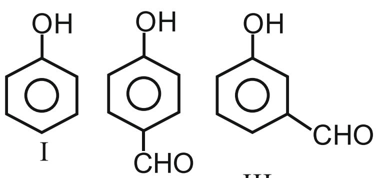
Hint : Benzene diazonium chloride on heating produces N_2 gas which escapes from the mixture. So, NaCN is not formed in Lassaigne's Test

52. The correct acidity order of phenol (I), 4-hydroxybenzaldehyde (II) and 3-hydroxybenzaldehyde (III) is

(A) I<II<III (B) I<III<II (C) II<III<I (D) III<II<I

Ans : (B)

Hint :



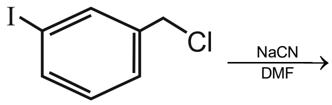
	I	II	III
pKa	9.95	7.61	8.98

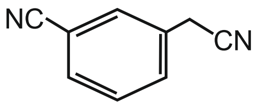
Acidic Strength II > III > I

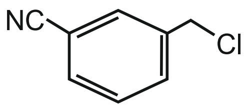
–CHO is electron withdrawing group which increases acidic strength of phenolic compound.

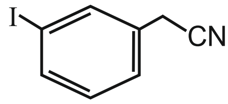
Also withdrawing effect of –CHO is more through para position as compound to meta position.

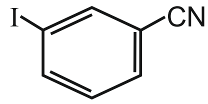
53. The major product of the following reaction is:



(A) 

(B) 

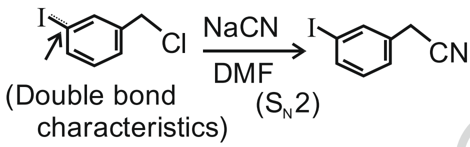
(C) 

(D) 

Ans : (C)

Hint : Halogen attached to benzene ring does not participate in S_N2 reaction.

Here, Cl attached to bezylic carbon can only be substituted.



54. Which of the following statements is correct for a spontaneous polymerization reaction ?

(A) $\Delta G < 0, \Delta H < 0, \Delta S < 0$ (B) $\Delta G < 0, \Delta H > 0, \Delta S > 0$

(C) $\Delta G > 0, \Delta H < 0, \Delta S > 0$ (D) $\Delta G > 0, \Delta H > 0, \Delta S > 0$

Ans : (A)

Hint : $nA \rightarrow A_n$

During spontaneous polymerisation reaction number of particles decreases, hence entropy decreases.

Being spontaneous ΔG of the reaction is less than zero.

$\Delta G = \Delta H - T\Delta S$

To make the ΔG negative, ΔH must be negative i.e. less than zero.

55. At 25°C, the ionic product of water is 10^{-14}. The free energy change for the self-ionization of water in kCal mol^{-1} is close to

(A) 20.5 (B) 14.0 (C) 19.1 (D) 25.3

Ans : (C)

Hint : $H_2O(\ell) \rightleftharpoons H^+(aq) + OH^-(aq)$ $K = 10^{-14}$

$\Delta G° = -2.303\ RT \log K$

$= -2.303 \times 1.987 \times 298 \log 10^{-14}$

$= 2.303 \times 1.987 \times 298 \times 14$

$= 19091$ Cal

≈ 19.1 KCal

56. Consider an electron moving in the first Bohr orbit of a He^+ ion with a velocity ν_1. If it is allowed to move in the third Bohr orbit with a velocity ν_3, then indicate the correct $\nu_3 : \nu_1$ ratio

(A) 3:1 (B) 2:1 (C) 1:3 (D) 1:2

Ans : (C)

Hint : $\nu = \nu_0 \frac{z}{n}$

For 1st orbit of He^+ $\nu_1 = \nu_0 \frac{2}{1}$

For 3rd orbit of He^+ $\nu_3 = \nu_0 \frac{2}{3}$

$\nu_3 : \nu_1 = 1 : 3$

57. The compressibility factor for a van der Waal gas at high pressure is

(A) $1+\frac{RT}{Pb}$ (B) $1+\frac{Pb}{RT}$ (C) $1-\frac{Pb}{RT}$ (D) 1

Ans : (B)

Hint : For 1 mol van der Waal's gas

$$\left(P+\frac{a}{V^2}\right)(v-b)=RT$$

At high pressure $P+\frac{a}{V^2} \approx P$

$\therefore$ P(V – b) = RT

$\therefore$ PV – Pb = RT

PV = RT + Pb

$$z=\frac{PV}{RT}=\frac{RT}{RT}+\frac{Pb}{RT}$$

$$\therefore\ z=1+\frac{Pb}{RT}$$

58. For a spontaneous process, the incorrect statement is

(A) $(\Delta G_{system})_{T,P} > 0$ (B) $(\Delta S_{system})_+ (\Delta S_{surroundings}) > 0$

(C) $(\Delta G_{system})_{T,P} < 0$ (D) $(\Delta U_{system})_{S,V} < 0$

Ans : (A)

Hint : For spontaneous process

* $(\Delta G_{system})_{P,T} < 0$
* $\Delta S_{universe} = \Delta S_{system} + \Delta S_{surrounding} > 0$
* $(\Delta U_{system})_{S,V} < 0$

59. Identify the incorrect statement among the following:

(A) Viscosity of liquid always decreases with increase in temperature

(B) Surface tension of liquid always decreases with increase in temperature

(C) Viscosity of liquid always increases in presence of impurity

(D) Surface tension of liquid always increases in presence of impurity

Ans : (C, D)

Hint : Both options C and D are incorrect, as Viscosity and Surface Tension may increase or decrease upon addition of impurities.

60. Which of the following statements is true about equilibrium constant and rate constant of a single step chemical reaction?

(A) Equilibrium constant may increase or decrease but rate constant always increases with temperature

(B) Both equilibrium constant and rate constant increase with temperature

(C) Rate constant may increase or decrease but equilibrium constant always increases with temperature

(D) Both equilibrium constant and rate constant decrease with temperature

Ans : (A)

Hint : According to Van't Hoff equation

$$\log\left(\frac{k_2}{k_1}\right)=\frac{\Delta H}{2.303R}\left[\frac{1}{T_1}-\frac{1}{T_2}\right]$$

Where k_1 and k_2 are Equilibrium Constants at T_1 and T_2 temperature respectively.

When $\Delta H > 0$ (Endothermic process)

then Equilibrium Constant increases with increase in temperature

When $\Delta H < 0$ (Exothermic process)

then Equilibrium Constant decreases with increases in temperature.

According to Arrhenius equation

$$\log\left(\frac{k_2}{k_1}\right)=\frac{E_a}{2.303R}\left[\frac{1}{T_1}-\frac{1}{T_2}\right]$$

where k_1 and k_2 are Rate Constants at T_1 and T_2 temperature respectively.

E_a (Activation energy) > 0

When temperature increases rate constant will increase

61. After the emission of a β-particle followed by an α-particle from ${}^{214}_{83}Bi$, the number of neurons in the atom is

(A) 210 (B) 128 (C) 129 (D) 82

Ans : (B)

Hint : ${}^{214}_{83}Bi \xrightarrow{\beta} {}^{214}_{84}Po \xrightarrow{\alpha} {}^{210}_{82}Pb$

No. of neutrons = Mass No. (A) – Atomic No. (Z) = 210 – 82 = 128

62. Which hydrogen like species will have the same radius as that of 1st Bohr orbit of hydrogen atom?

(A) n = 2, Li^{2+} (B) n = 2, Be^{3+} (C) n = 2, He^{+} (D) n = 3, Li^{2+}

Ans : (B)

Hint : Radius of n^{th} orbit = $0.529\ \frac{n^2}{Z}$ Å

Radius of 1st Bohr Orbit of Hydrogen atom = $0.529\times\frac{1^2}{1}=0.529$ Å

Radius of 2nd Bohr Orbit of Be^{3+} = $0.529\ \frac{2^2}{4}=0.529$ Å

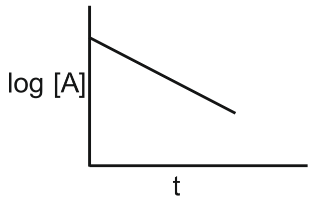
63. For a first order reaction with rate constant k, the slope of the plot of log (reactant concentration) against time is

(A) k/2.303 (B) k (C) –k/2.303 (D) –k

Ans : (C)

Hint : $\log[A]_t = \left(\frac{-k}{2.303}\right)t + \log[A]_0$ (Arrhenius Equation)

y mx C



Slope = $\frac{-k}{2.303}$

64. Equal volumes of aqueous solution of 0.1(M) HCl and 0.2 (M) H_2SO_4 are mixed. The concentration of H^+ ions in the resulting solution is

(A) 0.15 (M) (B) 0.30 (M) (C) 0.10 (M) (D) 0.25 (M)

Ans : (D)

Hint : Concentration of H^+ in molarity $= \frac{\text{Total no. of mole of } H^+}{\text{Total volume of solution (in L)}} = \frac{\sum M \times V\,(\text{in L}) \times \text{Basicity}}{\sum V}$

$$= \frac{V \times 0.1 \times 1 + V \times 0.2 \times 2}{V + V} = \frac{0.5}{2} = 0.25 \text{ M}$$

65. The correct order of boiling point of the given aqueous solution is

(A) 1 N KNO_3 > 1 N NaCl > 1 N CH_3COOH > 1 N sucrose

(B) 1 N KNO_3 = 1 N NaCl > 1 N CH_3COOH > 1 N sucrose

(C) Same for all

(D) 1 N KNO_3 = 1 N NaCl = 1 N CH_3COOH > 1 N sucrose

Ans : (B)

Hint : Elevation in boiling point $\propto$ No. of solute particles

KNO_3 (strong electrolyte); $KNO_{3(aq)} \rightarrow K^+_{(aq)} + NO^-_{3(aq)}$

NaCl (Strong electrolyte); $NaCl_{(aq)} \rightarrow Na^+_{(aq)} + Cl^-_{(aq)}$

CH_3COOH (Weak electrolyte); $CH_3COOH_{(aq)} \rightleftharpoons CH_3CO\bar{O}_{(aq)} + H^+_{(aq)}$

Sucrose (Non-electrolyte)

66. Correct solubility order of AgF, AgCl, AgBr, AgI in water is

(A) AgF < AgCl > AgBr > AgI

(B) AgI < AgBr < AgCl < AgF

(C) AgF < AgI < AgBr < AgCl

(D) AgCl > AgBr > AgF > AgI

Ans : (B)

Hint : AgF is soluble in water. The others are sparingly soluble and solubility decreases from chloride to iodide.

67. What will be the change in acidity if

(i) $CuSO_4$ is added in saturated $(NH_4)_2SO_4$ solution

(ii) SbF_5 is added in anhydrous HF

(A) increase, increase (B) decrease, decrease (C) increase, decrease (D) decrease, increase

Ans : (A)

Hint : i) $Cu^{2+}_{(aq)} + 4NH_4^+{}_{(aq)} \rightarrow [Cu(NH_3)_4]^{+2} + 4H^+{}_{(aq)}$

Formation of stable $[Cu(NH_3)_4]^{+2}$ complex increases H^+ ion concentration, thus acidity increases

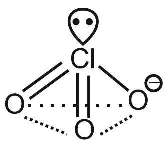
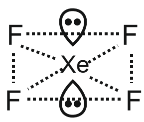
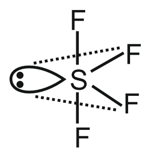
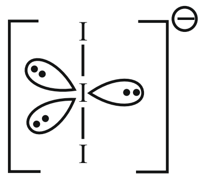
ii) $HF + HF \rightleftharpoons H_2F^+ + F^-$

SbF_5 removes F^- as SbF_6^- and shifts equilibrium forward to increase H_2F^+

68. Which of the following contains maximum number of lone pairs on the central atom?

(A) ClO_3^- (B) XeF_4 (C) SF_4 (D) I_3^-

Ans : (D)

Hint : a)  b)  c)  d) 

I_3^- has highest, i.e, 3 lone Pairs on central atom.

69. Number of moles of ions produced by complete dissociation of one mole of Mohr's salt in water is

(A) 3 (B) 4 (C) 5 (D) 6

Ans : (C)

Hint : Mohr's salt $FeSO_4.(NH_4)_2SO_4.6H_2O$

$$FeSO_4.(NH_4)_2SO_4.6H_2O \rightleftharpoons Fe^{+2} + 2NH_4^+ + 2SO_4^{-2}$$

5 ions produced on dissociation

70. Which of the following species exhibits both LMCT and paramagnetism?

(A) MnO_4^{2-} (B) MnO_4^- (C) $Cr_2O_7^{2-}$ (D) CrO_4^{2-}

Ans : (A)

Hint : MnO_4^{-2} ion exhibits colour due to charge transfer from O^{2-} ion to Mn^{+6} (LMCT) and also exhibits paramagnetism as it has $3d^1$ configuration (one unpaired electron)

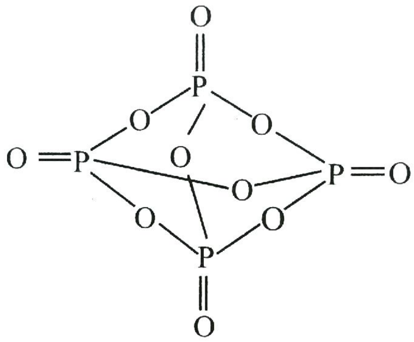
Category 2 (Q71 to Q 75)

(Carry 2 marks each. Only one option is correct. Negative marks :– ½)

71. How many P–O–P linkages are there in P_4O_{10}

(A) Six (B) Four (C) Five (D) One

Ans : (A)

Hint : 

72. $Me_3CCH_2CH_2OH \xleftarrow{Q} Me_3CCH = CH_2 \xrightarrow{R} Me_3C - \underset{}{\overset{OH}{\overset{|}{C}H}} - CH_3$. Q and R in the above reaction sequences are respectively

(A) $Hg(OAc)_2$, $NaBH_4/\overline{O}H$; B_2H_6,$H_2O_2/\overline{O}H$
(B) B_2H_6, $H_2O_2/\overline{O}H$; H^+/H_2O
(C) $Hg(OAc)_2$, $NaBH_4/\overline{O}H$; H^+/H_2O
(D) B_2H_6, $H_2O_2/\overline{O}H$; $Hg(OAc)_2$, $NaBH_4/\overline{O}H$

Ans : (D)

Hint : $Me_3CCH_2CH_2OH \xleftarrow[H_2O_2/\overline{O}H]{B_2H_6,} Me_3CCH{=}CH_2 \xrightarrow[NaBH_4/\overline{O}H]{Hg(OAC)_2,} Me_3C\overset{OH}{\overset{|}{C}}H{-}CH_3$

(Hydroboration - Oxidation) (Oxymercuration - Demercuration)

73. pH of 10^{-8} (M) HCl solution is

(A) 8
(B) greater than 7, less than 8
(C) greater than 8
(D) greater than 6, less than 7

Ans : (D)

Hint : For 10^{-8}(M) HCl solution, the solution being very dilute, H^+ ion conc. is not significant as compared to H^+ ion furnished from H_2O.

thus, $H^+_{total} = H^+_{HCl} + H^+_{H_2O}$

$= 10^{-8} + 10^{-7}$ (approx)

$= 1.1\times10^{-7}$ (approx)

$pH = -\log(1.1\times10^{-7})$

$= \quad 7 - \log(1.1) = 6.96$ (approx)

$\therefore$ Value of pH is greater than 6, less than 7.

74. The specific conductance (k) of 0.02 (M) aqueous acetic acid solution at 298K is 1.65×10^{-4} S cm^{-1}. The degree of dissociation of acetic acid is [$\lambda^\circ_{H^+} = 349.1$ S cm^2 mol^{-1} and $\lambda^\circ_{CH_3COO^-} = 40.9$ S cm^2 mol^{-1}]

(A) 0.021
(B) 0.21
(C) 0.012
(D) 0.12

Ans : (A)

Hint : Firstly we find λ_m (aq CH_3COOH)

$$\lambda_m = \frac{K\times1000}{C(M)} = \frac{1.65\times10^{-4}\times1000}{0.02} = 8.25\ Scm^2mol^{-1}$$

$\lambda_m(CH_3COOH \text{ at infinite dilution}) = \lambda_m^{\infty}(H^+) + \lambda_m^{\infty}(CH_3COO^-)$

Now degree of dissociation

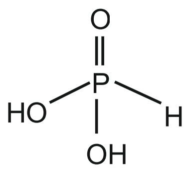
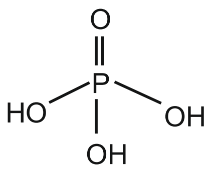
$$\alpha = \frac{\lambda_m[CH_3COOH \text{ at } 0.02(M)]}{\lambda_m[CH_3COOH \text{ at infinite dilution}]}$$

$$\alpha = \frac{8.25}{349.1 + 40.9} = 0.021$$

75. The number(s) of –OH group(s) present in H_3PO_3 and H_3PO_4 is/are

(A) 3 and 3 respectively (B) 3 and 4 respectively (C) 2 and 3 respectively (D) 1 and 3 respectively

Ans : (C)

Hint :  

Phosphorus acid — 2 OH groups

Orthophosphoric acid — 3 OH groups

Category 3 (Q76 to Q80)

(Carry 2 marks each. One or more options are correct. No negative marks)

76. Which of the following statements about the S_N2 reaction mechanism is/are true ?

(A) The rate of reaction increases with increasing nucleophilicity.

(B) The number 2 denotes a second order reaction

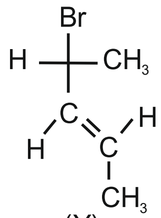
(C) Tertiary butyl substrates do not follow this mechanism

(D) The optical rotation of substrates always changes from (+) to (–) or from (–) to (+) in the products

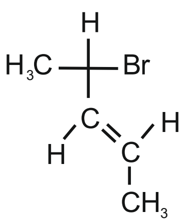
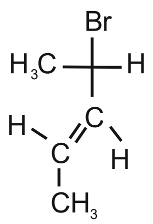
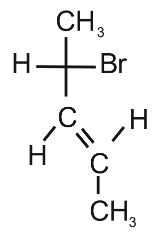
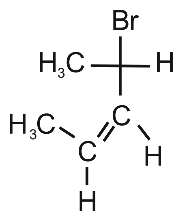
Ans : (A, C)

Hint : S_N2 reaction rate depends upon the nucleophilicity of the attacking nueleophile. Here the 2 in the notation stands for Bimolecularity. Tert-butyl substrate fails in S_N2 due to crowding. Optical rotation does not change necessarily from (+) to (–) or (–) to (+), as the product is not necessarily an isomer of reactant.

77. Which of the following represent(s) the enantiomer of Y ?



(Y)

(A)  (B)  (C)  (D) 

Ans : (B,C)

Hint : The configuration of the stereocentre in Y is 'R', while that of A, B and C are respectively R, S and S.

In option -D the configuration around the double bond is cis, so it can not be an enantiomer of Y.

78. Identify the correct statements(s) :

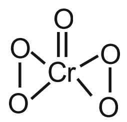
(A) The oxidation number of Cr in CrO_5 is + 6

(B) $\Delta H > \Delta U$ for the reaction $N_2O_4(g) \rightarrow 2\ NO_2(g)$, provided both gases behave ideally

(C) pH of 0.1 (N) H_2SO_4 is less than that of 0.1 (N) HCl at 25°C

(D) $\left(\frac{RT}{F}\right) = 0.0591$ volt at 25°C

Ans : (A,B)

Hint : A :  oxidation number of Cr is +6

B : $N_2O_4(g) \rightarrow 2\ NO_2(g)$

$\therefore\ \Delta n_g = 2-1=1$

and from $\Delta H = \Delta U + \Delta n_g RT$; $\Delta H > \Delta U$

C : For 0.1N HCl, pH is 1

$N_{H_2SO_4} = 0.1 \quad \therefore\ (H^+) = 0.1N \quad N_{HCl} = 0.1 \quad \therefore\ (H^+) = 0.1N$

Since both contains same concentration of H^+ ion, hence have same pH

D : $\frac{2.303\ RT}{F} = 0.0591$ at 25°C

79. Which of the following ion/ions is/are diamagnetic ?

(A) $[CoF_6]^{3-}$ (B) $[Co(NH_3)_6]^{3+}$ (C) $[Fe(OH_2)_6]^{2+}$ (D) $[Fe(CN)_6]^{4-}$

Ans : (B,D)

Hint : Co^{3+} is a d^6 ion. With strong field ligand like NH_3, $[Co(NH_3)_6]^{3+}$ has no unpaired electron. For Co^{3+}, F^- is a weak field ligand.

Fe^{2+} is a d^6 ion. With strong field ligand CN^-, $[Fe(CN)_6]^{4-}$ has no unpaired electron. For Fe^{2+}, H_2O is a weak field ligand.

80. Which of the following statement/statements is/are correct ?

(A) Solid I_2 is freely soluble in water

(B) Solid I_2 is freely soluble in water but only in presence of excess KI

(C) Solid I_2 is freely soluble in CCl_4

(D) Solid I_2 is freely soluble in hot water

Ans : (B,C)

Hint : I_2 dissolves in aqueous KI to form KI_3.

I_2 also dissolves in CCl_4 as both are non polar compounds.

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