Tutorial - 04

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D-30

1) $T(n) = 3T(n/2) + n^2$

$c = \log_2 3 = 1.58$

$n^2 = n^{1.58}$

$\therefore n^2 > n^{1.58}$

$\therefore f(n) > n^2$

$T(n) = \Theta(n^2)$

2) $T(n) = 4T(n/2) + n^2$

$c = \log_2 4 = 2$

$\therefore n^c = n^2$

$\therefore n^c = f(n)$

$T(n) = \Theta(n^2 \log n)$

3) $T(n) = T(n/2) + 2^n$

$c = \log_2 1 = 0$

$n^c = n^0 = 1$

$f(n) = 2^n$

$\therefore f(n) > n^c$

$T(n) = \Theta(2^n)$

4) $T(n) = 2^n T(n/2) + n^n$

$c = \log_2 2^n$

$n^c = n^{2n}$

$\because f(n) = n^n$

$\therefore n^c > f(n)$

$\therefore T(n) = \Theta(n^{2n})$

5)) $T(n) = 16T(n/4) + n$

$\therefore c = \log_4 16 = 2$

$n^c = n^2$

$f(n) = n$

$\therefore n^2 > n$

$n^c > f(n)$

$T(n) = \Theta(n^2)$

6) $T(n) = 2T(n/2) + n\log n$

$c = \log_2 2 = 1$

$n^c = n$

$f(n) = n\log n$

$\therefore n\log n > n$

$\therefore f(n) > n^c$

$\therefore T(n) = \Theta(n\log n)$

7) $T(n) = 2T(n/2) + \frac{n}{\log n}$

$c = \log_2 2 = 1$

$n^c = n$

$f(n) = \frac{n}{\log n}$

$\because \frac{n}{\log n} < n$

$\therefore f(n) < n^c$

$T(n) = \Theta(n)$

8) $T(n) = 2T(n/4) + n^{0.51}$

$c = \log_4 2 = 0.5$

$n^c = n^{0.5}$

$n^{0.5} < n^{0.51}$

$\therefore \quad n^c < f(n)$

$\therefore \quad T(n) = \Theta(n^{0.51})$

9) $T(n) = 0.5T(n/2) + 1/n$

$a = 0.5$

$\therefore$ $a < 1$, for applying master theorem condition should be $a \geq 1$

$\therefore$ Here, master theorem does not apply

10) $T(n) = 16T(n/4) + n!$

$c = \log_4 16 = 2$

$n^c = n^2$

$\because \quad n^c < n!$

$n^c < f(n)$

$\therefore \quad T(n) = \Theta(n!)$

11) $T(n) = 4T(n/2) + \log n$

$\therefore \quad c = \log_2 4 = 2$

$n^c = n^2$

$\therefore \quad n^2 > \log n$

$n^c > f(n)$

$\therefore \quad T(n) = \Theta(n^2)$

12) $T(n) = \sqrt{n}\, T(n/2) + \log n$

$c = \log_2 \sqrt{n}$

$\Rightarrow c = \log_2 n^{1/2}$

$c = \frac{1}{2} \log_2 n$

$\Rightarrow n^c = n^{(\log_2 n)/2}$

$\Rightarrow n^{(\log_2 n)/2} > \log n$

$\Rightarrow n^c > f(n)$

$\Rightarrow T(n) = \Theta(n^{(\log_2 n/2)})$

13) $T(n) = 3T(n/2) + n$

$\Rightarrow c = \log_2 3 = 1.584$

$n^{1.584} > n$

$n^c > f(n)$

$\Rightarrow T(n) = \Theta(n^{1.584})$

14) $T(n) = 3T(n/3) + \text{sqrt}(n)$

$\Rightarrow c = \log_3 3 = 1$

$n^c = n$, $f(n) = \sqrt{n}$

$\because n > \sqrt{n}$

$\Rightarrow n^c > f(n)$

$\Rightarrow T(n) = \Theta(n)$

15) $T(n) = 4T(n/2) + cn$

$\Rightarrow k = \log_2 4 = 2$

$n^k = n^2$

$n^2 > cn$ ($\because$ for any constant c)

$\Rightarrow T(n) = \Theta(n^2)$

16) $T(n) = 3T(n/4) + n\log n$

a) $a = 3, \; b = 4$

$c = \log_4 3 = 0.792$

$n^c = n^{0.792}$

$\because \; n^{0.792} < n\log n$

$\therefore \; T(n) = \Theta(n\log n)$

17) $T(n) = 3T(n/3) + n/2$

a) $a = 3, \; b = 3$

$c = \log_3 3 = 1$

$n^c = n$

$\because \; n > \frac{n}{2}$

$n^c > f(n)$

$\therefore \; T(n) = \Theta(n)$

18) $T(n) = 6T(n/3) + n^2\log n$

$a = 6, \; b = 3$

a) $c = \log_3 6 = 1.6309$

a) $n^c = n^{1.6309}$

$\because \; n^{1.6309} < n^2\log n$

$\therefore \; T(n) = \Theta(n^2\log n)$

19) $T(n) = 4T(n/2) + n/\log n$

$a = 4, \; b = 2$ a) $c = \log_2 4 = 2$

a) $n^c = n^2$

$\because \; n^2 = n/\log n$

$\therefore \; T(n) = \Theta(n^2)$

20) $T(n) = 64T(n/8) - n^2 \log n$

$a = 64$, $b = 8$

$c = \log_8 64 = 2$

$\Rightarrow n^c = n^2$, but

$\because f(n) = -n^2 \log n$

$f(n) < 0 \Rightarrow$ Master's theorem can't be applied.

21) $T(n) = 7T\left(\frac{n}{3}\right) + n^2$

$a = 7$, $b = 3$

$\Rightarrow c = \log_3 7 = 1.771$

$\Rightarrow n^c = n^{1.771}$

$n^2 > n^{1.771}$

$\Rightarrow f(n) > n^c$

$T(n) = \Theta(f(n)) \Rightarrow \Theta(n^2)$

22) $T(n) = T(n/2) + n(2 - \cos n)$

$a = 1$, $b = 2$

$c = \log_2 1 = 0$

$n^c = n^0 = 1$

now, $f(n) = n(2 - \cos n)$

$\because (2 - \cos n) \in [-1, 1]$

$\Rightarrow$ highest value of $n(2 - \cos n)$ can be,

$n(2 - (-1)) = 3n$.

$\Rightarrow 3n > 1$

$f(n) > 1$

$\Rightarrow T(n) = \Theta(n)$ //