question: Evaluate the improper integral ∫■^∞ x^2 * e^(-x^2) dx using the Gamma function.
type: brief answer format
difficulty: hard
expected_time: 8
marks: 6
answer: √(π) / 4
explanation: Let I = ∫■^∞ x^2 * e^(-x^2) dx. Substitute u = x^2, so x = √u and dx = (1/2)u^(-1/2) du. The integ
question: Which of the following integrals represents the Beta function B(m, n)?
type: MCQ with one correct answer
difficulty: hard
expected_time: 7
marks: 5
answer: ∫■¹ x^(m-1) (1-x)^(n-1) dx
explanation: The Beta function B(m, n) is defined as ∫■¹ x^(m-1) (1-x)^(n-1) dx. Other forms exist, but this is
type: brief answer format
difficulty: hard
expected_time: 10
marks: 8
options: ∫■^∞ x^(m-1) e^(-x) dx, ∫■¹ x^(m-1) (1-x)^(n-1) dx, ∫■^∞ e^(-x^2) dx, ∫■∞^∞ e^(-x^2) dx
answer: 5 molecules
explanation: The total number of molecules reacted is given by ∫■^∞ r(t) dt = ∫■^∞ t * e^(-0.1t^2) dt. Let u = 0
question: Evaluate the following integral: ∫■ sin^3(x) cos^3(x) dx
type: brief answer format
difficulty: hard
expected_time: 9
marks: 7
answer: 1/24
explanation: We can solve this using a substitution or by relating it to the Beta function. Let's use the Beta fu
question: A particle moves along the x-axis with velocity v(t) = t * cos(t) m/s. Calculate the total distance tra
type: brief answer format
difficulty: hard
expected_time: 10
marks: 8
answer: 4π meters
explanation: The total distance is given by ∫■^(2π) |t * cos(t)| dt. cos(t) is negative from π/2 to 3π/2. Therefore
common_mistakes: Forgetting to consider the absolute value for distance, Incorrect integration by parts, Sig
marking_scheme: Recognizing the need for absolute value and splitting the integral: 3 marks, Correct integra
prerequisites: Integration by parts, Definite Integrals, Trigonometry, Understanding of distance vs. displacem
visual_aids: None
common_mistakes: Incorrect substitution, Forgetting the limits of integration, When changing variables, Misap
marking_scheme: Substitution: 2 marks, Expressing in terms of Gamma function: 2 marks, Correct applicatio
prerequisites: Improper Integrals, Gamma Function properties, Substitution method of integration
visual_aids: None
common_mistakes: Incorrectly identifying Beta and Gamma functions, Incorrect manipulation of the exponents
marking_scheme: Correct answer: 5 marks
prerequisites: Beta Function definition, Integral definitions
visual_aids: None
common_mistakes: Incorrect substitution, Forgetting dx, When changing variables, Sign e
marking_scheme: Setting up the integral: 2 marks, Substitution: 3 marks, Evaluating the integral: 2 marks, F
prerequisites: Improper Integrals, Substitution method of integration, Understanding of rate equations
visual_aids: None
common_mistakes: Incorrect application of trigonometric identities, Errors in substitution, Miscalculation of li
marking_scheme: Correct trigonometric manipulation and/or substitution: 3 marks, Correct integration: 3 ma
prerequisites: Trigonometric identities, Substitution method of integration, Beta and Gamma functions (optio
visual_aids: None