

question: Evaluate the improper integral $\int_{-\infty}^{\infty} x^2 \cdot e^{-(x^2)} dx$ using the Gamma function.

type: brief answer format

difficulty: hard

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expected_time: 8
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marks: 6

answer: $\sqrt{\pi} / 4$

explanation: Let $I = \int_{-\infty}^{\infty} x^2 \cdot e^{-(x^2)} dx$. Substitute $u = x^2$, so $x = \sqrt{u}$ and $dx = (1/2)u^{(-1/2)} du$. The integ

question: Which of the following integrals represents the Beta function $\Gamma(a, b)$ when changing variables, Misapplied?

type: MC_S with 8 subquestions 2 marks, Expressing in terms of Gamma function: 2 marks, Correct application

difficulty has

Improper Integrals, Gamma Function properties, Substitution method of integration

expected_end\$inName

marks: 5

answer: $\int_0^1 x^{(m-1)} (1-x)^{(n-1)} dx$

explanation: The Beta function $B(m, n)$ is defined as $\int_0^1 x^{m-1} (1-x)^{n-1} dx$. Other forms exist, but this is

question: Aisha is a Confusing Beta and Gamma given by $n(t)$ (definition 5, 0.1 to 2) incidents per second. Explain

type: khibjes answer: 5 marks

different hard Beta Function definition, Integral definitions

expected in Neon10

$$\int_{-\infty}^{\infty} x^{m-1} e^{-x} dx, \int_0^1 x^{m-1} (1-x)^{n-1} dx, \int_{-\infty}^{\infty} e^{-x^2} dx, \int_{-\infty}^{\infty} e^{-x^2} dx$$

answer: 5 molecules

explanation: The total number of molecules reacted is given by $\int_0^\infty r(t) dt = \int_0^\infty t^* e^{-0.1t^2} dt$. Let $u = 0$

question: Evaluate the following integral, $\int_0^1 \frac{1}{\sqrt{1-x^2}} dx$. (2) Is $\sin(x)$ linear? (3) In integration when changing variables, Sign

type: integration Setting up the integral: 2 marks, Substitution: 3 marks, Evaluating the integral: 2 marks, Final answer: 1 mark

difficulty has: Improper Integrals, Substitution method of integration, Understanding of rate equations

~~expected_end\$inName~~

marks: 7

answer: $1/24$

explanation: We can solve this using a substitution or by relating it to the Beta function. Let's use the Beta function.

question: A particle moves along the x-axis with velocity $v(t)$ (in ft/s). Estimate the total distance of a

marking scheme: Correct trigonometric manipulation and/or substitution: 3 marks, Correct integration: 3 marks

difficulty has Trigonometric identities, Substitution method of integration, Beta and Gamma functions (optional)

expected in Neon10

marks: 8

answer: 4π meters

explanation: The total distance is given by $\int_0^{2\pi} |t \cdot \cos(t)| \, dt$. $\cos(t)$ is negative from $\pi/2$ to $3\pi/2$. Therefore

common_mistakes: Forgetting to consider the absolute value for distance, Incorrect integration by parts, Sig

marking_scheme: Recognizing the need for absolute value and splitting the integral: 3 marks, Correct integr

prerequisites: Integration by parts, Definite Integrals, Trigonometry, Understanding of distance vs. displacement

visual_aids: None