Quantitative Analysis of Financial Markets:

Essays on Multi-Asset Portfolio Management Topics

Youssef Louraoui

ESSEC Business School

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Abstract

This compilation of essays, titled *Quantitative Analysis of Financial Markets: Essays on Multi-Asset Portfolio Management Topics*, explores the multifaceted domain of quantitative finance, focusing on the critical aspects of portfolio management. The collection is structured into three comprehensive sections, each addressing a pivotal area of quantitative analysis in financial markets.

The first section, **Factor Investing**, delves into the theoretical foundations and practical applications of factor-based investment strategies. It begins with an introduction to the evolution of factor investing, tracing its origins from Markowitz's Modern Portfolio Theory to the Fama-French Five Factor Model. The section further discusses Smart Beta strategies, offering insights into both active and passive investment approaches. Additionally, it examines alternative weighting strategies and provides an in-depth analysis of the behavior of factor strategies during the COVID-19 pandemic, culminating in a detailed conclusion summarizing the key concepts.

The second section, **Portfolio Construction Techniques**, provides a rigorous exploration of methodologies for constructing robust investment portfolios. Highlighting a case study on the All-Weather Portfolio, this section covers various mathematical foundations, including Markowitz Portfolio Construction, GMV Portfolio Construction, PCA Analysis, and the Black-Litterman Approach. It also addresses the practical aspects of data cleaning and analysis, model implementation, and the inherent limitations of these models. The section concludes with comprehensive references and appendices to support further research.

The third section, **Analysis of Option-Based Strategies**, investigates the nuances of option-based investment strategies with a focus on the Constant Proportion Portfolio Insurance (CPPI) strategy. It includes a literature review, model descriptions, and implementation details, followed by an analysis comparing CPPI under different financial models. This section also explores equity derivatives strategies through a case study of Société Générale, detailing the analysis of the stock, the application of the Black-Scholes framework, and the implementation of an equity option strategy on Société Générale (SOGN.PA). The section concludes with a discussion on the findings and implications for future research.

In general, this compilation closes the gap in quantitative finance between theoretical models and real-world applications. It offers a comprehensive examination of factor investing, portfolio

construction methods, and option-based strategies, making it a helpful resource for professionals and students who want to improve their knowledge and application of quantitative analysis in financial markets.

Contents

Chapter 1

Introduction

Importance of quantitative analysis in financial markets

In today's fast-paced financial markets, the role of quantitative analysis has become more crucial than ever. Here's why it's indispensable:

Precision and objectivity

Quantitative analysis brings a level of precision to financial decision-making that's hard to achieve otherwise. By using mathematical models and statistical techniques, we can remove much of the guesswork and personal bias, leading to more objective and reliable insights.

Enhanced predictive power

By analyzing historical data, quantitative models can uncover patterns and trends that aren't immediately obvious. This predictive power helps investors forecast market movements, evaluate the potential impacts of economic events, and devise strategies to take advantage of future opportunities.

Risk management

Managing risk is a cornerstone of successful investing. Quantitative analysis equips investors with tools to measure and manage risk effectively. Techniques like Value at Risk (VaR) and stress testing allow for a detailed assessment of potential losses and the development of strategies to mitigate those risks.

Diversification and portfolio optimization

Creating a well-diversified portfolio is key to balancing risk and return. Quantitative methods, including Modern Portfolio Theory (MPT) and the Capital Asset Pricing Model (CAPM), provide frameworks for selecting a mix of assets that optimize returns for a given level of risk.

Evaluation of investment strategies

Quantitative analysis allows for the rigorous evaluation of investment strategies through back-testing. By testing strategies against historical data, investors can understand their performance, identify strengths and weaknesses, and make data-driven decisions about which strategies to implement.

Adaptation to market changes

Financial markets are constantly evolving. Quantitative models can be updated with new data, allowing them to adapt to changing market conditions. This flexibility ensures that investment strategies remain effective even as the market landscape shifts.

Data-driven decision making

In an age where data is abundant, quantitative analysis enables investors to make informed decisions based on solid evidence. Leveraging advanced analytics, machine learning, and artificial intelligence provides deeper insights into market behaviors and trends.

Transparency and accountability

Quantitative methods enhance transparency and accountability in the investment process. Using clear and replicable models allows for consistent evaluation of investment decisions and performance, which is crucial for maintaining investor confidence and meeting regulatory requirements.

Conclusion

In summary, quantitative analysis is a vital component of modern financial markets. It enhances precision and objectivity, boosts predictive capabilities, aids in risk management, and supports the development of optimized portfolios. As financial markets continue to evolve, the importance of quantitative analysis will only grow, making it an essential skill for anyone involved in finance.

2. Overview of the essays's structure and main themes

This essays is designed to provide a comprehensive understanding of quantitative analysis in financial markets, focusing on the practical application of theoretical models and strategies. The essays is structured into several key sections, each addressing a specific aspect of quantitative finance. Below is an overview of the main themes and structure of the essays:

1. **Factor investing:** This section begins with an introduction to factor investing, covering its origins from Markowitz's Modern Portfolio Theory to the Fama-French Five Factor Model. It also discusses Smart Beta strategies, alternative weighting strategies, and provides an analysis of factor strategies during the COVID-19 pandemic. The section concludes with a comprehensive summary of the discussed concepts.

2. **Portfolio construction:** Here, we delve into the methodologies for constructing robust investment portfolios. The section includes a detailed case study on the All-Weather Portfolio, discussing various mathematical foundations, data cleaning and analysis, and different modeling approaches such as the Black-Litterman model. It also addresses the limitations of these models and provides a concluding summary along with references and appendices.

3. **Analysis of option-based strategies:** This section covers option-based investment strategies with a focus on the Constant Proportion Portfolio Insurance (CPPI) strategy. It includes a review of relevant literature, model descriptions, implementation details, and an analysis of results comparing CPPI under different financial models. The section concludes with a discussion on the implications and potential future directions of research in this area. We also elaborate on equity derivatives strategies, specifically focusing on the Société Générale (SG) stock as a case study. It covers the analysis of the stock, the Black-Scholes framework, and the implementation of an equity option strategy. The section also includes references and an appendix for further reading.

4. **Conclusion:** The essays concludes with a summary of key findings, their implications for practitioners, and suggestions for future research directions.

   Each chapter is meant to elaborate on a specific topic, ensuring there is a cohesive flow of information. The essays combines theoretical foundations with practical applications, making it a valuable resource for both students and professionals in the field of finance.

Chapter 2

Factor Investing

2.1 An Introduction to Factor Investing

According to Blackrock [6], "factor investing" is an investment strategy that focuses on distinct drivers of performance across asset classes. The two most fundamental types of variables are macroeconomic and stylistic variables. Factor investing may help you enhance portfolio performance, reduce volatility, and diversify your holdings. Within some asset classes, style features may contribute to the explanation of returns. In a volatile market, investors seeking downside protection may increase their exposure to low-volatility strategies, while those willing to take on greater risk may opt for higher-return strategies such as momentum. Numerous studies—some of which were conducted by Nobel laureates—have shown how certain traits influence returns over the course of decades. These factors had an impact on returns for three different reasons: investors' propensity to take risks, structural obstacles, and the irrationality of some investors. Certain components generate greater returns due to the increased risk, but they may underperform in particular market conditions. Enhanced strategies employ variables in a more complex manner, including trading across asset classes and holding both long and short positions. Investors seeking absolute returns or as a complement to hedge funds and traditional active strategies utilise these enhanced factor methods [6].

The factors should have a rational foundation, and an investor's benchmark should only include those with the strongest academic foundation. To effectively explain why risk premiums are imposed, the study should provide either compelling logical reasons or convincing behavioural stories, or both. We don't need to agree on the method for calculating the risk premium, which, as everyone who has met a financial economist knows, is impossible. Under this criterion, value growth, momentum, and short volatility strategies all qualify as sufficient risk factors. A new study might uncover new variables, qualify past consensus on identified variables, or even rule out some, all of which could influence investing strategies [2].

Factors should have historically maintained large premiums and are expected to do so in the future. We need to not only understand why the risk premium existed in the past, but we should also have reason to think it will exist in the future (at least in the near run)[2]. Factors are systematic by definition: they originate from risk or behavioural patterns that are likely to persist (at least in the short term), even if everyone is aware of them and many investors use the same factor strategies [2].

Factor-risk premiums exist to encourage people to accept losses in tough circumstances. It's critical to have specific data points to assess risk-reward trade-offs and risk management. We also require a significant volume of data to carry out these tasks[2].

Factor techniques should be as low-cost as possible, which is best accomplished by using liquid securities. For institutional investors, scalability is a crucial criterion. In factoring procedures, leverage is frequently employed. Value stocks must be overweighted, whereas growth stocks must be underweighted or shorted. A dynamic leverage strategy involves taking a long-term perspective of value and a short-term view of growth. Factor methods are still effective even if shorting isn't an option: Even if an investor is unable to short, empirical study show that significant value and momentum factor premiums are still available, but the profitability of these factor strategies is decreased by 50 percent to 60 percent[2].

Several academic studies and years of investment experience have proven that some forms of stock, debt, and derivative assets pay out better than the overall market index. Stocks with low price-to-book ratios (value stocks) beat those with high price-to-book ratios (growth stocks) over long periods of time, resulting in a value-growth premium. Over time, winners (equities with a history of high returns) outperform losers (equities with a history of low or negative returns), leading to the creation of momentum strategies 4.3. Less liquid securities sell at a discount to their more liquid counterparts and, on average, generate a greater average excess return. As a result, illiquidity attracts a premium. Due to the credit risk premium, bonds with a higher chance of default generally have higher average returns. Furthermore, because investors are willing to pay for protection against periods of excessive volatility, when returns are likely to decline, sellers of volatility protection in option markets typically earn a high rate of return[2].

2.2 Origins of Factor Investing

2.2.1 Modern Portfolio Theory (MPT)

The beginnings of factor investing may be traced back to the 1950s. The most important aspect of Markowitz's model was his explanation of the influence of portfolio diversification on the number of stocks in a portfolio and their covariance relationships. His dissertation, titled "Portfolio Selection," was originally published in The Journal of Finance in 1952. These conclusions were considerably expanded with the publication of his book, Portfolio Selection: Efficient Diversification (1959). For his MPT contributions to both economics and corporate finance, Markowitz shared the Nobel Prize in economics and corporate finance over thirty years later. The holy grail of Markowitz's work is his estimate of the variance of a two-tier portfolio [28]:

$$\min \sigma_p^2 = w^2\sigma_a^2 + (1-w)^2\sigma_b^2 + 2w(1-w) \times \text{cov}(r_a, r_b) \tag{2.1}$$

where w and $(1-w)$ represent the asset weights of r_a and r_b, σ^2 represents the standard deviation of the assets r_a and r_b and $\text{cov}(r_a, r_b)$ represents the covariance of asset r_a and r_b.

As a result of Markowitz and Tobin's prior work, William Sharpe, John Lintner, and Jan Mossin created the Capital Asset Pricing Model (CAPM), a major capital markets theory. Because it allowed investors to correctly value assets in terms of systematic risk, the CAPM represented a significant evolutionary step forward in capital market equilibrium theory. Sharpe (1964) [30] made important contributions to the notions of the Efficient Frontier and Capital Market Line in his derivation of the CAPM. Sharpe's significant contributions earned him the Nobel Prize in Economics later in life [27].

For its theoretical ramifications, Markowitz's work is widely regarded as a pioneer in financial economics and corporate finance. Markowitz won the Nobel Prize in economics in 1990 for his contributions to these fields, which he outlined in his 1952 essay "Portfolio Selection" and expanded on in his 1959 book "Portfolio Selection: Efficient Diversification" [27]. His groundbreaking work established the foundation for what is now known as 'Modern Portfolio Theory' (MPT). Later, Markowitz's Nobel laureate colleague William Sharpe, well known for his 1964 Capital Asset Pricing Model work on the theory of financial asset price creation, built on the underpinnings of this theory. With the conclusion of his 1952 PhD dissertation in statistics, Harry Markowitz laid the foundation for modern portfolio theory ("MPT").

The most important aspect of Markowitz's model was his explanation of the influence of portfolio diversification on the number of stocks in a portfolio and their covariance relationships. His dissertation, "Portfolio Selection," was published in The Journal of Finance for the first time in 1952. These conclusions were considerably expanded with the publication of his book, Portfolio Selection: Efficient Diversification (1959). Markowitz shared the Nobel Prize for economics and corporate finance for his MPT contributions to both fields almost thirty years later. In his 1958 paper "Liquidity Preference as Risk-Taking Behaviour," economist James Tobin created the "Efficient Frontier" and "Capital Market Line" concepts based on Markowitz's theories. Market participants, regardless of their risk tolerance, would maintain identical stock portfolio proportions if they "have equal expectations about the future," according to Tobin's model. As a result, Tobin reasoned, their investment portfolios will be similar, except for their stock and bond allocations [27].

The Capital Asset Pricing Model (CAPM), created by William Sharpe, John Lintner, and Jan Mossin because of Markowitz and Tobin's prior research, is an important capital markets theory. The CAPM was a big step forward in capital market equilibrium theory, allowing investors to value assets more correctly in terms of systematic risk. In his derivation of the CAPM, Sharpe (1964) made significant contributions to the concepts of the Efficient Frontier and Capital Market Line. Sharpe will be awarded the Nobel Prize in Economics for his essential contributions [27].

MPT consists of two parts: Markowitz's Portfolio Selection theory, which was created in 1952, and William Sharpe's contributions to the theory of financial asset price development, which were published in 1964 and termed the Capital Asset Pricing Model ("CAPM"). MPT is based on a variety of assumptions about the market and investors. Investors are reasonable (they want to maximize profits while reducing risk), they will accept more risk only if the expected returns is higher, they have quick access to all relevant information on their investment decision and they can borrow or lend an infinite amount of money at a risk-free rate of interest.

MPT's core assumptions have been widely questioned. In Markowitz' portfolio selection theory, risk is equal to volatility—the higher the portfolio volatility, the higher the risk. Volatility is a word that refers to the degree of risk or uncertainty connected with the size of changes in a security's value.

This volatility is measured using several portfolio approaches, such as the ones listed below: (1) expected return computation; (2) expected return variance; (3) standard deviation from an expected return; (3) portfolio covariance; and (5) portfolio correlation. Systematic risk is a form of macroeconomic risk that has different degrees of influence on many assets. Systematic risk factors include inflation, interest rates, unemployment rates, currency exchange rates, and the amount of the Gross National Product. The current economic situation has a substantial impact on almost all securities. As a result, there is no way to eliminate systemic risk [27]. Unsystematic risk, on the other hand, is a form of risk that happens at the micro level, with risk variables affecting only one asset or a small group of assets [27]. It is a separate risk that is unconnected to other dangers and solely impacts certain securities or assets. A company's credit rating, negative media coverage, or a strike affecting a single company are all examples of unsystematic risk. Asset diversification can help reduce unsystematic risk in a portfolio. Unsystematic risk can never be completely avoided, regardless of the number of asset types pooled in a portfolio.

The term 'risk-reward trade-off' alludes to Markowitz's basic idea that the riskier an investment is, the greater the required potential return. Investors will usually keep a risky investment if the expected return is large enough to compensate them for taking the risk. Standard deviation is a technical measure of the probability that an investment's actual return will be less than expected.

A higher standard deviation indicates a greater risk and, as a result, a higher possible return. If investors are willing to take on risk, they expect to be rewarded with a risk premium. Risk premium is described as "the expected return on an investment that exceeds the risk-free rate of return." The greater the risk, the higher the risk premium required by investors. Certain risks may be avoided easily and cheaply, and hence have no expected reward. Only risks that are difficult to avoid are paid on average.

The risk-reward trade-off shows the possibility of a higher rate of return on investments, but it does not mean that a higher rate of return will be achieved. As a result, riskier investments do not always provide a better return than risk-free investments. It is for this reason that they are dangerous. However, historical data shows that taking on more risk is the only way for investors to get a higher rate of return. The terms 'diversification' and 'Diversification Effect' refer to portfolio risk and diversification. Diversification is a risk-reduction technique that includes allocating assets among a range of financial instruments, industries, and other asset classes, according to Markowitz's portfolio selection theory and MPT. Diversification may be achieved through investing in a range of firms, asset classes (such as bonds, real estate, and so on), and/or commodities such as gold or oil [27]. Diversification aims to boost returns while lowering risk by investing in several assets that react to the same event in various ways. The phrase "diversification effect" refers to the relationship between portfolio correlations and diversification. The diversification effect arises when there is an imperfect (positive or negative) link between assets. Risk mitigation may be performed without risking earnings, making it a crucial and successful risk reduction strategy. As a result, any wise 'risk averse' investor will diversify to some level [27].

Despite its theoretical importance, MPT has several critics who argue that its underlying assumptions and financial market modeling are usually out of sync with reality [27]. One may argue that none of them are completely true, and that each one weakens MPT in different ways.

Investors are rational and want to maximize profits while minimizing risk. This is the opposite of what market participants who get sucked into 'herd behavior' investing activities see. Speculative excesses, for example, cause investors to flock to 'hot' sectors, and markets to rise or bust because of speculative excesses.

Investor conduct consistently refutes the notion that they will only take on more risk in return for larger expected rewards. Investing strategies sometimes require investors to make a perceived risky investment (e.g., derivatives or futures) to reduce total risk without considerably boosting expected returns. Furthermore, investors may have utility functions that take precedence over concerns about return distribution.

MPT expects investors to receive all essential information about their investment in a timely and complete way. In truth, information asymmetry (one party has greater knowledge), insider trading, and investors who are just more aware than others define global markets. This might explain why stocks, commercial assets, and businesses are routinely bought at a lower price than their book or market value.

As previously stated, another key assumption is that investors have almost limitless borrowing capacity at a risk-free rate. In real-world markets, each investor has credit restrictions. Furthermore, the federal government is the only entity that may borrow at the zero-interest treasury bill rate on an ongoing basis.

Theoretically, Markowitz's contributions to MPT assume that markets are perfectly efficient [28]. MPT, on the other hand, is vulnerable to market whims such as environmental, personal, strategic, or social investment decision considerations because it is dependent on asset valuations. It also ignores potential market failures like as externalities (costs or benefits not reflected in pricing), information asymmetry, and public goods (a non-rivalrous, non-excludable commodity). From a different perspective, centuries of "rushes," "booms," "busts," "bubbles," and "market crises" show that markets are far from efficient.

Markowitz' theoretical contributions to MPT do not include taxes or transaction costs. Genuine investment products, on the other hand, are subject to both taxes and transaction costs (e.g., broker fees, administrative fees, and so on), and including these costs into portfolio selection may have a significant impact on the ideal portfolio composition.

MPT has established itself as the de facto orthodoxy of modern financial theory and practice. MPT's premise is that beating the market is difficult, and those that do it by appropriately diversifying their portfolios and accepting higher-than-average investment risks [27]. The most important thing to keep in mind is that the model is only a tool—albeit the most powerful hammer in one's financial toolbox. Markowitz invented MPT more than sixty years ago, and its popularity is unlikely to wane anytime soon. His theoretical contributions have paved the way for further theoretical research in the field of portfolio theory. Markowitz's portfolio theory, however, is vulnerable to and reliant on continuing 'probabilistic' development and expansion [27].

2.2.2 Capital Asset Pricing Model (CAPM)

The Capital Asset Pricing Model (CAPM), established by Henry Markowitz and William Sharpe in 1964, is the foundation of this classic approach. The CAPM is built around a set of market structure and investor assumptions. There are no intermediaries, there are no restrictions (short selling is possible), there are no transaction costs, the value of an investor's portfolio is maximized by maximizing the mean associated with expected returns while decreasing risk variation, investors have simultaneous access to information to carry out their investment strategies and investors are seen as "rational" and "risk averse".

The expected return of the asset is given by the equation:

$$E(r) = r_f + \beta \times (E(r_m) - r_f) \tag{2.2}$$

where $E(r)$ represents the expected return of the asset, r_f is the risk-free rate, β is a measure of the risk of the asset, $E(r_m)$ is the expected return of the market and $E(r_m) - r_f$ represents the Market Risk Premium.

In this model, the beta (β) parameter is a key parameter and is defined as:

$$\beta = \frac{\mathrm{Cov}(r, r_m)}{\sigma^2(r_m)} \tag{2.3}$$

where $\mathrm{Cov}(r, r_m)$ represents the covariance of the asset with the overall market and $\sigma^2(r_m)$ is the variance of market return.

Beta is a measure of how sensitive an asset is to market swings. This risk indicator helps investors estimate how their asset will perform with respect to the rest of the market. It relates the volatility of a certain asset to the market's systematic risk. The slope of a line generated by a regression of data points comparing stock returns to market returns is referred to as the beta. Investors can use it to figure out how the asset moves in relation to the market.

According to [17], the beta used in the CAPM can be interpreted in two ways: Beta may be conceived of statistically as the slope of the regression between the security's or predicted portfolio's return and the market return, according to the CAPM formula. As a result, beta measures how sensitive an asset or sensitivity portfolio is to changes in the market return. It may be thought of as the risk that each dollar invested in security/portfolio A contributes to the market portfolio, according to the beta calculation. This is an economic explanation based on the idea that the risk of the market portfolio is the weighted average of the covariance risks that are associated with the assets in the market portfolio. This means that beta is a measure of the covariance risk that is associated with a given security or portfolio compared to the return variance of the market portfolio. Furthermore, the CAPM distinguishes between two types of risk: systematic risk and specific risk.

Systematic risk refers to the potential impacts that arise from the fundamental structure, participants, and non-diversifiable elements of the market, including but not limited to monetary policy, political events, and natural disasters. Single risk, on the other hand, refers to the risk that is inherent in a specific security or portfolio and is hence diversifiable and hedgeable. As a result, the CAPM uses the beta measure to represent systematic risk, with the market's beta equal to one, lower-risk securities and portfolios having a beta less than one, and higher-risk securities and portfolios having a beta more than one. Finally, the key message of the CAPM is that when investors invest in a specific security or portfolio, they are rewarded twice: once via the time value of money effect (reflected in the risk-free component of the CAPM equation), and again through the benefit of taking on additional risk. However, due to an overly simplified set of assumptions and difficulties in creating validating tests when the model was initially introduced [17], the CAPM is not an empirically sound model. As a result, the CAPM has been altered and improved throughout time to solve both its flaws and to keep up with financial and economic changes. Sharpe (1990) highlights a number of changes to his fundamental model that other economists and financial experts have suggested in his evaluation of the CAPM.

The Capital Market Line (CML) depicts portfolios with the best balance of risk and return. In contrast to the more well-known efficient frontier, CML incorporates risk-free assets. The most efficient portfolio is the tangency portfolio, which is the intersection of the CML and the efficient frontier (CFA Institute, 2011). The risk-return relationship is maximized in portfolios that fall on the capital market line (CML). To earn returns above the risk-free asset, an investor should aim to raise his or her level of risk. In contrast to the more well-known efficient frontier, the CML contains risk-free assets. The most efficient portfolio is the tangency portfolio, which is the intersection of the CML and the efficient frontier.

The risk-return relationship is maximised in portfolios that fall on the capital market line (CML). The capital allocation line (CAL) represents an investor's risk-free asset and risky portfolio allocation. The risky asset in this situation is the market portfolio, which is a particular case of the CML. As a result, the Sharpe ratio of the market portfolio equals the slope of the CML. If the Sharpe ratio is larger than the CML, an investment strategy may be applied, such as buying assets if the Sharpe ratio is greater than the CML and selling assets if the Sharpe ratio is less than the CML. The SML is a spin-off from the CML. It's based on the Capital Asset Pricing Model (CAPM), which defines the risk-return trade-off for efficient portfolios. It's a theoretical notion that encompasses all portfolios that combine the risk-free rate of return with a market portfolio of risky assets in the best possible way.

All investors will pick a position on the capital market line by borrowing or lending at the risk-free rate, as this optimizes the return for a given degree of risk, according to the CAPM. The SML measures the risk and return of the market at a certain period and shows the projected returns of individual assets, whereas the CML represents the rates of return of a specific portfolio. In addition, whereas the standard deviation of returns (total risk) is used in the CML, the systematic risk, or beta, is used in the SML.

2.2.3 Fama-French Three-Factor Model

In response to the CAPM's shortcomings, Eugene Fama and Kenneth French developed the Fama- French Three-Factor model in 1993. It claims that, in addition to the CAPM's market risk component, two other factors influence the returns on securities and portfolios: market capitalization (often known as the "size" factor) and the book-to-market ratio (referred to as the "value" factor). The major reason for include these features, according to Fama and French, is because both size and book-to-market (BtM) ratios are connected to the economic fundamentals of the firm issuing the securities [16].

Earnings and book-to-market ratios are negatively related, with firms with low book-to-market ratios regularly reporting higher earnings. Size and average returns are negatively related due to a comparable risk component. This is based on their analysis of small business profits in the 1980s, which suggests that in the case of a recession in the economy in which they operate, small businesses endure longer periods of earnings depression than bigger businesses. They also highlighted that, following the 1982 recession, smaller businesses did not contribute to the economic expansion in the mid- and late-1980s. Profitability is linked to both size and BtM, and it is a common risk factor that highlights and explains the positive relationship between BtM ratios and average returns. The following equation provides the expected return of the asset or portfolio:

$$E(r) = r_f + \beta_A \times (E(r_m) - r_f) + \beta_S \times SMB + \beta_V \times HML + \alpha + \epsilon \tag{2.4}$$

where $E(r)$ is the expected return of the security or portfolio, r_f is the risk-free rate, β_A is the measure of the risk of the security or portfolio, $E(r_m)$ is the expected return of the market, β_S is the measure of the risk related to the size of the security or portfolio, β_V is the measure of the risk related to the value of the security or portfolio, SMB (which stands for "Small Minus Big") measures the difference in expected returns between small and big firms (in terms of market capitalization), HML (which stands for "High Minus Low") measures the difference in expected returns between value stocks and growth stocks,α is a regression intercept and ϵ is a measure of regression error.

Both SMB and HML are based on historical data as well as a combination of size and value-focused portfolios. Professor French posts these ideals on his own website on a regular basis. Meanwhile, linear regression is used to calculate the betas for both the size and value components, which can be positive or negative. The Fama-French Three-Factor approach, on the other hand, is not without faults. Griffin (2002) points out a major weakness in the model by claiming that when applied locally rather than globally, the Fama-French components of value and size are more accurate in explaining return disparities [15]. As a result, each of the elements should be addressed separately for each country (as professor French now does on his website, where he specifies the SMB and HML factors for each nation, such as the United Kingdom, France, and so on). While the Fama-French model breaks down security returns more thoroughly than the CAPM, it is still an incomplete model with geographically limited interpretation of its extra variables. Fama and French added two new factors, profitability and investment strategy, to the original Three-Factor model in 2015, while other academics, such as Carhart (1997)[8], added a fourth characteristic, momentum, to the original three-factor model in 1997.

2.2.4 Carhart Four-Factor Model

In 1997, Mark Carhart added a third element, momentum, to the Fama-French Three Factor model (1993)[16]. The apparent propensity for prices to continue increasing or dropping after an initial spike or decline is known as momentum. The Efficient Market Hypothesis says that there is no reason for security prices to continue rising or falling after an initial change in their value, therefore momentum is by definition an anomaly. While traditional financial theory is unable to precisely define what causes momentum in specific securities, behavioural finance offers some insight into why momentum exists; for example, Chan, Jegadeesh, and Lakonishok (1996) argue that momentum arises from the majority of investors' inability to react quickly and immediately to new market information and, as a result, integrate that information into their portfolios [15] . This argument illustrates investors' irrationality in valuing equities and making investment decisions. Because the Fama-French Three factor model was unable to account for return variation in momentum-sorted portfolios [16] [8] , Carhart was inspired to include the momentum component. As a result, Carhart used Jegadeesh and Titman's [24] one-year momentum variation in his model. When taking into account various factors, the expected return of the asset or portfolio is:

$$E(r) = r_f + \beta \times (E(r_m) - r_f) + \beta_S \times SMB + \beta_V \times HML + \beta_M \times UMD + \alpha + \epsilon \tag{2.5}$$

where β_M is the measure of risk related to the momentum factor of the security or portfolio and UMD (Up Minus Down) measures the difference in expected returns between "winning" and "losing" securities in terms of momentum.

The four-factor model, like the CAPM and the Fama-French Three-Factor, can be used to explain the sources of return on a given security/portfolio, according to Carhart's essay [8]. However, the model is most commonly used in asset management to assess the success of an actively managed portfolio and a mutual fund's overall performance.

2.2.5 Fama-French Five-Factor Model

In 2014, Fama and French claim that their 1993 three-factor model does not properly explain for some observed discrepancies in expected returns. As a result, Fama and French added two more variables to the three-factor model: profitability and investment. The theoretical implications of the dividend discount model (DDM), which argues that profitability and investment assist to explain the returns produced by the HML element in the first model, provide support for these two elements [18].

Surprisingly, the new Fama-French model does not include the momentum factor, unlike the Carhart model. This is mostly due to Fama's stance on momentum. While not disputing its existence, Fama believes that in an efficient market, the degree of risk exposed by securities cannot change so radically that it supports the need to recognize the momentum factor's involvement [18]. The expected return on any security is computed using the Fama-French five-factor model as follows:

$$E(r) = r_f + \beta \times (E(r_m) - r_f) + \beta_S \times SMB + \beta_V \times HML + \beta_P \times RMW + \beta_I \times CMA + \alpha + \epsilon \tag{2.6}$$

where β_P is the measure of risk related to the profitability factor of the security or portfolio, RMW (Robust Minus Weak) measures the difference in expected returns between securities with strong and inconsistent profitability levels, β_I is the measure of risk related to the investment factor of the security or portfolio and CMA (Conservative Minus Aggressive) measures the difference in expected returns between securities engaging in limited versus high levels of investment activities.

Fama and French developed a few portfolios with significant returns differences owing to size, value, profitability, and investment characteristics to validate the new approach. They also performed the following exercises. The first is a portfolio-results regression versus the modified model. This was done to see how well it explains the observed returns differences across the different portfolios. The second step is to compare the performance of the new model to the three-factor model. This was done to see if the new five-factor model properly accounted for the old three-factor model's observed returns disparities. Fama and French's results on the new model are summarized here. In terms of structure, the HML component is no longer necessary because any value contribution to a security's return can already be accounted for by market, size, investment, and profitability considerations. As a result, Fama and French urge investors and academics to ignore the HML impact if their primary goal is to understand unusual returns [18].

They do, however, suggest that when seeking to explain portfolio results that reflect size, value, profitability, and investment tilts, all five components should be included. Furthermore, the model accounts for between 69 and 93 percent of the return discrepancies seen after using the previous three- factor model [18]. However, there are several faults with this new model. In their 2016 study "Five challenges with the Five-Factor model," Blitz, Hanauer, Vidojevic, and van Vliet (hence referred to as BHVV) highlighted five issues with the new Fama-French five-factor model [15]. While two of these issues are related to some of the original Fama-French three factor model's original factors (most notably the continued existence within the model of the CAPM relationship between market risk and return, as well as the new model's overall acceptance by the academic community while some of the original factors remain contested), several of the others are related to other factors.

2.3 Smart Beta: Between Active and Passive Investing

Smart Beta techniques are often found somewhere in the middle between active and passive management. As a result, we'll look at how investors think about the opposition between these two tactics. These strategies explain their existence by claiming that capitalization-weighted indices would be inefficient, and that outperformance may be achieved via alternate weighting methods, therefore we'll look at the literature on capitalization-weighted indices' inefficiency.

Active management is an approach for going beyond matching a benchmark's performance and instead aiming to outperform it [15]. The great majority of active managers employ techniques that try to build an active portfolio based on strong historical data returns. It's called "momentum investing," and it's a trend-following strategy that assumes market movements repeat themselves. Stock picking is a method used by active managers to select shares based on a variety of variables such as growth rate, intrinsic value (the true worth of an asset), and so on. Market timing is a trading method that involves entering the market at precisely the appropriate time. The approach attempts to enter the market while it is in an upswing and exit when he believes the trend will reverse and the market will tend to a bullish market, allowing him to reinvest in other assets [30]. Discount Dividend Model (DDM) is a financial model for calculating the value of a company that is based on Gordon and Shapiro's (1960) work. It is predicated on the idea that future dividends are discounted to their net present value as cash flows (NPV). The stock is said to be undervalued if the net present value exceeds the actual stock price. Technical analysis, which is the examination of price fluctuations and volume movements over time to forecast short-term future evolution, and fundamental analysis, which is the examination of the macroeconomic and microeconomic landscape to forecast future asset prices, are the two assumptions upon which active managers base their decisions. Additionally, sectoral management is in place. It is predicated on a relationship between certain sectors and the economic cycle. The Efficient Market Hypothesis (EMH) asserts that markets are efficient, which underpins passive management. Passive managers attempt to replicate a standard[15].

In this circumstance, active management will be unable to outperform the benchmark, given benchmark performance is already difficult to accomplish. They do it by investing in the integrality of the assets that make up the benchmark (Vanilla ETF) or invest in a synthetic replication (where active managers employ other derivatives instruments to duplicate the index).

The research of Grossman and Stiglitz focused on the limitations of passive investment [21]. However, if the fund manager actively chooses assets for his portfolio rather than passively copying the benchmark, he may get larger anomalous returns. The difference between actual and expected returns is referred to as "abnormal returns." This "additional return" is referred to as alpha in the financial literature. One of the most highly monitored performance metrics by fund managers is the Greek. Research was carried out to show how passive management works in comparison to the market benchmark as reported in the literature.

There are two things to keep in mind when putting together a portfolio [25]. The ability of the fund management to predict the asset's price movement is the first factor, and the ability to reduce investment risk through diversification is the second. He started his research by devising a method for assessing how well fund managers predicted asset performance. He looked at the equivalent of 115 funds over a ten-year period in the second portion of his investigation. He concluded that there is no evidence to back up the idea that fund managers can accurately anticipate an asset's price evolution and benefit enormously.

Another study [26] looked at the performance of equities funds from 1971 to 1991 and came to the same result as Jensen: fund managers failed to produce excess returns over time. In this regard, he stated that the fund's performance, both with and without management fees, is below its benchmark. Furthermore, research shows that funds that have performed well in the past are likely to do well in the future. Survivor bias, a phenomenon in which investors overestimate the success rate of a fund that performed well as an exception rather than the norm, exemplifies this assumption. Malkiel advocates investing in low-cost index funds that track the benchmark rather than making active decisions. Given this, one would question why, despite financial literature demonstrating that actively managed funds underperform, investors continue to invest in them.

In recent years, smart beta funds have been a popular topic among investors. Smart beta is characterised as a revolutionary innovation that addresses a previously unmet need among consumers, namely, a higher return for less risk, net of transaction and administrative costs. In a way, these approaches create a new market. They even anticipate that active portfolio management will be split into two categories in the future: Smart Beta, which will charge lower costs, and "Pure Alpha," which will demand higher fees and be operated by a select few managers with superior research and financial engineering ability. They believe that rather than attempting to adopt both Smart Beta and Pure Alpha at the same time, an asset manager should concentrate on one of the two techniques. Indeed, during the previous decade, smart beta strategies have resulted in a fundamental shift in the hunt for performance, as investors seek a new source of financial success for their portfolios. As part of a category that spans the gap between passive and active investment, these alternative funds have exploded in popularity. By 2022, the factoring business is estimated to be valued $3.4 trillion, up from $1.9 trillion now [6]. Buy and hold investing, also known as position trading, is a passive investment technique in which an investor purchases a security and keeps it for an extended length of time, regardless of market changes.

A buy-and-hold investor actively chooses firms but is unconcerned with short-term market fluctuations or technical indications. Several renowned investors, like Warren Buffett and Jack Bogle, promote the buy-and-hold strategy to people seeking healthy long-term returns. Over longer time horizons and after expenses, buy-and-hold investors beat active management, and they can usually postpone capital gains taxes. Buy-and-hold investors, on the other hand, are not required to sell at the highest possible price, according to proponents. Smart beta investing is a more detailed approach to investing that goes beyond asset selection. It is a hybrid strategy because it will try to replicate the performance of a predetermined benchmark without engaging in market timing or stock picking, and an active strategy because investors will choose to gain exposure to a specific factor that enhances returns based on various investing variables, attempting to generate above market-cap returns. In principle, this will provide investors access to a strategy with systematically low costs due to the passive component of the investing methodology, while also aiming to beat traditional market-cap indexing strategies. Due to the active component of the smart beta strategy, management fees will automatically be higher than a passively managed fund due to the nature of the factor fund, which will rebalance, i.e., return the surplus obtained from an increase in one component of the portfolio to the component that has fallen in terms of portfolio value to maintain the same original weighting and thus ensure that the portfolio remains balanced.

2.3.1 Smart Beta 1.0

The phrase "Smart Beta" is commonly used in the financial industry to describe innovative indexing methods that are not dependent on capitalization-weighted market indexes. In terms of performance, smart beta "1.0" approaches outperform market capitalization-based strategies. According to Amenc et al (2016), the latter have a tendency for concentration and unrewarded risk, which makes them less appealing to investors. In finance, "unrewarded risk" refers to taking on more risk without receiving a return that is commensurate with the increased risk.

When smart beta techniques were first introduced, they were meant to improve portfolio diversification over highly concentrated and capitalization-weighted (e.g., equal weighting or equal risk contribution) techniques. They were also meant to take advantage of the factor premium that exists in the stock market, such as value indices or fundamentally weighted indices that try to take advantage of the value premium. While strengthening capitalization-weighted indices is critical, focusing just on boosting diversity or capturing factor exposure may provide less-than-ideal results. This is because diversification-based weighting methods usually result in implicit exposure to certain factors, which might have unexpected effects for investors who are ignorant of their implicit factor exposures. As a result, weighting methods based on diversification are not advised. The first generation of Smart Beta benchmarks are integrated systems that do not discriminate between stock selection and weighing methods, unlike the second generation of Smart Beta benchmarks. As a result, the investor must be exposed to systemic risks, which are the cause of the investor's bad performance. Because they deconcentrate cap-weighted indices, which are typically sensitive to momentum and big growth risk, the first-generation Smart Beta indexes are usually prone to value, small- or midcap, and occasionally contrarian biases. Furthermore, unique biases on risk indicators unrelated to deconcentration but critical to the scheme's goals may exacerbate these biases even more. Fundamentally weighted indexes, for example, have a value bias since they employ accounting metrics that are connected to the ratios used to build value indexes.

2.3.2 Smart Beta 2.0

To accomplish their factor tilts, factor-tilted strategies that do not consider a diversification-based aim may result in extremely concentrated portfolios. Using a flexible technique known as Smart Beta 2.0, investors have recently begun to combine factor tilts with diversification-based weighting methods to build well-diversified portfolios with well-defined factor tilts [13]. This technique allows for the construction of factor-tilted, well-diversified indexes (by using a diversification- based weighting scheme among companies with the necessary factor exposures). This technique is also known as "smart factor investment" since it combines the smart weighting scheme with the explicit factor tilt [1]. Investors are increasingly focused on allocation decisions across factor investing approaches to gain additional value-added [13].

Investors may use Smart Beta 2.0 to assess and control the risk of their Smart Beta stock indices investments. Rather than providing just pre-packaged options to alternative stock betas, the Smart Beta 2.0 technique allows investors to experiment with different Smart Beta index construction methodologies to come up with a benchmark that best suits their risk preferences. Smart Beta was supposed to replace cap-weighted indexes as a natural passive investment reference, but its success with institutional investors has been much greater than its original goal. For one thing, it's obvious that cap-weighted indices are unrivaled when it comes to capturing market movements; for another, it's as obvious that they're the simple benchmark that all investors and stakeholders in the investing industry recognize [1].

Even the toughest critics of cap-weighted indices use them to evaluate the performance of their own new indexes in the end [1]. Most investors, and their promoters, are likely to prefer the new indices over the previous cap-weighted indexes due to their superior performance. While everyone believes cap-weighted indexes give the most accurate representation of the market, they do not always provide an efficient benchmark that can be used as a guide for a smart allocation by a knowledgeable investor [1]. To put it another way, they don't give a starting point (for active investing) or an end goal (for passive investing) that delivers an acceptable reward for the risks that the investor assumes via diversification. Alternative Beta, also known as Advanced Beta or Smart Beta, is a market solution to a problem that has occupied Modern Portfolio Theory since Nobel Laureate Harry Markowitz's work [1]. These new sorts of benchmarks, like any other technique or paradigm, are not without risk. Smart Beta proponents emphasize the hazards of concentration to explain why cap-weighted indexes are no longer considered appropriate standards, which is understandable, but it's also important to recognize the risks that investors face when they pick alternative benchmarks [1].

According to the EDHEC-Risk Institute, this is one of the goals of the Smart Beta 2.0 approach. This new vision of Smart Beta investment, which the Institute has been studying for the past three years, aims to empower investors by allowing them to reduce the risk of investing in Smart Beta stock indexes while still reaping the full benefits of their performance [1]. Smart Beta 2.0 addresses the problem of estimating and reducing the risks connected with these new forms of indexes. Even though most Smart Beta indices have a strong chance of outperforming cap-weighted indices over the long term due to the latter's high level of concentration, it should be noted that these new benchmarks can sometimes underperform market indices due to their exposure to risk sources other than cap-weighted indices [1]. It's worth noting that smart beta 2.0 aims to close the gap in terms of exposure to variables from the first generation, but it doesn't guarantee outperformance over market capitalization indexation-based strategies. It's a product that must be utilized carefully if you want to reap the rewards of its upside potential [1].

2.4 Alternative Strategies to Market Capitalization Indices

The basic rule of applying a capitalization weighting methodology for the development of indexes has recently come under fire. As the demand for indices as investment vehicles has grown, different weighting systems and alternate definitions of sub-segments have emerged. There have also been several recent projects for non-cap-weighted ETFs. Since the first basic factor weighted ETF was released in May 2000, a slew of ETFs has been released to monitor non-market-cap-weighted indexes, including equal- weighted ETFs, minimal variance ETFs, characteristics-weighted ETFs, and so on. These are dubbed "Smart Beta ETFs" since they aim to outperform traditional market-capitalization-based indexes in terms of risk adjusted returns.

The categorization approach will be the same as [12], using either weighing techniques based on basic principles (heuristic weighting) or weight optimization solutions that are based on more advanced approaches and need the assistance of a solver to accomplish.

It's an arbitrary categorization system designed to make reading easier by differentiating between simpler and more complicated tactics.

2.4.1 Heuristic Weighting Strategies

The equal weighting method assigns equal weight to each share, making up the index. We can obtain the weightings from the following mathematical equation:

$$\text{Index} = \sum_{i=1}^{n} w_i X_i \quad \text{where} \quad w_i = \frac{1}{n_i} \tag{2.7}$$

where w_i, X_i represents the weighting of the asset in the index and X_i the asset selected for the index.

Because each component of the index has the same weight, equal weighting helps investors to obtain more exposure to smaller firms. Bigger firms will be more represented in capitalization-weighted indexes since capitalization will be larger. The benefit of this technique is that tiny capitalization risk-adjusted performance tends to be better than big capitalization. In their study, [4] created three distinct indices in terms of index composition. The first group consists of enterprises with a substantial market capitalization (as are capitalisation-weighted indices). Each business in the index is then given equal weight. This is how most equally weighted indexes are built (MSCI World Equal Index, S&P 500 Equal Weight Index).

The second way is to create an index based on basic criteria and then assign equal weight to each organization. The third strategy is a hybrid of the first two. It entails averaging the ranks from the two preceding approaches and then assigning equal weight to the remaining 1000 shares. There are versions that are equally weighted [12]. The risk-cluster equal weighting approach involves sorting equities by sector and nation, then assigning each cluster (sector or country) in the index same weight.

Fundamental weighting indexation

The fundamentals weighting approach divides companies into categories based on their basic size. Sales, cash flow, book value, and dividends are all considered. These four parameters are used to determine the top 1,000 firms, and each firm in the index is given a weight based on the magnitude of their individual components [12].

For a fundamental index that includes book value as a consideration, for example, the top 1000 companies in the market with the most extensive book values are chosen. Firm x_i is given a weight w_i, which is equal to the firm's book value divided by the total of the index components' book values. Fundamental indexation tries to address the following bias: in a cap-weighted index, if the market efficiency hypothesis is not validated and a share's price is, for example, overpriced (greater than its fair value), the share's weight in the index will be too high. Weighting by fundamentals will reduce the bias of overweighting or underweighting of companies based on criteria like sales, cash flows, book value, and dividends, which are not affected by market opinion unlike capitalization.

Low beta indexation

Low-beta strategies rely on the empirical result which tells that asset with a low beta have greater returns than those expected by the CAPM [12]. A beta of less than one indicates that the share price has tended to grow less than its benchmark index during bullish trends and to decrease less severely during negative trends throughout the observed timeframe. A low- beta index is created by selecting low-beta stocks and then giving each stock equal weight in the index. As a result, it's a hybrid of a low-beta and an equal-weighting method. On the other side, high beta strategies enable investors to profit from the amplification of favorable market moves.

Reverse capitalization weighting indexation

The weight of an asset capitalisation-weighted index can be defined as:

$$MC_{w_i} = \frac{MC_i}{\sum_{j=1}^{n} MC_j}$$

where MC stands for "Market Capitalisation", and w is the weighting of asset “i” in the index. In a reverse cap-weighted index, the weight of an asset can be defined as:

$$RCW_{w_i} = \frac{\frac{1}{MC_i}}{\sum_{i=1}^{500} \frac{1}{MC_i}}$$

"Reverse cap-weighted" is abbreviated as RCW. In a reverse cap-weighted index, an asset's weighting will be the opposite of its weighting in a capitalization-weighted index [7]. Consequently, in order to implement this strategy, a cap-weighted index is required. RCW methods, similar to equal-weight or low-beta strategies, are driven by the observation that risk-adjusted returns for small caps are higher than those for large caps. Such indexation necessitates constant rebalancing.

2.4.2 Weight optimisation strategies

The logic of Modern Portfolio Theory [28] is followed in Mean-Variance optimization. Theoretically, if we know the predicted returns of all stocks and their covariance matrix, we can construct risk-adjusted-performance-optimal portfolios. These two variables, on the other hand, are difficult to quantify. [10] shown that even little inaccuracies in these two parameters' estimates may have a large influence on risk-adjusted performance.

Maximum diversification

This technique aims to build a portfolio with as much diversification as feasible. A diversity index (DI) is employed to achieve the desired outcome, which is defined as the distance between the sum of the constituents' volatilities and the portfolio's volatility [1].

$$DI = \frac{\left(\sum_i W_i \sigma_i\right)}{\sqrt{\sum_{i,j} W_i W_j \sigma_{ij}}}$$

Where w_i is the weight of an asset in the portfolio, σ_i is its volatility and σ_{ij} is the covariance between assets i and j. Choueitafy and Coignard (2008) utilized this diversity index to develop a Maximum Diversification Ratio index as part of portfolio optimization [11].

Minimum Variance

[10] adopt the simple premise that all stocks have the same return expectation, since stock return expectations are difficult to quantify. As a result of this premise, the best portfolio is the one that minimizes risk. The goal of minimal variance strategies, which have been around since 1990, is to provide a better risk-return profile by lowering portfolio risk without modifying return expectations. The low volatility anomaly justifies this technique. Low-volatility stocks have historically outperformed high-volatility equities. These portfolios are built without using a benchmark as a guide. The portfolio variance minimization equation for a two-asset portfolio is as follows:

$$\min \sigma_p^2 = w^2\sigma_a^2 + (1-w)^2\sigma_b^2 + 2w(1-w) \times \mathrm{cov}(r_a, r_b) \tag{2.8}$$

where w and $(1-w)$ represent the asset weights of r_a and r_b, σ^2 represents the standard deviation of the assets r_a and r_b and $\mathrm{cov}(r_a, r_b)$ represents the covariance of asset r_a and r_b.

This method is used in the MSCI World Minimum Volatility Index, which was released in 2008. Global Minimum Variance, Maximum Decorrelation, and Diversified Minimum Variance are the three types of minimum variance techniques [1]. However, there are no indexes or exchange-traded funds (ETFs) based on the Maximum Decorrelation and Diversified Minimum Variance methods in actuality; they are still only theoretical notions.

2.5 Analysis of the Behaviour of Factor Strategies During the COVID-19

2.5.1 Data

The approach for analyzing factor performance is based on the EDHEC Risk Institute's research paper [22] . Given its reference nature in terms of liquidity, market breadth, and representativeness in the financial research community, we will focus on factorial techniques in the US market [22] . We compare the performance of the funds to the VIX level during the Covid-19 period. We used the Refinitiv Eikon data platform to extract the equivalent of one year of historical data, or 374 trading days. We chose MSCI factor funds as a benchmark in the financial sector, as well as for their openness and data availability [22] .

2.5.2 Timeframe

To make our comparison with previous research articles on the influence of covid-19 on the equity market more consistent, we used Pagano's [29] taxonomy, which divides the pandemic event into the following phases [22]:

- Incubation period: January 2nd to January 17th, 2020
- Dates of the outbreak: January 20, 2020, to February 21, 2020
- Fever: February 24th to March 20th, 2020
- Treatment: March 23rd to April 15th, 2020

The study of Ramelli and Wagner (2020) [29] , who documented the incidents connected to the epidemic crisis that defined the course of this worldwide pandemic, proposed this first-time breakdown[22] . The first information circulating on the very virulent characteristics of a strain of covid related to a case from China in Wuhan, the epicenter of the sanitary crisis, occurred in January. Various countries throughout the world made measures to battle the pandemic during the February breakout phase.

The scientific community keenly observed the unanimity with which many countries decided to quarantine their citizens during the March (fever) period in order to lessen the risk of contamination and flatten the epidemic curve. The treatment period, which lasts roughly a year, aims to find a solution to the issue, particularly via attempts to discover vaccinations and therapies that can lower the virus's mortality. The stock market has reacted positively to the first wave of efforts to develop and shorten the time it takes to develop a vaccine and treatment measures available to date, with prices rising sharply after one of the darkest stock market episodes on record, with major US indices falling to all-time lows. With over a hundred vaccine development research projects underway, the FDA, one of the major health and pharmaceuticals regulatory organizations in the United States, has recognized three of them (Moderna, Pfizer, AstraZeneca). The Covax vaccination effort is still ongoing, with the goal of distributing the vaccine internationally and putting a stop to the epidemic [22].

2.5.3 Methodology

GARCH modelling of US equity factor

GARCH, which stands for Generalized Autoregressive Conditional Heteroskedasticity, is a statistical model used to estimate the volatility of financial returns. It is an extension of the ARCH model, accommodating time-varying volatility clustering. The generalised form can be written in the following mathematical function:

$$\text{GARCH}(p,q): \begin{cases} \sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 \\ \alpha_0 > 0, \alpha_i \geq 0, \beta_j \geq 0 \end{cases} \tag{2.9}$$

The GARCH(1,1) model includes one lag of volatility (the GARCH term) and one lag of squared residuals (the ARCH term), with the following specification:

$$\sigma_t^2 = \omega + \alpha\epsilon_{t-1}^2 + \beta\sigma_{t-1}^2 \tag{2.10}$$

where σ_t^2 is the conditional variance at time t, ω is a constant term, α is the coefficient for the lagged squared residual, indicating short-run persistence of shocks, ϵ_{t-1}^2 is the squared residual from the mean equation at time $t-1$, β is the coefficient for the lagged conditional variance, indicating long-run persistence and σ_{t-1}^2 is the conditional variance at time $t-1$.

Parameters are estimated using maximum likelihood estimation (MLE). The GARCH model provides a dynamic forecast of volatility, where a large α suggests recent shocks significantly affect current volatility. A large β suggests volatility is persistent over time. Also, the sum of α and β near 1 indicates high persistence of volatility shocks.

Multiple Linear Regression

We assess the impact of each factor impact with respect to the S&P500, VIX and a dummy variable to capture the impact of COVID-19. We use a multiple linear regression framework to add a binary variable that captures the difference between the pre- and post-pandemic environments. This separates the effect of the pandemic from normal market movements. We run five different regressions across the full sample to assess the impact of the pandemic on US equity factors. Mathematically, we run the following regressions:

$$\text{Value} = \beta_0 + \beta_1 \times \text{SPX\_Index} + \beta_2 \times \text{VIX\_Index} + \beta_3 \times \text{COVID\_Impact} + \epsilon \tag{2.11}$$

$$\text{Size} = \beta_0 + \beta_1 \times \text{SPX\_Index} + \beta_2 \times \text{VIX\_Index} + \beta_3 \times \text{COVID\_Impact} + \epsilon \tag{2.12}$$

$$\text{Quality} = \beta_0 + \beta_1 \times \text{SPX\_Index} + \beta_2 \times \text{VIX\_Index} + \beta_3 \times \text{COVID\_Impact} + \epsilon \tag{2.13}$$

$$\text{Momentum} = \beta_0 + \beta_1 \times \text{SPX\_Index} + \beta_2 \times \text{VIX\_Index} + \beta_3 \times \text{COVID\_Impact} + \epsilon \tag{2.14}$$

$$\text{Minvol} = \beta_0 + \beta_1 \times \text{SPX\_Index} + \beta_2 \times \text{VIX\_Index} + \beta_3 \times \text{COVID\_Impact} + \epsilon \tag{2.15}$$

where β_0 represents the intercept of the model, β_1 represents the coefficient for the S&P Index returns, β_2 represents the coefficient for the VIX Index returns, β_3 represents the coefficient for the COVID_Impact dummy variable and ϵ represents the error term of the regression.

2.5.4 Results

GARCH analysis for US equity factors

Our study aimed to dissect the intricate volatility patterns of different equity factors, such as Minimum Volatility, Momentum, Quality, Size, and Value, during the tumultuous period of 2019 to 2021. By doing so, we sought to uncover the nuanced impacts of the pandemic on the financial markets.

Employing the GARCH(1,1) model allowed us to quantify and compare the evolving volatility levels across various equity factors. This model, renowned for its ability to model time-varying volatility, is particularly useful at capturing volatility clustering—a common characteristic in financial time series.

The GARCH analysis revealed several critical insights:. The is some volatility clustering, particularly during the early stages of the pandemic. This was characterized by significant peaks in the fitted volatility, aligning with actual market returns, thereby illustrating the heightened market sensitivity to the unfolding crisis. Our analysis delineated a differential response in volatility among the equity factors. Certain factors, like Minimum Volatility, exhibited resilience, as evidenced by their lower volatility levels compared to others like Momentum or Value, which displayed heightened volatility during the same periods.

We plotted the actual returns against the fitted volatility from the GARCH models. The visual analysis supported the numerical results by showing a strong alignment during times of high volatility. This proved that the GARCH model was good at capturing the volatility trends during the pandemic.

The empirical evidence from our GARCH models contributes to a more profound understanding of market behavior under stress. It underscores the varied impacts of the pandemic across different investment styles and raises pivotal considerations for risk management and investment strategies during periods of crisis.

The graph for MinVol illustrates the periods of elevated volatility, particularly noticeable during the early months of 2020, which corresponds with the onset of the COVID-19 pandemic 4.6a. This is shown by the fact that the actual returns spikes are very different from the fitted GARCH volatility. This shows that the model correctly predicts the overall trend but not the extreme changes that happen during stressful times. The subsequent reduction in volatility demonstrates the stabilizing effect of market interventions and the adaptation of investors to the 'new normal'.

The Momentum factor graph shows a similar pattern, with heightened volatility at the start of the pandemic 4.5d. However, the actual returns revert more quickly towards the model's fitted volatility, suggesting that the Momentum factor may have been more resilient or quicker to adjust to market changes induced by COVID-19. This could be due to the momentum strategy's characteristic of following recent trends, which might have aligned with the swift recovery in certain market segments post the initial shock.

For the Quality factor, the volatility spikes are pronounced but fewer compared to MinVol and Momentum, indicating that high-quality stocks, typically with strong balance sheets and profitability, may have provided some defensive characteristics during the market turmoil 4.5c. However, the factor still experienced considerable stress, as seen in the deviations from the fitted GARCH volatility, reflecting the widespread uncertainty that affected all market sectors.

The Size factor, often represented by small-cap stocks, shows substantial volatility that exceeds the fitted GARCH volatility during the pandemic 4.5b. This suggests that smaller companies were more vulnerable to the economic impacts of COVID-19, which is consistent with the higher risks associated with smaller enterprises during economic downturns.

Finally, the Value factor graph displays volatility that aligns closely with the GARCH model's fitted volatility, except during the pandemic's early stages 4.5a. The extreme negative returns during early 2020 imply that value stocks were not immune to the initial shock, possibly due to their sensitivity to economic cycles. However, the relatively quick convergence to the fitted volatility might indicate that the inherent 'value' proposition may have provided some degree of protection as markets processed the pandemic's potential long-term effects.

The analysis of these GARCH models over the period of the COVID-19 pandemic reveals the differential impact on various US equity factors. While all factors experienced increased volatility during the pandemic onset, the speed and extent of recovery varied, reflecting the diverse characteristics and investor sentiments associated with each factor. The pronounced spikes during the pandemic's early months are indicative of the market's initial reaction to an unprecedented global event, while the subsequent periods show the varying degrees of resilience and recovery across these factors. In conclusion, our study provides a granular view of the volatility landscape during one of the most disruptive periods in recent history. The insights from this research not only reinforce the critical role of dynamic volatility modeling in finance but also pave the way for future studies to explore the long-term implications of such global shocks on market stability.

MLR analysis for US equity factors

This study aimed to analyse the relationship between prevailing market conditions, captured by the S&P 500 and VIX indices, and the returns of various US equity factors during a period marked by the COVID-19 pandemic. We use a multiple linear regression framework to add a binary variable that captures the difference between the pre- and post-pandemic environments. This separates the effect of the pandemic from normal market movements. A robust positive correlation with the S&P 500 Index was observed, affirming the concomitant rise of the value factor with the market. Notably, the VIX Index and the pandemic dummy variable did not manifest as significant determinants within the regression model, indicating a negligible directional impact of market volatility and the pandemic on the value factor returns. 4.1 4.6b

The S&P 500 Index emerged as a potent predictor for the size factor, mirroring the relationship seen with the value factor. The VIX Index and the pandemic dummy variable were not significant in explaining the size factor returns, which suggests a degree of resilience or insensitivity to the pandemic's onset and associated market volatility 4.2 4.6c.

The S&P 500 Index also exerted a significant positive influence on the quality factor, suggesting that quality stocks have the propensity to outperform in tandem with market upswings. The VIX Index and the pandemic dummy variable did not exhibit a significant impact, indicating the potential stability or neutrality of the quality factor during the pandemic 4.3 4.6d.

Momentum returns displayed a positive association with the S&P 500 Index, devoid of significant perturbations from the VIX Index or the pandemic dummy variable. This finding suggests that momentum stocks may track market trends without being disproportionately affected by the heightened market volatility or the pandemic's initial shock 4.4 4.7a. Both the S&P 500 Index and the VIX Index were significant, with the latter displaying a positive coefficient. This outcome hints at the potential of minimum volatility stocks serving as a buffer during periods of heightened market turbulence. The pandemic dummy variable's non-significant coefficient suggests that the pandemic did not change the performance of minimum volatility stocks in a way that could be seen statistically in the model that was made 4.5 4.7b.

The calculated F-statistics and corresponding p-values substantiate the regression models' overall significance, with particularly compelling evidence for the value and size factors. The quality and momentum factors presented moderate significance, suggesting a nuanced interplay between market indices and factor returns. Overall, the quantitative analysis delineates a differential impact of market conditions and the COVID-19 pandemic across various equity factors. These nuanced insights contribute to the broader understanding of factor investing in the context of unprecedented global health crises and their attendant economic ramifications.

The correlation matrices for different periods—origins, incubation, outbreak, fever, and treatment—reveal how relationship between the equity factors and market indices (S&P 500 and VIX) evolved throughout the stages of the COVID-19 pandemic. During the origins phase, we observe a different correlation structure, with lower overall market correlations, as the pandemic's effects were not fully realized in the financial markets. This period show a more 'normal' market condition where factors behaved according to pre-pandemic expectations 4.4a.

As we move into the incubation and outbreak phases, the correlations between factors and the S&P 500 Index increased, suggesting a market-wide reaction to the unfolding crisis. It is during these times that the factors' returns started to move in tandem with the broader market, as investors react to the uncertainty and market sentiment 4.4c 4.4b.

The fever phase exhibit the highest correlations, particularly with the VIX Index, as this period likely represents the peak of market panic and volatility. Equity factors that typically have lower correlations with market movements might show increased correlations during this time, indicating that few assets were immune to the shocks caused by the pandemic. In the treatment phase, as the market adjusts to the 'new normal' and starts to price in the recovery, we expect the correlations with the VIX Index to decrease, reflecting a stabilization of market conditions. This is in line with the results obtained 4.4d.

The changes in correlation coefficients over these periods provide valuable insights into the risk characteristics of each factor and how they might be used to manage a portfolio during times of crisis. High correlations with the VIX Index during high volatility periods suggest that certain factors may carry higher systemic risk, while changes in the correlation with the S&P 500 Index could influence how these factors contribute to portfolio diversification. In a practical setting, these observations could guide portfolio construction and risk management decisions.

For instance, if a factor consistently shows high correlation with the VIX Index during volatile periods, it may not provide the desired diversification benefits and could be underweighted in a risk-averse portfolio. Conversely, factors that maintain lower correlations with market movements, even during a crisis, could be valuable for diversification purposes.

Analyzing these correlations can also provide deeper insight into the market dynamics and investor behavior during the pandemic. It could reveal how different equity factors are perceived in terms of risk and return in extraordinary market conditions, which is crucial information for asset managers and individual investors alike.

2.6 Conclusion

Our GARCH analysis has offered valuable insights into the volatility patterns of key equity factors during the 2019 to 2021 period, marked by the COVID-19 pandemic. We observed volatility clustering, especially during the pandemic's early stages, which was vividly captured by the GARCH(1,1) models. The alignment of actual returns with fitted GARCH volatility during these volatile times validated the model's efficacy in reflecting market sensitivity to the crisis.

Differential responses were noted among the factors; the Minimum Volatility factor demonstrated resilience, while Momentum and Value factors exhibited pronounced volatility. These findings are instrumental for investors in understanding the behavior of various investment styles during periods of heightened uncertainty and aid in strategic decision-making for portfolio management.

The multiple linear regression (MLR) analysis aimed to unravel the interplay between market conditions and US equity factor returns in the face of the pandemic. The S&P 500 Index emerged as a strong positive predictor for most factors, while the VIX Index's influence was more nuanced. Our regression model delineated how the pandemic's onset did not significantly skew factor returns, suggesting that market movements were primary drivers of factor performance during this period.

The MLR results offer a granular perspective on the behavior of equity factors under the pandemic's influence. The Value and Size factors' strong model significance implies a high correlation with market trends, while Quality and Momentum factors presented moderate significance, reflecting a more complex relationship with market indices and the pandemic.

The correlation matrices for various pandemic phases exposed the evolving dynamics between equity factors and market indices. As the pandemic unfolded, we observed an increase in correlations with the S&P 500 Index, indicating a collective market response to the crisis. The VIX Index correlations highlighted the factors' sensitivity to market volatility, which peaked during the fever phase of the pandemic. These correlation analyses underscore the imperative to adapt portfolio strategies in real-time, considering the fluid nature of equity factor correlations in response to macroeconomic shocks. They also emphasize the need for robust risk management frameworks capable of mitigating the adverse effects of such unprecedented events.

This study contributes to academic literature by providing empirical evidence of the disparate impacts of the COVID-19 pandemic on equity factor volatility and returns. It underscores the importance of incorporating dynamic volatility modeling into financial analysis and portfolio management. Future research may extend this work by examining the long-term implications of global health crises on market stability and the efficacy of different investment strategies during such periods.

Chapter 3

Portfolio Construction Techniques

3.1 Introduction

The concept of the All-Weather Portfolio, pioneered by Bridgewater Associates, embodies a strategy of asset diversification designed to generate resilient returns across various market conditions. Characterized by its diversified asset composition, this portfolio paradigm seeks to transcend traditional portfolio management by offering a shield against diverse market fluctuations.

Historical analyses of institutional portfolios reveal a recurring theme of suboptimal asset diversification, with an estimated 75% to 80% of portfolio risk typically tethered to equity securities. Such a configuration exposes these portfolios to the full brunt of stock market volatilities, a phenomenon markedly evident in portfolios with an equity concentration as high as 90% [37]. 4.8 represents the cumulative performance of the different stock market during the period 1900 – 2019. The obvious observation is that if in this period the investor had decided to follow a geographically diversified approach by investing equally on his portfolio in the five selected countries, the investor would have realized a cumulative performance of the portfolio approaching the best result on this analysis the return of an investment on the US equity market [38].

The principles of Modern Portfolio Theory, as formulated by Harry Markowitz, underscore the virtues of diversification. Markowitz's seminal work suggests that a well-diversified portfolio, characterized by maximum decorrelation among its constituents, can enhance performance. Neglecting such diversification could result in the forfeiture of substantial returns, measured in hundreds of basis points. The addition of geographic diversification further underscores this point. The 20th-century investment landscape offers poignant lessons, where countries like Russia and Germany faced systemic crises in the 1920s, leading to colossal investor losses. 4.9 represents the worst equity excess return drawdowns across countries Geographic diversification is an important vector of performance for the overall return of the portfolio, as evidenced by the results of previous centuries where investments concentrated on a single country, such as Russia in the early 1920s and Germany in the mid-1920s, both of which experienced systemic crises resulting in significant losses for investors and a total loss on a portfolio concentrated on a single country [33].

In contrast, a hypothetical portfolio evenly distributed across five selected countries during the same period would have yielded cumulative returns comparable to those of the robust U.S. equity market [38]. 4.10 represents the cumulative excess returns of an equally weighted geographically diversified portfolio of stocks and bond compared to a concentrated investment in the US and global set of countries. If we take the analysis a step further, the performance of a geographically diversified portfolio with an equal allocation has consistently outperformed over the last 70 years [38]. Extending this analysis reveals that for the past seven decades, a geographically diversified portfolio with equal allocation has consistently delivered superior performance [33].

This strategic allocation not only bolstered returns but also mitigated maximum potential losses, as reflected in the drawdown analysis of the portfolio's components [38]. 4.11 represents the maximum drawdown of an equally weighted geographically diversified portfolio of stocks and bond compared to a concentrated investment in the US and global set of countries. This consistency in the result is done while minimizing the maximum losses of the portfolio, as evidenced by the drawdowns between the different components of the analysis [38].

The conventional wisdom advocating a 60/40 asset-to-bond allocation ratio is predominantly driven by the asset component's performance, which demonstrates a staggering 95% correlation with the portfolio's overall returns. This relationship challenges the common investor assumption that a 40% bond allocation constitutes effective diversification [37]. 4.12 represents the correlation analysis of the drawdowns of a 60/40 portfolio. X-axis represent the period analysed, while the y-axis represents the overall drawdown of the portfolio analysed (Bridgewater Associates, 2009).

It is essential to revisit the foundational principles of portfolio construction, particularly the risk/return profiles of various asset classes and their alignment with an investor's risk tolerance. The quest for an 8% return traditionally directs investors towards a heavy reliance on equities. However, the All-Weather approach introduces a nuanced strategy, suggesting that leveraging assets can optimize the risk/return ratio without compromising diversification [37]. 4.13 represents the risk-return of different asset classes. X-axes represent the expected risk of each asset, while the y-axis represents the overall expected total return of each asset in the graph (Bridgewater Associates, 2009).

It is important to stress the key to getting the best possible return is to differentiate between combinations of slightly correlated assets and combinations of highly correlated assets, since a low correlation, or even a negative correlation, is a driver of portfolio return. 4.14 represents the risk-return of different asset classes. X-axis represent the period analysed, while the y-axis represents the performance of each asset selected in the analysis [37]. In this sense, we can analyse the effect of the correlation on the risk incurred by the portfolio, where a correlation of 60% would imply a probability of loss of funds of 38% annually, for a risk/return ratio of 0.31, while a portfolio with a neutral correlation will have a much lower probability of loss (of the order of 11%) while having an improved risk/return ratio (of the order of 1.25, i.e. practically five times the return of the former).

The All-Weather Portfolio approach aims at incorporating different asset classes in its allocation while scrupulously respecting the increase and decrease matrix in inflation and growth conditions to naturally neutralize the losses of one asset with the gains of another asset class and all this without forgetting the geographical diversification factor which is integrated into the portfolio construction [37].

3.2 Modeling of the All Weather portfolio

3.2.1 Mathematical foundations

This part structures the mathematical foundations used in the modeling part of this paper. All formulas are extracted from [35].

Markowitz portfolio construction

The return on an asset is based on its return and its portfolio weighting. The portfolio return can be calculated using the following formula:

$$\sum_{i=1}^{N} w_i \cdot r_i$$

The portfolio return expression is then used to calculate two critical portfolio features for investors: expected performance as indicated by the average return and risk as indicated by the standard deviation of returns.

The standard deviation of the portfolio's returns is calculated using the following formula:

$$\sigma_P = \sqrt{\sum_{i=1}^{N} \sum_{j=1}^{N} w_i \cdot w_j \cdot \rho_{ij} \cdot \sigma_i \cdot \sigma_j}$$

where wi is asset's weight i, wj is the asset's weight j, rho i is the asset's standard deviation, rho j is the asset's standard deviation and rho i, j is the correlation between the assets i, j.

Due to the computational burden associated with relying on many assets, we can employ the matrix form for easier implementation. Essentially, we multiply the weight vector by the variance-covariance matrix and the transpose of the weight vector:

$$\sigma_P = \sqrt{w \cdot \Sigma \cdot w'}$$

where w is the vector of weights, sigma is the matrix of variance-covariance and w' is the weight vector transposed.

GMV Portfolio construction

The GMV portfolio is only based on the covariance matrix estimation. This matrix estimator is inversed in the formula to determine the weights of the portfolio. The unbiased estimator of the inverse covariance matrix can be approached as:

$$\hat{\Sigma}\text{unbiased} = \frac{1}{T-n-2} \sum t = 1^T (r_t - \hat{\mu})(r_t - \hat{\mu})'$$

where $\hat{\mu} = \frac{1}{T} \sum_{t=1}^{T} r_t$ is an estimator for the mean returns, T the number of observations, n the number of assets and rt the vector of financial returns for the n assets observed at time t.

PCA analysis

Mathematically we can approach it as:

$$\hat{\Sigma}_{\text{3factors}} = \Phi_f \Lambda_f \Phi' + \Sigma\epsilon$$

The variance of the residuals ϵ_i can be computed as (Clauss, 2011):

$$\text{Var}(\epsilon_i) = \text{Var}(r_i) - \phi_{i1}^2 \lambda_1 - \phi_{i2}^2 \lambda_2 - \phi_{i3}^2 \lambda_3 \tag{3.1}$$

with Λ_f representing the diagonal matrix of the first three eigenvalues of the unbiased estimator of the covariance matrix, Φ_f the matrix with the first three eigenvectors, and Σ_ϵ the diagonal residual covariance matrix determined for each asset i.

Black-Litterman approach

The weights in a Black-Litterman can be approached as:

$$\omega = \frac{1}{A} \tilde{\Sigma}^{-1} \tilde{\mu}$$

with $\tilde{\Sigma}$ the covariance matrix between assets returns of length $n \times n$, $\tilde{\mu}$ the vector of the expected excess returns equal to $\mu - r_f e$ with μ the expected returns, r_f the risk-free rate and e a vector of 1 of length n and finally.

The unbiased estimator for mu and covariance can be approached as follows:

$$\tilde{\mu} = \frac{1}{T} \sum_{t=1}^{T} (r_t - r_f)$$

with rt is as the return of the asset at time t and rf as the risk-free rate asset return at time t.

The unbiased covariance estimator can be approached similarly as the the Global Minimum Variance (GMV) case [35]:

$$\tilde{\Sigma} = \frac{1}{T - n - 2} \sum_{t=1}^{T} (r_t - \hat{\mu})(r_t - \hat{\mu})'$$

Black-Litterman approach is quantitative approach that integrate investor views in a relevant way. This method adds to economic predictions statistical uncertainty. It is based on a Bayesian approach [35].

The Black-Litterman returns are the following mixed estimates:

$$\hat{\mu}_{mixed} = \left(\tau \tilde{\Sigma}^{-1} + \Omega^{-1}\right)^{-1} \left[\tilde{\Sigma}^{-1} \hat{\mu} + \Omega^{-1} Q\right]$$

with Q the economic views quantified by average returns, τ the confidence parameter in the views and Ω the matrix of uncertainty associated with the economic views; we assume that Ω is a diagonal matrix with diagonal elements equal to variances of assets returns.

3.2.2 Packages

The modeling is conducted in R program.

We import the necessary libraries to perform the first analysis.

- library(tidyverse): The tidyverse package constitutes a collection of R packages meticulously designed to work seamlessly in unison for data manipulation, visualization, and analysis. It encompasses packages like ggplot2 for data visualization, dplyr for data manipulation, tidyr for data tidying, and more. Loading tidyverse avails all these packages for utilization during your R session, streamlining your data analysis workflow.

- library(quantmod): The quantmod package primarily caters to quantitative financial modeling and analysis. It furnishes an array of functions and tools for handling financial data, such as downloading and managing stock price data, conducting technical analysis, and modeling financial time series.
- library(DataExplorer): The DataExplorer package serves the purpose of preliminary data exploration and analysis. It offers functions to generate summary statistics, visualize missing data, plot variable distributions, and more. It can aid in obtaining a quick overview of your data before delving into more extensive analyses.
- library(corrplot): The corrplot package specializes in visualizing correlation matrices. It provides functions for generating visually appealing and informative correlation plots, which prove valuable for comprehending relationships between variables within your data.
- library(scales): The scales package provides an assortment of scales and formatting functions tailored for R graphics. It is often employed in conjunction with other plotting packages, such as ggplot2, to customize the appearance of plots, including axes, labels, and color scales.

3.2.3 Investment universe

Fund selection

This investment universe is designed to provide diversified exposure across a range of asset classes, including broad market indices, emerging market equities, government bonds, commodities, and precious metals. This selection is intended to balance risk and return while capturing growth opportunities and providing a hedge against inflation.

- VTI (Vanguard Total Stock Market ETF): Broad exposure to the global stock market.
- EMGF (iShares MSCI Emerging Markets Multifactor ETF): Exposure to emerging markets with favorable exposure to value, quality, momentum, and size factors.
- IEF (iShares 7-10 Year Treasury Bond ETF): Exposure to intermediate-term government bonds.
- DBC (Invesco DB Commodity Index Tracking Fund): Exposure to a basket of commodities.
- GLD (SPDR Gold Trust): Exposure to gold.

Economic data

To connect our investment universe selection to the asset allocation, we can refer to market outlooks from prominent Wall Street's most reliable market commentaries. We take the economic data from Goldman Sachs Asset Management, BlackRock, and JPMorgan Asset Management. On average, the three financial institutions expect global GDP growth of +3.0% in 2023, US GDP growth of +2.0%, and S&P 500 earnings growth of +8%. They also expect the US 10-year

Treasury yield to reach 3.3%. In terms of asset allocation, all three institutions are overweight on equities, with Goldman Sachs Asset Management being the most overweight. They are also underweight on bonds, with JPMorgan Asset Management being the most underweight.

Overall, the market outlooks from these three financial institutions suggest that equities are still the preferred asset class for 2023, despite the expected slowdown in economic growth. Investors should consider diversifying their portfolios with other asset classes, such as bonds and commodities, to reduce risk.

This investment universe is a good option for investors who are looking for a diversified portfolio that offers the potential for both growth and income. The assets in this universe are supported by economic data, market outlook, and investment considerations.

Rationale

We have carefully chosen a set of five trackers (ETFs) to represent our investment universe. An ETF can be defined as a financial product that is based on a basket of different assets, to replicate the actual performance of each selected investment. An ETF has more or less the same proportion of the underlying components of the basket, depending on the style of management of the asset manager. Below is the Exchange-Traded Funds (ETF) chosen as the investment universe:

- Equities (VTI): VTI provides broad exposure to the global stock market, which is expected to benefit from economic growth.
- Emerging market equities (EMGF): EMGF provides exposure to emerging markets with favorable exposure to value, quality, momentum, and size factors. Emerging markets have the potential for higher growth than developed markets, but they also come with more risk. EMGF mitigates some of the risk by investing in companies with favorable exposure to specific factors.
- Government bonds (IEF): IEF provides exposure to intermediate-term government bonds, which are typically considered to be low-risk investments. This can provide stability to a portfolio in times of economic uncertainty.
- Commodities (DBC): DBC provides exposure to a basket of commodities, which can be a good way to diversify a portfolio and hedge against inflation. However, it is important to note that commodities are volatile investments.
- Precious metals (GLD): GLD provides exposure to gold, which is a traditional safe haven asset that can provide protection against inflation and market volatility.

Overall, this investment universe is well-diversified and offers a mix of assets with different risk and return profiles. This makes it a good choice for investors with a variety of investment objectives and time horizons.

Our selection strategy is based on several fundamental considerations:

- Investment universe: The investment universe aims at representing different asset classes in order to construct a multi-asset portfolio.
- Diversification strategy: Our commitment to constructing a well-rounded portfolio has been affirmed through diversification across various sectors and industries. Such diversification is a critical risk management tool, curbing overexposure to a single sector or asset class.
- Historical data availability: The selected trackers benefit from a rich repository of historical data and extensive research. The accessibility of such data enable these ETFs to rigorous portfolio management analysis and comprehensive research.

3.2.4 Data cleaning and analysis

Data cleaning

We initiate data cleaning, a crucial step in data analysis:

- Extracting adjusted closing prices: We use the lapply function to obtain adjusted closing prices, commonly used for return calculations, for a list of stock tickers. This code iterates through each ticker and fetches the adjusted closing prices.
- Combining stock returns: To consolidate the adjusted closing prices, we employ cbind, creating a data frame where each column represents a specific stock's adjusted closing prices.
- Renaming columns: We label the columns with their corresponding stock tickers, facilitating easy identification.
- Handling missing data: We remove rows with missing values from the dataset for quality and consistency.

Arithmetic returns are used for performance assessment, offering a clear view of gains and losses, suitable for short-term analysis, and ensuring transparency.

Data analysis

Data ranges from 01/01/2019 to 31/08/2023, capturing the period before and after the Covid 19 pandemic. We plot the rebased performance of the trackers selected in our investment universe. From the graph below, we can clearly see the out performance of equity tracker (VTI) by a considerable margin with respected to the other asset classes. The equity market has outperformed all of the other asset classes mentioned above since 2019 for a number of reasons.

- Strong economic growth: The global economy has grown at a healthy pace since 2019, which has supported corporate earnings growth. This has made equities more attractive to investors.
- Low interest rates: Interest rates have been low since 2019, which has made equities more attractive relative to other asset classes, such as bonds.
- Government stimulus: Governments around the world have enacted a number of stimulus measures to support the economy during the COVID-19 pandemic. This has boosted consumer spending and investment, which has benefited equities.
- Corporate earnings growth: Corporate earnings have grown strongly since 2019, as companies have benefited from the strong economy and low interest rates. This has made equities more attractive to investors.

The equity market is the riskiest of the asset classes mentioned above because it is more volatile and subject to larger swings in price. However, it has also been the most rewarding asset class over the long term, as it has generated higher returns than other asset classes, such as bonds and commodities.

Here are some additional insights on the outperformance of the equity market:

- The equity market has benefited from a number of technological trends, such as the rise of e-commerce and cloud computing. These trends have led to the growth of new companies and industries, which has created new investment opportunities for investors.
- The equity market has also benefited from the globalization of the economy. Companies are now able to operate in multiple countries, which has helped them to grow their businesses and increase their profits.
- The equity market has become more accessible to individual investors in recent years. This has led to an increase in demand for equities, which has helped to drive up prices.

Overall, the equity market has outperformed all of the other asset classes mentioned above since 2019 due to a number of factors, including strong economic growth, low interest rates, government stimulus, corporate earnings growth, technological trends, globalization, and increased access for individual investors.

To enhance our data analysis, we can perform statistical analysis on the monthly returns of each stock. This helps us evaluate their performance and behavior during the observed time frame. Please note that the figures derived from this analysis are presented in monthly returns.

Using monthly data to smooth the data and transform it to a close approximation to the Gaussian distribution has several advantages [35]:

- Reduced noise: Monthly data is less noisy than daily or weekly data. This is because it averages out the short-term fluctuations in the data.

- Improved normality: Monthly data is more likely to be normally distributed than daily or weekly data. This is because the Central Limit Theorem states that the distribution of the sum of a large number of independent and identically distributed random variables will be approximately normal, even if the original distribution is not normal.

- Easier to model: Gaussian distributions are easier to model than other types of distributions. This is because there are a number of well-established statistical methods for modeling Gaussian distributions.

There are also some disadvantages to using monthly data to smooth the data and transform it to a close approximation to the Gaussian distribution:

- Loss of information: Monthly data is less informative than daily or weekly data. This is because it averages out some of the detail in the data.

- Time lag: Monthly data has a time lag. This means that it does not reflect the latest changes in the underlying data.

Overall, the utility of using monthly data to smooth the data and transform it to a close approximation to the Gaussian distribution depends on the specific application. In order to reduce noise and improve normality, then using monthly data is a good option. However, if the goal is to capture the latest changes in the data or to preserve all of the information in the data, then using more frequent data is a better option.

Correlation is a fundamental concept related to diversification, which seeks to use poorly correlated and non correlated assets to construct a portfolio. Diversification aims to boost returns while minimizing risk by investing in assets that respond differently to various events. Prudent investors seeking risk mitigation will diversify their portfolios.

In our investment universe, opportunities for diversification arise. Defensive assets like gold (TICKER: GLD) can effectively diversify when combined with more aggressive ones. For instance, the Vanguard Total Index tracker (TICKER: VTI) has a poor correlation with gold (TICKER: GLD). iShares 7-10 years Treasuries Index tracker (TICKER: IEF) exhibits negative correlation with respect to international and emerging market equities 3.1.

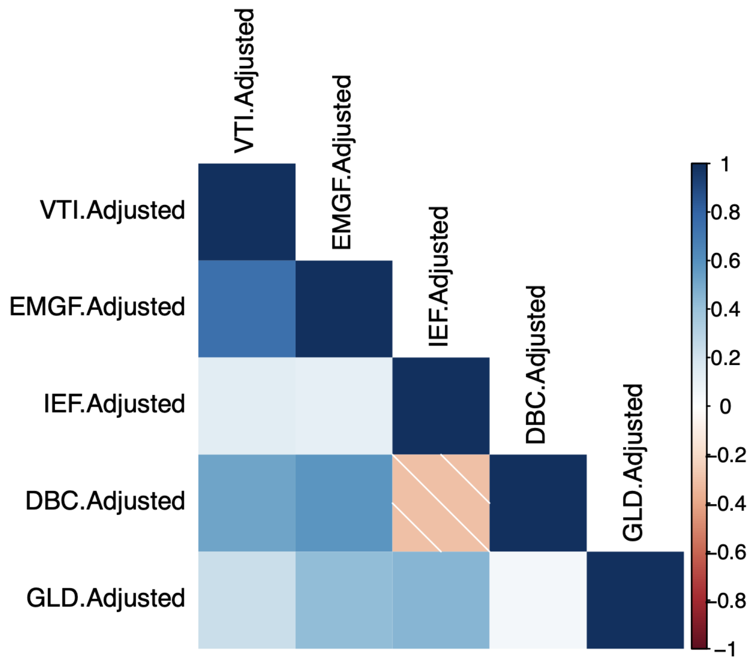

Figure 3.1: Correlation analysis of the funds returns.

3.2.5 Modelling of the portfolio

Unbiased Global Minimum Variance (GMV) portfolio

Modern Portfolio Theory (MPT) is founded on several market and investor assumptions. Several of these assumptions are stated explicitly, while others are implied. Markowitz's contributions to (MPT) in portfolio selection are based on the following basic assumptions [34]:

- Investors are rational (they seek to maximize returns while minimizing risk, or minimize risk while maximize return).
- Investors will accept increased risk only if compensated with higher expected returns.
- Investors receive all relevant information regarding their investment decision.
- Investors can borrow or lend an unlimited amount of capital at a risk-free rate of interest.

In the context of time series data analysis, one of the crucial applications is the division of the data set into a training set and a testing set based on temporal order. This approach is particularly important when dealing with data that evolves over time, such as stock prices in this instance.

The training set typically comprises historical data, while the testing set contains more recent observations. This temporal separation allows for the evaluation of predictive models and forecasting techniques. By using historical data to train the model and then assessing its performance on more recent data, analysts can gauge the model's ability to make accurate predictions and anticipate future trends. Time series data applications are essential in fields like finance, where understanding and forecasting trends over time is important

For the purpose of modelling, we will shrink the data set into two different sub-samples. We can define the following parameters:

- The first sub-sample will cover the first 20 trading month covered in the data set.
- The second sub-sample will cover the rest of the data set, covering the equivalent of 36 trading month.

We can see in this instance that the portfolio is mixed, with some long and short positions. The capital is fully invested, with some important exposure in the long part of the portfolio like the bond fund 3.2.

Figure 3.2: GMV portfolio weight. Each bar represents a fund weight from left to right: VTI, EMGF, IEF, DBC, GLD.

Implementation of Principal Component Analysis (PCA)

Principal Component Analysis (PCA) is a technique used to analyze and reduce the dimensionality of a dataset while retaining as much variance as possible [35]. We perform the PCA analysis using the second subsample starting from the trading month number 20 of the sample to the trading month number 56.

Principal component analysis (PCA) is an unsupervised machine learning technique that can be used to identify the underlying patterns in a dataset. PCA works by transforming the dataset into a new set of variables, called principal components, that are uncorrelated with each other. The principal components are ordered in terms of how much variance they explain in the dataset, with the first principal component explaining the most variance and the last principal component explaining the least variance.

PCA can be used for a variety of tasks, including dimensionality reduction, data visualization, and feature extraction. In the context of portfolio management, PCA can be used to [35]:

- Identify the underlying factors that drive the returns of the stocks in the portfolio.
- Construct more diversified portfolios by selecting stocks with low correlations with each other.
- Reduce the dimensionality of the portfolio by selecting the most important principal components.

The PCA results can be interpreted as follows:

- The first principal component can be thought of as a market factor. This factor represents the overall movement of the stock market. In other terms, the first factor explains half of the variance of the portfolio overall and can be assimilated as a proxy to the sensitivity to market fluctuations.

Figure 3.3: PCA portfolio weight. Each bar represent a fund weight (left to right): VTI, EMGF, IEF, DBC, GLD.

- The second and third principal components can be thought of as style factors. These factors represent the different styles of investing, such as value investing and growth investing. From the type of stocks retained, we have shortlisted an important number of growth stocks in the technology sector, which could explain part of the overall variance of the portfolio.
- The remaining principal components can be thought of as idiosyncratic factors. These factors are specific to the individual stocks in the portfolio.

3.3 represents the portfolio allocation using the PCA method.

Implementing the Tangency Portfolio (TP)

In time series data analysis, a crucial application involves splitting the dataset into training and testing sets based on temporal order. This is vital when working with data that evolves over time, such as stock prices or weather observations.

The training set consists of historical data, while the testing set contains more recent observations. This division allows analysts to assess predictive models and forecasting techniques. By training the model on historical data and evaluating it with recent data, we can determine its ability to make accurate predictions. This is especially important in finance for understanding and forecasting trends over time.

For modeling purposes, we'll create two sub-samples with defined parameters.

- The first sub-sample will cover the first 20 trading month covered in the data set.
- The second sub-sample will cover the rest of the data set, covering the equivalent of 36 trading months.

The portfolio allocation consists of both long positions, where investors expect favorable returns, and short positions, where they anticipate weaker performance. Here's a more detailed explanation of each asset class allocation.

The long segment of the portfolio is allocated to assets where positive performance is expected.

- International Equities (Ticker: VTI, Allocation: 17.54%): This segment of the portfolio is dedicated to international equities, with VTI making up 17.54% of the overall portfolio. Investors are optimistic about the performance of international equities, which is why they have allocated a significant portion of their capital to this asset class.
- US Short-Term Maturity Bond (Ticker: IEF, Allocation: 88.91%): The largest allocation in the long positions is in US short-term maturity bonds, represented by IEF. This allocation indicates a strong conviction in the stability and income potential of short-term bonds.
- Emerging Markets (Ticker: EMGF, Allocation: 5.7%): In the case of emerging markets, the allocation is bullish at 5.7%. This indicates that investors are taking a positive stance on the performance of emerging market assets, expecting their values to rise.
- Commodities (Ticker: DBC, Allocation: 0.06%): The allocation to commodities, represented by DBC, is even more conservative at 0.06%. This suggests a neutral outlook on commodities, which reflects a cautious approach towards commodities in the portfolio.

The short segment of the portfolio is allocated to assets where weak performance is expected.

- Gold (Ticker: GLD, Allocation: -12.83%): Gold is often considered a safe-haven asset, and this allocation suggests that investors still see value in holding gold as a hedge or store of value, albeit at a slightly reduced allocation.

In summary, the portfolio is strategically balanced between long positions, where investors have confidence in the assets' growth potential, and short positions, where they are expecting weaker performance. The specific allocations provide insights into the level of conviction and sentiment regarding each asset class 3.4.

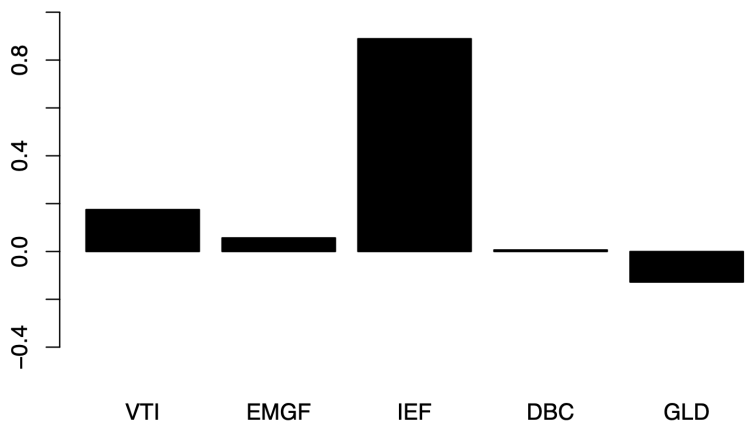

Figure 3.4: GMV portfolio weight. Each bar represent a fund weight (left to right): VTI, EMGF, IEF, DBC, GLD.

Implementing Black-Litterman Approach

In order to construct our views matrix, we can base our assumptions on the economic outlooks provided from a group of asset managers to confront outlooks and assess their perspective on markets. We will delve deeper into the outlooks from BlackRock [36], Goldman Sachs Asset Management (GSAM) [39], and J.P. Morgan Asset Management (JPMAM) [40] on gold, international equities, commodities, short-term US Treasuries, and emerging markets

International equity markets

International equity markets have underperformed US equity markets since 2019. However, they have started to outperform in 2023, as valuations have become more attractive and corporate earnings growth has picked up.

- BlackRock: BlackRock is neutral on international equities. The firm believes that international equities are undervalued, but it is cautious due to the risks associated with investing in international markets, such as currency fluctuations and political instability.
- Goldman Sachs Asset Management (GSAM): GSAM is neutral on international equities. The firm believes that international equities are undervalued, but it is cautious due to the risks associated with investing in international markets, such as currency fluctuations and political instability.
- J.P. Morgan Asset Management (JPMAM): JPMAM is overweight international equities. The firm believes that international equities are undervalued and that corporate earnings growth will pick up in the coming months.

Emerging market equity

Emerging market equity has underperformed US equity since 2019. However, it has started to outperform in 2023, as valuations have become more attractive and corporate earnings growth has picked up.

- BlackRock: BlackRock is neutral on emerging market equities. The firm believes that emerging market equities are undervalued, but it is cautious due to the risks associated with investing in emerging Currency fluctuations and political instability are examples of market risks.
- GSAM: GSAM is underweight emerging market equities. The firm believes that emerging markets are more risky than developed markets due to their higher exposure to China and other emerging economies.
- JPMAM: JPMAM is underweight emerging market equities. The firm believes that emerging markets are more risky than developed markets due to their higher exposure to China and other emerging economies.

US short-term treasuries

US short-term treasuries have underperformed most other asset classes since 2019. This is due to the fact that interest rates have been rising during this period.

- BlackRock: BlackRock is overweight short-term US Treasuries. The firm believes that the outlook for short-term US Treasuries is positive, as the Federal Reserve is expected to continue raising interest rates to combat inflation.
- GSAM: GSAM is overweight short-term US Treasuries. The firm believes that the outlook for short-term US Treasuries is positive, as the Federal Reserve is expected to continue raising interest rates to combat inflation.
- JPMAM: JPMAM is neutral on short-term US Treasury bonds. The firm believes that the outlook for short-term US Treasury bonds is neutral, as the Federal Reserve is expected to continue raising interest rates, but yields are already relatively high.

Commodities

Commodities have outperformed most other asset classes since 2019. This is due to a number of factors, including strong demand from China and the ongoing war in Ukraine.

- BlackRock: BlackRock is overweight commodities. The firm believes that commodity prices will remain supported by strong demand from China and the ongoing war in Ukraine.
- GSAM: GSAM is overweight commodities. The firm believes that commodity prices will remain supported by strong demand from China and the ongoing war in Ukraine.
- JPMAM: JPMAM is overweight commodities. The firm believes that commodity prices will remain supported by strong demand from China and the ongoing war in Ukraine.

Gold

Gold has outperformed most other asset classes since 2019. This is due to a number of factors, including concerns about global economic growth, geopolitical tensions, and inflation.

- BlackRock: BlackRock is overweight gold in its portfolios. The firm believes that gold is a good hedge against inflation and geopolitical risk.
- GSAM: GSAM is overweight gold in its portfolios. The firm believes that gold is a good hedge against inflation and geopolitical risk.
- JPMAM: JPMAM is overweight gold in its portfolios. The firm believes that gold is a good hedge against inflation and geopolitical risk.

Overall, there is a general convergence of outlooks on gold, commodities, and short-term US Treasuries. All three asset managers are overweight these asset classes. However, there is some divergence of outlooks on international equities and emerging markets. BlackRock and GSAM are neutral on international equities, while JPMAM is overweight. GSAM and JPMAM are underweight on emerging markets, while BlackRock is neutral.

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Asset class	**BlackRock**	**GSAM**	**JPMAM**
Gold	Overweight	Overweight	Overweight
International equities	Neutral	Neutral	Overweight
Commodities	Overweight	Overweight	Overweight
Short-term US Treasuries	Overweight	Overweight	Neutral
Emerging markets	Neutral	Underweight	Underweight

Table 3.1: Investment outlook from various firms. Data from BlackRock, Goldman Sachs Asset Management (GSAM), and J.P. Morgan Asset Management (JPMAM), 2023.

In the Black-Litterman model, the expected returns of individual assets are represented by the Q vector. The Q vector is defined by the investor, and it reflects their views on the future performance of each asset. In the code provided, the investor has specified strong overweight positions in international equities (VTI) and US short-term maturity bonds (IEF), a relatively overweight position in emerging markets (EMGF), and neutral and underweight positions in commodities (DBC) and gold (GLD), respectively.

Once the Q vector has been defined, the Black-Litterman model can be used to calculate the optimal portfolio weights. The optimal portfolio weights are the weights that will maximize the expected return of the portfolio, subject to the investor's risk constraints.

In summary, the Black-Litterman code you provided defines the expected returns of individual assets and uses this information to calculate the optimal portfolio weights.

In summary, the following table combines views suggesting the following outlook scenario for each asset class with an expected Q value in percentages.

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Asset Class	**Ticker**	**Position**	**Percentage**
International Equities	VTI	Strongly overweight	5
Emerging Markets	EMGF	Relatively overweight	1
US Short Term Maturity Bond	IEF	Strongly overweight	5
Commodities	DBC	Neutral	0
Gold	GLD	Relatively underweight	-2

Table 3.2: Outlook scenario for each asset class with expected Q value in percentages.

The Black-Litterman optimization approach is a widely-used method for improving the estimation of expected returns and making asset allocation decisions. It addresses the limitations of traditional mean-variance optimization by incorporating investor views and market equilibrium considerations [35].

- Mixed estimation of returns: The code starts by computing expected returns using mixed estimation. This approach allows investors to incorporate their subjective views on assets into the asset allocation process while considering the overall market dynamics.
- Tactical allocation with views directly: This part of the code calculates the tactical asset allocation weights based on views directly. These weights reflect the investor's views on specific assets and are adjusted according to their confidence in those views.
- Tactical allocation with mixed estimation: Similarly, this part calculates tactical asset allocation weights, but this time, it uses the mixed estimation approach. This allows investors to take a balanced approach, combining both market equilibrium considerations and their own views in the asset allocation process.

Black-Litterman [32] offers a flexible framework for investors to incorporate their insights and views while maintaining a connection to market equilibrium. It's a valuable tool for optimizing portfolio allocation decisions, especially when investors have specific expectations or insights about the market.

We can add the following comments on the new portfolio allocation under Black-Litterman approach, including asset views.

- International Equities (Ticker: VTI, Allocation: 18.4%): The allocation to international equities is 18.4%. This allocation represents an "overweight" position compared to the neutral stance of the BlackRock outlook, indicating a high level of confidence in the favorable performance of international equities. However, it's essential to note that there are differing views among asset managers. BlackRock and GSAM are neutral on international equities, neither optimistic nor pessimistic about their outlook, whereas JPMAM is "overweight," indicating a bullish perspective. This divergence in views reflects the varying assessments of international equity performance.
- Emerging Markets (Ticker: EMGF, Allocation: 5.9%): The allocation to emerging markets is 5.9%, indicating a "relatively overweight" position. This align poorly with the views formulated by or benchmark asset managers. BlackRock outlook takes a neutral stance on emerging markets. JPMAM shares this neutral view, while GSAM adopts a more cautious stance by being "underweight," indicating a bearish outlook. The divergence in views highlights the risk-return trade-off associated with emerging markets.

- US Short Term Maturity Bond (Ticker: IEF, Allocation: 92.4%): The allocation to US short-term maturity bonds is 92.4%, reflecting a "strongly overweight" position. This significant overweight allocation aligns with the views of both BlackRock and GSAM, which are bullish on short-term US Treasuries. In contrast, JPMAM maintains a "neutral" position, neither optimistic nor pessimistic. Short-term US Treasuries are seen as a favorable option for capital preservation and income generation, given the expected rise in yields.
- Commodities (Ticker: DBC, Allocation: 1.9%): The allocation to commodities is 1.9%, indicating a "neutral" position. This allocation is not consistent with the outlook of all three asset managers, which are "overweight" commodities. While commodity prices have exhibited volatility, they are expected to remain supported by strong demand from China and geopolitical tensions. Nevertheless, we take into account the downside potential in our assessment, reducing our exposure to commodities overall. We take into account the cyclical nature of commodity prices, which may not experience continuous growth.
- Gold (Ticker: GLD, Allocation: -18.22%): The allocation to gold is -18.22%, reflecting an "underweight" position. This is contrarian to the views of all three asset managers, who are collectively "overweight" on gold. This indicates a shared belief that gold is a sound investment at this time, particularly as it is considered a safe-haven asset sought after during periods of economic uncertainty. However, we believe, investor won't be compensated enough for holding gold in a diversified portfolio.

We can add comments on the weightings with respect to the benchmark views:

- The portfolio's strong overweight position in international equities represents a bullish stance, despite differing views among asset managers. BlackRock and GSAM maintain a neutral position, while JPMAM's overweight perspective suggests a higher degree of confidence in the outlook for international equities.
- The relatively overweight position in emerging markets aligns with the cautious approach recommended by BlackRock. Differing views from GSAM and JPMAM indicate varying risk appetites, acknowledging the challenges associated with investing in these markets.
- The significant overweight allocation to US short-term maturity bonds is a key feature of the portfolio allocation. This reflects confidence in the short-term Treasuries and is supported by BlackRock and GSAM's bullish stance, with JPMAM taking a neutral position.
- The relatively neutral allocation to commodities is consistent with the cyclical nature of these assets. The portfolio takes a more conservative stance, despite expectations of continued strength in commodity prices.
- The relatively underweight position in gold underscores the risk of the associated asset class in the short term. Our weighting shows a contrarian view on this particular asset class compared to all three asset managers.

Overall, the portfolio allocation under the Black-Litterman approach takes into account a broader spectrum of views from different asset managers and integrates them into a more specific allocation that reflects varying degrees of optimism and caution across different asset classes [32]. Incorporating views into the portfolio has resulted in diverse adjustments across asset classes, reflecting the investor's evolving outlook on each. These changes are indicative of a dynamic strategy that aims to capitalize on various market scenarios. It's crucial to consider these allocations within the context of the investor's overall investment objectives and risk tolerance, as they play a pivotal role in shaping the portfolio's risk-return profile.

Our updated asset class allocations are generally consistent with the economic views of the asset managers we have cited, with some exceptions.

- International Equities: We have increased our allocation to international equities from 17.54% to 18.4%. This remains overweight relative to the neutral stance of BlackRock and GSAM, but it is now more in line with the overweight stance of JPMAM.

- Emerging Markets: We have also increased our allocation to emerging markets from 5.7% to 5.9%. This remains a relatively overweight position, given that BlackRock and JPMAM are neutral on emerging markets, while GSAM is underweight. However, we have carefully considered the risks associated with emerging markets, such as currency fluctuations and political instability, and have determined that this level of overweight exposure is appropriate for our risk tolerance and investment goals.

- US Short-Term Treasuries: We have significantly increased our allocation to US short-term treasuries from 88.91% to 92.4%. This is now a strongly overweight position, consistent with the bullish views of BlackRock and GSAM. We believe that US short-term treasuries offer attractive yields and are a good way to preserve capital in the current uncertain economic environment.

- Commodities: We have increased our allocation to commodities from 0.06% to 1.9%. This is now a neutral position, but it is still below the overweight positions of all three asset managers. We have taken into account the cyclical nature of commodity prices and the potential for downside risk, and have determined that this level of exposure is appropriate for our overall portfolio.

- Gold: We have significantly reduced our allocation to gold from -12.83% to -18.22%. This is now a strongly underweight position, contrary to the overweight positions of all three asset managers. We believe that gold's safe-haven status is overvalued and that it is not a good investment for our portfolio at this time.

Our updated asset class allocations are more overweight international equities, emerging markets, and US short-term treasuries than the consensus view. We are also more underweight commodities and gold than the consensus view 3.5.

3.3 Limits and conclusion

3.3.1 Limits

As with any investment strategy, the All-Weather approach has potential limitations and underlying assumptions that must be acknowledged.

The sensitivity to changes in market conditions is a factor that can affect returns. The All-Weather approach implies that historical relationships between asset classes will persist in the future. However,

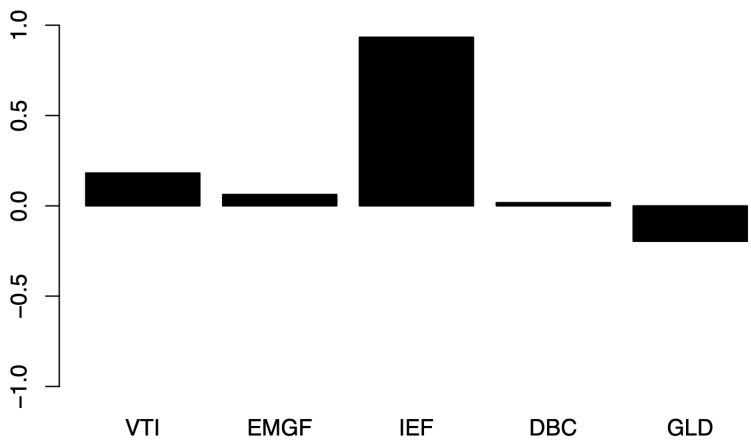

Figure 3.5: BL portfolio weight. Each bar represent a fund weight (left to right): VTI, EMGF, IEF, DBC, GLD.

market conditions may shift, resulting in alterations to correlations and performance dynamics. The relationships between asset classes can be disrupted by economic factors such as interest rate fluctuations, policy shifts, geopolitical events, and unanticipated disruptions. Investors should be aware that the All-Weather portfolio's historical performance may not be repeated under different market conditions.

The influence of transaction fees has also to be taken into consideration. Frequently, the All-Weather approach requires periodic rebalancing and reallocation of assets to maintain the desired asset allocation. There may be costs associated with these transactions, including brokerage fees, bid-ask spreads, and taxes. High transaction costs can diminish the portfolio's overall returns and impact its performance over time. It is crucial for investors to evaluate the impact of transaction costs in relation to the anticipated benefits of portfolio rebalancing.

Even though historical analysis and backtesting can provide valuable insights, it is essential to remember that past performance is not a guarantee of future results. In all market conditions, the All-Weather approach may not consistently outperform other strategies or benchmarks. Different economic cycles and market conditions may favor different investment approaches or asset allocations. Before implementing any investment strategy, including the All-Weather approach, investors should exercise extreme caution and undertake exhaustive research. Assumptions regarding the model and data can affect the overall profitability of the strategy. The All-Weather approach relies on numerous assumptions regarding asset class returns, volatilities, correlations, and risk measures. Typically, these assumptions are founded on historical information and statistical analysis. Nonetheless, data limitations, model selection, and estimation errors may affect the veracity of these assumptions. It is crucial to recognize that the effectiveness of the All-Weather approach is contingent on the quality and reliability of the underlying data and the robustness of the applied models.

Behavioral and psychological considerations can also influence the returns of the strategy. The All-Weather approach presupposes rational decision-making and strict adherence to the predetermined asset allocation. During times of market duress, investors may experience behavioral biases, emotional reactions to market volatility, or the temptation to deviate from the planned strategy. These psychological factors may have an impact on the efficacy of the All-Weather approach and its capacity to attain the intended risk-return objectives.

For investors interested in the All-Weather approach, addressing these limitations and assumptions is crucial. Understanding the potential drawbacks and undertaking ongoing monitoring and adjustments can help mitigate risks and improve the long-term efficacy of the strategy.

3.3.2 Conclusion

The All-Weather approach emphasizes diversification across asset classes with varying risk and return characteristics. Investors can implement this principle by constructing a diversified portfolio of stocks, bonds, commodities, and possibly additional asset classes. Careful consideration should be given to the selection of assets that have historically demonstrated minimal correlations, as this can enhance the portfolio's resilience during varying market conditions. The All-Weather approach is intended to perform reasonably well in a variety of market environments. Understanding that short-term fluctuations and volatility are a part of the investment voyage, investors should adopt a patient and disciplined mindset. It is essential to adhere to the selected asset allocation and avoid rash decisions based on short-term market fluctuations. When employing the All-Weather approach, it is crucial to conduct routine portfolio monitoring and rebalancing. Over time, market fluctuations can lead to asset allocation deviations. Periodic rebalancing is required to restore the original allocation percentages and preserve the portfolio's risk-return profile. Depending on individual preferences and circumstances, rebalancing may be performed annually, semiannually, or based on specific thresholds.

While the All-Weather approach provides a framework for portfolio management, it is essential to consider individual risk tolerance, financial objectives, and investment horizon. Investors may need to tailor their asset allocation to their circumstances. For instance, investors with a lower risk tolerance and a longer investment horizon may prefer a higher allocation to fixed income, while those with a higher risk tolerance and a longer investment horizon may prefer a higher allocation to equities. As market conditions evolve, it is essential to assess the All-Weather approach's continued suitability. The performance of asset classes may be impacted by economic trends, geopolitical events, and changes in monetary policies, so investors should remain informed of these factors. Regular evaluation of the All-Weather strategy's fundamental assumptions and performance drivers will ensure its continued relevance and efficacy.

Investors should carefully evaluate the impact of transaction costs, such as brokerage fees and bid-ask spreads, on the application of the All-Weather approach. In addition, tax implications, such as capital gains taxes, should be considered when rebalancing the portfolio. The advice of a financial advisor or tax expert can be invaluable for minimizing costs and maximizing tax efficiency. For investors to successfully implement the All-Weather approach, they must understand its underlying principles and rationale. Educating oneself on asset classes, risk management, and investment concepts can increase one's self-assurance and enhance decision-making. In addition, being cognizant of behavioral biases and maintaining discipline during periods of market volatility will enable investors to adhere to the chosen strategy and avoid emotionally motivated actions.

By considering these practical implications and implementing the All-Weather approach with care, investors can create robust, well-diversified portfolios designed to withstand a variety of market conditions and achieve their long-term investment goals. The key to the successful application of this approach is based on a regular monitoring, customization based on individual requirements, and a commitment to sound investment principles.

Chapter 4

Analysis of Option-Based Strategies

4.1 CPPI and OBPI strategies

Background

Capital preservation strategies have been developed to respond to the requirements of investors seeking portfolio protection methods that guarantee the value of their portfolio will not decline below a certain level at a given future date. Leland (1980) demonstrated that portfolio insurance maximises the expected utility of two distinct groups of investors [42]. The first ones can be denominated as safety-first investors, who are those who possess average market return expectations but have a higher degree of risk aversion that escalates more rapidly with wealth compared to the ordinary investor. The second group are investors with moderate risk aversion but above-average market return expectations who want well-diversified investment managers that anticipate generating positive alpha on average.

These investors have a hyperbolic absolute risk aversion (HARA) utility function, indicating a preference for spending over savings and a tendency to avoid risk. There are two categories of portfolio insurance strategies [42]. Static strategies involve a buy-and-hold approach, meaning that once an investment is made, it is held for a long period of time without making frequent changes.Dynamic methods, on the other hand, need regular adjustments to the portfolio based on market conditions and other factors 4.15b.

Portfolio insurance investment methods do not necessitate any form of forecasting skill. The utility function of the investor, which can be characterised by a number of factors like the bare minimum required wealth level and time horizon, is the only factor that influences portfolio insurance investing processes [42].

The significance of implementing a portfolio insurance investment procedure arises from the investor's having a risk preference that exhibits asymmetry around the mean, as depicted in 4.15c. Furthermore, the investor must indicate the maturity date, which is the desired date for asset recovery, the floor or protection level, which is the minimum value the portfolio should have at maturity, the performance asset, which is the chosen market in which the investor intends to be involved, and the return expectation or risk aversion, which refers to the anticipated level of commitment in the performance asset's fluctuations.

4.1.1 Literature Review

As mentioned in the introduction, the utility of implementing such portfolio protection strategies arises from investor's heterogeneous risk preferences. The most basic form of a Portfolio Insurance Investment Plan (PIIP) is known as the stop-loss strategy. This approach involves an initial investment entirely in a performance asset, with a switch to a risk-free asset if the portfolio's value falls below a predetermined threshold before the maturity date T. Mathematically, if the portfolio value at time t is less than the discounted floor value $V(t) = F(T) \cdot e^{-r(T-t)}$, then the assets are transferred into a risk-free investment. Proper execution of this strategy promises a specific payoff, as depicted in a designated figure [42].

However, this strategy is not without its downsides. The main drawback is the potential discontinuity in the performance asset's prices, which can impede the ability to sell at the floor price $F(T) \cdot e^{-r(T-t)}$, thus risking the guaranteed floor value at maturity. Moreover, if the performance asset's price falls below the floor's barrier, it ceases to contribute to the portfolio's returns. Brennan and Solanki (1981) have demonstrated that such a strategy is generally in conflict with the principle of maximizing expected utility [42].

An alternative elementary PIIP involves purchasing a zero-coupon bond for $F(T) \cdot e^{-rT}$, representing a risk-free investment, and allocating the remainder to the performance asset. This tactic ensures the floor value $F(T)$ at maturity, barring default by the bond issuer, while allowing limited participation in the returns of the performance asset. The portfolio's value at maturity is then given by the equation:

$$V(T) = F(T) + \frac{S(T)}{S(0)} \cdot (1 - e^{-rT}) \tag{4.1}$$

Regrettably, such a strategy is generally less than optimal concerning the maximization of expected utility [42].

Constant Proportion Portfolio Insurance (CPPI) strategy

The CPPI, credited to Perold and Sharpe (1988) for fixed-income instruments and Black and Jones (1987) for equity instruments, employs a straightforward strategy for dynamic asset allocation over time [42]. The investor initiates the process by establishing a floor, representing the lowest acceptable value for the portfolio. Subsequently, the cushion is calculated as the surplus of the portfolio value over the floor, and the allocation to the risky asset is determined by multiplying the cushion by a predetermined multiple [47]. The investor's risk tolerance has an impact on both the floor and the multiple, which are independent of the model. The combined amount allocated to the risky asset is referred to as the exposure, with the remaining funds directed towards the reserve asset, typically T-bills [42].

In the context of the portfolio at any point in time, the term 'cushion' $C(t)$ is described as follows [47]:

$$C(t) = \max\{V(t) - F(T) \cdot e^{-r \cdot (T-t)}, 0\} \tag{4.2}$$

The 'cushion' signifies that portion of the portfolio which can be reduced without negatively impacting the guaranteed capital at the time of maturity. Within the CPPI strategy, the multiplier m dictates the investment in the performance asset as $m \cdot C(t)$, with the remainder $V(t) - m \cdot C(t)$ allocated to the risk-free asset 4.15a. The decision to adjust the investment in the risk-free asset is based on the performance asset's return rate, which either escalates or diminishes the amount invested in it [47].

Option Based Portfolio Insurance (OBPI) strategy

The option based portfolio insurance (OBPI) investment process was introduced in 1976 by Leland and Rubinstein (1988)[45] [42]. It is based on the seminal work of Black and Scholes (1973) [48] on option pricing. The investor buys a zero coupon bond with a face value of the floor F (T) at maturity. The remaining proceeds are then used to buy at the money European call options on the performance asset.

The portfolio's value at any given time t can be represented as the sum of the present value of the floor and the value derived from a number of call options. The equation is given by[45][42]:

$$V(t) = F(T) \cdot e^{-r(T-t)} + n \cdot C(t, F(T), S(t), T, \sigma_S(t), r) \tag{4.3}$$

Here, $C(t, K, S(t), T, \sigma_S(t), r)$ denotes the price of a European call option on the underlying performance asset at time t, with a maturity T, strike price K, current price $S(t)$, implied volatility $\sigma_S(t)$, and the risk-free interest rate r. The parameter n is defined as the number of call options that can be acquired with the funds that are not invested in the zero-coupon bond, formulated as:

$$n = \frac{V(0) - F(T) \cdot e^{-rT}}{C(0, F(T), S(0), T, \sigma_S(0), r)} \tag{4.4}$$

This representation determines the quantity of call options that can be purchased with the amount leftover after buying the zero-coupon bond [45].

An analogous protection strategy may be created by retaining full ownership of the performance asset and acquiring a put option with a strike price equal to F(T) on the same asset. The put-call parity theorem establishes the equivalence between the bond plus call and equity plus put. The investing processes of OBPI have two primary benefits. Static structures necessitate trading only during the establishment of the strategy and are unaffected by fluctuations in the value of the performance asset [45][42].

However, the investor in an OBPI investing process faces certain drawbacks. The effectiveness of this investing strategy relies on the quantity, denoted as n, of call options that may be purchased using the remaining funds not allocated to the zero coupon bond [45][42]. The price of an option is determined by the volatility of the underlying asset's performance and the risk-free rate. Consequently, the potential performance participation rate is contingent upon the values of these two parameters at the time of setup. Excessive fluctuations and/or low interest rates have a negative impact on the anticipated rate of participation in the performance of asset returns [45][42]. Furthermore, since the volatility cannot be directly observed, the liquidity of the option market has a crucial significance. Greater market liquidity increases the likelihood that the volatility value used to price the option is closer to its fair value. For over-the-counter options, the investor is exposed to counterparty risk, which can become substantial as the option becomes more valuable. Acquiring options for the performance asset at the desired strike price and/or maturity might not be feasible. In this scenario, the investor has the ability to replicate the option's payoff in a dynamic manner [45][42]. Within the framework of comprehensive markets, this can be achieved by the implementation of a self-financing and duplicating strategy. Furthermore, it is impossible for a zero coupon bond to be available for the given maturity period. Instead, it is advisable to utilise shorter-term zero bonds or coupon bonds and mitigate reinvestment risk by employing swaptions [45][42].

It is crucial to acknowledge that OBPI techniques ensure the minimum value only when they reach maturity. The portfolio's value may decrease below the predetermined minimum value prior to maturity as a result of increasing interest rates, declining asset values, and/or reduced volatility of the performance asset [45][42].

4.1.2 Objective

A preliminary statistical analysis of the risky asset (the S&P 500 index, which represents the market portfolio as the performance asset) and the risk-free rate (captured by the US 10-year Treasury bill) is conducted in this report to examine the performance of the CPPI strategy. The second goal is to examine alternative portfolio insurance strategies, in this case the OBPI strategy. We analyse the performance of both portfolio insurance strategies, evaluate their sensitivity to various model parameters, and analyse their return distribution using a Monte Carlo simulation.

4.2 Methodology

The methodology section is organised into two main subsections: the first part explains the theoretical underpinnings of Levy Process jump modelling and Black-Scholes jump modelling, providing a strong foundation for the RStudio implementation, and the second part relates to the RStudio implementation and explains the main structure of the code to enhance comprehension.

4.2.1 Model Descriptions

Black-Scholes Model

Within the field of financial mathematics, numerous significant models have been developed to tackle the complex aspects of option pricing. One of the most influential models in finance is the Black & Scholes model, developed by Fischer Black, Myron Scholes, and Robert Merton. That model might be considered a significant milestone in the field, as it introduced an analytical approach for calculating the values of European-style options [48]. The fundamental equation of this model is the well-established Black-Scholes partial differential equation (PDE). The price of a European option can be determined by solving this equation under specific conditions [48]. The model is based on several underlying assumptions, including the presence of a constant and known risk-free rate, a constant level of volatility, the assumption that stock prices follow a geometric Brownian motion, the absence of any dividend payments throughout the lifespan of the option, the absence of any arbitrage opportunities, and the limitation that European-style options can only be exercised upon expiration. The uniqueness of the model lies in its ability to provide an analytical solution, enabling efficient calculations and establishing it as a fundamental tool in the field of financial engineering. Nevertheless, the subject under discussion has its share of critics. The primary focus of criticisms revolves around the assumptions made by the theory, specifically the assumption of consistent volatility. Empirical evidence suggests that volatility in real-world scenarios frequently exhibits stochastic behavior, perhaps manifesting in a "smile" or "skew" pattern [48].

Assumptions and characteristics

The model's derivation results in the well-known Black-Scholes Partial Differential Equation (PDE), which, under specific conditions, can be solved to ascertain the theoretical price of a European option.

An essential foundation of the Black-Scholes model is the assumption that the risk-free interest rate remains constant and is generally known [48].The interest rate serves as a means to adjust the future payoffs of the options to their current values, taking into account the concept of the time value of money. Another crucial assumption of the model is that the volatility of the returns of the underlying asset remains constant across time. Volatility, which quantifies the level of risk associated with the asset, is incorporated into the model as a key factor in determining the value of the option. By assuming that volatility remains constant, the calculations are simplified and a solution that can be expressed in a mathematical formula is obtained [48].The Black-Scholes model considers that the underlying stock prices adhere to a geometric Brownian motion, indicating that the logarithm of stock price returns follows a normal distribution and the stock prices themselves follow a log-normal distribution. The stochastic process determines the unpredictable trajectory that stock prices are believed to follow, which is characterized by constant and seamless price fluctuations.

Another simplification involves the omission of dividends. The Black-Scholes model postulates that the underlying asset remains dividend-free during the duration of the option [48]. This assumption is essential as dividends can impact the price of the underlying asset, hence influencing the pricing of the option. The model is based on the notion of the absence of arbitrage opportunities, indicating that it is impossible to make a riskless profit. The fundamental principle of arbitrage-free markets guarantees that the option pricing model is equitable and that the values of the underlying asset and the option are in alignment with each other [51].

The Black-Scholes model is exclusively tailored for European-style options, which can only be exercised on the option's expiration date. In contrast, American-style choices allow for exercising at any point up until the expiration date.

For a call option, the payoff is expressed as follows:

$$(S_T - K)^+ = \max(S_T - K; 0) \tag{4.5}$$

where K represents the strike price of the option and S_T represents the price of the underlying at time t.

The formula to price a European call under BS is given below:

$$C(S_0, K, r, T, \sigma) = S_0 N(d_1) - K e^{-rT} N(d_2) \tag{4.6}$$

where $N(d_1)$ and $N(d_2)$ are cumulative distribution functions of a standard normal distribution. The d_1 and d_2 functions in the Black-Scholes model are computed as:

$$d_1 = \frac{1}{\sigma\sqrt{T}} \left[\ln\left(\frac{S_0}{K}\right) + \left(r + \frac{1}{2}\sigma^2\right) T \right] \tag{4.7}$$

$$d_2 = d_1 - \sigma\sqrt{T} \tag{4.8}$$

The Gaussian distribution function is given by:

$$N(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}u^2}\, du \tag{4.9}$$

The payoff for a put option is given by:

$$(K - S_T)^+ = \max(K - S_T, 0) \tag{4.10}$$

where K is the strike price and S_T is the asset price at maturity.

For a put option, the Black-Scholes model can be rewritten as:

$$P(S_0, K, r, T, \sigma) = K e^{-rT} N(d_2) - S_0 N(d_1) \tag{4.11}$$

where T represents time to maturity, S_0 represents the underlying asset price at $t = 0$, K represents the strike price, σ represents the volatility of the option, and r represents the risk-free rate.

The model's beauty lies in its analytical solution, allowing for quick computations. It has become the foundational model in financial engineering. However, its assumptions, especially constant volatility, have been criticized. Market observations show that volatility can be stochastic and is often "smiled" or "skewed" [51].

In summary, the Black-Scholes model relies on a simplified set of assumptions to provide a theoretical framework for option pricing. While the model has been heavily used and is widely recognized as a reference in the financial industry, its assumptions have been subject to scrutiny, leading to the development of alternative models that attempt to relax these conditions and accommodate a wider range of market phenomena.

Jump Diffusion Model (Lévy Process)

We will elaborate on the Merton model as a foundation for jump diffusion modelling. The Merton represent significant improvement in financial modeling, expanding upon the classic Black-Scholes framework by incorporating a Poisson jump-diffusion process [44]. This integration marks a pivotal development in the understanding of asset pricing.

Developed by Robert C. Merton, this model skillfully combines the continuous path of geometric Brownian motion, inherent in the Black-Scholes model, with the stochastic and discrete nature of jump processes [53]. This combination allows the Merton model to reflect a more comprehensive picture of market behavior, acknowledging both the gradual, day-to-day price variations and the less frequent, yet impactful, abrupt price shifts that occur in financial markets. Its core assumptions posit that stock prices follow a blend of geometric Brownian motion and jump processes, and that jumps are log-normally distributed and occur at a Poisson rate. One of its main strengths is its capacity to account for abrupt, significant price fluctuations, or "jumps". However, it introduces a layer of complexity, and deriving analytical solutions becomes more intricate. Aligning it with market data can also present challenges [53].

Assumptions and characteristics

Jump diffusion models are a cornerstone in financial mathematics, offering a more realistic depiction of asset price dynamics compared to the standard Black-Scholes model. These models incorporate random jumps in asset prices, addressing the limitation of continuous paths assumed in classical models. A significant advancement in this area is the introduction of Lévy processes, particularly in the Merton jump-diffusion model.

Lévy processes extend the classical Brownian motion by including a jump component, capturing the empirical observation of sudden and significant changes in asset prices. The general form of a Lévy process $(X_t)_{t\geq 0}$ can be described as:

$$X_t = \mu t + \sigma W_t + J_t \tag{4.12}$$

where μ is the drift coefficient, σ is the volatility, W_t is a standard Brownian motion, and J_t represents the jump component, often modeled as a compound Poisson process.

The Merton jump-diffusion model, introduced by [53], is a pioneering work integrating jumps into asset pricing. In Merton's framework, the asset price S_t is given by the exponential of a Lévy process:

$$S_t = S_0 \exp\left(\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma W_t + \sum_{i=1}^{N_t}(Y_i - 1)\right) \tag{4.13}$$

where S_0 is the initial asset price, N_t is a Poisson process representing the number of jumps up to time t, and Y_i are i.i.d. random variables representing the relative jump sizes.

Lévy processes and jump diffusion models, particularly the Merton model, provide a powerful framework for capturing the complex behavior of asset prices. They address the limitations of continuous models and are essential for accurately pricing financial derivatives and assessing risk in financial markets.

A distinctive feature of the Merton model is its assumption that jumps are log-normally distributed and occur at a rate determined by a Poisson process. This assumption is crucial, as the log-normal distribution ensures that stock prices remain positive, while the Poisson process provides a probabilistic framework for the occurrence and frequency of these jumps. This framework is particularly adept at capturing the dynamics of market reactions to significant events or unexpected news that can dramatically influence the value of underlying assets [53].

The addition of a jump component introduces a layer of complexity to the model. While this enriches the model's capability to mirror real-world market phenomena, it also makes analytical solutions more intricate and challenging to derive [53]. The calibration of the Merton model to actual market data is a sophisticated task, demanding advanced numerical methods and a deep understanding of market dynamics and investor behavior. Despite these complexities, the Merton model has proven to be a valuable tool in financial markets, particularly in volatile environments where significant price jumps are more prevalent. Its ability to model these jumps provides a powerful mechanism for pricing complex financial derivatives and managing risks associated with abrupt market movements [53].

In summary, the Merton model's introduction of jump-diffusion processes to option pricing theory represents a significant leap forward from the Black-Scholes model, offering a more nuanced and comprehensive approach to understanding asset price dynamics. Its analytical complexity, while posing certain challenges, is a testament to the model's depth and its capacity to capture the multifaceted nature of financial markets. As a result, the Merton model remains a fundamental element in the toolkit of financial engineers and analysts, especially valuable in scenarios characterized by significant and rapid market changes.

4.2.2 Implementation in R

The implementation of portfolio insurance strategies, namely CPPI (Constant Proportion Portfolio Insurance) and OBPI (Option-Based Portfolio Insurance), involves the use of financial derivatives and dynamic portfolio allocation to provide a guaranteed level of protection to the investment portfolio.

First, the 'quantmod' package is used to get historical market data for a risky asset (the S&P 500 Index) and a risk-free rate (the U.S. 10-Year Treasury yield). The data is then aligned by date to make sure that the frequency is consistent, and it is processed to get daily returns. Market jumps are defined as changes in the index's returns that are greater than a certain threshold and are set to three times the standard deviation of the returns. These jumps can significantly impact the returns and risk profile of the investment strategy. A GARCH (Generalized Autoregressive Conditional Heteroskedasticity) model is fitted to the S&P 500 returns to estimate and forecast future volatility, an essential component in the risk management of investment strategies.

The CPPI strategy moves money around between a risky asset and a risk-free asset by setting a "cushion" as the difference between the asset value and a set floor. The risky asset exposure is a multiple of this cushion, and the multiplier shows how aggressive the protection strategy is. The OBPI strategy is a type of portfolio insurance that uses a European call option to mimic the payoffs of a put option, offering downside protection.

We use a Monte Carlo simulation to look at what might happen with the CPPI and OBPI strategies over time. The simulation creates a number of possible future equity paths by looking at the returns on the underlying assets and the way option prices change over time. It helps us come up with a series of random returns based on the risk-free rate, the risky asset's expected return, and its volatility. We then use these returns to simulate the equity price paths, pricing call options along the way.

The simulation and sensitivity analysis results are then visualised using histograms and line plots to show the distribution of final equity values. The sensitivity analysis involves varying key variables such as the risk-free rate, stress scenarios, volatility, expected returns, and strike prices to better understand how changes in market conditions affect the performance of portfolio insurance strategies.

4.3 Analysis and Results

4.3.1 CPPI under Black-Scholes vs. Jump Diffusion Model

The empirical analysis of the data performed over a ten-year dataframe from 2013-01-01 to 2023-01-01 tries to investigate on the performance of these two asset classes, reviewing historical trends and finding factors that influence their distinct risk and return characteristics.

The 10-year T-bill yield, which shows the interest rate paid on T-bills, fluctuated significantly over the time under consideration. In 2013, the yield averaged 2.15%, a significant decrease from its peak of 3.87% in 2011. This drop has been attributed to the Federal Reserve's (Fed) reduction of its quantitative easing (QE) programme and a better economic outlook. From 2014 to 2016, the yield remained reasonably constant between 1.50% and 2.00% as the Federal Reserve maintained its accommodating monetary policy. The Fed began to normalise its monetary policy in 2017, which resulted in a gradual rise in the 10-year Treasury bill rate. The yield peaked at 3.22% in 2019. However, the COVID-19 pandemic and the Fed's expansionary monetary policy response resulted in a dramatic drop in the yield in 2020, reaching a low of 0.52% in August. Since 2020, the yield has risen gradually, averaging 1.93% in 2021 and 2.97% in 2023, demonstrating the Fed's continuous attempts to control inflation.

In contrast, the performance of the S&P 500 index has been more erratic. In 2013, the index increased by 32.3%, owing to improved economic conditions and the Fed's supportive monetary policies. However, the index encountered more difficult times in 2014–2016, with negative effects from concerns about the global economic slowdown and the Fed's prospective interest rate hikes. The S&P 500 index began to recover in 2017, and it has continued to rise in consecutive years, reaching an all-time high of 3,386.16 in July 2019. However, the COVID-19 pandemic in 2020 resulted in a substantial reduction, with the index falling 33.9% in the first quarter. Despite this loss, the S&P 500 index showed resilience, rising rapidly in the second half of the year and finishing 2020 with a 16.3% gain. The index's increasing trend continued in 2021 and 2023, with an all-time high of 4,818.62 in January 2023 4.16a.

The performance of the 10-year US Treasury bill and the S&P 500 index from 2013 to 2023 is mixed. According to our empirical analysis of the risky asset and the risk-free asset used to construct the different portfolio insurance strategies, the US 10-year Treasury bill yield averaged across the period analysed had a level close to 2.15% and a volatility in terms of yield level of 0.07%. The S&P 500 index stayed on average at a level close to 2742.69 and returned on average 11.22% with a volatility of 17.83%. During the same time frame, there was a -0.023 correlation coefficient between the performance asset and the risk-free asset. This shows that having both assets in an investment portfolio could help diversify it.

In order to properly capture the volatility clustering and general behaviour of the performance asset, we additionally construct a volatility modelling to comprehend the behaviour of the asset in terms of jumps. This is done by applying a GARCH model. The S&P 500 index returns shown against the fitted volatility from a GARCH model offers a detailed look at the dynamics of the market across time. As captured in the plot 4.16b, we can see that there is an important volatility clustering around 2020, when the COVID pandemic hit the global financial market with its large scale and uncertain impacts. Significant changes emphasise periods of market stress or euphoria, and it depict actual return fluctuations. The red line, which shows the model's estimated conditional volatility, shows how volatility tends to cluster over time. This is a feature that makes it stand out: times of major market instability are often followed by times that are just as unstable. Additionally, this volatility clustering indicates when greater caution and plan adjustments may be required due to elevated risk. The volatility estimate reacts to spikes in real returns, demonstrating the model's reactivity to market events and providing insights on the market's reflexive behaviour in the face of new information.

In addition to being descriptive, the GARCH model is predictive in nature, as it uses the most recent volatility estimate as a foundation for estimating short-term market risk. Forecasts of this kind play a critical role in the execution of investment and hedging strategies, such as CPPI, where portfolio insurance mechanisms depend on an understanding of volatility dynamics. The model's ability to accurately reflect market behaviour during periods of volatility is particularly noteworthy, as it is essential for determining risk indicators such as Value at Risk (VaR) and Expected Shortfall that guide risk management and regulatory compliance. Taking into account these factors, one must also take into account the possibility of model constraints and the risk of misspecification, given the volatility and return distribution assumptions used in the model.The GARCH model is therefore an effective tool for analysing financial time series data, but its use needs to be balanced with an understanding of its limitations and the overall financial climate.

To construct the insurance portfolios, we rely on the results obtained from the preliminary statistical analysis to construct the models based on the following parameters:

- 'S0' as the index price at start is set as the average index level over the 10 year of data covered 2742.19.
- 'T' as the investment's time horizon, which covers the period analysed, equivalent to 10 years worth of data.
- 'r' as the risk-free interest rate, which take into account the average yield over the period analysed equivalent to 2.15%.
- 'Sigma' as the standard deviation or volatility, of the index's returns equivalent to the average volatility over the 10 years of data analysed (17.58%).
- 'lambda' as the jump process's intensity in Lévy process, equivalent in our empirical analysis to 3.81.
- 'k' as the logarithmic leap size on average (-0.011).
- 'delta' as the leap size's standard deviation (0.0539).
- 'multiplier' as the portfolio's aggressive response to variations in the cushion, or the gap between asset value and floor (4).
- 'floor' of the CPPI strategy's protection level is typically expressed as a percentage of the initial investment (80% of the initial capital).
- 'steps' which represent the simulation's total number of time steps, which typically correlates to the number of trading days in a year (252 trading days as we deal with daily data).

After simulating the CPPI strategy using two different modelling methods (Black-Scholes and Jump diffusion under Lévy process), we can highlight the following. The simulated portfolio values over time under the Black-Scholes model and the Lévy process are shown in Figure. Both models use a Constant Proportion Portfolio Insurance (CPPI) method to simulate the evolution of portfolio value 4.16c.

The Black-Scholes model's (blue line) volatility and fluctuations display a trajectory with notable fluctuations, indicating that continuous trading and a constant volatility parameter are underlying assumptions of the model. A smooth diffusion process is produced by these assumptions, which might not always be able to capture sharp market fluctuations. There seems to be more of a jagged trajectory with sharper climbs and falls in the Lévy model (red line). This behaviour is typical of Lévy processes, which are intended to include jumps to account for the possibility of abrupt, notable changes in the market that may be brought on by unforeseen events or news.

In terms of model differences, the Lévy process shows times of abrupt increases or declines in portfolio value in contrast to the Black-Scholes model. These spikes may result from the model's capacity to account for both minor and major moves, which aren't always predicted by historical volatility or price trends. In contrast, these jumps are not taken into consideration by the Black-Scholes model. Its reliance on a lognormal price distribution undervalues the likelihood of significant price fluctuations, which may lead to a smoother curve but also understate risk.

Keeping a dynamic allocation between a cash position and a hazardous asset is part of the CPPI approach. The exposure to the risky asset is determined by the "cushion," which is the difference between the asset value and a predefined floor. The CPPI strategy's multiplier increases the portfolio's reaction to profits and losses. The amplified patterns in both models demonstrate this, with gains having the potential to quickly boost portfolio value while losses have the potential to do the opposite.

In terms of risk consideration, the Lévy model's greater losses in portfolio value imply that this model might be more suited for stress-testing the portfolio against extreme market occurrences. Under typical market circumstances, the more optimistic Black-Scholes model might offer a forecast, but it might not be as reliable during market turbulence.

The significance of model selection in risk management is highlighted by the variations in the portfolio value trajectories. Capital protection methods can be more effectively informed by a model (such as the Lévy process) that accounts for the possibility of significant market movements. The accurate representation of real market behaviour by the underlying model is another factor that will determine how successful the CPPI strategy is. In the event that certain risks are not taken into account by the model, the CPPI strategy might not offer the anticipated degree of protection.

4.3.2 Comparison of CPPI and OBPI under Black-Scholes Model

After structuring the OBPI and CPPI strategies and estimate the different simulations to compare the different strategies, we implement a sensitivity analysis to understand how the models behave under a change of their key inputs. We look at the strike price (increase of two hundred points in terms of price level), the risk free rate change in yield (by a hundred basis point), the return of the performance asset (two hundred basis points), stress value (a hundred basis point increase).

We assess the strike price and its variation in portfolio value as the strike price changes. The strike price is a key component in options-based strategies such as CPPI and OBPI. The plot suggests that the equity path (blue line) is volatile and reflects the underlying asset's price movements. In contrast, the CPPI (red line) and OBPI (purple line) strategies are relatively stable, indicating that they provide a cushion against the volatility of the equity. The 'floor' (green line) represents the minimum guaranteed value that the CPPI aims to protect. It is constant across time, showing that the insurance aspect of the strategy is not affected by the strike price variation. This plot suggest that the strategies are effective in providing downside protection without significantly sacrificing the upside potential, as seen by the convergence of the CPPI and OBPI values with the equity line towards the end of the period 4.17d.

When assesing the sensitivity of the return parameter, the plot underline how the strategies perform when the expected return (mu) of the underlying asset varies by 200 price level increase. Similar to the strike price variation plot, the equity path shows high volatility. The CPPI and OBPI strategies again show stability, indicating that the strategies' protective mechanisms are robust against changes in expected returns. When the underlying asset's returns are higher, both CPPI and OBPI strategies appear to capture some of the upside, as indicated by the increase in their values towards the end of the period 4.17c.

The stress value in a CPPI strategy is indicative of the level of risk aversion and dictates how much exposure to the risky asset is taken. Here, the plot suggests that as the stress value varies, there is little impact on the portfolio values of CPPI and OBPI strategies, which is indicative of the strategies' resilience. Again, the floor value remains constant, and the equity line shows significant volatility compared to the more stable CPPI and OBPI lines 4.17b.

We also take a look on the impact of changing the risk-free rate on the portfolio values of the strategies. Typically, a higher risk-free rate would lead to a higher value of the risk-free asset component in the portfolio. However, the plot indicates that the variation in the risk-free rate does not significantly affect the CPPI and OBPI values, which could suggest that the asset mix is not overly sensitive to changes in the risk-free rate 4.17a.

Overall, these plots suggest that both CPPI and OBPI strategies are relatively stable across different market conditions and parameters. They offer downside protection while allowing participation in the upside, which is characteristic of portfolio insurance strategies. The equity curve's volatility emphasizes the need for such protective strategies in portfolio management. The constant floor value across all scenarios underlines the insurance feature of these strategies, ensuring that the portfolio value does not fall below a certain level.

The three histograms below represent the distribution of final values from a Monte Carlo simulation for an equity simulation and two investment strategies, CPPI (Constant Proportion Portfolio Insurance) and OBPI (Option-Based Portfolio Insurance). Here is an analysis of each.

This histogram displays the frequency distribution of the final values of a simulated equity investment over a given period. The distribution is bell-shaped and symmetric, suggesting a normal distribution of final equity prices. The central peak is around the 4000 price mark. There's a noticeable spread on both sides of the peak, which shows variability in the equity simulation payoffs 4.18c.

The OBPI histogram also presents a right-skewed distribution, similar to the CPPI strategy, but the peak is closer to the 3850 price mark and the distribution is more concentrated. This indicates that the OBPI strategy outcomes are more tightly clustered around the peak, with fewer instances of both higher and lower values compared to the CPPI strategy. The strategy seems to offer a balance between protecting against downside risk and capturing some upside potential 4.18a.

The CPPI histogram shows a right-skewed distribution, indicating that most of the final values are concentrated on the left side but there is a long tail towards higher values. The peak of the distribution is just above the 3850 price mark, and the spread is narrower than the equity histogram, which suggest less variability in the CPPI strategy payoff compared to the equity simulation. The right skewness might also reflect a floor mechanism characteristic of CPPI strategies that limit downside risk but allow for participation in upside potential 4.18b.

4.4 Discussion and conclusion

The empirical analysis, which covers the ten-year period from 2013 to 2023, provides an in-depth study of the dynamics of performance between the risky asset captured by the S&P 500 index and the risk-free asset captured by the 10-year Treasury bill yield. Inflation in the 10-year T-bill yield was caused by the Fed's tapering of QE and subsequent normalisation of monetary policy, two key economic events and policy changes that occurred during this time. A historic low was reached during the COVID-19 pandemic, and it peaked in 2019 at 3.22%. Contrarily, the S&P 500 shown tenacity and expansion, rising to unprecedented levels by 2023 despite the pandemic-caused decline.

From a risk-return standpoint, the S&P 500 yielded an average return of 11.22% and had volatility of 17.83%, while the 10-year T-bill had an average yield of 2.15% and volatility of 0.07%. The S&P 500 remained at an average level of roughly 2742.69. Because of their combined portfolio allocation's intrinsic benefit of diversity, the two assets have a negative correlation coefficient of -0.023.

A GARCH model was employed to represent the complex market behaviour of the performance asset (S&P 500 index), particularly the volatility clustering. The market's sharp reaction to unexpected shocks was evident in the considerable volatility that was centred around crucial occurrences such as the 2020 pandemic. The model's predicted conditional volatility (red line) depicted times of increased market volatility, highlighting the importance of careful risk management strategies like CPPI that depend on an awareness of volatility trends. Although the GARCH model has usefulness in both descriptive and predictive domains, it is crucial to acknowledge its constraints and make sure its implementation is moderated by the current state of the market.

A reasonable parameter set produced from the empirical data was employed in the development of the CPPI and OBPI strategies, which were founded on statistical analysis. Using the Black-Scholes and Lévy models to simulate these strategies, different portfolio value payoffs were found. While the Lévy model accommodated market jumps and showed sharp spikes that indicated its ability to capture sudden market swings, the Black-Scholes model showed a more consistent growth pattern. These tendencies emphasise how crucial model selection is to risk management; more resilient capital protection strategies can be derived from models, such as the Lévy process, that can explain large fluctuations in the market.

The impact of adjustments in the strike price, risk-free rate, stress value, and expected return on the CPPI and OBPI strategies was examined in the sensitivity analysis. The analysis verified that both strategies successfully reduce the risk of negative returns while maintaining the potential of positive results. The strategies appeared to be less vulnerable to fluctuations in interest rates, as evidenced by the fact that changes in the risk-free rate had no significant effect on them.

There are two types of risks that an investment manager who implements a CPPI or dynamically replicates an OBPI investment process must consider [42]. These are known as external risks, which are due to potential external occurrences such as market crashes or liquidity constraints that disturb the usual investment process, and internal risks, which are due to faults in the investment process such as valuation or model errors. All risks eventually culminate in the so-called gap risk. The gap risk is the possibility that the portfolio value will fall below the floor when it reaches maturity.

On the external risk side, the most common risks translating into gap risk are liquidity risk, which is the risk that the investment manager cannot sell the performance asset at the current market price in sufficient volume; discrete price risk, which is the risk that the investment manager is not able to sell at any given moment in time and at any given price; and extreme event risk, which is the risk that the performance asset's price drops to a value such that the portfolio falls below the floor when it reaches maturity.

It is possible to manage external risks by changing model parameters, such as multiple values, so that there is little chance that an external risk will have a significant negative impact on the value of the portfolio [42]. External risks are challenging to manage because conventional market theory does not cover them. Extreme event theories, together with Monte Carlo simulations, are typically necessary to obtain a realistic estimate of external risks. The investment manager can cover external risks by allocating some of his or her own funds to them, selling them to an insurance company, an investment bank, or bundling and selling them to the market[42].

Overall CPPI and OBPI strategies proved stable and capable of mitigating risk under a variety of market conditions and alteration of model parameters. The consistent floor value and the strategy adaptability to both expected and unforeseen market movements demonstrate how significant this resilience is in a portfolio construction. These results highlight the practical importance of incorporating advanced portfolio insurance and risk management strategies into financial planning and investment strategies.

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4.5 Equity derivatives strategies: Implementation on Société Générale (SG) stock

This case study aims to implement an equity option strategy and analyse its payoff. The report is structured as follows:

- **Introduction and analysis of Société Générale stock**: We analyse the stock of Société Générale during April 2018 using the data provided for the assignment derived from the research conducted at Deutsche Bank (DB) equity research and equity derivatives strategy group in order to structure our trade idea [49].
- **Black-Scholes-Merton framework**: We elaborate the underlying framework related to the Black-Scholes-Merton framework, looking at the underlying mathematical foundation and greek sensitivities in order understand the different parameters that affect the option value.
- **Implementation of an equity derivatives strategy**: We put in place two strategies to test against market data. The first strategy is based on a protective put. The second strategy is a short strangle. We wanted to reflect on our market view of the stock positioning our assumptions at the day of the trade and using the DB equity research data to structure the trade idea which could capture a bearish view on the SG stock price.

4.6 Analysis of Société Générale (SG) stock

According to the report from the Deutsch Bank equities research team (published on 11 December 2017, option market data from the equity derivatives group published later in April 2018), SG shares have underperformed year to date, with a recent acceleration in that underperformance. As of the close on December 7 2017, SG shares were down -8% year to date, underperforming the SX7P (up 7%, or -15% underperformance), the SX7E (up 12%, or -20% underperformance), and other French banks (BNP up +4% YTD, CASA +20%, and Natixis +26%) [49].

SG expects a revenue CAGR 16-20 of ">3%" (3.4% estimated), compared to DB of 2.0% and consensus of 2.3%. Revenues account for the majority of the increase in SG projections compared to forecasts. Revenue objectives exceed DB and consensus forecasts in all divisions (small proportion in the French retail banking). However, DB believe this is unlikely to be reached [49].

The report highlights that SG did not meet its revenue targets in the previous plan (which were close to the new ones), that revenue generation remains challenging due to low rates and volatility, and that converting the 2016-20 revenue CAGR to 2017-20 CAGR assuming Q4=Q3 revenues raises revenue targets even higher (">3%" 16-20 CAGR becomes ">4.2%" 17-20 CAGR)[49]. The equity research time gave a hold in its market outlook for SG stock price, stating that the mixed market and underlying fundamentals are making the positioning more tricky to express. To not fall in the look-back bias and structure a trade that use market data that were not available in early April 2018, we make our own judgement and position ourselves as if we are trading during that period and have not access the the data to make the trade idea more realistic and reasonable.

The main downside risks, according to the DB equity research report [49], include a worsening macro situation, weaker capital markets revenues, a stricter regulatory environment, execution risk on cost-cutting plans, some pressure on retail banking earnings due to the low rate environment, some exposure to the oil and gas sector due to the low oil price, and litigation. Upside risks include improved macroeconomic conditions, increased capital market income, a more relaxed regulatory environment, greater loan growth/NIM in retail firms (particularly in France), and strong asset quality in oil and gas exposure. SG may face similar risks as the banking sector, including credit risks, market risks, counterparty exposure, liquidity risks, operational risks, legal risks, and reputational risks. Given the relative underperformance of the SG stock compared to the banking sector benchmark [49], we believe the mean revesion of the stock to its fundamental value won't occur in the short term (less than 3-month). In this sense we decided to put in action to different strategies that would capture our underlying market view of the SG share price. We played the stock price decline by structuring a protective put and a short strangle strategy. The first method is centred on a protective put, which is going long on a put option while also taking a long position in the underlying to limit the option's downside loss. The second technique is a short strangle, which is employed for a limited volatility outlook in the short term, with downside plays potentially yielding a positive return as long as the underlying asset does not exceed the two strike level. It seeks to capture the premiums from selling a put and a call with differing strike prices.

4.7 Black-Scholes-Merton framework

Introduction

This model, developed by Robert Merton, Myron Scholes, and Fischer Black, is regarded as a fundamental model for option valuation because it provided an analytical method for determining the values of European-style options [51][48]. A number of fundamental assumptions underpin the model: the existence of a known and constant risk-free rate; the assumption that stock prices move in a geometric Brownian motion; the lack of dividend payments during the option's life; the absence of arbitrage opportunities; and the restriction that European-style options can only be exercised at expiration [48].

For a call option, the payoff is expressed as follows:

$$(S_T - K)^+ = \max(S_T - K; 0) \tag{4.14}$$

where K represents the strike price of the option and S_T represents the price of the underlying at time t.

Regarding the put option, the payoff is the following:

$$(K - S_T)^+ = \max(K - S_T; 0) \tag{4.15}$$

The formula to price a European call under BS is given below:

$$C(S_0, K, r, T, \sigma) = S_0 N(d_1) - Ke^{-rT} N(d_2) \tag{4.16}$$

For a put option, the Black-Scholes model can be rewritten as:

$$P(S_0, K, r, T, \sigma) = Ke^{-rT} N(d_2) - S_0 N(d_1) \tag{4.17}$$

where $N(d_1)$ and $N(d_2)$ are cumulative distribution functions for a standard normal distribution with the following Gaussian distribution function formula:

$$N(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}u^2} du \tag{4.18}$$

The d_1 and d_2 functions in the Black-Scholes model are computed as:

$$d_1 = \frac{1}{\sigma\sqrt{T}} \left[\ln\left(\frac{S_0}{K}\right) + \left(r + \frac{1}{2}\sigma^2\right) T \right] \tag{4.19}$$

$$d_2 = d_1 - \sigma\sqrt{T} \tag{4.20}$$

where T represents time to maturity, S_0 represents the underlying asset price at $t = 0$, K represents the strike price, σ represents the volatility of the option, and r represents the risk-free rate.

Greeks

We elaborate on the greeks for option pricing [51]. Delta measures the rate of change of the option price with respect to changes in the underlying asset's price.

For a call option:

$$\Delta_C = N(d_1) \tag{4.21}$$

For a put option:

$$\Delta_P = N(d_1) - 1 \tag{4.22}$$

where $N(\cdot)$ is the cumulative distribution function of the standard normal distribution, and d_1 is given by:

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(r - q + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \tag{4.23}$$

where S is the current price of the underlying asset, K is the strike price of the option, r is the risk-free interest rate, q is the continuous dividend yield, σ is the volatility of the underlying asset and T is the time to maturity of the option.

Theta measures the rate of change of the option price with respect to the passage of time [51].

For a call option:

$$\Theta_C = -\frac{S\sigma e^{-qT}N'(d_1)}{2\sqrt{T}} - rKe^{-rT}N(d_2) + qSe^{-qT}N(d_1) \tag{4.24}$$

For a put option:

$$\Theta_P = -\frac{S\sigma e^{-qT}N'(d_1)}{2\sqrt{T}} + rKe^{-rT}N(-d_2) - qSe^{-qT}N(-d_1) \tag{4.25}$$

where $N'(d_1)$ is the probability density function of the standard normal distribution.

Vega measures the sensitivity of the option price to changes in the volatility of the underlying asset [51]. When both call and put options are close to the money, their Vega values are the highest. This is because at-the-money options will experience the greatest price changes as implied volatility changes. An increase in volatility typically increases the price of a protective put option, which can give additional protection or increase the cost of hedging. Because selling options might result in losses if volatility increases, Vega risk is higher in a short strangle.

$$\nu = S\sqrt{T}e^{-qT}N'(d_1) \tag{4.26}$$

Gamma measures the rate of change of Delta with respect to changes in the underlying asset's price. It impacts in the same way calls and puts [51]. The high Gamma value in the plot shows that the Delta of the option is most sensitive when the stock price is near the strike price. For both protective put and short strangle strategies, managing Gamma is very important, as it can impact how quickly investor need to adjust the hedge as the stock price moves.

$$\Gamma = \frac{N'(d_1)}{S\sigma\sqrt{T}} \tag{4.27}$$

These are the primary Greeks used to measure the sensitivity of option prices to various factors. Each Greek captures a different aspect of risk and can be used to hedge portfolios against changes in market conditions.

4.8 Implementation of an equity option strategy on Société Générale (SOGN.PA)

In this part, we discuss the different strategies used and their underlying payoffs. We applied the approach in both Excel and Python. We structured the BSM valuation for calls and puts, the sensitivity of the key greeks, and the protective put structure in Excel but chose to switch to Python because it was easier to deal with. We reproduced the process in Python, utilising market prices downloaded from Refinitiv Eikon and Yahoo Finance for SG stock price and OAT 10 year yield, and saved them in an Excel file for further use. We were able to include the short strangle strategy as an extra strategy to analyse in the report.

For this trade idea, we have selected the following trade parameters. On the 6th April 2018 (the date of the last observed market data from the DB equity derivative report), we structure the trade with an anticipation of bearish outlook for Société Générale stock on a short term basis. From the equity derivative strategy group at DB, we rely on their data regarded the option market and decide to play the strategy using 3-month maturity OTM put option (strike price quoted 38.00) 4.23. We take the most traded put option traded in the market an to long on the underlying at the time of the trade (market price quoted at 44.52). We downloaded historical data for a one year time frame in order to derive the historical volatility measure which is a key input to compute in a Black-Scholes framework. We find a volatility for Societe generale on a 1y basis equivalent to 18.69% on an annual rate. For the risk free rate, we decided to take the yield observable from market data derived from the 10-year French government bond (OAT 10 year) and use it as the rate to discount option strike price to its present value (last observable yield on trading date at 0.72%). Since we operate on a BSM framework, dividend yield is not taken into account and we consider the stock will not deliver any dividends during the lifetime of the option. The trade maturity spans 100 days overall (from 06-04-2018 to 15-06-2018). We covered all the required parameters that we used to price the calls and puts required for the strategies.

Protective put on Société Générale (SOGN.PA)

Investors use the protective put method to hedge against future stock value losses [51]. Mathematically, it is simply the sum of the long position in the stock and the long put option's payout. The payment (P) for a protected put is given by:

$$P(S_T) = S_T + \max(K - S_T, 0) - \text{Premium paid} \tag{4.28}$$

where S_T is the stock price at maturity and K is the strike price of the put option.

The protective put ensures that the investor's losses are limited to the strike price of the put option, less the premium paid for the option. This is because any drop in the stock price below the strike price is offset by an equal increase in the value of the put option [51]. The technique is fundamentally positive or neutral because the investor owns the underlying stock and is protected from downside risk. As captured in the plot 4.1, we can see that the value of the strategy is caped at the put option strike price

(38.00 euros) minus the premium, while achieving a upside potential equivalent to the underlying share price increase above the strike price. The stock on 16-06-2018 was quoting at 36.26 euros, indicating the put option would be in the money and would appreciate from this decrease of stock price from 44.52 euros, even if the long underlying position would have incurred a loss. The final payoff of a protective put strategy $P(S_T)$ is given by the sum of the value of the stock at maturity S_T and the payoff from the put option, which is the maximum of $K - S_T$ and 0, minus the premium paid for the put option. The premium paid is the cost to purchase the put option, per share. Given that the strike price is $K = 38$ euros, initial stock price, $S_0 = 44.52$ euros, the option cost (premium paid), Premium $= 0.03$ euros per share and the final stock price at maturity (16-08-2018), $S_T = 36.26$ euros, the payoff from the put option is the maximum of $K - S_T$ and 0, which would be:

$$\max(K - S_T, 0) = \max(38 - 36.26, 0) = 1.74 \tag{4.29}$$

Now, we calculate the final payoff:

$$P(S_T) = S_T + \max(K - S_T, 0) - \text{Premium paid} \tag{4.30}$$

$$P(S_T) = 36.26 + 1.74 - 0.03 \tag{4.31}$$

$$P(S_T) = 36.26 + 1.74 - 0.03 \tag{4.32}$$

$$P(S_T) = 37.85 \tag{4.33}$$

The final payoff from the protective put strategy would be 37.85 euros per share.

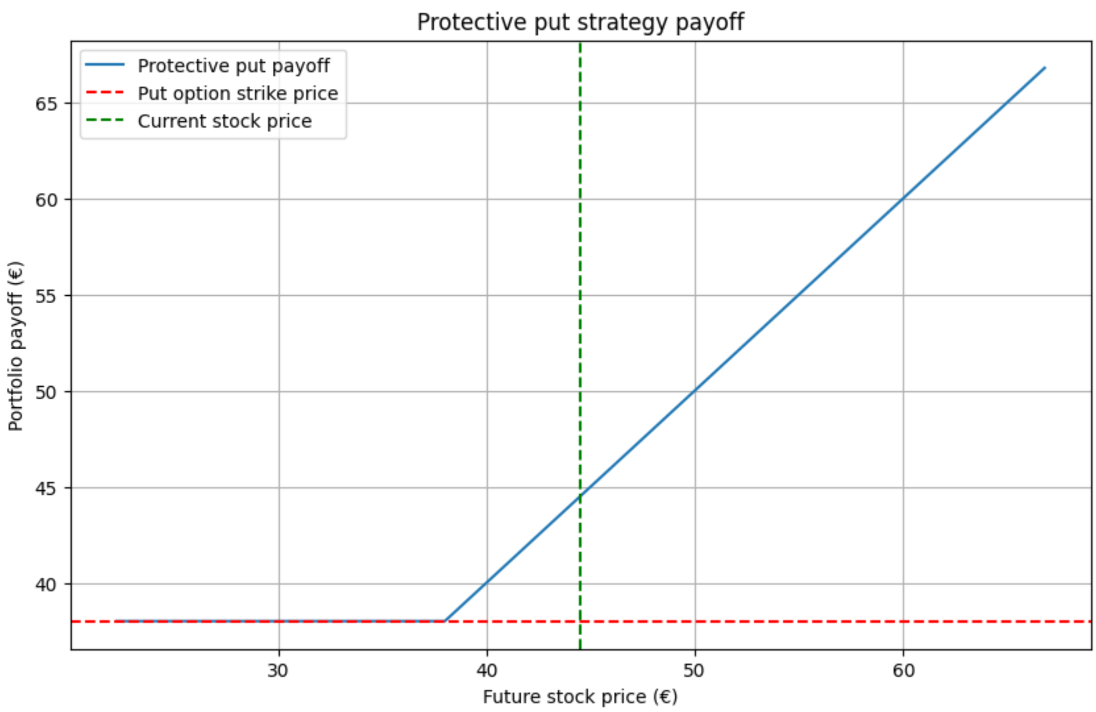


Figure 4.1: Protective put strategy payoff.

Short strangle on Société Générale (SOGN.PA)

The short strangle strategy includes selling both put and call options with the same maturity but different strike prices. It is a neutral-to-bearish strategy that benefits from lower implied volatility, or the stock price remaining between the two strike prices until the options expire. The overall profit is restricted to the premiums earned from selling the options. The payment (P) for a short strangle is given by:

$$P(S_T) = \begin{cases} K_p - S_T + \text{Premiums received}, & \text{if } S_T < K_p \\ \text{Premiums received}, & \text{if } K_p \leq S_T \leq K_c \\ S_T - K_c + \text{Premiums received}, & \text{if } S_T > K_c \end{cases} \tag{4.34}$$

where S_T is the stock price at maturity, K_p and K_c are the strike prices of the put and call options, respectively.

The short strangle strategy involves selling an out-of-the-money put and an out-of-the-money call. The strategy's profit is generally maximized if the stock price at maturity S_T remains between the strike prices of the sold options. If the final price at maturity is $S_T = 36.26$ euros, and the strike price for the call option is $K_c = 46.00$ euros, and the option was sold (short position), the call option will not be exercised since the strike price is higher than the stock price at maturity. Hence, the seller of the call option will keep the entire premium received. We know from the computation that we performed and the available market information that the premium received for selling the call option is $\text{Premium}_{\text{call}} = 0.8753$ euros, the premium received for selling the put option is $\text{Premium}_{\text{put}} = 0.032$ euros, the final stock price at maturity is $S_T = 36.26$ euros, the strike price of the call option: $K_c = 46.00$ euros (not exercised because $S_T < K_c$) and the strike price of the put option is $K_p = 38.00$ euros (exercised because $S_T < K_p$).

The total premiums received from selling both the call and put options:

$$\text{Total Premiums received} = \text{Premium}_{\text{call}} + \text{Premium}_{\text{put}} \tag{4.35}$$

$$\text{Total Premiums received} = 0.8753 + 0.032 \tag{4.36}$$

$$\text{Total Premiums received} = 0.9072 \tag{4.37}$$

Since $S_T < K_p$, the put option will be exercised, and the seller of the put will have to buy the stock at the strike price, so the payoff from the put option is:

$$\text{Payoff}_{\text{put}} = K_p - S_T \tag{4.38}$$

$$\text{Payoff}_{\text{put}} = 38.00 - 36.26 \tag{4.39}$$

$$\text{Payoff}_{\text{put}} = 1.74 \tag{4.40}$$

However, since this is a short position, this amount represents a loss, and the final payoff for the strategy considering both the call and put option components is:

$$P(S_T) = \text{Total Premiums received} - \text{Payoff}_{\text{put}} \tag{4.41}$$

$$P(S_T) = 0.9072 - 1.74 \tag{4.42}$$

$$P(S_T) = -0.8328 \tag{4.43}$$

This negative payoff indicates a loss for the seller of the short strangle. The seller had to pay out more to cover the exercised put option than the premiums received from selling both options. The plot represents the positive payoff derived from the premiums received (0.90 euros) from selling the call and put options. For a short strangle risk management, investor need to understand how the delta behaves in this type of strategies. Negative delta from the sold put and positive delta from the sold call mean the strategy benefits from the stock price remaining between the two strikes. Gamma indicates the risk as the stock price moves towards either strike price, vega shows the risk of increased volatility which could lead to losses, and theta represents the benefit of time decay to the strategy. As captured from the payoff 4.2 , the strategy is profitable (slight positive figure) only when the stock fluctuates around the two strike prices and start to incur losses when the underlying price starts to deviate far away from both strike prices. The idea of this trade is to capture the premiums obtained from selling both a call and a put option with different strike prices. However, as captured in the payoff plot, the downside loss is uncapped so proper risk management is mandatory to keep the losses under control.

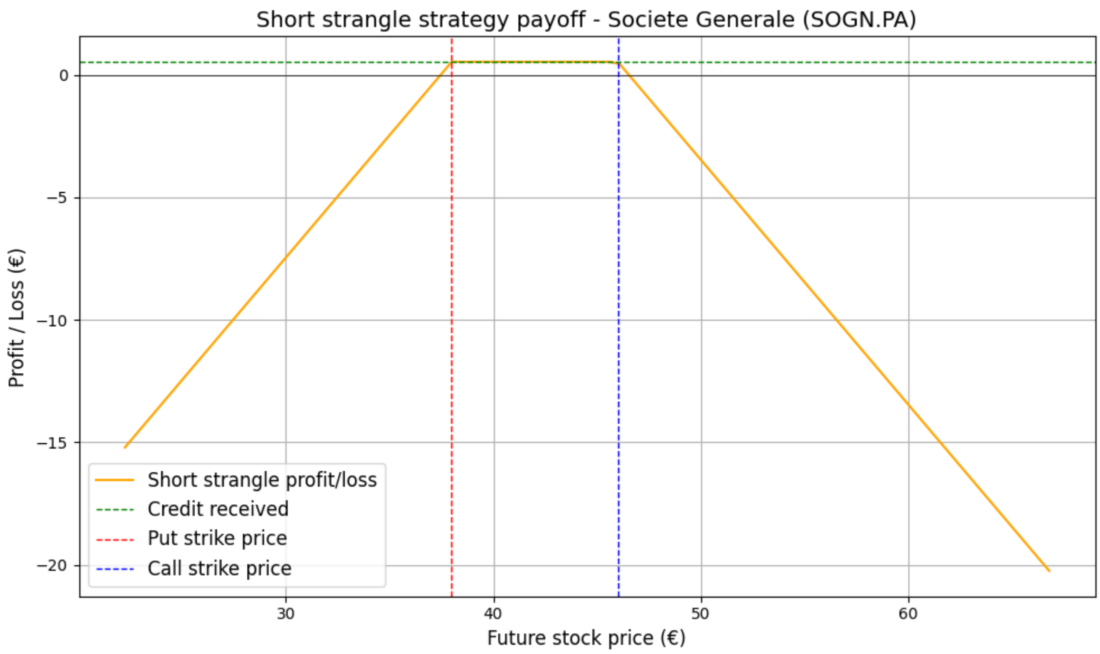


Figure 4.2: Short strangle strategy payoff.

Bibliography

[1] Amenc, Noël & Goltz, Felix. (2013). Smart Beta 2.0. The Journal of Index Investing. 4. 15-23. 10.3905/jii.2013.4.3.015.

[2] Ang, A., 2013. Factor Investing. SSRN Research Journal, pp. 1-72.

[3] Arnot, R.D., Hsu, J., & Moore, P. (2005). Fundamental Indexation. Financial Analysts Journal. Volume 61. Number 2. (83-98). Research Affiliates LLC.

[4] Arnott, Robert D. and Kalesnik, V.and Moghtader, P., Scholl, S (2010). Beyond Cap-Weight: The Search for Efficient Beta. Journal of Indexes, January-February 2010, Available at SSRN: https://ssrn.com/abstract=1499357.

[5] Asness, C. S. F. A. & Pedersen, L., 2018. Quality minus Junk. Review of Accounting studies.

[6] BlackRock research, 2021. What is Factor Investing?. [Online] Available at: https://www.blackrock.com/us/individual/investment-ideas/what-is-factor-investing [Accessed 2021].

[7] Blank, H.D., & Duan, Q., (2017). The Case for Reverse-Cap-Weighted Indexing.

[8] Carhart, M. (1997). On Persistence in Mutual Fund Performance. The Journal of Finance. 52(1), 57-82.

[9] CFA Institute, 2011. Investments: principles of portfolio and equity analysis. ISBN 978-0-470-91580-6 ed. CFA institute investment series.

[10] Chopra, V., & Ziemba, W. (1993). The Effect of Errors in Means, Variances, and Covariances on Optimal Portfolio Choice. Journal of Portfolio Management, Winter 1993 pg 6.

[11] Choueifaty Y., Coignard, Y. (2008). Toward Maximum Diversification. Journal of Portfolio Management. 35(1), 40-51. ISSN: 0095-4918.

[12] Chow, T., Hsu, J., Kalesnik, V., & Little, B. (2011). A Survey of Alternative Equity Index Strategies, Financial Analyst Journal, CFA Institute, vol. 67, no.5, pp 35-57.

[13] EDHEC Risk Institute, 2016. Investor perception about Smart Beta ETFs, s.l.: EDHEC Risk Institute Research Journal.

[14] Edwards, T. & Hamish, P., 2017. A practitioner guide to reading VIX, s.l.: S&P Global Research.

[15] El Lamti, N., 2017. Is Smart Beta really smart?. Wiley Research Journal.

[16] Eugene F., French, Kenneth R (1993). Common risk factors in the returns on stocks and bonds. Journal of Financial Economics. 33(1), 3-56.

[17] Eugene F., French, Kenneth R (2004). The Capital Asset Pricing Model: Theory and Evidence. The Journal of Economic Perspectives. 18(3), 25-46.

[18] Fama, Eugene F., French, Kenneth R (2015). A five-factor asset pricing model. Journal of Financial Economics. 116(1), 1-22.

[19] Foglia M., R. M. C. P. G., 2021. "Smart Beta Allocation and Macroeconomic Variables - The Impact of COVID-19". MDPI research journal.

[20] Glushkov D. (2015) "How Smart are 'Smart Beta' ETFs? Analysis of Relative Performance and Factor Exposure", SSRN Research Journal.

[21] Grossman, Sanford & Stiglitz, Joseph. (1980). On The Impossibility of Informationally Efficient Markets. American Economic Review. 70. 393-408.

[22] Hasaj, M. & Sherer, B., 2021. "Covid-19 and Smart-Beta: A Case Study on the Role of Sectors". EDHEC-Risk Institute Working Paper., pp. 1-35.

[23] Hodges, P., Hogan, K., Peterson, J. R. & Ang, A., 2017. Factor Timing with Cross-Sectional and Time-Series Predictors. The journal of portfolio management.

[24] Jegadeesh N., Titman, S. (1993). Returns to Buying Winners and Selling Losers: Implications for Stock Market Efficiency. The Journal of Finance. 48(1), 65-91.

[25] Jensen, Michael C., The Performance of Mutual Funds in the Period 1945-1964 (May 1, 1967). Journal of Finance, Vol. 23, No. 2, pp. 389-416, 1967.

[26] Malkiel, B. G., 2014. Is smart beta really smart? The Journal of Portfolio Management, Special 40th Anniversary. Issue 2014, 40 (5) 127-134. DOI: https://doi.org/10.3905/jpm.2014.40.5.127.

[27] Mangram, M. E., 2013. A simplified perspective of the Markowitz Portfolio Theory. SSRN Research Journal.

[28] Markowitz, H., 1952. Portfolio Selection. The Journal of Finance, 7(1), pp. 77-91.

[29] Pagano, Wagner & Zechner, J., 2020. Disaster Resilience and Asset Prices. SSRN Research Journal - http://dx.doi.org/10.2139/ssrn.3603666.

[30] Sharpe, W. F (1964). Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk. The Journal of Finance. 19(3), 425-442.

[31] Sharpe, W. F (1991). Capital Asset Prices with and without Negative Holdings. The Journal of Finance June 1991, 46(2), 489-509..

[32] Black, F., & Litterman, R. B. (1992). Global portfolio optimization. *Financial Analysts Journal*, 48(5), 28-43.

[33] Mangram, M. E. (2013). A simplified perspective of the Markowitz Portfolio Theory. *SSRN Research Journal*. Available at: https://papers.ssrn.com/sol3/papers.cfm?abstract_id=2147880.

[34] Markowitz, H. (1952). Portfolio Selection. *The Journal of Finance*, 7(1), 77-91.

[35] Clauss, P. (2011) *Gestion de Portfeuille: Une Approche Quantitative*. Edition Dunod.

[36] BlackRock Investment Institute. (2023). Midyear Global Outlook: Navigating a period of uncertainty, but not without opportunities. (June 2023).

[37] Bridgewater Associates, LP. (2009). The All-Weather Strategy. Bridgewater Associates, LP.

[38] Bridgewater Associates, LP. (2019). Geographic Diversification Can Be a Lifesaver, Yet Most Portfolios Are Highly Geographically Concentrated. Bridgewater Associates, LP.

[39] Goldman Sachs Global Investment Strategy Group. (2023). Midyear Outlook 2023: Resilience in the Face of Adversity. (June 2023).

[40] JPMorgan Asset Management. (2023). Investment Outlook for 2023: Slow-motion slowdown. (June 2023).

[41] Black, F., & Scholes, M. (1973). The Pricing of Options and Corporate Liabilities. Journal of Political Economy, 81(3), 637-654.

[42] Diderich, C. (2009). Positive Alpha Generation: Designing Sound Investment Processes. The Wiley Finance Series, Ch.17 (267-278). Wiley.

[43] Hull, J. C. (2017). Options, futures, and other derivatives (10th ed.). Pearson.

[44] Kou, S. G. (2002). A jump-diffusion model for option pricing. Management science, 48(8), 1086-1101.

[45] Leland, Hayne E. and Mark Rubinstein (1988) The evolution of portfolio insurance, in Portfolio Insurance: A Guide to Dynamic Hedging, Donald L. Luskin (ed.), John Wiley & Sons, Inc., Hoboken.

[46] Merton, R. C. (1976). Option pricing when underlying stock returns are discontinuous. Journal of financial economics, 3(1-2), 125-144.

[47] Perold, Andre F. (1986), Constant Portfolio Insurance, Harvard Business School, Harvard.

[48] Black, F., & Scholes, M. (1973). The Pricing of Options and Corporate Liabilities. Journal of Political Economy, 81(3), 637-654.

[49] Deutsche Bank (DB) Equity Research and Equity Derivatives Group market research: Société Générale (2018).

[50] Fabozzi, F.J. (2007). Fixed Income Analysis. Wiley Finance.

[51] Hull, J. C. (2017). Options, futures, and other derivatives (10th ed.). Pearson.

[52] Kalife, A. (2024). Equity Derivatives Strategies and Structuring module. Université Paris-Saclay, Évry.

[53] Merton, R. C. (1976). Option pricing when underlying stock returns are discontinuous. Journal of financial economics, 3(1-2), 125-144.

[54] Wilmott, P. (1998). Derivatives: The theory and practice of financial engineering. John Wiley & Sons.

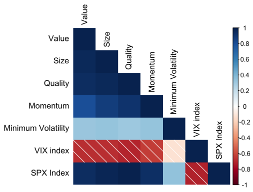
Appendix

Table 4.1: Multiple Linear Regression results for MSCI Value using full sample

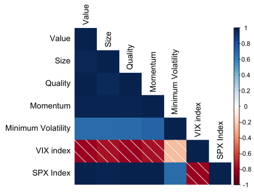
	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	-0.0009865	0.0008102	-1.218	0.224
SPX Index	1.0723200	0.0291386	36.801	<2e-16 ***
VIX index	0.0013397	0.0056796	0.236	0.814
COVID_Impact	0.0008592	0.0009214	0.933	0.352

Signif. level: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1.
Residual standard error: 0.007493 on 369 degrees of freedom
Multiple R-squared: 0.8742, Adjusted R-squared: 0.8732
F-statistic: 855 on 3 and 369 DF, p-value: < 2.2e-16.

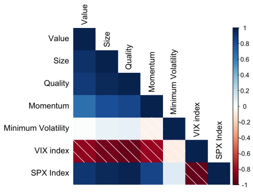
Figure 4.3: The figure represent the analysis of smart beta returns from June 1988 to September 2016. Research published in the Journal of Portfolio Management examines the performance of factor funds over a 30- year period, looking at the vectors of returns throughout that time [23]. From June 30, 1988, to September 30, 2016, panel A shows the average excess returns of each smart beta component (above the MSCI USA Index). Value, quality, momentum, and size all have positive average returns; momentum and value have the largest annual excess returns, with 3.4 percent and 1.5 percent, respectively. Minimum volatility has provided an average return that is consistent to the market (but with less risk), which is in line with Ang et al (2006) results [23]. While long-run excess premiums are positive, there is significant temporal variation throughout the sample panel B. Size, for example, moved from a negative 12-month mean return of -2.0 percent in 1999 to a positive 12-month mean return of 3.0 percent in the early 2000s. Panel C shows that the excess factor returns are not substantially correlated: the lowest correlation is -0.42, while the greatest is 0.67, between minimum volatility and size. Notably, the correlation coefficient between momentum and value is -0.22, which is consistent with their well-known negative connection.[23].



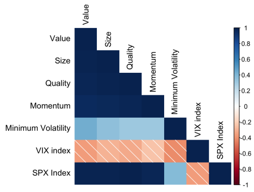
(a) Correlation of US equity factors across Origin subsample. Data: Refinitiv Eikon.



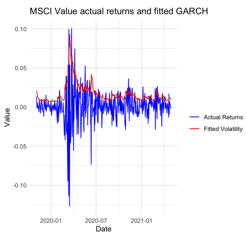
(b) Correlation of US equity factors across Outbreak subsample. Data: Refinitiv Eikon.



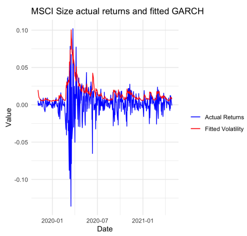
(c) Correlation of US equity factors across Incubation subsample. Data: Refinitiv Eikon.



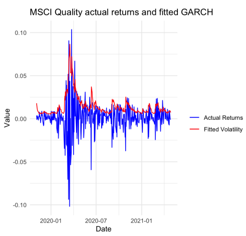
(d) Correlation of US equity factors across Treatment subsample. Data: Refinitiv Eikon.



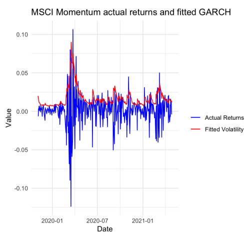
(a) GARCH(1,1) for MSCI Value to assess factor volatility. Data: Refinitiv Eikon.



(b) GARCH(1,1) for MSCI Size to assess factor volatility. Data: Refinitiv Eikon.



(c) GARCH(1,1) for MSCI Quality to assess factor volatility. Data: Refinitiv Eikon.

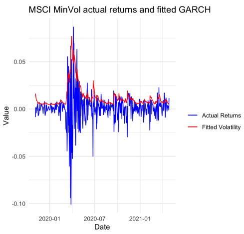


(d) GARCH(1,1) for MSCI Momentum to assess factor volatility. Data: Refinitiv Eikon.

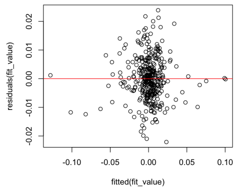
Table 4.2: Multiple Linear Regression results for MSCI Size using full sample

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	-0.0006684	0.0005327	-1.255	0.210
SPX Index	1.0637743	0.0191592	55.523	<2e-16 ***
VIX index	0.0057182	0.0037345	1.531	0.127
COVID_Impact	0.0007725	0.0006058	1.275	0.203

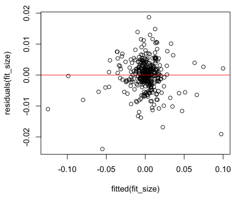
Signif. level: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1.
Residual standard error: 0.004927 on 369 degrees of freedom.
Multiple R-squared: 0.9389, Adjusted R-squared: 0.9384.
F-statistic: 1890 on 3 and 369 DF, p-value: < 2.2e-16.



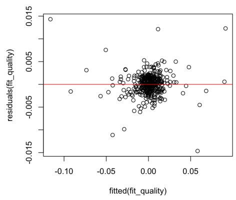
(a) GARCH(1,1) for MSCI Minimum Volatility to assess factor volatility. Data: Refinitiv Eikon.



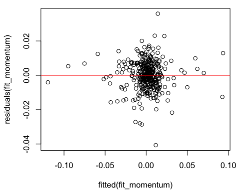
(b) Residuals for the MLR regression for MSCI Value factor.



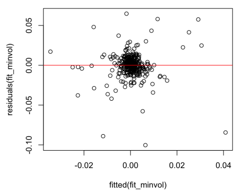
(c) Residuals for the MLR regression for MSCI Size factor.



(d) Residuals for the MLR regression for MSCI Quality factor.



(a) Residuals for the MLR regression for MSCI Momentum factor.



(b) Residuals for the MLR regression for MSCI Minimum Volatility factor.

Table 4.3: Multiple Linear Regression results for MSCI Quality using full sample

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	2.622e-05	2.736e-04	0.096	0.924
SPX Index	9.743e-01	9.840e-03	99.020	<2e-16 ***
VIX index	1.641e-03	1.918e-03	0.856	0.393
COVID_Impact	-9.621e-05	3.111e-04	-0.309	0.757

Signif. level: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1.
Residual standard error: 0.00253 on 369 degrees of freedom.
Multiple R-squared: 0.9804, Adjusted R-squared: 0.9803.
F-statistic: 6157 on 3 and 369 DF, p-value: < 2.2e-16.

Table 4.4: Multiple Linear Regression results for MSCI Momentum using full sample

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	0.0007613	0.0008947	0.851	0.395
SPX Index	0.9966607	0.0321783	30.973	<2e-16 ***
VIX index	-0.0003842	0.0062721	-0.061	0.951
COVID_Impact	-0.0007361	0.0010175	-0.723	0.470

Signif. level: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1.
Residual standard error: 0.008274 on 369 degrees of freedom.
Multiple R-squared: 0.8323, Adjusted R-squared: 0.831.
F-statistic: 610.6 on 3 and 369 DF, p-value: < 2.2e-16.

Table 4.5: Multiple Linear Regression results for MSCI Minimum Volatility using full sample

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	-0.0002917	0.0016468	-0.177	0.859493
SPX Index	0.4370503	0.0592295	7.379	1.07e-12 ***
VIX index	0.0421923	0.0115449	3.655	0.000295 ***
COVID_Impact	0.0001049	0.0018729	0.056	0.955347

Signif. level: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1.
Residual standard error: 0.01523 on 369 degrees of freedom.
Multiple R-squared: 0.1365, Adjusted R-squared: 0.1295.
F-statistic: 19.44 on 3 and 369 DF, p-value: 1.002e-11.

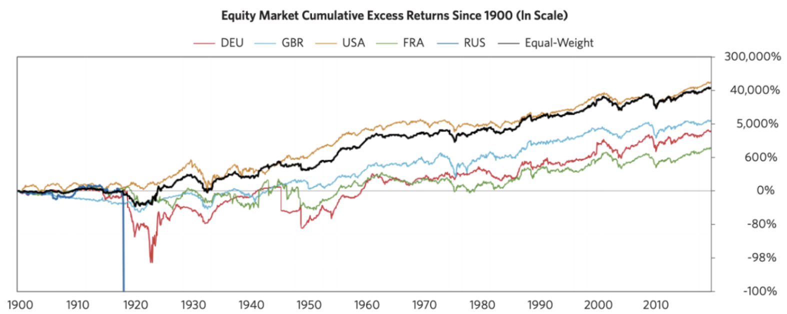


Figure 4.8: Appendix. Cumulative performance above of the stock market, 1900-2019.

Worst Equity Excess Return Drawdowns Across Countries (USD Terms)

Country	Data Starts	Period of Worst Drawdown	What Caused It To Happen	Years To Recover From Start of DD	Magnitude of Losses	Equal-Weight Returns During Country DD
Switzerland	Jan 1966	2007 - 2009	Global Financial Crisis	7	-51%	-49%
Equal-Weight	**Jan 1900**	**1929 - 1932**	**Great Depression**	**13**	-66%	--
Australia	Jun 1933	1969 - 1974	70s Inflation	10	-66%	-17%
UK	Jan 1900	1972 - 1974	70s Inflation	11	-72%	-20%
Norway	Feb 1970	1974 - 1978	70s Inflation	16	-74%	-17%
Japan	May 1949	1989 - 2003	Deflationary Grind	29 & Counting	-75%	-16%
Brazil	Aug 1994	1994 - 1998	Balance of Payments Crisis	24 & Counting	-77%	23%
Canada	Jan 1919	1929 - 1932	Great Depression	16	-79%	-65%
New Zealand	Dec 1984	1986 - 1990	Currency & Constitutional Crisis	32 & Counting	-81%	-10%
Sweden	Dec 1915	1917 - 1932	WWI and Great Depression	29	-81%	-30%
Spain	Dec 1915	1973 - 1982	Political Turmoil/70s Inflation	26	-83%	-19%
France	Jan 1900	1944 - 1950	WWII	15	-83%	41%
Taiwan	Jan 1988	1990 - 2001	Asian Financial Crisis	29 & Counting	-85%	0%
United States	**Jan 1900**	**1929 - 1932**	**Great Depression**	**16**	-85%	-64%
Italy	Jan 1948	1960 - 1977	Political Turmoil ("Years of Lead")	59 & Counting	-87%	49%
Korea	Jan 1965	1989 - 1998	Asian Financial Crisis	30 & Counting	-91%	33%
Germany	Jan 1900	1912 - 1923	WWI	47	-99%	-62%
Russia	Jan 1900	1912 - 1918	WWI and Bolshevik Revolution	Never	-100%	-31%

Figure 4.9: Appendix. Worst equity excess return drawdowns across countries.

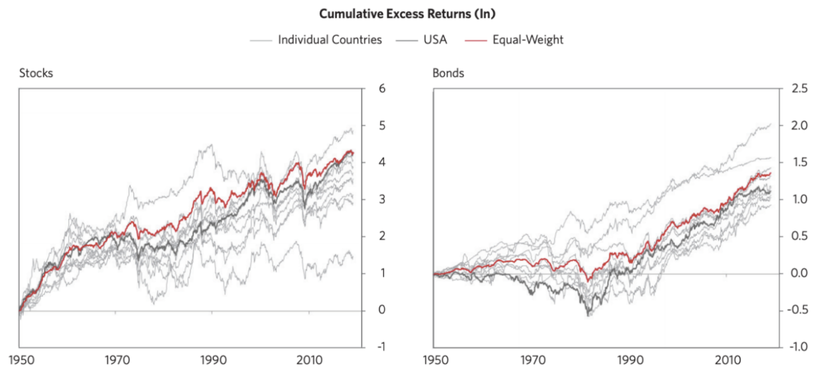


Figure 4.10: Appendix. Cumulative excess returns of an equally weighted geographically diversified portfolio.

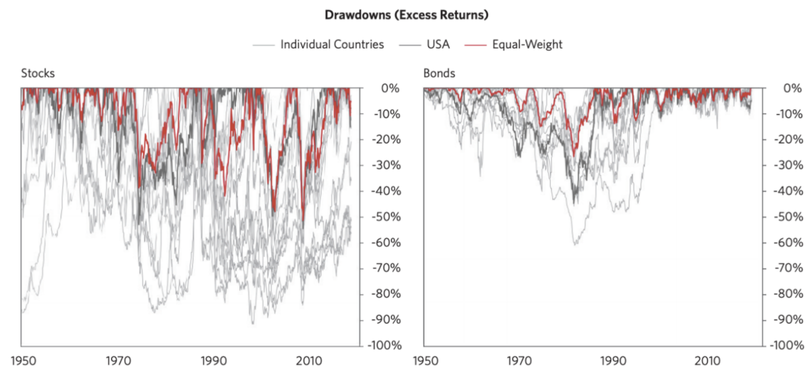


Figure 4.11: Appendix. Maximum drawdown of an equally weighted geographically diversified portfolio.

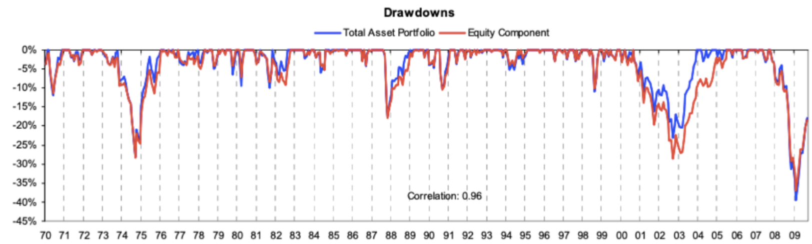


Figure 4.12: Appendix. Correlation analysis of the drawdowns of a 60% equity – 40% bond portfolio.

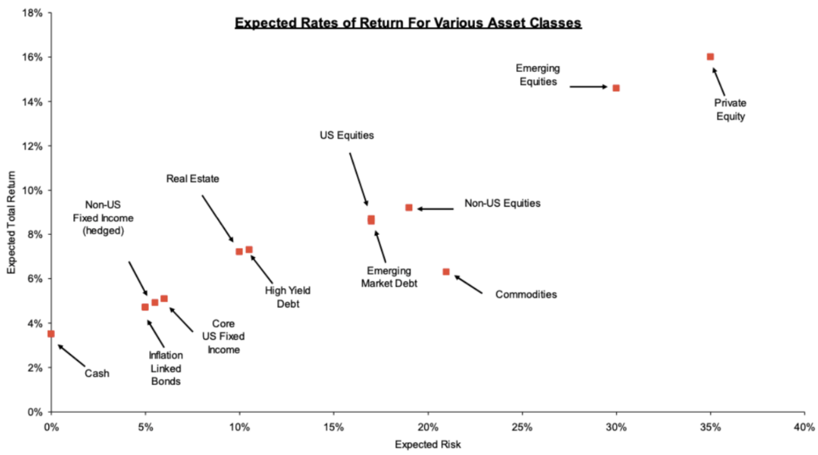


Figure 4.13: Appendix. Risk-return of different asset classes.

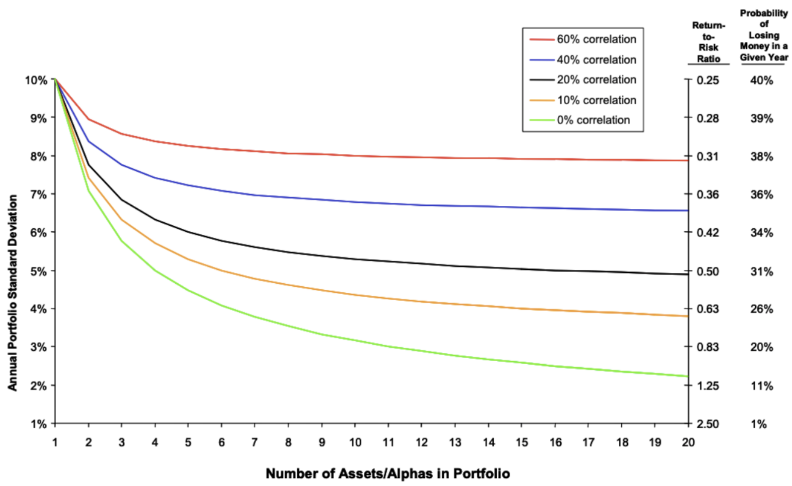
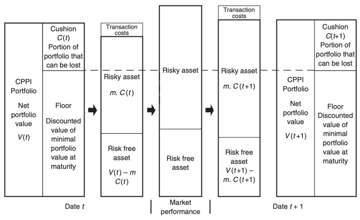
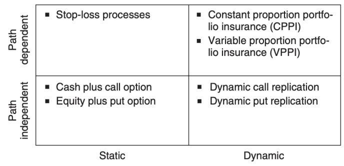


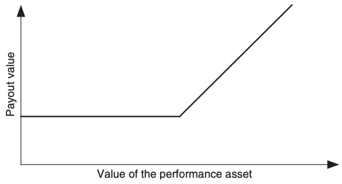
Figure 4.14: Appendix. Degree of correlation between assets and the impact on portfolio overall risk.



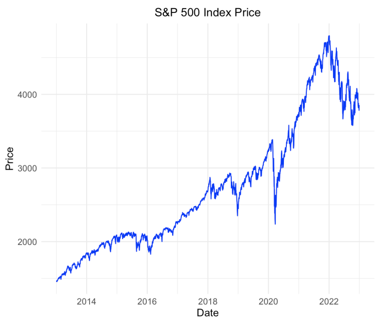
(a) Illustration of the CPPI investment process. Source:[42]



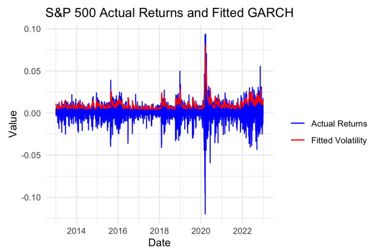
(b) Classification of the portfolio insurance investment processes along the two dimensions static versus dynamic processes and performance asset path dependent versus independent. Source:[42]



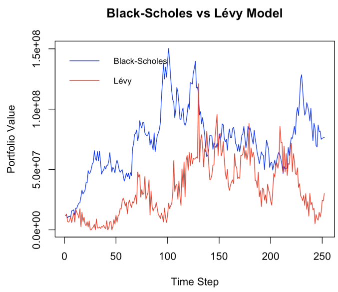
(c) Expected payoff diagram at maturity from a typical investor with a HARA utility function. Source:[42]



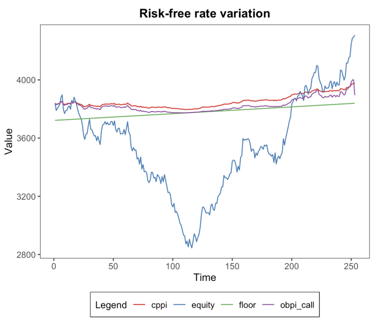
(a) Performance asset (S&P 500 index) price over the time frame analysed. Computation by the authors.



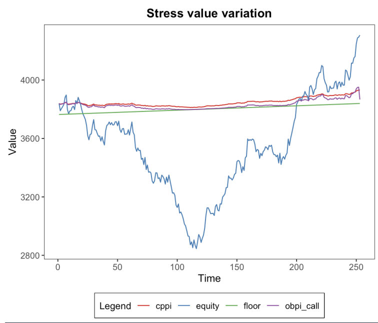
(b) GARCH model for the performance asset (S&P500 index) across the time frame analysed. Computation by the authors.



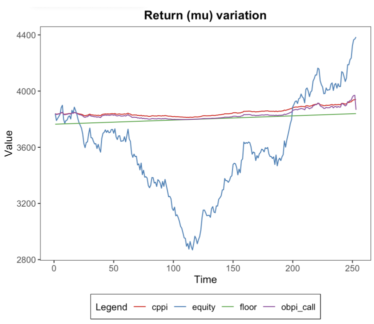
(c) Payoff of the simulated strategies using Black-Scholes and Lévy process to model a CPPI strategy. Computation by the authors.



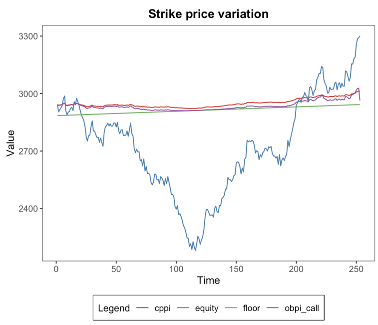
(a) Sensitivity analysis of the OBPI and CPPI strategy to a hundred basis point change in the risk-free asset yield (US 10-year Treasury yield).Computation by the authors.



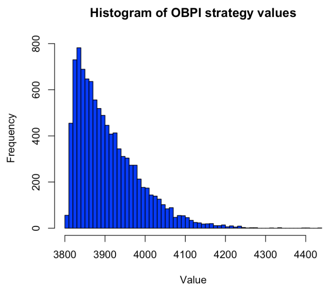
(b) Sensitivity analysis of the OBPI and CPPI strategies with respect to a change in the stress value of a hundred basis point.Computation by the authors.



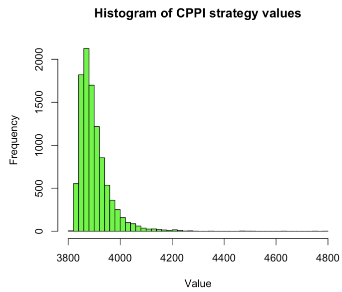
(c) Sensitivity analysis of the OBPI and CPPI strategies with respect to a change in the return value of two hundred basis point (2%).Computation by the authors.



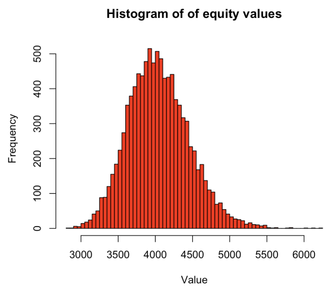
(d) Sensitivity analysis of the OBPI and CPPI strategies with respect to a change in the strike price of the performance asset (equivalent to a two hundred price change in the index level).Computation by the authors.



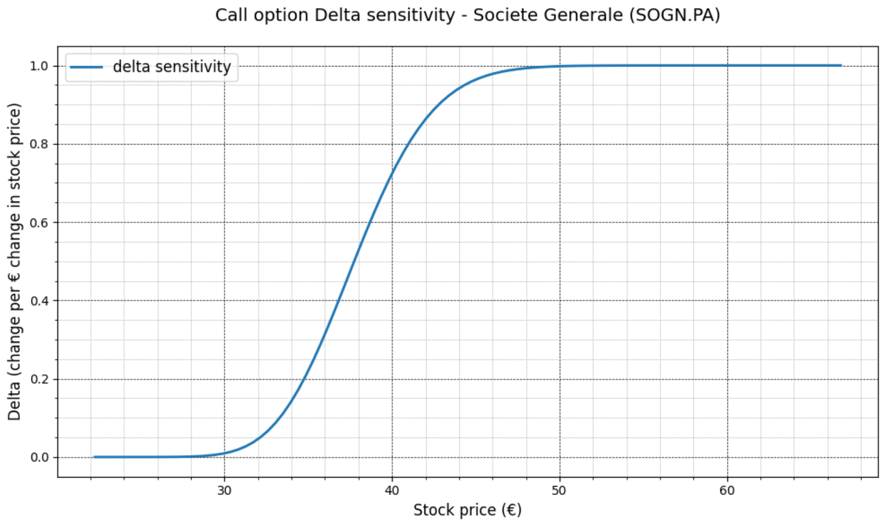
(a) Histogram of the OBPI strategy values.Computation by the authors.



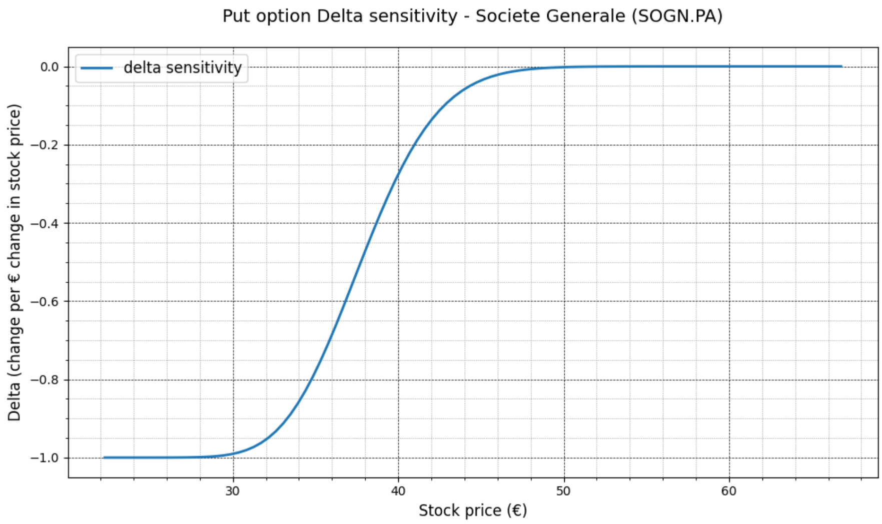
(b) Histogram of the CPPI strategy values.Computation by the authors.



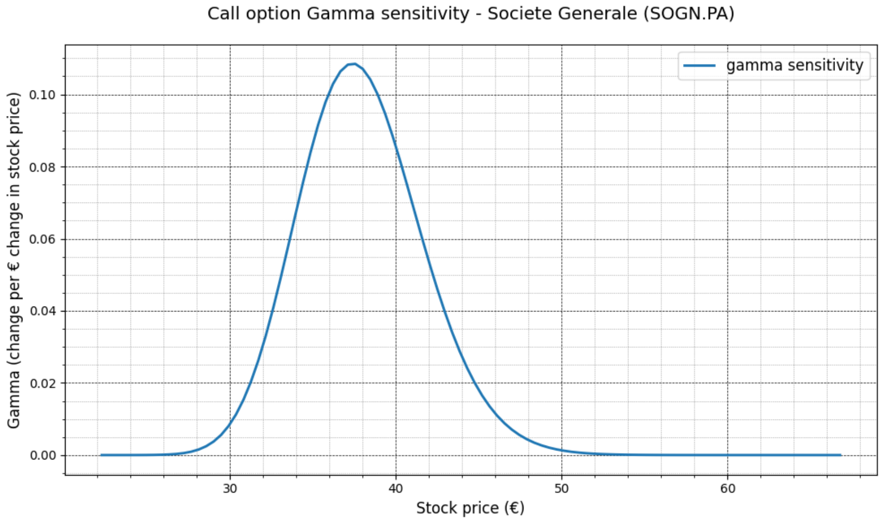
(c) Histogram of the equity price values.Computation by the authors.



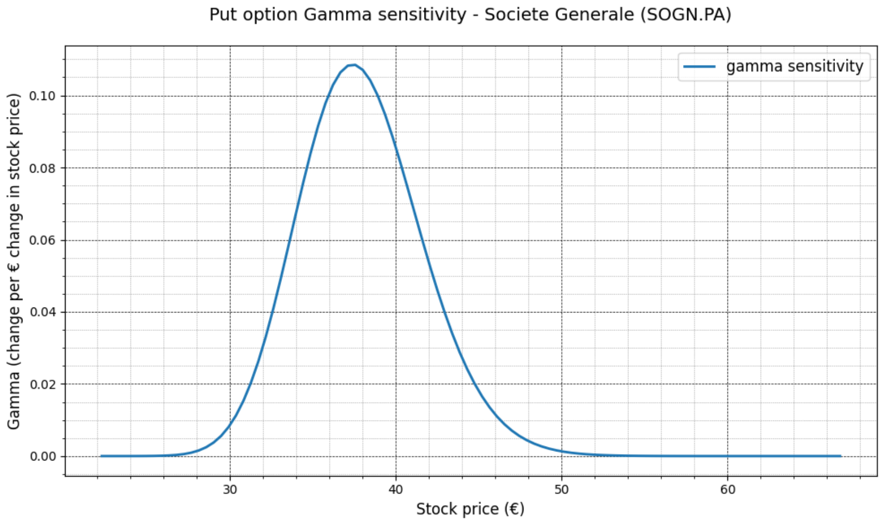
(a) Greek sensitivity: Delta for a call option for Societe Generale (SOGN.PA).



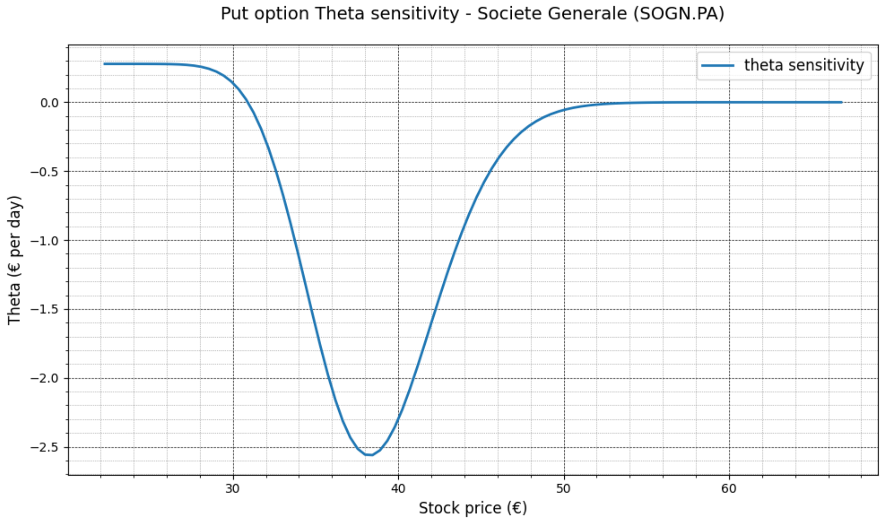
(b) Greek sensitivity: Delta for a put option on the stock Societe Generale (SOGN.PA)



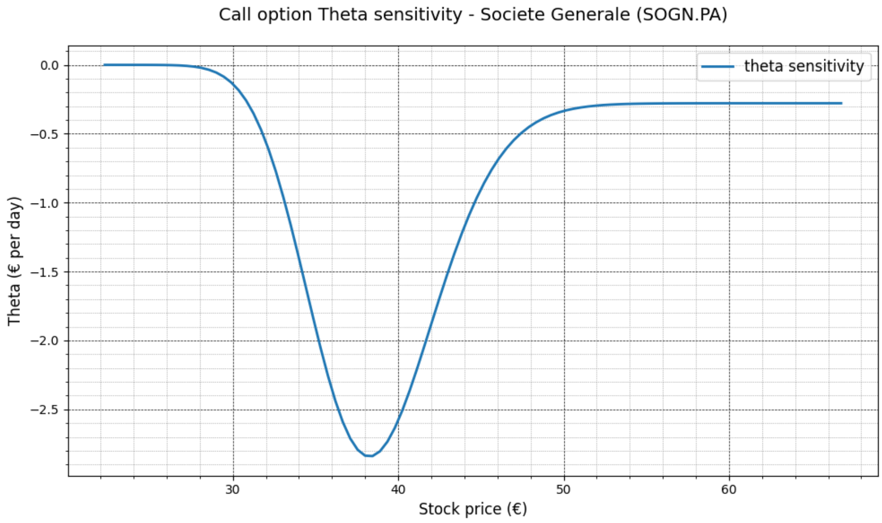
(a) Greek sensitivity: Gamma for a call option for Societe Generale (SOGN.PA).



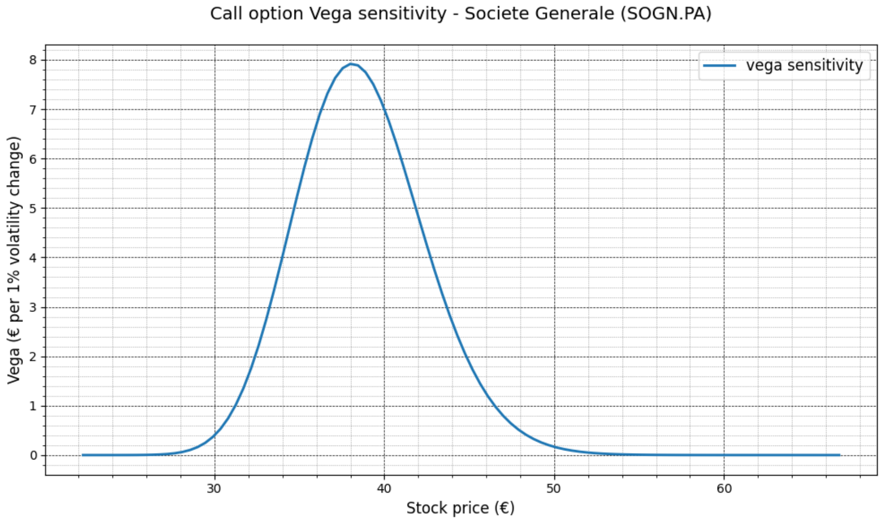
(b) Greek sensitivity: Gamma for a put option for Societe Generale (SOGN.PA).



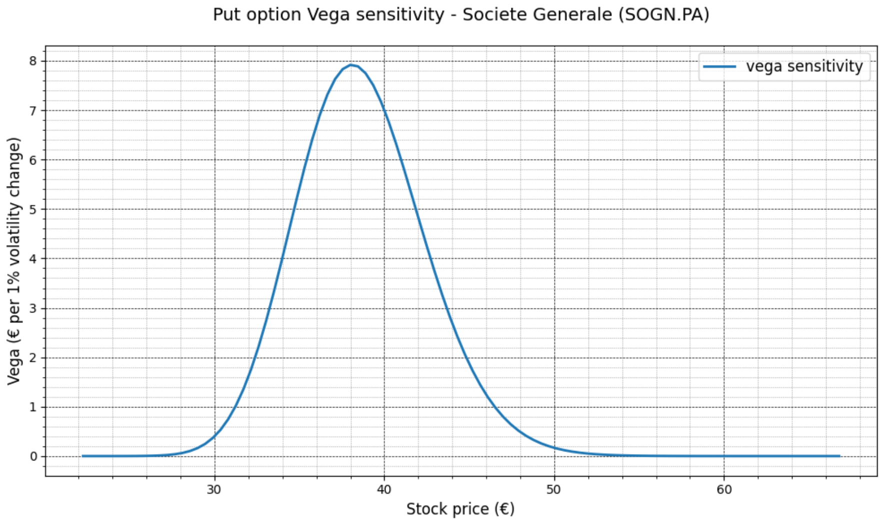
(a) Greek sensitivity: Theta for a put option for Societe Generale (SOGN.PA).



(b) Greek sensitivity: Theta for a call option for Societe Generale (SOGN.PA).



(a) Greek sensitivity: Vega for a call option for Societe Generale (SOGN.PA).



(b) Greek sensitivity: Vega for a put option for Societe Generale (SOGN.PA).

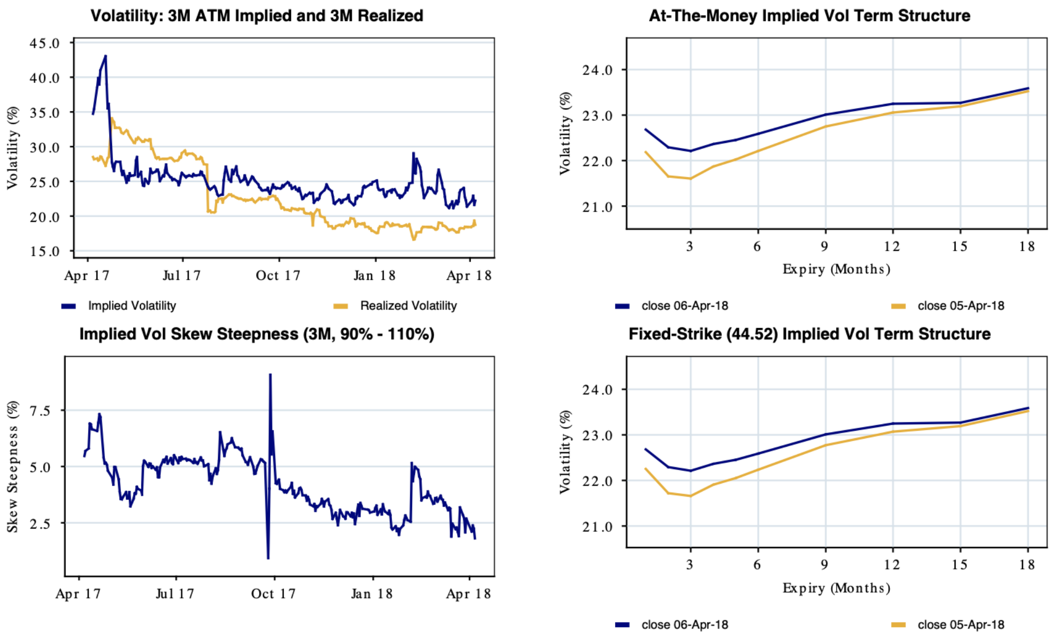


Figure 4.23: Option overview. Extracted from the Deutsche Bank equity report ("Equity Derivatives Strategy Group").