Assignment:1

Monday, September 19, 2022 3:43 PM

Question – 1 (Kirk-3.5)

Given-

Difference equation:

$$x(K+1) = 0.75x(K) + u(K)$$

$$\left(\begin{array}{c}\text{Target set for system :-}\\ 0.0 \leq x(2) \leq 2\end{array}\right)$$

Cost function to minimize:

$$J = u^2(0) + u^2(1)$$

Allowable state and control value constraints:

$$0.0 \leq x(K) \leq 6$$
$$-1.0 \leq u(K) \leq 1.0$$

Quantize state values in levels:

$$x(K) = 0, 2.0, 4.0, 6.0 \text{ for } K = 0, 1, 2$$
$$u(K) = -1.0, -0.5, 0.0, 0.5, 1.0 \text{ for } K = 0, 1$$

(a)

Current state $x(1)$	Control $u(1)$	Next state $x(2) = 0.75x(1) + u(1)$	Cost $J_{12}(x(1), u(1)) = u^2(1)$	Minimum Cost $J_{12}^*(x(1))$	Optimal Control applied at K=1 $u^*(x(1),1)$
0	-1.0	~~-1~~	–		
	-0.5	~~-0.5~~	–		
	0.0	0.0	0	$J_{12}^*(0) = 0$	$u^*(0,1) = 0$
	0.5	0.5	0.25		
	1.0	1.0	1		
2.0	-1.0	0.5	1		
	-0.5	1	0.25		
	0.0	1.5	0	$J_{12}^*(2) = 0$	$u^*(2,1) = 0$
	0.5	2	0.25		
	1.0	~~2.5~~	1		
4.0	-1.0	2	1		
	-0.5	~~2.5~~	–		
	0.0	~~3~~	–	$J_{12}^*(4) = 1$	$u^*(4,1) = -1$
	0.5	~~3.5~~	–		
	1.0	~~4~~	–		
6.0	-1.0	~~3.5~~			
	-0.5	~~4~~	N/A	N/A	N/A
	0.0	~~4.5~~			
	0.5	~~5~~			
	1.0	~~5.5~~			

Current state $x(0)$	Control $u(0)$	Next state $x(1) = 0.75x(0) + u(0)$	Min cost over last two stages for $u(0)$ $J_{01}(x(0), u(0)) + J_{12}^*(x(1)) = u^2(0) + J_{12}^* x(1) = C_{02}^*(x(0), u(0))$	Min. cost over last two stages $J_{02}^*(x(0))$	Optimal Control applied at K=0 $u^*(x(0),0)$
0	-1.0	~~-1~~	–		
	-0.5	~~-0.5~~	–		
	0.0	0.0	$(0)^2 + 0$	$J_{02}^*(0) = 0$	$u^*(0,0) = 0$
	0.5	0.5	$(0.5)^2 + 0$		
	1.0	1.0	$(1)^2 + 0$		
2.0	-1.0	0.5	$(-1)^2 + 0 = 1$		
	-0.5	1	$(-0.5)^2 + 0 = 0.25$		
	0.0	1.5	$(0)^2 + 0 = 0$	$J_{02}^*(2) = 0$	$u^*(2,0) = 0$
	0.5	2	$(0.5)^2 + 0$		

	-0.5	1	$(-0.5)^2+0=0.25$		
	0.0	1.5	$(0)^2+0=0$	$J_{02}^*(2)=0$	$u^*(2,0)=0$
	0.5	2	$(0.5)^2+0=0.25$		
	1.0	2.5	$(1)^2+0=1$		
4.0	-1.0	2	$(-1)^2+0=1$		
	-0.5	2.5	$(-0.5)^2+0.25=0.5$	$J_{02}^*(4)=\begin{Bmatrix}0.5\\0.5\end{Bmatrix}$	$u^*(4,0)=\begin{Bmatrix}-0.5\\0\end{Bmatrix}$
	0.0	3	$(0)^2+0.5=0.5$		
	0.5	3.5	$(0.5)^2+0.75=1$		
	1.0	4	$(1)^2+1=2$		
6.0	-1.0	3.5	$(-1)^2+(0.75)=1.75$		
	-0.5	4	$(-0.5)^2+(1)=1.25$		
	0.0	~~4.5~~	–	$J_{02}^*(6)=1.25$	$u^*(6,0)=-0.5$
	0.5	~~5~~	–		
	1.0	~~5.5~~	–		

(b) Control sequence for $\{u^*(0), u^*(1)\}$ if $x(0)=6.0$

$$x(0)=6 \rightarrow u^*(6,0)=-0.5 \rightarrow x(1)=4 \rightarrow u^*(4,1)=-1 \rightarrow x(2)=2$$

$$\boxed{J^*=1.25}$$

Assignment-1

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Question-2

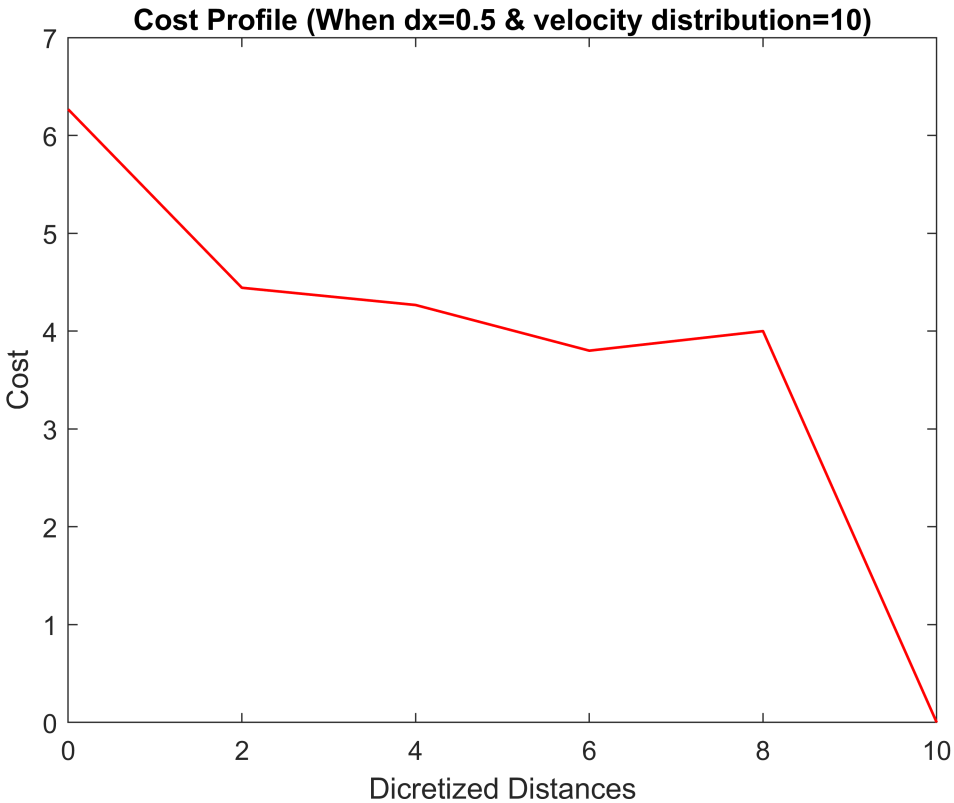
Case-1) When dx=2 & velocity distribution=6

```
clc;
clear;
v_grid=linspace(0,5,6);
dx=2; % Position increament 0.2
X=[0:dx:10];
Ns=numel(X)-1; % Number of stages
Nv=numel(v_grid); % Number of velocity distributions
amin=-2; % Acceleration (accel) lower bound
amax=2; % Acceleration (accel) upper bound
count=1;
cost = zeros(Nv,Ns); % Initialized cost matrix with zeroes

for i=Ns:-1:1
    for j=1:Nv;
        for k=1:Nv;
            dt(j,k)=2*dx/(v_grid(j)+v_grid(k));
            a(j,k)=(v_grid(j)^2-v_grid(k)^2)/(2*dx);
            if a(j,k) < amin || a(j,k) > amax
                dt(j,k) = inf;
            end
        end
    end
    if i==Ns && count==1 % for last stage taking only the '0' velocity in consideration
        cost(:,i) = flip(dt(1,:));
        accel(:,i) = flip(a(1,:));
        count = 0;
    end
    % for other stages except last stage
    if i~=Ns
        prev_cost = flip(transpose(cost(:,i+1))); % taking stage cost into a column vector
        [least_costs,indices] = min(dt + meshgrid(prev_cost,prev_cost),[],2); % indices -> stor
        cost(:,i) = flip(least_costs);
        % indices -> stores column indices of least cost for every node in a stage
        l=1;
        while l<(Nv+1)
            accel((Nv+1)-l,i) = a(indices(l,1),l); % filling the value of acceleration starting
            l = l+1;
        end
    end
end

%% Printing the 'cost' and 'acceleration' matrix for
% disp('Cost and Acceleration matrix :-')
cost;
```

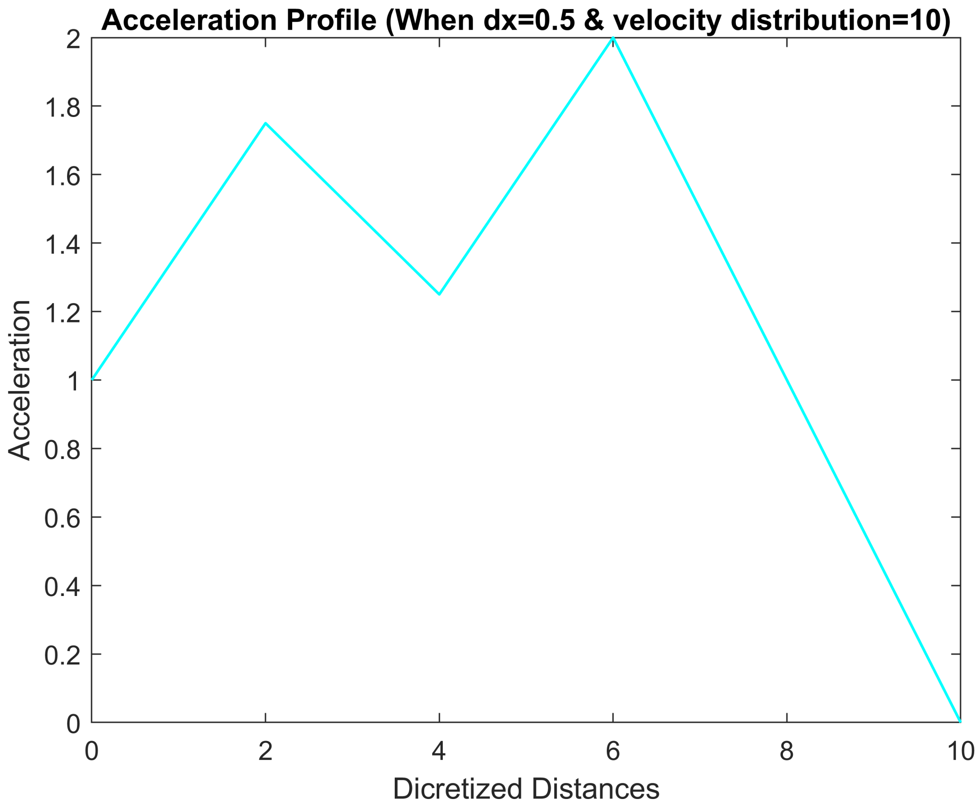
```
accel;

%%%% Forward pass to find values of cost, acceleration and velocity profile

% Starting from rest (velocity = 0)
velocity_profile(1)=0;
accel_profile(1) = accel(end,1);
cost_profile(1) = cost(end,1);

% Finding nodes based on the difference of velocity at every stage from the Velocity grid
veldiff = 0;
for i = 2:Ns
    velocity_profile(i) = real(sqrt(velocity_profile(i-1)^2 + 2*accel_profile(i-1)*dx));
    for s = 1:Nv
        veldiff(s) =  velocity_profile(i)-v_grid(s);
    end
% Saving index of node with least positive velocity difference
    [minDiff, indexOfMin] = min(abs(veldiff));
    velocity_profile(i) = v_grid(indexOfMin);
    accel_profile(i) = accel(indexOfMin,i-1);
    cost_profile(i) = cost(indexOfMin,i-1);
end

% Final stage of rest (cost, velocity and acceleration at last stage should be zero)
velocity_profile(Ns+1)=0;
cost_profile(Ns+1) = 0;
accel_profile(Ns+1) = 0
```

```
accel_profile = 1×6
    1.0000    1.7500    1.2500    2.0000    1.0000         0
```

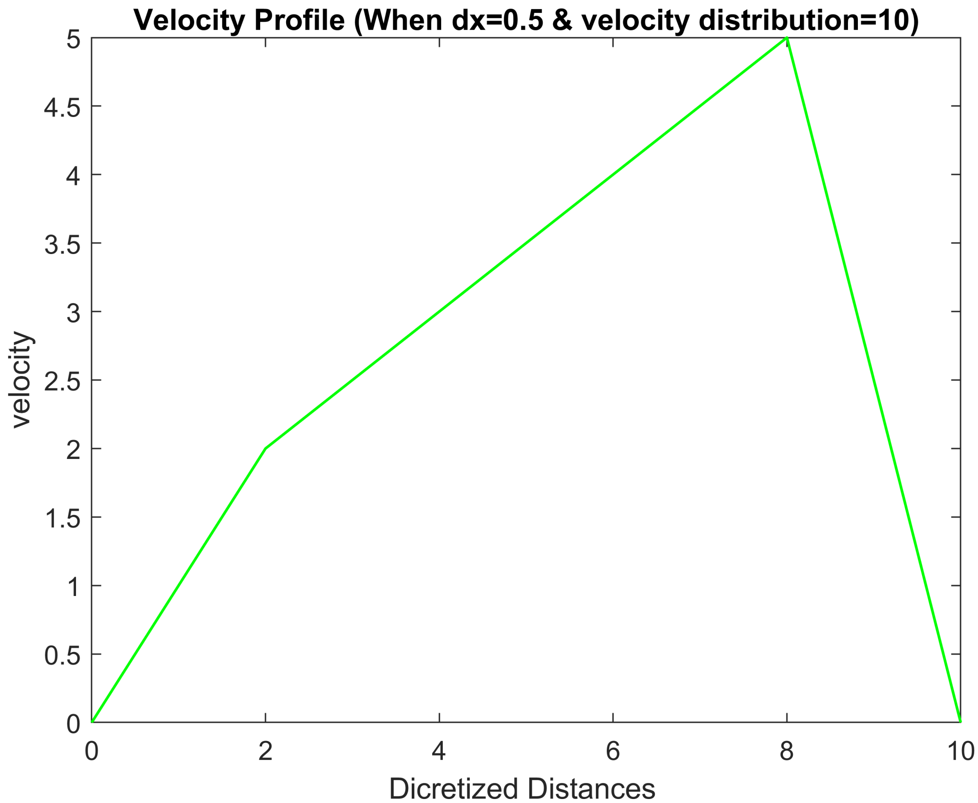
```

%% Cost profile plot
figure
plot(X, cost_profile, LineWidth=1, Color="r")
title("Cost Profile (When dx=0.5 & velocity distribution=10)")
xlabel("Dicretized Distances")
ylabel("Cost")
```




```
%% Acceleration profile plot
figure
plot(X, accel_profile, LineWidth=1, Color="c")
title("Acceleration Profile (When dx=0.5 & velocity distribution=10)")
xlabel("Dicretized Distances")
ylabel("Acceleration")
```




```
%% Velocity profile plot
figure
plot(X, velocity_profile, LineWidth=1, Color="g")
title("Velocity Profile (When dx=0.5 & velocity distribution=10)")
xlabel("Dicretized Distances ")
ylabel("velocity")
```



Case-2) When dx=0.1 and velocity distribution = 50

```
clc;
clear;
v_grid=linspace(0,5,50);
dx = 0.1; % Position increament
X = [0:dx:10];
Ns=numel(X)-1; % Number of stages
Nv=numel(v_grid); % Number of velocity distributions
amin=-3; % Acceleration (accel) lower bound
amax=3; % Acceleration (accel) upper bound
count=1;
cost = zeros(Nv,Ns); % Initialized cost matrix with zeroes

for i=Ns:-1:1
    for j=1:Nv;
        for k=1:Nv;
            dt(j,k)=2*dx/(v_grid(j)+v_grid(k));
            a(j,k)=(v_grid(j)^2-v_grid(k)^2)/(2*dx);
            if a(j,k) < amin || a(j,k) > amax
                dt(j,k) = inf;
            end
        end
    end
    if i==Ns && count==1 % for last stage taking only the '0' velocity in consideration
        cost(:,i) = flip(dt(1,:));
        accel(:,i) = flip(a(1,:));
```

```
        count = 0;
    end
    % for other stages except last stage
    if i~=Ns
        prev_cost = flip(transpose(cost(:,i+1))); % taking stage cost into a column vector
        [least_costs,indices] = min(dt + meshgrid(prev_cost,prev_cost),[],2); % indices -> stor
        cost(:,i) = flip(least_costs);
        % indices -> stores column indices of least cost for every node in a stage
        l=1;
        while l<(Nv+1)
            accel((Nv+1)-l,i) = a(indices(l,1),l); % filling the value of acceleration starting
            l = l+1;
        end
    end
end

%% Printing the 'cost' and 'acceleration' matrix for
% disp('Cost and Acceleration matrix :-')
cost;
accel;

%%%% Forward pass to find values of cost, acceleration and velocity profile
% Cost
[M,I] = min(cost(:,2:Ns),[],1, "linear"); % Taking the least cost and its index
disp('Cost profile :-')
```

```
Cost profile :-
```

```
cost_profile = [cost(Nv,1) M 0]
```

```
cost_profile = 1×101
    4.6417    3.9767    3.9429    3.9091    3.8754    3.8416    3.8078    3.7740 ⋯
```

```

% Acceleration
disp('Acceleration profile :-')
```

```
Acceleration profile :-
```

```
accel_profile = [accel(Nv,1) accel(I+Nv)  0] % Taking the acceleration values corresponding to
```

```
accel_profile = 1×101
    2.5510         0         0         0         0         0         0         0 ⋯
```

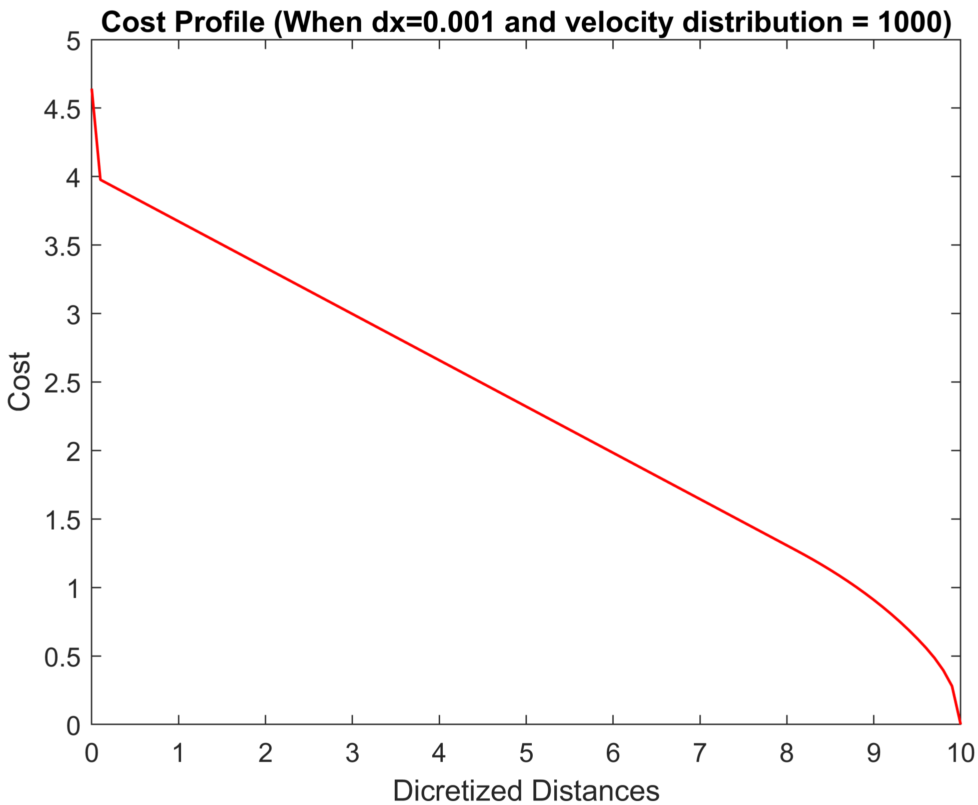
```

% Velocity
velocity_profile(1) = 0;
for m = 2:Ns
    velocity_profile(m) = real(sqrt(velocity_profile(m-1)^2 + 2*accel_profile(m-1)*dx));
end
velocity_profile(Ns+1) = 0;
disp('Velocity profile :-')
```

```
Velocity profile :-
```

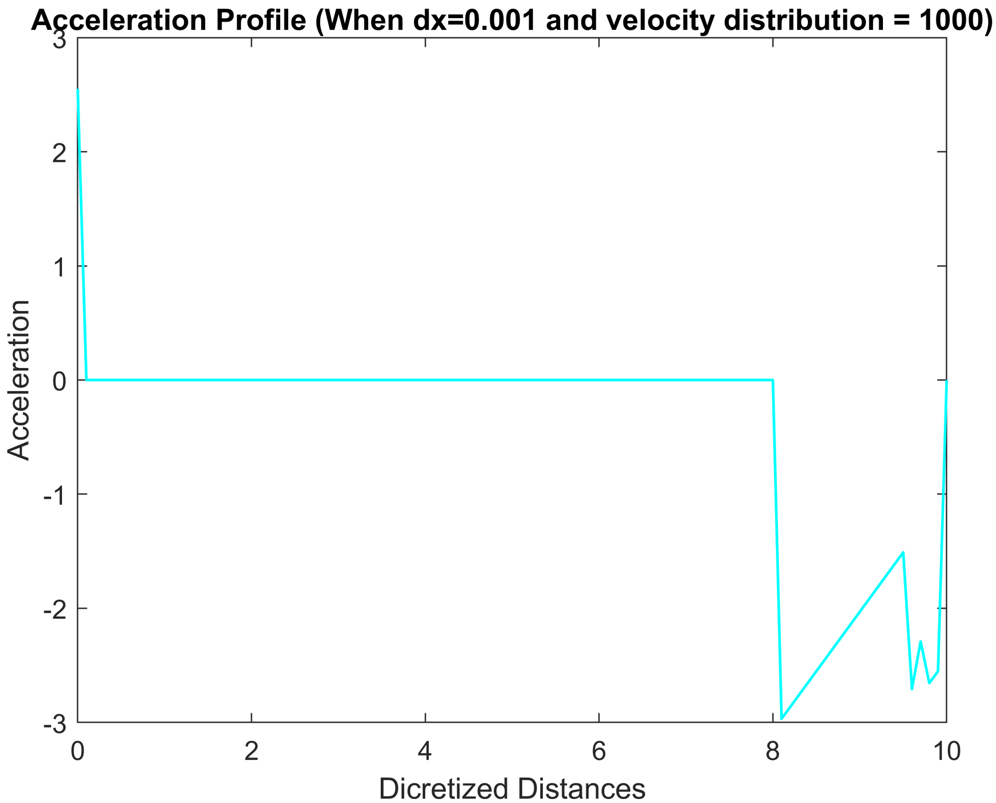
```
velocity_profile
```

```
velocity_profile = 1×101
         0    0.7143    0.7143    0.7143    0.7143    0.7143    0.7143    0.7143 ···
```

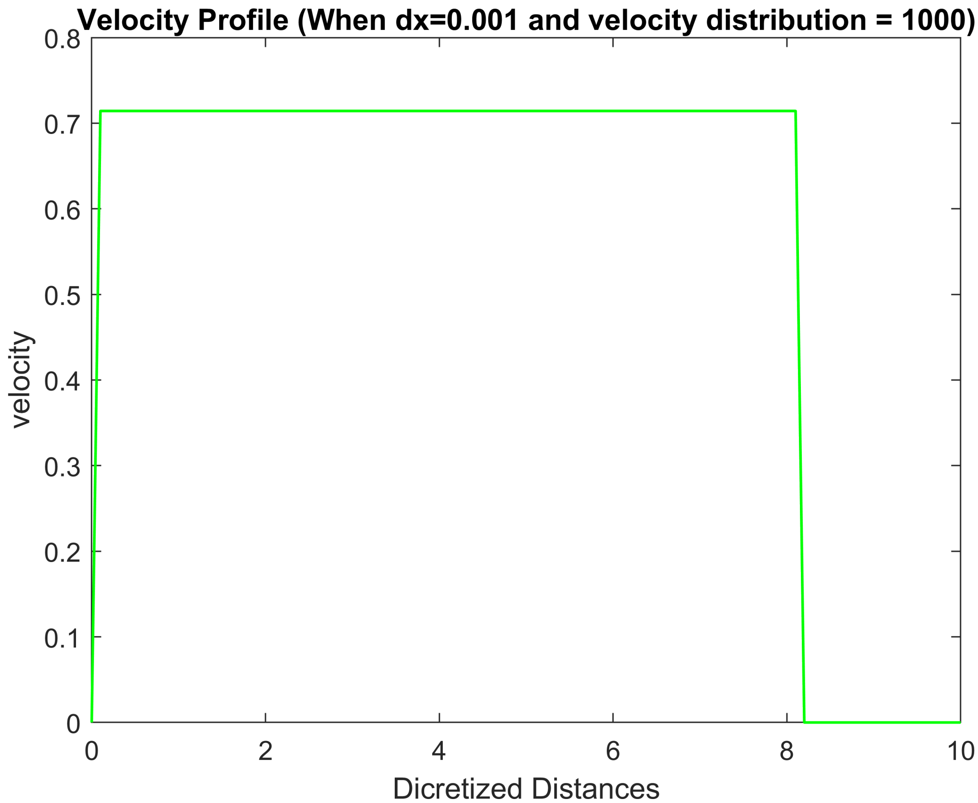
```
%% Cost profile plot
figure
plot(X, cost_profile, LineWidth=1, Color="r")
title("Cost Profile (When dx=0.001 and velocity distribution = 1000)")
xlabel("Dicretized Distances")
ylabel("Cost")
xticks(0:Ns);
```




```
%% Acceleration profile plot
figure
plot(X, accel_profile, LineWidth=1, Color="c")
title("Acceleration Profile (When dx=0.001 and velocity distribution = 1000)")
xlabel("Dicretized Distances")
ylabel("Acceleration")
```




```
%% Velocity profile plot
figure
plot(X, velocity_profile, LineWidth=1, Color="g")
title("Velocity Profile (When dx=0.001 and velocity distribution = 1000)")
xlabel("Dicretized Distances")
ylabel("velocity")
```



Case-3) When dx=0.01 and velocity distribution = 50

```
clc;
clear;
v_grid=linspace(0,5,50);
dx = 0.01; % Position increament
X = [0:dx:10];
Ns=numel(X)-1; % Number of stages
Nv=numel(v_grid); % Number of velocity distributions
amin=-3; % Acceleration (accel) lower bound
amax=3; % Acceleration (accel) upper bound
count=1;
cost = zeros(Nv,Ns); % Initialized cost matrix with zeroes

for i=Ns:-1:1
    for j=1:Nv;
        for k=1:Nv;
            dt(j,k)=2*dx/(v_grid(j)+v_grid(k));
            a(j,k)=(v_grid(j)^2-v_grid(k)^2)/(2*dx);
            if a(j,k) < amin || a(j,k) > amax
                dt(j,k) = inf;
            end
        end
    end
    if i==Ns && count==1 % for last stage taking only the '0' velocity in consideration
        cost(:,i) = flip(dt(1,:));
        accel(:,i) = flip(a(1,:));
```

```
        count = 0;
    end
    % for other stages except last stage
    if i~=Ns
        prev_cost = flip(transpose(cost(:,i+1))); % taking stage cost into a column vector
        [least_costs,indices] = min(dt + meshgrid(prev_cost,prev_cost),[],2); % indices -> stor
        cost(:,i) = flip(least_costs);
        % indices -> stores column indices of least cost for every node in a stage
        l=1;
        while l<(Nv+1)
            accel((Nv+1)-l,i) = a(indices(l,1),l); % filling the value of acceleration starting
            l = l+1;
        end
    end
end

%% Printing the 'cost' and 'acceleration' matrix for
% disp('Cost and Acceleration matrix :-')
cost;
accel;

%% Forward pass to find values of cost, acceleration and velocity profile
% Cost
[M,I] = min(cost(:,2:Ns),[],1, "linear"); % Taking the least cost and its index
disp('Cost profile :-')
```

```
Cost profile :-
```

```
cost_profile = [cost(Nv,1) M 0]
```

```
cost_profile = 1×1001
   32.8104    32.7059    32.6732    32.6405    32.6079    32.5752    32.5425    32.5099 ⋯
```

```

% Acceleration
disp('Acceleration profile :-')
```

```
Acceleration profile :-
```

```
accel_profile = [accel(Nv,1) accel(I+Nv)  0] % Taking the acceleration values corresponding to
```

```
accel_profile = 1×1001
    2.0825         0         0         0         0         0         0         0 ⋯
```

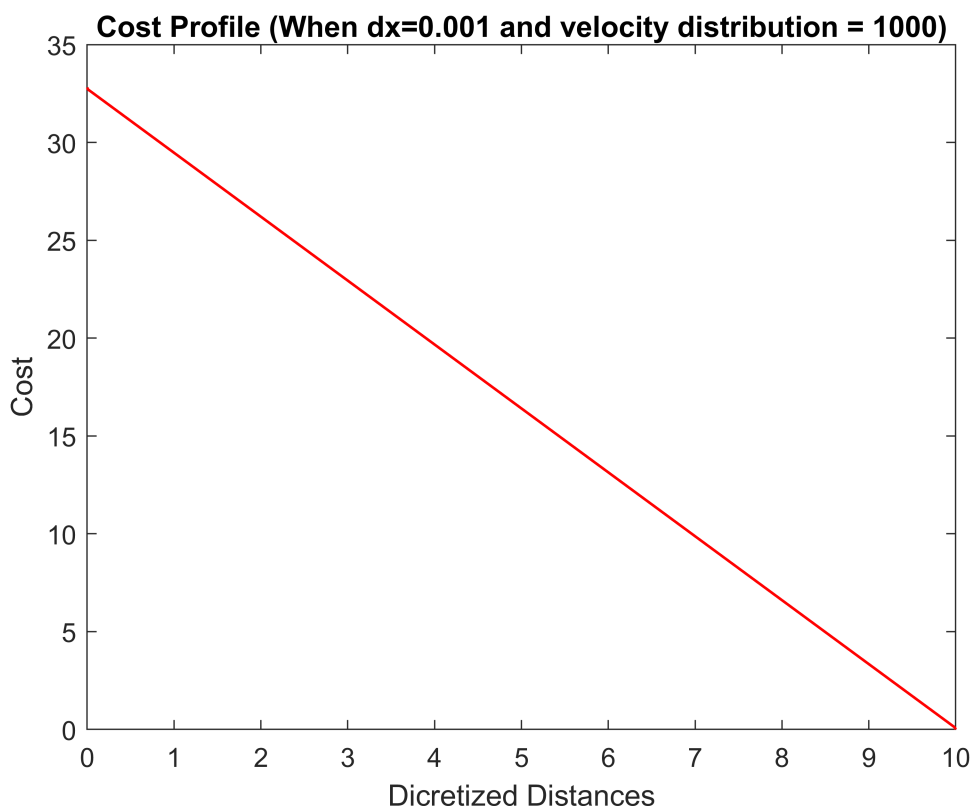
```

% Velocity
velocity_profile(1) = 0;
for m = 2:Ns+1
    velocity_profile(m) = real(sqrt(velocity_profile(m-1)^2 + 2*accel_profile(m-1)*dx));
end
velocity_profile(Ns+1) = 0;
disp('Velocity profile :-')
```

```
Velocity profile :-
```

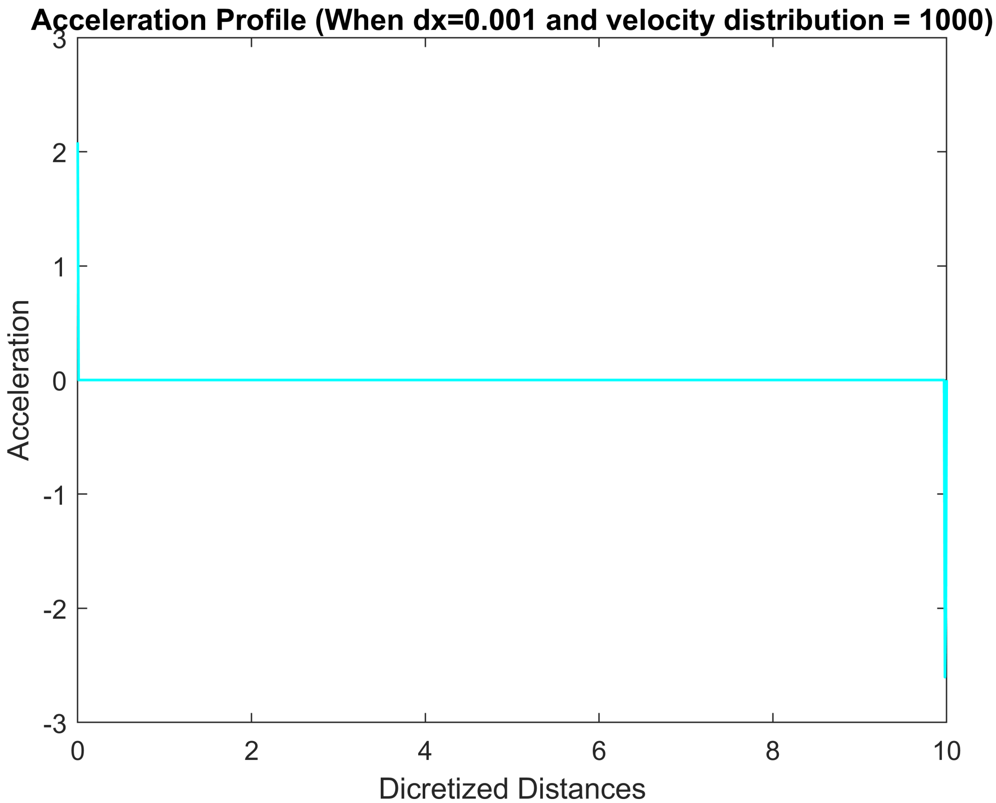
```
velocity_profile
```

```
velocity_profile = 1×1001
         0    0.2041    0.2041    0.2041    0.2041    0.2041    0.2041    0.2041 ···
```

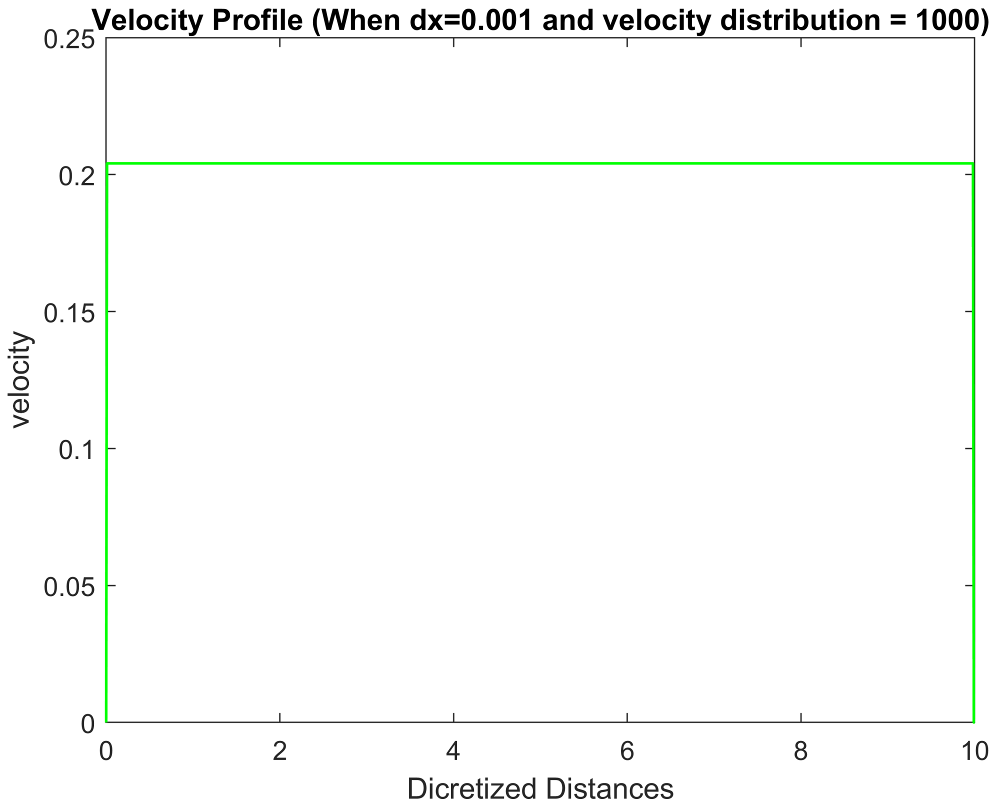
```
%% Cost profile plot
figure
plot(X, cost_profile, LineWidth=1, Color="r")
title("Cost Profile (When dx=0.001 and velocity distribution = 1000)")
xlabel("Dicretized Distances")
ylabel("Cost")
xticks(0:Ns);
```




```
%% Acceleration profile plot
figure
plot(X, accel_profile, LineWidth=1, Color="c")
title("Acceleration Profile (When dx=0.001 and velocity distribution = 1000)")
xlabel("Dicretized Distances")
ylabel("Acceleration")
```




```
%% Velocity profile plot
figure
plot(X, velocity_profile, LineWidth=1, Color="g")
title("Velocity Profile (When dx=0.001 and velocity distribution = 1000)")
xlabel("Dicretized Distances")
ylabel("velocity")
```



Question-3

Taking dx=2 and velocity distribution = 6

(Assuming regeneration; therefore considering the negative cost values)

Case-1) When q=0; i.e., cost function does not depend on the trip time

```
clc;
clear;
v_grid=linspace(0,5,6);
dx=2; % position increament
X=[0:dx:10];
Ns=numel(X)-1; % number of stages
Nv=numel(v_grid); % number of velocity distributions
amin=-3; % acceleration (accel) lower bound
amax=3; % acceleration (accel) upper bound
m=2; % mass of the cart
A=0.1; % frontal area
Cd=0.4; % drag coefficient
rho=1.204; % air density
mu=0.2; % rolling resistance coefficient
g=10; % acceleration due to gravity
cost = zeros(Nv,Ns); % Initialized cost matrix with zeroes
C1 = 1/2*Cd*rho*A;
```

```
q=0;
count=1;

for i=Ns:-1:1
    for j=1:Nv;
        for k=1:Nv;
            a(j,k)=(v_grid(j)^2-v_grid(k)^2)/(2*dx);
            dE(j,k)= (m*a(j,k)+mu*m*g)*dx + C1*(1/2*dx)*(v_grid(j).^2+v_grid(k).^2) + q*2*dx/(v
            if a(j,k) < amin || a(j,k) > amax
                dE(j,k) = inf;
            end
        end
    end
    if i==Ns && count==1 % for last stage taking only the '0' velocity in consideration
        cost(:,i) = flip(dE(1,:));
        accel(:,i) = flip(a(1,:));
        count = 0;
    end
    % for other stages except last stage
    if i~=Ns
        prev_cost = flip(transpose(cost(:,i+1))); % taking stage cost into a column vector
        [least_costs,indices] = min(dE + meshgrid(prev_cost,prev_cost),[],2); % indices -> stor
        cost(:,i) = flip(least_costs);
        % indices -> stores column indices of least cost for every node in a stage
        l=1;
        while l<(Nv+1)
            accel((Nv+1)-l,i) = a(indices(l,1),l); % filling the value of acceleration starting
            l = l+1;
        end
    end
end

disp('Q3) When q=0 :-')
```

```
Q3) When q=0 :-
```

```
% disp('Cost and Acceleration matrix :-')
% cost;
% accel;

%% Forward pass to find values of cost, acceleration and velocity profile
% Cost
[M,I] = min(cost(:,2:Ns),[],1, "linear"); % Taking the least cost and its index
disp('Cost profile(q=0) :-')
```

```
Cost profile(q=0) :-
```

```
cost_profile1 = [cost(Nv,1) M 0]
```

```
cost_profile1 = 1×6
   22.5298   14.4816    6.4816   -1.5666   -0.7833         0
```

```

% Acceleration
disp('Acceleration profile(q=0) :-')
```

```
Acceleration profile(q=0) :-
```

```
accel_profile1 = [accel(Nv,1) accel(I+Nv)  0] % Taking the acceleration values corresponding to
```

```
accel_profile1 = 1×6
    0.2500    0.2500    0.2500    2.2500   -2.2500         0
```

```
% Velocity
velocity_profile1(1) = 0;
% accel_profile1 = [accel(Nv,1) v_grid(I+Nv)  0]
for m = 2:Ns+1
    velocity_profile1(m) = real(sqrt(velocity_profile1(m-1)^2 + 2*accel_profile1(m-1)*dx));
end
velocity_profile1(Ns+1) = 0;
disp('Velocity profile(q=0) :-')
```

```
Velocity profile(q=0) :-
```

```
velocity_profile1
```

```
velocity_profile1 = 1×6
         0    1.0000    1.4142    1.7321    3.4641         0
```

Case-2) When q=2; i.e., cost function does not depend on the trip time

```
v_grid=linspace(0,5,6);
dx=2; % position increament
X=[0:dx:10];
Ns=numel(X)-1; % number of stages
Nv=numel(v_grid); % number of velocity distributions
amin=-3; % acceleration (accel) lower bound
amax=3; % acceleration (accel) upper bound
m=2; % mass of the cart
A=0.1; % frontal area
Cd=0.4; % drag coefficient
rho=1.204; % air density
mu=0.2; % rolling resistance coefficient
g=10; % acceleration due to gravity
cost = zeros(Nv,Ns); % Initialized cost matrix with zeroes
C1 = 1/2*Cd*rho*A;
q=2;
count=1;

for i=Ns:-1:1
    for j=1:Nv;
        for k=1:Nv;
            a(j,k)=(v_grid(j)^2-v_grid(k)^2)/(2*dx);
            dE(j,k)= (m*a(j,k)+mu*m*g)*dx + C1*(1/2*dx)*(v_grid(j).^2+v_grid(k).^2) + q*2*dx/(v
            if a(j,k) < amin || a(j,k) > amax
                dE(j,k) = inf;
            end
        end
    end
    if i==Ns && count==1 % for last stage taking only the '0' velocity in consideration
```

```
        cost(:,i) = flip(dE(1,:));
        accel(:,i) = flip(a(1,:));
        count = 0;
    end
    % for other stages except last stage
    if i~=Ns
        prev_cost = flip(transpose(cost(:,i+1))); % taking stage cost into a column vector
        [least_costs,indices] = min(dE + meshgrid(prev_cost,prev_cost),[],2); % indices -> stor
        cost(:,i) = flip(least_costs);
        % indices -> stores column indices of least cost for every node in a stage
        l=1;
        while l<(Nv+1)
            accel((Nv+1)-l,i) = a(indices(l,1),l); % filling the value of acceleration starting
            l = l+1;
        end
    end
end

disp('Q3) When q=2 :-')
```

```
Q3) When q=2 :-
```

```
% disp('Cost and Acceleration matrix :-')
% cost;
% accel;

%% Forward pass to find values of cost, acceleration and velocity profile
% Cost
[M,I] = min(cost(:,2:Ns),[],1, "linear"); % Taking the least cost and its index during forward
disp('Cost profile(q=2) :-')
```

```
Cost profile(q=2) :-
```

```
cost_profile2 = [cost(Nv,1) M 0]
```

```
cost_profile2 = 1×6
   33.0233   23.2565   13.5335    3.7668    1.8834         0
```

```

% Acceleration
disp('Acceleration profile(q=2) :-')
```

```
Acceleration profile(q=2) :-
```

```
accel_profile2 = [accel(Nv,1) accel(I+Nv)  0] % Taking the acceleration values corresponding to
```

```
accel_profile2 = 1×6
    2.2500    2.2500    2.2500    2.2500   -2.2500         0
```

```

% Velocity
velocity_profile2(1) = 0;
for m = 2:Ns+1
    velocity_profile2(m) = real(sqrt(velocity_profile2(m-1)^2 + 2*accel_profile2(m-1)*dx));
end
velocity_profile2(Ns+1) = 0;
```

```
disp('Velocity profile(q=2) :-')
```

```
Velocity profile(q=2) :-
```

```
velocity_profile2
```

```
velocity_profile2 = 1×6
         0    3.0000    4.2426    5.1962    6.0000         0
```

Case-4) When q=4; i.e., cost function does not depend on the trip time

```
v_grid=linspace(0,5,6);
dx=2; % position increament
X=[0:dx:10];
Ns=numel(X)-1; % number of stages
Nv=numel(v_grid); % number of velocity distributions
amin=-3; % acceleration (accel) lower bound
amax=3; % acceleration (accel) upper bound
m=2; % mass of the cart
A=0.1; % frontal area
Cd=0.4; % drag coefficient
rho=1.204; % air density
mu=0.2; % rolling resistance coefficient
g=10; % acceleration due to gravity
cost = zeros(Nv,Ns); % Initialized cost matrix with zeroes
C1 = 1/2*Cd*rho*A;
q=4;
count=1;

for i=Ns:-1:1
    for j=1:Nv;
        for k=1:Nv;
            a(j,k)=(v_grid(j)^2-v_grid(k)^2)/(2*dx);
            dE(j,k)= (m*a(j,k)+mu*m*g)*dx + C1*(1/2*dx)*(v_grid(j).^2+v_grid(k).^2) + q*2*dx/(v
            if a(j,k) < amin || a(j,k) > amax
                dE(j,k) = inf;
            end
        end
    end
    if i==Ns && count==1 % for last stage taking only the '0' velocity in consideration
        cost(:,i) = flip(dE(1,:));
        accel(:,i) = flip(a(1,:));
        count = 0;
    end
    % for other stages except last stage
    if i~=Ns
        prev_cost = flip(transpose(cost(:,i+1))); % taking stage cost into a column vector
        [least_costs,indices] = min(dE + meshgrid(prev_cost,prev_cost),[],2); % indices -> stor
        cost(:,i) = flip(least_costs);
        % indices -> stores column indices of least cost for every node in a stage
        l=1;
        while l<(Nv+1)
            accel((Nv+1)-l,i) = a(indices(l,1),l); % filling the value of acceleration starting
            l = l+1;
```

```
        end
    end
end

disp('Q3) When q=4 :-')
```

```
Q3) When q=4 :-
```

```
% disp('Cost and Acceleration matrix :-')
% cost;
% accel;

%% Forward pass to find values of cost, acceleration and velocity profile
% Cost
[M,I] = min(cost(:,2:Ns),[],1, "linear"); % Taking the least cost and its index
disp('Cost profile(q=4) :-')
```

```
Cost profile(q=4) :-
```

```
cost_profile3 = [cost(Nv,1) M 0]
```

```
cost_profile3 = 1×6
   41.6461   30.5663   19.8910    8.7909    4.5501         0
```

```

% Acceleration
disp('Acceleration profile(q=4) :-')
```

```
Acceleration profile(q=4) :-
```

```
accel_profile3 = [accel(Nv,1) accel(I+Nv)  0] % Taking the acceleration values corresponding to
```

```
accel_profile3 = 1×6
    2.2500    2.0000    2.0000    2.0000   -2.2500         0
```

```

% Velocity
velocity_profile3(1) = 0;
for m = 2:Ns+1
    velocity_profile3(m) = real(sqrt(velocity_profile3(m-1)^2 + 2*accel_profile3(m-1)*dx));
end
velocity_profile3(Ns+1) = 0;
disp('Velocity profile(q=4) :-')
```

```
Velocity profile(q=4) :-
```

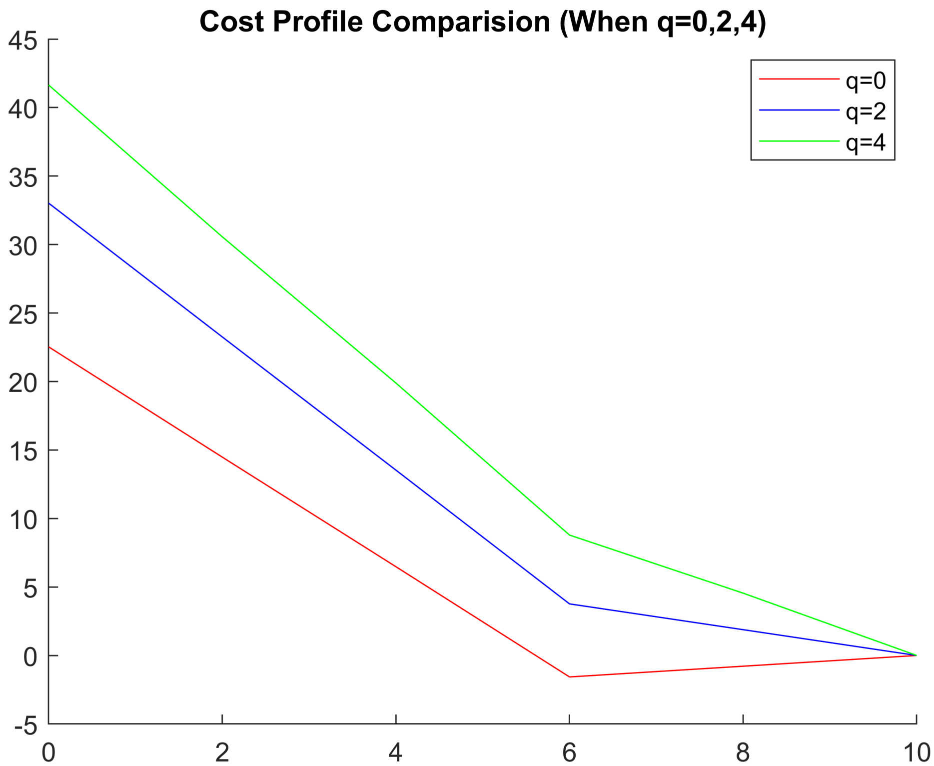
```
velocity_profile3
```

```
velocity_profile3 = 1×6
         0    3.0000    4.1231    5.0000    5.7446         0
```

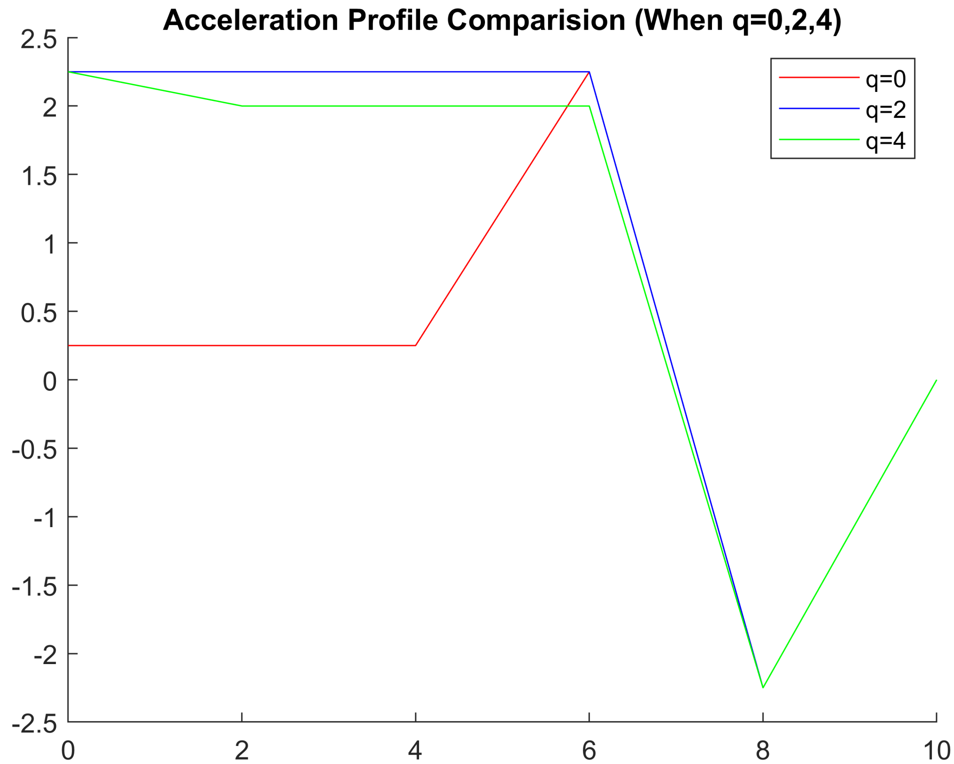
Comparing cost, acceleration and velocity profiles (when q=1,2,4)

```
%% Cost profile plot comparision
figure
hold on
plot(X,cost_profile1, 'r')
```

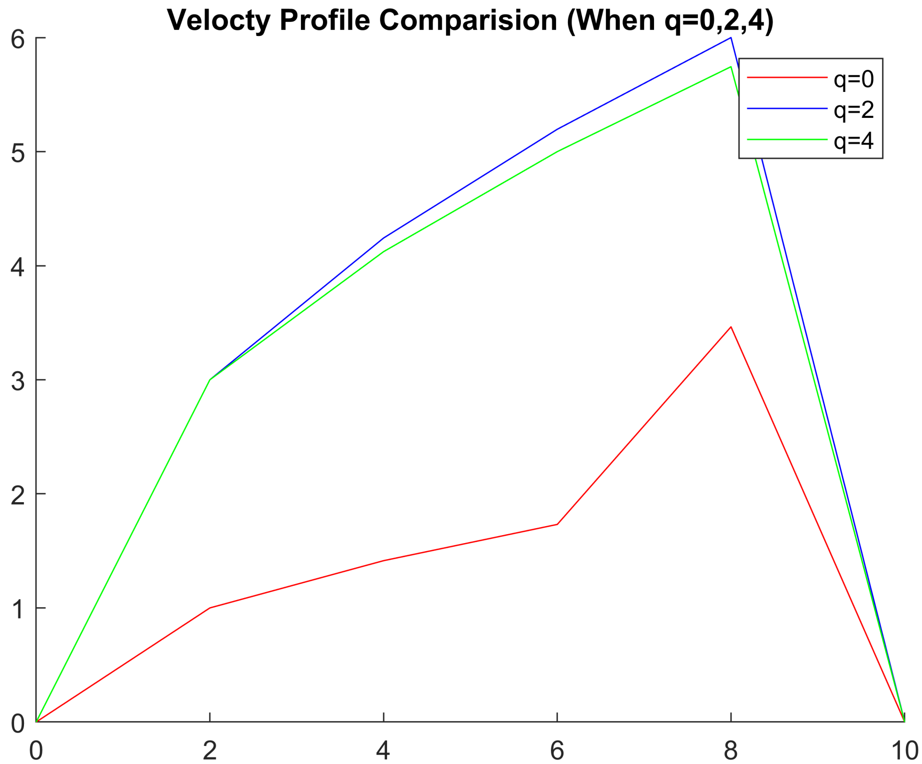
```
title('Cost Profile Comparision (When q=0,2,4)')
plot(X,cost_profile2, 'b')
plot(X,cost_profile3, 'g')
legend("q=0","q=2","q=4")
hold off
```




```
%% Cost acceleration plot comparision
figure
hold on
plot(X,accel_profile1, 'r')
title('Acceleration Profile Comparision (When q=0,2,4)')
plot(X,accel_profile2, 'b')
plot(X,accel_profile3, 'g')
legend("q=0","q=2","q=4")
hold off
```




```
%% Cost velocity plot comparision
figure
hold on
plot(X,velocity_profile1, 'r')
title('Velocty Profile Comparision (When q=0,2,4)')
plot(X,velocity_profile2, 'b')
plot(X,velocity_profile3, 'g')
legend("q=0","q=2","q=4")
hold off
```



As seen in the graphs above;

1. There was a increase in cost with the increase in the value of `q`, which is reasonable since including the 'total trip time' into the cost function should increase the value of the cost function, as correspondigly velocity and acceleration is also increased.
2. In the velocity graph, it can be seen that velocities in the cases where `q` is non-zero are higher compared to when the value of `q` is equal to `0` and 'total trip time' was ignored.
3. Similarly, from the acceleration graph , it can be seen that including the 'total trip time' in the cost function resulted in a higher value of acceleration.
4. Lastly, when `q` was equal to `0`, as accelration went down the value of cost function went below `0` impying the regeneration capability.

Question-4

```
clc;
clear;
v_grid=linspace(0,5,6); % Velocity distribution of 100
dx=2; % Position increament 0.5
X=[0:dx:10];
Ns=numel(X)+1; % Number of discretized distance
Nv=numel(v_grid); % Number of velocity distributions

%% Taking v^2 as the variable to optimze\
```

```
x = zeros(1,Ns)';

% Initial point
x0 = zeros(Ns,1);

% Equality constraints
Aeq = zeros(2,Ns);
Aeq(1,1)=1;
Aeq(2,Ns)=1;
Beq = [0; 0];

% Linear inequality constraints
A = zeros(2*length(x)-2,length(x));
b = ones(length(A),1);
b = b.*(3*2*dx);

% Lower and upper bound limits for v^2 (i.e., 0 and 25)
lb = zeros(1,Ns);
ub = (ones(1,Ns)).*25;

j = 1;
for i=2:1:length(A)/2+1
    A(j,i-1) = 1;
    A(j,i) = -1;
    j =j+1;
end
for k=2:1:size(A,1)/2+1
    A(j,k-1) = -1;
    A(j,k) = 1;
    j=j+1;
end

x = fmincon(@cal,x0,A,b,Aeq,Beq,lb,ub)
```

```
Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in
feasible directions, to within the value of the optimality tolerance,
and constraints are satisfied to within the value of the constraint tolerance.

<stopping criteria details>
x = 7×1
    0.0000
    5.5651
    5.6636
    5.6649
    5.6636
    5.5651
    0.0000
```

```
display('Velocity values - ')
```

```
Velocity values -
```

```
% Taking square root of output from fmincon to find values for 'v'
velocity = sqrt(x)
```

```
velocity = 7×1
    0.0008
    2.3590
    2.3798
    2.3801
    2.3798
    2.3590
    0.0008
```

```
function cost = cal(x)
    v=sqrt(x);
    v_i = v(1:length(x)-1);
    v_i_2 = v(2:length(x));
    cost = 0;
    for j = 1:length(v_i)
        temp = v_i(j)^2 - v_i_2(j)^2;
        temp = temp/(2*2);
        cost = cost + temp;
    end
end
```