Mathematics

Textbook for Class VII

0756

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NATIONAL COUNCIL OF EDUCATIONAL RESEARCH AND TRAINING

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Foreword

The National Curriculum Framework (NCF), 2005, recommends that children's life at school must be linked to their life outside the school. This principle marks a departure from the legacy of bookish learning which continues to shape our system and causes a gap between the school, home and community. The syllabi and textbooks developed on the basis of NCF signify an attempt to implement this basic idea. They also attempt to discourage rote learning and the maintenance of sharp boundaries between different subject areas. We hope these measures will take us significantly further in the direction of a child-centred system of education outlined in the National Policy on Education (1986).

The success of this effort depends on the steps that school principals and teachers will take to encourage children to reflect on their own learning and to pursue imaginative activities and questions. We must recognise that, given space, time and freedom, children generate new knowledge by engaging with the information passed on to them by adults. Treating the prescribed textbook as the sole basis of examination is one of the key reasons why other resources and sites of learning are ignored. Inculcating creativity and initiative is possible if we perceive and treat children as participants in learning, not as receivers of a fixed body of knowledge.

These aims imply considerable change in school routines and mode of functioning. Flexibility in the daily time-table is as necessary as rigour in implementing the annual calendar so that the required number of teaching days are actually devoted to teaching. The methods used for teaching and evaluation will also determine how effective this textbook proves for making children's life at school a happy experience, rather than a source of stress or boredom. Syllabus designers have tried to address the problem of curricular burden by restructuring and reorienting knowledge at different stages with greater consideration for child psychology and the time available for teaching. The textbook attempts to enhance this endeavour by giving higher priority and space to opportunities for contemplation and wondering, discussion in small groups, and activities requiring hands-on experience.

The National Council of Educational Research and Training (NCERT) appreciates the hard work done by the textbook development committee responsible for this book. We wish to thank the Chairperson of the advisory group in science and mathematics, Professor J.V. Narlikar and the Chief Advisor for this book, Dr H.K. Dewan for guiding the work of this committee. Several teachers contributed to the development of this textbook; we are grateful to their principals for making this possible. We are indebted to the institutions and organisations which have generously permitted us to draw upon their resources, material and personnel. We are especially grateful to the members of the National Monitoring Committee, appointed by the Department of Secondary and Higher Education, Ministry of Human Resource Development under the Chairpersonship of Professor Mrinal Miri and Professor G.P. Deshpande, for their valuable time anc contribution. As an organisation committed to systemic reform and continuous improvement in the quality of its products, NCERT welcomes comments and suggestions which will enable us to undertake further revision and refinement.

Director
National Council of Educational
Research and Training

New Delhi
20 November 2006

Rationalisation of Content in the Textbooks

In view of the COVID-19 pandemic, it is imperative to reduce content load on students. The National Education Policy 2020, also emphasises reducing the content load and providing opportunities for experiential learning with creative mindset. In this background, the NCERT has undertaken the exercise to rationalise the textbooks across all classes. Learning Outcomes already developed by the NCERT across classes have been taken into consideration in this exercise.

Contents of the textbooks have been rationalised in view of the following:

- Overlapping with similar content included in other subject areas in the same class
- Similar content included in the lower or higher class in the same subject
- Difficulty level
- Content, which is easily accessible to students without much interventions from teachersand can be learned by children through self-learning or peer-learning
- Content, which is irrelevant in the present context

This present edition, is a reformatted version after carrying out the changes given above.

Preface

The National Curriculum Framework (NCF), 2005 suggests the need for developing the ability for mathematisation in the child. It points out that the aim of learning mathematics is not merely being able to do quantitative calculations but also to develop abilities in the child that would enable her/him to redefine her/his relationship with the World. The NCF-2005 also lays emphasis on development in the children logical abilities as well as abilities to comprehend space, spatial transformations and develop the ability to visualise both these. It recommends that mathematics needs to slowly move towards abstraction even though it starts from concrete experiences and models. The ability to generalise and perceive patterns is an important step in being able to relate to the abstract and logic governed nature of the subject.

We also know that most children in upper primary and secondary classes develop a fear of mathematics and it is one of the reasons for students not being able to continue in schools. NCF-2005 has also mentioned this problem and has therefore emphasised the need to develop a programme which is relevant and meaningful. The need for conceptualising mathematics teaching allows children to explore concepts as well as develop their own ways of solving problems. This also forms corner-stone of the principles highlighted in the NCF-2005.

In Class VI we have begun the process of developing a programme which would help children understand the abstract nature of mathematics while developing in them the ability to construct their own concepts. As suggested by NCF-2005, an attempt has been made to allow multiple ways of solving problems and encouraging children to develop strategies different from each other. There is an emphasis on working with basic principles rather than on memorisation of algorithms and short-cuts.

The Class VII textbook has continued that spirit and has attempted to use language which the children can read and understand themselves. This reading can be in groups or individual and at some places require help and support by the teacher. We also tried to include a variety of examples and opportunities for children to set problems. The appearance of the book has sought to be made pleasant by including many illustrations. The book attempts to engage the mind of the child actively and provides opportunities to use concepts and develop her/his own structures rather than struggling with unnecessarily complicated terms and numbers.

We hope that this book would help all children in their attempt to learn mathematics and would build in them the ability to appreciate its power and beauty. We also hope that this would enable to revisit and consolidate concepts and skills that they have learnt in the primary school. We hope to strengthen the foundation of mathematics, on which further engagement with studies as well as her daily life would become possible in an enriched manner.

The team in developing the textbook consists of many teachers who are experienced and brought to the team the view point of the child and the school. We also had people who have done research in learning of mathematics and those who have been writing textbooks for mathematics for many years. The team has tried to make an effort to remove fear of mathematics from the minds of children and make it a part of their daily routine even outside the school. We had many discussions and a review process with some other teachers of schools across the country. The effort by the team has been to accommodate all the comments.

In the end, I would like to place on record our gratefulness to Prof Krishna Kumar, Director, NCERT, Prof G. Ravindra, Joint Director, NCERT and Prof Hukum Singh, Head, DESM, for giving opportunity to me and the team to work on this challenging task. I am also grateful to

Prof J.V. Narlikar, Chairperson of the Advisory Group in Science and Mathematics for his suggestions. I am also grateful for the support of all those who were part of this team including Prof S.K. Singh Gautam, Dr V.P. Singh and Dr Ashutosh K. Wazalwar from NCERT, who have worked very hard to make this possible. In the end I must thank the Publication Department of NCERT for its support and advice and those from Vidya Bhawan who helped produce the book.

The process of developing materials is a continuous one and we would hope to make this book better. Suggestions and comments on the book are most welcome.

Dr H.K. Dewan
Chief Advisor
Textbook Development Committee

A Note for the Teachers

This book is a continuation of the process and builds on what was initiated in Class VI. We had shared with you the main points reflected in NCF-2005. These include relating mathematics to a wider development of abilities in children, moving away from complex calculations and algorithms following, to understanding and constructing a framework of understanding. The mathematical ideas in the mind of the child grow neither by telling nor by merely giving explanations. For children to learn mathematics, to be confident in it and understand the foundational ideas, they need to develop their own framework of concepts. This would require a classroom where children discuss ideas, look for solutions of problems, set new problems and find not only their own ways of solving problems but also their own definitions with the language they can use and understand. These definitions need not be as general and complete as the standard ones.

In the mathematics class it is important to help children read with understanding the textbook and other references. The reading of materials is not normally considered to be related to learning of mathematics but learning mathematics any further would require the child to comprehend the text. The text in mathematics uses a language that has brevity. It requires the ability to deal with terseness and with symbols, to follow logical arguments and appreciate the need for keeping certain factors and constraints. Children need practice in translating mathematical statements into normal statements expressing ideas in words and vice-a-versa. We would require children to become confident of using language in words and also being able to communicate through mathematical statements.

Mathematics at the upper primary stage is a major challenge and has to perform the dual role of being both close to the experience and environment of the child and being abstract. Children often are not able to work in terms of ideas alone. They need the comfort of context and/or models linked to their experience to find meaning. This stage presents before us the challenge of engaging the children while using the contexts but gradually moving them away from such dependence. So while children should be able to identify the principles to be used in a contextual situation, they should not be dependent or be limited to contexts. As we progress further in the middle school there would be greater requirement from the child to be able to do this.

Learning mathematics is not about remembering solutions or methods but knowing how to solve problems. Problem-solving strategies give learners opportunities to think rationally, enabling them to understand and create methods as well as processes; they become active participants in the construction of new knowledge rather than being passive receivers. Learners need to identify and define a problem, select or design possible solutions and revise or redesign the steps, if required. The role of a teacher gets modified to that of a guide and facilitator. Students need to be provided with activities and challenging problems, along with sets of many problem-solving experiences.

On being presented a problem, children first need to decode it. They need to identify the knowledge required for attempting it and build a model for it. This model could be in the form of an illustration or a situation construct. We must remember that for generating proofs in geometry the figures constructed are also models of the ideal dimensionless figure. These diagrams are, however, more abstract than the concrete models required for attempting problems in arithmetic and algebra. Helping children to develop the ability to construct appropriate models by breaking up the problems and evolving their own strategies and analysis of problems is extremely important. This should replace prescriptive algorithms to solve problems.

Teachers are expected to encourage cooperative learning. Children learn a lot in purposeful conversation with each other. Our classrooms should develop in the students the desire and capacity to learn from each other rather than compete. Conversation is not noise and consultation is not cheating. It is a challenge to make possible classroom groups that benefit the most from being with each other and in which each child contributes to the learning of the group. Teachers must recognise that different children and different groups will use distinct strategies. Some of these strategies would appear to be more efficient and some not as efficient. They would reflect the modelling done by each group and would indicate the process of thinking used. It is inappropriate to identify the best strategy or pull down incorrect strategies. We need to record all strategies adopted and analyse them. During this, it is crucial to discuss why some of the strategies are unsuccessful. The class as a group can improve upon the ineffective and unsuccessful strategies and correct them. This implies that we need to complete each strategy rather than discard some as incorrect or inappropriate. Exposures to a variety of strategies would deepen mathematical understanding and ability to learn from others. This would also help them to understand the importance of being aware of what one is doing.

Enquiry to understand is one of the natural ways by which students acquire and construct knowledge. The process can even begin with casual observations and end in generation and acquisition of knowledge. This can be aided by providing examples for different forms of questioning-explorative, open-ended, contextual, error detection etc. Students need to get exposed to challenging investigations. For example in geometry there could be things like, experimenting with suitable nets for solids, visualising solids through shadow play, slicing and elevations etc. In arithmetic we can make them explore relationships among members, generalise the relationships, discover patterns and rules and then form algebraic relations etc.

Children need the opportunity to follow logical arguments and find loopholes in the arguments presented. This will lead them to understand the requirement of a proof.

At this stage topics like Geometry are poised to enter a formal stage. Provide activities that encourage students to exercise creativity and imagination while discovering geometric vocabulary and relationships using simple geometric tools.

Mathematics has to emerge as a subject of exploration and creation rather than an exercise of finding answers to old and complicated problems. There is a need to encourage children to find many different ways to solve problems. They also need to appreciate the use of many alternative algorithms and strategies that may be adopted to solve a problem.

Topics like Integers, Fractions and Decimals, Symmetry have been presented here by linking them with their introductory parts studied in earlier classes. An attempt has been made to link chapters with each other and the ideas introduced in the initial chapters have been used to evolve concepts in the subsequent chapters. Please devote enough time to the ideas of negative integers, rational numbers, exploring statements in Geometry and visualising solids shapes.

We hope that the book will help children learn to enjoy mathematics and be confident in the concepts introduced. We want to recommend the creation of opportunity for thinking individually and collectively. Group discussions need to become a regular feature of mathematics classroom thereby making learners confident about mathematics and make the fear of mathematics a thing of past.

We look forward to your comments and suggestions regarding the book and hope that you will send interesting exercises, activities and tasks that you develop during the course of teaching, to be included in the future editions.

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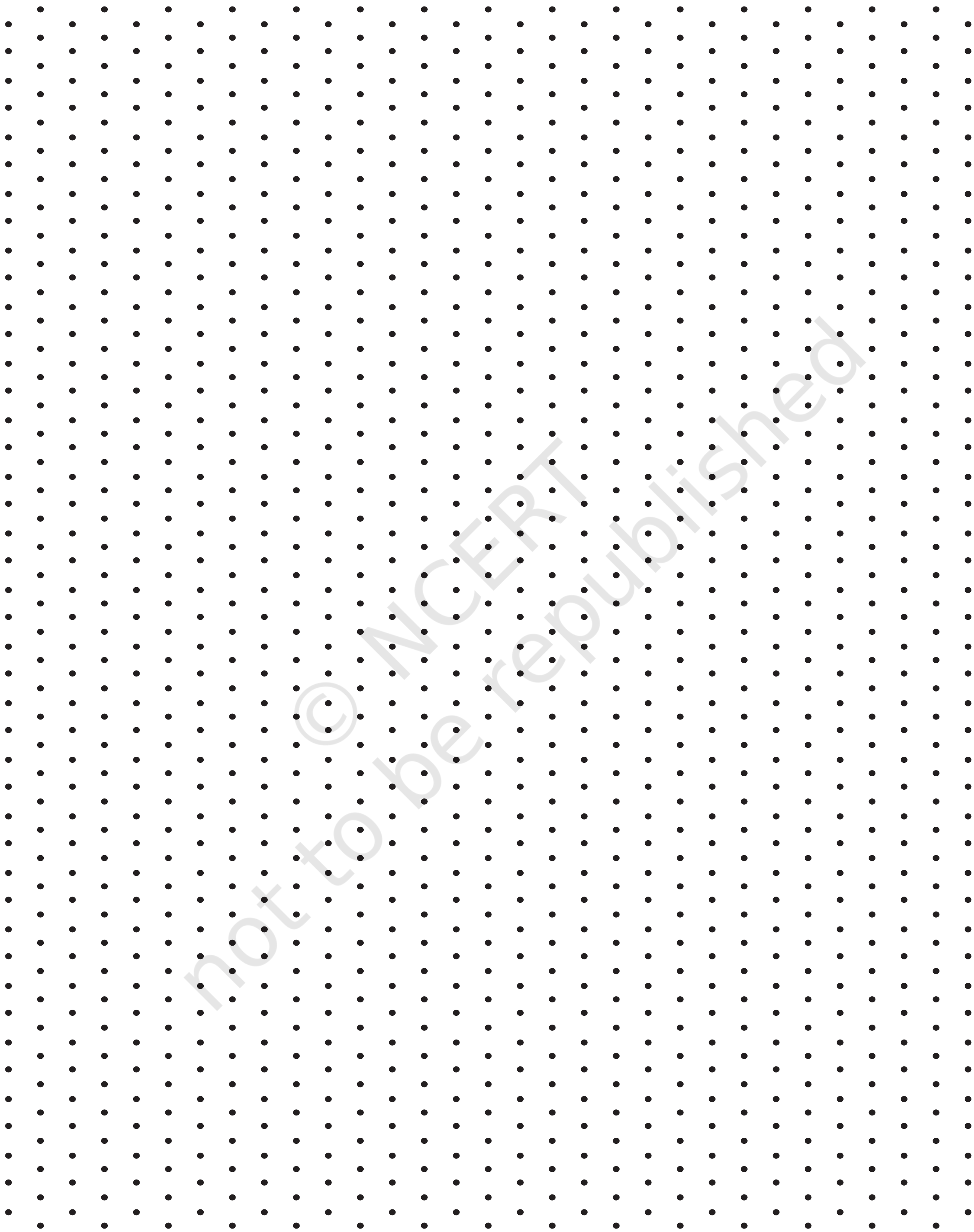

Chapter 1

Integers

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1.1 Properties of Addition and Subtraction of Integers

We have learnt about whole numbers and integers in Class VI. We have also learnt about addition and subtraction of integers.

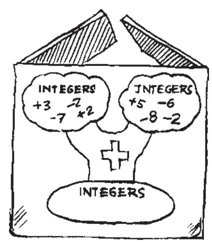
1.1.1 Closure under Addition

We have learnt that sum of two whole numbers is again a whole number. For example, $17 + 24 = 41$ which is again a whole number. We know that, this property is known as the closure property for addition of the whole numbers.

Let us see whether this property is true for integers or not.

Following are some pairs of integers. Observe the following table and complete it.

Statement	Observation
(i) $17 + 23 = 40$	Result is an integer
(ii) $(–10) + 3 =$ _____	__________
(iii) $(– 75) + 18 =$ _____	__________
(iv) $19 + (– 25) = – 6$	Result is an integer
(v) $27 + (– 27) =$ _____	__________
(vi) $(– 20) + 0 =$ _____	__________
(vii) $(– 35) + (– 10) =$ _____	__________



What do you observe? Is the sum of two integers always an integer?

Did you find a pair of integers whose sum is not an integer?

Since addition of integers gives integers, we say **integers are closed under addition**.

In general, **for any two integers *a* and *b*, *a* + *b* is an integer.**

1.1.2 Closure under Subtraction

What happens when we subtract an integer from another integer? Can we say that their difference is also an integer?

Observe the following table and complete it:

Statement	Observation
(i) $7 - 9 = -2$	Result is an integer
(ii) $17 - (-21) =$ ________	________
(iii) $(-8) - (-14) = 6$	Result is an integer
(iv) $(-21) - (-10) =$ ________	________
(v) $32 - (-17) =$ ________	________
(vi) $(-18) - (-18) =$ ________	________
(vii) $(-29) - 0 =$ ________	________

What do you observe? Is there any pair of integers whose difference is not an integer? Can we say integers are closed under subtraction? Yes, we can see that *integers are closed under subtraction*.

Thus, *if a and b are two integers then a – b is also an intger*. Do the whole numbers satisfy this property?

1.1.3 Commutative Property

We know that $3 + 5 = 5 + 3 = 8$, that is, the whole numbers can be added in any order. In other words, addition is commutative for whole numbers.

Can we say the same for integers also?

We have $5 + (-6) = -1$ and $(-6) + 5 = -1$

So, $5 + (-6) = (-6) + 5$

Are the following equal?

(i) $(-8) + (-9)$ and $(-9) + (-8)$

(ii) $(-23) + 32$ and $32 + (-23)$

(iii) $(-45) + 0$ and $0 + (-45)$

Try this with five other pairs of integers. Do you find any pair of integers for which the sums are different when the order is changed? Certainly not. We say that *addition is commutative for integers.*

In general, for any two integers a and b, we can say

$$a + b = b + a$$

- We know that subtraction is not commutative for whole numbers. Is it commutative for integers?

  Consider the integers 5 and (–3).

  Is 5 – (–3) the same as (–3) –5? No, because 5 – (–3) = 5 + 3 = 8, and (–3) – 5 = – 3 – 5 = – 8.

  Take atleast five different pairs of integers and check this.

  We conclude that subtraction is not commutative for integers.

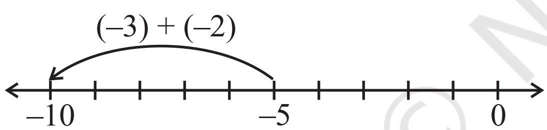
1.1.4 Associative Property

Observe the following examples:

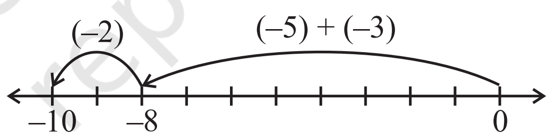
Consider the integers –3, –2 and –5.

Look at (–5) + [(–3) + (–2)] and [(–5) + (–3)] + (–2).

In the first sum (–3) and (–2) are grouped together and in the second (–5) and (–3) are grouped together. We will check whether we get different results.



(–5) + [(–3) + (–2)]



[(–5) + (–3)] + (–2)

In both the cases, we get –10.

i.e., (–5) + [(–3) + (–2)] = [(–5) + (–2)] + (–3)

Similarly consider –3 , 1 and –7.

(–3) + [1 + (–7)] = –3 + ___________ = ___________

[(–3) + 1] + (–7) = –2 + ___________ = ___________

Is (–3) + [1 + (–7)] same as [(–3) + 1] + (–7)?

Take five more such examples. You will not find any example for which the sums are different. *Addition is associative for integers*.

In general for any integers a, b and c, we can say

$$a + (b + c) = (a + b) + c$$

1.1.5 Additive Identity

When we add zero to any whole number, we get the same whole number. Zero is an additive identity for whole numbers. Is it an additive identity again for integers also?

Observe the following and fill in the blanks:

(i) $(-8) + 0 = -8$ (ii) $0 + (-8) = -8$

(iii) $(-23) + 0 =$ ______ (iv) $0 + (-37) = -37$

(v) $0 + (-59) =$ ______ (vi) $0 +$ ______ $= -43$

(vii) $-61 +$ ______ $= -61$ (viii) ______ $+ 0 =$ ______

The above examples show that zero is an additive identity for integers.

You can verify it by adding zero to any other five integers.

In general, for any integer a

$$a + 0 = a = 0 + a$$

Try These

1. Write a pair of integers whose sum gives

(a) a negative integer (b) zero

(c) an integer smaller than both the integers. (d) an integer smaller than only one of the integers.

(e) an integer greater than both the integers.

2. Write a pair of integers whose difference gives

(a) a negative integer. (b) zero.

(c) an integer smaller than both the integers. (d) an integer greater than only one of the integers.

(e) an integer greater than both the integers.

Example 1 Write down a pair of integers whose

(a) sum is -3 (b) difference is -5

(c) difference is 2 (d) sum is 0

Solution (a) $(-1) + (-2) = -3$ or $(-5) + 2 = -3$

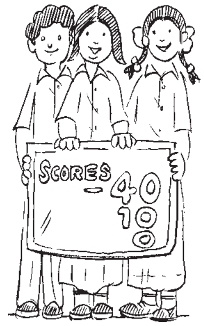
(b) $(-9) - (-4) = -5$ or $(-2) - 3 = -5$

(c) $(-7) - (-9) = 2$ or $1 - (-1) = 2$

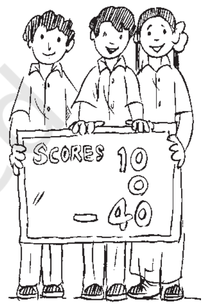
(d) $(-10) + 10 = 0$ or $5 + (-5) = 0$

Can you write more pairs in these examples?

EXERCISE 1.1



1. Write down a pair of integers whose:
   (a) sum is –7 (b) difference is –10 (c) sum is 0
2. (a) Write a pair of negative integers whose difference gives 8.
   (b) Write a negative integer and a positive integer whose sum is –5.
   (c) Write a negative integer and a positive integer whose difference is –3.
3. In a quiz, team A scored – 40, 10, 0 and team B scored 10, 0, – 40 in three successive rounds. Which team scored more? Can we say that we can add integers in any order?



4. Fill in the blanks to make the following statements true:
   (i) (–5) + (– 8) = (– 8) + (............)
   (ii) –53 + ... = –53
   (iii) 17 + ... = 0
   (iv) [13 + (– 12)] + (............) = 13 + [(–12) + (–7)]
   (v) (– 4) + [15 + (–3)] = [– 4 + 15] + ...

1.2 MULTIPLICATION OF INTEGERS

We can add and subtract integers. Let us now learn how to multiply integers.

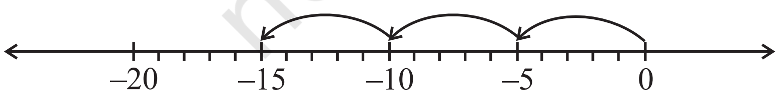
1.2.1 Multiplication of a Positive and a Negative Integer

We know that multiplication of whole numbers is repeated addition. For example,

$$5 + 5 + 5 = 3 \times 5 = 15$$

Can you represent addition of integers in the same way?

We have from the following number line, (–5) + (–5) + (–5) = –15



But we can also write

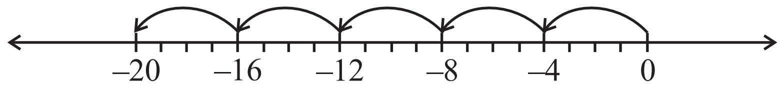
$$(–5) + (–5) + (–5) = 3 \times (–5)$$

Therefore, $3 \times (–5) = –15$

TRY THESE

Find:
$4 \times (– 8)$,
$8 \times (–2)$,
$3 \times (–7)$,
$10 \times (–1)$
using number line.

Similarly $(-4)+(-4)+(-4)+(-4)+(-4) = 5 \times (-4) = -20$



And $(-3)+(-3)+(-3)+(-3) =$ __________ = __________

Also, $(-7)+(-7)+(-7) =$ __________ = __________

Let us see how to find the product of a positive integer and a negative integer without using number line.

Let us find $3 \times (-5)$ in a different way. First find 3×5 and then put minus sign (–) before the product obtained. You get –15. That is we find $-(3 \times 5)$ to get –15.

Similarly, $5 \times (-4) = -(5 \times 4) = -20$.

Find in a similar way,

$4 \times (-8) =$ _____ = _____, $3 \times (-7) =$ _____ = _____

$6 \times (-5) =$ _____ = _____, $2 \times (-9) =$ _____ = _____

TRY THESE

Find:

(i) $6 \times (-19)$

(ii) $12 \times (-32)$

(iii) $7 \times (-22)$

Using this method we thus have,

$10 \times (-43) =$ _____ $-(10 \times 43) = -430$

Till now we multiplied integers as (positive integer) × (negative integer).

Let us now multiply them as (negative integer) × (positive integer).

We first find -3×5.

To find this, observe the following pattern:

We have, $3 \times 5 = 15$

$2 \times 5 = 10 = 15 - 5$

$1 \times 5 = 5 = 10 - 5$

$0 \times 5 = 0 = 5 - 5$

So, $-1 \times 5 = 0 - 5 = -5$

$-2 \times 5 = -5 - 5 = -10$

$-3 \times 5 = -10 - 5 = -15$

We already have $3 \times (-5) = -15$

So we get $(-3) \times 5 = -15 = 3 \times (-5)$

Using such patterns, we also get $(-5) \times 4 = -20 = 5 \times (-4)$

Using patterns, find $(-4) \times 8$, $(-3) \times 7$, $(-6) \times 5$ and $(-2) \times 9$

Check whether, $(-4) \times 8 = 4 \times (-8)$, $(-3) \times 7 = 3 \times (-7)$, $(-6) \times 5 = 6 \times (-5)$

and $(-2) \times 9 = 2 \times (-9)$

Using this we get, $(-33) \times 5 = 33 \times (-5) = -165$

We thus find that while *multiplying a positive integer and a negative integer, we multiply them as whole numbers and put a minus sign (–) before the product. We thus get a negative integer.*

TRY THESE

1. Find: (a) $15 \times (-16)$ (b) $21 \times (-32)$
   (c) $(-42) \times 12$ (d) -55×15
2. Check if (a) $25 \times (-21) = (-25) \times 21$ (b) $(-23) \times 20 = 23 \times (-20)$
   Write five more such examples.

In general, for any two positive integers a and b we can say

$$a \times (-b) = (-a) \times b = -(a \times b)$$

1.2.2 Multiplication of two Negative Integers

Can you find the product $(-3) \times (-2)$?

Observe the following:

$-3 \times 4 = -12$

$-3 \times 3 = -9 = -12 - (-3)$

$-3 \times 2 = -6 = -9 - (-3)$

$-3 \times 1 = -3 = -6 - (-3)$

$-3 \times 0 = 0 = -3 - (-3)$

$-3 \times -1 = 0 - (-3) = 0 + 3 = 3$

$-3 \times -2 = 3 - (-3) = 3 + 3 = 6$

Do you see any pattern? Observe how the products change.

Based on this observation, complete the following:

$-3 \times -3 =$ _____ $-3 \times -4 =$ _____

Now observe these products and fill in the blanks:

$-4 \times 4 = -16$

$-4 \times 3 = -12 = -16 + 4$

$-4 \times 2 =$ _____ $= -12 + 4$

$-4 \times 1 =$ _____

$-4 \times 0 =$ _____

$-4 \times (-1) =$ _____

$-4 \times (-2) =$ _____

$-4 \times (-3) =$ _____

TRY THESE

(i) Starting from $(-5) \times 4$, find $(-5) \times (-6)$

(ii) Starting from $(-6) \times 3$, find $(-6) \times (-7)$

From these patterns we observe that,

$(-3) \times (-1) = 3 = 3 \times 1$

$(-3) \times (-2) = 6 = 3 \times 2$

$(-3) \times (-3) = 9 = 3 \times 3$

and $(-4) \times (-1) = 4 = 4 \times 1$

So, $(-4) \times (-2) = 4 \times 2 =$ _____

$(-4) \times (-3) =$ _____ $=$ _____

So observing these products we can say that the *product of two negative integers is a positive integer. We multiply the two negative integers as whole numbers and put the positive sign before the product.*

Thus, we have $(-10) \times (-12) = +120 = 120$

Similarly $(-15) \times (-6) = +90 = 90$

In general, for any two positive integers a and b,

$$(-a) \times (-b) = a \times b$$

TRY THESE Find: $(-31) \times (-100)$, $(-25) \times (-72)$, $(-83) \times (-28)$

Game 1

(i) Take a board marked from –104 to 104 as shown in the figure.

(ii) Take a bag containing two blue and two red dice. Number of dots on the blue dice indicate positive integers and number of dots on the red dice indicate negative integers.

(iii) Every player will place his/her counter at zero.

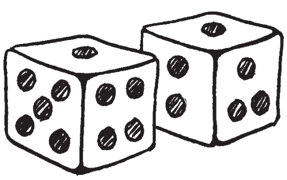
(iv) Each player will take out two dice at a time from the bag and throw them.

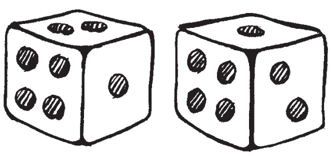
(v) After every throw, the player has to multiply the numbers marked on the dice.

104	103	102	101	100	99	98	97	96	95	94
83	84	85	86	87	88	89	90	91	92	93
82	81	80	79	78	77	76	75	74	73	72
61	62	63	64	65	66	67	68	69	70	71
60	59	58	57	56	55	54	53	52	51	50
39	40	41	42	43	44	45	46	47	48	49
38	37	36	35	34	33	32	31	30	29	28
17	18	19	20	21	22	23	24	25	26	27
16	15	14	13	12	11	10	9	8	7	6
–5	–4	–3	–2	–1	0	1	2	3	4	5
–6	–7	–8	–9	–10	–11	–12	–13	–14	–15	–16
–27	–26	–25	–24	–23	–22	–21	–20	–19	–18	–17
–28	–29	–30	–31	–32	–33	–34	–35	–36	–37	–38
–49	–48	–47	–46	–45	–44	–43	–42	–41	–40	–39
–50	–51	–52	–53	–54	–55	–56	–57	–58	–59	–60
–71	–70	–69	–68	–67	–66	–65	–64	–63	–62	–61
–72	–73	–74	–75	–76	–77	–78	–79	–80	–81	–82
–93	–92	–91	–90	–89	–88	–87	–86	–85	–84	–83
–94	–95	–96	–97	–98	–99	–100	–101	–102	–103	–104

(vi) If the product is a positive integer then the player will move his counter towards 104; if the product is a negative integer then the player will move his counter towards –104.

(vii) The player who reaches either -104 or 104 first is the winner.





1.3 Properties of Multiplication of Integers

1.3.1 Closure under Multiplication

1. Observe the following table and complete it:

Statement	Inference
(–20) × (–5) = 100	Product is an integer
(–15) × 17 = – 255	Product is an integer
(–30) × 12 = _____	
(–15) × (–23) = _____	
(–14) × (–13) = _____	
12 × (–30) = _____	

What do you observe? Can you find a pair of integers whose product is not an integer? No. This gives us an idea that the product of two integers is again an integer. So we can say that *integers are closed under multiplication*.

In general,

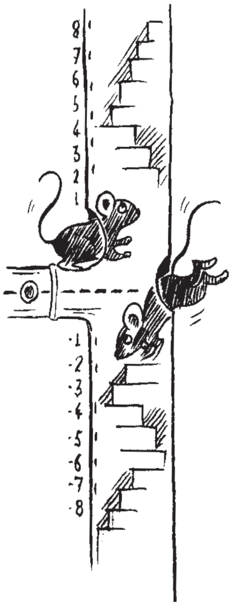
$a \times b$ is an integer, for all integers a and b.

Find the product of five more pairs of integers and verify the above statement.

1.3.2 Commutativity of Multiplication

We know that multiplication is commutative for whole numbers. Can we say, multiplication is also commutative for integers?

Observe the following table and complete it:



Statement 1	Statement 2	Inference
3 × (– 4) = –12	(– 4) × 3 = –12	3 × (– 4) = (– 4) × 3
(–30) × 12 = _____	12 × (–30) = _____	
(–15) × (–10) = 150	(–10) × (–15) = 150	
(–35) × (–12) = _____	(–12) × (–35) =	
(–17) × 0 = _____		
__________ = _____	(–1) × (–15) =	

What are your observations? The above examples suggest *multiplication is commutative for integers*. Write five more such examples and verify.

In general, for any two integers a and b,

$$a \times b = b \times a$$

1.3.3 Multiplication by Zero

We know that any whole number when multiplied by zero gives zero. Observe the following products of negative integers and zero. These are obtained from the patterns done earlier.

$(-3) \times 0 = 0$

$0 \times (-4) = 0$

$-5 \times 0 =$ _____

$0 \times (-6) =$ _____

This shows that the product of a negative integer and zero is zero.

In general, for any integer a,

$$a \times 0 = 0 \times a = 0$$

1.3.4 Multiplicative Identity

We know that 1 is the multiplicative identity for whole numbers.

Check that 1 is the multiplicative identity for integers as well. Observe the following products of integers with 1.

$(-3) \times 1 = -3$	$1 \times 5 = 5$
$(-4) \times 1 =$ _____	$1 \times 8 =$ _____
$1 \times (-5) =$ _____	$3 \times 1 =$ _____
$1 \times (-6) =$ _____	$7 \times 1 =$ _____

This shows that 1 is the multiplicative identity for integers also.

In general, for any integer a we have,

$$a \times 1 = 1 \times a = a$$

What happens when we multiply any integer with -1? Complete the following:

$(-3) \times (-1) = 3$

$3 \times (-1) = -3$

$(-6) \times (-1) =$ _____

$(-1) \times 13 =$ _____

$(-1) \times (-25) =$ _____

$18 \times (-1) =$ _____

0 is the additive identity whereas 1 is the multiplicative identity for integers. We get additive inverse of an integer a when we multiply (–1) to a, i.e. $a \times (-1) = (-1) \times a = -a$

What do you observe?

Can we say -1 is a multiplicative identity of integers? No.

1.3.5 Associativity for Multiplication

Consider –3, –2 and 5.

Look at $[(-3) \times (-2)] \times 5$ and $(-3) \times [(-2) \times 5]$.

In the first case (–3) and (–2) are grouped together and in the second (–2) and 5 are grouped together.

We see that $[(-3) \times (-2)] \times 5 = 6 \times 5 = 30$

and $\quad (-3) \times [(-2) \times 5] = (-3) \times (-10) = 30$

So, we get the same answer in both the cases.

Thus, $\quad [(-3) \times (-2)] \times 5 = (-3) \times [(-2) \times 5]$

Look at this and complete the products:

$[(7) \times (-6)] \times 4 =$ __________ $\times 4 =$ __________

$7 \times [(-6) \times 4] = 7 \times$ __________ $=$ __________

Is $[7 \times (-6)] \times 4 = 7 \times [(-6) \times 4]$?

Does the grouping of integers affect the product of integers? No.

In general, for any three integers *a*, *b* and *c*

$$(a \times b) \times c = a \times (b \times c)$$

Take any five values for *a*, *b* and *c* each and verify this property.

Thus, like whole numbers, *the product of three integers does not depend upon the grouping of integers and this is called the associative property for multiplication of integers.*

1.3.6 Distributive Property

We know

$16 \times (10 + 2) = (16 \times 10) + (16 \times 2)$ [Distributivity of multiplication over addition]

Let us check if this is true for integers also.

Observe the following:

(a) $(-2) \times (3 + 5) = -2 \times 8 = -16$

and $\quad [(-2) \times 3] + [(-2) \times 5] = (-6) + (-10) = -16$

So, $\quad (-2) \times (3 + 5) = [(-2) \times 3] + [(-2) \times 5]$

(b) $(-4) \times [(-2) + 7] = (-4) \times 5 = -20$

and $\quad [(-4) \times (-2)] + [(-4) \times 7] = 8 + (-28) = -20$

So, $\quad (-4) \times [(-2) + 7] = [(-4) \times (-2)] + [(-4) \times 7]$

(c) $(-8) \times [(-2) + (-1)] = (-8) \times (-3) = 24$

and $\quad [(-8) \times (-2)] + [(-8) \times (-1)] = 16 + 8 = 24$

So, $\quad (-8) \times [(-2) + (-1)] = [(-8) \times (-2)] + [(-8) \times (-1)]$

Can we say that the distributivity of multiplication over addition is true for integers also? Yes.

In general, for any integers *a*, *b* and *c*,

$$a \times (b + c) = a \times b + a \times c$$

Take atleast five different values for each of *a*, *b* and *c* and verify the above Distributive property.

Try These

(i) Is $10 \times [(6 + (-2)] = 10 \times 6 + 10 \times (-2)$?

(ii) Is $(-15) \times [(-7) + (-1)] = (-15) \times (-7) + (-15) \times (-1)$?

Now consider the following:

Can we say $4 \times (3 - 8) = 4 \times 3 - 4 \times 8$?

Let us check:

$4 \times (3 - 8) = 4 \times (-5) = -20$

$4 \times 3 - 4 \times 8 = 12 - 32 = -20$

So, $4 \times (3 - 8) = 4 \times 3 - 4 \times 8$.

Look at the following:

$(-5) \times [(-4) - (-6)] = (-5) \times 2 = -10$

$[(-5) \times (-4)] - [(-5) \times (-6)] = 20 - 30 = -10$

So, $(-5) \times [(-4) - (-6)] = [(-5) \times (-4)] - [(-5) \times (-6)]$

Check this for $(-9) \times [10 - (-3)]$ and $[(-9) \times 10] - [(-9) \times (-3)]$

You will find that these are also equal.

In general, for any three integers *a*, *b* and *c*,

$$a \times (b - c) = a \times b - a \times c$$

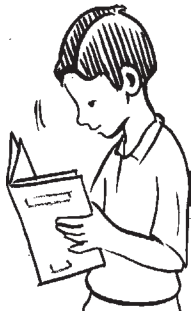
Take atleast five different values for each of *a*, *b* and *c* and verify this property.

Try These

(i) Is $10 \times (6 - (-2)] = 10 \times 6 - 10 \times (-2)$?

(ii) Is $(-15) \times [(-7) - (-1)] = (-15) \times (-7) - (-15) \times (-1)$?

EXERCISE 1.2



1. Find each of the following products:

(a) $3 \times (-1)$ (b) $(-1) \times 225$

(c) $(-21) \times (-30)$ (d) $(-316) \times (-1)$

(e) $(-15) \times 0 \times (-18)$ (f) $(-12) \times (-11) \times (10)$

(g) $9 \times (-3) \times (-6)$ (h) $(-18) \times (-5) \times (-4)$

(i) $(-1) \times (-2) \times (-3) \times 4$ (j) $(-3) \times (-6) \times (-2) \times (-1)$

2. Verify the following:

(a) $18 \times [7 + (-3)] = [18 \times 7] + [18 \times (-3)]$

(b) $(-21) \times [(-4) + (-6)] = [(-21) \times (-4)] + [(-21) \times (-6)]$

3. (i) For any integer a, what is $(-1) \times a$ equal to?

(ii) Determine the integer whose product with (-1) is

(a) -22 (b) 37 (c) 0

4. Starting from $(-1) \times 5$, write various products showing some pattern to show $(-1) \times (-1) = 1$.

1.4 DIVISION OF INTEGERS

We know that division is the inverse operation of multiplication. Let us see an example for whole numbers.

Since $3 \times 5 = 15$

So $15 \div 5 = 3$ and $15 \div 3 = 5$

Similarly, $4 \times 3 = 12$ gives $12 \div 4 = 3$ and $12 \div 3 = 4$

We can say for each multiplication statement of whole numbers there are two division statements.

Can you write multiplication statement and its corresponding divison statements for integers?

- Observe the following and complete it.

Multiplication Statement	Corresponding Division Statements	
$2 \times (-6) = (-12)$	$(-12) \div (-6) = 2$,	$(-12) \div 2 = (-6)$
$(-4) \times 5 = (-20)$	$(-20) \div 5 = (-4)$,	$(-20) \div (-4) = 5$
$(-8) \times (-9) = 72$	$72 \div$ ______ $=$ ______ ,	$72 \div$ ______ $=$ ______
$(-3) \times (-7) =$ ______	______ $\div (-3) =$ ______ ,	______________
$(-8) \times 4 =$ _____	______________ ,	______________
$5 \times (-9) =$ ______	______________ ,	______________
$(-10) \times (-5) =$	______________ ,	______________

From the above we observe that :

$(-12) \div 2 = (-6)$

$(-20) \div 5 = (-4)$

$(-32) \div 4 = (-8)$

$(-45) \div 5 = (-9)$

TRY THESE

Find:

(a) $(-100) \div 5$ (b) $(-81) \div 9$

(c) $(-75) \div 5$ (d) $(-32) \div 2$

We observe that *when we divide a negative integer by a positive integer, we divide them as whole numbers and then put a minus sign (–) before the quotient.*

- We also observe that:

$72 \div (-8) = -9$ and $50 \div (-10) = -5$

$72 \div (-9) = -8$ $50 \div (-5) = -10$

So we can say that *when we divide a positive integer by a negative integer, we first divide them as whole numbers and then put a minus sign (–) before the quotient.*

In general, for any two positive integers a and b

$$a \div (-b) = (-a) \div b \quad \text{where } b \neq 0$$

Can we say that

$(-48) \div 8 = 48 \div (-8)$?

Let us check. We know that

$(-48) \div 8 = -6$

and $48 \div (-8) = -6$

So $(-48) \div 8 = 48 \div (-8)$

Check this for

(i) $90 \div (-45)$ and $(-90) \div 45$

(ii) $(-136) \div 4$ and $136 \div (-4)$

TRY THESE

Find: (a) $125 \div (-25)$ (b) $80 \div (-5)$ (c) $64 \div (-16)$

- Lastly, we observe that

$(-12) \div (-6) = 2; (-20) \div (-4) = 5; (-32) \div (-8) = 4; (-45) \div (-9) = 5$

So, we can say that when we divide a negative integer by a negative integer, we first divide them as whole numbers and then put a positive sign (+).

In general, for any two positive integers a and b

$$(-a) \div (-b) = a \div b \quad \text{where } b \neq 0$$

TRY THESE

Find: (a) $(-36) \div (-4)$ (b) $(-201) \div (-3)$ (c) $(-325) \div (-13)$

1.5 PROPERTIES OF DIVISION OF INTEGERS

Observe the following table and complete it:

What do you observe? We observe that integers are not closed under division.

Statement	Inference	Statement	Inference
$(-8) \div (-4) = 2$	Result is an integer	$(-8) \div 3 = \frac{-8}{3}$	________________
$(-4) \div (-8) = \frac{-4}{-8}$	Result is not an integer	$3 \div (-8) = \frac{3}{-8}$	________________

Justify it by taking five more examples of your own.

- We know that division is not commutative for whole numbers. Let us check it for integers also.

  You can see from the table that $(-8) \div (-4) \neq (-4) \div (-8)$.

  Is $(-9) \div 3$ the same as $3 \div (-9)$?

  Is $(-30) \div (-6)$ the same as $(-6) \div (-30)$?

  Can we say that division is commutative for integers? No.

  You can verify it by taking five more pairs of integers.

- Like whole numbers, any integer divided by zero is meaningless and zero divided by an integer other than zero is equal to zero i.e., *for any integer a,* $a \div 0$ *is not defined but* $0 \div a = 0$ *for* $a \neq 0$.

- When we divide a whole number by 1 it gives the same whole number. Let us check whether it is true for negative integers also.

  Observe the following :

$(-8) \div 1 = (-8)$	$(-11) \div 1 = -11$	$(-13) \div 1 = -13$
$(-25) \div 1 =$ ____	$(-37) \div 1 =$ ____	$(-48) \div 1 =$ ____

  This shows that negative integer divided by 1 gives the same negative integer. So, *any integer divided by 1 gives the same integer.*

  In general, for any integer a,

  $$\boxed{a \div 1 = a}$$

- What happens when we divide any integer by (-1)? Complete the following table

$(-8) \div (-1) = 8$	$11 \div (-1) = -11$	$13 \div (-1) =$ ____
$(-25) \div (-1) =$ ____	$(-37) \div (-1) =$ ____	$-48 \div (-1) =$ ____

  What do you observe?

  We can say that if any integer is divided by (-1) it does not give the same integer.

- Can we say $[(-16) \div 4] \div (-2)$ is the same as $(-16) \div [4 \div (-2)]$?

  We know that $[(-16) \div 4] \div (-2) = (-4) \div (-2) = 2$

  and $(-16) \div [4 \div (-2)] = (-16) \div (-2) = 8$

  So $[(-16) \div 4] \div (-2) \neq (-16) \div [4 \div (-2)]$

  Can you say that division is associative for integers? No.

  Verify it by taking five more examples of your own.

TRY THESE

Is (i) $1 \div a = 1$?

(ii) $a \div (-1) = -a$? for any integer a.

Take different values of a and check.

EXAMPLE 2 In a test (+5) marks are given for every correct answer and (–2) marks are given for every incorrect answer. (i) Radhika answered all the questions and scored 30 marks though she got 10 correct answers. (ii) Jay also answered all the questions and scored (–12) marks though he got 4 correct answers. How many incorrect answers had they attempted?

SOLUTION

(i) Marks given for one correct answer = 5

So, marks given for 10 correct answers = 5 × 10 = 50

Radhika's score = 30

Marks obtained for incorrect answers = 30 – 50 = – 20

Marks given for one incorrect answer = (–2)

Therefore, number of incorrect answers = (–20) ÷ (–2) = 10

(ii) Marks given for 4 correct answers = 5 × 4 = 20

Jay's score = –12

Marks obtained for incorrect answers = –12 – 20 = – 32

Marks given for one incorrect answer = (–2)

Therefore number of incorrect answers = (–32) ÷ (–2) = 16

EXAMPLE 3 A shopkeeper earns a profit of ₹ 1 by selling one pen and incurs a loss of 40 paise per pencil while selling pencils of her old stock.

(i) In a particular month she incurs a loss of ₹ 5. In this period, she sold 45 pens. How many pencils did she sell in this period?

(ii) In the next month she earns neither profit nor loss. If she sold 70 pens, how many pencils did she sell?

SOLUTION

(i) Profit earned by selling one pen = ₹ 1

Profit earned by selling 45 pens = ₹ 45, which we denote by + ₹ 45

Total loss given = ₹ 5, which we denote by – ₹ 5

Profit earned + Loss incurred = Total loss

Therefore, Loss incurred = Total Loss – Profit earned

= ₹ (– 5 – 45) = ₹ (–50) = –5000 paise

Loss incurred by selling one pencil = 40 paise which we write as – 40 paise

So, number of pencils sold = (–5000) ÷ (– 40) = 125

(ii) In the next month there is neither profit nor loss.

So, Profit earned + Loss incurred = 0

i.e., Profit earned = – Loss incurred.

Now, profit earned by selling 70 pens = ₹ 70

Hence, loss incurred by selling pencils = ₹ 70 which we indicate by – ₹ 70 or – 7,000 paise.

Total number of pencils sold = (–7000) ÷ (– 40) = 175 pencils.

EXERCISE 1.3

1. Evaluate each of the following:

(a) (–30) ÷ 10 (b) 50 ÷ (–5) (c) (–36) ÷ (–9)

(d) (– 49) ÷ (49) (e) 13 ÷ [(–2) + 1] (f) 0 ÷ (–12)

(g) (–31) ÷ [(–30) + (–1)]

(h) [(–36) ÷ 12] ÷ 3 (i) [(– 6) + 5)] ÷ [(–2) + 1]

2. Verify that $a \div (b + c) \neq (a \div b) + (a \div c)$ for each of the following values of a, b and c.

(a) $a = 12, b = -4, c = 2$ (b) $a = (-10), b = 1, c = 1$

3. Fill in the blanks:

(a) 369 ÷ _____ = 369 (b) (–75) ÷ _____ = –1

(c) (–206) ÷ _____ = 1 (d) – 87 ÷ _____ = 87

(e) _____ ÷ 1 = – 87 (f) _____ ÷ 48 = –1

(g) 20 ÷ _____ = –2 (h) _____ ÷ (4) = –3

4. Write five pairs of integers (a, b) such that $a \div b = -3$. One such pair is (6, –2) because 6 ÷ (–2) = (–3).

5. The temperature at 12 noon was 10°C above zero. If it decreases at the rate of 2°C per hour until midnight, at what time would the temperature be 8°C below zero? What would be the temperature at mid-night?

6. In a class test (+ 3) marks are given for every correct answer and (–2) marks are given for every incorrect answer and no marks for not attempting any question. (i) Radhika scored 20 marks. If she has got 12 correct answers, how many questions has she attempted incorrectly? (ii) Mohini scores –5 marks in this test, though she has got 7 correct answers. How many questions has she attempted incorrectly?

7. An elevator descends into a mine shaft at the rate of 6 m/min. If the descent starts from 10 m above the ground level, how long will it take to reach – 350 m.

What have We Discussed?

1. We now study the properties satisfied by addition and subtraction.

(a) Integers are closed for addition and subtraction both. That is, $a + b$ and $a - b$ are again integers, where a and b are any integers.

(b) Addition is commutative for integers, i.e., $a + b = b + a$ for all integers a and b.

(c) Addition is associative for integers, i.e., $(a + b) + c = a + (b + c)$ for all integers a, b and c.

(d) Integer 0 is the identity under addition. That is, $a + 0 = 0 + a = a$ for every integer a.

2. We studied, how integers could be multiplied, and found that product of a positive and a negative integer is a negative integer, whereas the product of two negative integers is a positive integer. For example, $-2 \times 7 = -14$ and $-3 \times -8 = 24$.

3. Product of even number of negative integers is positive, whereas the product of odd number of negative integers is negative.

4. Integers show some properties under multiplication.

(a) Integers are closed under multiplication. That is, $a \times b$ is an integer for any two integers a and b.

(b) Multiplication is commutative for integers. That is, $a \times b = b \times a$ for any integers a and b.

(c) The integer 1 is the identity under multiplication, i.e., $1 \times a = a \times 1 = a$ for any integer a.

(d) Multiplication is associative for integers, i.e., $(a \times b) \times c = a \times (b \times c)$ for any three integers a, b and c.

5. Under addition and multiplication, integers show a property called distributive property. That is, $a \times (b + c) = a \times b + a \times c$ for any three integers a, b and c.

6. The properties of commutativity, associativity under addition and multiplication, and the distributive property help us to make our calculations easier.

7. We also learnt how to divide integers. We found that,

(a) When a positive integer is divided by a negative integer, the quotient obtained is negative and vice-versa.

(b) Division of a negative integer by another negative integer gives positive as quotient.

8. For any integer a, we have

(a) $a \div 0$ is not defined (b) $a \div 1 = a$

Chapter 2

Fractions and Decimals

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2.1 Multiplication of Fractions

You know how to find the area of a rectangle. It is equal to length × breadth. If the length and breadth of a rectangle are 7 cm and 4 cm respectively, then what will be its area? Its area would be $7 \times 4 = 28$ cm^2.

What will be the area of the rectangle if its length and breadth are $7\frac{1}{2}$ cm and $3\frac{1}{2}$ cm respectively? You will say it will be $7\frac{1}{2} \times 3\frac{1}{2} = \frac{15}{2} \times \frac{7}{2}$ cm^2. The numbers $\frac{15}{2}$ and $\frac{7}{2}$ are fractions. To calculate the area of the given rectangle, we need to know how to multiply fractions. We shall learn that now.

2.1.1 Multiplication of a Fraction by a Whole Number

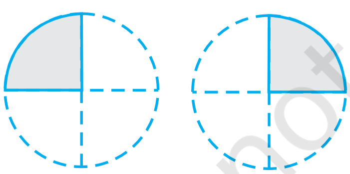


Fig 2.1

Observe the pictures at the left (Fig 2.1). Each shaded part is $\frac{1}{4}$ part of a circle. How much will the two shaded parts represent together? They will represent $\frac{1}{4} + \frac{1}{4} = 2 \times \frac{1}{4}$.

Combining the two shaded parts, we get Fig 2.2 . What part of a circle does the shaded part in Fig 2.2 represent? It represents $\frac{2}{4}$ part of a circle .

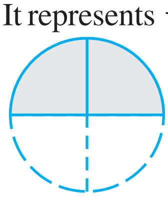


Fig 2.2

The shaded portions in Fig 2.1 taken together are the same as the shaded portion in Fig 2.2, i.e., we get Fig 2.3.

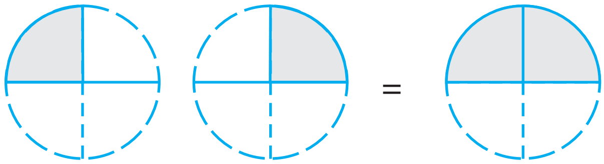


Fig 2.3

or $$2\times\frac{1}{4}=\frac{2}{4}.$$

Can you now tell what this picture will represent? (Fig 2.4)

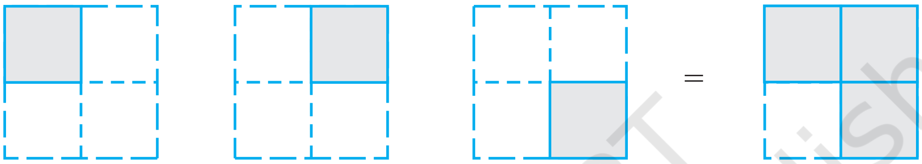


Fig 2.4

And this? (Fig 2.5)

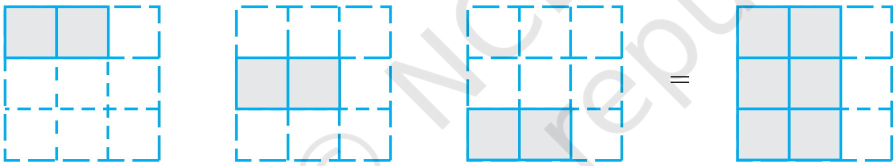


Fig 2.5

Let us now find $3\times\frac{1}{2}$.

We have $$3\times\frac{1}{2}=\frac{1}{2}+\frac{1}{2}+\frac{1}{2}=\frac{3}{2}$$

We also have $$\frac{1}{2}+\frac{1}{2}+\frac{1}{2}=\frac{1+1+1}{2}=\frac{3\times 1}{2}=\frac{3}{2}$$

So $$3\times\frac{1}{2}=\frac{3\times 1}{2}=\frac{3}{2}$$

Similarly $$\frac{2}{3}\times 5=\frac{2\times 5}{3}=?$$

Can you tell $3\times\frac{2}{7}=?$ $\quad 4\times\frac{3}{5}=?$

The fractions that we considered till now, i.e., $\frac{1}{2},\frac{2}{3},\frac{2}{7}$ and $\frac{3}{5}$ were proper fractions.

For improper fractions also we have,

$$2\times\frac{5}{3} = \frac{2\times 5}{3} = \frac{10}{3}$$

Try, $3\times\frac{8}{7} = ?\quad 4\times\frac{7}{5} = ?$

Thus, *to multiply a whole number with a proper or an improper fraction, we multiply the whole number with the numerator of the fraction, keeping the denominator same.*

Try These

1. Find: (a) $\frac{2}{7}\times 3$ (b) $\frac{9}{7}\times 6$ (c) $3\times\frac{1}{8}$ (d) $\frac{13}{11}\times 6$

   If the product is an improper fraction express it as a mixed fraction.

2. Represent pictorially : $2\times\frac{2}{5} = \frac{4}{5}$

Try These

Find: (i) $5\times 2\frac{3}{7}$

(ii) $1\frac{4}{9}\times 6$

To multiply a mixed fraction to a whole number, first convert the mixed fraction to an improper fraction and then multiply.

Therefore, $3\times 2\frac{5}{7} = 3\times\frac{19}{7} = \frac{57}{7} = 8\frac{1}{7}$.

Similarly, $2\times 4\frac{2}{5} = 2\times\frac{22}{5} = ?$

Fraction as an operator 'of'

Observe these figures (Fig 2.6)

The two squares are exactly similar.

Each shaded portion represents $\frac{1}{2}$ of 1.

So, both the shaded portions together will represent $\frac{1}{2}$ of 2.

Combine the 2 shaded $\frac{1}{2}$ parts. It represents 1.

So, we say $\frac{1}{2}$ of 2 is 1. We can also get it as $\frac{1}{2}\times 2 = 1$.

Thus, $\frac{1}{2}$ of $2 = \frac{1}{2}\times 2 = 1$

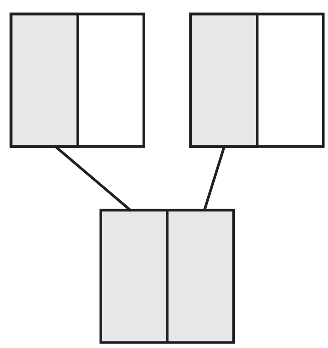


Fig 2.6

Also, look at these similar squares (Fig 2.7).

Each shaded portion represents $\frac{1}{2}$ of 1.

So, the three shaded portions represent $\frac{1}{2}$ of 3.

Combine the 3 shaded parts.

It represents $1\frac{1}{2}$ i.e., $\frac{3}{2}$.

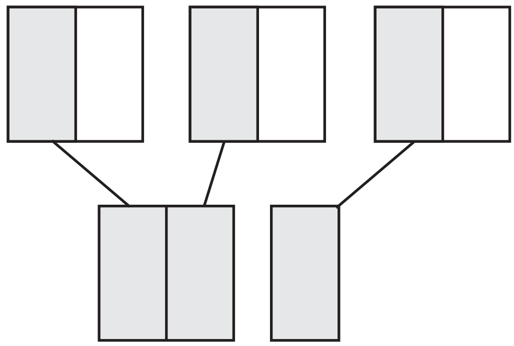


Fig 2.7

So, $\frac{1}{2}$ of 3 is $\frac{3}{2}$. Also, $\frac{1}{2} \times 3 = \frac{3}{2}$.

Thus, $\frac{1}{2}$ of $3 = \frac{1}{2} \times 3 = \frac{3}{2}$.

So we see that **'of' represents multiplication.**

Farida has 20 marbles. Reshma has $\frac{1}{5}$th of the number of marbles what Farida has. How many marbles Reshma has? As, 'of' indicates multiplication, so, Reshma has $\frac{1}{5} \times 20 = 4$ marbles.

Similarly, we have $\frac{1}{2}$ of 16 is $\frac{1}{2} \times 16 = \frac{16}{2} = 8$.

TRY THESE

Can you tell, what is (i) $\frac{1}{2}$ of 10?, (ii) $\frac{1}{4}$ of 16?, (iii) $\frac{2}{5}$ of 25?

EXAMPLE 1 In a class of 40 students $\frac{1}{5}$ of the total number of studetns like to study English, $\frac{2}{5}$ of the total number like to study Mathematics and the remaining students like to study Science.

(i) How many students like to study English?

(ii) How many students like to study Mathematics?

(iii) What fraction of the total number of students like to study Science?

SOLUTION Total number of students in the class = 40.

(i) Of these $\frac{1}{5}$ of the total number of students like to study English.

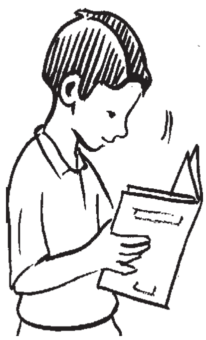
Thus, the number of students who like to study English $= \frac{1}{5}$ of $40 = \frac{1}{5} \times 40 = 8$.

(ii) Try yourself.

(iii) The number of students who like English and Mathematics $= 8 + 16 = 24$. Thus, the number of students who like Science $= 40 - 24 = 16$.

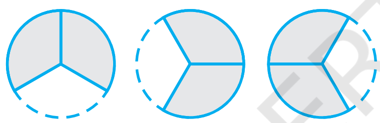
Thus, the required fraction is $\frac{16}{40}$.

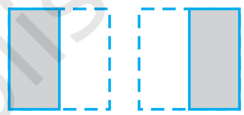
Exercise 2.1

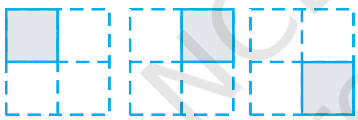


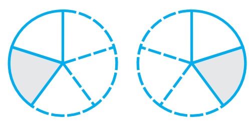
1. Which of the drawings (a) to (d) show :

(i) $2 \times \frac{1}{5}$ (ii) $2 \times \frac{1}{2}$ (iii) $3 \times \frac{2}{3}$ (iv) $3 \times \frac{1}{4}$

(a) 

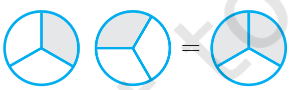
(b) 

(c) 

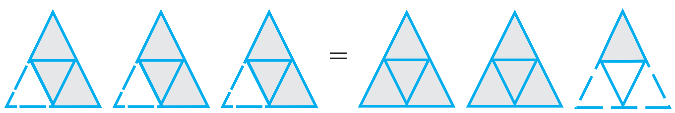
(d) 

2. Some pictures (a) to (c) are given below. Tell which of them show:

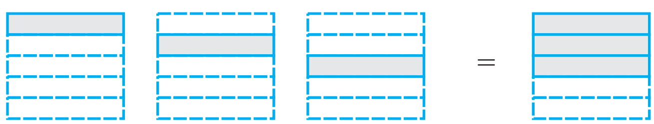
(i) $3 \times \frac{1}{5} = \frac{3}{5}$ (ii) $2 \times \frac{1}{3} = \frac{2}{3}$ (iii) $3 \times \frac{3}{4} = 2\frac{1}{4}$



(a)



(b)



(c)

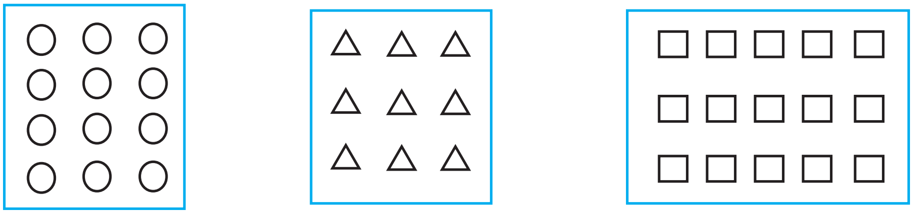
3. Multiply and reduce to lowest form and convert into a mixed fraction:

(i) $7 \times \frac{3}{5}$ (ii) $4 \times \frac{1}{3}$ (iii) $2 \times \frac{6}{7}$ (iv) $5 \times \frac{2}{9}$ (v) $\frac{2}{3} \times 4$

(vi) $\frac{5}{2} \times 6$ (vii) $11 \times \frac{4}{7}$ (viii) $20 \times \frac{4}{5}$ (ix) $13 \times \frac{1}{3}$ (x) $15 \times \frac{3}{5}$

4. Shade: (i) $\frac{1}{2}$ of the circles in box (a) (ii) $\frac{2}{3}$ of the triangles in box (b)

(iii) $\frac{3}{5}$ of the squares in box (c).



(a) (b) (c)

5. Find:

(a) $\frac{1}{2}$ of (i) 24 (ii) 46 (b) $\frac{2}{3}$ of (i) 18 (ii) 27

(c) $\frac{3}{4}$ of (i) 16 (ii) 36 (d) $\frac{4}{5}$ of (i) 20 (ii) 35

6. Multiply and express as a mixed fraction :

(a) $3 \times 5\frac{1}{5}$ (b) $5 \times 6\frac{3}{4}$ (c) $7 \times 2\frac{1}{4}$

(d) $4 \times 6\frac{1}{3}$ (e) $3\frac{1}{4} \times 6$ (f) $3\frac{2}{5} \times 8$

7. Find: (a) $\frac{1}{2}$ of (i) $2\frac{3}{4}$ (ii) $4\frac{2}{9}$ (b) $\frac{5}{8}$ of (i) $3\frac{5}{6}$ (ii) $9\frac{2}{3}$

8. Vidya and Pratap went for a picnic. Their mother gave them a water bottle that contained 5 litres of water. Vidya consumed $\frac{2}{5}$ of the water. Pratap consumed the remaining water.

(i) How much water did Vidya drink?

(ii) What fraction of the total quantity of water did Pratap drink?

2.1.2 Multiplication of a Fraction by a Fraction

Farida had a 9 cm long strip of ribbon. She cut this strip into four equal parts. How did she do it? She folded the strip twice. What fraction of the total length will each part represent?

Each part will be $\frac{9}{4}$ of the strip. She took one part and divided it in two equal parts by folding the part once. What will one of the pieces represent? It will represent $\frac{1}{2}$ of $\frac{9}{4}$ or $\frac{1}{2} \times \frac{9}{4}$.

Let us now see how to find the product of two fractions like $\frac{1}{2} \times \frac{9}{4}$.

To do this we first learn to find the products like $\frac{1}{2} \times \frac{1}{3}$.

Fig 2.8

(a) How do we find $\frac{1}{3}$ of a whole? We divide the whole in three equal parts. Each of the three parts represents $\frac{1}{3}$ of the whole. Take one part of these three parts, and shade it as shown in Fig 2.8.

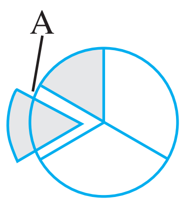


Fig 2.9

(b) How will you find $\frac{1}{2}$ of this shaded part? Divide this one-third ($\frac{1}{3}$) shaded part into two equal parts. Each of these two parts represents $\frac{1}{2}$ of $\frac{1}{3}$ i.e., $\frac{1}{2} \times \frac{1}{3}$ (Fig 2.9).

Take out 1 part of these two and name it 'A'. 'A' represents $\frac{1}{2} \times \frac{1}{3}$.

(c) What fraction is 'A' of the whole? For this, divide each of the remaining $\frac{1}{3}$ parts also in two equal parts. How many such equal parts do you have now?

There are six such equal parts. 'A' is one of these parts.

So, 'A' is $\frac{1}{6}$ of the whole. Thus, $\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$.

How did we decide that 'A' was $\frac{1}{6}$ of the whole? The whole was divided in $6 = 2 \times 3$ parts and $1 = 1 \times 1$ part was taken out of it.

Thus, $$\frac{1}{2} \times \frac{1}{3} = \frac{1}{6} = \frac{1\times1}{2\times3}$$

or $$\frac{1}{2} \times \frac{1}{3} = \frac{1\times1}{2\times3}$$

The value of $\frac{1}{3} \times \frac{1}{2}$ can be found in a similar way. Divide the whole into two equal parts and then divide one of these parts in three equal parts. Take one of these parts. This will represent $\frac{1}{3} \times \frac{1}{2}$ i.e., $\frac{1}{6}$.

Therefore $\quad \frac{1}{3} \times \frac{1}{2} = \frac{1}{6} = \frac{1 \times 1}{3 \times 2}$ as discussed earlier.

Hence $\quad \frac{1}{2} \times \frac{1}{3} = \frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$

Find $\frac{1}{3} \times \frac{1}{4}$ and $\frac{1}{4} \times \frac{1}{3}$; $\frac{1}{2} \times \frac{1}{5}$ and $\frac{1}{5} \times \frac{1}{2}$ and check whether you get

$$\frac{1}{3} \times \frac{1}{4} = \frac{1}{4} \times \frac{1}{3};\ \frac{1}{2} \times \frac{1}{5} = \frac{1}{5} \times \frac{1}{2}$$

TRY THESE

Fill in these boxes:

(i) $\frac{1}{2} \times \frac{1}{7} = \frac{1 \times 1}{2 \times 7} = \square$ (ii) $\frac{1}{5} \times \frac{1}{7} = \square = \square$

(iii) $\frac{1}{7} \times \frac{1}{2} = \square = \square$ (iv) $\frac{1}{7} \times \frac{1}{5} = \square = \square$

EXAMPLE 2 Sushant reads $\frac{1}{3}$ part of a book in 1 hour. How much part of the book will he read in $2\frac{1}{5}$ hours?

SOLUTION The part of the book read by Sushant in 1 hour $= \frac{1}{3}$.

So, the part of the book read by him in $2\frac{1}{5}$ hours $= 2\frac{1}{5} \times \frac{1}{3}$

$$= \frac{11}{5} \times \frac{1}{3} = \frac{11 \times 1}{5 \times 3} = \frac{11}{15}$$

Let us now find $\frac{1}{2} \times \frac{5}{3}$. We know that $\frac{5}{3} = \frac{1}{3} \times 5$.

So, $\frac{1}{2} \times \frac{5}{3} = \frac{1}{2} \times \frac{1}{3} \times 5 = \frac{1}{6} \quad 5 = \frac{5}{6}$

Also, $\frac{5}{6} = \frac{1 \times 5}{2 \times 3}$. Thus, $\frac{1}{2} \times \frac{5}{3} = \frac{1 \times 5}{2 \times 3} = \frac{5}{6}$.

This is also shown by the figures drawn below. Each of these five equal shapes (Fig 2.10) are parts of five similar circles. Take one such shape. To obtain this shape we first divide a circle in three equal parts. Further divide each of these three parts in two equal parts. One part out of it is the shape we considered. What will it represent?

It will represent $\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$. The total of such parts would be $5 \times \frac{1}{6} = \frac{5}{6}$.

Fig 2.10

TRY THESE

Find: $\frac{1}{3} \times \frac{4}{5}$; $\frac{2}{3} \times \frac{1}{5}$

Similarly $\frac{3}{5} \times \frac{1}{7} = \frac{3 \times 1}{5 \times 7} = \frac{3}{35}$.

We can thus find $\frac{2}{3} \times \frac{7}{5}$ as $\frac{2}{3} \times \frac{7}{5} = \frac{2 \times 7}{3 \times 5} = \frac{14}{15}$.

So, we find that we multiply two fractions as $\frac{\text{Product of Numerators}}{\text{Product of Denominators}}$.

TRY THESE

Find: $\frac{8}{3} \times \frac{4}{7}$; $\frac{3}{4} \times \frac{2}{3}$.

Value of the Products

You have seen that the product of two whole numbers is bigger than each of the two whole numbers. For example, $3 \times 4 = 12$ and $12 > 4$, $12 > 3$. What happens to the value of the product when we multiply two fractions?

Let us first consider the product of two proper fractions.

We have,

$\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$	$\frac{8}{15} < \frac{2}{3}, \frac{8}{15} < \frac{4}{5}$	Product is less than each of the fractions
$\frac{1}{5} \times \frac{2}{7}$ = ---------	--------,--------	-------------------------------------
$\frac{3}{5} \times \frac{\square}{8}$ =	--------,--------	-------------------------------------
$\frac{2}{\square} \times \frac{4}{9} = \frac{8}{45}$	--------,--------	-------------------------------------

You will find that *when two proper fractions are multiplied, the product is less than each of the fractions*. Or, *we say the value of the product of two proper fractions is smaller than each of the two fractions.*

Check this by constructing five more examples.

Let us now multiply two improper fractions.

$\frac{7}{3}\times\frac{5}{2}=\frac{35}{6}$	$\frac{35}{6}>\frac{7}{3},\frac{35}{6}>\frac{5}{2}$	Product is greater than each of the fractions
$\frac{6}{5}\times\frac{\square}{3}=\frac{24}{15}$	--------,--------	--
$\frac{9}{2}\times\frac{7}{\square}=\frac{63}{8}$	--------,--------	--
$\frac{3}{\square}\times\frac{8}{7}=\frac{24}{14}$	--------,--------	--

We find that *the product of two improper fractions is greater than each of the two fractions.*

Or, *the value of the product of two improper fractions is more than each of the two fractions*.

Construct five more examples for yourself and verify the above statement.

Let us now multiply a proper and an improper fraction, say $\frac{2}{3}$ and $\frac{7}{5}$.

We have $\frac{2}{3}\times\frac{7}{5}=\frac{14}{15}$. Here, $\frac{14}{15}<\frac{7}{5}$ and $\frac{14}{15}>\frac{2}{3}$

The product obtained is less than the improper fraction and greater than the proper fraction involved in the multiplication.

Check it for $\frac{6}{5}\times\frac{2}{8}$, $\frac{8}{3}\times\frac{4}{5}$.

EXERCISE 2.2

1. Find:

   (i) $\frac{1}{4}$ of (a) $\frac{1}{4}$ (b) $\frac{3}{5}$ (c) $\frac{4}{3}$

   (ii) $\frac{1}{7}$ of (a) $\frac{2}{9}$ (b) $\frac{6}{5}$ (c) $\frac{3}{10}$

2. Multiply and reduce to lowest form (if possible) :

(i) $\frac{2}{3} \times 2\frac{2}{3}$ (ii) $\frac{2}{7} \times \frac{7}{9}$ (iii) $\frac{3}{8} \times \frac{6}{4}$ (iv) $\frac{9}{5} \times \frac{3}{5}$

(v) $\frac{1}{3} \times \frac{15}{8}$ (vi) $\frac{11}{2} \times \frac{3}{10}$ (vii) $\frac{4}{5} \times \frac{12}{7}$

3. Multiply the following fractions:

(i) $\frac{2}{5} \times 5\frac{1}{4}$ (ii) $6\frac{2}{5} \times \frac{7}{9}$ (iii) $\frac{3}{2} \times 5\frac{1}{3}$ (iv) $\frac{5}{6} \times 2\frac{3}{7}$

(v) $3\frac{2}{5} \times \frac{4}{7}$ (vi) $2\frac{3}{5} \times 3$ (vii) $3\frac{4}{7} \times \frac{3}{5}$

4. Which is greater:

(i) $\frac{2}{7}$ of $\frac{3}{4}$ or $\frac{3}{5}$ of $\frac{5}{8}$ (ii) $\frac{1}{2}$ of $\frac{6}{7}$ or $\frac{2}{3}$ of $\frac{3}{7}$

5. Saili plants 4 saplings, in a row, in her garden. The distance between two adjacent saplings is $\frac{3}{4}$ m. Find the distance between the first and the last sapling.

6. Lipika reads a book for $1\frac{3}{4}$ hours everyday. She reads the entire book in 6 days. How many hours in all were required by her to read the book?

7. A car runs 16 km using 1 litre of petrol. How much distance will it cover using $2\frac{3}{4}$ litres of petrol.

8. (a) (i) Provide the number in the box $\square$, such that $\frac{2}{3} \times \square = \frac{10}{30}$.

(ii) The simplest form of the number obtained in $\square$ is _____.

(b) (i) Provide the number in the box $\square$, such that $\frac{3}{5} \times \square = \frac{24}{75}$.

(ii) The simplest form of the number obtained in $\square$ is _____.

2.2 Division of Fractions

John has a paper strip of length 6 cm. He cuts this strip in smaller strips of length 2 cm each. You know that he would get 6 ÷ 2 = 3 strips.

John cuts another strip of length 6 cm into smaller strips of length $\frac{3}{2}$ cm each. How many strips will he get now? He will get $6 \div \frac{3}{2}$ strips.

A paper strip of length $\frac{15}{2}$ cm can be cut into smaller strips of length $\frac{3}{2}$ cm each to give $\frac{15}{2} \div \frac{3}{2}$ pieces.

So, we are required to divide a whole number by a fraction or a fraction by another fraction. Let us see how to do that.

2.2.1 Division of Whole Number by a Fraction

Let us find $1 \div \frac{1}{2}$.

We divide a whole into a number of equal parts such that each part is half of the whole. The number of such half ($\frac{1}{2}$) parts would be $1 \div \frac{1}{2}$. Observe the figure (Fig 2.11). How many half parts do you see?

There are two half parts.

So, $1 \div \frac{1}{2} = 2$. Also, $1 \times \frac{2}{1} = 1 \times 2 = 2$. Thus, $1 \div \frac{1}{2} = 1 \times \frac{2}{1}$

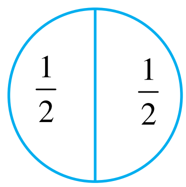


Fig 2.11

Similarly, $3 \div \frac{1}{4}$ = number of $\frac{1}{4}$ parts obtained when each of the 3 whole, are divided into $\frac{1}{4}$ equal parts = 12 (From Fig 2.12)

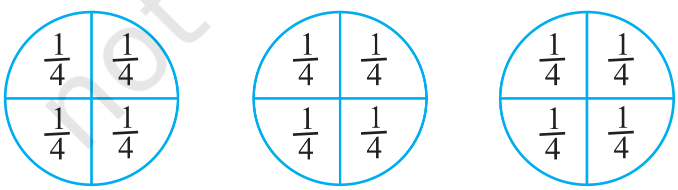


Fig 2.12

Observe also that, $3 \times \frac{4}{1} = 3 \times 4 = 12$. Thus, $3 \div \frac{1}{4} = 3 \times \frac{4}{1} = 12$.

Find in a similar way, $3 \div \frac{1}{2}$ and $3 \times \frac{2}{1}$.

Reciprocal of a fraction

The number $\frac{2}{1}$ can be obtained by interchanging the numerator and denominator of $\frac{1}{2}$ or by inverting $\frac{1}{2}$. Similarly, $\frac{3}{1}$ is obtained by inverting $\frac{1}{3}$.
Let us first see about the inverting of such numbers.
Observe these products and fill in the blanks :

$7 \times \frac{1}{7} = 1$	$\frac{5}{4} \times \frac{4}{5} = $ ---------
$\frac{1}{9} \times 9 = $ ------	$\frac{2}{7} \times$ ------- $= 1$
$\frac{2}{3} \times \frac{3}{2} = \frac{2 \times 3}{3 \times 2} = \frac{6}{6} = 1$	------ $\times \frac{5}{9} = 1$

Multiply five more such pairs.

The non-zero numbers whose product with each other is 1, are called the reciprocals of each other. So reciprocal of $\frac{5}{9}$ is $\frac{9}{5}$ and the reciprocal of $\frac{9}{5}$ is $\frac{5}{9}$. What is the receiprocal of $\frac{1}{9}$? $\frac{2}{7}$?

You will see that the reciprocal of $\frac{2}{3}$ is obtained by inverting it. You get $\frac{3}{2}$.

THINK, DISCUSS AND WRITE

(i) Will the reciprocal of a proper fraction be again a proper fraction?

(ii) Will the reciprocal of an improper fraction be again an improper fraction?

Therefore, we can say that

$$1 \div \frac{1}{2} = 1 \times \frac{2}{1} = 1 \times \text{ reciprocal of } \frac{1}{2}.$$

$$3 \div \frac{1}{4} = 3 \times \frac{4}{1} = 3 \times \text{ reciprocal of } \frac{1}{4}.$$

$$3 \div \frac{1}{2} = \text{------} = \text{----------------------}.$$

$$\text{So, } 2 \div \frac{3}{4} = 2 \times \text{ reciprocal of } \frac{3}{4} = 2 \times \frac{4}{3}.$$

$$5 \div \frac{2}{9} = 5 \times \text{ ------------------- } = 5 \times \text{ -------------}$$

Thus, *to divide a whole number by any fraction, multiply that whole number by the reciprocal of that fraction.*

TRY THESE

Find : (i) $7 \div \frac{2}{5}$ (ii) $6 \div \frac{4}{7}$ (iii) $2 \div \frac{8}{9}$

- *While dividing a whole number by a mixed fraction, first convert the mixed fraction into improper fraction and then solve it.*

Thus, $4 \div 2\frac{2}{5} = 4 \div \frac{12}{5} = \ ?$ Also, $5 \div 3\frac{1}{3} = 3 \div \frac{10}{3} = ?$

TRY THESE

Find: (i) $6 \div 5\frac{1}{3}$

(ii) $7 \div 2\frac{4}{7}$

2.2.2 Division of a Fraction by a Whole Number

- What will be $\frac{3}{4} \div 3$?

Based on our earlier observations we have: $\frac{3}{4} \div 3 = \frac{3}{4} \div \frac{3}{1} = \frac{3}{4} \times \frac{1}{3} = \frac{3}{12} = \frac{1}{4}$

So, $\frac{2}{3} \div 7 = \frac{2}{3} \times \frac{1}{7} = \quad ?$ What is $\frac{5}{7} \div 6$, $\frac{2}{7} \div 8$?

- While dividing mixed fractions by whole numbers, convert the mixed fractions into improper fractions. That is,

$2\frac{2}{3} \div 5 = \frac{8}{3} \div 5 =$ ------ ; $4\frac{2}{5} \div 3 =$ ------ $=$ ------; $2\frac{3}{5} \div 2 =$ ------ $=$ ------

2.2.3 Division of a Fraction by Another Fraction

We can now find $\frac{1}{3} \div \frac{6}{5}$.

$\frac{1}{3} \div \frac{6}{5} = \frac{1}{3} \times$ reciprocal of $\frac{6}{5} = \frac{1}{3} \times \frac{5}{6} = \frac{5}{18}$.

Similarly, $\frac{8}{5} \div \frac{2}{3} = \frac{8}{5} \times$ reciprocal of $\frac{2}{3} = ?$ and, $\frac{1}{2} \div \frac{3}{4} = ?$

TRY THESE

Find: (i) $\frac{3}{5} \div \frac{1}{2}$ (ii) $\frac{1}{2} \div \frac{3}{5}$ (iii) $2\frac{1}{2} \div \frac{3}{5}$ (iv) $5\frac{1}{6} \div \frac{9}{2}$

EXERCISE 2.3

1. Find:

(i) $12 \div \frac{3}{4}$ (ii) $14 \div \frac{5}{6}$ (iii) $8 \div \frac{7}{3}$ (iv) $4 \div \frac{8}{3}$

(v) $3 \div 2\frac{1}{3}$ (vi) $5 \div 3\frac{4}{7}$

2. Find the reciprocal of each of the following fractions. Classify the reciprocals as proper fractions, improper fractions and whole numbers.

(i) $\frac{3}{7}$ (ii) $\frac{5}{8}$ (iii) $\frac{9}{7}$ (iv) $\frac{6}{5}$

(v) $\frac{12}{7}$ (vi) $\frac{1}{8}$ (vii) $\frac{1}{11}$

3. Find:

(i) $\frac{7}{3} \div 2$ (ii) $\frac{4}{9} \div 5$ (iii) $\frac{6}{13} \div 7$ (iv) $4\frac{1}{3} \div 3$

(v) $3\frac{1}{2} \div 4$ (vi) $4\frac{3}{7} \div 7$

4. Find:

(i) $\frac{2}{5} \div \frac{1}{2}$ (ii) $\frac{4}{9} \div \frac{2}{3}$ (iii) $\frac{3}{7} \div \frac{8}{7}$ (iv) $2\frac{1}{3} \div \frac{3}{5}$ (v) $3\frac{1}{2} \div \frac{8}{3}$

(vi) $\frac{2}{5} \div 1\frac{1}{2}$ (vii) $3\frac{1}{5} \div 1\frac{2}{3}$ (viii) $2\frac{1}{5} \div 1\frac{1}{5}$

2.3 MULTIPLICATION OF DECIMAL NUMBERS

Reshma purchased 1.5kg vegetable at the rate of ₹ 8.50 per kg. How much money should she pay? Certainly it would be ₹ (8.50 × 1.50). Both 8.5 and 1.5 are decimal numbers. So, we have come across a situation where we need to know how to multiply two decimals. Let us now learn the multiplication of two decimal numbers.

First we find 0.1 × 0.1.

Now, $0.1 = \frac{1}{10}$. So, $0.1 \times 0.1 = \frac{1}{10} \times \frac{1}{10} = \frac{1 \times 1}{10 \times 10} = \frac{1}{100} = 0.01$.

Let us see it's pictorial representation (Fig 2.13)

The fraction $\frac{1}{10}$ represents 1 part out of 10 equal parts.

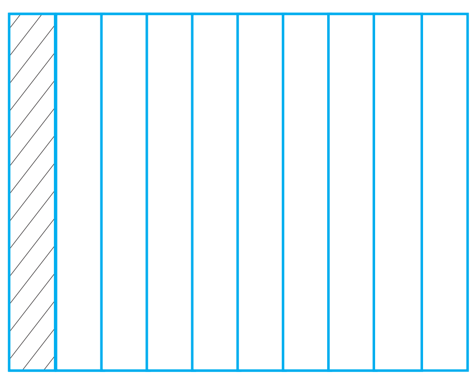


Fig 2.13

The shaded part in the picture represents $\frac{1}{10}$.

We know that,

$\frac{1}{10} \times \frac{1}{10}$ means $\frac{1}{10}$ of $\frac{1}{10}$. So, divide this $\frac{1}{10}^{\text{th}}$ part into 10 equal parts and take one part out of it.

Thus, we have, (Fig 2.14).

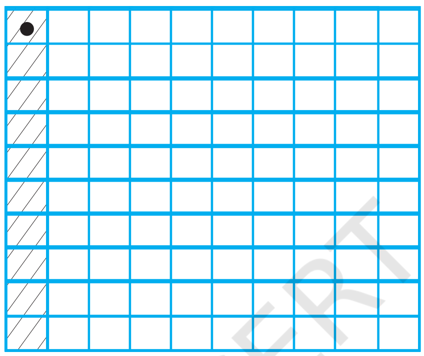


Fig 2.14

The dotted square is one part out of 10 of the $\frac{1}{10}^{\text{th}}$ part. That is, it represents $\frac{1}{10} \times \frac{1}{10}$ or 0.1×0.1.

Can the dotted square be represented in some other way?

How many small squares do you find in Fig 2.14?

There are 100 small squares. So the dotted square represents one out of 100 or 0.01.

Hence, $0.1 \times 0.1 = 0.01$.

Note that 0.1 occurs two times in the product. In 0.1 there is one digit to the right of the decimal point. In 0.01 there are two digits (i.e., 1 + 1) to the right of the decimal point.

Let us now find 0.2×0.3.

We have, $0.2 \times 0.3 = \frac{2}{10} \times \frac{3}{10}$

As we did for $\frac{1}{10} \times \frac{1}{10}$, let us divide the square into 10 equal parts and take three parts out of it, to get $\frac{3}{10}$. Again divide each

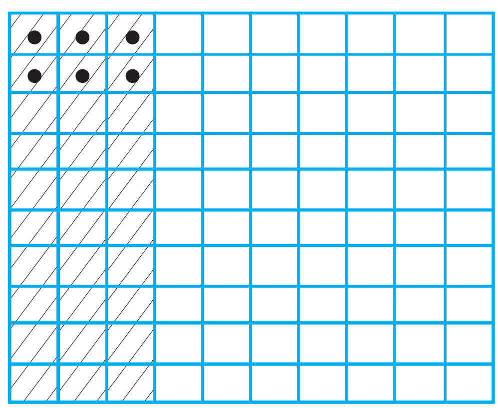


Fig 2.15

of these three equal parts into 10 equal parts and take two from each. We get $\frac{2}{10} \times \frac{3}{10}$.

The dotted squares represent $\frac{2}{10} \times \frac{3}{10}$ or 0.2×0.3. (Fig 2.15)

Since there are 6 dotted squares out of 100, so they also reprsent 0.06.

Thus, $0.2 \times 0.3 = 0.06$.

Observe that $2 \times 3 = 6$ and the number of digits to the right of the decimal point in 0.06 is 2 (= 1 + 1).

Check whether this applies to 0.1×0.1 also.

Find 0.2×0.4 by applying these observations.

While finding 0.1×0.1 and 0.2×0.3, you might have noticed that first we multiplied them as whole numbers ignoring the decimal point. In 0.1×0.1, we found 01×01 or 1×1. Similarly in 0.2×0.3 we found 02×03 or 2×3.

Then, we counted the number of digits starting from the rightmost digit and moved towards left. We then put the decimal point there. The number of digits to be counted is obtained by adding the number of digits to the right of the decimal point in the decimal numbers that are being multiplied.

Let us now find 1.2×2.5.

Multiply 12 and 25. We get 300. Both, in 1.2 and 2.5, there is 1 digit to the right of the decimal point. So, count $1 + 1 = 2$ digits from the rightmost digit (i.e., 0) in 300 and move towards left. We get 3.00 or 3.

Find in a similar way 1.5×1.6, 2.4×4.2.

While multiplying 2.5 and 1.25, you will first multiply 25 and 125. For placing the decimal in the product obtained, you will count $1 + 2 = 3$ (Why?) digits starting from the rightmost digit. Thus, $2.5 \times 1.25 = 3.225$

Find 2.7×1.35.

TRY THESE

1. Find: (i) 2.7×4 (ii) 1.8×1.2 (iii) 2.3×4.35
2. Arrange the products obtained in (1) in descending order.

EXAMPLE 3 The side of an equilateral triangle is 3.5 cm. Find its perimeter.

SOLUTION All the sides of an equilateral triangle are equal.

So, length of each side = 3.5 cm

Thus, perimeter = 3 × 3.5 cm = 10.5 cm

Example 4 The length of a rectangle is 7.1 cm and its breadth is 2.5 cm. What is the area of the rectangle?

Solution Length of the rectangle = 7.1 cm

Breadth of the rectangle = 2.5 cm

Therefore, area of the rectangle = 7.1 × 2.5 cm^2 = 17.75 cm^2

2.3.1 Multiplication of Decimal Numbers by 10, 100 and 1000

Reshma observed that $2.3 = \frac{23}{10}$ whereas $2.35 = \frac{235}{100}$. Thus, she found that depending on the position of the decimal point the decimal number can be converted to a fraction with denominator 10 or 100. She wondered what would happen if a decimal number is multiplied by 10 or 100 or 1000.

Let us see if we can find a pattern of multiplying numbers by 10 or 100 or 1000.

Have a look at the table given below and fill in the blanks:

$1.76 \times 10 = \frac{176}{100} \times 10 = 17.6$	2.35 ×10 =___	12.356 × 10 =___
$1.76 \times 100 = \frac{176}{100} \times 100 = 176$ or 176.0	2.35 ×100 = ___	12.356 × 100 =___
$1.76 \times 1000 = \frac{176}{100} \times 1000 = 1760$ or 1760.0	2.35 ×1000 = ___	12.356 × 1000 = ___
$0.5 \times 10 = \frac{5}{10} \times 10 = 5$; 0.5 × 100 = ___ ; 0.5 × 1000 = ___		

Observe the shift of the decimal point of the products in the table. Here the numbers are multiplied by 10,100 and 1000. In 1.76 × 10 = 17.6, the digits are same i.e., 1, 7 and 6. Do you observe this in other products also? Observe 1.76 and 17.6. To which side has the decimal point shifted, right or left? The decimal point has shifted to the right by one place. Note that 10 has one zero over 1.

In 1.76×100 = 176.0, observe 1.76 and 176.0. To which side and by how many digits has the decimal point shifted? The decimal point has shifted to the right by two places.

Note that 100 has two zeros over one.

Do you observe similar shifting of decimal point in other products also?

So we say, *when a decimal number is multiplied by 10, 100 or 1000, the digits in the product are same as in the decimal number but the decimal point in the product is shifted to the right by as, many of places as there are zeros over one.*

TRY THESE

Find: (i) 0.3×10
(ii) 1.2×100
(iii) 56.3×1000

Based on these observations we can now say

$$0.07 \times 10 = 0.7,\ 0.07 \times 100 = 7 \text{ and } 0.07 \times 1000 = 70.$$

Can you now tell $2.97 \times 10 = ?$ $\quad 2.97 \times 100 = ?$ $\quad 2.97 \times 1000 = ?$

Can you now help Reshma to find the total amount i.e., ₹ 8.50×150, that she has to pay?

EXERCISE 2.4

1. Find:
   (i) 0.2×6 (ii) 8×4.6 (iii) 2.71×5 (iv) 20.1×4
   (v) 0.05×7 (vi) 211.02×4 (vii) 2×0.86
2. Find the area of rectangle whose length is 5.7cm and breadth is 3 cm.
3. Find:
   (i) 1.3×10 (ii) 36.8×10 (iii) 153.7×10 (iv) 168.07×10
   (v) 31.1×100 (vi) 156.1×100 (vii) 3.62×100 (viii) 43.07×100
   (ix) 0.5×10 (x) 0.08×10 (xi) 0.9×100 (xii) 0.03×1000
4. A two-wheeler covers a distance of 55.3 km in one litre of petrol. How much distance will it cover in 10 litres of petrol?
5. Find:
   (i) 2.5×0.3 (ii) 0.1×51.7 (iii) 0.2×316.8 (iv) 1.3×3.1
   (v) 0.5×0.05 (vi) 11.2×0.15 (vii) 1.07×0.02
   (viii) 10.05×1.05 (ix) 101.01×0.01 (x) 100.01×1.1

2.4 DIVISION OF DECIMAL NUMBERS

Savita was preparing a design to decorate her classroom. She needed a few coloured strips of paper of length 1.9 cm each. She had a strip of coloured paper of length 9.5 cm. How many pieces of the required length will she get out of this strip? She thought it would be $\frac{9.5}{1.9}$ cm. Is she correct?

Both 9.5 and 1.9 are decimal numbers. So we need to know the division of decimal numbers too!

2.4.1 Division by 10, 100 and 1000

Let us find the division of a decimal number by 10, 100 and 1000.
Consider $31.5 \div 10$.

$$31.5 \div 10 = \frac{315}{10} \times \frac{1}{10} = \frac{315}{100} = 3.15$$

Similarly, $31.5 \div 100 = \frac{315}{10} \times \frac{1}{100} = \frac{315}{1000} = 0.315$

Let us see if we can find a pattern for dividing numbers by 10, 100 or 1000. This may help us in dividing numbers by 10, 100 or 1000 in a shorter way.

31.5 ÷ 10 = 3.15	231.5 ÷ 10 =___	1.5 ÷ 10 =___	29.36 ÷ 10 =___
31.5 ÷ 100 = 0.315	231.5 ÷ 10 =___	1.5 ÷ 100 =___	29.36 ÷ 100 =___
31.5 ÷1000 = 0.0315	231.5 ÷ 1000 =___	1.5 ÷ 1000 =___	29.36 ÷1000 =___

Take 31.5 ÷ 10 = 3.15. In 31.5 and 3.15, the digits are same i.e., 3, 1, and 5 but the decimal point has shifted in the quotient. To which side and by how many digits? The decimal point has shifted to the left by one place. Note that 10 has one zero over 1.

TRY THESE

Find: (i) 235.4 ÷ 10

(ii) 235.4 ÷100

(iii) 235.4 ÷ 1000

Consider now 31.5 ÷ 100 = 0.315. In 31.5 and 0.315 the digits are same, but what about the decimal point in the quotient? It has shifted to the left by two places. Note that 100 has two zeros over1.

So we can say that, *while dividing a number by 10, 100 or 1000, the digits of the number and the quotient are same but the decimal point in the quotient shifts to the left by as many places as there are zeros over 1.* Using this observation let us now quickly find: 2.38 ÷ 10 = 0.238, 2.38 ÷ 100 = 0.0238, 2.38 ÷ 1000 = 0.00238

2.4.2 Division of a Decimal Number by a Whole Number

TRY THESE

(i) 35.7 ÷ 3 = ?;

(ii) 25.5 ÷ 3 = ?

Let us find $\frac{6.4}{2}$. Remember we also write it as 6.4 ÷ 2.

So, $6.4 \div 2 = \frac{64}{10} \div 2 = \frac{64}{10} \times \frac{1}{2}$ as learnt in fractions.

$$= \frac{64 \times 1}{10 \times 2} = \frac{1 \times 64}{10 \times 2} = \frac{1}{10} \times \frac{64}{2} = \frac{1}{10} \times 32 = \frac{32}{10} = 3.2$$

Or, let us first divide 64 by 2. We get 32. There is one digit to the right of the decimal point in 6.4. Place the decimal in 32 such that there would be one digit to its right. We get 3.2 again.

To find 19.5 ÷ 5, first find 195 ÷5. We get 39. There is one digit to the right of the decimal point in 19.5. Place the decimal point in 39 such that there would be one digit to its right. You will get 3.9.

TRY THESE

(i) 43.15 ÷ 5 = ?;

(ii) 82.44 ÷ 6 = ?

TRY THESE

Find: (i) $15.5 \div 5$

(ii) $126.35 \div 7$

Now, $12.96 \div 4 = \frac{1296}{100} \div 4 = \frac{1296}{100} \times \frac{1}{4} = \frac{1}{100} \times \frac{1296}{4} = \frac{1}{100} \times 324 = 3.24$

Or, divide 1296 by 4. You get 324. There are two digits to the right of the decimal in 12.96. Making similar placement of the decimal in 324, you will get 3.24.

Note that here and in the next section, we have considered only those divisions in which, ignoring the decimal, the number would be completely divisible by another number to give remainder zero. Like, in $19.5 \div 5$, the number 195 when divided by 5, leaves remainder zero.

However, there are situations in which the number may not be completely divisible by another number, i.e., we may not get remainder zero. For example, $195 \div 7$. We deal with such situations in later classes.

EXAMPLE 5 Find the average of 4.2, 3.8 and 7.6.

SOLUTION The average of 4.2, 3.8 and 7.6 is $\frac{4.2 + 3.8 + 7.6}{3} = \qquad = 5.2$.

2.4.3 Division of a Decimal Number by another Decimal Number

Let us find $\frac{25.5}{0.5}$ i.e., $25.5 \div 0.5$.

We have $25.5 \div 0.5 = \frac{255}{10} \div \frac{5}{10} = \frac{255}{10} \times \frac{10}{5} = 51$. Thus, $25.5 \div 0.5 = 51$

What do you observe? For $\frac{25.5}{0.5}$, we find that there is one digit to the right of the decimal in 0.5. This could be converted to whole number by dividing by 10. Accordingly 25.5 was also converted to a fraction by dividing by 10.

Or, we say the decimal point was shifted by one place to the right in 0.5 to make it 5. So, there was a shift of one decimal point to the right in 25.5 also to make it 255.

Thus, $22.5 \div 1.5 = \frac{22.5}{1.5} = \frac{225}{15} = 15$

Find $\frac{20.3}{0.7}$ and $\frac{15.2}{0.8}$ in a similar way.

Let us now find $20.55 \div 1.5$.

TRY THESE

Find: (i) $\frac{7.75}{0.25}$ (ii) $\frac{42.8}{0.02}$ (iii) $\frac{5.6}{1.4}$

We can write it is as $205.5 \div 15$, as discussed above. We get 13.7. Find $\frac{3.96}{0.4}$, $\frac{2.31}{0.3}$.

Consider now, $\frac{33.725}{0.25}$. We can write it as $\frac{3372.5}{25}$ (How?) and we get the quotient as 134.9. How will you find $\frac{27}{0.03}$? We know that 27 can be written as 27.00.

So, $\frac{27}{0.03} = \frac{27.00}{0.03} = \frac{2700}{3} = 900$

EXAMPLE 6 Each side of a regular polygon is 2.5 cm in length. The perimeter of the polygon is 12.5cm. How many sides does the polygon have?

SOLUTION The perimeter of a regular polygon is the sum of the lengths of all its equal sides = 12.5 cm.

Length of each side = 2.5 cm. Thus, the number of sides = $\frac{12.5}{2.5} = \frac{125}{25} = 5$

The polygon has 5 sides.

EXAMPLE 7 A car covers a distance of 89.1 km in 2.2 hours. What is the average distance covered by it in 1 hour?

SOLUTION Distance covered by the car = 89.1 km.

Time required to cover this distance = 2.2 hours.

So distance covered by it in 1 hour $= \frac{89.1}{2.2} = \frac{891}{22} = 40.5$ km.

EXERCISE 2.5

1. Find:
(i) 0.4 ÷ 2 (ii) 0.35 ÷ 5 (iii) 2.48 ÷ 4 (iv) 65.4 ÷ 6
(v) 651.2 ÷ 4 (vi) 14.49 ÷ 7 (vii) 3.96 ÷ 4 (viii) 0.80 ÷ 5

2. Find:
(i) 4.8 ÷ 10 (ii) 52.5 ÷ 10 (iii) 0.7 ÷ 10 (iv) 33.1 ÷ 10
(v) 272.23 ÷ 10 (vi) 0.56 ÷ 10 (vii) 3.97 ÷10

3. Find:
(i) 2.7 ÷ 100 (ii) 0.3 ÷ 100 (iii) 0.78 ÷ 100
(iv) 432.6 ÷ 100 (v) 23.6 ÷100 (vi) 98.53 ÷ 100

4. Find:
(i) 7.9 ÷ 1000 (ii) 26.3 ÷ 1000 (iii) 38.53 ÷ 1000
(iv) 128.9 ÷ 1000 (v) 0.5 ÷ 1000

5. Find:

(i) $7 \div 3.5$ (ii) $36 \div 0.2$ (iii) $3.25 \div 0.5$ (iv) $30.94 \div 0.7$

(v) $0.5 \div 0.25$ (vi) $7.75 \div 0.25$ (vii) $76.5 \div 0.15$ (viii) $37.8 \div 1.4$

(ix) $2.73 \div 1.3$

6. A vehicle covers a distance of 43.2 km in 2.4 litres of petrol. How much distance will it cover in one litre of petrol?

What have We Discussed?

1. We have learnt how to multiply fractions. Two fractions are multiplied by multiplying their numerators and denominators seperately and writing the product as $\frac{\text{product of numerators}}{\text{product of denominators}}$. For example, $\frac{2}{3} \times \frac{5}{7} = \frac{2 \times 5}{3 \times 7} = \frac{10}{21}$.

2. A fraction acts as an operator 'of'. For example, $\frac{1}{2}$ of 2 is $\frac{1}{2} \times 2 = 1$.

3. (a) The product of two proper fractions is less than each of the fractions that are multiplied.

(b) The product of a proper and an improper fraction is less than the improper fraction and greater than the proper fraction.

(c) The product of two imporper fractions is greater than the two fractions.

4. A reciprocal of a fraction is obtained by inverting it upside down.

5. We have seen how to divide two fractions.

(a) While dividing a whole number by a fraction, we multiply the whole number with the reciprocal of that fraction.

For example, $2 \div \frac{3}{5} = 2 \times \frac{5}{3} = \frac{10}{3}$

(b) While dividing a fraction by a whole number we multiply the fraction by the reciprocal of the whole number.

For example, $\frac{2}{3} \div 7 = \frac{2}{3} \times \frac{1}{7} = \frac{2}{21}$

(c) While dividing one fraction by another fraction, we multuiply the first fraction by the reciprocal of the other. So, $\frac{2}{3} \div \frac{5}{7} = \frac{2}{3} \times \frac{7}{5} = \frac{14}{15}$.

6. We also learnt how to multiply two decimal numbers. While multiplying two decimal numbers, first multiply them as whole numbers. Count the number of digits to the right of the decimal point in both the decimal numbers. Add the number of digits counted. Put the decimal point in the product by counting the digits from its rightmost place. The count should be the sum obtained earlier.

For example, $0.5 \times 0.7 = 0.35$

7. To multiply a decimal number by 10, 100 or 1000, we move the decimal point in the number to the right by as many places as there are zeros over 1.

Thus $0.53 \times 10 = 5.3$, $0.53 \times 100 = 53$, $0.53 \times 1000 = 530$

8. We have seen how to divide decimal numbers.

(a) To divide a decimal number by a whole number, we first divide them as whole numbers. Then place the decimal point in the quotient as in the decimal number.

For example, $8.4 \div 4 = 2.1$

Note that here we consider only those divisions in which the remainder is zero.

(b) To divide a decimal number by 10, 100 or 1000, shift the digits in the decimal number to the left by as many places as there are zeros over 1, to get the quotient.

So, $23.9 \div 10 = 2.39, 23.9 \div 100 = 0\ .239$, $23.9 \div 1000 = 0.0239$

(c) While dividing two decimal numbers, first shift the decimal point to the right by equal number of places in both, to convert the divisor to a whole number. Then divide. Thus, $2.4 \div 0.2 = 24 \div 2 = 12$.

Data Handling

Chapter 3

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3.1 Representative Values

You might be aware of the term *average* and would have come across statements involving the term 'average' in your day-to-day life:

- Isha spends on an average of about 5 hours daily for her studies.
- The average temperature at this time of the year is about 40 degree celsius.
- The average age of pupils in my class is 12 years.
- The average attendance of students in a school during its final examination was 98 per cent.

Many more of such statements could be there. Think about the statements given above.

Do you think that the child in the first statement studies exactly for 5 hours daily?

Or, is the temperature of the given place during that particular time always 40 degrees?

Or, is the age of each pupil in that class 12 years? Obviously not.

Then what do these statements tell you?

By average we understand that Isha, usually, studies for 5 hours. On some days, she may study for less number of hours and on the other days she may study longer.

Similarly, the average temperature of 40 degree celsius, means that, very often, the temperature at this time of the year is around 40 degree celsius. Sometimes, it may be less than 40 degree celsius and at other times, it may be more than 40°C.

Thus, we realise that average is a number that represents or shows the central tendency of a group of observations or data. Since average lies between the highest and the lowest value of the given data so, we say average is a measure of the central tendency of the group of data. Different forms of data need different forms of representative or central value to describe it. One of these representative values is the "**Arithmetic mean**". You will learn about the other representative values in the later part of the chapter.

3.2 Arithmetic Mean

The most common representative value of a group of data is the **arithmetic mean** or the **mean**. To understand this in a better way, let us look at the following example:

Two vessels contain 20 litres and 60 litres of milk respectively. What is the amount that each vessel would have, if both share the milk equally? When we ask this question we are seeking the arithmetic mean.

In the above case, the average or the arithmetic mean would be

$$\frac{\text{Total quantity of milk}}{\text{Number of vessels}} = \frac{20+60}{2} \text{ litres} = 40 \text{ litres.}$$

Thus, each vessels would have 40 litres of milk.

The average or Arithmetic Mean (A.M.) or simply mean is defined as follows:

$$\text{mean} = \frac{\text{Sum of all observations}}{\text{number of observations}}$$

Consider these examples.

Example 1 Ashish studies for 4 hours, 5 hours and 3 hours respectively on three consecutive days. How many hours does he study daily on an average?

Solution The average study time of Ashish would be

$$\frac{\text{Total number of study hours}}{\text{Number of days for which he studied}} = \frac{4+5+3}{3} \text{ hours} = 4 \text{ hours per day}$$

Thus, we can say that Ashish studies for 4 hours daily on an average.

Example 2 A batsman scored the following number of runs in six innings:

36, 35, 50, 46, 60, 55

Calculate the mean runs scored by him in an inning.

Solution Total runs = 36 + 35 + 50 + 46 + 60 + 55 = 282.

To find the mean, we find the sum of all the observations and divide it by the number of observations.

Therefore, in this case, mean $= \frac{282}{6} = 47$. Thus, the mean runs scored in an inning are 47.

Where does the arithmetic mean lie

Try These

How would you find the average of your study hours for the whole week?

Think, Discuss and Write

Consider the data in the above examples and think on the following:

- Is the mean bigger than each of the observations?
- Is it smaller than each observation?

Discuss with your friends. Frame one more example of this type and answer the same questions.

You will find that the mean lies inbetween the greatest and the smallest observations.

In particular, the mean of two numbers will always lie between the two numbers. For example the mean of 5 and 11 is $\frac{5+11}{2} = 8$, which lies between 5 and 11.

Can you use this idea to show that between any two fractional numbers, you can find as many fractional numbers as you like. For example between $\frac{1}{2}$ and $\frac{1}{4}$ you have their average $\frac{\frac{1}{2}+\frac{1}{4}}{2} = \frac{3}{8}$ and then between $\frac{1}{2}$ and $\frac{3}{8}$, you have their average $\frac{7}{16}$ and so on.

Try These

1. Find the mean of your sleeping hours during one week.
2. Find atleast 5 numbers between $\frac{1}{2}$ and $\frac{1}{3}$.

3.2.1 Range

The difference between the highest and the lowest observation gives us an idea of the spread of the observations. This can be found by subtracting the lowest observation from the highest observation. We call the result the **range** of the observation. Look at the following example:

Example 3 The ages in years of 10 teachers of a school are:

32, 41, 28, 54, 35, 26, 23, 33, 38, 40

(i) What is the age of the oldest teacher and that of the youngest teacher?

(ii) What is the range of the ages of the teachers?

(iii) What is the mean age of these teachers?

Solution

(i) Arranging the ages in ascending order, we get:

23, 26, 28, 32, 33, 35, 38, 40, 41, 54

We find that the age of the oldest teacher is 54 years and the age of the youngest teacher is 23 years.

(ii) Range of the ages of the teachers = (54 – 23) years = 31 years

(iii) Mean age of the teachers

$$= \frac{23+26+28+32+33+35+38+40+41+54}{10} \text{ years}$$

$$= \frac{350}{10} \text{ years} = 35 \text{ years}$$

EXERCISE 3.1

1. Find the range of heights of any ten students of your class.

2. Organise the following marks in a class assessment, in a tabular form.

4, 6, 7, 5, 3, 5, 4, 5, 2, 6, 2, 5, 1, 9, 6, 5, 8, 4, 6, 7

(i) Which number is the highest? (ii) Which number is the lowest?

(iii) What is the range of the data? (iv) Find the arithmetic mean.

3. Find the mean of the first five whole numbers.

4. A cricketer scores the following runs in eight innings:

58, 76, 40, 35, 46, 45, 0, 100.

Find the mean score.

5. Following table shows the points of each player scored in four games:

Player	Game 1	Game 2	Game 3	Game 4
A	14	16	10	10
B	0	8	6	4
C	8	11	Did not Play	13

Now answer the following questions:

(i) Find the mean to determine A's average number of points scored per game.

(ii) To find the mean number of points per game for C, would you divide the total points by 3 or by 4? Why?

(iii) B played in all the four games. How would you find the mean?

(iv) Who is the best performer?

6. The marks (out of 100) obtained by a group of students in a science test are 85, 76, 90, 85, 39, 48, 56, 95, 81 and 75. Find the:

(i) Highest and the lowest marks obtained by the students.

(ii) Range of the marks obtained.

(iii) Mean marks obtained by the group.

7. The enrolment in a school during six consecutive years was as follows:
1555, 1670, 1750, 2013, 2540, 2820
Find the mean enrolment of the school for this period.

8. The rainfall (in mm) in a city on 7 days of a certain week was recorded as follows:

Day	**Mon**	**Tue**	**Wed**	**Thurs**	**Fri**	**Sat**	**Sun**
Rainfall (in mm)	0.0	12.2	2.1	0.0	20.5	5.5	1.0

(i) Find the range of the rainfall in the above data.

(ii) Find the mean rainfall for the week.

(iii) On how many days was the rainfall less than the mean rainfall.

9. The heights of 10 girls were measured in cm and the results are as follows:
135, 150, 139, 128, 151, 132, 146, 149, 143, 141.

(i) What is the height of the tallest girl? (ii) What is the height of the shortest girl?

(iii) What is the range of the data? (iv) What is the mean height of the girls?

(v) How many girls have heights more than the mean height.

3.3 Mode

As we have said Mean is not the only measure of central tendency or the only form of representative value. For different requirements from a data, other measures of central tendencies are used.

Look at the following example

To find out the weekly demand for different sizes of shirt, a shopkeeper kept records of sales of sizes 90 cm, 95 cm, 100 cm, 105 cm, 110 cm. Following is the record for a week:

Size (in inches)	90 cm	95 cm	100 cm	105 cm	110 cm	**Total**
Number of Shirts Sold	8	22	32	37	6	**105**

If he found the mean number of shirts sold, do you think that he would be able to decide which shirt sizes to keep in stock?

$$\text{Mean of total shirts sold} = \frac{\text{Total number of shirts sold}}{\text{Number of different sizes of shirts}} = \frac{105}{5} = 21$$

Should he obtain 21 shirts of each size? If he does so, will he be able to cater to the needs of the customers?

The shopkeeper, on looking at the record, decides to procure shirts of sizes 95 cm, 100 cm, 105 cm. He decided to postpone the procurement of the shirts of other sizes because of their small number of buyers.

Look at another example

The owner of a readymade dress shop says, "The most popular size of dress I sell is the size 90 cm.

Observe that here also, the owner is concerned about the number of shirts of different sizes sold. She is however looking at the shirt size that is sold the most. This is another representative value for the data. The highest occuring event is the sale of size 90 cm.This representative value is called the **mode** of the data.

The mode of a set of observations is the observation that occurs most often.

EXAMPLE 4 Find the mode of the given set of numbers: 1, 1, 2, 4, 3, 2, 1, 2, 2, 4

SOLUTION Arranging the numbers with same values together, we get

1, 1, 1, 2, 2, 2, 2, 3, 4, 4

Mode of this data is 2 because it occurs more frequently than other observations.

3.3.1 Mode of Large Data

Putting the same observations together and counting them is not easy if the number of observations is large. In such cases we tabulate the data. Tabulation can begin by putting tally marks and finding the frequency, as you did in your previous class.

Look at the following example:

EXAMPLE 5 Following are the margins of victory in the football matches of a league.

1, 3, 2, 5, 1, 4, 6, 2, 5, 2, 2, 2, 4, 1, 2, 3, 1, 1, 2, 3, 2,
6, 4, 3, 2, 1, 1, 4, 2, 1, 5, 3, 3, 2, 3, 2, 4, 2, 1, 2

Find the mode of this data.

SOLUTION Let us put the data in a tabular form:

Margins of Victory	Tally Bars	Number of Matches
1	卌 ||||	9
2	卌 卌 ||||	14
3	卌 ||	7
4	卌	5
5	|||	3
6	||	2
	Total	**40**

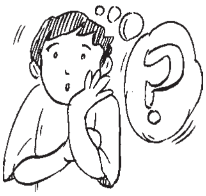
TRY THESE

Find the mode of

(i) 2, 6, 5, 3, 0, 3, 4, 3, 2, 4, 5, 2, 4

(ii) 2, 14, 16, 12, 14, 14, 16, 14, 10, 14, 18, 14

Looking at the table, we can quickly say that 2 is the 'mode' since 2 has occured the highest number of times. Thus, most of the matches have been won with a victory margin of 2 goals.



Think, Discuss and Write

Can a set of numbers have more than one mode?

Example 6 Find the mode of the numbers: 2, 2, 2, 3, 3, 4, 5, 5, 5, 6, 6, 8

Solution Here, 2 and 5 both occur three times. Therefore, they both are modes of the data.

Do This

1. Record the age in years of all your classmates. Tabulate the data and find the mode.
2. Record the heights in centimetres of your classmates and find the mode.

Try These

1. Find the mode of the following data:
12, 14, 12, 16, 15, 13, 14, 18, 19, 12, 14, 15, 16, 15, 16, 16, 15,
17, 13, 16, 16, 15, 15, 13, 15, 17, 15, 14, 15, 13, 15, 14

2. Heights (in cm) of 25 children are given below:
168, 165, 163, 160, 163, 161, 162, 164, 163, 162, 164, 163, 160, 163, 160,
165, 163, 162, 163, 164, 163, 160, 165, 163, 162
What is the mode of their heights? What do we understand by mode here?

Whereas mean gives us the average of all observations of the data, the mode gives that observation which occurs most frequently in the data.

Let us consider the following examples:

(a) You have to decide upon the number of chapattis needed for 25 people called for a feast.

(b) A shopkeeper selling shirts has decided to replenish her stock.

(c) We need to find the height of the door needed in our house.

(d) When going on a picnic, if only one fruit can be bought for everyone, which is the fruit that we would get.

In which of these situations can we use the mode as a good estimate?

Consider the first statement. Suppose the number of chapattis needed by each person is 2, 3, 2, 3, 2, 1, 2, 3, 2, 2, 4, 2, 2, 3, 2, 4, 4, 2, 3, 2, 4, 2, 4, 3, 5

The mode of the data is 2 chapattis. If we use mode as the representative value for this data, then we need 50 chapattis only, 2 for each of the 25 persons. However the total number would clearly be inadequate. Would **mean** be an appropriate representative value?

For the third statement the height of the door is related to the height of the persons using that door. Suppose there are 5 children and 4 adults using the door and the height of each of 5 children is around 135 cm. The mode for the heights is 135 cm. Should we get a door that is 144 cm high? Would all the adults be able to go through that door? It is clear that mode is not the appropriate representative value for this data. Would **mean** be an appropriate representative value here?

Why not? Which representative value of height should be used to decide the doorheight?

Similarly analyse the rest of the statements and find the representative value useful for that issue.

TRY THESE

Discuss with your friends and give

(a) Two situations where mean would be an appropriate representative value to use, and

(b) Two situations where mode would be an appropriate representative value to use.

3.4 MEDIAN

We have seen that in some situations, arithmetic mean is an appropriate measure of central tendency whereas in some other situations, mode is the appropriate measure of central tendency.

Let us now look at another example. Consider a group of 17 students with the following heights (in cm): 106, 110, 123, 125, 117, 120, 112, 115, 110, 120, 115, 102, 115, 115, 109, 115, 101.

The games teacher wants to divide the class into two groups so that each group has equal number of students, one group has students with height lesser than a particular height and the other group has students with heights greater than the particular height. How would she do that?

Let us see the various options she has:

(i) She can find the mean. The mean is

$$\frac{106+110+123+125+117+120+112+115+110+120+115+102+115+115+109+115+101}{17}$$

$$=\frac{1930}{17}=113.5$$

So, if the teacher divides the students into two groups on the basis of this mean height, such that one group has students of height less than the mean height and the other group has students with height more than the mean height, then the groups would be of unequal size. They would have 7 and 10 members respectively.

(ii) The second option for her is to find mode. The observation with highest frequency is 115 cm, which would be taken as mode.

There are 7 children below the mode and 10 children at the mode and above the mode. Therefore, we cannot divide the group into equal parts.

Let us therefore think of an alternative representative value or measure of central tendency. For doing this we again look at the given heights (in cm) of students and arrange them in ascending order. We have the following observations:

101, 102, 106, 109, 110, 110, 112, 115, 115, 115, 115, 115, 117, 120, 120, 123, 125

The middle value in this data is 115 because this value divides the students into two equal groups of 8 students each. This value is called as **Median**. Median refers to the value which lies in the middle of the data (when arranged in an increasing or decreasing order) with half of the observations above it and the other half below it. The games teacher decides to keep the middle student as a refree in the game.

Here, we consider only those cases where number of observations is odd.

TRY THESE

Your friend found the median and the mode of a given data. Describe and correct your friends error if any:

35, 32, 35, 42, 38, 32, 34

Median = 42, Mode = 32

Thus, in a given data, arranged in ascending or descending order, the **median** gives us the middle observation.

Note that in general, we may not get the same value for median and mode.

Thus we realise that mean, mode and median are the numbers that are the representative values of a group of observations or data. They lie between the minimum and maximum values of the data. They are also called the measures of the central tendency.

EXAMPLE 7 Find the median of the data: 24, 36, 46, 17, 18, 25, 35

SOLUTION We arrange the data in ascending order, we get 17, 18, 24, 25, 35, 36, 46
Median is the middle observation. Therefore 25 is the median.

EXERCISE 3.2

1. The scores in mathematics test (out of 25) of 15 students is as follows:

   19, 25, 23, 20, 9, 20, 15, 10, 5, 16, 25, 20, 24, 12, 20

   Find the mode and median of this data. Are they same?

2. The runs scored in a cricket match by 11 players is as follows:

   6, 15, 120, 50, 100, 80, 10, 15, 8, 10, 15

   Find the mean, mode and median of this data. Are the three same?

3. The weights (in kg.) of 15 students of a class are:

   38, 42, 35, 37, 45, 50, 32, 43, 43, 40, 36, 38, 43, 38, 47

   (i) Find the mode and median of this data.

   (ii) Is there more than one mode?

4. Find the mode and median of the data: 13, 16, 12, 14, 19, 12, 14, 13, 14

5. Tell whether the statement is true or false:

   (i) The mode is always one of the numbers in a data.

   (ii) The mean is one of the numbers in a data.

   (iii) The median is always one of the numbers in a data.

   (iv) The data 6, 4, 3, 8, 9, 12, 13, 9 has mean 9.

3.5 Use of Bar Graphs with a Different Purpose

We have seen last year how information collected could be first arranged in a frequency distribution table and then this information could be put as a visual representation in the form of pictographs or bar graphs. You can look at the bar graphs and make deductions about the data. You can also get information based on these bar graphs. For example, you can say that the mode is the longest bar if the bar represents the frequency.

3.5.1 Choosing a Scale

We know that a bar graph is a representation of numbers using bars of uniform width and the lengths of the bars depend upon the frequency and the scale you have chosen. For example, in a bar graph where numbers in units are to be shown, the graph represents one unit length for one observation and if it has to show numbers in tens or hundreds, one unit length can represent 10 or 100 observations. Consider the following examples:

Example 8 Two hundred students of 6th and 7th classes were asked to name their favourite colour so as to decide upon what should be the colour of their school building. The results are shown in the following table. Represent the given data on a bar graph.

Favourite Colour	Red	Green	Blue	Yellow	Orange
Number of Students	43	19	55	49	34

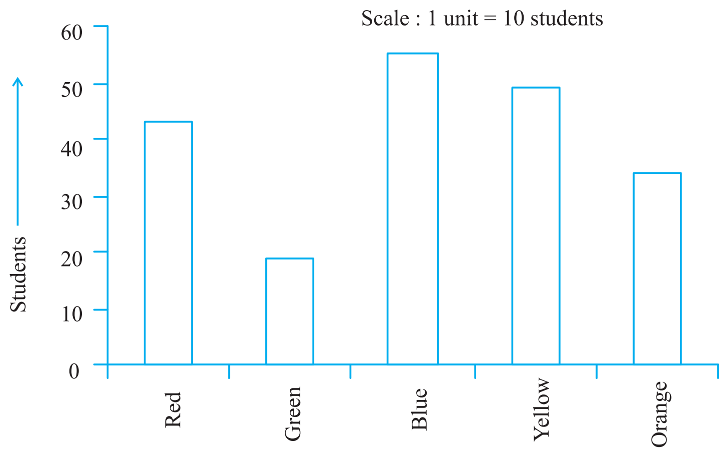
Answer the following questions with the help of the bar graph:

(i) Which is the most preferred colour and which is the least preferred?

(ii) How many colours are there in all? What are they?

Solution Choose a suitable scale as follows:

Start the scale at 0. The greatest value in the data is 55, so end the scale at a value greater than 55, such as 60. Use equal divisions along the axes, such as increments of 10. You



know that all the bars would lie between 0 and 60. We choose the scale such that the length between 0 and 60 is neither too long nor too small. Here we take 1 unit for 10 students.

We then draw and label the graph as shown.

From the bar graph we conclude that

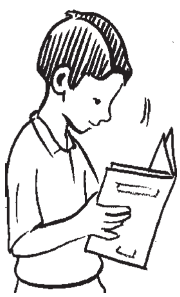
(i) Blue is the most preferred colour (Because the bar representing Blue is the tallest).

(ii) Green is the least preferred colour. (Because the bar representing Green is the shortest).

(iii) There are five colours. They are Red, Green, Blue, Yellow and Orange. (These are observed on the horizontal line)

EXAMPLE 9 Following data gives total marks (out of 600) obtained by six children of a particular class. Represent the data on a bar graph.

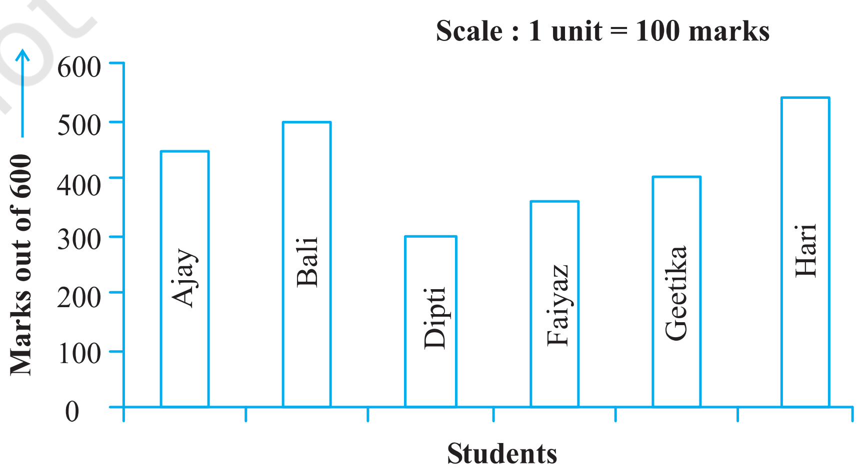
Students	Ajay	Bali	Dipti	Faiyaz	Geetika	Hari
Marks Obtained	450	500	300	360	400	540



SOLUTION

(i) To choose an appropriate scale we make equal divisions taking increments of 100. Thus 1 unit will represent 100 marks. (What would be the difficulty if we choose one unit to represent 10 marks?)

(ii) Now represent the data on the bar graph.



Drawing double bar graph

Consider the following two collections of data giving the average daily hours of sunshine in two cities Aberdeen and Margate for all the twelve months of the year. These cities are near the south pole and hence have only a few hours of sunshine each day.

In Margate												
	Jan.	**Feb.**	**Mar.**	**April**	**May**	**June**	**July**	**Aug.**	**Sept.**	**Oct.**	**Nov.**	**Dec.**
Average hours of Sunshine	2	$3\frac{1}{4}$	4	4	$7\frac{3}{4}$	8	$7\frac{1}{2}$	7	$6\frac{1}{4}$	6	4	2
In Aberdeen												
Average hours of Sunshine	$1\frac{1}{2}$	3	$3\frac{1}{2}$	6	$5\frac{1}{2}$	$6\frac{1}{2}$	$5\frac{1}{2}$	5	$4\frac{1}{2}$	4	3	$1\frac{3}{4}$

By drawing individual bar graphs you could answer questions like

(i) In which month does each city has maximum sunlight? or

(ii) In which months does each city has minimum sunlight?

However, to answer questions like "In a particular month, which city has more sunshine hours", we need to compare the average hours of sunshine of both the cities. To do this we will learn to draw what is called a double bar graph giving the information of both cities side-by-side.

This bar graph (Fig 3.1) shows the average sunshine of both the cities.

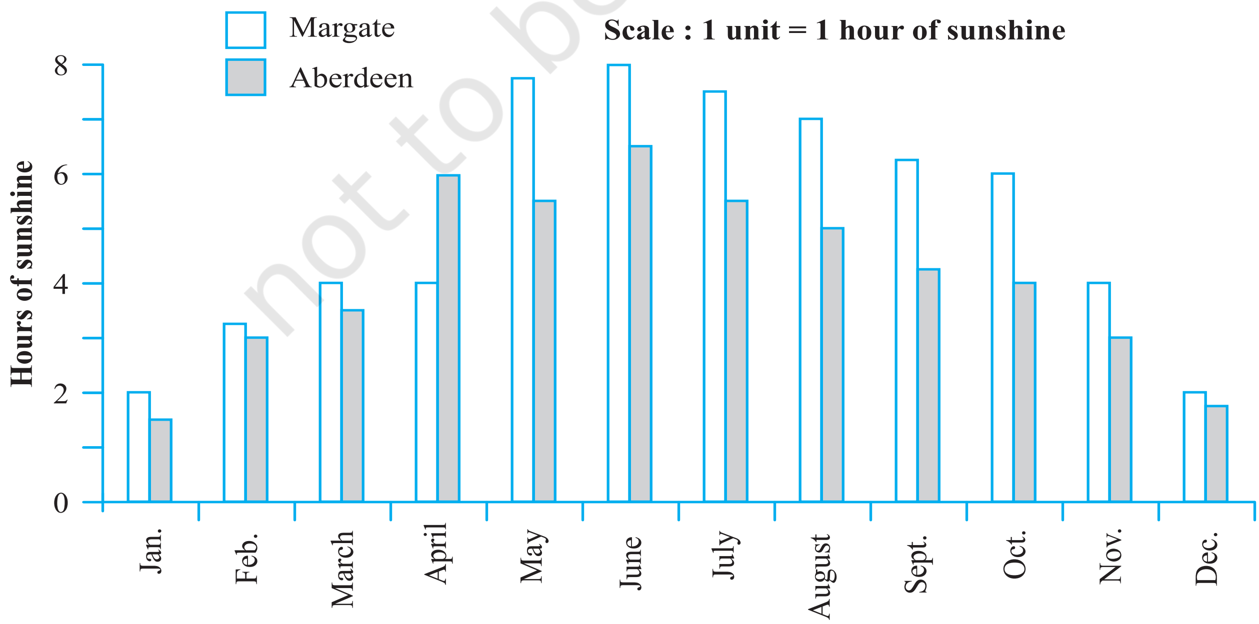


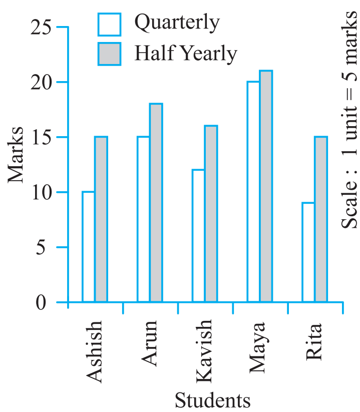
Fig 3.1

For each month we have two bars, the heights of which give the average hours of sunshine in each city. From this we can infer that except for the month of April, there is always more sunshine in Margate than in Aberdeen. You could put together a similiar bar graph for your area or for your city.

Let us look at another example more related to us.

Example 10 A mathematics teacher wants to see, whether the new technique of teaching she applied after quarterly test was effective or not. She takes the scores of the 5 weakest children in the quarterly test (out of 25) and in the half yearly test (out of 25):

Students	Ashish	Arun	Kavish	Maya	Rita
Quarterly	10	15	12	20	9
Half yearly	15	18	16	21	15



Solution She draws the adjoining double bar graph and finds a marked improvement in most of the students, the teacher decides that she should continue to use the new technique of teaching.

Can you think of a few more situations where you could use double bar graphs?

Try These

1. The bar graph (Fig 3.2) shows the result of a survey to test water resistant watches made by different companies.

   Each of these companies claimed that their watches were water resistant. After a test the above results were revealed.

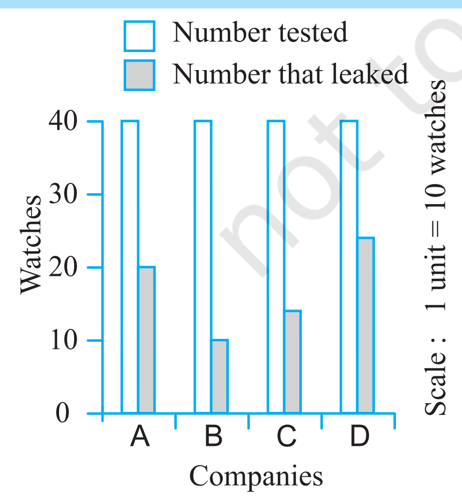


Fig 3.2

(a) Can you work out a fraction of the number of watches that leaked to the number tested for each company?

(b) Could you tell on this basis which company has better watches?

2. Sale of English and Hindi books in the years 1995, 1996, 1997 and 1998 are given below:

Years	1995	1996	1997	1998
English	350	400	450	620
Hindi	500	525	600	650

Draw a double bar graph and answer the following questions:

(a) In which year was the difference in the sale of the two language books least?.

(b) Can you say that the demand for English books rose faster? Justify.

EXERCISE 3.3

1. Use the bar graph (Fig 3.3) to answer the following questions.

(a) Which is the most popular pet? (b) How many students have dog as a pet?

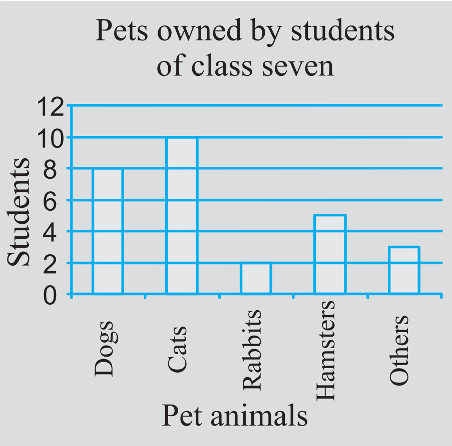


Fig 3.3

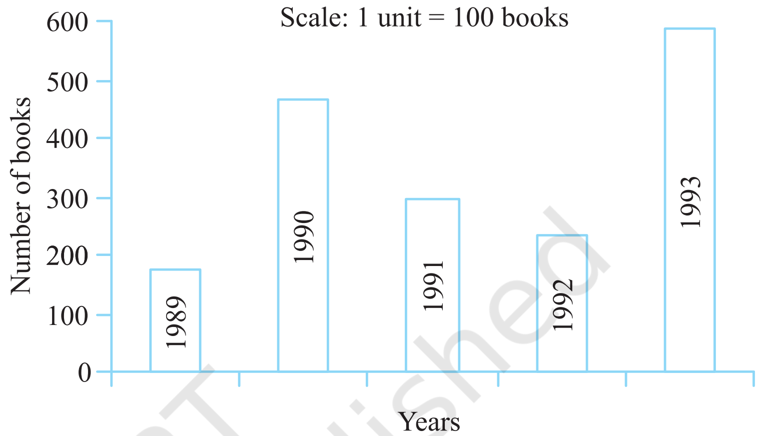


Fig 3.4

2. Read the bar graph (Fig 3.4) which shows the number of books sold by a bookstore during five consecutive years and answer the following questions:

(i) About how many books were sold in 1989? 1990? 1992?

(ii) In which year were about 475 books sold? About 225 books sold?

(iii) In which years were fewer than 250 books sold?

(iv) Can you explain how you would estimate the number of books sold in 1989?

3. Number of children in six different classes are given below. Represent the data on a bar graph.

Class	**Fifth**	**Sixth**	**Seventh**	**Eighth**	**Ninth**	**Tenth**
Number of Children	135	120	95	100	90	80

(a) How would you choose a scale?

(b) Answer the following questions:

(i) Which class has the maximum number of children? And the minimum?

(ii) Find the ratio of students of class sixth to the students of class eight.

4. The performance of a student in 1st Term and 2nd Term is given. Draw a double bar graph choosing appropriate scale and answer the following:

Subject	**English**	**Hindi**	**Maths**	**Science**	**S. Science**
1st Term (M.M. 100)	67	72	88	81	73
2nd Term (M.M. 100)	70	65	95	85	75

(i) In which subject, has the child improved his performance the most?

(ii) In which subject is the improvement the least?

(iii) Has the performance gone down in any subject?

5. Consider this data collected from a survey of a colony.

Favourite Sport	Cricket	Basket Ball	Swimming	Hockey	Athletics
Watching	1240	470	510	430	250
Participating	620	320	320	250	105

(i) Draw a double bar graph choosing an appropriate scale.
What do you infer from the bar graph?

(ii) Which sport is most popular?

(iii) Which is more preferred, watching or participating in sports?

6. Take the data giving the minimum and the maximum temperature of various cities given in the beginning of this Chapter (Table 3.1). Plot a double bar graph using the data and answer the following:

(i) Which city has the largest difference in the minimum and maximum temperature on the given date?

(ii) Which is the hottest city and which is the coldest city?

(iii) Name two cities where maximum temperature of one was less than the minimum temperature of the other.

(iv) Name the city which has the least difference between its minimum and the maximum temperature.

What have We Discussed?

1. Average is a number that represents or shows the central tendency of a group of observations or data.

2. Arithmetic mean is one of the representative values of data.

3. Mode is another form of central tendency or representative value. The mode of a set of observations is the observation that occurs most often.

4. Median is also a form of representative value. It refers to the value which lies in the middle of the data with half of the observations above it and the other half below it.

5. A bar graph is a representation of numbers using bars of uniform widths.

6. Double bar graphs help to compare two collections of data at a glance.

Chapter 4

Simple Equations

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4.1 A Mind-reading Game!

The teacher has said that she would be starting a new chapter in mathematics and it is going to be simple equations. Appu, Sarita and Ameena have revised what they learnt in algebra chapter in Class VI. Have you? Appu, Sarita and Ameena are excited because they have constructed a game which they call mind reader and they want to present it to the whole class.

The teacher appreciates their enthusiasm and invites them to present their game. Ameena begins; she asks Sara to think of a number, multiply it by 4 and add 5 to the product. Then, she asks Sara to tell the result. She says it is 65. Ameena instantly declares that the number Sara had thought of is 15. Sara nods. The whole class including Sara is surprised.

It is Appu's turn now. He asks Balu to think of a number, multiply it by 10 and subtract 20 from the product. He then asks Balu what his result is? Balu says it is 50. Appu immediately tells the number thought by Balu. It is 7, Balu confirms it.

Everybody wants to know how the 'mind reader' presented by Appu, Sarita and Ameena works. Can you see how it works? After studying this chapter and chapter 12, you will very well know how the game works.

4.2 Setting up of an Equation

Let us take Ameena's example. Ameena asks Sara to think of a number. Ameena does not know the number. For her, it could be anything 1, 2, 3, . . ., 11, . . . , 100, ... Let us denote this unknown number by a letter, say x. You may use y or t or some other letter in place of x. It does not matter which letter we use to denote the unknown number Sara has thought of. When Sara multiplies the number by 4, she gets $4x$. She then adds 5 to the product, which gives $4x + 5$. The value of $(4x + 5)$ depends on the value of x. Thus if $x = 1$, $4x + 5 = 4 \times 1 + 5 = 9$. This means that if Sara had 1 in her mind, her result would have been 9. Similarly, if she thought of 5, then for $x = 5$, $4x + 5 = 4 \times 5 + 5 = 25$; Thus if Sara had chosen 5, the result would have been 25.

To find the number thought by Sara let us work backward from her answer 65. We have to find x such that

$$4x + 5 = 65 \qquad (4.1)$$

Solution to the equation will give us the number which Sara held in her mind.

Let us similarly look at Appu's example. Let us call the number Balu chose as y. Appu asks Balu to multiply the number by 10 and subtract 20 from the product. That is, from y, Balu first gets $10y$ and from there $(10y - 20)$. The result is known to be 50.

Therefore, $$10y - 20 = 50 \qquad (4.2)$$

The solution of this equation will give us the number Balu had thought of.

4.3 Review of what We Know

Note, (4.1) and (4.2) are equations. Let us recall what we learnt about equations in Class VI. *An equation is a condition on a variable.* In equation (4.1), the variable is x; in equation (4.2), the variable is y.

The word *variable* means something that can vary, i.e. change. A **variable** *takes on different numerical values*; *its value is not fixed. Variables are denoted usually by letters of the alphabets, such as x, y, z, l, m, n, p, etc. From variables*, *we form expressions*. The expressions are formed by performing operations like addition, subtraction, multiplication and division on the variables. From x, we formed the expression $(4x + 5)$. For this, first we multiplied x by 4 and then added 5 to the product. Similarly, from y, we formed the expression $(10y - 20)$. For this, we multiplied y by 10 and then subtracted 20 from the product. All these are examples of expressions.

The value of an expression thus formed depends upon the chosen value of the variable. As we have already seen, when $x = 1$, $4x + 5 = 9$; when $x = 5$, $4x + 5 = 25$. Similarly,

when $x = 15$, $4x + 5 = 4 \times 15 + 5 = 65$;

when $x = 0$, $4x + 5 = 4 \times 0 + 5 = 5$; and so on.

Equation (4.1) is a condition on the variable x. It states that the value of the expression $(4x + 5)$ is 65. The condition is satisfied when $x = 15$. It is the solution to the equation $4x + 5 = 65$. When $x = 5$, $4x + 5 = 25$ and not 65. Thus $x = 5$ is not a solution to the equation. Similarly, $x = 0$ is not a solution to the equation. No value of x other than 15 satisfies the condition $4x + 5 = 65$.

Try These

The value of the expression $(10y - 20)$ depends on the value of y. Verify this by giving five different values to y and finding for each y the value of $(10\,y - 20)$. From the different values of $(10y - 20)$ you obtain, do you see a solution to $10y - 20 = 50$? If there is no solution, try giving more values to y and find whether the condition $10y - 20 = 50$ is met.

4.4 What Equation is?

In an equation there is always an **equality** sign. The equality sign shows that the value of the expression to the left of the sign (the left hand side or LHS) is equal to the value of the expression to the right of the sign (the right hand side or RHS). In equation (4.1), the LHS is $(4x + 5)$ and the RHS is 65. In equation (4.2), the LHS is $(10y - 20)$ and the RHS is 50.

> If there is some sign other than the equality sign between the LHS and the RHS, it is not an equation. Thus, $4x + 5 > 65$ is not an equation.
>
> It says that, the value of $(4x + 5)$ is greater than 65.
>
> Similarly, $4x + 5 < 65$ is not an equation. It says that the value of $(4x + 5)$ is smaller than 65.

In equations, we often find that the RHS is just a number. In Equation (4.1), it is 65 and in equation (4.2), it is 50. But this need not be always so. The RHS of an equation may be an expression containing the variable. For example, the equation

$$4x + 5 = 6x - 25$$

has the expression $(4x + 5)$ on the left and $(6x - 25)$ on the right of the equality sign.

In short, an equation is a condition on a variable. The condition is that two expressions should have equal value. Note that at least one of the two expressions must contain the variable.

We also note a simple and useful property of equations. The equation $4x + 5 = 65$ is the same as $65 = 4x + 5$. Similarly, the equation $6x - 25 = 4x + 5$ is the same as $4x + 5 = 6x - 25$. *An equation remains the same, when the expressions on the left and on the right are interchanged.* This property is often useful in solving equations.

Example 1 Write the following statements in the form of equations:

(i) The sum of three times x and 11 is 32.

(ii) If you subtract 5 from 6 times a number, you get 7.

(iii) One fourth of m is 3 more than 7.

(iv) One third of a number plus 5 is 8.

Solution

(i) Three times x is $3x$.

Sum of $3x$ and 11 is $3x + 11$. The sum is 32.

The equation is $3x + 11 = 32$.

(ii) Let us say the number is z; z multiplied by 6 is $6z$.

Subtracting 5 from $6z$, one gets $6z - 5$. The result is 7.

The equation is $6z - 5 = 7$

(iii) One fourth of m is $\frac{m}{4}$.

It is greater than 7 by 3. This means the difference ($\frac{m}{4} - 7$) is 3.

The equation is $\frac{m}{4} - 7 = 3$.

(iv) Take the number to be n. One third of n is $\frac{n}{3}$.

This one-third plus 5 is $\frac{n}{3} + 5$. It is 8.

The equation is $\frac{n}{3} + 5 = 8$.

EXAMPLE 2 Convert the following equations in statement form:

(i) $x - 5 = 9$ (ii) $5p = 20$ (iii) $3n + 7 = 1$ (iv) $\frac{m}{5} - 2 = 6$

SOLUTION (i) Taking away 5 from x gives 9.

(ii) Five times a number p is 20.

(iii) Add 7 to three times n to get 1.

(iv) You get 6, when you subtract 2 from one-fifth of a number m.

What is important to note is that for a given equation, **not just one, but many** statement forms can be given. For example, for Equation (i) above, you can say:

Subtract 5 from x, you get 9.

or The number x is 5 more than 9.

or The number x is greater by 5 than 9.

or The difference between x and 5 is 9, and so on.

TRY THESE

Write atleast one other form for each equation (ii), (iii) and (iv).

EXAMPLE 3 Consider the following situation:

Raju's father's age is 5 years more than three times Raju's age. Raju's father is 44 years old. Set up an equation to find Raju's age.

SOLUTION We do not know Raju's age. Let us take it to be y years. Three times Raju's age is $3y$ years. Raju's father's age is 5 years more than $3y$; that is, Raju's father is $(3y + 5)$ years old. It is also given that Raju's father is 44 years old.

Therefore, $$3y + 5 = 44 \qquad (4.3)$$

This is an equation in y. It will give Raju's age when solved.

EXAMPLE 4 A shopkeeper sells mangoes in two types of boxes, one small and one large. A large box contains as many as 8 small boxes plus 4 loose mangoes. Set up an equation which gives the number of mangoes in each small box. The number of mangoes in a large box is given to be 100.

SOLUTION Let a small box contain m mangoes. A large box contains 4 more than 8 times m, that is, 8m + 4 mangoes. But this is given to be 100. Thus

$$8m + 4 = 100 \qquad (4.4)$$

You can get the number of mangoes in a small box by solving this equation.

EXERCISE 4.1

1. Complete the last column of the table.

S. No.	Equation	Value	Say, whether the Equation is Satisfied. (Yes/ No)
(i)	$x + 3 = 0$	$x = 3$	
(ii)	$x + 3 = 0$	$x = 0$	
(iii)	$x + 3 = 0$	$x = -3$	
(iv)	$x - 7 = 1$	$x = 7$	
(v)	$x - 7 = 1$	$x = 8$	
(vi)	$5x = 25$	$x = 0$	
(vii)	$5x = 25$	$x = 5$	
(viii)	$5x = 25$	$x = -5$	
(ix)	$\frac{m}{3} = 2$	$m = -6$	
(x)	$\frac{m}{3} = 2$	$m = 0$	
(xi)	$\frac{m}{3} = 2$	$m = 6$	

2. Check whether the value given in the brackets is a solution to the given equation or not:

(a) $n + 5 = 19$ $(n = 1)$ (b) $7n + 5 = 19$ $(n = -2)$ (c) $7n + 5 = 19$ $(n = 2)$

(d) $4p - 3 = 13$ $(p = 1)$ (e) $4p - 3 = 13$ $(p = -4)$ (f) $4p - 3 = 13$ $(p = 0)$

3. Solve the following equations by trial and error method:

(i) $5p + 2 = 17$ (ii) $3m - 14 = 4$

4. Write equations for the following statements:

(i) The sum of numbers x and 4 is 9. (ii) 2 subtracted from y is 8.

(iii) Ten times a is 70. (iv) The number b divided by 5 gives 6.

(v) Three-fourth of t is 15. (vi) Seven times m plus 7 gets you 77.

(vii) One-fourth of a number x minus 4 gives 4.

(viii) If you take away 6 from 6 times y, you get 60.

(ix) If you add 3 to one-third of z, you get 30.

5. Write the following equations in statement forms:

(i) $p + 4 = 15$ (ii) $m - 7 = 3$ (iii) $2m = 7$ (iv) $\frac{m}{5} = 3$

(v) $\frac{3m}{5} = 6$ (vi) $3p + 4 = 25$ (vii) $4p - 2 = 18$ (viii) $\frac{p}{2} + 2 = 8$

6. Set up an equation in the following cases:

(i) Irfan says that he has 7 marbles more than five times the marbles Parmit has. Irfan has 37 marbles. (Take m to be the number of Parmit's marbles.)

(ii) Laxmi's father is 49 years old. He is 4 years older than three times Laxmi's age. (Take Laxmi's age to be y years.)

(iii) The teacher tells the class that the highest marks obtained by a student in her class is twice the lowest marks plus 7. The highest score is 87. (Take the lowest score to be l.)

(iv) In an isosceles triangle, the vertex angle is twice either base angle. (Let the base angle be b in degrees. Remember that the sum of angles of a triangle is 180 degrees).

4.4.1 Solving an Equation

Consider an equality $\qquad 8 - 3 = 4 + 1 \qquad$ (4.5)

The equality (4.5) holds, since both its sides are equal (each is equal to 5).

- Let us now add 2 to both sides; as a result

  LHS $= 8 - 3 + 2 = 5 + 2 = 7$ $\qquad$ RHS $= 4 + 1 + 2 = 5 + 2 = 7$.

  Again the equality holds (i.e., its LHS and RHS are equal).

  Thus *if we add the same number to both sides of an equality, it still holds.*

- Let us now subtract 2 from both the sides; as a result,

  LHS $= 8 - 3 - 2 = 5 - 2 = 3$ $\qquad$ RHS $= 4 + 1 - 2 = 5 - 2 = 3$.

  Again, the equality holds.

  Thus *if we subtract the same number from both sides of an equality, it still holds.*

- Similarly, *if we multiply or divide both sides of the equality by the same non-zero number, it still holds.*

  For example, let us multiply both the sides of the equality by 3, we get

  LHS $= 3 \times (8 - 3) = 3 \times 5 = 15$, $\quad$ RHS $= 3 \times (4 + 1) = 3 \times 5 = 15$.

  The equality holds.

  Let us now divide both sides of the equality by 2.

  LHS $= (8 - 3) \div 2 = 5 \div 2 = \frac{5}{2}$

  RHS $= (4+1) \div 2 = 5 \div 2 = \frac{5}{2}$ = LHS

  Again, the equality holds.

If we take any other equality, we shall find the same conclusions.

Suppose, we do not observe these rules. Specificially, suppose we add different numbers, to the two sides of an equality. We shall find in this case that the equality does not hold (i.e., its both sides are not equal). For example, let us take again equality (4.5),

$$8 - 3 = 4 + 1$$

add 2 to the LHS and 3 to the RHS. The new LHS is $8 - 3 + 2 = 5 + 2 = 7$ and the new RHS is $4 + 1 + 3 = 5 + 3 = 8$. The equality does not hold, because the new LHS and RHS are not equal.

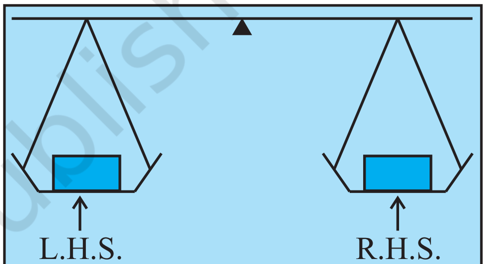
Thus if we fail to do the same mathematical operation with same number on both sides of an equality, the equality may not hold.

The equality that involves variables is an equation.

These conclusions are also valid for equations, as in each equation variable represents a number only.

Often an equation is said to be like a weighing balance. Doing a mathematical operation on an equation is like adding weights to or removing weights from the pans of a weighing balance.

An equation is like a weighing balance with equal weights on both its pans, in which case the arm of the balance is exactly horizontal. If we add the same weights to both the pans, the arm remains horizontal. Similarly, if we remove the same weights from both the pans, the arm remains horizontal. On the other hand if we add different weights to the pans or remove different weights from them, the balance is tilted; that is, the arm of the balance does not remain horizontal.



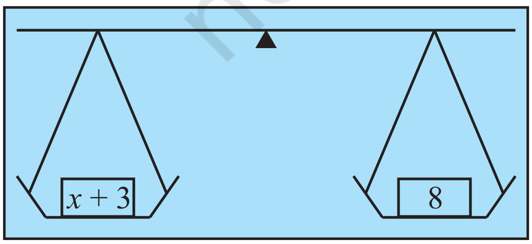
A balanced equation is like a weighing balance with equal weights in the two pans.

We use this principle for solving an equation. Here, ofcourse, the balance is imaginary and numbers can be used as weights that can be physically balanced against each other. This is the real purpose in presenting the principle. Let us take some examples.

- Consider the equation: $x + 3 = 8$ (4.6)

  We shall subtract 3 from both sides of this equation.

  The new LHS is $x + 3 - 3 = x$ and the new RHS is $8 - 3 = 5$



Why should we subtract 3, and not some other number? Try adding 3. Will it help? Why not?

It is because subtracting 3 reduces the LHS to x.

Since this does not disturb the balance, we have

$$\text{New LHS} = \text{New RHS} \quad \text{or} \quad x = 5$$

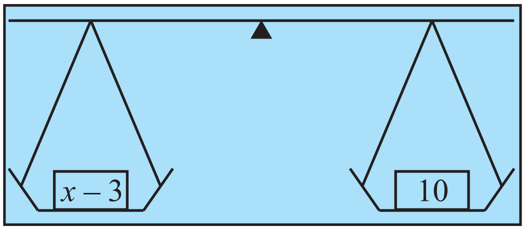
which is exactly what we want, the solution of the equation (4.6).

To confirm whether we are right, we shall put $x = 5$ in the original equation. We get LHS $= x + 3 = 5 + 3 = 8$, which is equal to the RHS as required.

By doing the right mathematical operation (i.e., subtracting 3) on both the sides of the equation, we arrived at the solution of the equation.

- Let us look at another equation $\quad x - 3 = 10 \quad$ (4.7)

What should we do here? We should add 3 to both the sides, By doing so, we shall retain the balance and also the LHS will reduce to just x.



New LHS $= x - 3 + 3 = x$, New RHS $= 10 + 3 = 13$

Therefore, $x = 13$, which is the required solution.

By putting $x = 13$ in the original equation (4.7) we confirm that the solution is correct:

LHS of original equation $= x - 3 = 13 - 3 = 10$

This is equal to the RHS as required.

- Similarly, let us look at the equations

$$5y = 35 \quad (4.8)$$

$$\frac{m}{2} = 5 \quad (4.9)$$

In the first case, we shall divide both the sides by 5. This will give us just y on LHS

$$\text{New LHS} = \frac{5y}{5} = \frac{5 \times y}{5} = y, \qquad \text{New RHS} = \frac{35}{5} = \frac{5 \times 7}{5} = 7$$

Therefore, $\quad y = 7$

This is the required solution. We can substitute $y = 7$ in Eq. (4.8) and check that it is satisfied.

In the second case, we shall multiply both sides by 2. This will give us just m on the LHS

The new LHS $= \frac{m}{2} \times 2 = m$. The new RHS $= 5 \times 2 = 10$.

Hence, $m = 10$ (It is the required solution. You can check whether the solution is correct).

One can see that in the above examples, the operation we need to perform depends on the equation. Our attempt should be to get the variable in the equation separated. Sometimes, for doing so we may have to carry out more than one mathematical operation. Let us solve some more equations with this in mind.

EXAMPLE 5 Solve: (a) $3n + 7 = 25$ (4.10)

(b) $2p - 1 = 23$ (4.11)

SOLUTION

(a) We go stepwise to separate the variable n on the LHS of the equation. The LHS is $3n + 7$. We shall first subtract 7 from it so that we get $3n$. From this, in the next step we shall divide by 3 to get n. Remember we must do the same operation on both sides of the equation. Therefore, subtracting 7 from both sides,

$$3n + 7 - 7 = 25 - 7 \quad \text{(Step 1)}$$

or $\quad 3n = 18$

Now divide both sides by 3,

$$\frac{3n}{3} = \frac{18}{3} \qquad \text{(Step 2)}$$

or $\quad n = 6$, which is the solution.

(b) What should we do here? First we shall add 1 to both the sides:

$$2p - 1 + 1 = 23 + 1 \qquad \text{(Step 1)}$$

or $\quad 2p = 24$

Now divide both sides by 2, we get $\frac{2p}{2} = \frac{24}{2}$ (Step 2)

or $\quad p = 12$, which is the solution.

One good practice you should develop is to check the solution you have obtained. Although we have not done this for (a) above, let us do it for this example.

Let us put the solution $p = 12$ back into the equation.

$$\text{LHS} = 2p - 1 = 2 \times 12 - 1 = 24 - 1$$
$$= 23 = \text{RHS}$$

The solution is thus checked for its correctness.

Why do you not check the solution of (a) also?

We are now in a position to go back to the mind-reading game presented by Appu, Sarita, and Ameena and understand how they got their answers. For this purpose, let us look at the equations (4.1) and (4.2) which correspond respectively to Ameena's and Appu's examples.

- First consider the equation $4x + 5 = 65$. (4.1)

  Subtracting 5 from both sides, $4x + 5 - 5 = 65 - 5$.

  i.e. $\quad 4x = 60$

  Divide both sides by 4; this will separate x. We get $\frac{4x}{4} = \frac{60}{4}$

  or $\quad x = 15$, which is the solution. (Check, if it is correct.)

- Now consider, $10y - 20 = 50$ (4.2)

  Adding 20 to both sides, we get $10y - 20 + 20 = 50 + 20$ or $10y = 70$

  Dividing both sides by 10, we get $\frac{10y}{10} = \frac{70}{10}$

  or $\quad y = 7$, which is the solution. (Check if it is correct.)

You will realise that exactly these were the answers given by Appu, Sarita and Ameena. They had learnt to set up equations and solve them. That is why they could construct their mind reader game and impress the whole class. We shall come back to this in Section 4.7.

Exercise 4.2

1. Give first the step you will use to separate the variable and then solve the equation:

(a) $x - 1 = 0$ (b) $x + 1 = 0$ (c) $x - 1 = 5$ (d) $x + 6 = 2$

(e) $y - 4 = -7$ (f) $y - 4 = 4$ (g) $y + 4 = 4$ (h) $y + 4 = -4$

2. Give first the step you will use to separate the variable and then solve the equation:

(a) $3l = 42$ (b) $\frac{b}{2} = 6$ (c) $\frac{p}{7} = 4$ (d) $4x = 25$

(e) $8y = 36$ (f) $\frac{z}{3} = \frac{5}{4}$ (g) $\frac{a}{5} = \frac{7}{15}$ (h) $20t = -10$

3. Give the steps you will use to separate the variable and then solve the equation:

(a) $3n - 2 = 46$ (b) $5m + 7 = 17$ (c) $\frac{20p}{3} = 40$ (d) $\frac{3p}{10} = 6$

4. Solve the following equations:

(a) $10p = 100$ (b) $10p + 10 = 100$ (c) $\frac{p}{4} = 5$ (d) $\frac{-p}{3} = 5$

(e) $\frac{3p}{4} = 6$ (f) $3s = -9$ (g) $3s + 12 = 0$ (h) $3s = 0$

(i) $2q = 6$ (j) $2q - 6 = 0$ (k) $2q + 6 = 0$ (l) $2q + 6 = 12$

4.5 More Equations

Let us practise solving some more equations. While solving these equations, we shall learn about transposing a number, i.e., moving it from one side to the other. We can transpose a number instead of adding or subtracting it from both sides of the equation.

Example 6 Solve: $12p - 5 = 25$ (4.12)

Solution

- Adding 5 on both sides of the equation,

  $12p - 5 + 5 = 25 + 5$ or $12p = 30$

- Dividing both sides by 12,

  $\frac{12p}{12} = \frac{30}{12}$ or $p = \frac{5}{2}$

> *Note, adding 5 to both sides is the same as changing side of (– 5).*
>
> $12p - 5 = 25$
>
> $12p = 25 + 5$
>
> *Changing side is called* ***transposing****. While transposing a number, we change its sign.*

Check Putting $p = \frac{5}{2}$ in the LHS of equation 4.12,

$$\text{LHS} = 12 \times \frac{5}{2} - 5 = 6 \times 5 - 5$$
$$= 30 - 5 = 25 = \text{RHS}$$

As we have seen, while solving equations one commonly used operation is adding or subtracting the same number on both sides of the equation. *Transposing a number (i.e., changing the side of the number) is the same as adding or subtracting the number from both sides.* In doing so, the sign of the number has to be changed. What applies to numbers also applies to expressions. Let us take two more examples of transposing.

Adding or Subtracting on both sides	**Transposing**
(i) $3p - 10 = 5$	(i) $3p - 10 = 5$
Add 10 to both sides	Transpose (–10) from LHS to RHS
$3p - 10 + 10 = 5 + 10$	(On transposing – 10 becomes + 10).
or $3p = 15$	$3p = 5 + 10$ or $3p = 15$
(ii) $5x + 12 = 27$	(ii) $5x + 12 = 27$
Subtract 12 from both sides	Transposing + 12
	(On transposing + 12 becomes – 12)
$5x + 12 - 12 = 27 - 12$	$5x = 27 - 12$
or $5x = 15$	or $5x = 15$

We shall now solve two more equations. As you can see they involve brackets, which have to be solved before proceeding.

EXAMPLE 7 Solve

(a) $4(m + 3) = 18$ (b) $-2(x + 3) = 8$

SOLUTION

(a) $4(m + 3) = 18$

Let us divide both the sides by 4. This will remove the brackets in the LHS We get,

$$m+3=\frac{18}{4} \quad \text{or} \quad m+3=\frac{9}{2}$$

or $m=\frac{9}{2}-3$ (transposing 3 to RHS)

or $m=\frac{3}{2}$ (required solution) $\left(\text{as } \frac{9}{2}-3=\frac{9}{2}-\frac{6}{2}=\frac{3}{2}\right)$

Check LHS $= 4\left[\frac{3}{2}+3\right]=4\times\frac{3}{2}+4\times3=2\times3+4\times3$ [put $m=\frac{3}{2}$]

$= 6 + 12 = 18 = $ RHS

(b) $-2(x + 3) = 8$

We divide both sides by (– 2), so as to remove the brackets in the LHS, we get,

$$x + 3 = -\frac{8}{2} \quad \text{or} \quad x + 3 = -4$$

i.e., $x = -4 - 3$ (transposing 3 to RHS) or $x = -7$ (required solution)

Check LHS = $-2(-7+3)=-2(-4)$

$= 8 = \text{RHS}$ as required.

4.6 Applications of Simple Equations to Practical Situations

We have already seen examples in which we have taken statements in everyday language and converted them into simple equations. We also have learnt how to solve simple equations. Thus we are ready to solve puzzles/problems from practical situations. The method is first to form equations corresponding to such situations and then to solve those equations to give the solution to the puzzles/problems. We begin with what we have already seen [Example 1 (i) and (iii), Section 4.2].

Example 8 The sum of three times a number and 11 is 32. Find the number.

Solution

- If the unknown number is taken to be x, then three times the number is $3x$ and the sum of $3x$ and 11 is 32. That is, $3x + 11 = 32$
- To solve this equation, we transpose 11 to RHS, so that

  $3x = 32 - 11$ or $3x = 21$

  Now, divide both sides by 3

  So $x = \frac{21}{3} = 7$

  The required number is 7. (We may check it by taking 3 times 7 and adding 11 to it. It gives 32 as required.)

> This equation was obtained earlier in Section 4.2, Example 1.

Example 9 Find a number, such that one-fourth of the number is 3 more than 7.

> **Try These**
>
> (i) When you multiply a number by 6 and subtract 5 from the product, you get 7. Can you tell what the number is?
>
> (ii) What is that number one third of which added to 5 gives 8?

Solution

- Let us take the unknown number to be y; one-fourth of y is $\frac{y}{4}$.

  This number $\left(\frac{y}{4}\right)$ is more than 7 by 3.

  Hence we get the equation for y as $\frac{y}{4} - 7 = 3$

- To solve this equation, first transpose 7 to RHS We get, $\frac{y}{4} = 3 + 7 = 10.$

  We then multiply both sides of the equation by 4, to get

  $\frac{y}{4} \times 4 = 10 \times 4$ or $y = 40$ (the required number)

Let us check the equation formed. Putting the value of y in the equation,

$$\text{LHS} = \frac{40}{4} - 7 = 10 - 7 = 3 = \text{RHS}, \quad \text{as required.}$$

Example 10 Raju's father's age is 5 years more than three times Raju's age. Find Raju's age, if his father is 44 years old.

Solution

- As given in Example 3 earlier, the equation that gives Raju's age is

$$3y + 5 = 44$$

- To solve it, we first transpose 5, to get $\quad 3y = 44 - 5 = 39$

Dividing both sides by 3, we get $\quad y = 13$

That is, Raju's age is 13 years. (You may check the answer.)

Try These

There are two types of boxes containing mangoes. Each box of the larger type contains 4 more mangoes than the number of mangoes contained in 8 boxes of the smaller type. Each larger box contains 100 mangoes. Find the number of mangoes contained in the smaller box?

Exercise 4.3

1. Set up equations and solve them to find the unknown numbers in the following cases:
   (a) Add 4 to eight times a number; you get 60.
   (b) One-fifth of a number minus 4 gives 3.
   (c) If I take three-fourths of a number and add 3 to it, I get 21.
   (d) When I subtracted 11 from twice a number, the result was 15.
   (e) Munna subtracts thrice the number of notebooks he has from 50, he finds the result to be 8.
   (f) Ibenhal thinks of a number. If she adds 19 to it and divides the sum by 5, she will get 8.
   (g) Anwar thinks of a number. If he takes away 7 from $\frac{5}{2}$ of the number, the result is 23.
2. Solve the following:
   (a) The teacher tells the class that the highest marks obtained by a student in her class is twice the lowest marks plus 7. The highest score is 87. What is the lowest score?

(b) In an isosceles triangle, the base angles are equal. The vertex angle is 40°. What are the base angles of the triangle? (Remember, the sum of three angles of a triangle is 180°).

(c) Sachin scored twice as many runs as Rahul. Together, their runs fell two short of a double century. How many runs did each one score?

3. Solve the following:

(i) Irfan says that he has 7 marbles more than five times the marbles Parmit has. Irfan has 37 marbles. How many marbles does Parmit have?

(ii) Laxmi's father is 49 years old. He is 4 years older than three times Laxmi's age. What is Laxmi's age?

(iii) People of Sundargram planted trees in the village garden. Some of the trees were fruit trees. The number of non-fruit trees were two more than three times the number of fruit trees. What was the number of fruit trees planted if the number of non-fruit trees planted was 77?

4. Solve the following riddle:

I am a number,
Tell my identity!
Take me seven times over
And add a fifty!
To reach a triple century
You still need forty!

What have We Discussed?

1. An equation is a condition on a variable such that two expressions in the variable should have equal value.

2. The value of the variable for which the equation is satisfied is called the solution of the equation.

3. An equation remains the same if the LHS and the RHS are interchanged.

4. In case of the balanced equation, if we

(i) add the same number to both the sides, or (ii) subtract the same number from both the sides, or (iii) multiply both sides by the same number, or (iv) divide both sides by the same number, the balance remains undisturbed, i.e., the value of the LHS remains equal to the value of the RHS

5. The above property gives a systematic method of solving an equation. We carry out a series of identical mathematical operations on the two sides of the equation in such a way that on one of the sides we get just the variable. The last step is the solution of the equation.

6. Transposing means moving to the other side. Transposition of a number has the same effect as adding same number to (or subtracting the same number from) both sides of the equation. When you transpose a number from one side of the equation to the other side, you change its sign. For example, transposing +3 from the LHS to the RHS in equation $x + 3 = 8$ gives $x = 8 - 3$ (= 5). We can carry out the transposition of an expression in the same way as the transposition of a number.

7. We also learnt how, using the technique of doing the same mathematical operation (for example adding the same number) on both sides, we could build an equation starting from its solution. Further, we also learnt that we could relate a given equation to some appropriate practical situation and build a practical word problem/puzzle from the equation.

Chapter 5

Lines and Angles

5.1 Introduction

You already know how to identify different lines, line segments and angles in a given shape. Can you identify the different line segments and angles formed in the following figures? (Fig 5.1)

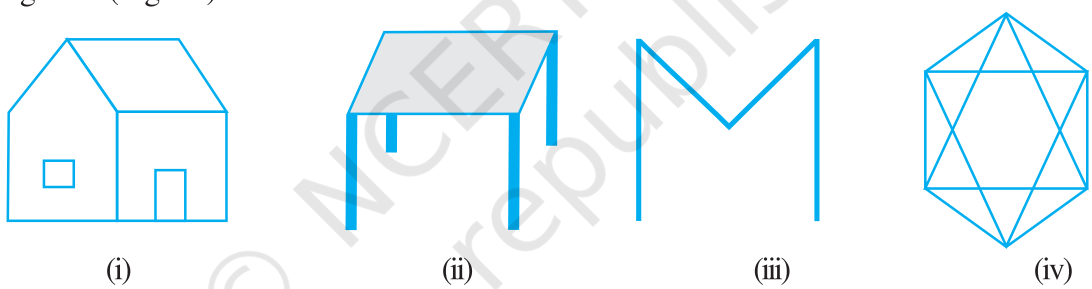


Fig 5.1

Can you also identify whether the angles made are acute or obtuse or right?

Recall that a **line segment** has two end **points**. If we extend the two end points in either direction endlessly, we get a **line**. Thus, we can say that a line has no end points. On the other hand, recall that a ray has one end point (namely its starting point). For example, look at the figures given below:

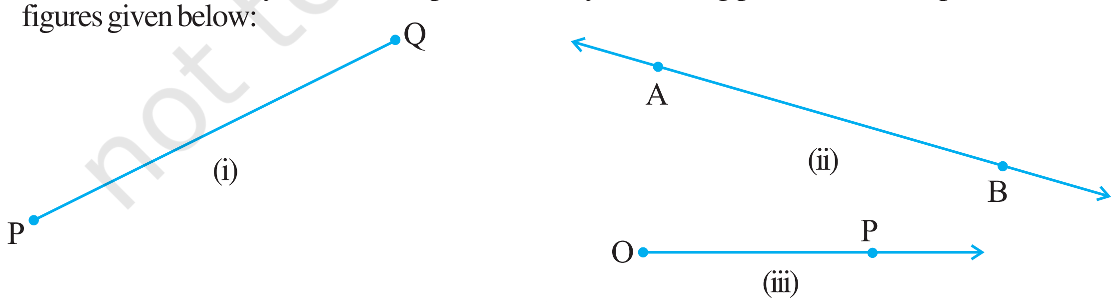


Fig 5.2

Here, Fig 5.2 (i) shows a **line segment,** Fig 5.2 (ii) shows a **line** and Fig 5.2 (iii) is that of a **ray**. A line segment PQ is generally denoted by the symbol $\overline{PQ}$, a line AB is denoted by the symbol $\overleftrightarrow{AB}$ and the ray OP is denoted by $\overrightarrow{OP}$. Give some examples of line segments and rays from your daily life and discuss them with your friends.

Again recall that an **angle** is formed when lines or line segments meet. In Fig 5.1, observe the corners. These corners are formed when two lines or line segments intersect at a point. For example, look at the figures given below:

(i) (ii)

Fig 5.3

In Fig 5.3 (i) line segments AB and BC intersect at B to form angle ABC, and again line segments BC and AC intersect at C to form angle ACB and so on. Whereas, in Fig 5.3 (ii) lines PQ and RS intersect at O to form four angles POS, SOQ, QOR and ROP. An angle ABC is represented by the symbol ∠ABC. Thus, in Fig 5.3 (i), the three angles formed are ∠ABC, ∠BCA and ∠BAC, and in Fig 5.3 (ii), the four angles formed are ∠POS, ∠SOQ, ∠QOR and ∠POR. You have already studied how to classify the angles as acute, obtuse or right angle.

Try These

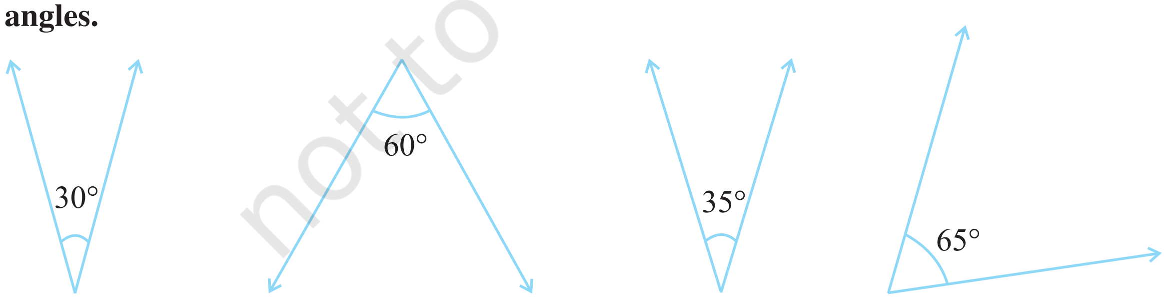
List ten figures around you and identify the acute, obtuse and right angles found in them.

Note: While referring to the measure of an angle ABC, we shall write m∠ABC as simply ∠ABC. The context will make it clear, whether we are referring to the angle or its measure.

5.2 Related Angles

5.2.1 Complementary Angles

When the sum of the measures of two angles is 90°, the angles are called **complementary angles.**



(i) (ii) (iii) (iv)

Are these two angles complementary? Yes

Are these two angles complementary? No

Fig 5.4

Whenever two angles are complementary, each angle is said to be the **complement** of the other angle. In the above diagram (Fig 5.4), the '30° angle' is the complement of the '60° angle' and vice versa.

Think, Discuss and Write

1. Can two acute angles be complement to each other?
2. Can two obtuse angles be complement to each other?
3. Can two right angles be complement to each other?

Try These

1. Which pairs of following angles are complementary? (Fig 5.5)

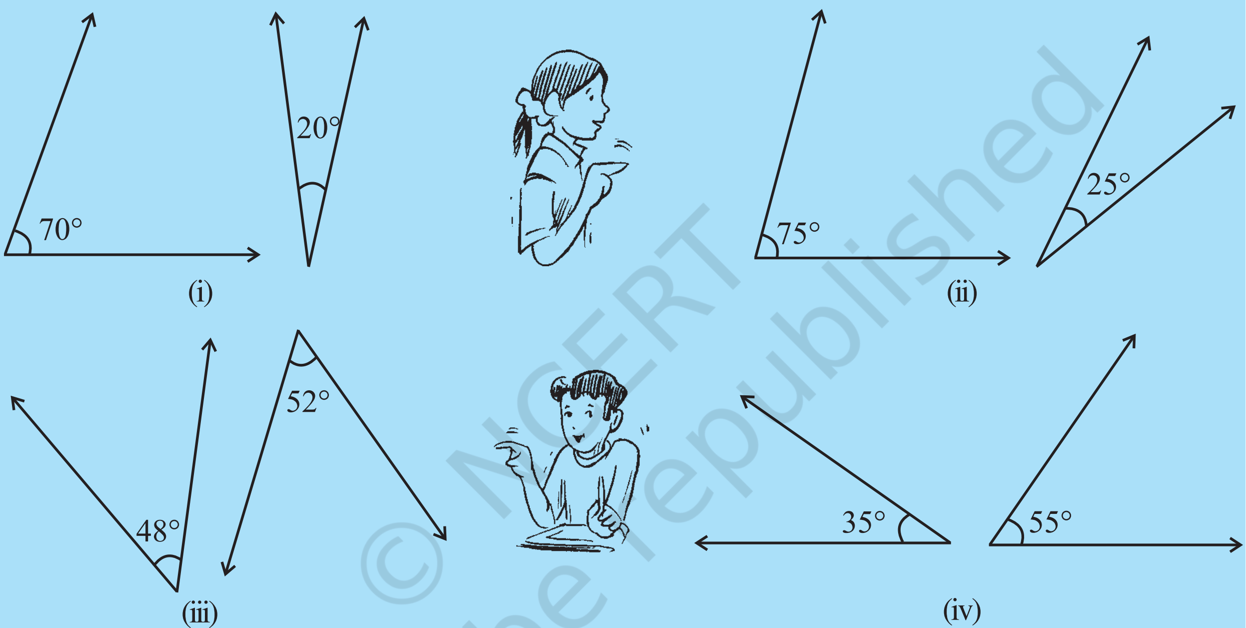
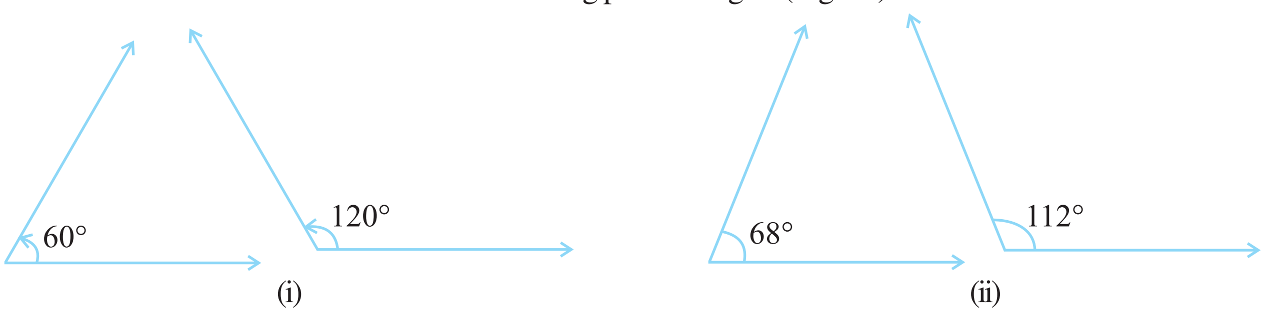


Fig 5.5

2. What is the measure of the complement of each of the following angles?
   (i) 45° (ii) 65° (iii) 41° (iv) 54°
3. The difference in the measures of two complementary angles is 12°. Find the measures of the angles.

5.2.2 Supplementary Angles

Let us now look at the following pairs of angles (Fig 5.6):



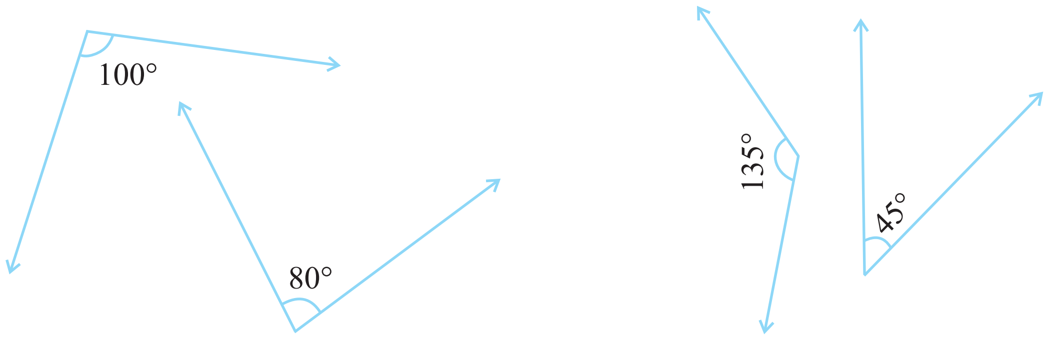


Fig 5.6

Do you notice that the sum of the measures of the angles in each of the above pairs (Fig 5.6) comes out to be 180º? Such pairs of angles are called **supplementary angles**. When two angles are supplementary, each angle is said to be the **supplement** of the other.

THINK, DISCUSS AND WRITE

1. Can two obtuse angles be supplementary?
2. Can two acute angles be supplementary?
3. Can two right angles be supplementary?

TRY THESE

1. Find the pairs of supplementary angles in Fig 5.7:

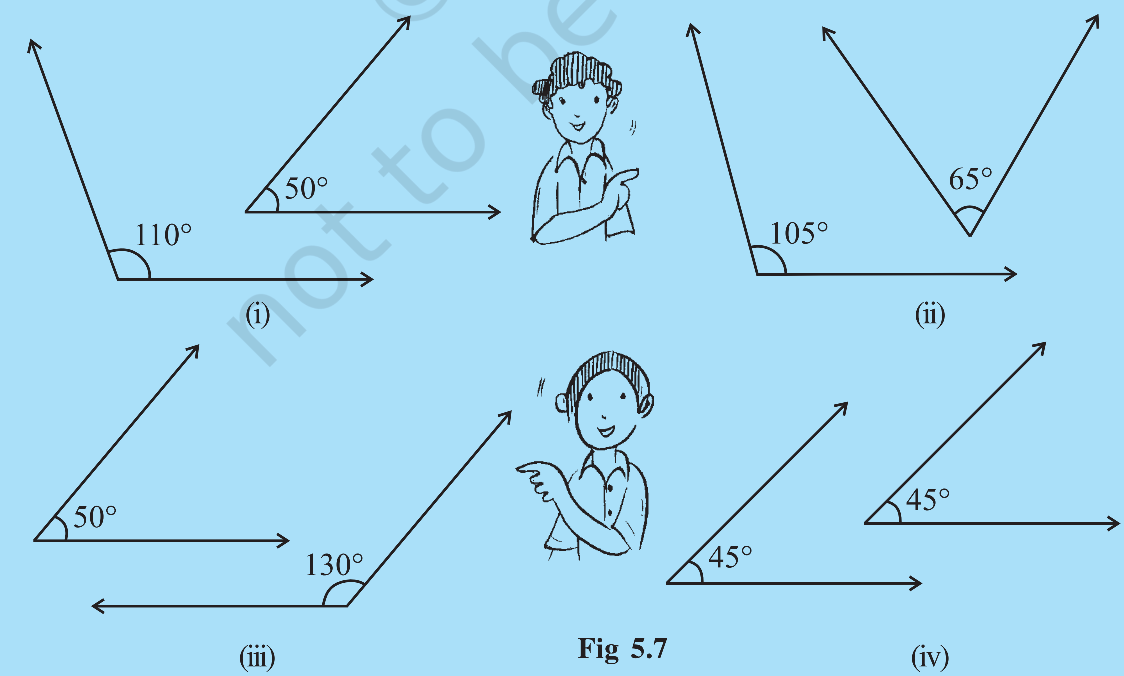
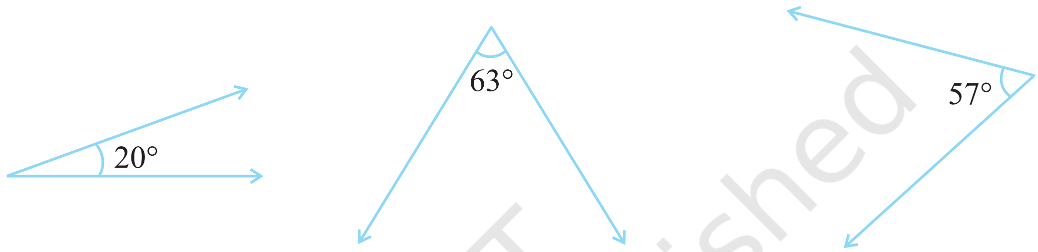


Fig 5.7

2. What will be the measure of the supplement of each one of the following angles?
   (i) 100° (ii) 90° (iii) 55° (iv) 125°
3. Among two supplementary angles the measure of the larger angle is 44° more than the measure of the smaller. Find their measures.

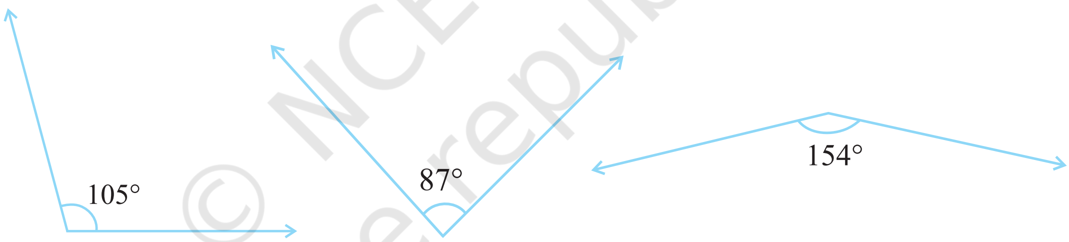
EXERCISE 5.1

1. Find the complement of each of the following angles:



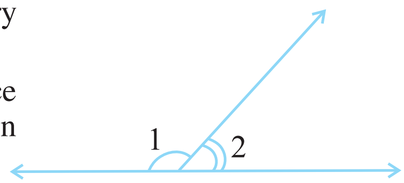
(i) (ii) (iii)

2. Find the supplement of each of the following angles:



(i) (ii) (iii)

3. Identify which of the following pairs of angles are complementary and which are supplementary.
   (i) 65°, 115° (ii) 63°, 27° (iii) 112°, 68°
   (iv) 130°, 50° (v) 45°, 45° (vi) 80°, 10°
4. Find the angle which is equal to its complement.
5. Find the angle which is equal to its supplement.
6. In the given figure, ∠1 and ∠2 are supplementary angles.
   If ∠1 is decreased, what changes should take place in ∠2 so that both the angles still remain supplementary.



7. Can two angles be supplementary if both of them are:
   (i) acute? (ii) obtuse? (iii) right?

8. An angle is greater than 45°. Is its complementary angle greater than 45° or equal to 45° or less than 45°?

9. Fill in the blanks:

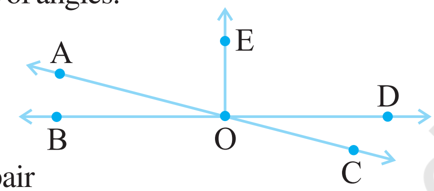
(i) If two angles are complementary, then the sum of their measures is _______.

(ii) If two angles are supplementary, then the sum of their measures is ______.

(iii) If two adjacent angles are supplementary, they form a ___________.

10. In the adjoining figure, name the following pairs of angles.

(i) Obtuse vertically opposite angles
(ii) Adjacent complementary angles
(iii) Equal supplementary angles
(iv) Unequal supplementary angles
(v) Adjacent angles that do not form a linear pair



5.3 Pairs of Lines

5.3.1 Intersecting Lines

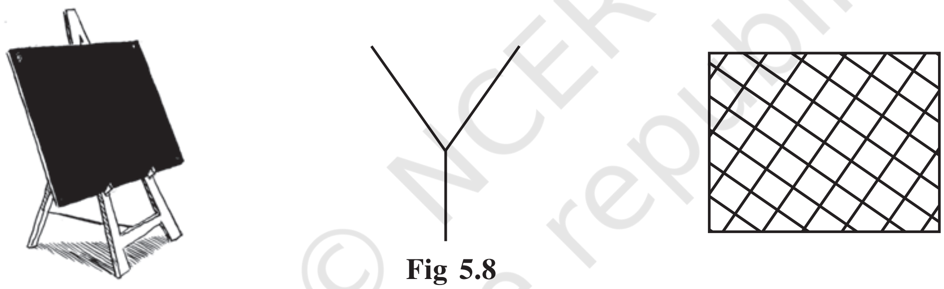
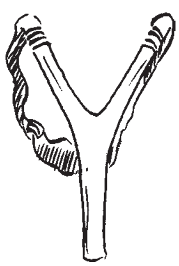


Fig 5.8

The blackboard on its stand, the letter Y made up of line segments and the grill-door of a window (Fig 5.8), what do all these have in common? They are examples of **intersecting lines**.

Two lines *l* and m intersect if they have a point in common. This common point O is their **point of intersection**.

Think, discuss and write



In Fig 5.9, AC and BE intersect at P.

AC and BC intersect at C, AC and EC intersect at C.

Try to find another ten pairs of intersecting line segments.

Should any two lines or line segments necessarily intersect? Can you find two pairs of non-intersecting line segments in the figure?

Can two lines intersect in more than one point? Think about it.

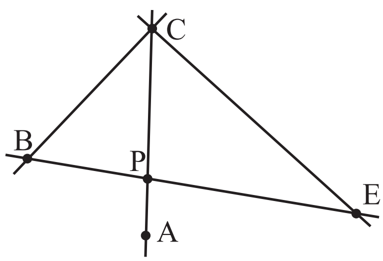


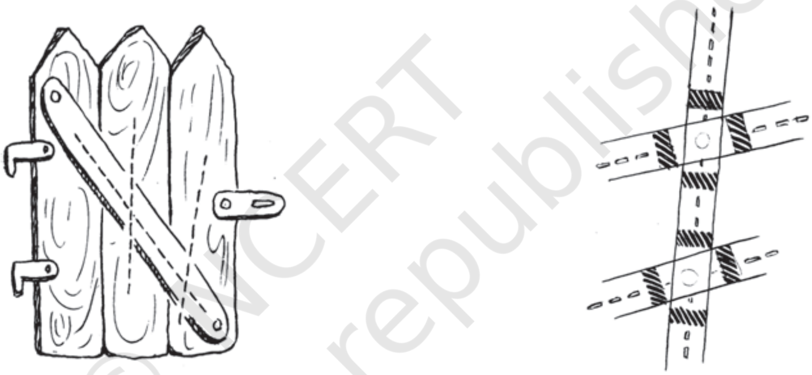
Fig 5.9

TRY THESE

1. Find examples from your surroundings where lines intersect at right angles.
2. Find the measures of the angles made by the intersecting lines at the vertices of an equilateral triangle.
3. Draw any rectangle and find the measures of angles at the four vertices made by the intersecting lines.
4. If two lines intersect, do they always intersect at right angles?

5.3.2 Transversal

You might have seen a road crossing two or more roads or a railway line crossing several other lines (Fig 5.10). These give an idea of a transversal.



(i) **Fig 5.10** (ii)

A line that intersects two or more lines at **distinct** points is called a **transversal**.

In the Fig 5.11, p is a transversal to the lines l and m.

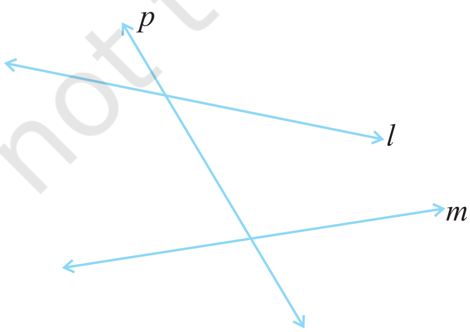


Fig 5.11

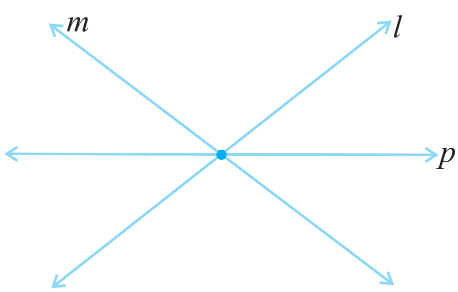


Fig 5.12

In Fig 5.12 the line p is not a transversal, although it cuts two lines l and m. Can you say, 'why'?

5.3.3. Angles made by a Transversal

In Fig 5.13, you see lines l and m cut by transversal p. The eight angles marked 1 to 8 have their special names:

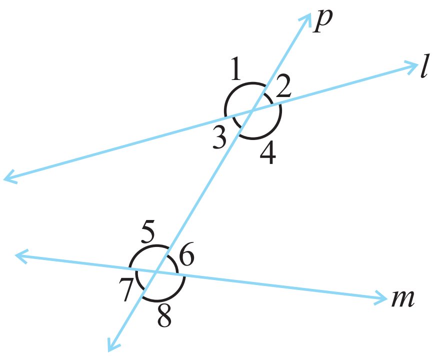


Fig 5.13

> **TRY THESE**
>
> 1. Suppose two lines are given. How many transversals can you draw for these lines?
> 2. If a line is a transversal to three lines, how many points of intersections are there?
> 3. Try to identify a few transversals in your surroundings.

Interior angles	$\angle 3$, $\angle 4$, $\angle 5$, $\angle 6$
Exterior angles	$\angle 1$, $\angle 2$, $\angle 7$, $\angle 8$
Pairs of Corresponding angles	$\angle 1$ and $\angle 5$, $\angle 2$ and $\angle 6$, $\angle 3$ and $\angle 7$, $\angle 4$ and $\angle 8$
Pairs of Alternate interior angles	$\angle 3$ and $\angle 6$, $\angle 4$ and $\angle 5$
Pairs of Alternate exterior angles	$\angle 1$ and $\angle 8$, $\angle 2$ and $\angle 7$
Pairs of interior angles on the same side of the transversal	$\angle 3$ and $\angle 5$, $\angle 4$ and $\angle 6$

Note: Corresponding angles (like $\angle 1$ and $\angle 5$ in Fig 5.14) include

(i) different vertices (ii) are on the same side of the transversal and

(iii) are in 'corresponding' positions (above or below, left or right) relative to the two lines.

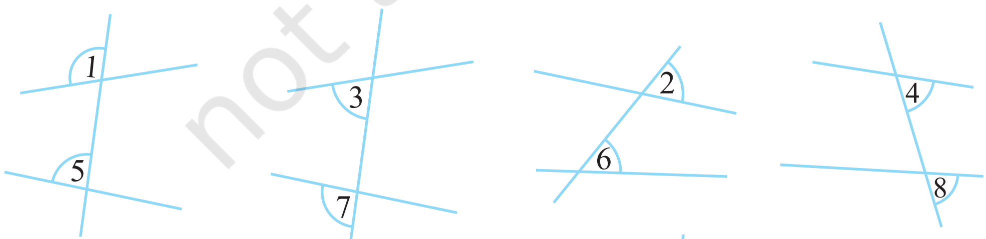


Fig 5.14

Alternate interior angles (like $\angle 3$ and $\angle 6$ in Fig 5.15)

(i) have different vertices

(ii) are on opposite sides of the transversal and

(iii) lie 'between' the two lines.

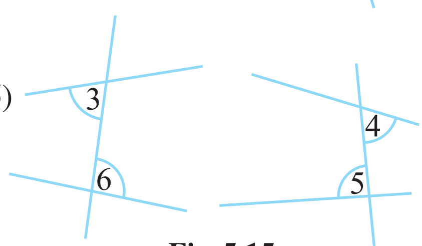
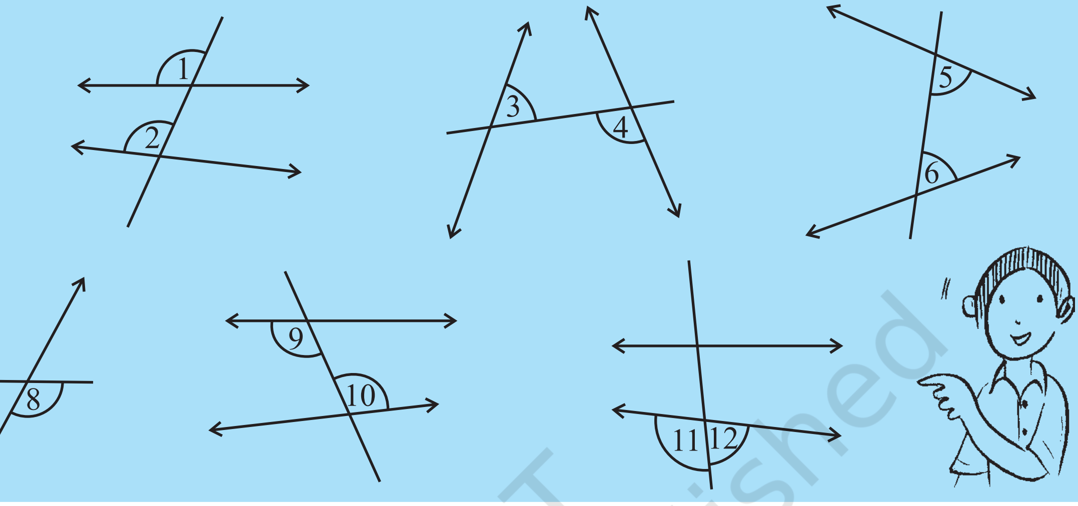


Fig 5.15

Try These

Name the pairs of angles in each figure:



5.3.4 Transversal of Parallel Lines

Do you remember what parallel lines are? They are lines on a plane that do not meet anywhere. Can you identify parallel lines in the following figures? (Fig 5.16)

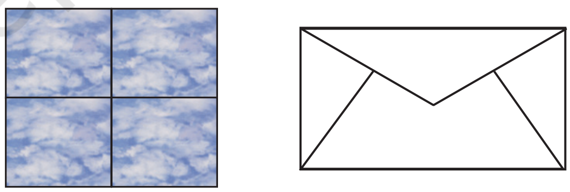


Fig 5.16

Transversals of parallel lines give rise to quite interesting results.

Do This

Take a ruled sheet of paper. Draw (in thick colour) two parallel lines *l* and *m*.

Draw a transversal *t* to the lines *l* and *m*. Label ∠1 and ∠2 as shown [Fig 5.17(i)].

Place a tracing paper over the figure drawn. Trace the lines *l*, *m* and *t*.

Slide the tracing paper along *t*, until *l* coincides with *m*.

You find that ∠1 on the traced figure coincides with ∠2 of the original figure.

In fact, you can see all the following results by similar tracing and sliding activity.

(i) ∠1 = ∠2 (ii) ∠3 = ∠4 (iii) ∠5 = ∠6 (iv) ∠7 = ∠8

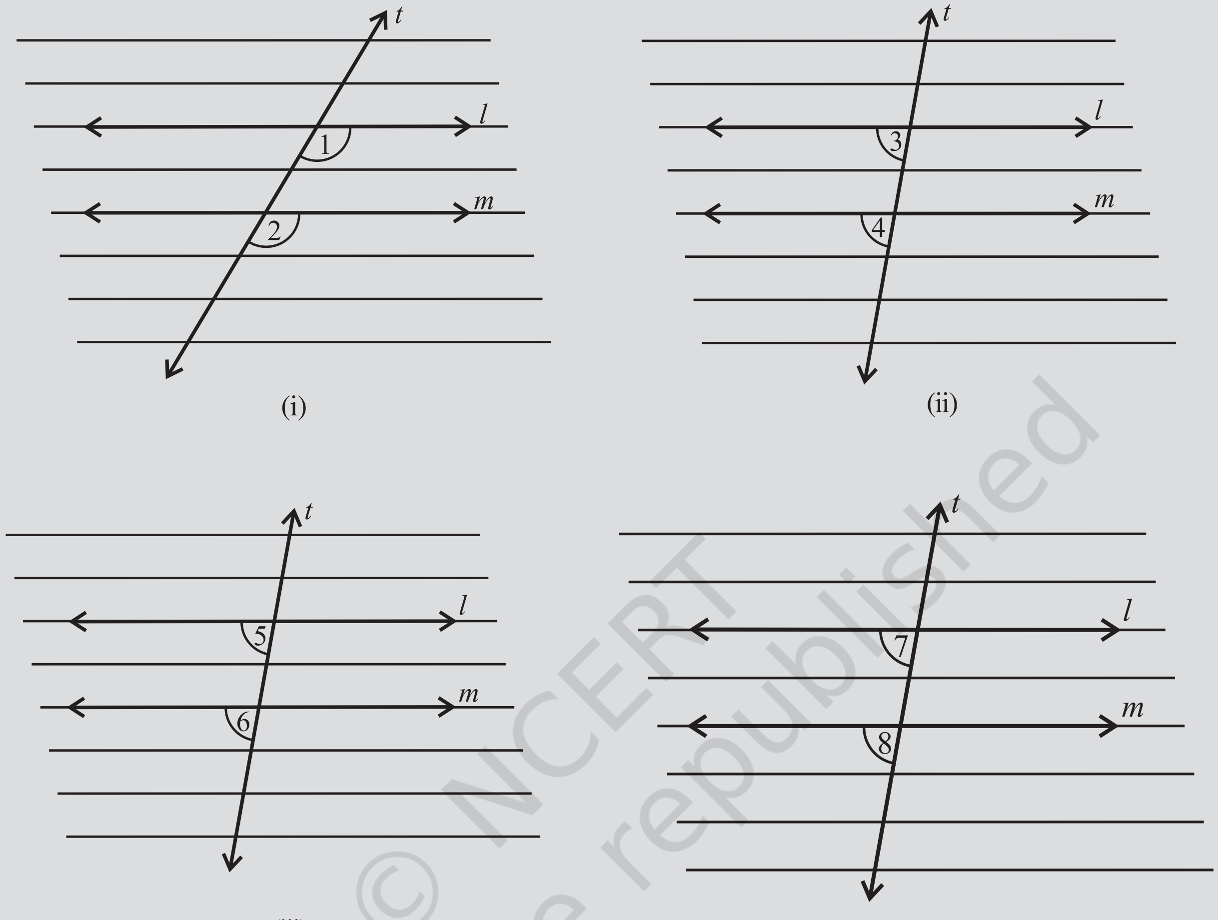


Fig 5.17

This activity illustrates the following fact:

> If two parallel lines are cut by a transversal, each pair of corresponding angles are equal in measure.

We use this result to get another interesting result. Look at Fig 5.18.

When t cuts the parallel lines, l, m, we get, $\angle 3 = \angle 7$ (vertically opposite angles).

But $\angle 7 = \angle 8$ (corresponding angles). Therefore, $\angle 3 = \angle 8$

You can similarly show that $\angle 1 = \angle 6$. Thus, we have the following result :

> If two parallel lines are cut by a transversal, each pair of alternate interior angles are equal.

This second result leads to another interesting property. Again, from Fig 5.18.

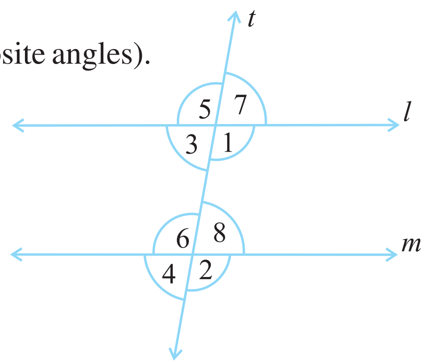


Fig 5.18

$\angle 3 + \angle 1 = 180°$ ($\angle 3$ and $\angle 1$ form a linear pair)

But $\angle 1 = \angle 6$ (A pair of alternate interior angles)

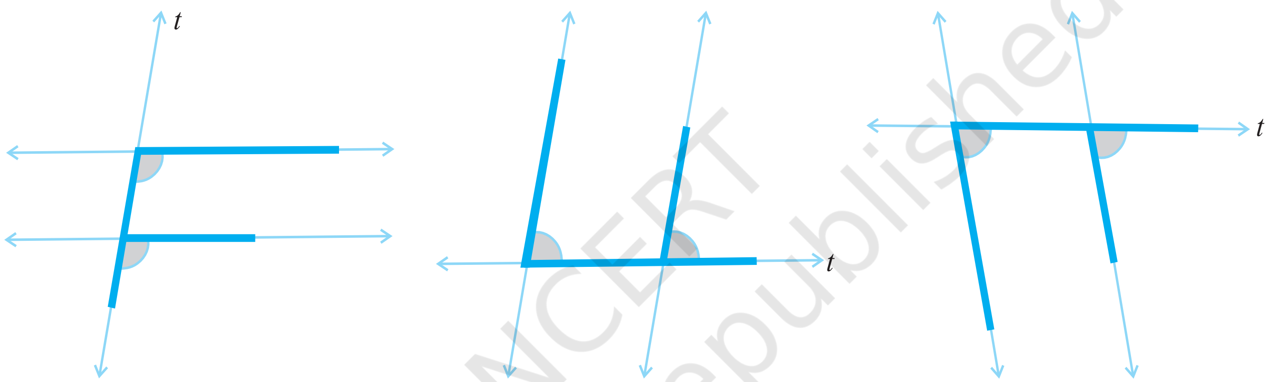
Therefore, we can say that $\angle 3 + \angle 6 = 180°$.

Similarly, $\angle 1 + \angle 8 = 180°$. Thus, we obtain the following result:

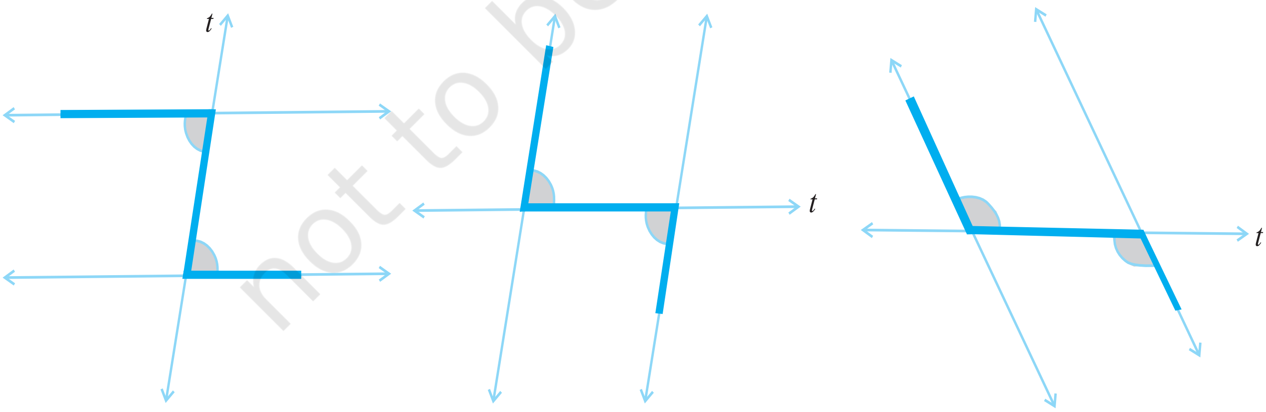
> If two parallel lines are cut by a transversal, then each pair of interior angles on the same side of the transversal are supplementary.

You can very easily remember these results if you can look for relevant 'shapes'.

The F-shape stands for corresponding angles:



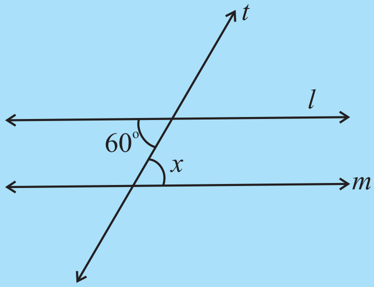
The Z - shape stands for alternate angles.



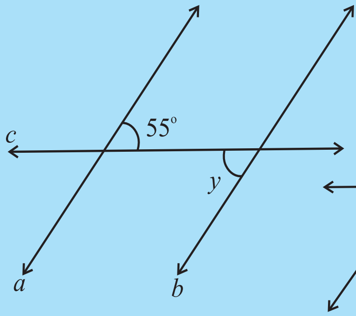
Do This

Draw a pair of parallel lines and a transversal. Verify the above three statements by actually measuring the angles.

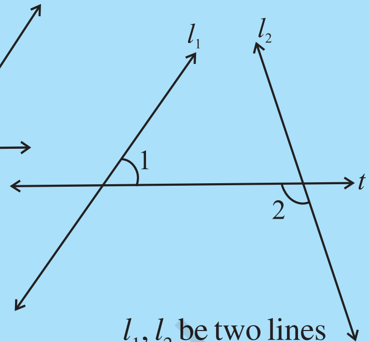
TRY THESE



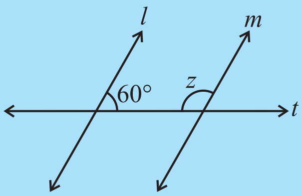
Lines $l \parallel m$;
t is a transversal
$\angle x = ?$



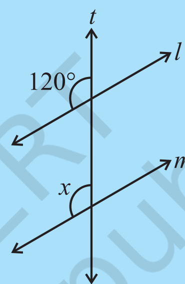
Lines $a \parallel b$;
c is a transversal
$\angle y = ?$



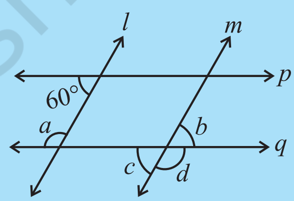
l_1, l_2 be two lines
t is a transversal
Is $\angle 1 = \angle 2$?



Lines $l \parallel m$;
t is a transversal
$\angle z = ?$



Lines $l \parallel m$;
t is a transversal
$\angle x = ?$



Lines $l \parallel m$, $p \parallel q$;
Find a, b, c, d

5.4 CHECKING FOR PARALLEL LINES

If two lines are parallel, then you know that a transversal gives rise to pairs of equal corresponding angles, equal alternate interior angles and interior angles on the same side of the transversal being supplementary.

When two lines are given, is there any method to check if they are parallel or not? You need this skill in many life-oriented situations.

A draftsman uses a carpenter's square and a straight edge (ruler) to draw these segments (Fig 5.19). He claims they are parallel. How?

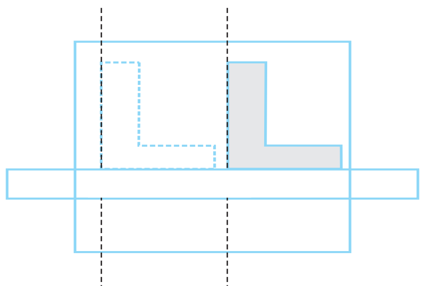


Fig 5.19

Are you able to see that he has kept the corresponding angles to be equal? (What is the transversal here?)

Thus, *when a transversal cuts two lines, such that pairs of corresponding angles are equal, then the lines have to be parallel.*

Look at the letter Z(Fig 5.20). The horizontal segments here are parallel, because the alternate angles are equal.

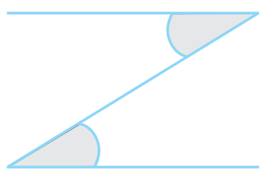


Fig 5.20

When a transversal cuts two lines, such that pairs of alternate interior angles are equal, the lines have to be parallel.

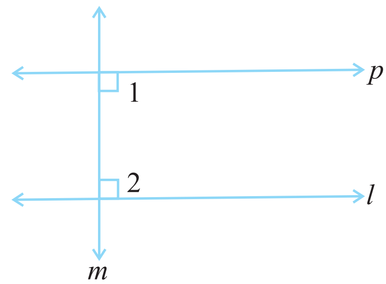


Fig 5.21

Draw a line l (Fig 5.21).

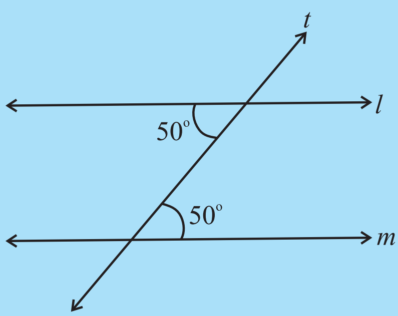
Draw a line m, perpendicular to l. Again draw a line p, such that p is perpendicular to m.

Thus, p is perpendicular to a perpendicular to l.

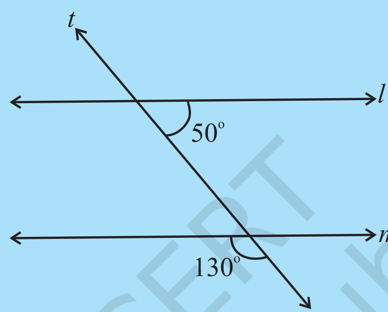
You find $p \parallel l$. How? This is because you draw p such that $\angle 1 + \angle 2 = 180^\circ$.

Thus, *when a transversal cuts two lines, such that pairs of interior angles on the same side of the transversal are supplementary, the lines have to be parallel.*

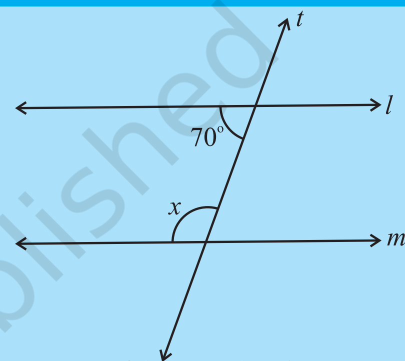
Try These



Is $l \parallel m$? Why?



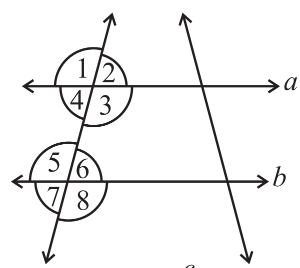
Is $l \parallel m$? Why?



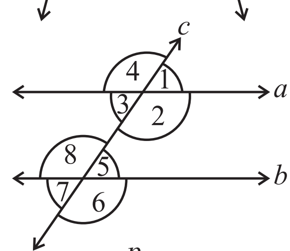
If $l \parallel m$, what is $\angle x$?

Exercise 5.2

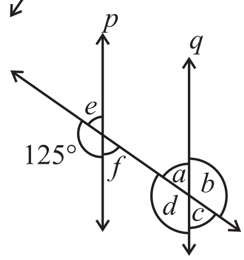
1. State the property that is used in each of the following statements?
   (i) If $a \parallel b$, then $\angle 1 = \angle 5$.
   (ii) If $\angle 4 = \angle 6$, then $a \parallel b$.
   (iii) If $\angle 4 + \angle 5 = 180^\circ$, then $a \parallel b$.



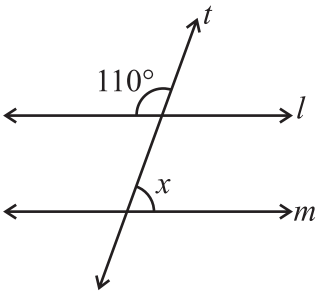
2. In the adjoining figure, identify
   (i) the pairs of corresponding angles.
   (ii) the pairs of alternate interior angles.
   (iii) the pairs of interior angles on the same side of the transversal.
   (iv) the vertically opposite angles.



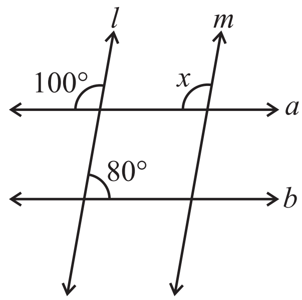
3. In the adjoining figure, $p \parallel q$. Find the unknown angles.



4. Find the value of x in each of the following figures if $l \parallel m$.

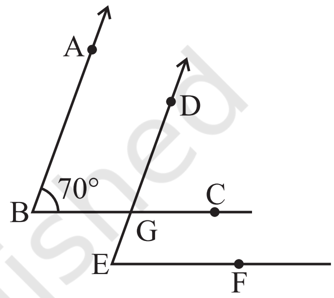


(i)

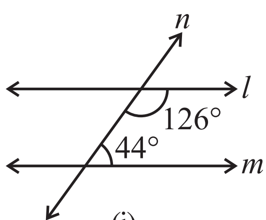


(ii)

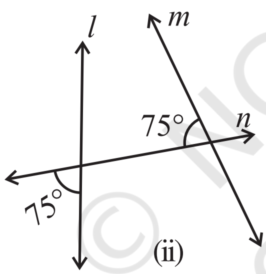
5. In the given figure, the arms of two angles are parallel.
If ∠ABC = 70°, then find
(i) ∠DGC
(ii) ∠DEF



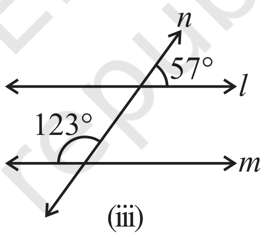
6. In the given figures below, decide whether l is parallel to m.



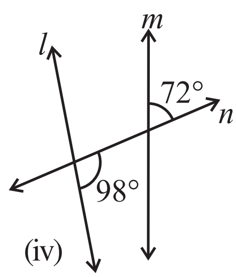
(i)



(ii)



(iii)



(iv)

What have We Discussed?

1. We recall that (i) A line-segment has two end points.
(ii) A ray has only one end point (its initial point); and
(iii) A line has no end points on either side.

2. When two lines l and m meet, we say they *intersect*; the meeting point is called the point of intersection.
When lines drawn on a sheet of paper do not meet, however far produced, we call them to be *parallel* lines.

Chapter 6

The Triangle and its Properties

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6.1 Introduction

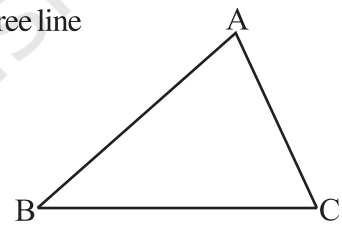


Fig 6.1

A triangle, you have seen, is a simple closed curve made of three line segments. It has three vertices, three sides and three angles.

Here is ΔABC (Fig 6.1). It has

Sides: $\overline{AB}$, $\overline{BC}$, $\overline{CA}$

Angles: $\angle$BAC, $\angle$ABC, $\angle$BCA

Vertices: A, B, C

The side opposite to the vertex A is BC. Can you name the angle opposite to the side AB?

You know how to classify triangles based on the (i) sides (ii) angles.

(i) Based on Sides: Scalene, Isosceles and Equilateral triangles.

(ii) Based on Angles: Acute-angled, Obtuse-angled and Right-angled triangles.

Make paper-cut models of the above triangular shapes. Compare your models with those of your friends and discuss about them.

Try These

1. Write the six elements (i.e., the 3 sides and the 3 angles) of ΔABC.
2. Write the:
   (i) Side opposite to the vertex Q of ΔPQR
   (ii) Angle opposite to the side LM of ΔLMN
   (iii) Vertex opposite to the side RT of ΔRST
3. Look at Fig 6.2 and classify each of the triangles according to its
   (a) Sides
   (b) Angles

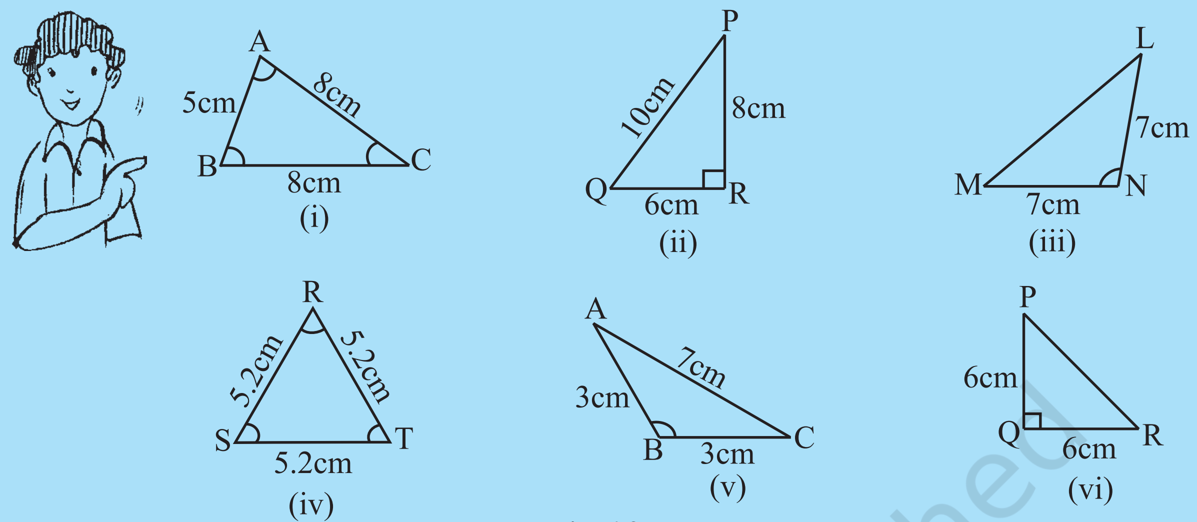


Fig 6.2

Now, let us try to explore something more about triangles.

6.2 Medians of a Triangle

Given a line segment, you know how to find its perpendicular bisector by paper folding. Cut out a triangle ABC from a piece of paper (Fig 6.3). Consider any one of its sides, say, $\overline{BC}$. By paper-folding, locate the perpendicular bisector of $\overline{BC}$. The folded crease meets $\overline{BC}$ at D, its mid-point. Join $\overline{AD}$.

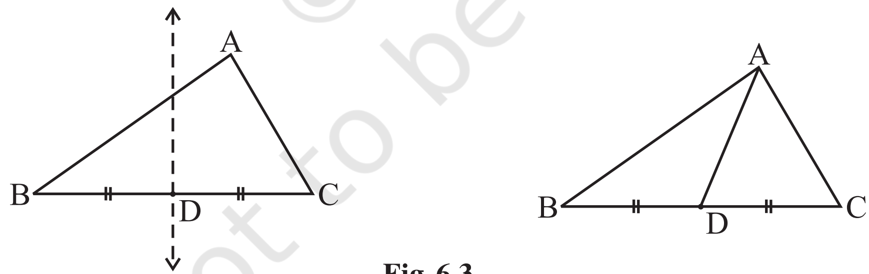


Fig 6.3

The line segment AD, joining the mid-point of $\overline{BC}$ to its opposite vertex A is called a **median** of the triangle.

Consider the sides $\overline{AB}$ and $\overline{CA}$ and find two more medians of the triangle.
A median connects a vertex of a triangle to the mid-point of the opposite side.

Think, Discuss and Write

1. How many medians can a triangle have?
2. Does a median lie wholly in the interior of the triangle? (If you think that this is not true, draw a figure to show such a case).

6.3 Altitudes of a Triangle

Make a triangular shaped cardboard ABC. Place it upright on a table. How 'tall' is the triangle? The **height** is the distance from vertex A (in the Fig 6.4) to the base $\overline{BC}$.

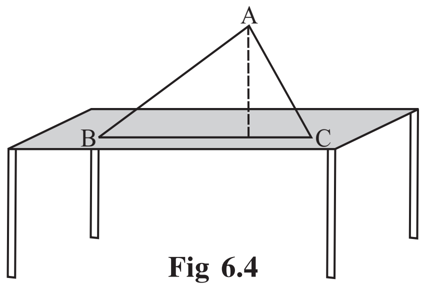


Fig 6.4

From A to $\overline{BC}$, you can think of many line segments (see the next Fig 6.5). Which among them will represent its height?

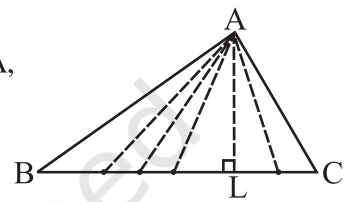


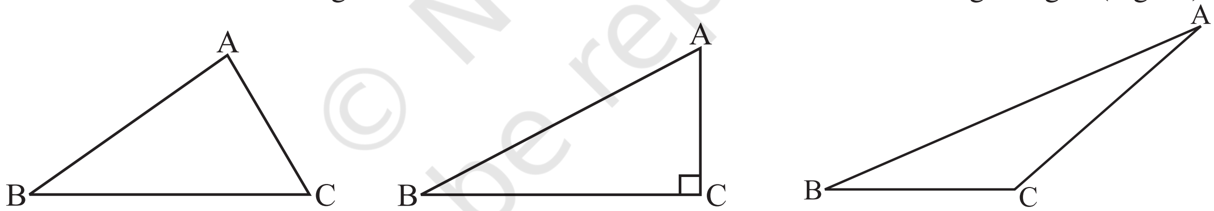
Fig 6.5

The **height** is given by the line segment that starts from A, comes straight down to $\overline{BC}$, and is perpendicular to $\overline{BC}$. This line segment $\overline{AL}$ is an **altitude** of the triangle.

An altitude has one end point at a vertex of the triangle and the other on the line containing the opposite side. Through each vertex, an altitude can be drawn.

Think, Discuss and Write

1. How many altitudes can a triangle have?
2. Draw rough sketches of altitudes from A to $\overline{BC}$ for the following triangles (Fig 6.6):



Acute-angled (i) | Right-angled (ii) | Obtuse-angled (iii)

Fig 6.6

3. Will an altitude always lie in the interior of a triangle? If you think that this need not be true, draw a rough sketch to show such a case.
4. Can you think of a triangle in which two altitudes of the triangle are two of its sides?
5. Can the altitude and median be same for a triangle?

(Hint: For Q.No. 4 and 5, investigate by drawing the altitudes for every type of triangle).

Do This

Take several cut-outs of

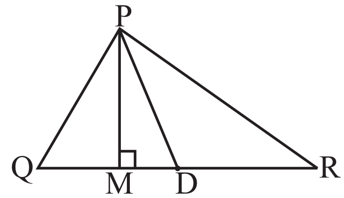
(i) an equilateral triangle (ii) an isosceles triangle and

(iii) a scalene triangle.

Find their altitudes and medians. Do you find anything special about them? Discuss it with your friends.

Exercise 6.1

1. In Δ PQR, D is the mid-point of $\overline{QR}$.



$\overline{PM}$ is __________________.

PD is __________________.

Is QM = MR?

2. Draw rough sketches for the following:

(a) In ΔABC, BE is a median.

(b) In ΔPQR, PQ and PR are altitudes of the triangle.

(c) In ΔXYZ, YL is an altitude in the exterior of the triangle.

3. Verify by drawing a diagram if the median and altitude of an isosceles triangle can be same.

6.4 Exterior Angle of a Triangle and its Property

Do This

1. Draw a triangle ABC and produce one of its sides, say BC as shown in Fig 6.7. Observe the angle ACD formed at the point C. This angle lies in the exterior of ΔABC. We call it an **exterior angle** of the ΔABC formed at vertex C.

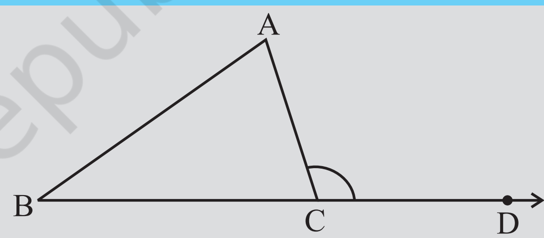


Fig 6.7

Clearly $\angle$BCA is an adjacent angle to $\angle$ACD. The remaining two angles of the triangle namely $\angle$A and $\angle$B are called the two **interior opposite angles** or the two remote interior angles of $\angle$ACD. Now cut out (or make trace copies of) $\angle$A and $\angle$B and place them adjacent to each other as shown in Fig 6.8.

Do these two pieces together entirely cover $\angle$ACD?

Can you say that

$m\angle ACD = m\angle A + m\angle B$?

2. As done earlier, draw a triangle ABC and form an exterior angle ACD. Now take a protractor and measure $\angle$ACD, $\angle$A and $\angle$B.

Find the sum $\angle A + \angle B$ and compare it with the measure of $\angle$ACD. Do you observe that $\angle$ACD is equal (or nearly equal, if there is an error in measurement) to $\angle A + \angle B$?

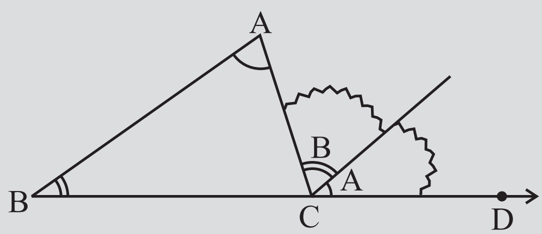


Fig 6.8

You may repeat the two activities as mentioned by drawing some more triangles along with their exterior angles. Every time, you will find that the exterior angle of a triangle is equal to the sum of its two interior opposite angles.

A logical step-by-step argument can further confirm this fact.

An exterior angle of a triangle is equal to the sum of its interior opposite angles.

Given: Consider ΔABC.

$\angle$ACD is an exterior angle.

To Show: $m\angle ACD = m\angle A + m\angle B$

Through C draw $\overline{CE}$, parallel to $\overline{BA}$.

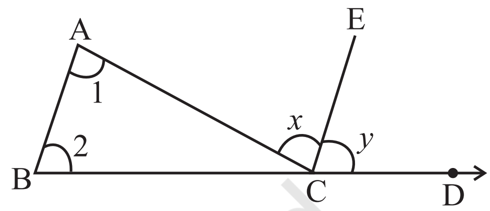


Fig 6.9

Justification

	Steps	**Reasons**
(a)	$\angle 1 = \angle x$	$\overline{BA} \parallel \overline{CE}$ and $\overline{AC}$ is a transversal. Therefore, alternate angles should be equal.
(b)	$\angle 2 = \angle y$	$\overline{BA} \parallel \overline{CE}$ and $\overline{BD}$ is a transversal. Therefore, corresponding angles should be equal.
(c)	$\angle 1 + \angle 2 = \angle x + \angle y$	
(d)	Now, $\angle x + \angle y = m\ \angle ACD$	From Fig 6.9
	Hence, $\angle 1 + \angle 2 = \angle ACD$	

The above relation between an exterior angle and its two interior opposite angles is referred to as the **Exterior Angle Property of a triangle.**

THINK, DISCUSS AND WRITE

1. Exterior angles can be formed for a triangle in many ways. Three of them are shown here (Fig 6.10)

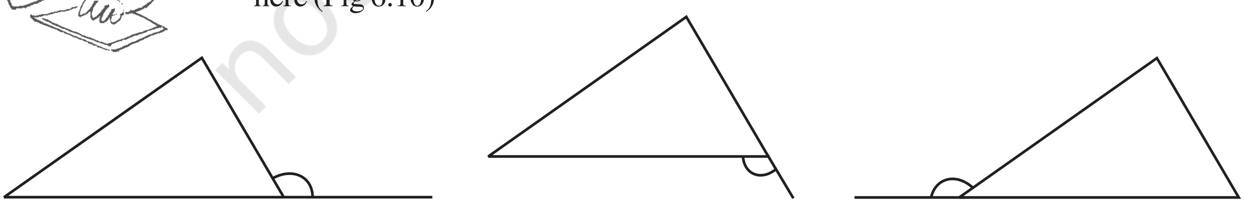


Fig 6.10

There are three more ways of getting exterior angles. Try to produce those rough sketches.

2. Are the exterior angles formed at each vertex of a triangle equal?

3. What can you say about the sum of an exterior angle of a triangle and its adjacent interior angle?

EXAMPLE 1 Find angle x in Fig 6.11.

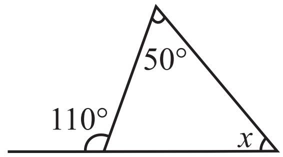


Fig 6.11

SOLUTION Sum of interior opposite angles = Exterior angle

or $$50° + x = 110°$$

or $$x = 60°$$

THINK, DISCUSS AND WRITE

1. What can you say about each of the interior opposite angles, when the exterior angle is
   (i) a right angle? (ii) an obtuse angle? (iii) an acute angle?
2. Can the exterior angle of a triangle be a straight angle?

TRY THESE

1. An exterior angle of a triangle is of measure 70º and one of its interior opposite angles is of measure 25º. Find the measure of the other interior opposite angle.
2. The two interior opposite angles of an exterior angle of a triangle are 60º and 80º. Find the measure of the exterior angle.
3. Is something wrong in this diagram (Fig 6.12)? Comment.

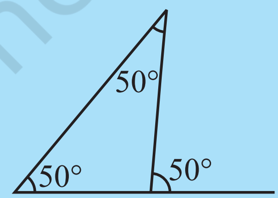
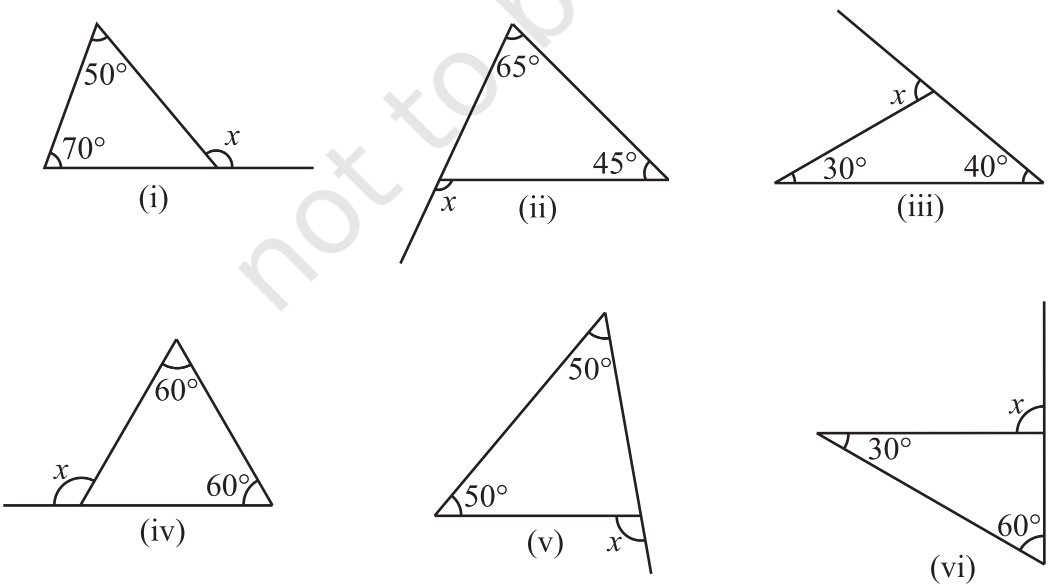


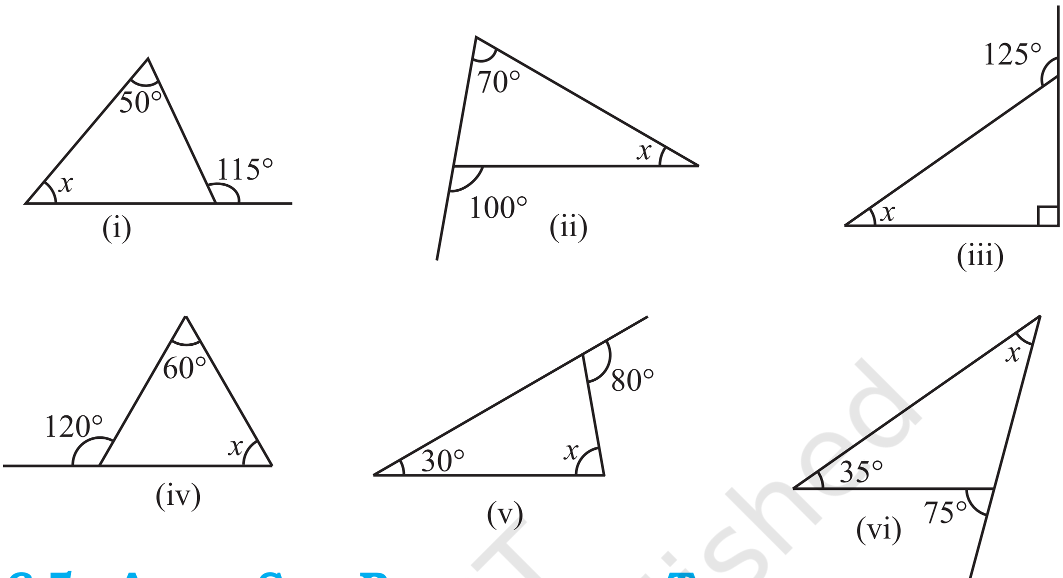
Fig 6.12

EXERCISE 6.2

1. Find the value of the unknown exterior angle x in the following diagrams:



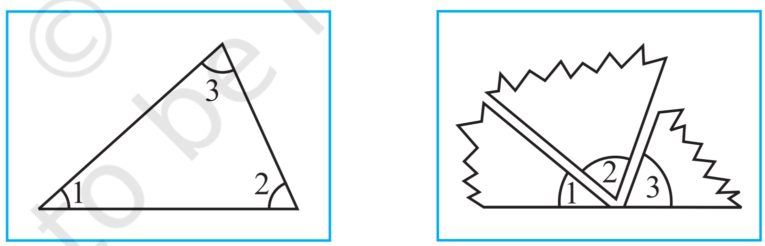
2. Find the value of the unknown interior angle x in the following figures:



6.5 Angle Sum Property of a Triangle

There is a remarkable property connecting the three angles of a triangle. You are going to see this through the following four activities.

1. Draw a triangle. Cut on the three angles. Rearrange them as shown in Fig 6.13 (i), (ii). The three angles now constitute one angle. This angle is a straight angle and so has measure 180°.



(i)

(ii) **Fig 6.13**

Thus, the sum of the measures of the three angles of a triangle is 180°.

2. The same fact you can observe in a different way also. Take three copies of any triangle, say ΔABC (Fig 6.14).

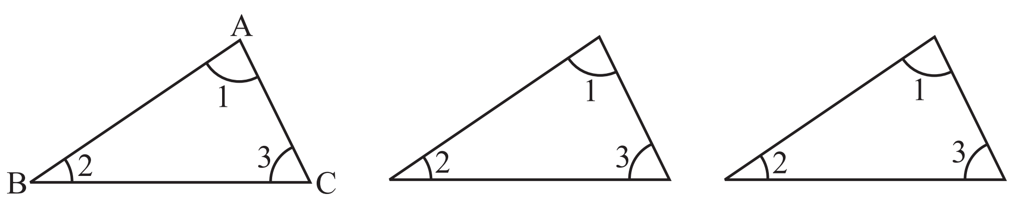


Fig 6.14

Arrange them as in Fig 6.15.

What do you observe about $\angle 1 + \angle 2 + \angle 3$?

(Do you also see the 'exterior angle property'?)

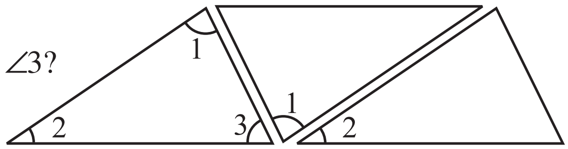


Fig 6.15

3. Take a piece of paper and cut out a triangle, say, ΔABC (Fig 6.16).

   Make the altitude AM by folding ΔABC such that it passes through A.

   Fold now the three corners such that all the three vertices A, B and C touch at M.

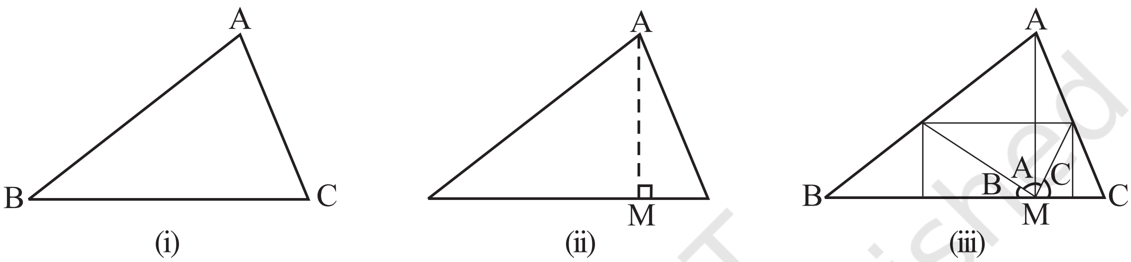


Fig 6.16

You find that all the three angles form together a straight angle. This again shows that the sum of the measures of the three angles of a triangle is 180°.

4. Draw any three triangles, say ΔABC, ΔPQR and ΔXYZ in your notebook.

   Use your protractor and measure each of the angles of these triangles.

   Tabulate your results

Name of Δ	Measures of Angles			Sum of the Measures of the three Angles
ΔABC	m∠A =	m∠B =	m∠C =	m∠A + m∠B + m∠C =
ΔPQR	m∠P =	m∠Q =	m∠R =	m∠P + m∠Q + m∠R =
ΔXYZ	m∠X =	m∠Y =	m∠Z =	m∠X + m∠Y + m∠Z =

Allowing marginal errors in measurement, you will find that the last column always gives 180° (or nearly 180°).

When perfect precision is possible, this will also show that the sum of the measures of the three angles of a triangle is 180°.

You are now ready to give a formal justification of your assertion through logical argument.

Statement **The total measure of the three angles of a triangle is 180°.**

To justify this let us use the exterior angle property of a triangle.

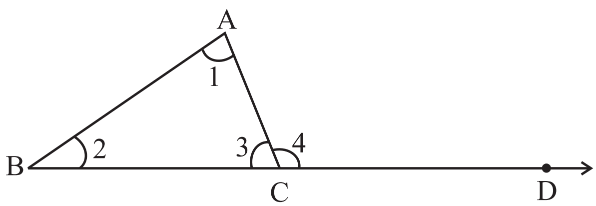


Fig 6.17

Given ∠1, ∠2, ∠3 are angles of ΔABC (Fig 6.17).

∠4 is the exterior angle when BC is extended to D.

Justification ∠1 + ∠2 = ∠4 (by exterior angle property)

∠1 + ∠2 + ∠3 = ∠4 + ∠3 (adding ∠3 to both the sides)

But ∠4 and ∠3 form a linear pair so it is 180°. Therefore, ∠1 + ∠2 + ∠3 = 180°.

Let us see how we can use this property in a number of ways.

EXAMPLE 2 In the given figure (Fig 6.18) find m∠P.

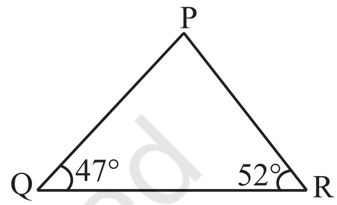


Fig 6.18

SOLUTION By angle sum property of a triangle,

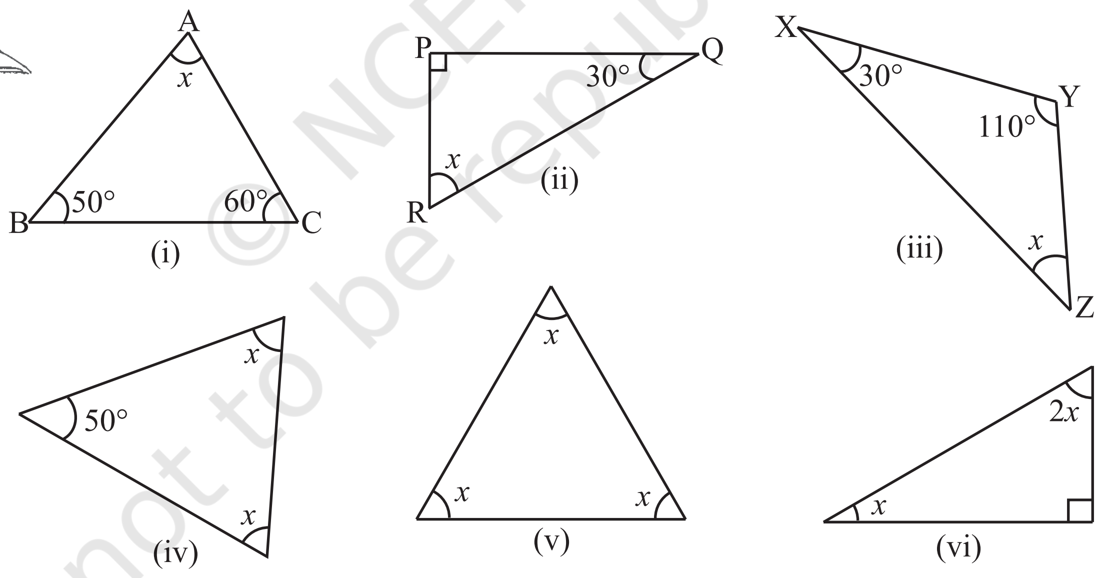
m∠P + 47° + 52° = 180°

Therefore m∠P = 180° – 47° – 52°

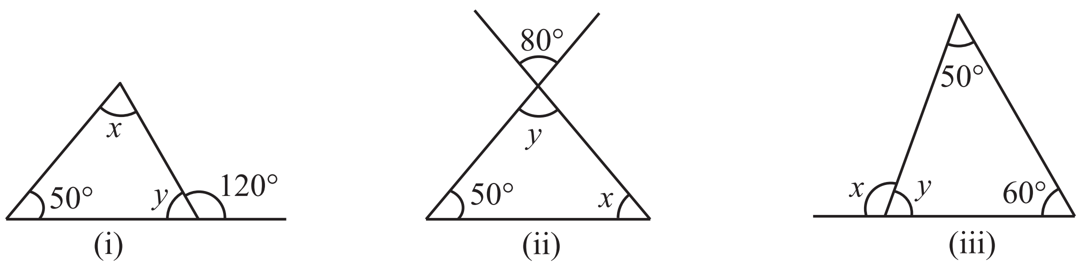
= 180° – 99° = 81°

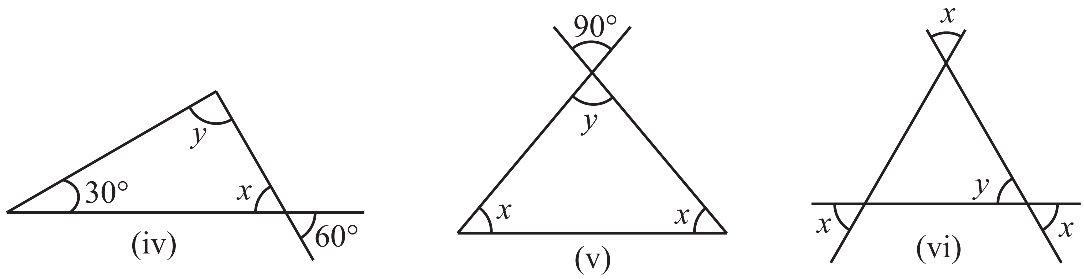
EXERCISE 6.3

1. Find the value of the unknown x in the following diagrams:



2. Find the values of the unknowns x and y in the following diagrams:





Try These

1. Two angles of a triangle are 30º and 80º. Find the third angle.
2. One of the angles of a triangle is 80º and the other two angles are equal. Find the measure of each of the equal angles.
3. The three angles of a triangle are in the ratio 1:2:1. Find all the angles of the triangle. Classify the triangle in two different ways.

Think, Discuss and Write

1. Can you have a triangle with two right angles?
2. Can you have a triangle with two obtuse angles?
3. Can you have a triangle with two acute angles?
4. Can you have a triangle with all the three angles greater than 60º?
5. Can you have a triangle with all the three angles equal to 60º?
6. Can you have a triangle with all the three angles less than 60º?

6.6 Two Special Triangles : Equilateral and Isosceles

A triangle in which all the three sides are of equal lengths is called an equilateral triangle.

Take two copies of an equilateral triangle ABC (Fig 6.19). Keep one of them fixed. Place the second triangle on it. It fits exactly into the first. Turn it round in any way and still they fit with one another exactly. Are you able to see that when the three sides of a triangle have equal lengths then the three angles are also of the same size?

We conclude that in an equilateral triangle:

(i) all sides have same length.

(ii) each angle has measure 60°.

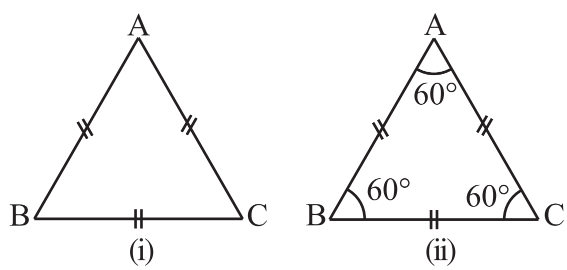


Fig 6.19

A triangle in which two sides are of equal lengths is called an isosceles triangle.

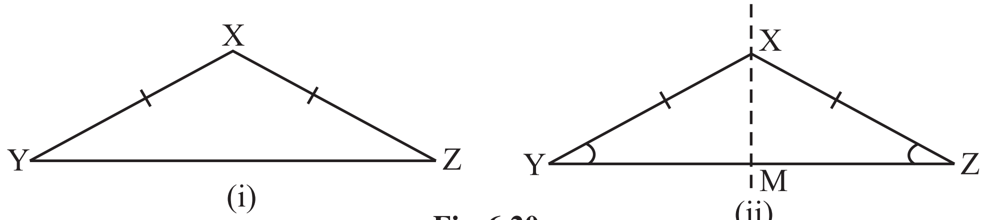


Fig 6.20

From a piece of paper cut out an isosceles triangle XYZ, with XY=XZ (Fig 6.20). Fold it such that Z lies on Y. The line XM through X is now the axis of symmetry (which you will read in Chapter 14). You find that $\angle$Y and $\angle$Z fit on each other exactly. XY and XZ are called equal sides; YZ is called the base; $\angle$Y and $\angle$Z are called base angles and these are also equal.

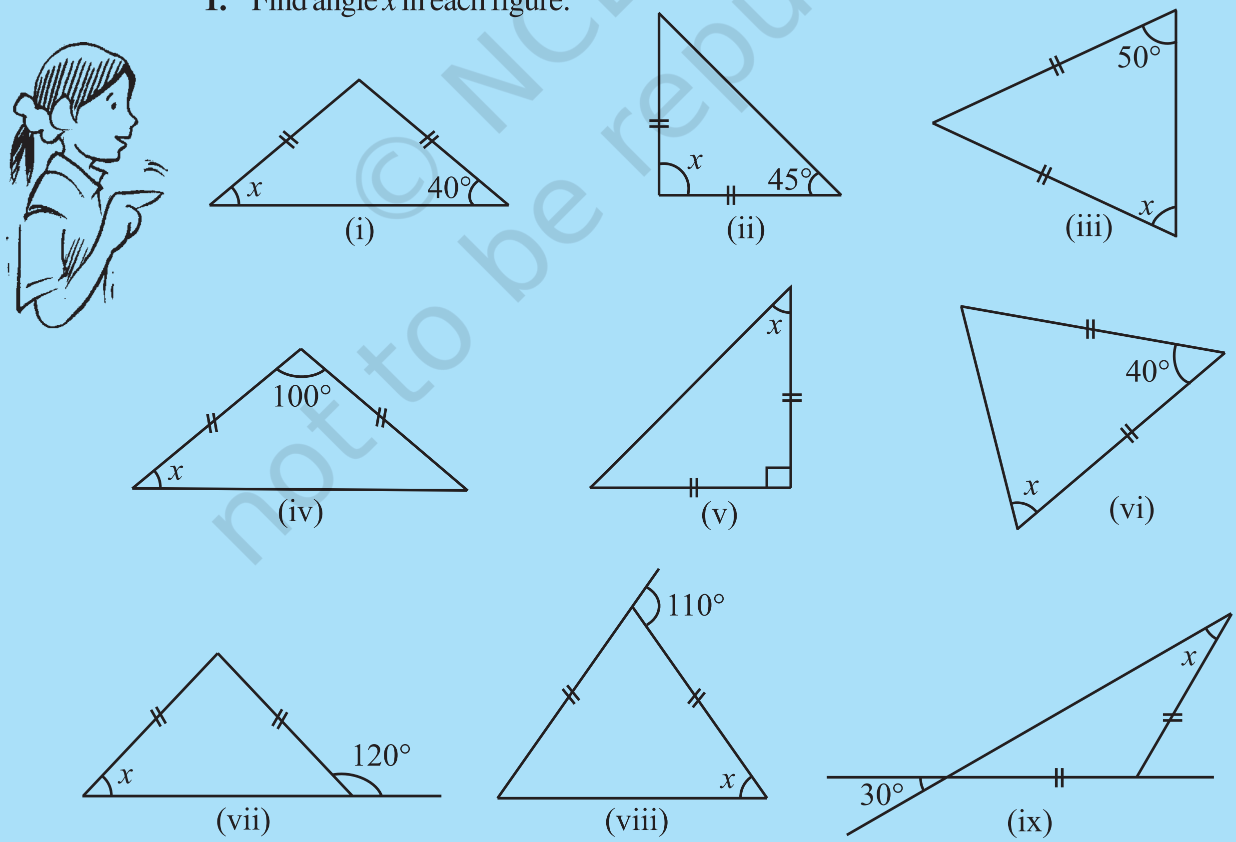
Thus, in an isosceles triangle:

(i) two sides have same length.

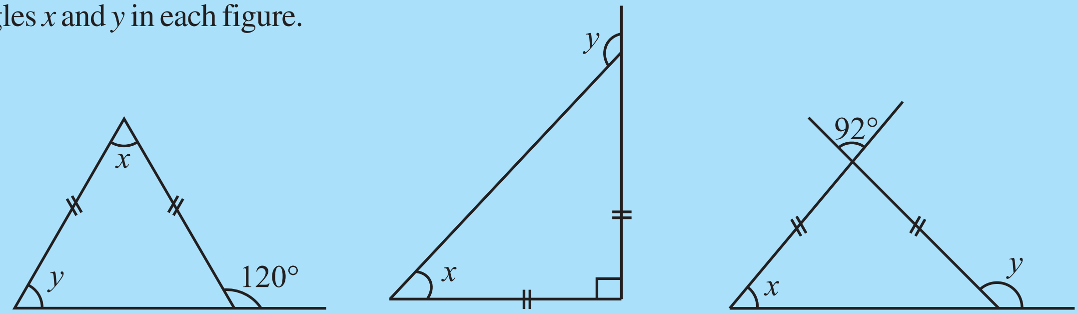
(ii) base angles opposite to the equal sides are equal.

TRY THESE

1. Find angle x in each figure:



2. Find angles x and y in each figure.



(i) (ii) (iii)

6.7 Sum of the Lengths of Two Sides of a Triangle

1. Mark three non-collinear spots A, B and C in your playground. Using lime powder mark the paths AB, BC and AC.

   Ask your friend to start from A and reach C, walking along one or more of these paths. She can, for example, walk first along $\overline{AB}$ and then along $\overline{BC}$ to reach C; or she can walk straight along $\overline{AC}$. She will naturally prefer the direct path AC. If she takes the other path ($\overline{AB}$ and then $\overline{BC}$), she will have to walk more. In other words,

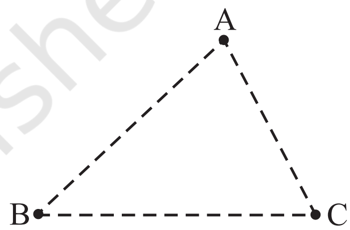


Fig 6.21

$$AB + BC > AC \qquad \text{(i)}$$

   Similarly, if one were to start from B and go to A, he or she will not take the route $\overline{BC}$ and $\overline{CA}$ but will prefer $\overline{BA}$ This is because

$$BC + CA > AB \qquad \text{(ii)}$$

   By a similar argument, you find that

$$CA + AB > BC \qquad \text{(iii)}$$

   These observations suggest that **the sum of the lengths of any two sides of a triangle is greater than the third side**.

2. Collect fifteen small sticks (or strips) of different lengths, say, 6 cm, 7 cm, 8 cm, 9 cm, ..., 20 cm.

   Take any three of these sticks and try to form a triangle. Repeat this by choosing different combinations of three sticks.

   Suppose you first choose two sticks of length 6 cm and 12 cm. Your third stick has to be of length more than 12 – 6 = 6 cm and less than 12 + 6 = 18 cm. Try this and find out why it is so.

   To form a triangle you will need any three sticks such that the sum of the lengths of any two of them will always be greater than the length of the third stick.

   This also suggests that the sum of the lengths of any two sides of a triangle is greater than the third side.

3. Draw any three triangles, say ΔABC, ΔPQR and ΔXYZ in your notebook (Fig 6.22).

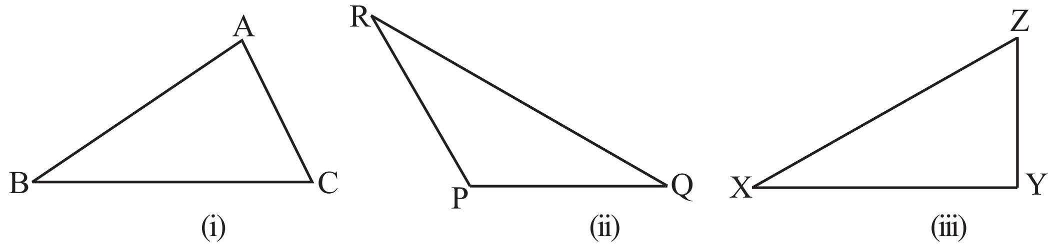


Fig 6.22

Use your ruler to find the lengths of their sides and then tabulate your results as follows:

Name of Δ	Lengths of Sides	Is this True?	
Δ ABC	AB ___	AB – BC < CA	(Yes/No)
		___ + ___ > ___	
	BC ___	BC – CA < AB	(Yes/No)
		___ + ___ > ___	
	CA ___	CA – AB < BC	(Yes/No)
		___ + ___ > ___	
Δ PQR	PQ ___	PQ – QR < RP	(Yes/No)
		___ + ___ > ___	
	QR ___	QR – RP < PQ	(Yes/No)
		___ + ___ > ___	
	RP ___	RP – PQ < QR	(Yes/No)
		___ + ___ > ___	
Δ XYZ	XY ___	XY – YZ < ZX	(Yes/No)
		___ + ___ > ___	
	YZ ___	YZ – ZX < XY	(Yes/No)
		___ + ___ > ___	
	ZX ___	ZX – XY < YZ	(Yes/No)
		___ + ___ > ___	

This also strengthens our earlier guess. Therefore, we conclude that **sum of the lengths of any two sides of a triangle is greater than the length of the third side.**

We also find that the difference between the length of any two sides of a triangle is smaller than the length of the third side.

EXAMPLE 3 Is there a triangle whose sides have lengths 10.2 cm, 5.8 cm and 4.5 cm?

SOLUTION Suppose such a triangle is possible. Then the sum of the lengths of any two sides would be greater than the length of the third side. Let us check this.

Is $4.5 + 5.8 > 10.2$?	Yes
Is $5.8 + 10.2 > 4.5$?	Yes
Is $10.2 + 4.5 > 5.8$?	Yes

Therefore, the triangle is possible.

EXAMPLE 4 The lengths of two sides of a triangle are 6 cm and 8 cm. Between which two numbers can length of the third side fall?

SOLUTION We know that the sum of two sides of a triangle is always greater than the third.

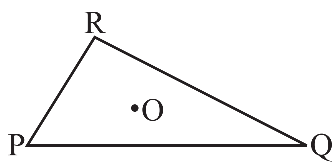
Therefore, third side has to be less than the sum of the two sides. The third side is thus, less than $8 + 6 = 14$ cm.

The side cannot be less than the difference of the two sides. Thus, the third side has to be more than $8 - 6 = 2$ cm.

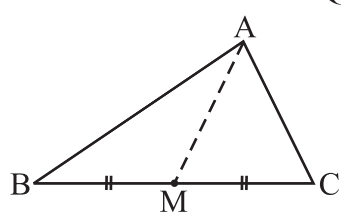
The length of the third side could be any length greater than 2 and less than 14 cm.

EXERCISE 6.4

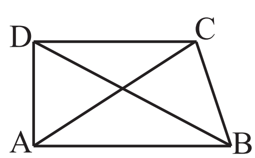
1. Is it possible to have a triangle with the following sides?
   (i) 2 cm, 3 cm, 5 cm (ii) 3 cm, 6 cm, 7 cm
   (iii) 6 cm, 3 cm, 2 cm



2. Take any point O in the interior of a triangle PQR. Is
   (i) OP + OQ > PQ?
   (ii) OQ + OR > QR?
   (iii) OR + OP > RP?



3. AM is a median of a triangle ABC.
   Is AB + BC + CA > 2 AM?
   (Consider the sides of triangles ΔABM and ΔAMC.)



4. ABCD is a quadrilateral.
   Is AB + BC + CD + DA > AC + BD?
5. ABCD is quadrilateral. Is
   AB + BC + CD + DA < 2 (AC + BD)?

6. The lengths of two sides of a triangle are 12 cm and 15 cm. Between what two measures should the length of the third side fall?

THINK, DISCUSS AND WRITE

1. Is the sum of any two angles of a triangle always greater than the third angle?

6.8 RIGHT-ANGLED TRIANGLES AND PYTHAGORAS PROPERTY

Pythagoras, a Greek philosopher of sixth century B.C. is said to have found a very important and useful property of right-angled triangles given in this section. The property is, hence, named after him. In fact, this property was known to people of many other countries too. The Indian mathematician Baudhayan has also given an equivalent form of this property. We now try to explain the Pythagoras property.

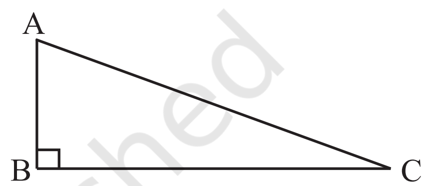


Fig 6.23

In a right-angled triangle, the sides have some special names. The side opposite to the right angle is called the **hypotenuse**; the other two sides are known as the **legs** of the right-angled triangle.

In ΔABC (Fig 6.23), the right-angle is at B. So, AC is the hypotenuse. $\overline{AB}$ and $\overline{BC}$ are the legs of ΔABC.

Make eight identical copies of right angled triangle of any size you prefer. For example, you make a right-angled triangle whose hypotenuse is a units long and the legs are of lengths b units and c units (Fig 6.24).

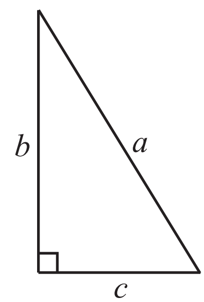


Fig 6.24

Draw two identical squares on a sheet with sides of lengths $b + c$.

You are to place four triangles in one square and the remaining four triangles in the other square, as shown in the following diagram (Fig 6.25).

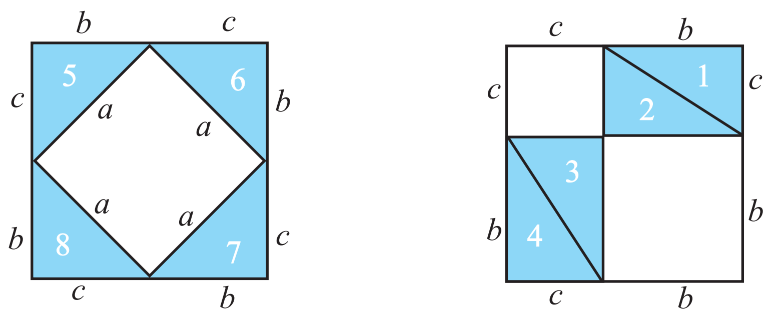


Fig 6.25

The squares are identical; the eight triangles inserted are also identical.

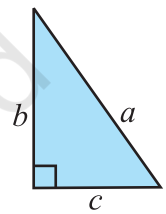
Hence the uncovered area of square A = Uncovered area of square B.

i.e., Area of inner square of square A = The total area of two uncovered squares in square B.

$$a^2 = b^2 + c^2$$

This is Pythagoras property. It may be stated as follows:

> In a right-angled triangle,
> the square on the hypotenuse = sum of the squares on the legs.



Pythagoras property is a very useful tool in mathematics. It is formally proved as a theorem in later classes. You should be clear about its meaning.

It says that for any right-angled triangle, the area of the square on the hypotenuse is equal to the sum of the areas of the squares on the legs.

Draw a right triangle, preferably on a square sheet, construct squares on its sides, compute the area of these squares and verify the theorem practically (Fig 6.26).

If you have a right-angled triangle, the Pythagoras property holds. If the Pythagoras property holds for some triangle, will the triangle be right-angled? (Such problems are known as converse problems). We will try to answer this. Now, we will show that, if there is a triangle such that sum of the squares on two of its sides is equal to the square of the third side, it must be a right-angled triangle.

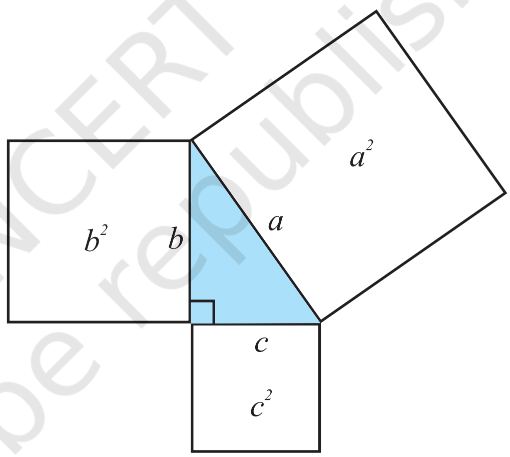


Fig 6.26

DO THIS

1. Have cut-outs of squares with sides 4 cm, 5 cm, 6 cm long. Arrange to get a triangular shape by placing the corners of the squares suitably as shown in the figure (Fig 6.27). Trace out the triangle formed. Measure each angle of the triangle. You find that there is no right angle at all.

In fact, in this case each angle will be acute! Note that $4^2 + 5^2 \neq 6^2$, $5^2 + 6^2 \neq 4^2$ and $6^2 + 4^2 \neq 5^2$.

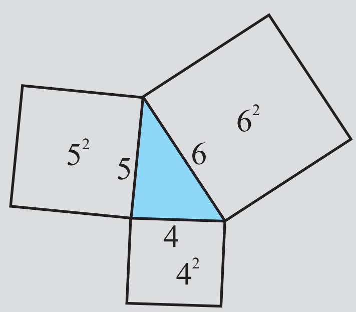


Fig 6.27

2. Repeat the above activity with squares whose sides have lengths 4 cm, 5 cm and 7 cm. You get an obtuse-angled triangle! Note that

$$4^2 + 5^2 \neq 7^2 \text{ etc.}$$

This shows that Pythagoras property holds if and only if the triangle is right-angled. Hence we get this fact:

> If the Pythagoras property holds, the triangle must be right-angled.

Example 5 Determine whether the triangle whose lengths of sides are 3 cm, 4 cm, 5 cm is a right-angled triangle.

Solution $3^2 = 3 \times 3 = 9$; $4^2 = 4 \times 4 = 16$; $5^2 = 5 \times 5 = 25$

We find $3^2 + 4^2 = 5^2$.

Therefore, the triangle is right-angled.

Note: In any right-angled triangle, the hypotenuse happens to be the longest side. In this example, the side with length 5 cm is the hypotenuse.

Example 6 Δ ABC is right-angled at C. If AC = 5 cm and BC = 12 cm find the length of AB.

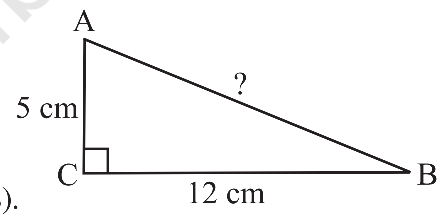


Fig 6.28

Solution A rough figure will help us (Fig 6.28).

By Pythagoras property,

$$AB^2 = AC^2 + BC^2$$
$$= 5^2 + 12^2 = 25 + 144 = 169 = 13^2$$

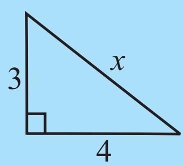
or $\quad AB^2 = 13^2$. So, $\quad AB = 13$

or the length of AB is 13 cm.

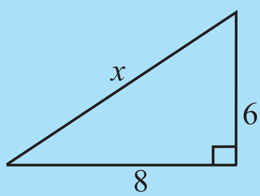
Note: To identify perfect squares, you may use prime factorisation technique.

Try These

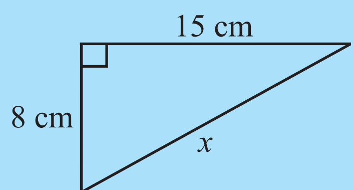
Find the unknown length x in the following figures (Fig 6.29):



(i)



(ii)



(iii)

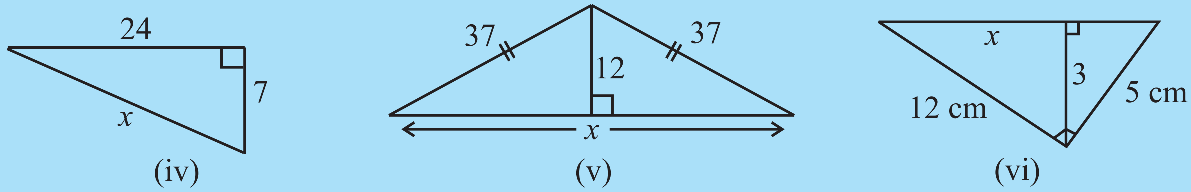
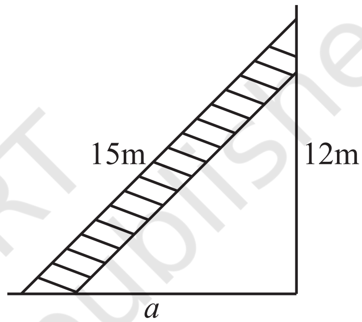


Fig 6.29

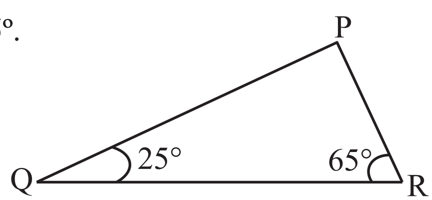
EXERCISE 6.5

1. PQR is a triangle, right-angled at P. If PQ = 10 cm and PR = 24 cm, find QR.
2. ABC is a triangle, right-angled at C. If AB = 25 cm and AC = 7 cm, find BC.
3. A 15 m long ladder reached a window 12 m high from the ground on placing it against a wall at a distance *a*. Find the distance of the foot of the ladder from the wall.



4. Which of the following can be the sides of a right triangle?
   (i) 2.5 cm,6.5 cm, 6 cm.
   (ii) 2 cm, 2 cm, 5 cm.
   (iii) 1.5 cm, 2cm, 2.5 cm.

   In the case of right-angled triangles, identify the right angles.
5. A tree is broken at a height of 5 m from the ground and its top touches the ground at a distance of 12 m from the base of the tree. Find the original height of the tree.
6. Angles Q and R of a ΔPQR are 25º and 65º. Write which of the following is true:
   (i) $PQ^2 + QR^2 = RP^2$
   (ii) $PQ^2 + RP^2 = QR^2$
   (iii) $RP^2 + QR^2 = PQ^2$



7. Find the perimeter of the rectangle whose length is 40 cm and a diagonal is 41 cm.
8. The diagonals of a rhombus measure 16 cm and 30 cm. Find its perimeter.

Think, Discuss and Write

1. Which is the longest side in the triangle PQR, right-angled at P?
2. Which is the longest side in the triangle ABC, right-angled at B?
3. Which is the longest side of a right triangle?
4. 'The diagonal of a rectangle produce by itself the same area as produced by its length and breadth' – This is Baudhayan Theorem. Compare it with the Pythagoras property.

Do This

Enrichment activity

There are many proofs for Pythagoras theorem, using 'dissection' and 'rearrangement' procedure. Try to collect a few of them and draw charts explaining them.

What have We Discussed?

1. The **six elements** of a triangle are its **three angles** and the **three sides**.
2. The line segment joining a vertex of a triangle to the mid point of its opposite side is called a **median** of the triangle. A triangle has 3 medians.
3. The perpendicular line segment from a vertex of a triangle to its opposite side is called an **altitude** of the triangle. A triangle has 3 altitudes.
4. An **exterior angle** of a triangle is formed, when a side of a triangle is produced. At each vertex, you have two ways of forming an exterior angle.
5. A property of exterior angles:

   The measure of any exterior angle of a triangle is equal to the sum of the measures of its interior opposite angles.
6. The angle sum property of a triangle:

   The total measure of the three angles of a triangle is 180°.
7. A triangle is said to be **equilateral,** if each one of its sides has the same length.

   In an equilateral triangle, each angle has measure 60°.
8. A triangle is said to be **isosceles,** if atleast any two of its sides are of same length.

   The non-equal side of an isosceles triangle is called its **base**; the base angles of an isosceles triangle have equal measure.
9. Property of the lengths of sides of a triangle:

   The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

   The difference between the lengths of any two sides is smaller than the length of the third side.

This property is useful to know if it is possible to draw a triangle when the lengths of the three sides are known.

10. In a right angled triangle, the side opposite to the right angle is called the **hypotenuse** and the other two sides are called its **legs**.

11. **Pythagoras property**:

In a right-angled triangle,

the square on the hypotenuse = the sum of the squares on its legs.

If a triangle is not right-angled, this property does not hold good. This property is useful to decide whether a given triangle is right-angled or not.

Chapter 7

Comparing Quantities

0756CH08

7.1 Percentage – Another Way of Comparing Quantities

Anita's Report
Total 320/400
Percentage: 80

Rita's Report
Total 300/360
Percentage: 83.3

Anita said that she has done better as she got 320 marks whereas Rita got only 300. Do you agree with her? Who do you think has done better?

Mansi told them that they cannot decide who has done better by just comparing the total marks obtained because the maximum marks out of which they got the marks are not the same.

She said why don't you see the Percentages given in your report cards?

Anita's Percentage was 80 and Rita's was 83.3. So, this shows Rita has done better.

Do you agree?

Percentages are numerators of fractions with denominator 100 and have been used in comparing results. Let us try to understand in detail about it.

7.1.1 Meaning of Percentage

Per cent is derived from Latin word 'per centum' meaning 'per hundred'.

Per cent is represented by the symbol % and means hundredths too. That is 1% means 1 out of hundred or one hundredth. It can be written as: $1\% = \frac{1}{100} = 0.01$

To understand this, let us consider the following example.

Rina made a table top of 100 different coloured tiles. She counted yellow, green, red and blue tiles separately and filled the table below. Can you help her complete the table?

Colour	Number of Tiles	Rate per Hundred	Fraction	Written as	Read as
Yellow	14	14	$\frac{14}{100}$	14%	14 per cent
Green	26	26	$\frac{26}{100}$	26%	26 per cent
Red	35	35	----	----	----
Blue	25	--------	----	----	----
Total	**100**				

Try These

1. Find the Percentage of children of different heights for the following data.

Height	Number of Children	In Fraction	In Percentage
110 cm	22		
120 cm	25		
128 cm	32		
130 cm	21		
Total	**100**		

2. A shop has the following number of shoe pairs of different sizes.

   Size 2 : 20 Size 3 : 30 Size 4 : 28

   Size 5 : 14 Size 6 : 8

   Write this information in tabular form as done earlier and find the Percentage of each shoe size available in the shop.

Percentages when total is not hundred

In all these examples, the total number of items add up to 100. For example, Rina had 100 tiles in all, there were 100 children and 100 shoe pairs. How do we calculate Percentage of an item if the total number of items do not add up to 100? In such cases, we need to convert the fraction to an equivalent fraction with denominator 100. Consider the following example. You have a necklace with twenty beads in two colours.

Colour	Number of Beads	Fraction	Denominator Hundred	In Percentage
Red	8	$\frac{8}{20}$	$\frac{8}{20}\times\frac{100}{100}=\frac{40}{100}$	40%
Blue	12	$\frac{12}{20}$	$\frac{12}{20}\times\frac{100}{100}=\frac{60}{100}$	60%
Total	**20**			

Anwar found the Percentage of red beads like this

Out of 20 beads, the number of red beads is 8.

Hence, out of 100, the number of red beads is

$\frac{8}{20}\times 100 = 40$ (out of hundred) $= 40\%$

Asha does it like this

$\frac{8}{20} = \frac{8\times 5}{20\times 5}$

$= \frac{40}{100} = 40\%$

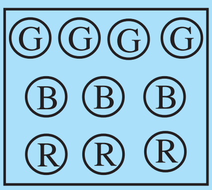
We see that these three methods can be used to find the Percentage when the total does not add to give 100. In the method shown in the table, we multiply the fraction by $\frac{100}{100}$. This does not change the value of the fraction. Subsequently, only 100 remains in the denominator.

Anwar has used the unitary method. Asha has multiplied by $\frac{5}{5}$ to get 100 in the denominator. You can use whichever method you find suitable. May be, you can make your own method too.

The method used by Anwar can work for all ratios. Can the method used by Asha also work for all ratios? Anwar says Asha's method can be used only if you can find a natural number which on multiplication with the denominator gives 100. Since denominator was 20, she could multiply it by 5 to get 100. If the denominator was 6, she would not have been able to use this method. Do you agree?

TRY THESE

1. A collection of 10 chips with different colours is given .



Colour	Number	Fraction	Denominator Hundred	In Percentage
Green				
Blue				
Red				
Total				

Fill the table and find the percentage of chips of each colour.

2. Mala has a collection of bangles. She has 20 gold bangles and 10 silver bangles. What is the percentage of bangles of each type? Can you put it in the tabular form as done in the above example?

Think, Discuss and Write

1. Look at the examples below and in each of them, discuss which is better for comparison.

In the atmosphere, 1 g of air contains:

.78 g Nitrogen
.21 g Oxygen
.01 g Other gas

or

78% Nitrogen
21% Oxygen
1% Other gas

2. A shirt has:

$\frac{3}{5}$ Cotton
$\frac{2}{5}$ Polyster

or

60% Cotton
40% Polyster

7.1.2 Converting Fractional Numbers to Percentage

Fractional numbers can have different denominator. To compare fractional numbers, we need a common denominator and we have seen that it is more convenient to compare if our denominator is 100. That is, we are converting the fractions to Percentages. Let us try converting different fractional numbers to Percentages.

Example 1 Write $\frac{1}{3}$ as per cent.

Solution We have, $\frac{1}{3} = \frac{1}{3} \times \frac{100}{100} = \frac{1}{3} \times 100\%$

$$= \frac{100}{3}\% = 33\frac{1}{3}\%$$

Example 2 Out of 25 children in a class, 15 are girls. What is the percentage of girls?

Solution Out of 25 children, there are 15 girls.

Therefore, percentage of girls $= \frac{15}{25} \times 100 = 60$. There are 60% girls in the class.

Example 3 Convert $\frac{5}{4}$ to per cent.

Solution We have, $\frac{5}{4} = \frac{5}{4} \times 100\% = 125\%$

From these examples, we find that the percentages related to proper fractions are less than 100 whereas percentages related to improper fractions are more than 100.

Think, Discuss and Write

(i) Can you eat 50% of a cake? Can you eat 100% of a cake?
Can you eat 150% of a cake?

(ii) Can a price of an item go up by 50%? Can a price of an item go up by 100%?
Can a price of an item go up by 150%?

7.1.3 Converting Decimals to Percentage

We have seen how fractions can be converted to per cents. Let us now find how decimals can be converted to per cents.

Example 4 Convert the given decimals to per cents:

(a) 0.75 (b) 0.09 (c) 0.2

Solution

(a) $0.75 = 0.75 \times 100\ \%$

$= \frac{75}{100} \times 100\ \% = 75\%$

(b) $0.09 = \frac{9}{100} = 9\ \%$

(c) $0.2 = \frac{2}{10} \times 100\% = 20\ \%$

Try These

1. Convert the following to per cents:

(a) $\frac{12}{16}$ (b) 3.5 (c) $\frac{49}{50}$ (d) $\frac{2}{2}$ (e) 0.05

2. (i) Out of 32 students, 8 are absent. What per cent of the students are absent?
(ii) There are 25 radios, 16 of them are out of order. What per cent of radios are out of order?
(iii) A shop has 500 items, out of which 5 are defective. What per cent are defective?
(iv) There are 120 voters, 90 of them voted yes. What per cent voted yes?

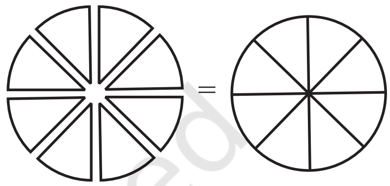
7.1.4 Converting Percentages to Fractions or Decimals

We have so far converted fractions and decimals to percentages. We can also do the reverse. That is, given per cents, we can convert them to decimals or fractions. Look at the table, observe and complete it:

Per cent	1%	10%	25%	50%	90%	125%	250%
Fraction	$\frac{1}{100}$	$\frac{10}{100} = \frac{1}{10}$					
Decimal	0.01	0.10					

Make some more such examples and solve them.

Parts always add to give a whole



In the examples for coloured tiles, for the heights of children and for gases in the air, we find that when we add the Percentages we get 100. All the parts that form the whole when added together gives the whole or 100%. So, if we are given one part, we can always find out the other part. Suppose, 30% of a given number of students are boys.

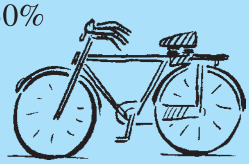
This means that if there were 100 students, 30 out of them would be boys and the remaining would be girls.

Then girls would obviously be (100 – 30)% = 70%.

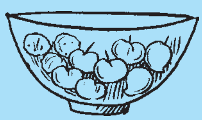
Try These

1. 35% + ________% = 100%, 64% + 20% + ________ % = 100%

   45% = 100% – ________ %, 70% = ______% – 30%



2. If 65% of students in a class have a bicycle, what per cent of the student do not have bicycles?



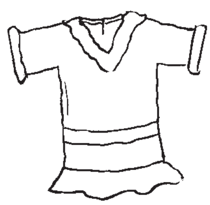
3. We have a basket full of apples, oranges and mangoes. If 50% are apples, 30% are oranges, then what per cent are mangoes?

Think, Discuss and Write

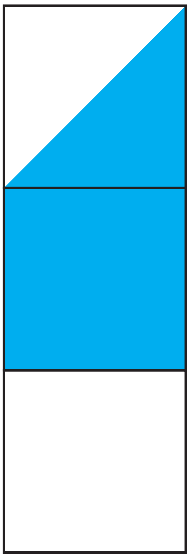
Consider the expenditure made on a dress

20% on embroidery, 50% on cloth, 30% on stitching.

Can you think of more such examples?



7.1.5 Fun with Estimation



Percentages help us to estimate the parts of an area.

Example 5 What per cent of the adjoining figure is shaded?

Solution We first find the fraction of the figure that is shaded. From this fraction, the percentage of the shaded part can be found.

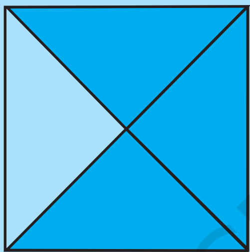
You will find that half of the figure is shaded. And, $\frac{1}{2} = \frac{1}{2} \times 100\,\% = 50\,\%$

Thus, 50 % of the figure is shaded.

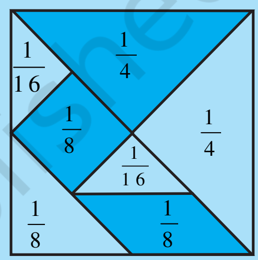
Try These

What per cent of these figures are shaded?

(i)



(ii)



Tangram

You can make some more figures yourself and ask your friends to estimate the shaded parts.

7.2 Use of Percentages

7.2.1 Interpreting Percentages

We saw how percentages were helpful in comparison. We have also learnt to convert fractional numbers and decimals to percentages. Now, we shall learn how percentages can be used in real life. For this, we start with interpreting the following statements:

— 5 % of the income is saved by Ravi. — 20 % of Meera's dresses are blue in colour.

— Rekha gets 10 % on every book sold by her.

What can you infer from each of these statements?

By 5 % we mean 5 parts out of 100 or we write it as $\frac{5}{100}$. It means Ravi is saving ₹ 5 out of every ₹ 100 that he earns. In the same way, interpret the rest of the statements given above.

7.2.2 Converting Percentages to "How Many"

Consider the following examples:

Example 6 A survey of 40 children showed that 25% liked playing football. How many children liked playing football?

Solution Here, the total number of children are 40. Out of these, 25% like playing football. Meena and Arun used the following methods to find the number. You can choose either method.

Arun does it like this

Out of 100, 25 like playing football

So out of 40, number of children who like playing football $= \frac{25}{100} \times 40 = 10$

Meena does it like this

25% of $40 = \frac{25}{100} \times 40$

$= 10$

Hence, 10 children out of 40 like playing football.

Try These

1. Find:

   (a) 50% of 164 (b) 75% of 12 (c) $12\frac{1}{2}$% of 64

2. 8 % children of a class of 25 like getting wet in the rain. How many children like getting wet in the rain.

Example 7 Rahul bought a sweater and saved ₹ 200 when a discount of 25% was given. What was the price of the sweater before the discount?

Solution Rahul has saved ₹ 200 when price of sweater is reduced by 25%. This means that 25% reduction in price is the amount saved by Rahul. Let us see how Mohan and Abdul have found the original cost of the sweater.

Mohan's solution

25% of the original price = ₹ 200

Let the price (in ₹) be P

So, 25% of $P = 200$ or $\frac{25}{100} \times P = 200$

or, $\frac{P}{4} = 200$ or $P = 200 \times 4$

Therefore, $P = 800$

Abdul's solution

₹ 25 is saved for every ₹ 100

Amount for which ₹ 200 is saved

$= \frac{100}{25} \times 200 =$ ₹ 800

Thus both obtained the original price of sweater as ₹ 800.

Try These

1. 9 is 25% of what number?
2. 75% of what number is 15?

Exercise 7.1

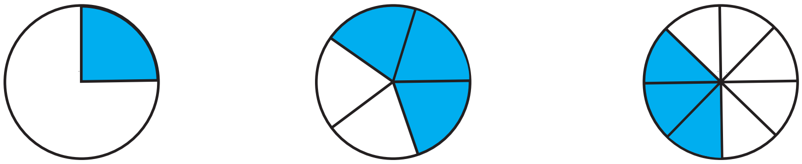
1. Convert the given fractional numbers to per cents.

   (a) $\frac{1}{8}$ (b) $\frac{5}{4}$ (c) $\frac{3}{40}$ (d) $\frac{2}{7}$

2. Convert the given decimal fractions to per cents.

(a) 0.65 (b) 2.1 (c) 0.02 (d) 12.35

3. Estimate what part of the figures is coloured and hence find the per cent which is coloured.



(i) (ii) (iii)

4. Find:

(a) 15% of 250 (b) 1% of 1 hour (c) 20% of ₹ 2500 (d) 75% of 1 kg

5. Find the whole quantity if

(a) 5% of it is 600. (b) 12% of it is ₹ 1080. (c) 40% of it is 500 km.

(d) 70% of it is 14 minutes. (e) 8% of it is 40 litres.

6. Convert given per cents to decimal fractions and also to fractions in simplest forms:

(a) 25% (b) 150% (c) 20% (d) 5%

7. In a city, 30% are females, 40% are males and remaining are children. What per cent are children?

8. Out of 15,000 voters in a constituency, 60% voted. Find the percentage of voters who did not vote. Can you now find how many actually did not vote?

9. Meeta saves ₹ 4000 from her salary. If this is 10% of her salary. What is her salary?

10. A local cricket team played 20 matches in one season. It won 25% of them. How many matches did they win?

7.2.3 Ratios to Percents

Sometimes, parts are given to us in the form of ratios and we need to convert those to percentages. Consider the following example:

Example 8 Reena's mother said, to make *idlis*, you must take two parts rice and one part *urad dal*. What percentage of such a mixture would be rice and what percentage would be *urad dal*?

Solution In terms of ratio we would write this as Rice : *Urad dal* = 2 : 1.

Now, 2 + 1=3 is the total of all parts. This means $\frac{2}{3}$ part is rice and $\frac{1}{3}$ part is *urad dal*.

Then, percentage of rice would be $\frac{2}{3} \times 100\ \% = \frac{200}{3} = 66\frac{2}{3}\ \%$.

Percentage of *urad dal* would be $\frac{1}{3} \times 100\ \% = \frac{100}{3} = 33\frac{1}{3}\ \%$.

EXAMPLE 9 If ₹ 250 is to be divided amongst Ravi, Raju and Roy, so that Ravi gets two parts, Raju three parts and Roy five parts. How much money will each get? What will it be in percentages?

SOLUTION The parts which the three boys are getting can be written in terms of ratios as 2 : 3 : 5. Total of the parts is 2 + 3 + 5 = 10.

Amounts received by each	Percentages of money for each
$\frac{2}{10} \times ₹\ 250 = ₹\ 50$	Ravi gets $\frac{2}{10} \times 100\ \% = 20\ \%$
$\frac{3}{10} \times ₹\ 250 = ₹\ 75$	Raju gets $\frac{3}{10} \times 100\ \% = 30\ \%$
$\frac{5}{10} \times ₹\ 250 = ₹\ 125$	Roy gets $\frac{5}{10} \times 100\ \% = 50\ \%$

TRY THESE

1. Divide 15 sweets between Manu and Sonu so that they get 20 % and 80 % of them respectively.
2. If angles of a triangle are in the ratio 2 : 3 : 4. Find the value of each angle.

7.2.4 Increase or Decrease as Per Cent

There are times when we need to know the increase or decrease in a certain quantity as percentage. For example, if the population of a state increased from 5,50,000 to 6,05,000. Then the increase in population can be understood better if we say, the population increased by 10 %.

How do we convert the increase or decrease in a quantity as a percentage of the initial amount? Consider the following example.

EXAMPLE 10 A school team won 6 games this year against 4 games won last year. What is the per cent increase?

SOLUTION The increase in the number of wins (or amount of change) = 6 – 4 = 2.

$$\text{Percentage increase} = \frac{\text{amount of change}}{\text{original amount or base}} \times 100$$

$$= \frac{\text{increase in the number of wins}}{\text{original number of wins}} \times 100 = \frac{2}{4} \times 100 = 50$$

EXAMPLE 11 The number of illiterate persons in a country decreased from 150 lakhs to 100 lakhs in 10 years. What is the percentage of decrease?

SOLUTION Original amount = the number of illiterate persons initially = 150 lakhs.

Amount of change = decrease in the number of illiterate persons = 150 – 100 = 50 lakhs
Therefore, the percentage of decrease

$$= \frac{\text{amount of change}}{\text{original amount}} \times 100 = \frac{50}{150} \times 100 = 33\frac{1}{3}$$

Try These

1. Find Percentage of increase or decrease:
   – Price of shirt decreased from ₹ 280 to ₹ 210.
   – Marks in a test increased from 20 to 30.
2. My mother says, in her childhood petrol was ₹ 1 a litre. It is ₹ 52 per litre today. By what Percentage has the price gone up?

7.3 Prices Related to an Item or Buying and Selling

I bought it for ₹ 600

and will sell it for ₹ 610

The buying price of any item is known as its **cost price**. It is written in short as CP.

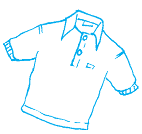
The price at which you sell is known as the **selling price** or in short SP.

What would you say is better, to you sell the item at a lower price, same price or higher price than your buying price? You can decide whether the sale was profitable or not depending on the CP and SP. If CP < SP then you made a profit = SP – CP.

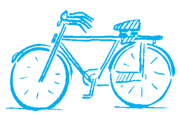
If CP = SP then you are in a no profit no loss situation.

If CP > SP then you have a loss = CP – SP.

Let us try to interpret the statements related to prices of items.



- A toy bought for ₹ 72 is sold at ₹ 80.
- A T-shirt bought for ₹ 120 is sold at ₹ 100.
- A cycle bought for ₹ 800 is sold for ₹ 940.



Let us consider the first statement.

The buying price (or CP) is ₹ 72 and the selling price (or SP) is ₹ 80. This means SP is more than CP. Hence profit made = SP – CP = ₹ 80 – ₹ 72 = ₹ 8

Now try interpreting the remaining statements in a similar way.

7.3.1 Profit or Loss as a Percentage

The profit or loss can be converted to a percentage. It is always calculated on the CP.

For the above examples, we can find the profit % or loss %.

Let us consider the example related to the toy. We have CP = ₹ 72, SP = ₹ 80, Profit = ₹ 8. To find the percentage of profit, Neha and Shekhar have used the following methods.

Neha does it this way

Profit per cent = $\frac{\text{Profit}}{\text{CP}} \times 100 = \frac{8}{72} \times 100$

$= \frac{1}{9} \times 100 = 11\frac{1}{9}$

Thus, the profit is ₹ 8 and profit Per cent is $11\frac{1}{9}$.

Shekhar does it this way

On ₹ 72 the profit is ₹ 8

On ₹ 100, profit = $\frac{8}{72} \times 100$

$= 11\frac{1}{9}$. Thus, profit per cent = $11\frac{1}{9}$

Similarly you can find the loss per cent in the second situation. Here, CP = ₹ 120, SP = ₹ 100.

Therefore, Loss = ₹ 120 – ₹ 100 = ₹ 20

Loss per cent = $\frac{\text{Loss}}{\text{CP}} \times 100$

$= \frac{20}{120} \times 100$

$= \frac{50}{3} = 16\frac{2}{3}$

On ₹ 120, the loss is ₹ 20
So on ₹ 100, the loss

$= \frac{20}{120} \times 100 = \frac{50}{3} = 16\frac{2}{3}$

Thus, loss per cent is $16\frac{2}{3}$

Try the last case.

Now we see that given any two out of the three quantities related to prices that is, CP, SP, amount of Profit or Loss or their percentage, we can find the rest.

EXAMPLE 12 The cost of a flower vase is ₹ 120. If the shopkeeper sells it at a loss of 10%, find the price at which it is sold.

SOLUTION We are given that CP = ₹ 120 and Loss per cent = 10. We have to find the SP.

Sohan does it like this

Loss of 10% means if CP is ₹ 100, Loss is ₹ 10

Therefore, SP would be

₹ (100 – 10) = ₹ 90

When CP is ₹ 100, SP is ₹ 90.
Therefore, if CP were ₹ 120 then

SP = $\frac{90}{100} \times 120$ = ₹ 108

Anandi does it like this

Loss is 10% of the cost price
= 10% of ₹ 120

$= \frac{10}{100} \times 120$ = ₹ 12

Therefore

SP = CP – Loss
= ₹ 120 – ₹ 12 = ₹ 108

Thus, by both methods we get the SP as ₹ 108.

EXAMPLE 13 Selling price of a toy car is ₹ 540. If the profit made by shopkeeper is 20%, what is the cost price of this toy?

SOLUTION We are given that SP = ₹ 540 and the Profit = 20%. We need to find the CP.

Amina does it like this

20% profit will mean if CP is ₹ 100, profit is ₹ 20

Therefore, SP = 100 + 20 = 120

Now, when SP is ₹ 120,

then CP is ₹ 100.

Therefore, when SP is ₹ 540,

then CP = $\frac{100}{120} \times 540$ = ₹ 450

Arun does it like this

Profit = 20% of CP and SP = CP + Profit

So, 540 = CP + 20% of CP

$= \text{CP} + \frac{20}{100} \times \text{CP} = \left[1 + \frac{1}{5}\right]\text{CP}$

$= \frac{6}{5}\text{CP}$. Therefore, $540 \times \frac{5}{6} = \text{CP}$

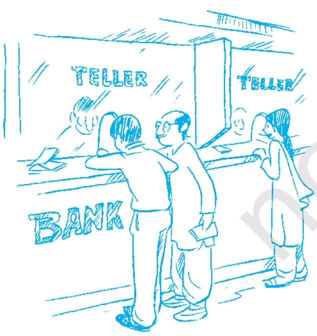
or ₹ 450 = CP

Thus, by both methods, the cost price is ₹ 450.

TRY THESE

1. A shopkeeper bought a chair for ₹ 375 and sold it for ₹ 400. Find the gain Percentage.
2. Cost of an item is ₹ 50. It was sold with a profit of 12%. Find the selling price.
3. An article was sold for ₹ 250 with a profit of 5%. What was its cost price?
4. An item was sold for ₹ 540 at a loss of 5%. What was its cost price?

7.4 CHARGE GIVEN ON BORROWED MONEY OR SIMPLE INTEREST



Sohini said that they were going to buy a new scooter. Mohan asked her whether they had the money to buy it. Sohini said her father was going to take a loan from a bank. The money you borrow is known as **sum** borrowed or **principal**.

This money would be used by the borrower for some time before it is returned. For keeping this money for some time the borrower has to pay some extra money to the bank. This is known as **Interest.**

You can find the amount you have to pay at the end of the year by adding the sum borrowed and the interest. That is, **Amount = Principal + Interest.**

Interest is generally given in per cent for a period of one year. It is written as say 10% per year or per annum or in short as 10% p.a. (per annum).

10% p.a. means on every ₹ 100 borrowed, ₹ 10 is the interest you have to pay for one year. Let us take an example and see how this works.

EXAMPLE 14 Anita takes a loan of ₹ 5,000 at 15% per year as rate of interest. Find the interest she has to pay at the end of one year.

SOLUTION The sum borrowed = ₹ 5,000, Rate of interest = 15% per year.

This means if ₹ 100 is borrowed, she has to pay ₹ 15 as interest for one year. If she has borrowed ₹ 5,000, then the interest she has to pay for one year

$$= ₹\ \frac{15}{100} \times 5000 = ₹\ 750$$

So, at the end of the year she has to give an amount of ₹ 5,000 + ₹ 750 = ₹ 5,750.

We can write a general relation to find interest for one year. Take P as the principal or sum and R % as Rate per cent per annum.

Now on every ₹ 100 borrowed, the interest paid is ₹ R

Therefore, on ₹ P borrowed, the interest paid for one year would be $\frac{R \times P}{100} = \frac{P \times R}{100}$.

7.4.1 Interest for Multiple Years

If the amount is borrowed for more than one year the interest is calculated for the period the money is kept for. For example, if Anita returns the money at the end of two years and the rate of interest is the same then she would have to pay twice the interest i.e., ₹ 750 for the first year and ₹ 750 for the second. This way of calculating interest where principal is not changed is known as **simple interest**. As the number of years increase the interest also increases. For ₹ 100 borrowed for 3 years at 18%, the interest to be paid at the end of 3 years is 18 + 18 + 18 = 3 × 18 = ₹ 54.

We can find the general form for simple interest for more than one year.

We know that on a principal of ₹ P at R% rate of interest per year, the interest paid for one year is $\frac{R \times P}{100}$. Therefore, interest I paid for T years would be

$$\frac{T \times R \times P}{100} = \frac{P \times R \times T}{100} \text{ or } \frac{PRT}{100}$$

And amount you have to pay at the end of T years is $A = P + I$

TRY THESE

1. ₹ 10,000 is invested at 5% interest rate p.a. Find the interest at the end of one year.
2. ₹ 3,500 is given at 7% p.a. rate of interest. Find the interest which will be received at the end of two years.
3. ₹ 6,050 is borrowed at 6.5% rate of interest p.a.. Find the interest and the amount to be paid at the end of 3 years.
4. ₹ 7,000 is borrowed at 3.5% rate of interest p.a. borrowed for 2 years. Find the amount to be paid at the end of the second year.

Just as in the case of prices related to items, if you are given any two of the three quantities in the relation $I = \frac{P \times T \times R}{100}$, you could find the remaining quantity.

EXAMPLE 15 If Manohar pays an interest of ₹ 750 for 2 years on a sum of ₹ 4,500, find the rate of interest.

Solution 1

$$I = \frac{P \times T \times R}{100}$$

Therefore, $750 = \frac{4500 \times 2 \times R}{100}$

or $\frac{750}{45 \times 2} = R$

Therefore, Rate $= 8\frac{1}{3}\%$

Solution 2

For 2 years, interest paid is ₹ 750

Therefore, for 1 year, interest paid ₹ $\frac{750}{2}$ = ₹ 375

On ₹ 4,500, interest paid is ₹ 375

Therefore, on ₹ 100, rate of interest paid

$$= \frac{375 \times 100}{4500} = 8\frac{1}{3}\%$$

TRY THESE

1. You have ₹ 2,400 in your account and the interest rate is 5%. After how many years would you earn ₹ 240 as interest.
2. On a certain sum the interest paid after 3 years is ₹ 450 at 5% rate of interest per annum. Find the sum.

EXERCISE 7.2

1. Tell what is the profit or loss in the following transactions. Also find profit per cent or loss per cent in each case.
   (a) Gardening shears bought for ₹ 250 and sold for ₹ 325.
   (b) A refrigerater bought for ₹ 12,000 and sold at ₹ 13,500.
   (c) A cupboard bought for ₹ 2,500 and sold at ₹ 3,000.
   (d) A skirt bought for ₹ 250 and sold at ₹ 150.
2. Convert each part of the ratio to percentage:
   (a) 3 : 1 (b) 2 : 3 : 5 (c) 1:4 (d) 1 : 2 : 5
3. The population of a city decreased from 25,000 to 24,500. Find the percentage decrease.
4. Arun bought a car for ₹ 3,50,000. The next year, the price went upto ₹ 3,70,000. What was the Percentage of price increase?
5. I buy a T.V. for ₹ 10,000 and sell it at a profit of 20%. How much money do I get for it?
6. Juhi sells a washing machine for ₹ 13,500. She loses 20% in the bargain. What was the price at which she bought it?
7. (i) Chalk contains calcium, carbon and oxygen in the ratio 10:3:12. Find the percentage of carbon in chalk.
   (ii) If in a stick of chalk, carbon is 3g, what is the weight of the chalk stick?

8. Amina buys a book for ₹ 275 and sells it at a loss of 15%. How much does she sell it for?

9. Find the amount to be paid at the end of 3 years in each case:

(a) Principal = ₹ 1,200 at 12% p.a. (b) Principal = ₹ 7,500 at 5% p.a.

10. What rate gives ₹ 280 as interest on a sum of ₹ 56,000 in 2 years?

11. If Meena gives an interest of ₹ 45 for one year at 9% rate p.a.. What is the sum she has borrowed?

What have We Discussed?

1. A way of comparing quantities is percentage. Percentages are numerators of fractions with denominator 100. Per cent means per hundred.

For example 82% marks means 82 marks out of hundred.

2. Fractions can be converted to percentages and vice-versa.

For example, $\frac{1}{4} = \frac{1}{4} \times 100\,\%$ whereas, $75\% = \frac{75}{100} = \frac{3}{4}$

3. Decimals too can be converted to percentages and vice-versa.

For example, 0.25 = 0.25 × 100% = = 25%

4. Percentages are widely used in our daily life,

(a) We have learnt to find exact number when a certain per cent of the total quantity is given.

(b) When parts of a quantity are given to us as ratios, we have seen how to convert them to percentages.

(c) The increase or decrease in a certain quantity can also be expressed as percentage.

(d) The profit or loss incurred in a certain transaction can be expressed in terms of percentages.

(e) While computing interest on an amount borrowed, the rate of interest is given in terms of per cents. For example, ₹ 800 borrowed for 3 years at 12% per annum.

Chapter 8

Rational Numbers

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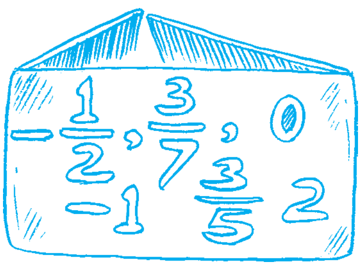
8.1 Introduction



You began your study of numbers by counting objects around you. The numbers used for this purpose were called counting numbers or natural numbers. They are 1, 2, 3, 4, ... By including 0 to natural numbers, we got the whole numbers, i.e., 0, 1, 2, 3, ... The negatives of natural numbers were then put together with whole numbers to make up integers. Integers are ..., –3, –2, –1, 0, 1, 2, 3, ... We, thus, extended the number system, from natural numbers to whole numbers and from whole numbers to integers.

You were also introduced to fractions. These are numbers of the form $\frac{\text{numerator}}{\text{denominator}}$, where the numerator is either 0 or a positive integer and the denominator, a positive integer. You compared two fractions, found their equivalent forms and studied all the four basic operations of addition, subtraction, multiplication and division on them.

In this Chapter, we shall extend the number system further. We shall introduce the concept of rational numbers alongwith their addition, subtraction, multiplication and division operations.

8.2 Need for Rational Numbers

Earlier, we have seen how integers could be used to denote opposite situations involving numbers. For example, if the distance of 3 km to the right of a place was denoted by 3, then the distance of 5 km to the left of the same place could be denoted by –5. If a profit of ₹ 150 was represented by 150 then a loss of ₹ 100 could be written as –100.

There are many situations similar to the above situations that involve fractional numbers. You can represent a distance of 750m above sea level as $\frac{3}{4}$ km. Can we represent 750m below sea level in km? Can we denote the distance of $\frac{3}{4}$ km below sea level by $\frac{-3}{4}$? We can see $\frac{-3}{4}$ is neither an integer, nor a fractional number. We need to extend our number system to include such numbers.

8.3 What are Rational Numbers?

The word 'rational' arises from the term 'ratio'. You know that a ratio like 3:2 can also be written as $\frac{3}{2}$. Here, 3 and 2 are natural numbers.

Similarly, the ratio of two integers p and q $(q \neq 0)$, i.e., $p:q$ can be written in the form $\frac{p}{q}$. This is the form in which rational numbers are expressed.

A rational number is defined as a number that can be expressed in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

Thus, $\frac{4}{5}$ is a rational number. Here, $p = 4$ and $q = 5$.

Is $\frac{-3}{4}$ also a rational number? Yes, because $p = -3$ and $q = 4$ are integers.

- You have seen many fractions like $\frac{3}{8}, \frac{4}{8}, 1\frac{2}{3}$ etc. All fractions are rational numbers. Can you say why?

How about the decimal numbers like 0.5, 2.3, etc.? Each of such numbers can be written as an ordinary fraction and, hence, are rational numbers. For example, $0.5 = \frac{5}{10}$, $0.333 = \frac{333}{1000}$ etc.

Try These

1. Is the number $\frac{2}{-3}$ rational? Think about it.
2. List ten rational numbers.

Numerator and Denominator

In $\frac{p}{q}$, the integer p is the numerator, and the integer q $(\neq 0)$ is the denominator.

Thus, in $\frac{-3}{7}$, the numerator is -3 and the denominator is 7.

Mention five rational numbers each of whose

(a) Numerator is a negative integer and denominator is a positive integer.
(b) Numerator is a positive integer and denominator is a negative integer.
(c) Numerator and denominator both are negative integers.
(d) Numerator and denominator both are positive integers.

- Are integers also rational numbers?

Any integer can be thought of as a rational number. For example, the integer -5 is a rational number, because you can write it as $\frac{-5}{1}$. The integer 0 can also be written as $0 = \frac{0}{2}$ or $\frac{0}{7}$ etc. Hence, it is also a rational number.

Thus, rational numbers include integers and fractions.

Equivalent rational numbers

A rational number can be written with different numerators and denominators. For example, consider the rational number $\frac{-2}{3}$.

$$\frac{-2}{3} = \frac{-2\times 2}{3\times 2} = \frac{-4}{6}. \text{ We see that } \frac{-2}{3} \text{ is the same as } \frac{-4}{6}.$$

Also, $$\frac{-2}{3} = \frac{(-2)\times(-5)}{3\times(-5)} = \frac{10}{-15}. \text{ So, } \frac{-2}{3} \text{ is also the same as } \frac{10}{-15}.$$

Thus, $\frac{-2}{3} = \frac{-4}{6} = \frac{10}{-15}$. Such rational numbers that are equal to each other are said to be equivalent to each other.

Again, $\frac{10}{-15} = \frac{-10}{15}$ (How?)

By multiplying the numerator and denominator of a rational number by the same non zero integer, we obtain another rational number equivalent to the given rational number. This is exactly like obtaining equivalent fractions.

Just as multiplication, the division of the numerator and denominator by the same non zero integer, also gives equivalent rational numbers. For example,

$$\frac{10}{-15} = \frac{10\div(-5)}{-15\div(-5)} = \frac{-2}{3}, \quad \frac{-12}{24} = \frac{-12\div 12}{24\div 12} = \frac{-1}{2}$$

> We write $\frac{-2}{3}$ as $-\frac{2}{3}$, $\frac{-10}{15}$ as $-\frac{10}{15}$, etc.

TRY THESE

Fill in the boxes:

(i) $\frac{5}{4} = \frac{\square}{16} = \frac{25}{\square} = \frac{-15}{\square}$

(ii) $\frac{-3}{7} = \frac{\square}{14} = \frac{9}{\square} = \frac{-6}{\square}$

8.4 Positive and Negative Rational Numbers

Consider the rational number $\frac{2}{3}$. Both the numerator and denominator of this number are positive integers. Such a rational number is called a **positive rational number**. So, $\frac{3}{8}, \frac{5}{7}, \frac{2}{9}$ etc. are positive rational numbers.

TRY THESE

1. Is 5 a positive rational number?
2. List five more positive rational numbers.

The numerator of $\frac{-3}{5}$ is a negative integer, whereas the denominator is a positive integer. Such a rational number is called a **negative rational number**. So, $\frac{-5}{7}, \frac{-3}{8}, \frac{-9}{5}$ etc. are negative rational numbers.

- Is $\frac{8}{-3}$ a negative rational number? We know that $\frac{8}{-3} = \frac{8 \times -1}{-3 \times -1} = \frac{-8}{3}$, and $\frac{-8}{3}$ is a negative rational number. So, $\frac{8}{-3}$ is a negative rational number.

  Similarly, $\frac{5}{-7}, \frac{6}{-5}, \frac{2}{-9}$ etc. are all negative rational numbers. Note that their numerators are positive and their denominators negative.

Try These

1. Is – 8 a negative rational number?
2. List five more negative rational numbers.

- The number 0 is neither a positive nor a negative rational number.
- What about $\frac{-3}{-5}$?

  You will see that $\frac{-3}{-5} = \frac{-3 \times (-1)}{-5 \times (-1)} = \frac{3}{5}$. So, $\frac{-3}{-5}$ is a positive rational number.

  Thus, $\frac{-2}{-5}, \frac{-5}{-3}$ etc. are positive rational numbers.

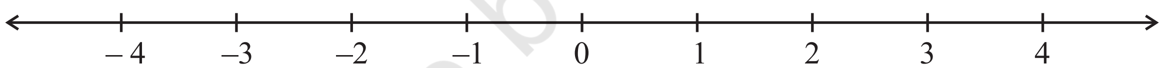
Try These

Which of these are negative rational numbers?

(i) $\frac{-2}{3}$ (ii) $\frac{5}{7}$ (iii) $\frac{3}{-5}$ (iv) 0 (v) $\frac{6}{11}$ (vi) $\frac{-2}{-9}$

8.5 Rational Numbers on a Number Line

You know how to represent integers on a number line. Let us draw one such number line.



The points to the right of 0 are denoted by + sign and are positive integers. The points to the left of 0 are denoted by – sign and are negative integers.

Representation of fractions on a number line is also known to you.

Let us see how the rational numbers can be represented on a number line.

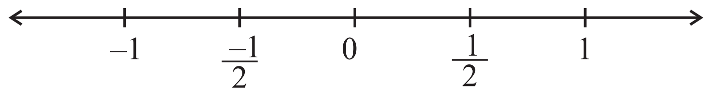
Let us represent the number $-\frac{1}{2}$ on the number line.

As done in the case of positive integers, the positive rational numbers would be marked on the right of 0 and the negative rational numbers would be marked on the left of 0.

To which side of 0 will you mark $-\frac{1}{2}$? Being a negative rational number, it would be marked to the left of 0.

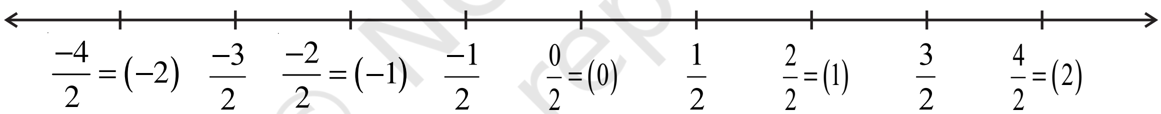
You know that while marking integers on the number line, successive integers are marked at equal intervels. Also, from 0, the pair 1 and –1 is equidistant. So are the pairs 2 and –2, 3 and –3.

In the same way, the rational numbers $\frac{1}{2}$ and $-\frac{1}{2}$ would be at equal distance from 0. We know how to mark the rational number $\frac{1}{2}$. It is marked at a point which is half the distance between 0 and 1. So, $-\frac{1}{2}$ would be marked at a point half the distance between 0 and –1.



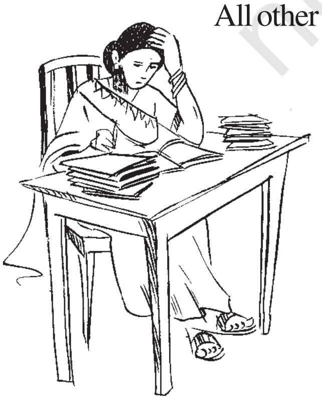
We know how to mark $\frac{3}{2}$ on the number line. It is marked on the right of 0 and lies halfway between 1 and 2. Let us now mark $\frac{-3}{2}$ on the number line. It lies on the left of 0 and is at the same distance as $\frac{3}{2}$ from 0.

In decreasing order, we have, $\frac{-1}{2}, \frac{-2}{2}(=-1), \frac{-3}{2}, \frac{-4}{2}(=-2)$. This shows that $\frac{-3}{2}$ lies between – 1 and – 2. Thus, $\frac{-3}{2}$ lies halfway between – 1 and – 2.



Mark $\frac{-5}{2}$ and $\frac{-7}{2}$ in a similar way.

Similarly, $-\frac{1}{3}$ is to the left of zero and at the same distance from zero as $\frac{1}{3}$ is to the right. So as done above, $-\frac{1}{3}$ can be represented on the number line. Once we know how to represent $-\frac{1}{3}$ on the number line, we can go on representing $-\frac{2}{3}, -\frac{4}{3}, -\frac{5}{3}$ and so on. All other rational numbers with different denominators can be represented in a similar way.



8.6 Rational Numbers in Standard Form

Observe the rational numbers $\frac{3}{5}, \frac{-5}{8}, \frac{2}{7}, \frac{-7}{11}$.

The denominators of these rational numbers are positive integers and 1 is the only common factor between the numerators and denominators. Further, the negative sign occurs only in the numerator.

Such rational numbers are said to be in **standard form**.

A rational number is said to be in the standard form if its denominator is a positive integer and the numerator and denominator have no common factor other than 1.

If a rational number is not in the standard form, then it can be reduced to the standard form.

Recall that for reducing fractions to their lowest forms, we divided the numerator and the denominator of the fraction by the same non zero positive integer. We shall use the same method for reducing rational numbers to their standard form.

EXAMPLE 1 Reduce $\frac{-45}{30}$ to the standard form.

SOLUTION We have, $\frac{-45}{30} = \frac{-45 \div 3}{30 \div 3} = \frac{-15}{10} = \frac{-15 \div 5}{10 \div 5} = \frac{-3}{2}$

We had to divide twice. First time by 3 and then by 5. This could also be done as

$$\frac{-45}{30} = \frac{-45 \div 15}{30 \div 15} = \frac{-3}{2}$$

In this example, note that 15 is the HCF of 45 and 30.

Thus, to reduce the rational number to its standard form, we divide its numerator and denominator by their HCF ignoring the negative sign, if any. (The reason for ignoring the negative sign will be studied in Higher Classes)

If there is negative sign in the denominator, divide by '– HCF'.

EXAMPLE 2 Reduce to standard form:

(i) $\frac{36}{-24}$ (ii) $\frac{-3}{-15}$

SOLUTION

(i) The HCF of 36 and 24 is 12.

Thus, its standard form would be obtained by dividing by –12.

$$\frac{36}{-24} = \frac{36 \div (-12)}{-24 \div (-12)} = \frac{-3}{2}$$

(ii) The HCF of 3 and 15 is 3.

Thus, $\frac{-3}{-15} = \frac{-3 \div (-2)}{-15 \div (-3)} = \frac{1}{5}$

TRY THESE

Find the standard form of (i) $\frac{-18}{45}$ (ii) $\frac{-12}{18}$

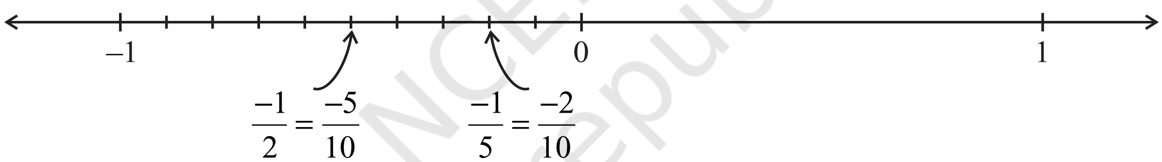
8.7 Comparison of Rational Numbers

We know how to compare two integers or two fractions and tell which is smaller or which is greater among them. Let us now see how we can compare two rational numbers.

- Two positive rational numbers, like $\frac{2}{3}$ and $\frac{5}{7}$ can be compared as studied earlier in the case of fractions.
- Mary compared two negative rational numbers $-\frac{1}{2}$ and $-\frac{1}{5}$ using number line. She knew that the integer which was on the right side of the other integer, was the greater integer.

For example, 5 is to the right of 2 on the number line and $5 > 2$. The integer -2 is on the right of -5 on the number line and $-2 > -5$.

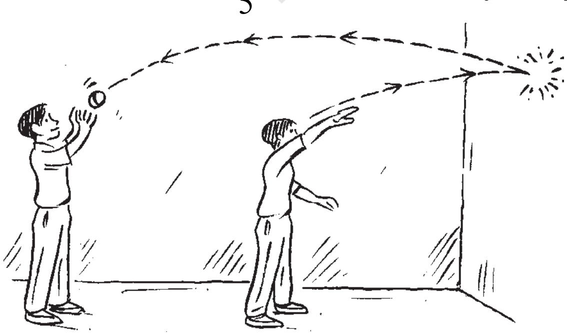
She used this method for rational numbers also. She knew how to mark rational numbers on the number line. She marked $-\frac{1}{2}$ and $-\frac{1}{5}$ as follows:



Has she correctly marked the two points? How and why did she convert $-\frac{1}{2}$ to $-\frac{5}{10}$ and $-\frac{1}{5}$ to $-\frac{2}{10}$? She found that $-\frac{1}{5}$ is to the right of $-\frac{1}{2}$. Thus, $-\frac{1}{5} > -\frac{1}{2}$ or $-\frac{1}{2} < -\frac{1}{5}$.

Can you compare $-\frac{3}{4}$ and $-\frac{2}{3}$? $-\frac{1}{3}$ and $-\frac{1}{5}$?

We know from our study of fractions that $\frac{1}{5} < \frac{1}{2}$. And what did Mary get for $-\frac{1}{2}$ and $-\frac{1}{5}$? Was it not exactly the opposite?



You will find that, $\frac{1}{2} > \frac{1}{5}$ but $-\frac{1}{2} < -\frac{1}{5}$.

Do you observe the same for $-\frac{3}{4}$, $-\frac{2}{3}$ and $-\frac{1}{3}$, $-\frac{1}{5}$?

Mary remembered that in integers she had studied $4 > 3$ but $-4 < -3$, $5 > 2$ but $-5 < -2$ etc.

- The case of pairs of negative rational numbers is similar. *To compare two negative rational numbers, we compare them ignoring their negative signs and then reverse the order.*

For example, to compare $-\frac{7}{5}$ and $-\frac{5}{3}$, we first compare $\frac{7}{5}$ and $\frac{5}{3}$.

We get $\frac{7}{5} < \frac{5}{3}$ and conclude that $\frac{-7}{5} > \frac{-5}{3}$.

Take five more such pairs and compare them.

Which is greater $-\frac{3}{8}$ or $-\frac{2}{7}$?; $-\frac{4}{3}$ or $-\frac{3}{2}$?

- Comparison of a negative and a positive rational number is obvious. A negative rational number is to the left of zero whereas a positive rational number is to the right of zero on a number line. So, a negative rational number will always be less than a positive rational number.

Thus, $-\frac{2}{7} < \frac{1}{2}$.

- To compare rational numbers $\frac{-3}{-5}$ and $\frac{-2}{-7}$ reduce them to their standard forms and then compare them.

Example 3 Do $\frac{4}{-9}$ and $\frac{-16}{36}$ represent the same rational number?

Solution Yes, because $\frac{4}{-9} = \frac{4 \times (-4)}{9 \times (-4)} = \frac{-16}{36}$ or $\frac{-16}{36} = \frac{-16 \div -4}{35 \div -4} = \frac{-4}{-9}$.

8.8 Rational Numbers Between Two Rational Numbers

Reshma wanted to count the whole numbers between 3 and 10. From her earlier classes, she knew there would be exactly 6 whole numbers between 3 and 10. Similarly, she wanted to know the total number of integers between –3 and 3. The integers between –3 and 3 are –2, –1, 0, 1, 2. Thus, there are exactly 5 integers between –3 and 3.

Are there any integers between –3 and –2? No, there is no integer between –3 and –2. Between two successive integers the number of integers is 0.

Thus, we find that number of integers between two integers are limited (finite). Will the same happen in the case of rational numbers also?

Reshma took two rational numbers $\frac{-3}{5}$ and $\frac{-1}{3}$.

She converted them to rational numbers with same denominators.

So $\quad \frac{-3}{5} = \frac{-9}{15}$ and $\frac{-1}{3} = \frac{-5}{15}$

We have $\quad \frac{-9}{15} < \frac{-8}{15} < \frac{-7}{15} < \frac{-6}{15} < \frac{-5}{15}$ or $\frac{-3}{5} < \frac{-8}{15} < \frac{-7}{15} < \frac{-6}{15} < \frac{-1}{3}$

She could find rational numbers $\frac{-8}{15} < \frac{-7}{15} < \frac{-6}{15}$ between $\frac{-3}{5}$ and $\frac{-1}{3}$.

Are the numbers $\frac{-8}{15}, \frac{-7}{15}, \frac{-6}{15}$ the only rational numbers between $-\frac{3}{5}$ and $-\frac{1}{3}$?

We have $\quad \frac{-3}{5} < \frac{-18}{30}$ and $\frac{-8}{15} < \frac{-16}{30}$

And $\quad \frac{-18}{30} < \frac{-17}{30} < \frac{-16}{30}$. i.e., $\frac{-3}{5} < \frac{-17}{30} < \frac{-8}{15}$

Hence $\quad \frac{-3}{5} < \frac{-17}{30} < \frac{-8}{15} < \frac{-7}{15} < \frac{-6}{15} < \frac{-1}{3}$

So, we could find one more rational number between $\frac{-3}{5}$ and $\frac{-1}{3}$.

By using this method, you can insert as many rational numbers as you want between two different rational numbers.

For example, $\quad \frac{-3}{5} = \frac{-3 \times 30}{5 \times 30} = \frac{-90}{150}$ and $\frac{-1}{3} = \frac{-1 \times 50}{3 \times 50} = \frac{-50}{150}$

We get 39 rational numbers $\left(\frac{-89}{150}, \ldots, \frac{-51}{150}\right)$ between $\frac{-90}{150}$ and $\frac{-50}{150}$

i.e., between $\frac{-3}{5}$ and $\frac{-1}{3}$. You will find that the list is unending.

Can you list five rational numbers between $\frac{-5}{3}$ and $\frac{-8}{7}$?

We can find unlimited number of rational numbers between any two rational numbers.

TRY THESE

Find five rational numbers between $\frac{-5}{7}$ and $\frac{-3}{8}$.

EXAMPLE 4 List three rational numbers between – 2 and – 1.

SOLUTION Let us write –1 and –2 as rational numbers with denominator 5. (Why?)

We have, $-1 = \frac{-5}{5}$ and $-2 = \frac{-10}{5}$

So, $$\frac{-10}{5} < \frac{-9}{5} < \frac{-8}{5} < \frac{-7}{5} < \frac{-6}{5} < \frac{-5}{5} \text{ or } -2 < \frac{-9}{5} < \frac{-8}{5} < \frac{-7}{5} < \frac{-6}{5} < -1$$

The three rational numbers between –2 and –1 would be, $\frac{-9}{5}, \frac{-8}{5}, \frac{-7}{5}$

(You can take any three of $\frac{-9}{5}, \frac{-8}{5}, \frac{-7}{5}, \frac{-6}{5}$)

EXAMPLE 5 Write four more numbers in the following pattern:

$$\frac{-1}{3}, \frac{-2}{6}, \frac{-3}{9}, \frac{-4}{12}, \ldots$$

SOLUTION We have,

$$\frac{-2}{6} = \frac{-1 \times 2}{3 \times 2}, \frac{-3}{9} = \frac{-1 \times 3}{3 \times 3}, \frac{-4}{12} = \frac{-1 \times 4}{3 \times 4}$$

or $$\frac{-1 \times 1}{3 \times 1} = \frac{-1}{3}, \frac{-1 \times 2}{3 \times 2} = \frac{-2}{6}, \frac{-1 \times 3}{3 \times 3} = \frac{-3}{9}, \frac{-1 \times 4}{3 \times 4} = \frac{-4}{12}$$

Thus, we observe a pattern in these numbers.

The other numbers would be $\frac{-1 \times 5}{3 \times 5} = \frac{-5}{15}, \frac{-1 \times 6}{3 \times 6} = \frac{-6}{18}, \frac{-1 \times 7}{3 \times 7} = \frac{-7}{21}$.

EXERCISE 8.1

1. List five rational numbers between:

   (i) –1 and 0 (ii) –2 and –1 (iii) $\frac{-4}{5}$ and $\frac{-2}{3}$ (iv) $-\frac{1}{2}$ and $\frac{2}{3}$

2. Write four more rational numbers in each of the following patterns:

   (i) $\frac{-3}{5}, \frac{-6}{10}, \frac{-9}{15}, \frac{-12}{20}, \ldots\ldots$ (ii) $\frac{-1}{4}, \frac{-2}{8}, \frac{-3}{12}, \ldots\ldots$

(iii) $\frac{-1}{6}, \frac{2}{-12}, \frac{3}{-18}, \frac{4}{-24}, \ldots$ (iv) $\frac{-2}{3}, \frac{2}{-3}, \frac{4}{-6}, \frac{6}{-9}, \ldots$

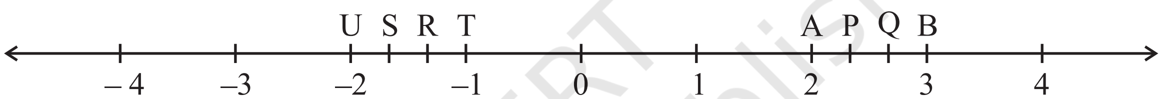
3. Give four rational numbers equivalent to:

(i) $\frac{-2}{7}$ (ii) $\frac{5}{-3}$ (iii) $\frac{4}{9}$

4. Draw the number line and represent the following rational numbers on it:

(i) $\frac{3}{4}$ (ii) $\frac{-5}{8}$ (iii) $\frac{-7}{4}$ (iv) $\frac{7}{8}$

5. The points P, Q, R, S, T, U, A and B on the number line are such that, TR = RS = SU and AP = PQ = QB. Name the rational numbers represented by P, Q, R and S.



6. Which of the following pairs represent the same rational number?

(i) $\frac{-7}{21}$ and $\frac{3}{9}$ (ii) $\frac{-16}{20}$ and $\frac{20}{-25}$ (iii) $\frac{-2}{-3}$ and $\frac{2}{3}$

(iv) $\frac{-3}{5}$ and $\frac{-12}{20}$ (v) $\frac{8}{-5}$ and $\frac{-24}{15}$ (vi) $\frac{1}{3}$ and $\frac{-1}{9}$

(vii) $\frac{-5}{-9}$ and $\frac{5}{-9}$

7. Rewrite the following rational numbers in the simplest form:

(i) $\frac{-8}{6}$ (ii) $\frac{25}{45}$ (iii) $\frac{-44}{72}$ (iv) $\frac{-8}{10}$

8. Fill in the boxes with the correct symbol out of >, <, and =.

(i) $\frac{-5}{7} \boxed{} \frac{2}{3}$ (ii) $\frac{-4}{5} \boxed{} \frac{-5}{7}$ (iii) $\frac{-7}{8} \boxed{} \frac{14}{-16}$

(iv) $\frac{-8}{5} \boxed{} \frac{-7}{4}$ (v) $\frac{1}{-3} \boxed{} \frac{-1}{4}$ (vi) $\frac{5}{-11} \boxed{} \frac{-5}{11}$

(vii) $0 \boxed{} \frac{-7}{6}$

9. Which is greater in each of the following:

(i) $\frac{2}{3}, \frac{5}{2}$ (ii) $\frac{-5}{6}, \frac{-4}{3}$ (iii) $\frac{-3}{4}, \frac{2}{-3}$

(iv) $\frac{-1}{4}, \frac{1}{4}$ (v) $-3\frac{2}{7}, -3\frac{4}{5}$

10. Write the following rational numbers in ascending order:

(i) $\frac{-3}{5}, \frac{-2}{5}, \frac{-1}{5}$ (ii) $\frac{-1}{3}, \frac{-2}{9}, \frac{-4}{3}$ (iii) $\frac{-3}{7}, \frac{-3}{2}, \frac{-3}{4}$

8.9 Operations on Rational Numbers

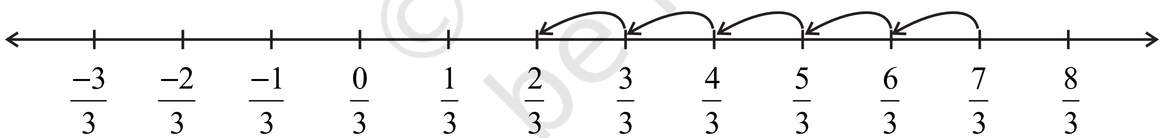
You know how to add, subtract, multiply and divide integers as well as fractions. Let us now study these basic operations on rational numbers.

8.9.1 Addition

- Let us add two rational numbers with same denominators, say $\frac{7}{3}$ and $\frac{-5}{3}$.

We find $\frac{7}{3} + \left(\frac{-5}{3}\right)$

On the number line, we have:



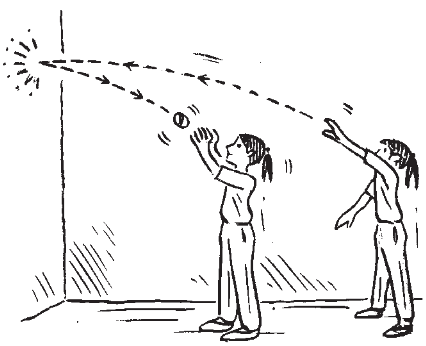
The distance between two consecutive points is $\frac{1}{3}$. So adding $\frac{-5}{3}$ to $\frac{7}{3}$ will mean, moving to the left of $\frac{7}{3}$, making 5 jumps. Where do we reach? We reach at $\frac{2}{3}$.

So, $\frac{7}{3} + \left(\frac{-5}{3}\right) = \frac{2}{3}$.

Let us now try this way:

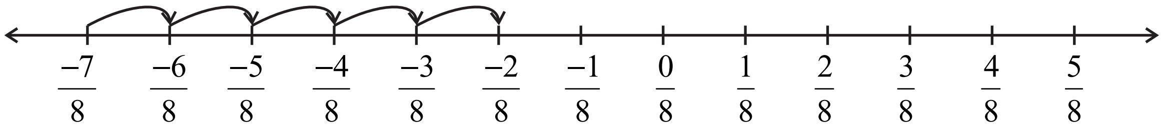
$$\frac{7}{3} + \frac{(-5)}{3} = \frac{7 + (-5)}{3} = \frac{2}{3}$$

We get the same answer.



Find $\frac{6}{5} + \frac{(-2)}{5}$, $\frac{3}{7} + \frac{(-5)}{7}$ in both ways and check if you get the same answers.

Similarly, $\frac{-7}{8}+\frac{5}{8}$ would be



What do you get?

Also, $\frac{-7}{8}+\frac{5}{8}=\frac{-7+5}{8}=?$ Are the two values same?

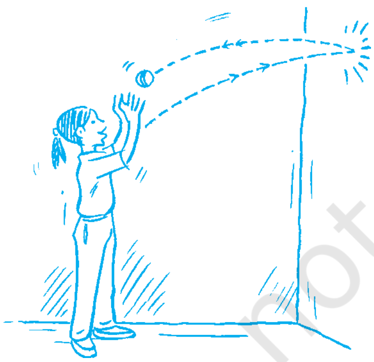
Try These

Find: $\frac{-13}{7}+\frac{6}{7}$, $\frac{19}{5}+\left(\frac{-7}{5}\right)$

So, we find that *while adding rational numbers with same denominators, we add the numerators keeping the denominators same.*

Thus, $\frac{-11}{5}+\frac{7}{5}=\frac{-11+7}{5}=\frac{-4}{5}$

- How do we add rational numbers with different denominators? As in the case of fractions, we first find the LCM of the two denominators. Then, we find the equivalent rational numbers of the given rational numbers with this LCM as the denominator. Then, add the two rational numbers.



For example, let us add $\frac{-7}{5}$ and $\frac{-2}{3}$.

LCM of 5 and 3 is 15.

So, $\frac{-7}{5}=\frac{-21}{15}$ and $\frac{-2}{3}=\frac{-10}{15}$

Thus, $\frac{-7}{5}+\frac{(-2)}{3}=\frac{-21}{15}+\frac{(-10)}{15}=\frac{-31}{15}$

Try These

Find:

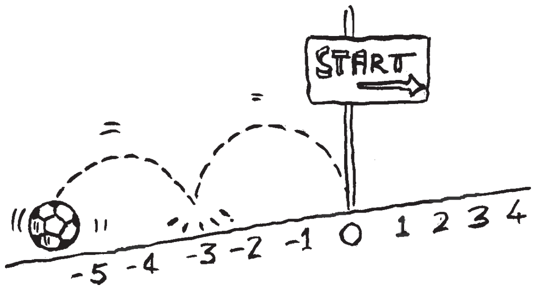
(i) $\frac{-3}{7}+\frac{2}{3}$

(ii) $\frac{-5}{6}+\frac{-3}{11}$

Additive Inverse

What will be $\frac{-4}{7}+\frac{4}{7}=?$

$\frac{-4}{7}+\frac{4}{7}=\frac{-4+4}{7}=0$. Also, $\frac{4}{7}+\left(\frac{-4}{7}\right)=0$.



Similarly, $\frac{-2}{3}+\frac{2}{3}=0=\frac{2}{3}+\left(\frac{-2}{3}\right)$.

In the case of integers, we call – 2 as the additive inverse of 2 and 2 as the additive inverse of – 2.

For rational numbers also, we call $\frac{-4}{7}$ as the **additive inverse** of $\frac{4}{7}$ and $\frac{4}{7}$ as the additive inverse of $\frac{-4}{7}$. Similarly, $\frac{-2}{3}$ is the additive inverse of $\frac{2}{3}$ and $\frac{2}{3}$ is the additive inverse of $\frac{-2}{3}$.

TRY THESE

What will be the additive inverse of $\frac{-3}{9}$?, $\frac{-9}{11}$?, $\frac{5}{7}$?

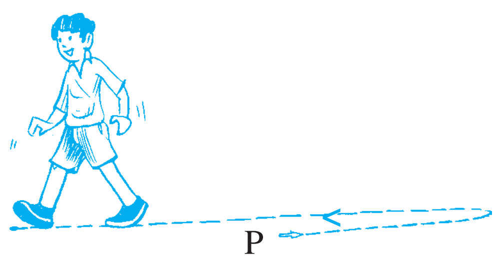
EXAMPLE 6 Satpal walks $\frac{2}{3}$ km from a place P, towards east and then from there $1\frac{5}{7}$ km towards west. Where will he be now from P?

SOLUTION Let us denote the distance travelled towards east by positive sign. So, the distances towards west would be denoted by negative sign.

Thus, distance of Satpal from the point P would be

$$\frac{2}{3}+\left(-1\frac{5}{7}\right)=\frac{2}{3}+\frac{(-12)}{7}=\frac{2\times 7}{3\times 7}+\frac{(-12)\times 3}{7\times 3}$$

$$=\frac{14-36}{21}=\frac{-22}{21}=-1\frac{1}{21}$$



Since it is negative, it means Satpal is at a distance $1\frac{1}{21}$ km towards west of P.

8.9.2 Subtraction

Savita found the difference of two rational numbers $\frac{5}{7}$ and $\frac{3}{8}$ in this way:

$$\frac{5}{7}-\frac{3}{8}=\frac{40-21}{56}=\frac{19}{56}$$

Farida knew that for two integers a and b she could write $a-b=a+(-b)$

She tried this for rational numbers also and found, $\frac{5}{7}-\frac{3}{8}=\frac{5}{7}+\frac{(-3)}{8}=\frac{19}{56}$.

Both obtained the same difference.

Try to find $\frac{7}{8}-\frac{5}{9}$, $\frac{3}{11}-\frac{8}{7}$ in both ways. Did you get the same answer?

So, we say *while subtracting two rational numbers, we add the additive inverse of the rational number that is being subtracted, to the other rational number.*

Thus, $1\frac{2}{3}-2\frac{4}{5}=\frac{5}{3}-\frac{14}{5}=\frac{5}{3}+$ additive inverse of $\frac{14}{5}=\frac{5}{3}+\frac{(-14)}{5}$

$=\frac{-17}{15}=-1\frac{2}{15}$.

TRY THESE

Find:

(i) $\frac{7}{9}-\frac{2}{5}$ (ii) $2\frac{1}{5}-\frac{(-1)}{3}$

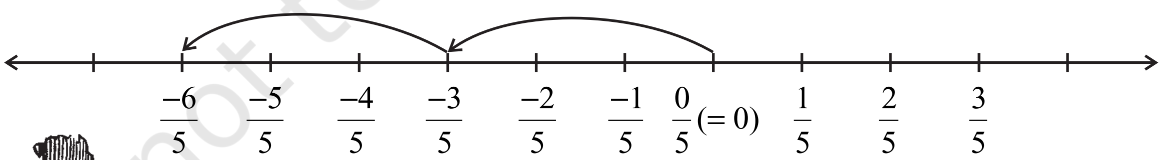
What will be $\frac{2}{7}-\left(\frac{-5}{6}\right)$?

$$\frac{2}{7}-\left(\frac{-5}{6}\right)=\frac{2}{7}+\text{ additive inverse of }\left(\frac{-5}{6}\right)=\frac{2}{7}+\frac{5}{6}=\frac{47}{42}=1\frac{5}{42}$$

8.9.3 Multiplication

Let us multiply the rational number $\frac{-3}{5}$ by 2, i.e., we find $\frac{-3}{5}\times 2$.

On the number line, it will mean two jumps of $\frac{3}{5}$ to the left.



Where do we reach? We reach at $\frac{-6}{5}$. Let us find it as we did in fractions.

$$\frac{-3}{5}\times 2=\frac{-3\times 2}{5}=\frac{-6}{5}$$

We arrive at the same rational number.

Find $\frac{-4}{7}\times 3$, $\frac{-6}{5}\times 4$ using both ways. What do you observe?

So, *we find that while multiplying a rational number by a positive integer, we multiply the numerator by that integer, keeping the denominator unchanged.*

Let us now multiply a rational number by a negative integer,

$$\frac{-2}{9}\times(-5) = \frac{-2\times(-5)}{9} = \frac{10}{9}$$

Remember, –5 can be written as $\frac{-5}{1}$.

So, $\frac{-2}{9}\times\frac{-5}{1} = \frac{10}{9} = \frac{-2\times(-5)}{9\times 1}$

Similarly, $\frac{3}{11}\times(-2) = \frac{3\times(-2)}{11\times 1} = \frac{-6}{11}$

TRY THESE

What will be

(i) $\frac{-3}{5}\times 7$? (ii) $\frac{-6}{5}\times(-2)$?

Based on these observations, we find that, $\frac{-3}{8}\times\frac{5}{7} = \frac{-3\times 5}{8\times 7} = \frac{-15}{56}$

So, as we did in the case of fractions, we multiply two rational numbers in the following way:

Step 1 Multiply the numerators of the two rational numbers.

Step 2 Multiply the denominators of the two rational numbers.

Step 3 Write the product as $\frac{\text{Result of Step 1}}{\text{Result of Step 2}}$

Thus, $\frac{-3}{5}\times\frac{2}{7} = \frac{-3\times 2}{5\times 7} = \frac{-6}{35}$.

Also, $\frac{-5}{8}\times\frac{-9}{7} = \frac{-5\times(-9)}{8\times 7} = \frac{45}{56}$

TRY THESE

Find:

(i) $\frac{-3}{4}\times\frac{1}{7}$

(ii) $\frac{2}{3}\times\frac{-5}{9}$

8.9.4 Division

We have studied reciprocals of a fraction earlier. What is the reciprocal of $\frac{2}{7}$? It will be $\frac{7}{2}$. We extend this idea of reciprocals to non-zero rational numbers also.

The reciprocal of $\frac{-2}{7}$ will be $\frac{7}{-2}$ i.e., $\frac{-7}{2}$; that of $\frac{-3}{5}$ would be $\frac{-5}{3}$.

TRY THESE

What will be the reciprocal of $\frac{-6}{11}$? and $\frac{-8}{5}$?

Product of reciprocals

The product of a rational number with its reciprocal is always 1.

For example, $\frac{-4}{9} \times \left(\text{reciprocal of } \frac{-4}{9}\right)$

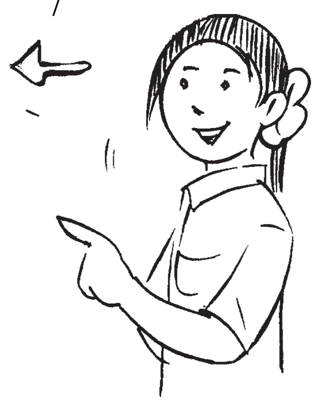
$= \frac{-4}{9} \times \frac{-9}{4} = 1$

Similarly, $\frac{-6}{13} \times \frac{-13}{6} = 1$

Try some more examples and confirm this observation.

Savita divided a rational number $\frac{4}{9}$ by another rational number $\frac{-5}{7}$ as,

$$\frac{4}{9} \div \frac{-5}{7} = \frac{4}{9} \times \frac{7}{-5} = \frac{-28}{45}.$$



She used the idea of reciprocal as done in fractions.

Arpit first divided $\frac{4}{9}$ by $\frac{5}{7}$ and got $\frac{28}{45}$.

He finally said $\frac{4}{9} \div \frac{-5}{7} = \frac{-28}{45}$. How did he get that?

He divided them as fractions, ignoring the negative sign and then put the negative sign in the value so obtained.

Both of them got the same value $\frac{-28}{45}$. Try dividing $\frac{2}{3}$ by $\frac{-5}{7}$ both ways and see if you get the same answer.

This shows, *to divide one rational number by the other non-zero rational number we multiply the rational number by the reciprocal of the other.*

Thus, $\frac{-6}{5} \div \frac{-2}{3} = \frac{6}{-5} \times \text{reciprocal of } \left(\frac{-2}{3}\right) = \frac{6}{-5} \times \frac{3}{-2} = \frac{18}{10}$

TRY THESE

Find: (i) $\frac{2}{3}\times\frac{-7}{8}$ (ii) $\frac{-6}{7}\times\frac{5}{7}$

EXERCISE 8.2

1. Find the sum:

(i) $\frac{5}{4}+\left(\frac{-11}{4}\right)$ (ii) $\frac{5}{3}+\frac{3}{5}$ (iii) $\frac{-9}{10}+\frac{22}{15}$

(iv) $\frac{-3}{-11}+\frac{5}{9}$ (v) $\frac{-8}{19}+\frac{(-2)}{57}$ (vi) $\frac{-2}{3}+0$

(vii) $-2\frac{1}{3}+4\frac{3}{5}$

2. Find

(i) $\frac{7}{24}-\frac{17}{36}$ (ii) $\frac{5}{63}-\left(\frac{-6}{21}\right)$ (iii) $\frac{-6}{13}-\left(\frac{-7}{15}\right)$

(iv) $\frac{-3}{8}-\frac{7}{11}$ (v) $-2\frac{1}{9}-6$

3. Find the product:

(i) $\frac{9}{2}\times\left(\frac{-7}{4}\right)$ (ii) $\frac{3}{10}\times(-9)$ (iii) $\frac{-6}{5}\times\frac{9}{11}$

(iv) $\frac{3}{7}\times\left(\frac{-2}{5}\right)$ (v) $\frac{3}{11}\times\frac{2}{5}$ (vi) $\frac{3}{-5}\times\frac{-5}{3}$

4. Find the value of:

(i) $(-4)\div\frac{2}{3}$ (ii) $\frac{-3}{5}\div 2$ (iii) $\frac{-4}{5}\div(-3)$

(iv) $\frac{-1}{8}\div\frac{3}{4}$ (v) $\frac{-2}{13}\div\frac{1}{7}$ (vi) $\frac{-7}{12}\div\left(\frac{-2}{13}\right)$

(vii) $\frac{3}{13}\div\left(\frac{-4}{65}\right)$

What have We Discussed?

1. A number that can be expressed in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$, is called a rational number. The numbers $\frac{-2}{7}, \frac{3}{8}, 3$ etc. are rational numbers.
2. All integers and fractions are rational numbers.
3. If the numerator and denominator of a rational number are multiplied or divided by a non-zero integer, we get a rational number which is said to be equivalent to the given rational number. For example $\frac{-3}{7} = \frac{-3 \times 2}{7 \times 2} = \frac{-6}{14}$. So, we say $\frac{-6}{14}$ is the equivalent form of $\frac{-3}{7}$. Also note that $\frac{-6}{14} = \frac{-6 \div 2}{14 \div 2} = \frac{-3}{7}$.
4. Rational numbers are classified as Positive and Negative rational numbers. When the numerator and denominator, both, are positive integers, it is a positive rational number. When either the numerator or the denominator is a negative integer, it is a negative rational number. For example, $\frac{3}{8}$ is a positive rational number whereas $\frac{-8}{9}$ is a negative rational number.
5. The number 0 is neither a positive nor a negative rational number.
6. A rational number is said to be in the standard form if its denominator is a positive integer and the numerator and denominator have no common factor other than 1. The numbers $\frac{-1}{3}, \frac{2}{7}$ etc. are in standard form.
7. There are unlimited number of rational numbers between two rational numbers.
8. Two rational numbers with the same denominator can be added by adding their numerators, keeping the denominator same. Two rational numbers with different denominators are added by first taking the LCM of the two denominators and then converting both the rational numbers to their equivalent forms having the LCM as the denominator. For example, $\frac{-2}{3} + \frac{3}{8} = \frac{-16}{24} + \frac{9}{24} = \frac{-16+9}{24} = \frac{-7}{24}$. Here, LCM of 3 and 8 is 24.
9. While subtracting two rational numbers, we add the additive inverse of the rational number to be subtracted to the other rational number.

   Thus, $\frac{7}{8} - \frac{2}{3} = \frac{7}{8} + \text{additive inverse of } \frac{2}{3} = \frac{7}{8} + \frac{(-2)}{3} = \frac{21+(-16)}{24} = \frac{5}{24}$.

10. To multiply two rational numbers, we multiply their numerators and denominators separately, and write the product as $\frac{\text{product of numerators}}{\text{product of denominators}}$.

11. To divide one rational number by the other non-zero rational number, we multiply the rational number by the reciprocal of the other. Thus,

$$\frac{-7}{2} \div \frac{4}{3} = \frac{-7}{2} \times (\text{reciprocal of } \frac{4}{3}) = \frac{-7}{2} \times \frac{3}{4} = \frac{-21}{8}.$$

Chapter 9

Perimeter and Area

9.1 Area of a Parallelogram

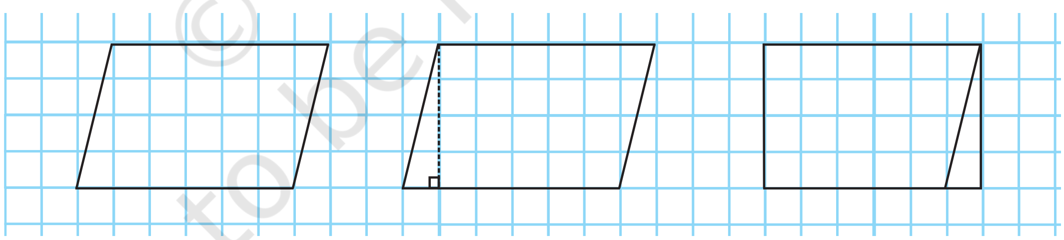
We come across many shapes other than squares and rectangles.

How will you find the area of a land which is a parallelogram in shape?

Let us find a method to get the area of a parallelogram.

Can a parallelogram be converted into a rectangle of equal area?

Draw a parallelogram on a graph paper as shown in Fig 9.1(i). Cut out the parallelogram. Draw a line from one vertex of the parallelogram perpendicular to the opposite side [Fig 9.1(ii)]. Cut out the triangle. Move the triangle to the other side of the parallelogram.



(i) (ii) (iii)

Fig 9.1

What shape do you get? You get a rectangle.

Is the area of the parallelogram equal to the area of the rectangle formed?

Yes, area of the parallelogram = area of the rectangle formed

What are the length and the breadth of the rectangle?

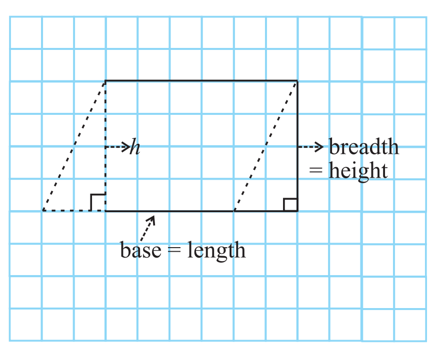


Fig 9.2

We find that the length of the rectangle formed is equal to the base of the parallelogram and the breadth of the rectangle is equal to the height of the parallelogram (Fig 9.2).

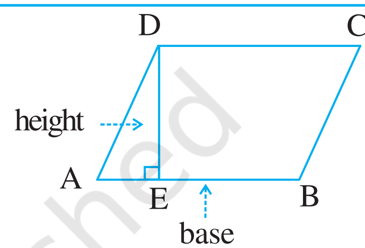
Now, Area of parallelogram = Area of rectangle

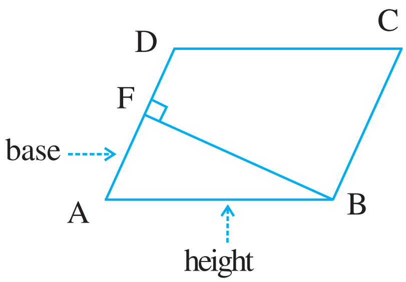
= length × breadth = $l \times b$

But the length l and breadth b of the rectangle are exactly the base b and the height h, respectively of the parallelogram.

Thus, the area of parallelogram = base × height = $b \times h$.

Any side of a parallelogram can be chosen as **base** of the parallelogram. The perpendicular dropped on that side from the opposite vertex is known as **height** (altitude). In the parallelogram ABCD, DE is perpendicular to AB. Here AB is the base and DE is the height of the parallelogram.





In this parallelogram ABCD, BF is the perpendicular to opposite side AD. Here AD is the **base** and BF is the **height**.

Consider the following parallelograms (Fig 9.2).

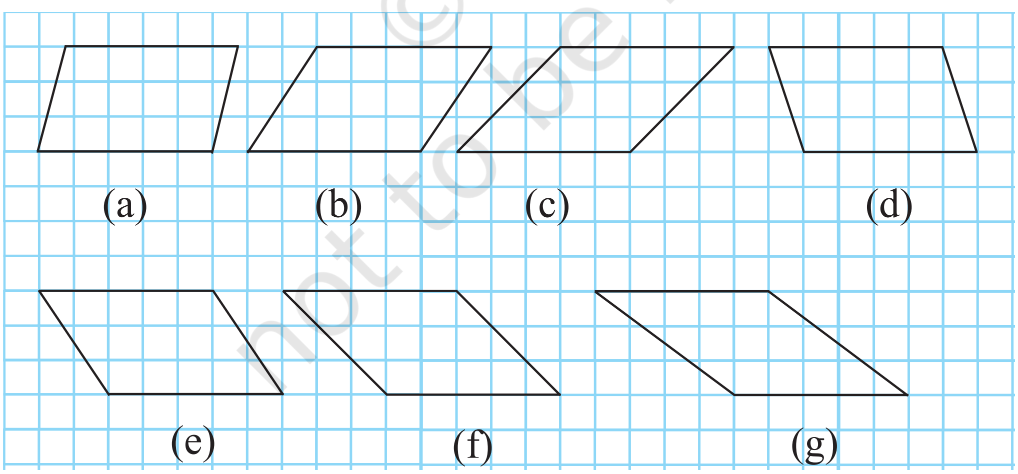


Fig 9.3

Find the areas of the parallelograms by counting the squares enclosed within the figures and also find the perimeters by measuring the sides.

Complete the following table:

Parallelogram	Base	Height	Area	Perimeter
(a)	5 units	3 units	15 sq units	
(b)				
(c)				
(d)				
(e)				
(f)				
(g)				

You will find that all these parallelograms have equal areas but different perimeters. Now, consider the following parallelograms with sides 7 cm and 5 cm (Fig 9.4).

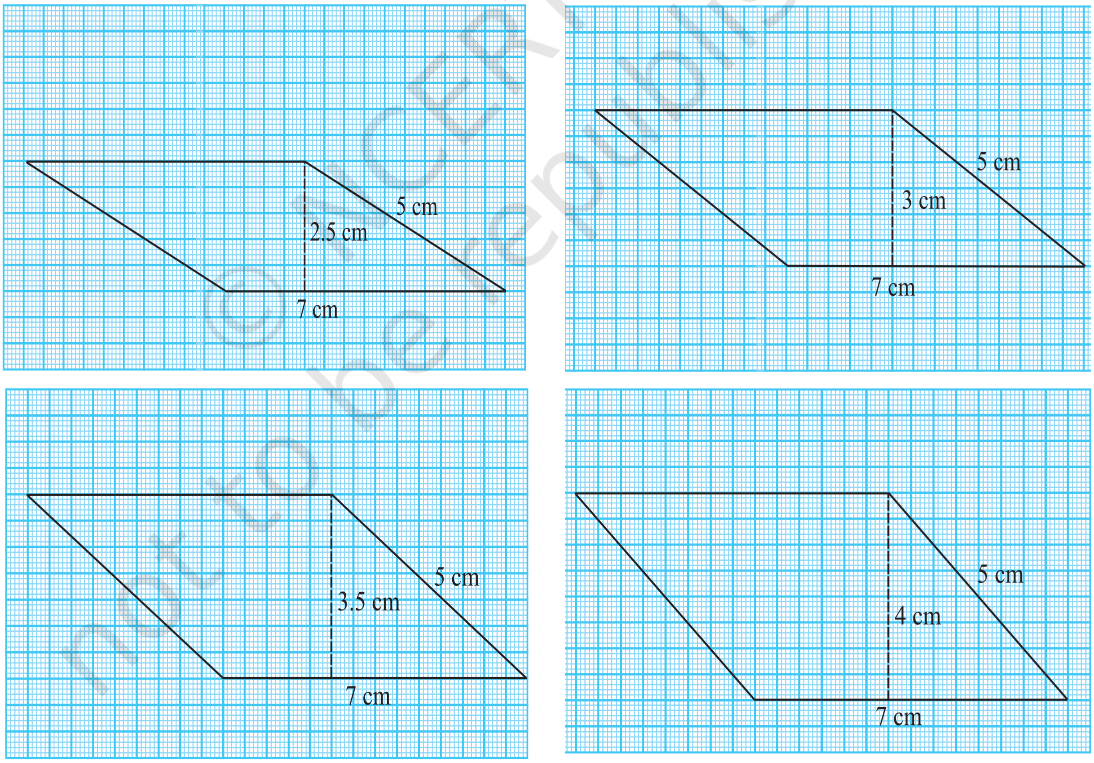


Fig 9.4

Find the perimeter and area of each of these parallelograms. Analyse your results.

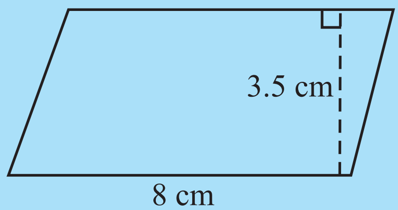
You will find that these parallelograms have different areas but equal perimeters.

To find the area of a parallelogram, you need to know only the base and the corresponding height of the parallelogram.

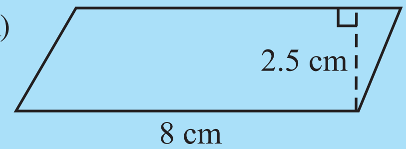
TRY THESE

Find the area of following parallelograms:

(i)

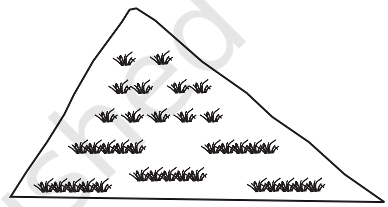


(ii)



(iii) In a parallelogram ABCD, AB = 7.2 cm and the perpendicular from C on AB is 4.5 cm.

9.2 AREA OF A TRIANGLE



A gardener wants to know the cost of covering the whole of a triangular garden with grass.

In this case we need to know the area of the triangular region.

Let us find a method to get the area of a triangle.

Draw a scalene triangle on a piece of paper. Cut out the triangle.

Place this triangle on another piece of paper and cut out another triangle of the same size.

So now you have two scalene triangles of the same size.

Are both the triangles congruent?

Superpose one triangle on the other so that they match. You may have to rotate one of the two triangles.

Now place both the triangles such that a pair of corresponding sides is joined as shown in Fig 9.5.

Is the figure thus formed a parallelogram?

Compare the area of each triangle to the area of the parallelogram.

Compare the base and height of the triangles with the base and height of the parallelogram.

You will find that the sum of the areas of both the triangles is equal to the area of the parallelogram. The base and the height of the triangle are the same as the base and the height of the parallelogram, respectively.

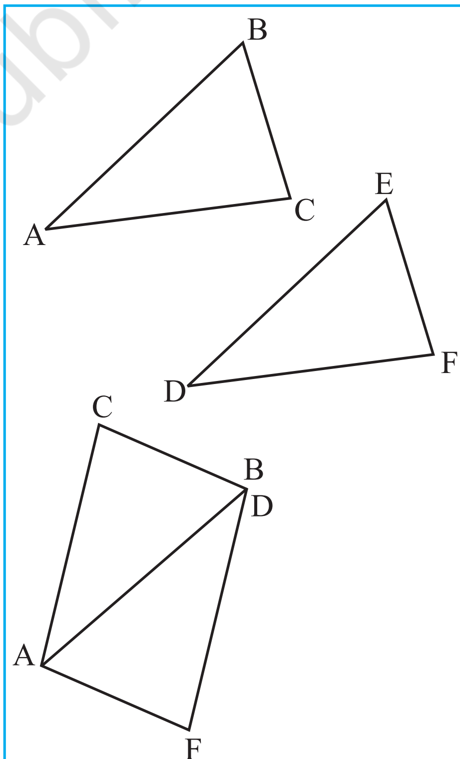


Fig 9.5

$$\text{Area of each triangle} = \frac{1}{2}(\text{Area of parallelogram})$$

$$= \frac{1}{2}(\text{base} \times \text{height}) \text{ (Since area of a parallelogram = base} \times \text{height)}$$

$$= \frac{1}{2}(b \times h) \text{ (or } \frac{1}{2}bh \text{, in short)}$$

Try These

1. Try the above activity with different types of triangles.
2. Take different parallelograms. Divide each of the parallelograms into two triangles by cutting along any of its diagonals. Are the triangles congruent?

In the figure (Fig 9.6) all the triangles are on the base AB = 6 cm.

What can you say about the height of each of the triangles corresponding to the base AB?

Can we say all the triangles are equal in area? Yes.

Are the triangles congruent also? No.

We conclude that **all the congruent triangles are equal in area but the triangles equal in area need not be congruent.**

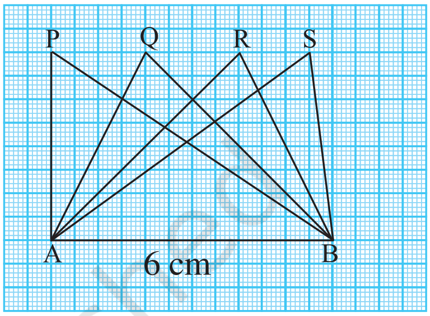


Fig 9.6

Consider the obtuse-angled triangle ABC of base 6 cm (Fig 9.7).

Its height AD which is perpendicular from the vertex A is outside the triangle.

Can you find the area of the triangle?

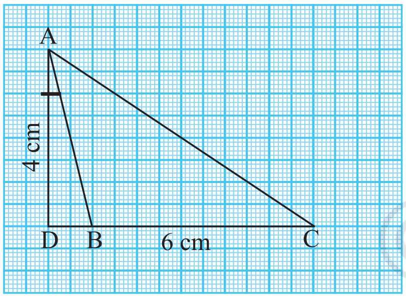


Fig 9.7

Example 1 One of the sides and the corresponding height of a parallelogram are 4 cm and 3 cm respectively. Find the area of the parallelogram (Fig 9.8).

Solution Given that length of base (b) = 4 cm, height (h) = 3 cm

Area of the parallelogram = $b \times h$

$= 4 \text{ cm} \times 3 \text{ cm} = 12 \text{ cm}^2$

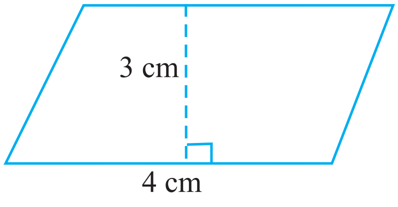


Fig 9.8

Example 2 Find the height 'x' if the area of the parallelogram is 24 cm^2 and the base is 4 cm.

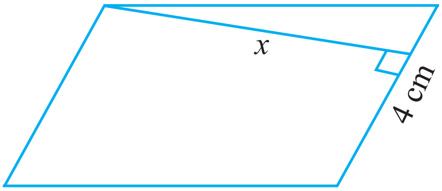


Fig 9.9

Solution Area of parallelogram = $b \times h$

Therefore, $24 = 4 \times x$ (Fig 9.9)

or $\dfrac{24}{4} = x$ or $x = 6$ cm

So, the height of the parallelogram is 6 cm.

EXAMPLE 3 The two sides of the parallelogram ABCD are 6 cm and 4 cm. The height corresponding to the base CD is 3 cm (Fig 9.10). Find the

(i) area of the parallelogram. (ii) the height corresponding to the base AD.

SOLUTION

(i) Area of parallelogram = $b \times h$

$$= 6 \text{ cm} \times 3 \text{ cm} = 18 \text{ cm}^2$$

(ii) base (b) = 4 cm, height = x (say),

Area = 18 cm^2

Area of parallelogram = $b \times x$

$$18 = 4 \times x$$

$$\frac{18}{4} = x$$

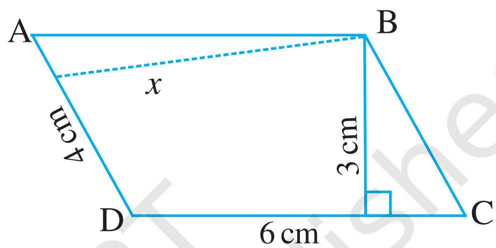


Fig 9.10

Therefore, $x = 4.5$ cm

Thus, the height corresponding to base AD is 4.5 cm.

EXAMPLE 4 Find the area of the following triangles (Fig 9.11).

(i) **Fig 9.11** (ii)

SOLUTION

(i) Area of triangle $= \frac{1}{2}bh = \frac{1}{2} \times \text{QR} \times \text{PS}$

$$= \frac{1}{2} \times 4 \text{ cm} \times 2 \text{ cm} = 4 \text{ cm}^2$$

(ii) Area of triangle $= \frac{1}{2}bh = \frac{1}{2} \times \text{MN} \times \text{LO}$

$$= \frac{1}{2} \times 3 \text{ cm} \times 2 \text{ cm} = 3 \text{ cm}^2$$

EXAMPLE 5 Find BC, if the area of the triangle ABC is 36 cm² and the height AD is 3 cm (Fig 9.12).

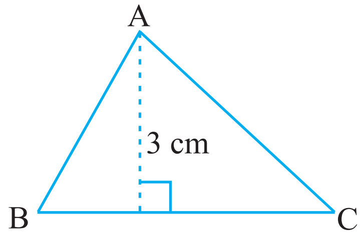


Fig 9.12

SOLUTION Height = 3 cm, Area = 36 cm²

$$\text{Area of the triangle ABC} = \frac{1}{2}bh$$

or $$36 = \frac{1}{2}\times b\times 3 \text{ i.e., } b = \frac{36\times 2}{3} = 24 \text{ cm}$$

So, $$\text{BC} = 24 \text{ cm}$$

EXAMPLE 6 In ΔPQR, PR = 8 cm, QR = 4 cm and PL = 5 cm (Fig 9.13). Find:

(i) the area of the ΔPQR

(ii) QM

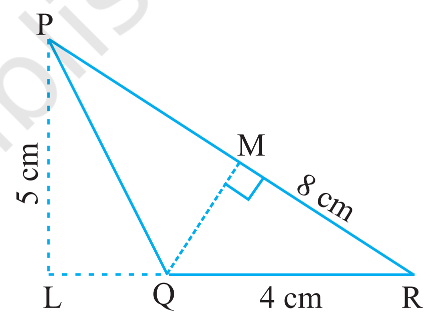


Fig 9.13

SOLUTION

(i) QR = base =4 cm, PL = height = 5 cm

$$\text{Area of the triangle PQR} = \frac{1}{2}bh$$

$$= \frac{1}{2}\times 4 \text{ cm}\times 5 \text{ cm} = 10 \text{ cm}^2$$

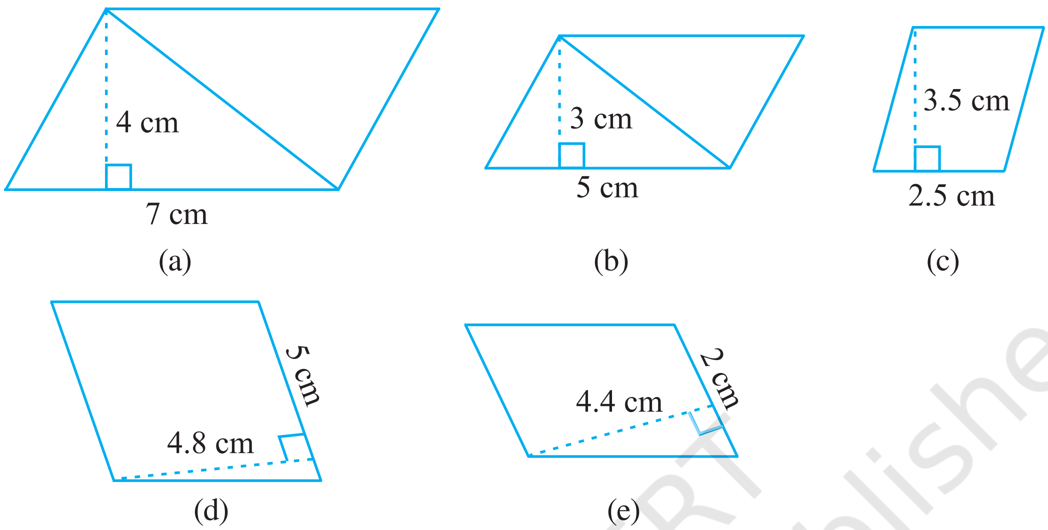
(ii) PR = base = 8 cm QM = height = ? Area = 10 cm²

$$\text{Area of triangle} = \frac{1}{2}\times b\times h \quad \text{i.e.,} \quad 10 = \frac{1}{2}\times 8\times h$$

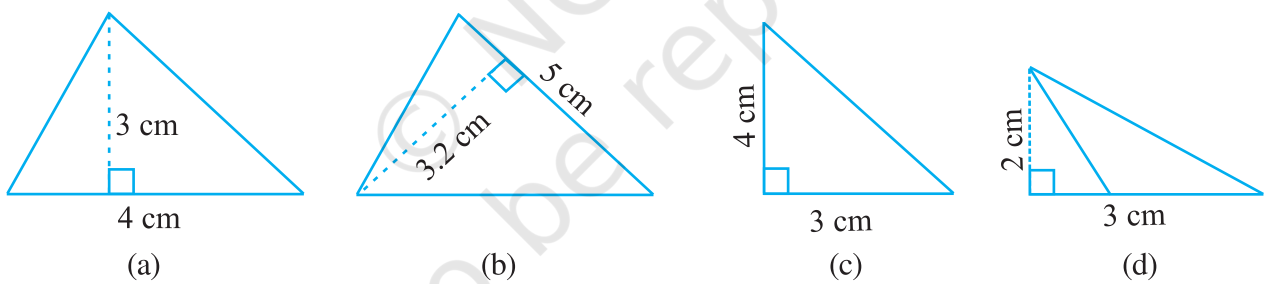
$$h = \frac{10}{4} = \frac{5}{2} = 2.5. \quad \text{So,} \quad \text{QM} = 2.5 \text{ cm}$$

Exercise 9.1

1. Find the area of each of the following parallelograms:



2. Find the area of each of the following triangles:



3. Find the missing values:

S.No.	Base	Height	Area of the Parallelogram
a.	20 cm		246 cm^2
b.		15 cm	154.5 cm^2
c.		8.4 cm	48.72 cm^2
d.	15.6 cm		16.38 cm^2

4. Find the missing values:

Base	Height	Area of Triangle
15 cm	______	87 cm^2
_____	31.4 mm	1256 mm^2
22 cm	______	170.5 cm^2

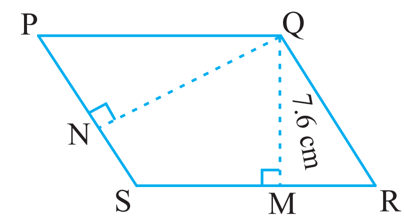


Fig 9.14

5. PQRS is a parallelogram (Fig 9.14). QM is the height from Q to SR and QN is the height from Q to PS. If SR = 12 cm and QM = 7.6 cm. Find:
   (a) the area of the parallegram PQRS
   (b) QN, if PS = 8 cm

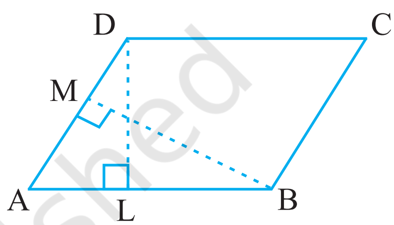


Fig 9.15

6. DL and BM are the heights on sides AB and AD respectively of parallelogram ABCD (Fig 9.15). If the area of the parallelogram is 1470 cm^2, AB = 35 cm and AD = 49 cm, find the length of BM and DL.

7. ΔABC is right angled at A (Fig 9.16). AD is perpendicular to BC. If AB = 5 cm, BC = 13 cm and AC = 12 cm, Find the area of ΔABC. Also find the length of AD.

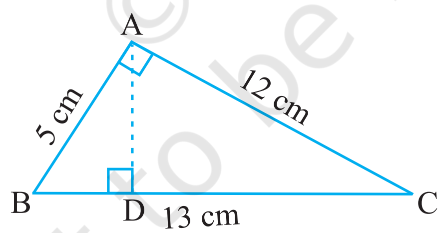


Fig 9.16

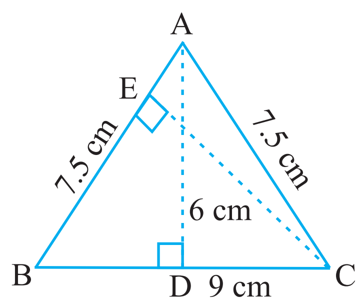


Fig 9.17

8. ΔABC is isosceles with AB = AC = 7.5 cm and BC = 9 cm (Fig 9.17). The height AD from A to BC, is 6 cm. Find the area of ΔABC. What will be the height from C to AB i.e., CE?

9.3 Circles

A racing track is semi-circular at both ends (Fig 9.18).

Can you find the distance covered by an athlete if he takes two rounds of a racing track? We need to find a method to find the distances around when a shape is circular.

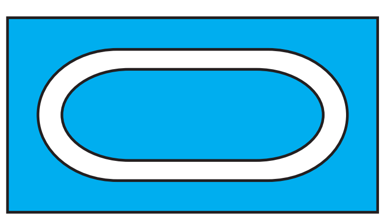
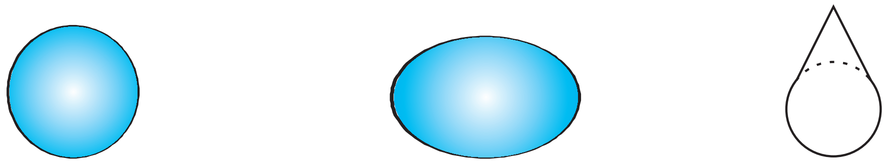


Fig 9.18

9.3.1 Circumference of a Circle

Tanya cut different cards, in curved shape from a cardboard. She wants to put lace around to decorate these cards. What length of the lace does she require for each? (Fig 9.19)



(a) (b) (c)

Fig 9.19

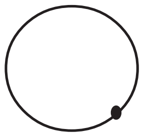


Fig 9.20

You cannot measure the curves with the help of a ruler, as these figures are not "straight". What can you do?

Here is a way to find the length of lace required for shape in Fig 9.19(a). Mark a point on the edge of the card and place the card on the table. Mark the position of the point on the table also (Fig 9. 20).

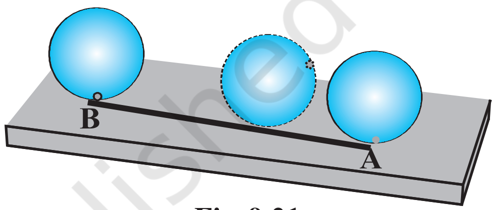


Fig 9.21

Now roll the circular card on the table along a straight line till the marked point again touches the table. Measure the distance along the line. This is the length of the lace required (Fig 9.21). It is also the distance along the edge of the card from the marked point back to the marked point.

You can also find the distance by putting a string on the edge of the circular object and taking all round it.

The distance around a circular region is known as its circumference.

DO THIS

Take a bottle cap, a bangle or any other circular object and find the circumference.

Now, can you find the distance covered by the athlete on the track by this method?

Still, it will be very difficult to find the distance around the track or any other circular object by measuring through string. Moreover, the measurement will not be accurate.

So, we need some formula for this, as we have for rectilinear figures or shapes.

Let us see if there is any relationship between the diameter and the circumference of the circles.

Consider the following table: Draw six circles of different radii and find their circumference by using string. Also find the ratio of the circumference to the diameter.

Circle	Radius	Diameter	Circumference	Ratio of Circumference to Diameter
1.	3.5 cm	7.0 cm	22.0 cm	$\frac{22}{7} = 3.14$

2.	7.0 cm	14.0 cm	44.0 cm	$\frac{44}{14} = 3.14$
3.	10.5 cm	21.0 cm	66.0 cm	$\frac{66}{21} = 3.14$
4.	21.0 cm	42.0 cm	132.0 cm	$\frac{132}{42} = 3.14$
5.	5.0 cm	10.0 cm	32.0 cm	$\frac{32}{10} = 3.2$
6.	15.0 cm	30.0 cm	94.0 cm	$\frac{94}{30} = 3.13$

What do you infer from the above table? Is this ratio approximately the same? Yes.

Can you say that the circumference of a circle is always more than three times its diameter? Yes.

This ratio is a constant and is denoted by π (pi). Its approximate value is $\frac{22}{7}$ or 3.14.

So, we can say that $\frac{C}{d} = \pi$, where 'C' represents circumference of the circle and 'd' its diameter.

or $\qquad C = \pi d$

We know that diameter (d) of a circle is twice the radius (r) i.e., $d = 2r$

So, $\qquad C = \pi d = \pi \times 2r \qquad$ or $\qquad C = 2\pi r$.

Try These

In Fig 9.22,

(a) Which square has the larger perimeter?

(b) Which is larger, perimeter of smaller square or the circumference of the circle?

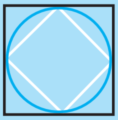
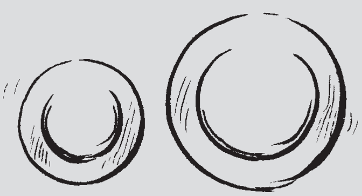


Fig 9.22

Do This



Take one each of quarter plate and half plate. Roll once each of these on a table-top. Which plate covers more distance in one complete revolution? Which plate will take less number of revolutions to cover the length of the table-top?

EXAMPLE 7 What is the circumference of a circle of diameter 10 cm (Take $\pi = 3.14$)?

SOLUTION Diameter of the circle (d) = 10 cm

Circumference of circle = πd

$= 3.14 \times 10$ cm = 31.4 cm

So, the circumference of the circle of diameter 10 cm is 31.4 cm.

EXAMPLE 8 What is the circumference of a circular disc of radius 14 cm?

$$\left(\text{Use } \pi = \frac{22}{7}\right)$$

SOLUTION Radius of circular disc (r) = 14 cm

Circumference of disc = $2\pi r$

$= 2 \times \frac{22}{7} \times 14$ cm = 88 cm

So, the circumference of the circular disc is 88 cm.

EXAMPLE 9 The radius of a circular pipe is 10 cm. What length of a tape is required to wrap once around the pipe ($\pi = 3.14$)?

SOLUTION Radius of the pipe (r) = 10 cm

Length of tape required is equal to the circumference of the pipe.

Circumference of the pipe = $2\pi r$

$= 2 \times 3.14 \times 10$ cm

$= 62.8$ cm

Therefore, length of the tape needed to wrap once around the pipe is 62.8 cm.

EXAMPLE 10 Find the perimeter of the given shape (Fig 9.23) (Take $\pi = \frac{22}{7}$).

SOLUTION In this shape we need to find the circumference of semicircles on each side of the square. Do you need to find the perimeter of the square also? No. The outer boundary, of this figure is made up of semicircles. Diameter of each semicircle is 14 cm.

We know that:

Circumference of the circle = πd

Circumference of the semicircle = $\frac{1}{2}\pi d$

$= \frac{1}{2} \times \frac{22}{7} \times 14$ cm = 22 cm

Circumference of each of the semicircles is 22 cm

Therefore, perimeter of the given figure = 4×22 cm = 88 cm

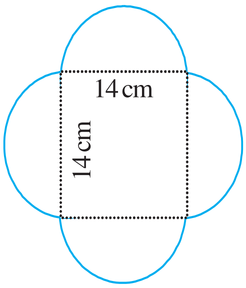


Fig 9.23

Example 11 Sudhanshu divides a circular disc of radius 7 cm in two equal parts.

What is the perimeter of each semicircular shape disc? (Use $\pi = \frac{22}{7}$)

Solution To find the perimeter of the semicircular disc (Fig 9.24), we need to find

(i) Circumference of semicircular shape (ii) Diameter

Given that radius (r) = 7 cm. We know that the circumference of circle = $2\pi r$

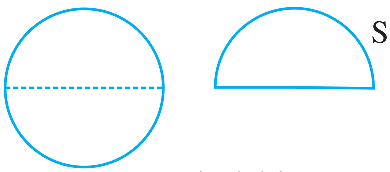


Fig 9.24

So, the circumference of the semicircle $= \frac{1}{2} \times 2\pi r = \pi r$

$= \frac{22}{7} \times 7 \text{ cm} = 22 \text{ cm}$

So, the diameter of the circle $= 2r = 2 \times 7 \text{ cm} = 14 \text{ cm}$

Thus, perimeter of each semicircular disc = 22 cm + 14 cm = 36 cm

9.3.2 Area of Circle

Consider the following:

- A farmer dug a flower bed of radius 7 m at the centre of a field. He needs to purchase fertiliser. If 1 kg of fertiliser is required for 1 square metre area, how much fertiliser should he purchase?
- What will be the cost of polishing a circular table-top of radius 2 m at the rate of ₹ 10 per square metre?

Can you tell what we need to find in such cases, Area or Perimeter? In such cases we need to find the area of the circular region. Let us find the area of a circle, using graph paper.

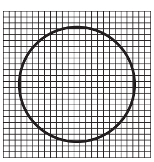
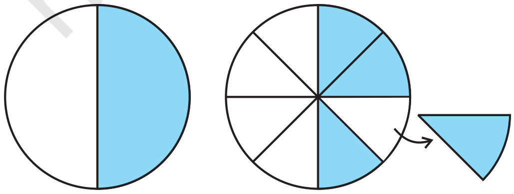


Fig 9.25

Draw a circle of radius 4 cm on a graph paper (Fig 9.25). Find the area by counting the number of squares enclosed.

As the edges are not straight, we get a rough estimate of the area of circle by this method. There is another way of finding the area of a circle.

Draw a circle and shade one half of the circle [Fig 9.26(i)]. Now fold the circle into **eighths** and cut along the folds [Fig 9.26(ii)].



(i) (ii)

Fig 9.26

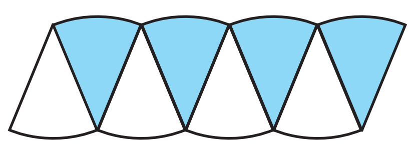


Fig 9.27

Arrange the separate pieces as shown, in Fig 9.27, which is roughly a parallelogram.

The more sectors we have, the nearer we reach an appropriate parallelogram.

As done above if we divide the circle in 64 sectors, and arrange these sectors. It gives nearly a rectangle (Fig 9.28).

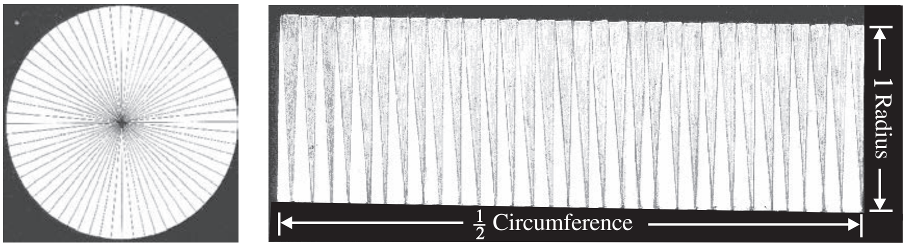


Fig 9.28

What is the breadth of this rectangle? The breadth of this rectangle is the radius of the circle, i.e., 'r'.

As the whole circle is divided into 64 sectors and on each side we have 32 sectors, the length of the rectangle is the length of the 32 sectors, which is half of the circumference. (Fig 9.28)

Area of the circle = Area of rectangle thus formed = $l \times b$

= (Half of circumference) × radius $= \left(\frac{1}{2} \times 2\pi r\right) \times r = \pi r^2$

So, the area of the circle = πr^2

Try These

Draw circles of different radii on a graph paper. Find the area by counting the number of squares. Also find the area by using the formula. Compare the two answers.

Example 12 Find the area of a circle of radius 30 cm (use $\pi = 3.14$).

Solution Radius, $r = 30$ cm

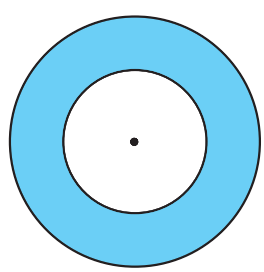
Area of the circle = $\pi r^2 = 3.14 \times 30^2 = 2{,}826\ \text{cm}^2$

Example 13 Diameter of a circular garden is 9.8 m. Find its area.

Solution Diameter, $d = 9.8$ m. Therefore, radius $r = 9.8 \div 2 = 4.9$ m

Area of the circle = $\pi r^2 = \frac{22}{7} \times (4.9)^2\ \text{m}^2 = \frac{22}{7} \times 4.9 \times 4.9\ \text{m}^2 = 75.46\ \text{m}^2$

Example 14 The adjoining figure shows two circles with the same centre. The radius of the larger circle is 10 cm and the radius of the smaller circle is 4 cm.



Find: (a) the area of the larger circle

(b) the area of the smaller circle

(c) the shaded area between the two circles. ($\pi = 3.14$)

Solution

(a) Radius of the larger circle $= 10$ cm

So, area of the larger circle $= \pi r^2$

$= 3.14 \times 10 \times 10 = 314\ \text{cm}^2$

(b) Radius of the smaller circle $= 4$ cm

Area of the smaller circle $= \pi r^2$

$= 3.14 \times 4 \times 4 = 50.24\ \text{cm}^2$

(c) Area of the shaded region $= (314 - 50.24)\ \text{cm}^2 = 263.76\ \text{cm}^2$

Exercise 9.2

1. Find the circumference of the circles with the following radius: (Take $\pi = \frac{22}{7}$)

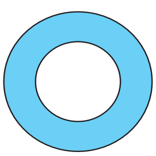
(a) 14 cm (b) 28 mm (c) 21 cm

2. Find the area of the following circles, given that:

(a) radius = 14 mm (Take $\pi = \frac{22}{7}$) (b) diameter = 49 m

(c) radius = 5 cm

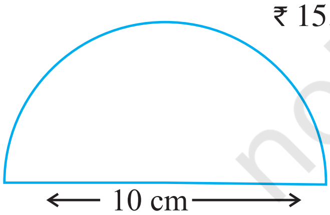
3. If the circumference of a circular sheet is 154 m, find its radius. Also find the area of the sheet. (Take $\pi = \frac{22}{7}$)



4. A gardener wants to fence a circular garden of diameter 21m. Find the length of the rope he needs to purchase, if he makes 2 rounds of fence. Also find the cost of the rope, if it costs ₹ 4 per meter. (Take $\pi = \frac{22}{7}$)

5. From a circular sheet of radius 4 cm, a circle of radius 3 cm is removed. Find the area of the remaining sheet. (Take $\pi = 3.14$)

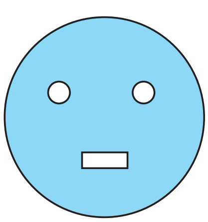
6. Saima wants to put a lace on the edge of a circular table cover of diameter 1.5 m. Find the length of the lace required and also find its cost if one meter of the lace costs ₹ 15. (Take $\pi = 3.14$)



7. Find the perimeter of the adjoining figure, which is a semicircle including its diameter.

8. Find the cost of polishing a circular table-top of diameter 1.6 m, if the rate of polishing is ₹ 15/m². (Take $\pi = 3.14$)

9. Shazli took a wire of length 44 cm and bent it into the shape of a circle. Find the radius of that circle. Also find its area. If the same wire is bent into the shape of a square, what will be the length of each of its sides? Which figure encloses more area, the circle or the square? (Take $\pi = \frac{22}{7}$)

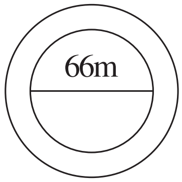


10. From a circular card sheet of radius 14 cm, two circles of radius 3.5 cm and a rectangle of length 3 cm and breadth 1cm are removed. (as shown in the adjoining figure). Find the area of the remaining sheet. (Take $\pi = \frac{22}{7}$)

11. A circle of radius 2 cm is cut out from a square piece of an aluminium sheet of side 6 cm. What is the area of the left over aluminium sheet? (Take $\pi = 3.14$)

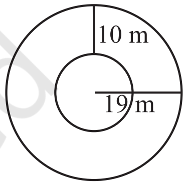
12. The circumference of a circle is 31.4 cm. Find the radius and the area of the circle? (Take $\pi = 3.14$)

13. A circular flower bed is surrounded by a path 4 m wide. The diameter of the flower bed is 66 m. What is the area of this path? ($\pi = 3.14$)



14. A circular flower garden has an area of 314 m^2. A sprinkler at the centre of the garden can cover an area that has a radius of 12 m. Will the sprinkler water the entire garden? (Take $\pi = 3.14$)

15. Find the circumference of the inner and the outer circles, shown in the adjoining figure? (Take $\pi = 3.14$)



16. How many times a wheel of radius 28 cm must rotate to go 352 m? (Take $\pi = \frac{22}{7}$)

17. The minute hand of a circular clock is 15 cm long. How far does the tip of the minute hand move in 1 hour. (Take $\pi = 3.14$)

What have We Discussed?

1. Area of a parallelogram = base × height

2. Area of a triangle = $\frac{1}{2}$ (area of the parallelogram generated from it)

$$= \frac{1}{2} \times \text{base} \times \text{height}$$

3. The distance around a circular region is known as its circumference.

Circumference of a circle = πd, where d is the diameter of a circle and $\pi = \frac{22}{7}$ or 3.14 (approximately).

4. Area of a circle = πr^2, where r is the radius of the circle.

Chapter 10

Algebraic Expressions

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10.1 Introduction

We have already come across simple algebraic expressions like $x + 3$, $y - 5$, $4x + 5$, $10y - 5$ and so on. In Class VI, we have seen how these expressions are useful in formulating puzzles and problems. We have also seen examples of several expressions in the chapter on simple equations.

Expressions are a central concept in algebra. This Chapter is devoted to algebraic expressions. When you have studied this Chapter, you will know how algebraic expressions are formed, how they can be combined, how we can find their values and how they can be used.

10.2 How are Expressions Formed?

We now know very well what a variable is. We use letters x, y, l, m, ... etc. to denote variables. A **variable** can take various values. Its value is not fixed. On the other hand, a **constant** has a fixed value. Examples of constants are: 4, 100, –17, etc.

We combine variables and constants to make algebraic expressions. For this, we use the operations of addition, subtraction, multiplication and division. We have already come across expressions like $4x + 5$, $10y - 20$. The expression $4x + 5$ is obtained from the variable x, first by multiplying x by the constant 4 and then adding the constant 5 to the product. Similarly, $10y - 20$ is obtained by first multiplying y by 10 and then subtracting 20 from the product.

The above expressions were obtained by combining variables with constants. We can also obtain expressions by combining variables with themselves or with other variables. Look at how the following expressions are obtained:

$$x^2,\ 2y^2,\ 3x^2 - 5,\ xy,\ 4xy + 7$$

(i) The expression x^2 is obtained by multiplying the variable x by itself;

$$x \times x = x^2$$

Just as 4×4 is written as 4^2, we write $x \times x = x^2$. It is commonly read as x squared.

(Later, when you study the chapter 'Exponents and Powers' you will realise that x^2 may also be read as x raised to the power 2).

In the same manner, we can write $\qquad x \times x \times x = x^3$

Commonly, x^3 is read as 'x cubed'. Later, you will realise that x^3 may also be read as x raised to the power 3.

$x, x^2, x^3, \ldots$ are all algebraic expressions obtained from x.

(ii) The expression $2y^2$ is obtained from y: $\quad 2y^2 = 2 \times y \times y$

Here by multiplying y with y we obtain y^2 and then we multiply y^2 by the constant 2.

(iii) In $(3x^2 - 5)$ we first obtain x^2, and multiply it by 3 to get $3x^2$. From $3x^2$, we subtract 5 to finally arrive at $3x^2 - 5$.

(iv) In xy, we multiply the variable x with another variable y. Thus, $x \times y = xy$.

(v) In $4xy + 7$, we first obtain xy, multiply it by 4 to get $4xy$ and add 7 to $4xy$ to get the expression.

TRY THESE

Describe how the following expressions are obtained:

$7xy + 5,\ x^2y,\ 4x^2 - 5x$

10.3 TERMS OF AN EXPRESSION

We shall now put in a systematic form what we have learnt above about how expressions are formed. For this purpose, we need to understand what **terms** of an expression and their **factors** are.

Consider the expression $(4x + 5)$. In forming this expression, we first formed $4x$ separately as a product of 4 and x and then added 5 to it. Similarly consider the expression $(3x^2 + 7y)$. Here we first formed $3x^2$ separately as a product of 3, x and x. We then formed $7y$ separately as a product of 7 and y. Having formed $3x^2$ and $7y$ separately, we added them to get the expression.

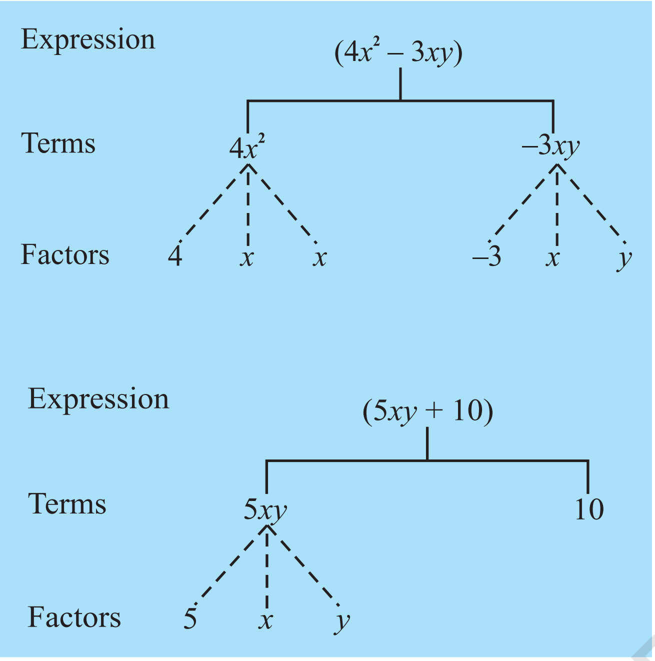
You will find that the expressions we deal with can always be seen this way. They have parts which are formed separately and then added. Such parts of an expression which are formed separately first and then added are known as **terms**. Look at the expression $(4x^2 - 3xy)$. We say that it has two terms, $4x^2$ and $-3xy$. The term $4x^2$ is a product of 4, x and x, and the term $(-3xy)$ is a product of (-3), x and y.

Terms are added to form expressions. Just as the terms $4x$ and 5 are added to form the expression $(4x + 5)$, the terms $4x^2$ and $(-3xy)$ are added to give the expression $(4x^2 - 3xy)$. This is because $4x^2 + (-3xy) = 4x^2 - 3xy$.

> Note, the minus sign (–) is included in the term. In the expression $4x^2 - 3xy$, we took the term as $(-3xy)$ and not as $(3xy)$. That is why we do not need to say that terms are 'added or subtracted' to form an expression; just 'added' is enough.

Factors of a term

We saw above that the expression $(4x^2 - 3xy)$ consists of two terms $4x^2$ and $-3xy$. The term $4x^2$ is a product of 4, x and x; we say that 4, x and x are the factors of the term $4x^2$. A term is a product of its factors. The term $-3xy$ is a product of the factors -3, x and y.



We can represent the terms and factors of the terms of an expression conveniently and elegantly by a tree diagram. The tree for the expression $(4x^2 - 3xy)$ is as shown in the adjacent figure.

> Note, in the tree diagram, we have used dotted lines for factors and continuous lines for terms. This is to avoid mixing them.

Let us draw a tree diagram for the expression $5xy + 10$.

The factors are such that they cannot be further factorised. Thus we do not write $5xy$ as $5 \times xy$, because xy can be further factorised. Similarly, if x^3 were a term, it would be written as $x \times x \times x$ and not $x^2 \times x$. Also, remember that 1 is not taken as a separate factor.

Try These

1. What are the terms in the following expressions? Show how the terms are formed. Draw a tree diagram for each expression:

   $8y + 3x^2$, $7mn - 4$, $2x^2y$.

2. Write three expression each having 4 terms.

Coefficients

We have learnt how to write a term as a product of factors. One of these factors may be numerical and the others algebraic (i.e., they contain variables). The numerical factor is said to be the numerical coefficient or simply the **coefficient** of the term. It is also said to be the coefficient of the rest of the term (which is obviously the product of algebraic factors of the term). Thus in $5xy$, 5 is the coefficient of the term. It is also the coefficient of xy. In the term $10xyz$, 10 is the coefficient of xyz, in the term $-7x^2y^2$, -7 is the coefficient of x^2y^2.

When the coefficient of a term is $+1$, it is usually omitted. For example, $1x$ is written as x; $1\,x^2y^2$ is written as x^2y^2 and so on. Also, the coefficient (-1) is indicated only by the minus sign. Thus $(-1)\,x$ is written as $-x$; $(-1)\,x^2\,y^2$ is written as $-x^2\,y^2$ and so on.

Try These

Identify the coefficients of the terms of following expressions:

$4x - 3y$, $a + b + 5$, $2y + 5$, $2xy$

Sometimes, the word 'coefficient' is used in a more general way. Thus we say that in the term $5xy$, 5 is the coefficient of xy, x is the coefficient of $5y$ and y is the coefficient of $5x$. In $10xy^2$, 10 is the coefficient of xy^2, x is the coefficient of $10y^2$ and y^2 is the coefficient of $10x$. Thus, in this more general way, a coefficient may be either a numerical factor or an algebraic factor or a product of two or more factors. It is said to be the coefficient of the product of the remaining factors.

Example 1 Identify, in the following expressions, terms which are not constants. Give their numerical coefficients:

$$xy + 4,\ 13 - y^2,\ 13 - y + 5y^2,\ 4p^2q - 3pq^2 + 5$$

SOLUTION

S. No.	Expression	Term (which is not a Constant)	Numerical Coefficient
(i)	$xy + 4$	xy	1
(ii)	$13 - y^2$	$-y^2$	-1
(iii)	$13 - y + 5y^2$	$-y$ $5y^2$	-1 5
(iv)	$4p^2q - 3pq^2 + 5$	$4p^2q$ $-3pq^2$	4 -3

EXAMPLE 2

(a) What are the coefficients of x in the following expressions?

$$4x - 3y,\ 8 - x + y,\ y^2x - y,\ 2z - 5xz$$

(b) What are the coefficients of y in the following expressions?

$$4x - 3y,\ 8 + yz,\ yz^2 + 5,\ my + m$$

SOLUTION

(a) In each expression we look for a term with x as a factor. The remaining part of that term is the coefficient of x.

S. No.	Expression	Term with Factor x	Coefficient of x
(i)	$4x - 3y$	$4x$	4
(ii)	$8 - x + y$	$-x$	-1
(iii)	$y^2x - y$	y^2x	y^2
(iv)	$2z - 5xz$	$-5xz$	$-5z$

(b) The method is similar to that in (a) above.

S. No.	Expression	Term with factor y	Coefficient of y
(i)	$4x - 3y$	$-3y$	-3
(ii)	$8 + yz$	yz	z
(iii)	$yz^2 + 5$	yz^2	z^2
(iv)	$my + m$	my	m

10.4 LIKE AND UNLIKE TERMS

When terms have the same algebraic factors, they are **like** terms. When terms have different algebraic factors, they are **unlike** terms. For example, in the expression $2xy - 3x + 5xy - 4$, look at the terms $2xy$ and $5xy$. The factors of $2xy$ are 2, x and y. The factors of $5xy$ are 5, x and y. Thus their algebraic (i.e., those which contain variables) factors are the same and

TRY THESE

Group the like terms together from the following:

$12x$, 12, $-25x$, -25, $-25y$, 1, x, $12y$, y

hence they are **like** terms. On the other hand the terms $2xy$ and $-3x$, have different algebraic factors. They are **unlike** terms. Similarly, the terms, $2xy$ and 4, are unlike terms. Also, the terms $-3x$ and 4 are unlike terms.

10.5 Monomials, Binomials, Trinomials and Polynomials

An expression with only one term is called a **monomial**; for example, $7xy$, $-5m$, $3z^2$, 4 etc.

TRY THESE

Classify the following expressions as a monomial, a binomial or a trinomial: a, $a + b$, $ab + a + b$, $ab + a + b - 5$, xy, $xy + 5$, $5x^2 - x + 2$, $4pq - 3q + 5p$, 7, $4m - 7n + 10$, $4mn + 7$.

An expression which contains two unlike terms is called a **binomial**; for example, $x + y$, $m - 5$, $mn + 4m$, $a^2 - b^2$ are binomials. The expression $10pq$ is not a binomial; it is a monomial. The expression $(a + b + 5)$ is not a binomial. It contains three terms.

An expression which contains three terms is called a **trinomial**; for example, the expressions $x + y + 7$, $ab + a + b$, $3x^2 - 5x + 2$, $m + n + 10$ are trinomials. The expression $ab + a + b + 5$ is, however not a trinomial; it contains four terms and not three. The expression $x + y + 5x$ is not a trinomial as the terms x and $5x$ are like terms.

In general, an expression with one or more terms is called a **polynomial**. Thus a monomial, a binomial and a trinomial are all polynomials.

EXAMPLE 3 State with reasons, which of the following pairs of terms are of like terms and which are of unlike terms:

(i) $7x$, $12y$ (ii) $15x$, $-21x$ (iii) $-4ab$, $7ba$ (iv) $3xy$, $3x$

(v) $6xy^2$, $9x^2y$ (vi) pq^2, $-4pq^2$ (vii) mn^2, $10mn$

SOLUTION

S. No.	Pair	Factors	Algebraic factors same or different	Like/ Unlike terms	Remarks
(i)	$7x$ $12y$	$7, x$ $12, y$	Different	Unlike	The variables in the terms are different.
(ii)	$15x$ $-21x$	$15, x$ $-21, x$	Same	Like	
(iii)	$-4ab$ $7ba$	$-4, a, b$ $7, a, b$	Same	Like	Remember $ab = ba$

(iv)	$3xy$ $3x$	$\left.\begin{matrix}3, x, y\\ 3, x\end{matrix}\right\}$	Different	Unlike	The variable y is only in one term.
(v)	$6xy^2$ $9x^2y$	$\left.\begin{matrix}6, x, y, y\\ 9, x, x, y\end{matrix}\right\}$	Different	Unlike	The variables in the two terms match, but their powers do not match.
(vi)	pq^2 $-4pq^2$	$\left.\begin{matrix}1, p, q, q\\ -4, p, q, q\end{matrix}\right\}$	Same	Like	Note, numerical factor 1 is not shown

Following simple steps will help you to decide whether the given terms are **like** *or* **unlike** *terms:*

(i) Ignore the numerical coefficients. Concentrate on the algebraic part of the terms.

(ii) Check the variables in the terms. They must be the same.

(iii) Next, check the powers of each variable in the terms. They must be the same.

Note that in deciding like terms, two things do not matter (1) the numerical coefficients of the terms and (2) the order in which the variables are multiplied in the terms.

EXERCISE 10.1

1. Get the algebraic expressions in the following cases using variables, constants and arithmetic operations.

(i) Subtraction of z from y.

(ii) One-half of the sum of numbers x and y.

(iii) The number z multiplied by itself.

(iv) One-fourth of the product of numbers p and q.

(v) Numbers x and y both squared and added.

(vi) Number 5 added to three times the product of numbers m and n.

(vii) Product of numbers y and z subtracted from 10.

(viii) Sum of numbers a and b subtracted from their product.

2. (i) Identify the terms and their factors in the following expressions

Show the terms and factors by tree diagrams.

(a) $x - 3$ (b) $1 + x + x^2$ (c) $y - y^3$

(d) $5xy^2 + 7x^2y$ (e) $-ab + 2b^2 - 3a^2$

(ii) Identify terms and factors in the expressions given below:

(a) $-4x + 5$ (b) $-4x + 5y$ (c) $5y + 3y^2$

(d) $xy + 2x^2y^2$ (e) $pq + q$ (f) $1.2\ ab - 2.4\ b + 3.6\ a$

(g) $\frac{3}{4}x + \frac{1}{4}$ (h) $0.1\ p^2 + 0.2\ q^2$

3. Identify the numerical coefficients of terms (other than constants) in the following expressions:

(i) $5 - 3t^2$ (ii) $1 + t + t^2 + t^3$ (iii) $x + 2xy + 3y$

(iv) $100m + 1000n$ (v) $-p^2q^2 + 7pq$ (vi) $1.2\ a + 0.8\ b$

(vii) $3.14\ r^2$ (viii) $2\ (l + b)$ (ix) $0.1\ y + 0.01\ y^2$

4. (a) Identify terms which contain x and give the coefficient of x.

(i) $y^2x + y$ (ii) $13y^2 - 8yx$ (iii) $x + y + 2$

(iv) $5 + z + zx$ (v) $1 + x + xy$ (vi) $12xy^2 + 25$

(vii) $7x + xy^2$

(b) Identify terms which contain y^2 and give the coefficient of y^2.

(i) $8 - xy^2$ (ii) $5y^2 + 7x$ (iii) $2x^2y - 15xy^2 + 7y^2$

5. Classify into monomials, binomials and trinomials.

(i) $4y - 7z$ (ii) y^2 (iii) $x + y - xy$ (iv) 100

(v) $ab - a - b$ (vi) $5 - 3t$ (vii) $4p^2q - 4pq^2$ (viii) $7mn$

(ix) $z^2 - 3z + 8$ (x) $a^2 + b^2$ (xi) $z^2 + z$

(xii) $1 + x + x^2$

6. State whether a given pair of terms is of like or unlike terms.

(i) $1, 100$ (ii) $-7x, \frac{5}{2}x$ (iii) $-29x, -29y$

(iv) $14xy, 42yx$ (v) $4m^2p, 4mp^2$ (vi) $12xz, 12x^2z^2$

7. Identify like terms in the following:

(a) $-xy^2, -4yx^2, 8x^2, 2xy^2, 7y, -11x^2, -100x, -11yx, 20x^2y,$
$-6x^2, y, 2xy, 3x$

(b) $10pq, 7p, 8q, -p^2q^2, -7qp, -100q, -23, 12q^2p^2, -5p^2, 41, 2405p, 78qp,$
$13p^2q, qp^2, 701p^2$

10.6 Finding the Value of an Expression

We know that the value of an algebraic expression depends on the values of the variables forming the expression. There are a number of situations in which we need to find the value of an expression, such as when we wish to check whether a particular value of a variable satisfies a given equation or not.

We find values of expressions, also, when we use formulas from geometry and from everyday mathematics. For example, the area of a square is l^2, where l is the length of a side of the square. If $l = 5$ cm., the area is 5^2 cm^2 or 25 cm^2; if the side is 10 cm, the area is 10^2 cm^2 or 100 cm^2 and so on. We shall see more such examples in the next section.

EXAMPLE 4 Find the values of the following expressions for $x = 2$.

(i) $x + 4$ (ii) $4x - 3$ (iii) $19 - 5x^2$

(iv) $100 - 10x^3$

SOLUTION Putting $x = 2$

(i) In $x + 4$, we get the value of $x + 4$, i.e.,

$x + 4 = 2 + 4 = 6$

(ii) In $4x - 3$, we get

$4x - 3 = (4 \times 2) - 3 = 8 - 3 = 5$

(iii) In $19 - 5x^2$, we get

$19 - 5x^2 = 19 - (5 \times 2^2) = 19 - (5 \times 4) = 19 - 20 = -1$

(iv) In $100 - 10x^3$, we get

$100 - 10x^3 = 100 - (10 \times 2^3) = 100 - (10 \times 8)$ (Note $2^3 = 8$)

$= 100 - 80 = 20$

EXAMPLE 5 Find the value of the following expressions when $n = -2$.

(i) $5n - 2$ (ii) $5n^2 + 5n - 2$ (iii) $n^3 + 5n^2 + 5n - 2$

SOLUTION

(i) Putting the value of $n = -2$, in $5n - 2$, we get,

$5(-2) - 2 = -10 - 2 = -12$

(ii) In $5n^2 + 5n - 2$, we have,

for $n = -2$, $5n - 2 = -12$

and $5n^2 = 5 \times (-2)^2 = 5 \times 4 = 20$ [as $(-2)^2 = 4$]

Combining,

$5n^2 + 5n - 2 = 20 - 12 = 8$

(iii) Now, for $n = -2$,

$5n^2 + 5n - 2 = 8$ and

$n^3 = (-2)^3 = (-2) \times (-2) \times (-2) = -8$

Combining,

$n^3 + 5n^2 + 5n - 2 = -8 + 8 = 0$

We shall now consider expressions of two variables, for example, $x + y$, xy. To work out the numerical value of an expression of two variables, we need to give the values of both variables. For example, the value of $(x + y)$, for $x = 3$ and $y = 5$, is $3 + 5 = 8$.

EXAMPLE 6 Find the value of the following expressions for $a = 3, b = 2$.

(i) $a + b$ (ii) $7a - 4b$ (iii) $a^2 + 2ab + b^2$

(iv) $a^3 - b^3$

SOLUTION Substituting $a = 3$ and $b = 2$ in

(i) $a + b$, we get

$a + b = 3 + 2 = 5$

(ii) $7a - 4b$, we get

$7a - 4b = 7 \times 3 - 4 \times 2 = 21 - 8 = 13.$

(iii) $a^2 + 2ab + b^2$, we get

$a^2 + 2ab + b^2 = 3^2 + 2 \times 3 \times 2 + 2^2 = 9 + 2 \times 6 + 4 = 9 + 12 + 4 = 25$

(iv) $a^3 - b^3$, we get

$a^3 - b^3 = 3^3 - 2^3 = 3 \times 3 \times 3 - 2 \times 2 \times 2 = 9 \times 3 - 4 \times 2 = 27 - 8 = 19$

EXERCISE 10.2

1. If $m = 2$, find the value of:

(i) $m - 2$ (ii) $3m - 5$ (iii) $9 - 5m$

(iv) $3m^2 - 2m - 7$ (v) $\frac{5m}{2}$ 4

2. If $p = -2$, find the value of:

(i) $4p + 7$ (ii) $-3p^2 + 4p + 7$ (iii) $-2p^3 - 3p^2 + 4p + 7$

3. Find the value of the following expressions, when $x = -1$:

(i) $2x - 7$ (ii) $-x + 2$ (iii) $x^2 + 2x + 1$

(iv) $2x^2 - x - 2$

4. If $a = 2, b = -2$, find the value of:

(i) $a^2 + b^2$ (ii) $a^2 + ab + b^2$ (iii) $a^2 - b^2$

5. When $a = 0, b = -1$, find the value of the given expressions:

(i) $2a + 2b$ (ii) $2a^2 + b^2 + 1$ (iii) $2a^2b + 2ab^2 + ab$

(iv) $a^2 + ab + 2$

6. Simplify the expressions and find the value if x is equal to 2

(i) $x + 7 + 4(x - 5)$ (ii) $3(x + 2) + 5x - 7$

(iii) $6x + 5(x - 2)$ (iv) $4(2x - 1) + 3x + 11$

7. Simplify these expressions and find their values if $x = 3, a = -1, b = -2$.

(i) $3x - 5 - x + 9$ (ii) $2 - 8x + 4x + 4$

(iii) $3a + 5 - 8a + 1$ (iv) $10 - 3b - 4 - 5b$

(v) $2a - 2b - 4 - 5 + a$

8. (i) If $z = 10$, find the value of $z^3 - 3(z - 10)$.

(ii) If $p = -10$, find the value of $p^2 - 2p - 100$

9. What should be the value of a if the value of $2x^2 + x - a$ equals to 5, when $x = 0$?

10. Simplify the expression and find its value when $a = 5$ and $b = -3$.

$$2(a^2 + ab) + 3 - ab$$

What have We Discussed?

1. **Algebraic expressions** are formed from **variables** and **constants**. We use the operations of **addition, subtraction, multiplication** and **division** on the variables and constants to form expressions. For example, the expression $4xy + 7$ is formed from the variables x and y and constants 4 and 7. The constant 4 and the variables x and y are multiplied to give the product $4xy$ and the constant 7 is added to this product to give the expression.

2. Expressions are made up of **terms**. Terms are **added** to make an expression. For example, the addition of the terms $4xy$ and 7 gives the expression $4xy + 7$.

3. A term is a **product of factors**. The term $4xy$ in the expression $4xy + 7$ is a product of factors x, y and 4. Factors containing variables are said to be **algebraic factors**.

4. The **coefficient** is the numerical factor in the term. Sometimes anyone factor in a term is called the coefficient of the remaining part of the term.

5. Any expression with one or more terms is called a **polynomial**. Specifically a one term expression is called a **monomial**; a two-term expression is called a **binomial**; and a three-term expression is called a **trinomial**.

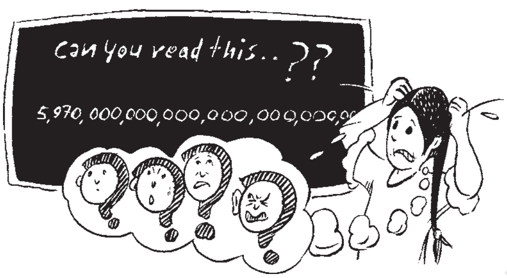
6. Terms which have the same algebraic factors are **like** terms. Terms which have different algebraic factors are **unlike** terms. Thus, terms $4xy$ and $-3xy$ are like terms; but terms $4xy$ and $-3x$ are not like terms.

7. In situations such as solving an equation and using a formula, we have to **find the value of an expression**. The value of the expression depends on the value of the variable from which the expression is formed. Thus, the value of $7x - 3$ for $x = 5$ is 32, since $7(5) - 3 = 35 - 3 = 32$.

Chapter 11

Exponents and Powers

11.1 Introduction



Do you know what the mass of earth is? It is 5,970,000,000,000,000,000,000,000 kg!
Can you read this number?
Mass of Uranus is 86,800,000,000,000,000,000,000,000 kg.
Which has greater mass, Earth or Uranus?

Distance between Sun and Saturn is 1,433,500,000,000 m and distance between Saturn and Uranus is 1,439,000,000,000 m. Can you read these numbers? Which distance is less?

These very large numbers are difficult to read, understand and compare. To make these numbers easy to read, understand and compare, we use exponents. In this Chapter, we shall learn about exponents and also learn how to use them.

11.2 Exponents

We can write large numbers in a shorter form using exponents.

Observe $\quad 10,000 = 10 \times 10 \times 10 \times 10 = 10^4$

The short notation 10^4 stands for the product $10\times10\times10\times10$. Here '10' is called the **base** and '4' the **exponent**. **The number 10^4 is read as 10 raised to the power of 4** or simply as **fourth power of 10. 10^4** is called the **exponential form** of 10,000.

We can similarly express 1,000 as a power of 10. Note that

$$1000 = 10 \times 10 \times 10 = 10^3$$

Here again, 10^3 is the exponential form of 1,000.

Similarly, $\quad 1,00,000 = 10 \times 10 \times 10 \times 10 \times 10 = 10^5$

10^5 is the exponential form of 1,00,000

In both these examples, the base is 10; in case of 10^3, the exponent is 3 and in case of 10^5 the exponent is 5.

We have used numbers like 10, 100, 1000 etc., while writing numbers in an expanded form. For example, $47561 = 4 \times 10000 + 7 \times 1000 + 5 \times 100 + 6 \times 10 + 1$

This can be written as $4 \times 10^4 + 7 \times 10^3 + 5 \times 10^2 + 6 \times 10 + 1$.

Try writing these numbers in the same way 172, 5642, 6374.

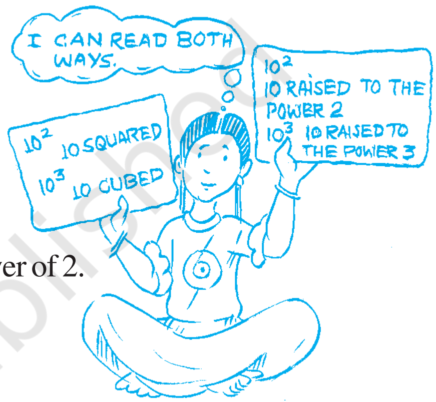
In all the above given examples, we have seen numbers whose base is 10. However the base can be any other number also. For example:

$81 = 3 \times 3 \times 3 \times 3$ can be written as $81 = 3^4$, here 3 is the base and 4 is the exponent.

Some powers have special names. For example,

10^2, which is 10 raised to the power 2, also read as '10 squared' and

10^3, which is 10 raised to the power 3, also read as '10 cubed'.



Can you tell what 5^3 (5 cubed) means?

$$5^3 = 5 \times 5 \times 5 = 125$$

So, we can say 125 is the third power of 5.

What is the exponent and the base in 5^3?

Similarly, $2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$, which is the fifth power of 2.

In 2^5, 2 is the base and 5 is the exponent.

In the same way, $243 = 3 \times 3 \times 3 \times 3 \times 3 = 3^5$

$$64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^6$$

$$625 = 5 \times 5 \times 5 \times 5 = 5^4$$

TRY THESE

Find five more such examples, where a number is expressed in exponential form. Also identify the base and the exponent in each case.

You can also extend this way of writing when the base is a negative integer.

What does $(-2)^3$ mean?

It is $(-2)^3 = (-2) \times (-2) \times (-2) = -8$

Is $(-2)^4 = 16$? Check it.

Instead of taking a fixed number let us take any integer a as the base, and write the numbers as,

$a \times a = a^2$ (read as 'a squared' or 'a raised to the power 2')

$a \times a \times a = a^3$ (read as 'a cubed' or 'a raised to the power 3')

$a \times a \times a \times a = a^4$ (read as a raised to the power 4 or the 4th power of a)

....

$a \times a \times a \times a \times a \times a \times a = a^7$ (read as a raised to the power 7 or the 7th power of a) and so on.

$a \times a \times a \times b \times b$ can be expressed as a^3b^2 (read as a cubed b squared)

TRY THESE

Express:

(i) 729 as a power of 3

(ii) 128 as a power of 2

(iii) 343 as a power of 7

$a \times a \times b \times b \times b \times b$ can be expressed as a^2b^4 (read as a squared into b raised to the power of 4).

EXAMPLE 1 Express 256 as a power 2.

SOLUTION We have $256 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$.

So we can say that $256 = 2^8$

EXAMPLE 2 Which one is greater 2^3 or 3^2?

SOLUTION We have, $2^3 = 2 \times 2 \times 2 = 8$ and

$3^2 = 3 \times 3 = 9$.

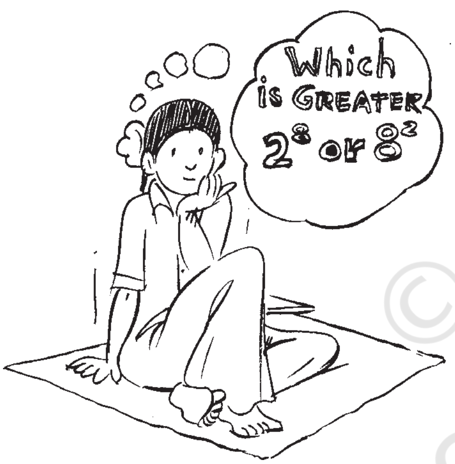
Since $9 > 8$, so, 3^2 is greater than 2^3

EXAMPLE 3 Which one is greater 8^2 or 2^8?

SOLUTION $8^2 = 8 \times 8 = 64$

$2^8 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 256$

Clearly, $2^8 > 8^2$



EXAMPLE 4 Expand $a^3 b^2$, $a^2 b^3$, $b^2 a^3$, $b^3 a^2$. Are they all same?

SOLUTION $a^3 b^2 = a^3 \times b^2$

$= (a \times a \times a) \times (b \times b)$

$= a \times a \times a \times b \times b$

$a^2 b^3 = a^2 \times b^3$

$= a \times a \times b \times b \times b$

$b^2 a^3 = b^2 \times a^3$

$= b \times b \times a \times a \times a$

$b^3 a^2 = b^3 \times a^2$

$= b \times b \times b \times a \times a$

Note that in the case of terms $a^3 b^2$ and $a^2 b^3$ the powers of a and b are different. Thus $a^3 b^2$ and $a^2 b^3$ are different.

On the other hand, $a^3 b^2$ and $b^2 a^3$ are the same, since the powers of a and b in these two terms are the same. The order of factors does not matter.

Thus, $a^3 b^2 = a^3 \times b^2 = b^2 \times a^3 = b^2 a^3$. Similarly, $a^2 b^3$ and $b^3 a^2$ are the same.

EXAMPLE 5 Express the following numbers as a product of powers of prime factors:

(i) 72 (ii) 432 (iii) 1000 (iv) 16000

SOLUTION

2	72
2	36
2	18
3	9
	3

(i) $72 = 2 \times 36 = 2 \times 2 \times 18$

$= 2 \times 2 \times 2 \times 9$

$= 2 \times 2 \times 2 \times 3 \times 3 = 2^3 \times 3^2$

Thus, $72 = 2^3 \times 3^2$ (required prime factor product form)

(ii) $432 = 2 \times 216 = 2 \times 2 \times 108 = 2 \times 2 \times 2 \times 54$

$= 2 \times 2 \times 2 \times 2 \times 27 = 2 \times 2 \times 2 \times 2 \times 3 \times 9$

$= 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3$

or $432 = 2^4 \times 3^3$ (required form)

(iii) $1000 = 2 \times 500 = 2 \times 2 \times 250 = 2 \times 2 \times 2 \times 125$

$= 2 \times 2 \times 2 \times 5 \times 25 = 2 \times 2 \times 2 \times 5 \times 5 \times 5$

or $1000 = 2^3 \times 5^3$

> Atul wants to solve this example in another way:
>
> $1000 = 10 \times 100 = 10 \times 10 \times 10$
>
> $= (2 \times 5) \times (2 \times 5) \times (2 \times 5)$ (Since $10 = 2 \times 5$)
>
> $= 2 \times 5 \times 2 \times 5 \times 2 \times 5 = 2 \times 2 \times 2 \times 5 \times 5 \times 5$
>
> or $1000 = 2^3 \times 5^3$
>
> Is Atul's method correct?

(iv) $16{,}000 = 16 \times 1000 = (2 \times 2 \times 2 \times 2) \times 1000 = 2^4 \times 10^3$ (as $16 = 2 \times 2 \times 2 \times 2$)

$= (2 \times 2 \times 2 \times 2) \times (2 \times 2 \times 2 \times 5 \times 5 \times 5) = 2^4 \times 2^3 \times 5^3$

(Since $1000 = 2 \times 2 \times 2 \times 5 \times 5 \times 5$)

$= (2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2) \times (5 \times 5 \times 5)$

or, $16{,}000 = 2^7 \times 5^3$

EXAMPLE 6 Work out $(1)^5$, $(-1)^3$, $(-1)^4$, $(-10)^3$, $(-5)^4$.

SOLUTION

(i) We have $(1)^5 = 1 \times 1 \times 1 \times 1 \times 1 = 1$

In fact, you will realise that 1 raised to any power is 1.

(ii) $(-1)^3 = (-1) \times (-1) \times (-1) = 1 \times (-1) = -1$

(iii) $(-1)^4 = (-1) \times (-1) \times (-1) \times (-1) = 1 \times 1 = 1$

You may check that (-1) raised to any **odd** power is (-1), and (-1) raised to any **even** power is $(+1)$.

> $(-1)^{\text{odd number}} = -1$
>
> $(-1)^{\text{even number}} = +1$

(iv) $(-10)^3 = (-10) \times (-10) \times (-10) = 100 \times (-10) = -1000$

(v) $(-5)^4 = (-5) \times (-5) \times (-5) \times (-5) = 25 \times 25 = 625$

EXERCISE 11.1

1. Find the value of:

(i) 2^6 (ii) 9^3 (iii) 11^2 (iv) 5^4

2. Express the following in exponential form:

(i) $6 \times 6 \times 6 \times 6$ (ii) $t \times t$ (iii) $b \times b \times b \times b$

(iv) $5 \times 5 \times 7 \times 7 \times 7$ (v) $2 \times 2 \times a \times a$ (vi) $a \times a \times a \times c \times c \times c \times c \times d$

3. Express each of the following numbers using exponential notation:

(i) 512 (ii) 343 (iii) 729 (iv) 3125

4. Identify the greater number, wherever possible, in each of the following?

(i) 4^3 or 3^4 (ii) 5^3 or 3^5 (iii) 2^8 or 8^2

(iv) 100^2 or 2^{100} (v) 2^{10} or 10^2

5. Express each of the following as product of powers of their prime factors:

(i) 648 (ii) 405 (iii) 540 (iv) 3,600

6. Simplify :

(i) 2×10^3 (ii) $7^2 \times 2^2$ (iii) $2^3 \times 5$ (iv) 3×4^4

(v) 0×10^2 (vi) $5^2 \times 3^3$ (vii) $2^4 \times 3^2$ (viii) $3^2 \times 10^4$

7. Simplify :

(i) $(-4)^3$ (ii) $(-3) \times (-2)^3$ (iii) $(-3)^2 \times (-5)^2$

(iv) $(-2)^3 \times (-10)^3$

8. Compare the following numbers:

(i) 2.7×10^{12} ; 1.5×10^8 (ii) 4×10^{14} ; 3×10^{17}

11.3 Laws of Exponents

11.3.1 Multiplying Powers with the Same Base

(i) Let us calculate $2^2 \times 2^3$

$$2^2 \times 2^3 = (2 \times 2) \times (2 \times 2 \times 2)$$
$$= 2 \times 2 \times 2 \times 2 \times 2 = 2^5 = 2^{2+3}$$

Note that the base in 2^2 and 2^3 is same and the sum of the exponents, i.e., 2 and 3 is 5

(ii) $(-3)^4 \times (-3)^3 = [(-3) \times (-3) \times (-3) \times (-3)] \times [(-3) \times (-3) \times (-3)]$

$$= (-3) \times (-3) \times (-3) \times (-3) \times (-3) \times (-3) \times (-3)$$
$$= (-3)^7$$
$$= (-3)^{4+3}$$

Again, note that the base is same and the sum of exponents, i.e., 4 and 3, is 7

(iii) $a^2 \times a^4 = (a \times a) \times (a \times a \times a \times a)$

$$= a \times a \times a \times a \times a \times a = a^6$$

(Note: the base is the same and the sum of the exponents is $2 + 4 = 6$)

Similarly, verify:

$$4^2 \times 4^2 = 4^{2+2}$$
$$3^2 \times 3^3 = 3^{2+3}$$

Can you write the appropriate number in the box.

$$(-11)^2 \times (-11)^6 = \quad (-11)^{\square}$$

$$b^2 \times b^3 = b^{\square} \text{ (Remember, base is same; } b \text{ is any integer).}$$

$$c^3 \times c^4 = c^{\square} \text{ (} c \text{ is any integer)}$$

$$d^{10} \times d^{20} = d^{\square}$$

From this we can generalise that for any non-zero integer a, where m and n are whole numbers,

$$a^m \times a^n = a^{m+n}$$

TRY THESE

Simplify and write in exponential form:

(i) $2^5 \times 2^3$

(ii) $p^3 \times p^2$

(iii) $4^3 \times 4^2$

(iv) $a^3 \times a^2 \times a^7$

(v) $5^3 \times 5^7 \times 5^{12}$

(vi) $(-4)^{100} \times (-4)^{20}$

Caution!

Consider $2^3 \times 3^2$

Can you add the exponents? No! Do you see 'why'? The base of 2^3 is 2 and base of 3^2 is 3. The bases are not same.

11.3.2 Dividing Powers with the Same Base

Let us simplify $3^7 \div 3^4$?

$$3^7 \div 3^4 = \frac{3^7}{3^4} = \frac{3\times3\times3\times3\times3\times3\times3}{3\times3\times3\times3}$$

$$= 3 \times 3 \times 3 = 3^3 = 3^{7-4}$$

Thus $\quad 3^7 \div 3^4 = 3^{7-4}$

(Note, in 3^7 and 3^4 the base is same and $3^7 \div 3^4$ becomes 3^{7-4})

Similarly,

$$5^6 \div 5^2 = \frac{5^6}{5^2} = \frac{5\times5\times5\times5\times5\times5}{5\times5}$$

$$= 5 \times 5 \times 5 \times 5 = 5^4 = 5^{6-2}$$

or $\quad 5^6 \div 5^2 = 5^{6-2}$

Let a be a non-zero integer, then,

$$a^4 \div a^2 = \frac{a^4}{a^2} = \frac{a\times a\times a\times a}{a\times a} = a \times a = a^2 = a^{4\;\;2}$$

or $\quad a^4 \div a^2 = a^{4-2}$

Now can you answer quickly?

$$10^8 \div 10^3 = 10^{8-3} = 10^5$$

$$7^9 \div 7^6 = 7^{\square}$$

$$a^8 \div a^5 = a^{\square}$$

Try These

Simplify and write in exponential form: (eg., $11^6 \div 11^2 = 11^4$)

(i) $2^9 \div 2^3$ (ii) $10^8 \div 10^4$
(iii) $9^{11} \div 9^7$ (iv) $20^{15} \div 20^{13}$
(v) $7^{13} \div 7^{10}$

For non-zero integers b and c,

$$b^{10} \div b^5 = b^{\square}$$

$$c^{100} \div c^{90} = c^{\square}$$

In general, for any non-zero integer a,

$$a^m \div a^n = a^{m-n}$$

where m and n are whole numbers and $m > n$.

11.3.3 Taking Power of a Power

Consider the following

Simplify $\left(2^3\right)^2$; $\left(3^2\right)^4$

Now, $\left(2^3\right)^2$ means 2^3 is multiplied two times with itself.

$$\left(2^3\right)^2 = 2^3 \times 2^3$$

$$= 2^{3+3} \text{ (Since } a^m \times a^n = a^{m+n}\text{)}$$

$$= 2^6 = 2^{3 \times 2}$$

Thus $\left(2^3\right)^2 = 2^{3\times2}$

Similarly $\left(3^2\right)^4 = 3^2 \times 3^2 \times 3^2 \times 3^2$

$$= 3^{2+2+2+2}$$

$$= 3^8 \text{ (Observe 8 is the product of 2 and 4).}$$

$$= 3^{2 \times 4}$$

Can you tell what would $\left(7^2\right)^{10}$ would be equal to?

So $\left(2^3\right)^2 = 2^{3 \times 2} = 2^6$

$$\left(3^2\right)^4 = 3^{2 \times 4} = 3^8$$

$$= 7^{2 \times 10} = 7^{20}$$

$$\left(a^2\right)^3 = a^{2 \times 3} = a^6$$

$$= a^{m \times 3} = a^{3m}$$

From this we can generalise for any non-zero integer 'a', where 'm' and 'n' are whole numbers,

$$\left(a^m\right)^n = a^{mn}$$

Try These

Simplify and write the answer in exponential form:

(i) $\left(6^2\right)^4$ (ii) $\left(2^2\right)^{100}$

(iii) $\left(7^{50}\right)^2$ (iv) $\left(5^3\right)^7$

Example 7 Can you tell which one is greater $(5^2) \times 3$ or $\left(5^2\right)^3$?

Solution $(5^2) \times 3$ means 5^2 is multiplied by 3 i.e., $5 \times 5 \times 3 = 75$

but $\left(5^2\right)^3$ means 5^2 is multiplied by itself three times i.e. ,

$$5^2 \times 5^2 \times 5^2 = 5^6 = 15{,}625$$

Therefore $\qquad (5^2)^3 > (5^2) \times 3$

11.3.4 Multiplying Powers with the Same Exponents

Can you simplify $2^3 \times 3^3$? Notice that here the two terms 2^3 and 3^3 have different bases, but the same exponents.

Now,
$$\begin{aligned}
2^3 \times 3^3 &= (2 \times 2 \times 2) \times (3 \times 3 \times 3)\\
&= (2 \times 3) \times (2 \times 3) \times (2 \times 3)\\
&= 6 \times 6 \times 6\\
&= 6^3 \quad \text{(Observe 6 is the product of bases 2 and 3)}
\end{aligned}$$

Consider $4^4 \times 3^4$
$$\begin{aligned}
&= (4 \times 4 \times 4 \times 4) \times (3 \times 3 \times 3 \times 3)\\
&= (4 \times 3) \times (4 \times 3) \times (4 \times 3) \times (4 \times 3)\\
&= 12 \times 12 \times 12 \times 12\\
&= 12^4
\end{aligned}$$

Consider, also, $3^2 \times a^2$
$$\begin{aligned}
&= (3 \times 3) \times (a \times a)\\
&= (3 \times a) \times (3 \times a)\\
&= (3 \times a)^2\\
&= (3a)^2 \qquad \text{(Note: } 3 \times a = 3a \text{)}
\end{aligned}$$

Similarly, $a^4 \times b^4$
$$\begin{aligned}
&= (a \times a \times a \times a) \times (b \times b \times b \times b)\\
&= (a \times b) \times (a \times b) \times (a \times b) \times (a \times b)\\
&= (a \times b)^4\\
&= (ab)^4 \qquad \text{(Note } a \times b = ab\text{)}
\end{aligned}$$

Try These

Put into another form using $a^m \times b^m = (ab)^m$:

(i) $4^3 \times 2^3$ (ii) $2^5 \times b^5$

(iii) $a^2 \times t^2$ (iv) $5^6 \times (-2)^6$

(v) $(-2)^4 \times (-3)^4$

In general, for any non-zero integer a

$$\boxed{a^m \times b^m = (ab)^m} \qquad \text{(where } m \text{ is any whole number)}$$

Example 8 Express the following terms in the exponential form:

(i) $(2 \times 3)^5$ (ii) $(2a)^4$ (iii) $(-4m)^3$

Solution

(i) $(2 \times 3)^5 = (2 \times 3) \times (2 \times 3) \times (2 \times 3) \times (2 \times 3) \times (2 \times 3)$
$= (2 \times 2 \times 2 \times 2 \times 2) \times (3 \times 3 \times 3 \times 3 \times 3)$
$= 2^5 \times 3^5$

(ii) $(2a)^4 = 2a \times 2a \times 2a \times 2a$

$= (2 \times 2 \times 2 \times 2) \times (a \times a \times a \times a)$

$= 2^4 \times a^4$

(iii) $(-4m)^3 = (-4 \times m)^3$

$= (-4 \times m) \times (-4 \times m) \times (-4 \times m)$

$= (-4) \times (-4) \times (-4) \times (m \times m \times m) = (-4)^3 \times (m)^3$

11.3.5 Dividing Powers with the Same Exponents

Observe the following simplifications:

(i) $$\frac{2^4}{3^4} = \frac{2\times2\times2\times2}{3\times3\times3\times3} = \frac{2}{3}\times\frac{2}{3}\times\frac{2}{3}\times\frac{2}{3} = \left(\frac{2}{3}\right)^4$$

(ii) $$\frac{a^3}{b^3} = \frac{a\times a\times a}{b\times b\times b} = \frac{a}{b}\times\frac{a}{b}\times\frac{a}{b} = \left(\frac{a}{b}\right)^3$$

From these examples we may generalise

$$a^m \div b^m = \frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m$$ where a and b are any non zero integers and m is a whole number.

TRY THESE

Put into another form using $a^m \div b^m = \left(\frac{a}{b}\right)^m$:

(i) $4^5 \div 3^5$
(ii) $2^5 \div b^5$
(iii) $(-2)^3 \div b^3$
(iv) $p^4 \div q^4$
(v) $5^6 \div (-2)^6$

EXAMPLE 9 Expand: (i) $\left(\frac{3}{5}\right)^4$ (ii) $\left(\frac{-4}{7}\right)^5$

SOLUTION

(i) $$\left(\frac{3}{5}\right)^4 = \frac{3^4}{5^4} = \frac{3\times3\times3\times3}{5\times5\times5\times5}$$

(ii) $$\left(\frac{-4}{7}\right)^5 = \frac{(-4)^5}{7^5} = \frac{(-4)\times(-4)\times(-4)\times(-4)\times(-4)}{7\times7\times7\times7\times7}$$

- **Numbers with exponent zero**

Can you tell what $\frac{3^5}{3^5}$ equals to?

$$\frac{3^5}{3^5} = \frac{3\times3\times3\times3\times3}{3\times3\times3\times3\times3} = 1$$

by using laws of exponents

What is a^0?

Obeserve the following pattern:

$2^6 = 64$
$2^5 = 32$
$2^4 = 16$
$2^3 = 8$
$2^2 = ?$
$2^1 = ?$
$2^0 = ?$

You can guess the value of 2^0 by just studying the pattern!

You find that $2^0 = 1$

If you start from $3^6 = 729$, and proceed as shown above finding 3^5, 3^4, 3^3,... etc, what will be $3^0 = ?$

$$3^5 \div 3^5 = 3^{5-5} = 3^0$$

So $3^0 = 1$

Can you tell what 7^0 is equal to?

$$7^3 \div 7^3 = 7^{3-3} = 7^0$$

And $\dfrac{7^3}{7^3} = \dfrac{7 \times 7 \times 7}{7 \times 7 \times 7} = 1$

Therefore $7^0 = 1$

Similarly $a^3 \div a^3 = a^{3-3} = a^0$

And $a^3 \div a^3 = \dfrac{a^3}{a^3} = \dfrac{a \times a \times a}{a \times a \times a} = 1$

Thus $a^0 = 1$ (for any non-zero integer a)

So, we can say that any number (except 0) raised to the power (or exponent) 0 is 1.

11.4 Miscellaneous Examples using the Laws of Exponents

Let us solve some examples using rules of exponents developed.

Example 10 Write exponential form for $8 \times 8 \times 8 \times 8$ taking base as 2.

Solution We have, $8 \times 8 \times 8 \times 8 = 8^4$

But we know that $8 = 2 \times 2 \times 2 = 2^3$

Therefore $8^4 = (2^3)^4 = 2^3 \times 2^3 \times 2^3 \times 2^3$

$= 2^{3 \times 4}$ [You may also use $(a^m)^n = a^{mn}$]

$= 2^{12}$

Example 11 Simplify and write the answer in the exponential form.

(i) $\left(\dfrac{3^7}{3^2}\right) \times 3^5$ (ii) $2^3 \times 2^2 \times 5^5$ (iii) $(6^2 \times 6^4) \div 6^3$

(iv) $[(2^2)^3 \times 3^6] \times 5^6$ (v) $8^2 \div 2^3$

Solution

(i) $\left(\dfrac{3^7}{3^2}\right) \times 3^5 = \left(3^{7-2}\right) \times 3^5$

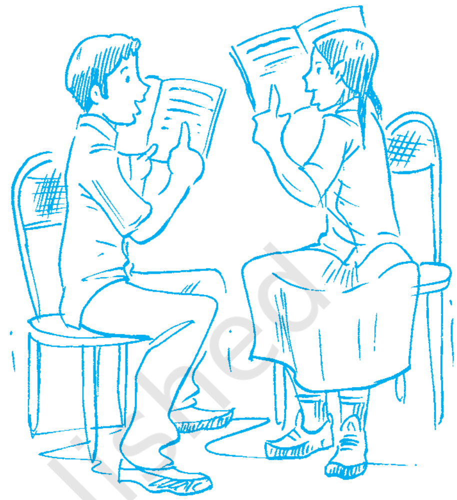
$= 3^5 \times 3^5 = 3^{5+5} = 3^{10}$

(ii) $2^3 \times 2^2 \times 5^5 = 2^{3+2} \times 5^5$

$= 2^5 \times 5^5 = (2 \times 5)^5 = 10^5$

(iii) $\left(6^2 \times 6^4\right) \div 6^3 = 6^{2+4} \div 6^3$

$$= \frac{6^6}{6^3} = 6^{6-3} = 6^3$$



(iv) $\left[\left(2^2\right)^3 \times 3^6\right] \times 5^6 = [2^6 \times 3^6] \times 5^6$

$= \left(2 \times 3\right)^6 \times 5^6$

$= \left(2 \times 3 \times 5\right)^6 = 30^6$

(v) $8 = 2 \times 2 \times 2 = 2^3$

Therefore $8^2 \div 2^3 = (2^3)^2 \div 2^3$

$= 2^6 \div 2^3 = 2^{6-3} = 2^3$

Example 12 Simplify:

(i) $\dfrac{12^4 \times 9^3 \times 4}{6^3 \times 8^2 \times 27}$ (ii) $2^3 \times a^3 \times 5a^4$ (iii) $\dfrac{2 \times 3^4 \times 2^5}{9 \times 4^2}$

Solution

(i) We have

$$\frac{12^4 \times 9^3 \times 4}{6^3 \times 8^2 \times 27} = \frac{(2^2 \times 3)^4 \times (3^2)^3 \times 2^2}{(2 \times 3)^3 \times (2^3)^2 \times 3^3}$$

$$= \frac{\left(2^2\right)^4 \times (3)^4 \times 3^{2\times3} \times 2^2}{2^3 \times 3^3 \times 2^{2\times3} \times 3^3} = \frac{2^8 \times 2^2 \times 3^4 \times 3^6}{2^3 \times 2^6 \times 3^3 \times 3^3}$$

$$= \frac{2^{8+2} \times 3^{4+6}}{2^{3+6} \times 3^{3+3}} = \frac{2^{10} \times 3^{10}}{2^9 \times 3^6}$$

$= 2^{10-9} \times 3^{10-6} = 2^1 \times 3^4$

$= 2 \times 81 = 162$

(ii) $2^3 \times a^3 \times 5a^4 = 2^3 \times a^3 \times 5 \times a^4$

$= 2^3 \times 5 \times a^3 \times a^4 = 8 \times 5 \times a^{3+4}$

$= 40\, a^7$

(ii) $\dfrac{2\times 3^4\times 2^5}{9\times 4^2} = \dfrac{2\times 3^4\times 2^5}{3^2\times\left(2^2\right)^2} = \dfrac{2\times 2^5\times 3^4}{3^2\times 2^{2\times 2}}$

$= \dfrac{2^{1+5}\times 3^4}{2^4\times 3^2} = \dfrac{2^6\times 3^4}{2^4\times 3^2} = 2^{6-4}\times 3^{4-2}$

$= 2^2\times 3^2 = 4\times 9 = 36$

Note: In most of the examples that we have taken in this Chapter, the base of a power was taken an integer. But all the results of the chapter apply equally well to a base which is a rational number.

EXERCISE 11.2

1. Using laws of exponents, simplify and write the answer in exponential form:

(i) $3^2\times 3^4\times 3^8$ (ii) $6^{15}\div 6^{10}$ (iii) $a^3\times a^2$

(iv) $7^x\times 7^2$ (v) $\left(5^2\right)^3\div 5^3$ (vi) $2^5\times 5^5$

(vii) $a^4\times b^4$ (viii) $\left(3^4\right)^3$ (ix) $\left(2^{20}\div 2^{15}\right)\times 2^3$

(x) $8^t\div 8^2$

2. Simplify and express each of the following in exponential form:

(i) $\dfrac{2^3\times 3^4\times 4}{3\times 32}$ (ii) $\left(\left(5^2\right)^3\times 5^4\right)\div 5^7$ (iii) $25^4\div 5^3$

(iv) $\dfrac{3\times 7^2\times 11^8}{21\times 11^3}$ (v) $\dfrac{3^7}{3^4\times 3^3}$ (vi) $2^0+3^0+4^0$

(vii) $2^0\times 3^0\times 4^0$ (viii) $(3^0+2^0)\times 5^0$ (ix) $\dfrac{2^8\times a^5}{4^3\times a^3}$

(x) $\left(\dfrac{a^5}{a^3}\right)\times a^8$ (xi) $\dfrac{4^5\times a^8b^3}{4^5\times a^5b^2}$ (xii) $\left(2^3\times 2\right)^2$

3. Say true or false and justify your answer:

(i) $10\times 10^{11} = 100^{11}$ (ii) $2^3 > 5^2$ (iii) $2^3\times 3^2 = 6^5$

(iv) $3^0 = (1000)^0$

4. Express each of the following as a product of prime factors only in exponential form:

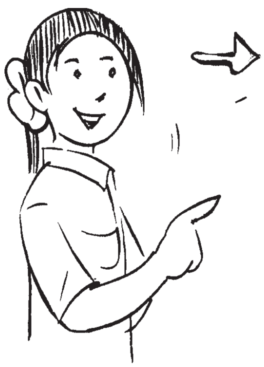
(i) 108×192 (ii) 270 (iii) 729×64

(iv) 768

5. Simplify:

(i) $\dfrac{(2^5)^2 \times 7^3}{8^3 \times 7}$ (ii) $\dfrac{25 \times 5^2 \times t^8}{10^3 \times t^4}$ (iii) $\dfrac{3^5 \times 10^5 \times 25}{5^7 \times 6^5}$

11.5 Decimal Number System



Let us look at the expansion of 47561, which we already know:

$$47561 = 4 \times 10000 + 7 \times 1000 + 5 \times 100 + 6 \times 10 + 1$$

We can express it using powers of 10 in the exponent form:

Therefore, $47561 = 4 \times 10^4 + 7 \times 10^3 + 5 \times 10^2 + 6 \times 10^1 + 1 \times 10^0$

(Note $10,000 = 10^4$, $1000 = 10^3$, $100 = 10^2$, $10 = 10^1$ and $1 = 10^0$)

Let us expand another number:

$$\begin{aligned} 104278 &= 1 \times 100,000 + 0 \times 10,000 + 4 \times 1000 + 2 \times 100 + 7 \times 10 + 8 \times 1 \\ &= 1 \times 10^5 + 0 \times 10^4 + 4 \times 10^3 + 2 \times 10^2 + 7 \times 10^1 + 8 \times 10^0 \\ &= 1 \times 10^5 + 4 \times 10^3 + 2 \times 10^2 + 7 \times 10^1 + 8 \times 10^0 \end{aligned}$$

Notice how the exponents of 10 start from a maximum value of 5 and go on decreasing by 1 at a step from the left to the right upto 0.

11.6 Expressing Large Numbers in the Standard Form

Let us now go back to the beginning of the chapter. We said that large numbers can be conveniently expressed using exponents. We have not as yet shown this. We shall do so now.

1. Sun is located 300,000,000,000,000,000,000 m from the centre of our Milky Way Galaxy.
2. Number of stars in our Galaxy is 100,000,000,000.
3. Mass of the Earth is 5,976,000,000,000,000,000,000,000 kg.

These numbers are not convenient to write and read. To make it convenient we use powers.

Observe the following:

$$59 = 5.9 \times 10 = 5.9 \times 10^1$$
$$590 = 5.9 \times 100 = 5.9 \times 10^2$$
$$5900 = 5.9 \times 1000 = 5.9 \times 10^3$$
$$59000 = 5.9 \times 10000 = 5.9 \times 10^4 \text{ and so on.}$$

Try These

Expand by expressing powers of 10 in the exponential form:

(i) 172
(ii) 5,643
(iii) 56,439
(iv) 1,76,428

We have expressed all these numbers in the **standard form**. Any number can be expressed as a decimal number between 1.0 and 10.0 including 1.0 multiplied by a power of 10. Such a form of a number is called its **standard form**. Thus,

$5,985 = 5.985 \times 1,000 = 5.985 \times 10^3$ is the standard form of 5,985.

Note, 5,985 can also be expressed as 59.85×100 or 59.85×10^2. But these are not the standard forms, of 5,985. Similarly, $5,985 = 0.5985 \times 10,000 = 0.5985 \times 10^4$ is also not the standard form of 5,985.

We are now ready to express the large numbers we came across at the beginning of the chapter in this form.

The, distance of Sun from the centre of our Galaxy i.e.,

300,000,000,000,000,000,000 m can be written as

$3.0 \times 100,000,000,000,000,000,000 = 3.0 \times 10^{20}$ m

Now, can you express 40,000,000,000 in the similar way?

Count the number of zeros in it. It is 10.

So, $40,000,000,000 = 4.0 \times 10^{10}$

Mass of the Earth = 5,976,000,000,000,000,000,000,000 kg

$= 5.976 \times 10^{24}$ kg

Do you agree with the fact, that the number when written in the standard form is much easier to read, understand and compare than when the number is written with 25 digits?

Now,

Mass of Uranus = 86,800,000,000,000,000,000,000,000 kg

$= 8.68 \times 10^{25}$ kg

Simply by comparing the powers of 10 in the above two, you can tell that the mass of Uranus is greater than that of the Earth.

The distance between Sun and Saturn is 1,433,500,000,000 m or 1.4335×10^{12} m. The distance betwen Saturn and Uranus is 1,439,000,000,000 m or 1.439×10^{12} m. The distance between Sun and Earth is 149, 600,000,000 m or 1.496×10^{11} m.

Can you tell which of the three distances is smallest?

EXAMPLE 13 Express the following numbers in the standard form:

(i) 5985.3 (ii) 65,950

(iii) 3,430,000 (iv) 70,040,000,000

SOLUTION

(i) $5985.3 = 5.9853 \times 1000 = 5.9853 \times 10^3$

(ii) $65,950 = 6.595 \times 10,000 = 6.595 \times 10^4$

(iii) $3,430,000 = 3.43 \times 1,000,000 = 3.43 \times 10^6$

(iv) $70,040,000,000 = 7.004 \times 10,000,000,000 = 7.004 \times 10^{10}$

A point to remember is that one less than the digit count (number of digits) to the left of the decimal point in a given number is the exponent of 10 in the standard form. Thus, in 70,040,000,000 there is no decimal point shown; we assume it to be at the (right) end. From there, the count of the places (digits) to the left is 11. The exponent of 10 in the standard form is 11 – 1 = 10. In 5985.3 there are 4 digits to the left of the decimal point and hence the exponent of 10 in the standard form is 4 – 1 = 3.

EXERCISE 11.3

1. Write the following numbers in the expanded forms:

279404, 3006194, 2806196, 120719, 20068

2. Find the number from each of the following expanded forms:

(a) $8 \times 10^4 + 6 \times 10^3 + 0 \times 10^2 + 4 \times 10^1 + 5 \times 10^0$

(b) $4 \times 10^5 + 5 \times 10^3 + 3 \times 10^2 + 2 \times 10^0$

(c) $3 \times 10^4 + 7 \times 10^2 + 5 \times 10^0$

(d) $9 \times 10^5 + 2 \times 10^2 + 3 \times 10^1$

3. Express the following numbers in standard form:

(i) 5,00,00,000 (ii) 70,00,000 (iii) 3,18,65,00,000

(iv) 3,90,878 (v) 39087.8 (vi) 3908.78

4. Express the number appearing in the following statements in standard form.

(a) The distance between Earth and Moon is 384,000,000 m.

(b) Speed of light in vacuum is 300,000,000 m/s.

(c) Diameter of the Earth is 1,27,56,000 m.

(d) Diameter of the Sun is 1,400,000,000 m.

(e) In a galaxy there are on an average 100,000,000,000 stars.

(f) The universe is estimated to be about 12,000,000,000 years old.

(g) The distance of the Sun from the centre of the Milky Way Galaxy is estimated to be 300,000,000,000,000,000,000 m.

(h) 60,230,000,000,000,000,000,000 molecules are contained in a drop of water weighing 1.8 gm.

(i) The earth has 1,353,000,000 cubic km of sea water.

(j) The population of India was about 1,027,000,000 in March, 2001.

What have We Discussed?

1. Very large numbers are difficult to read, understand, compare and operate upon. To make all these easier, we use exponents, converting many of the large numbers in a shorter form.
2. The following are exponential forms of some numbers?

$10{,}000 = 10^4$ (read as 10 raised to 4)

$243 = 3^5$, $128 = 2^7$.

Here, 10, 3 and 2 are the bases, whereas 4, 5 and 7 are their respective exponents. We also say, 10,000 is the 4th power of 10, 243 is the 5th power of 3, etc.

3. Numbers in exponential form obey certain laws, which are:

For any non-zero integers a and b and whole numbers m and n,

(a) $a^m \times a^n = a^{m+n}$

(b) $a^m \div a^n = a^{m-n}$, $\quad m > n$

(c) $(a^m)^n = a^{mn}$

(d) $a^m \times b^m = (ab)^m$

(e) $a^m \div b^m = \left(\dfrac{a}{b}\right)^m$

(f) $a^0 = 1$

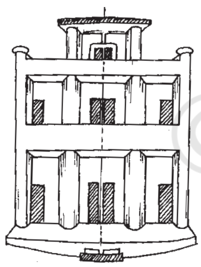
(g) $(-1)^{\text{even number}} = 1$

$(-1)^{\text{odd number}} = -1$

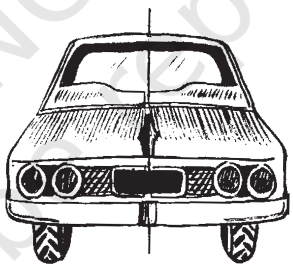
Symmetry

12.1 Introduction

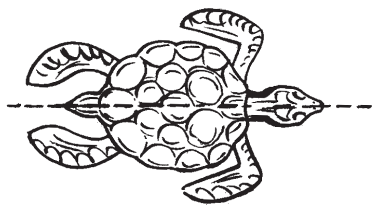
Symmetry is an important geometrical concept, commonly exhibited in nature and is used almost in every field of activity. Artists, professionals, designers of clothing or jewellery, car manufacturers, architects and many others make use of the idea of symmetry. The beehives, the flowers, the tree-leaves, religious symbols, rugs, and handkerchiefs — everywhere you find symmetrical designs.



Architecture



Engineering

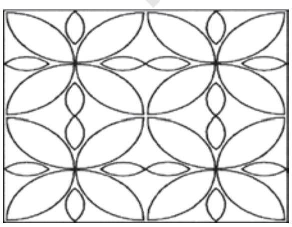


Nature

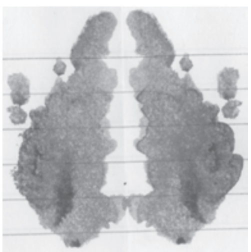
You have already had a 'feel' of **line symmetry** in your previous class.

A figure has a line symmetry, if there is a line about which the figure may be folded so that the two parts of the figure will coincide.

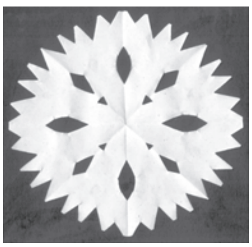
You might like to recall these ideas. Here are some activities to help you.



Compose a picture-album showing symmetry.



Create some colourful Ink-dot devils



Make some symmetrical paper-cut designs.

Enjoy identifying lines (also called axes) of symmetry in the designs you collect.

Let us now strengthen our ideas on symmetry further. Study the following figures in which the lines of symmetry are marked with dotted lines. [Fig 12.1 (i) to (iv)]

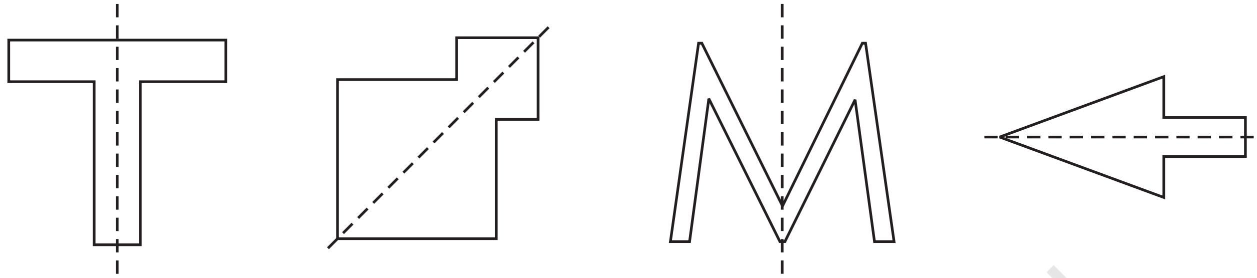


Fig 12.1

12.2 Lines of Symmetry for Regular Polygons

You know that a polygon is a closed figure made of several line segments. The polygon made up of the least number of line segments is the triangle. (Can there be a polygon that you can draw with still fewer line segments? Think about it).

A polygon is said to be regular if all its sides are of equal length and all its angles are of equal measure. Thus, an equilateral triangle is a regular polygon of three sides. Can you name the regular polygon of four sides?

An equilateral triangle is regular because each of its sides has same length and each of its angles measures 60° (Fig 12.2).

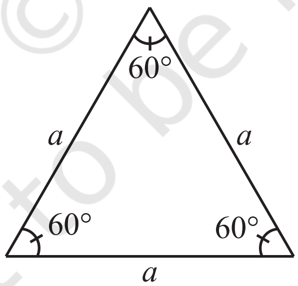


Fig 12.2

A square is also regular because all its sides are of equal length and each of its angles is a right angle (i.e., 90°). Its diagonals are seen to be perpendicular bisectors of one another (Fig 12.3).

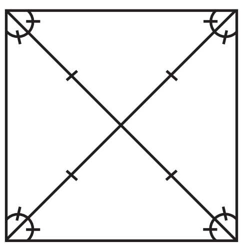


Fig 12.3

If a pentagon is regular, naturally, its sides should have equal length. You will, later on, learn that the measure of each of its angles is 108° (Fig 12.4).

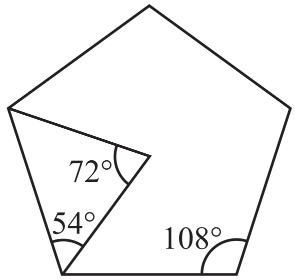


Fig 12.4

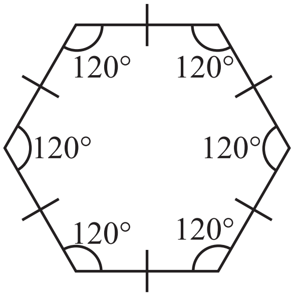


Fig 12.5

A regular hexagon has all its sides equal and each of its angles measures 120°. You will learn more of these figures later (Fig 12.5).

The regular polygons are symmetrical figures and hence their lines of symmetry are quite interesting,

Each regular polygon has as many lines of symmetry as it has sides [Fig 12.6 (i) - (iv)]. We say, they have multiple lines of symmetry.

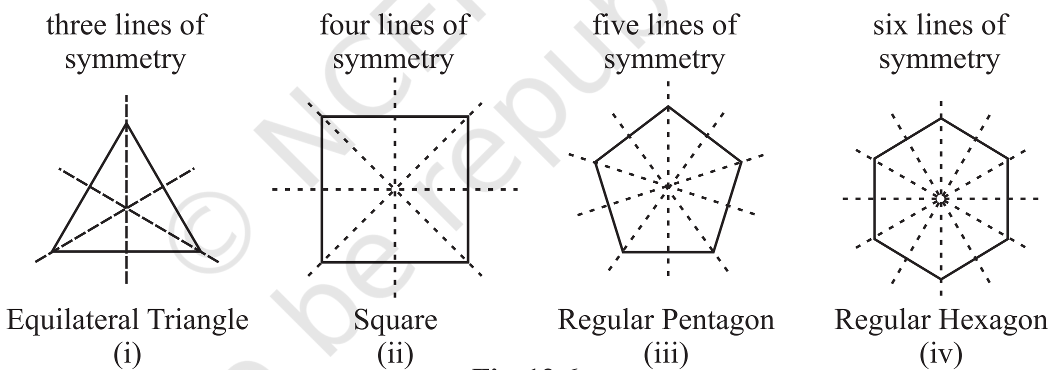


Fig 12.6

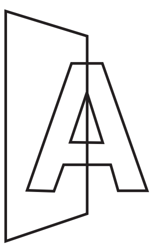


Fig 12.7

Perhaps, you might like to investigate this by paper folding. Go ahead!

The concept of line symmetry is closely related to mirror reflection. A shape has line symmetry when one half of it is the mirror image of the other half (Fig 12.7). A mirror line, thus, helps to visualise a line of symmetry (Fig 12.8).

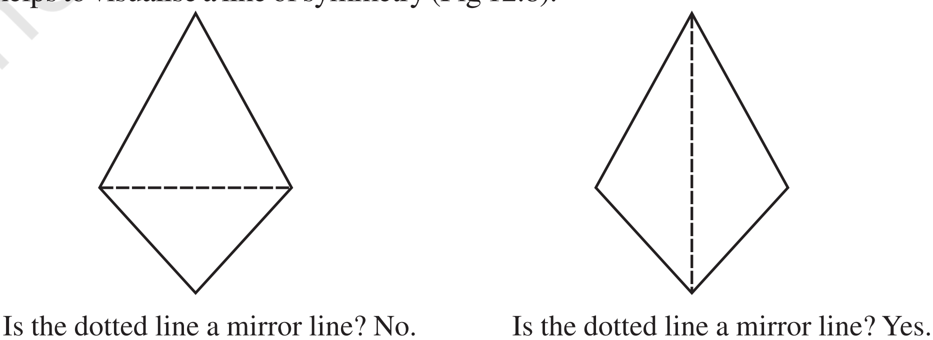
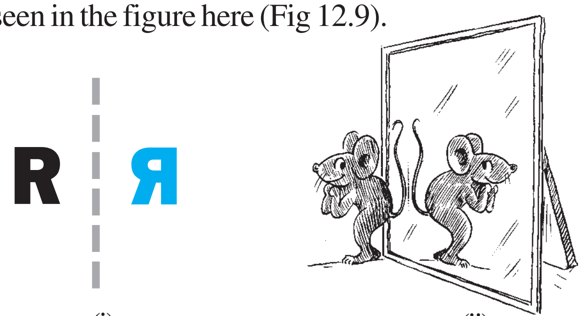


Fig 12.8

While dealing with mirror reflection, care is needed to note down the left-right changes in the orientation, as seen in the figure here (Fig 12.9).

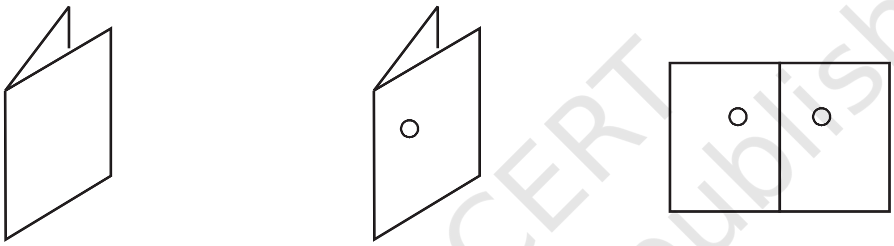


(i) (ii)

Fig 12.9

The shape is same, but the other way round!

Play this punching game!



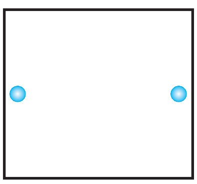
Fold a sheet into two halves | Punch a hole | two holes about the symmetric fold.

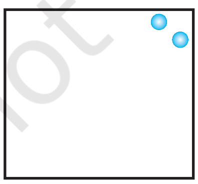
Fig 12.10

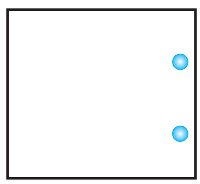
The fold is a line (or axis) of symmetry. Study about punches at different locations on the folded paper and the corresponding lines of symmetry (Fig 12.10).

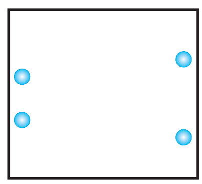
EXERCISE 12.1

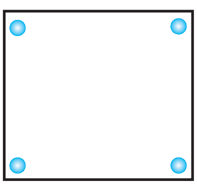
1. Copy the figures with punched holes and find the axes of symmetry for the following:

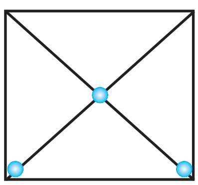

(a)

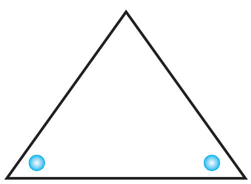

(b)

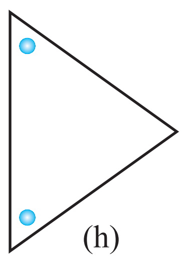

(c)

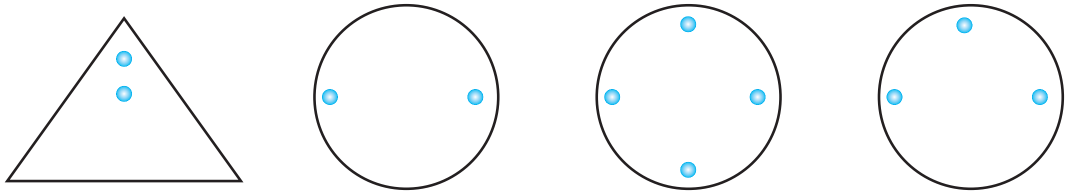

(d)


(e)


(f)

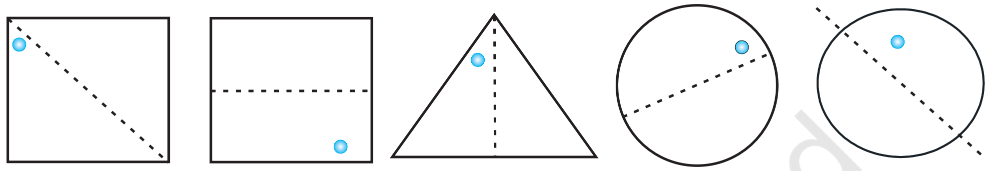

(g)


(h)



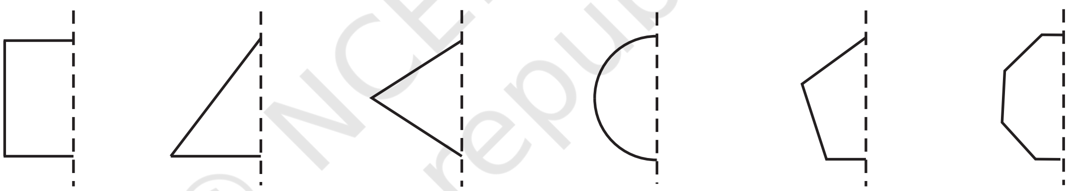
(i) (j) (k) (l)

2. Given the line(s) of symmetry, find the other hole(s):



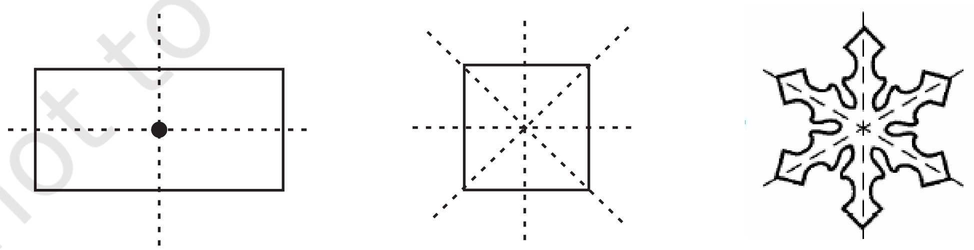
(a) (b) (c) (d) (e)

3. In the following figures, the mirror line (i.e., the line of symmetry) is given as a dotted line. Complete each figure performing reflection in the dotted (mirror) line. (You might perhaps place a mirror along the dotted line and look into the mirror for the image). Are you able to recall the name of the figure you complete?



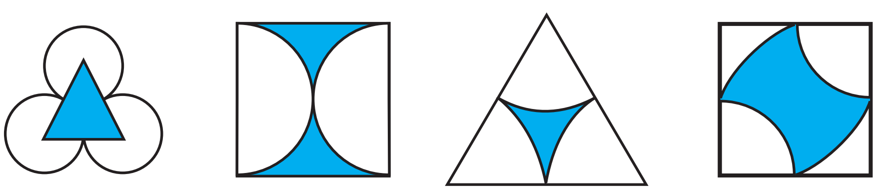
(a) (b) (c) (d) (e) (f)

4. The following figures have more than one line of symmetry. Such figures are said to have multiple lines of symmetry.

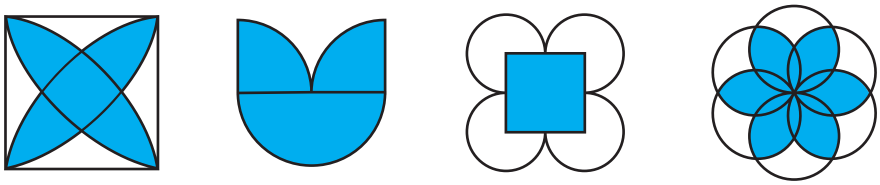


(a) (b) (c)

Identify multiple lines of symmetry, if any, in each of the following figures:

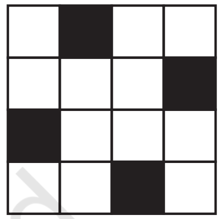


(a) (b) (c) (d)



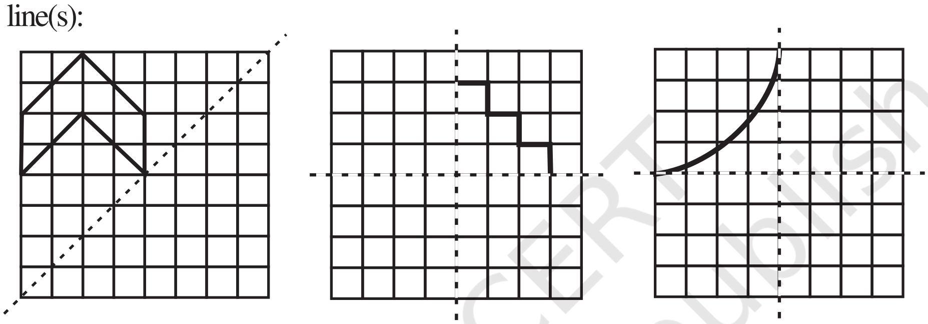
(e) (f) (g) (h)

5. Copy the figure given here.



Take any one diagonal as a line of symmetry and shade a few more squares to make the figure symmetric about a diagonal. Is there more than one way to do that? Will the figure be symmetric about both the diagonals?

6. Copy the diagram and complete each shape to be symmetric about the mirror line(s):



(a) (b) (c)

7. State the number of lines of symmetry for the following figures:

(a) An equilateral triangle (b) An isosceles triangle (c) A scalene triangle

(d) A square (e) A rectangle (f) A rhombus

(g) A parallelogram (h) A quadrilateral (i) A regular hexagon

(j) A circle

8. What letters of the English alphabet have reflectional symmetry (i.e., symmetry related to mirror reflection) about.

(a) a vertical mirror (b) a horizontal mirror

(c) both horizontal and vertical mirrors

9. Give three examples of shapes with no line of symmetry.

10. What other name can you give to the line of symmetry of

(a) an isosceles triangle? (b) a circle?

12.3 Rotational Symmetry

What do you say when the hands of a clock go round?

You say that they rotate. The hands of a clock rotate in only one direction, about a fixed point, the centre of the clock-face.

Rotation, like movement of the hands of a clock, is called a clockwise rotation; otherwise it is said to be anticlockwise.

What can you say about the rotation of the blades of a ceiling fan? Do they rotate clockwise or anticlockwise? Or do they rotate both ways?

If you spin the wheel of a bicycle, it rotates. It can rotate in either way: both clockwise and anticlockwise. Give three examples each for (i) a clockwise rotation and (ii) anticlockwise rotation.

When an object rotates, its shape and size do not change. The rotation turns an object about a fixed point. This fixed point is the **centre of rotation**. What is the centre of rotation of the hands of a clock? Think about it.

Fig 12.11

The angle of turning during rotation is called the **angle of rotation**. A full turn, you know, means a rotation of 360°. What is the degree measure of the angle of rotation for (i) a half-turn? (ii) a quarter-turn?

A half-turn means rotation by 180°; a quarter-turn is rotation by 90°.

When it is 12 O'clock, the hands of a clock are together. By 3 O'clock, the minute hand would have made three complete turns; but the hour hand would have made only a quarter-turn. What can you say about their positions at 6 O'clock?

Have you ever made a paper windmill? The Paper windmill in the picture looks symmetrical (Fig 12.11); but you do not find any line of symmetry. No folding can help you to have coincident halves. However if you rotate it by 90° about the fixed point, the windmill will look exactly the same. We say the windmill has a **rotational symmetry.**

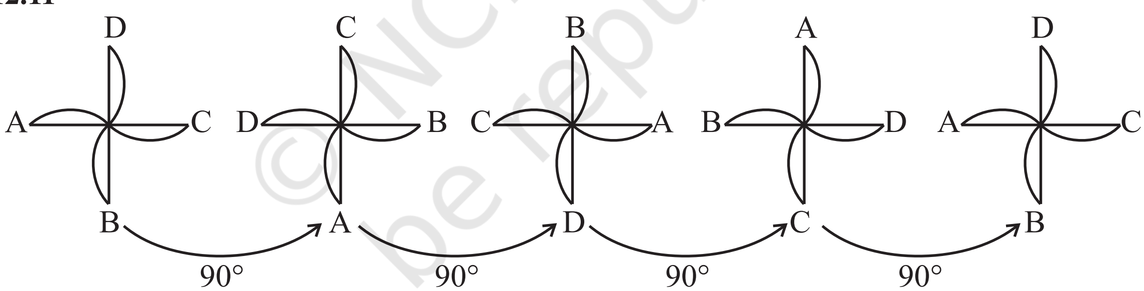


Fig 12.12

In a full turn, there are precisely **four positions** (on rotation through the angles 90°, 180°, 270° and 360°) when the windmill looks exactly the same. Because of this, we say it has a rotational symmetry of order 4.

Here is one more example for rotational symmetry.

Consider a square with P as one of its corners (Fig 12.13).

Let us perform quarter-turns about the centre of the square marked ×.

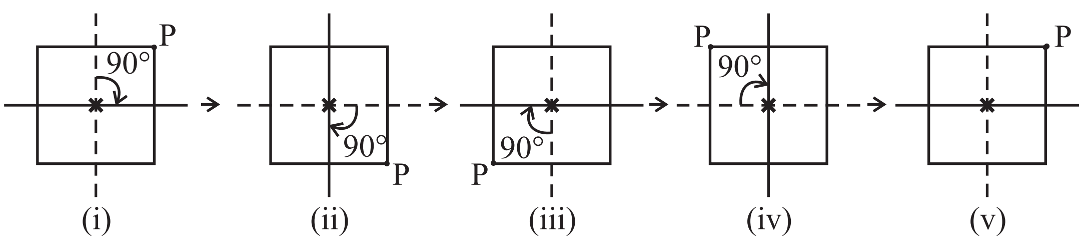


Fig 12.13

Fig 12.13 (i) is the initial position. Rotation by 90° about the centre leads to Fig 12.13 (ii). Note the position of P now. Rotate again through 90° and you get Fig 12.13 (iii). In this way, when you complete four quarter-turns, the square reaches its original position. It now looks the same as Fig12.13 (i). This can be seen with the help of the positions taken by P.

Thus a square has a **rotational symmetry of order 4** about its centre. Observe that in this case,

(i) The centre of rotation is the centre of the square.

(ii) The angle of rotation is 90°.

(iii) The direction of rotation is clockwise.

(iv) The order of rotational symmetry is 4.

Try These

1. (a) Can you now tell the order of the rotational symmetry for an equilateral triangle? (Fig 12.14)

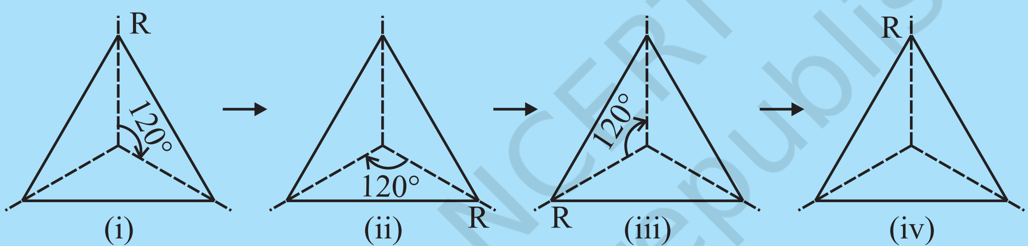


Fig 12.14

(b) How many positions are there at which the triangle looks exactly the same, when rotated about its centre by 120°?

2. Which of the following shapes (Fig 12.15) have rotational symmetry about the marked point.

(i) (ii) (iii) (iv)

Fig 12.15

Do This

Draw two identical parallelograms, one-ABCD on a piece of paper and the other A' B' C' D' on a transparent sheet. Mark the points of intersection of their diagonals, O and O' respectively (Fig 12.16).

Place the parallelograms such that A' lies on A, B' lies on B and so on. O' then falls on O.

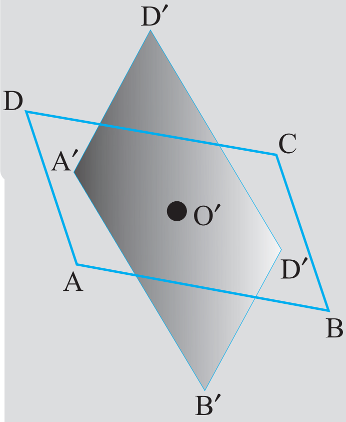


Fig 12.16

Stick a pin into the shapes at the point O.

Now turn the transparent shape in the clockwise direction.

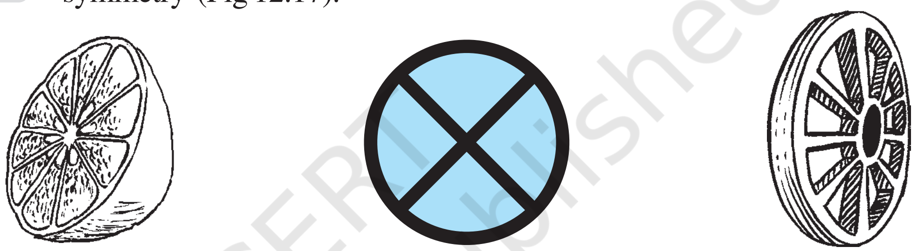
How many times do the shapes coincide in one full round?

What is the order of rotational symmetry?

The point where we have the pin is the centre of rotation. It is the intersecting point of the diagonals in this case.

Every object has a rotational symmetry of order 1, as it occupies same position after a rotation of 360° (i.e., one complete revolution). Such cases have no interest for us.

You have around you many shapes, which possess rotational symmetry (Fig 12.17).



Fruit (i) Road sign (ii) Wheel (iii)

Fig 12.17

For example, when you slice certain fruits, the cross-sections are shapes with rotational symmetry. This might surprise you when you notice them [Fig 12.17(i)].

Then there are many road signs that exhibit rotational symmetry. Next time when you walk along a busy road, try to identify such road signs and find about the order of rotational symmetry [Fig 12.17(ii)].

Think of some more examples for rotational symmetry. Discuss in each case:

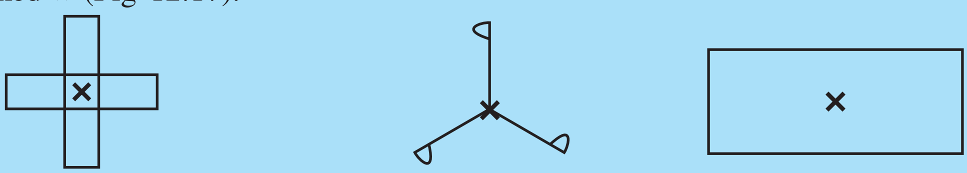
(i) the centre of rotation (ii) the angle of rotation

(iii) the direction in which the rotation is affected and

(iv) the order of the rotational symmetry.

TRY THESE

Give the order of the rotational symmetry of the given figures about the point marked × (Fig 12.17).

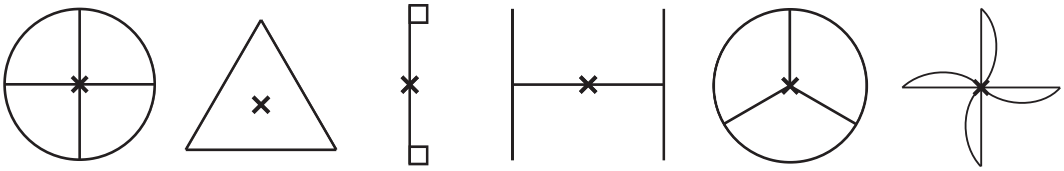


(i) (ii) (iii)

Fig 12.18

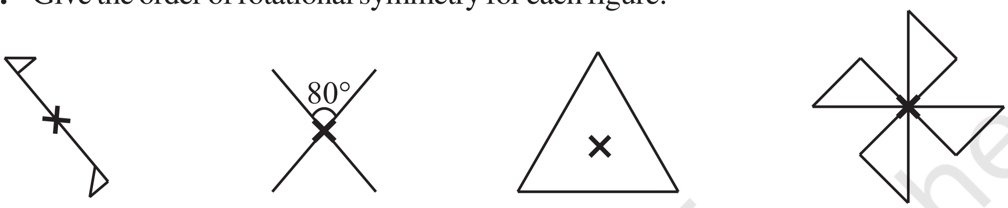
EXERCISE 12.2

1. Which of the following figures have rotational symmetry of order more than 1:



(a) (b) (c) (d) (e) (f)

2. Give the order of rotational symmetry for each figure:



(a) (b) (c) (d)

(e) (f) (g) (h)

12.4 LINE SYMMETRY AND ROTATIONAL SYMMETRY

You have been observing many shapes and their symmetries so far. By now you would have understood that some shapes have only line symmetry, some have only rotational symmetry and some have both line symmetry and rotational symmetry.

For example, consider the square shape (Fig 12.19).

How many lines of symmetry does it have?

Does it have any rotational symmetry?

If 'yes', what is the order of the rotational symmetry?

Think about it.

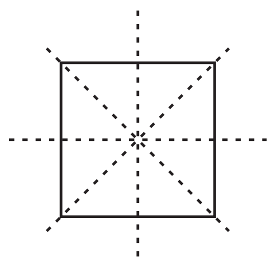


Fig 12.19

The circle is the most perfect symmetrical figure, because it can be rotated around its centre through any angle and at the same time it has unlimited number of lines of symmetry. Observe any circle pattern. Every line through the centre (that is every diameter) forms a line of (reflectional) symmetry and it has rotational symmetry around the centre for every angle.

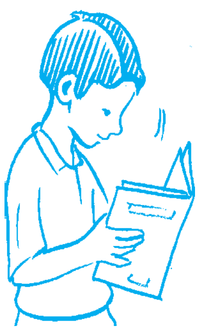
Do This

Some of the English alphabets have fascinating symmetrical structures. Which capital letters have just one line of symmetry (like **E**)? Which capital letters have a rotational symmetry of order 2 (like **I**)?

By attempting to think on such lines, you will be able to fill in the following table:

Alphabet Letters	Line Symmetry	Number of Lines of Symmetry	Rotational Symmetry	Order of Rotational Symmetry
Z	No	0	Yes	2
S				
H	Yes		Yes	
O	Yes		Yes	
E	Yes			
N			Yes	
C				

Exercise 12.3



1. Name any two figures that have both line symmetry and rotational symmetry.
2. Draw, wherever possible, a rough sketch of
   (i) a triangle with both line and rotational symmetries of order more than 1.
   (ii) a triangle with only line symmetry and no rotational symmetry of order more than 1.
   (iii) a quadrilateral with a rotational symmetry of order more than 1 but not a line symmetry.
   (iv) a quadrilateral with line symmetry but not a rotational symmetry of order more than 1.
3. If a figure has two or more lines of symmetry, should it have rotational symmetry of order more than 1?
4. Fill in the blanks:

Shape	Centre of Rotation	Order of Rotation	Angle of Rotation
Square			
Rectangle			
Rhombus			
Equilateral Triangle			
Regular Hexagon			
Circle			
Semi-circle			

5. Name the quadrilaterals which have both line and rotational symmetry of order more than 1.
6. After rotating by 60° about a centre, a figure looks exactly the same as its original position. At what other angles will this happen for the figure?
7. Can we have a rotational symmetry of order more than 1 whose angle of rotation is
   (i) 45°? (ii) 17°?

What have We Discussed?

1. A figure has **line symmetry**, if there is a line about which the figure may be folded so that the two parts of the figure will coincide.
2. Regular polygons have equal sides and equal angles. They have multiple (i.e., more than one) lines of symmetry.
3. Each regular polygon has as many lines of symmetry as it has sides.

Regular Polygon	Regular hexagon	Regular pentagon	Square	Equilateral triangle
Number of lines of symmetry	6	5	4	3

4. Mirror reflection leads to symmetry, under which the left-right orientation have to be taken care of.
5. Rotation turns an object about a fixed point.

   This fixed point is the **centre of rotation**.

   The angle by which the object rotates is the **angle of rotation**.

   A half-turn means rotation by 180°; a quarter-turn means rotation by 90°. Rotation may be clockwise or anticlockwise.
6. If, after a rotation, an object looks exactly the same, we say that it has a **rotational symmetry**.
7. In a complete turn (of 360°), the number of times an object looks exactly the same is called the **order of rotational symmetry**. The order of symmetry of a square, for example, is 4 while, for an equilateral triangle, it is 3.
8. Some shapes have only one line of symmetry, like the letter E; some have only rotational symmetry, like the letter S; and some have both symmetries like the letter H.

   The study of symmetry is important because of its frequent use in day-to-day life and more because of the beautiful designs it can provide us.

Chapter 13

Visualising Solid Shapes

13.1 Introduction: Plane Figures and Solid Shapes

In this chapter, you will classify figures you have seen in terms of what is known as *dimension*.

In our day to day life, we see several objects like books, balls, ice-cream cones etc., around us which have different shapes. One thing common about most of these objects is that they all have some length, breadth and height or depth.

That is, they all occupy space and have three dimensions.

Hence, they are called three dimensional shapes.

Do you remember some of the three dimensional shapes (i.e., solid shapes) we have seen in earlier classes?

Try These

Match the shape with the name:

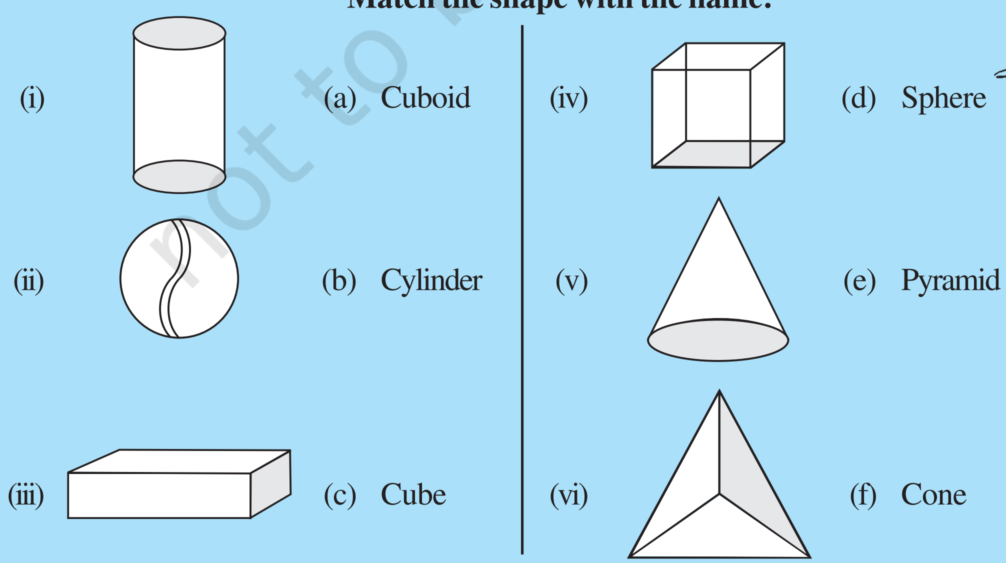


Fig 13.1

Try to identify some objects shaped like each of these.

By a similar argument, we can say figures drawn on paper which have only length and breadth are called two dimensional (i.e., plane) figures. We have also seen some two dimensional figures in the earlier classes.

Match the 2 dimensional figures with the names (Fig 13.2):

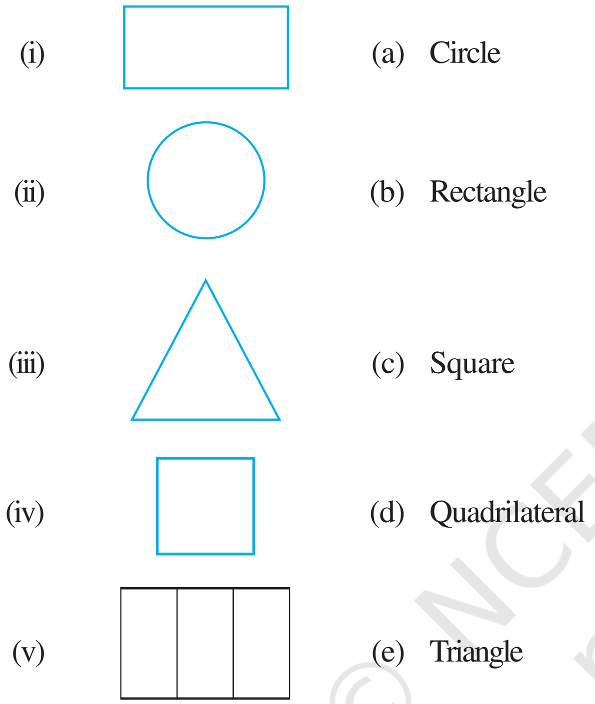


Fig 13.2

Note: We can write 2-D in short for 2-dimension and 3-D in short for 3-dimension.

13.2 Faces, Edges and Vertices

Do you remember the Faces, Vertices and Edges of solid shapes, which you studied earlier? Here you see them for a cube:

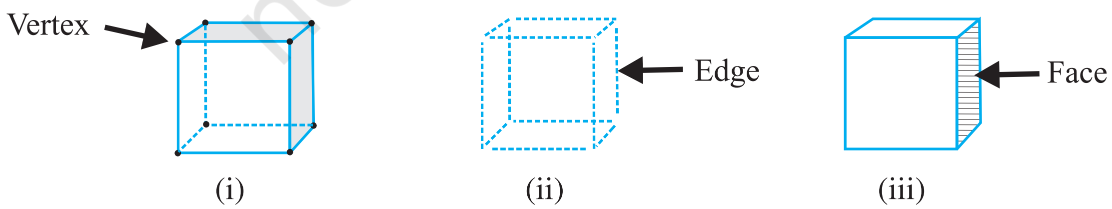


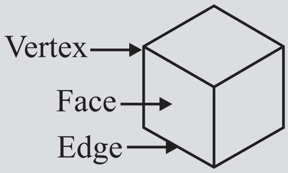
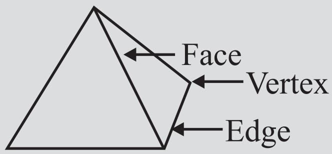
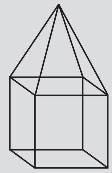
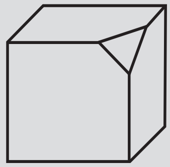
Fig 13.3

The 8 corners of the cube are its **vertices**. The 12 line segments that form the skeleton of the cube are its **edges**. The 6 flat square surfaces that are the skin of the cube are its **faces**.

Do This

Complete the following table:

Table 13.1

				
Faces **(F)**	6	4		
Edges **(E)**	12			
Vertices **(V)**	8	4		

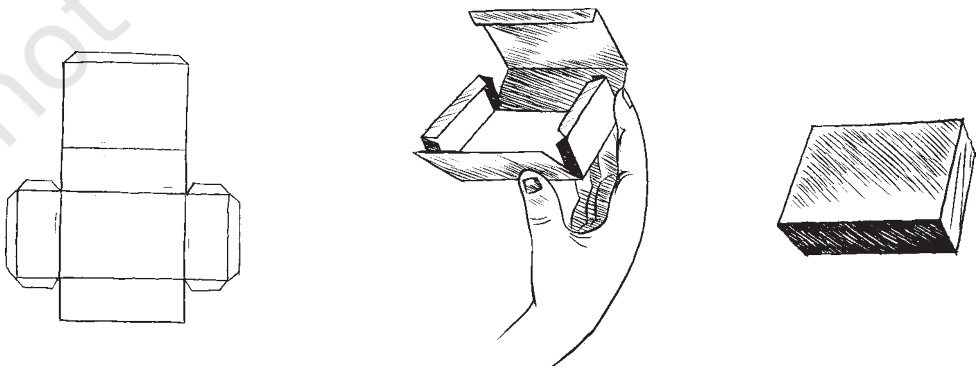
Can you see that, the two dimensional figures can be identified as the faces of the three dimensional shapes? For example a cylinder  has two faces which are circles, and a pyramid, shaped like this  has triangles as its faces.

We will now try to see how some of these 3-D shapes can be visualised on a 2-D surface, that is, on paper.

In order to do this, we would like to get familiar with three dimensional objects closely. Let us try forming these objects by making what are called nets.

13.3 Nets for Building 3-D Shapes

Take a cardboard box. Cut the edges to lay the box flat. You have now a **net** for that box. A net is a sort of skeleton-outline in 2-D [Fig13.4 (i)], which, when folded [Fig13.4 (ii)], results in a 3-D shape [Fig13.4 (iii)].



(i) (ii) (iii)

Fig 13.4

Here you got a **net** by suitably separating the edges. Is the reverse process possible?

Here is a net pattern for a box (Fig 13.5). Copy an enlarged version of the net and try to make the box by suitably folding and gluing together. (You may use suitable units). The box is a solid. It is a 3-D object with the shape of a cuboid.

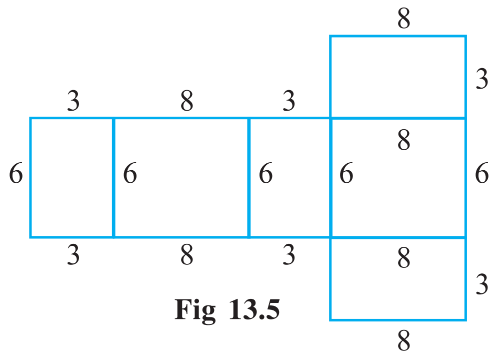


Fig 13.5

Similarly, you can get a net for a cone by cutting a slit along its slant surface (Fig 13.6).

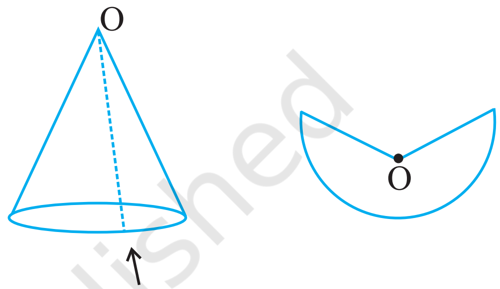


Fig 13.6

You have different nets for different shapes. Copy enlarged versions of the nets given (Fig 13.7) and try to make the 3-D shapes indicated. (You may also like to prepare skeleton models using strips of cardboard fastened with paper clips).

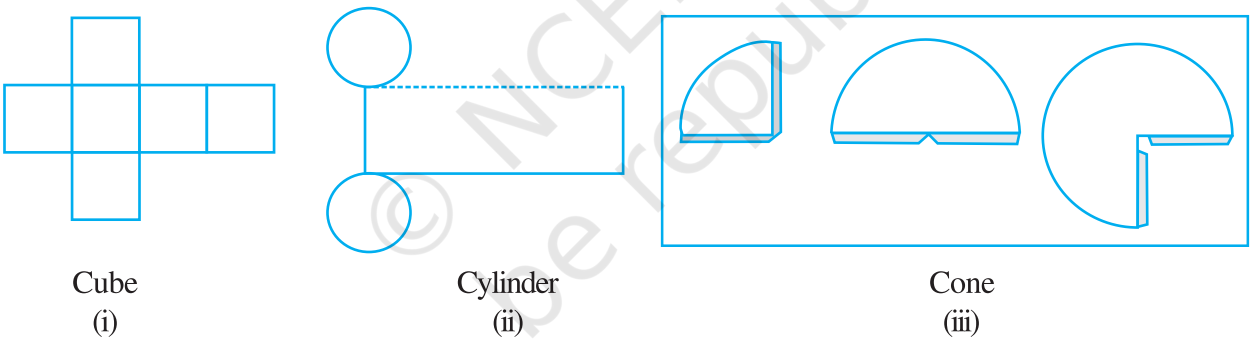


Fig 13.7

We could also try to make a net for making a pyramid like the Great Pyramid in Giza (Egypt) (Fig 13.8). That pyramid has a square base and triangles on the four sides.

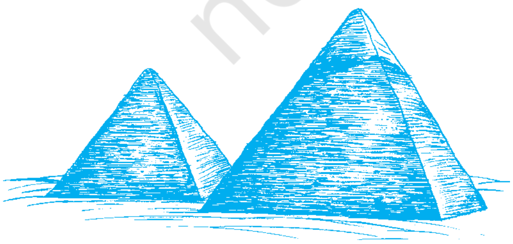


Fig 13.8

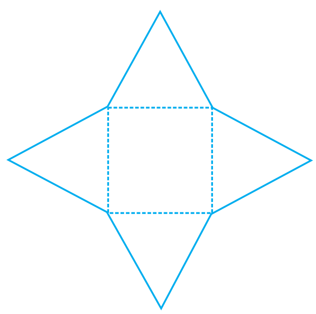


Fig 13.9

See if you can make it with the given net (Fig 13.9).

Try These

Here you find four nets (Fig 13.10). There are two *correct* nets among them to make a tetrahedron. See if you can work out which nets will make a tetrahedron.

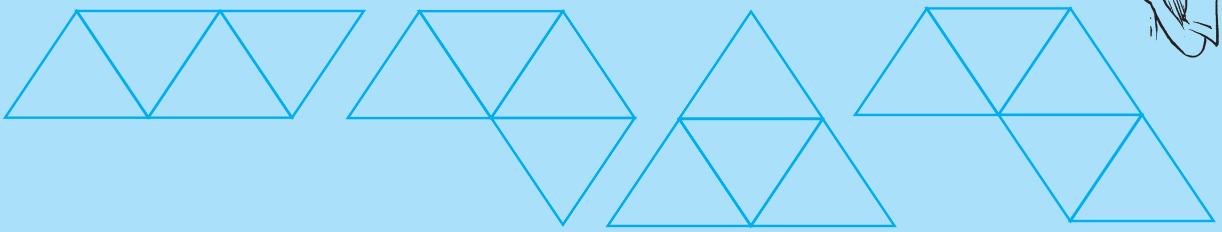
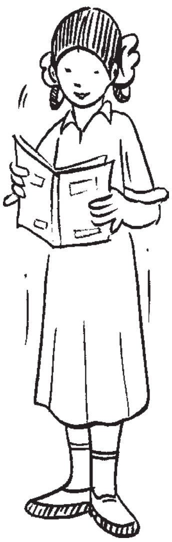
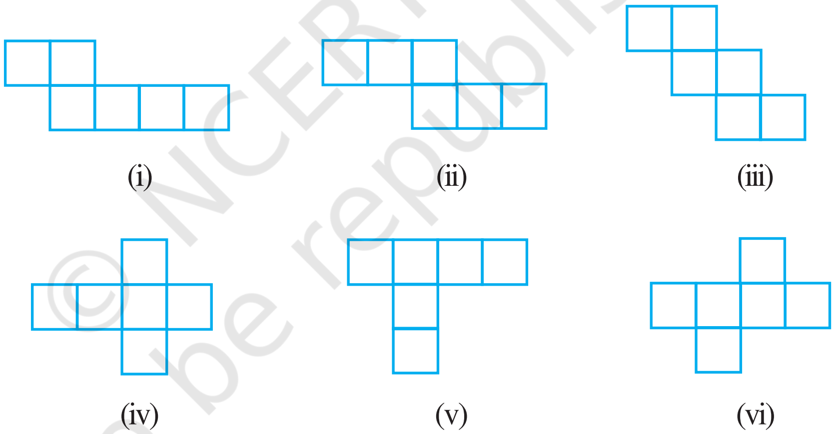


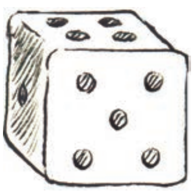
Fig 13.10



Exercise 13.1

1. Identify the nets which can be used to make cubes (cut out copies of the nets and try it):





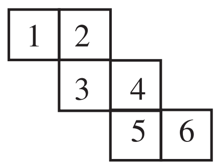
2. Dice are cubes with dots on each face. Opposite faces of a die always have a total of seven dots on them.

Here are two nets to make dice (cubes); the numbers inserted in each square indicate the number of dots in that box.

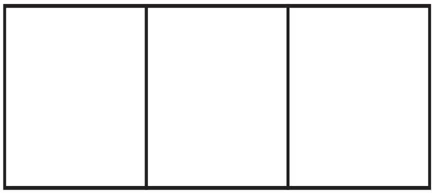
Insert suitable numbers in the blanks, remembering that the number on the opposite faces should total to 7.

3. Can this be a net for a die?

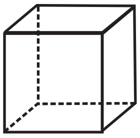
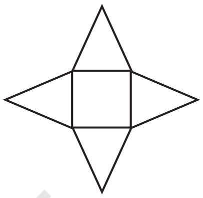
Explain your answer.

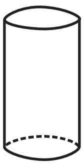
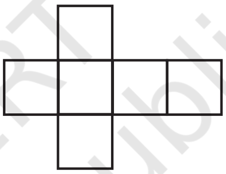


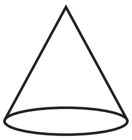
4. Here is an incomplete net for making a cube. Complete it in at least two different ways. Remember that a cube has six faces. How many are there in the net here? (Give two separate diagrams. If you like, you may use a squared sheet for easy manipulation.)

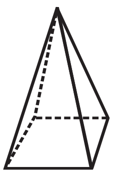
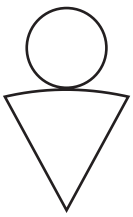


5. Match the nets with appropriate solids:

(a)  (i) 

(b)  (ii) 

(c)  (iii) 

(d)  (iv) 

Play this game

You and your friend sit back-to-back. One of you reads out a net to make a 3-D shape, while the other attempts to copy it and sketch or build the described 3-D object.

13.4 Drawing Solids on a Flat Surface

Your drawing surface is paper, which is flat. When you draw a solid shape, the images are somewhat distorted to make them appear three-dimensional. It is a visual illusion. You will find here two techniques to help you.

13.4.1 Oblique Sketches

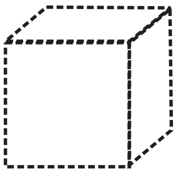
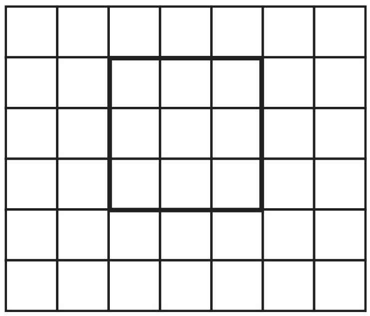


Fig 13.11

Here is a picture of a cube (Fig 13.11). It gives a clear idea of how the cube looks like, when seen from the front. You do not see certain faces. In the drawn picture, the lengths are not equal, as they should be in a cube. Still, you are able to recognise it as a cube. Such a sketch of a solid is called an **oblique sketch**.

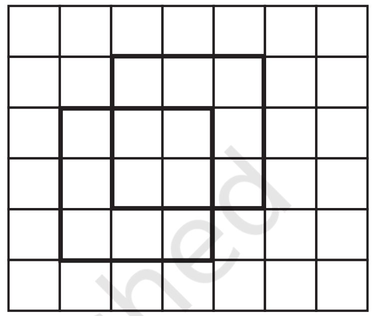
How can you draw such sketches? Let us attempt to learn the technique.

You need a squared (lines or dots) paper. Initially practising to draw on these sheets will later make it easy to sketch them on a plain sheet (without the aid of squared lines or dots!) Let us attempt to draw an oblique sketch of a $3 \times 3 \times 3$ (each edge is 3 units) cube (Fig 13.12).



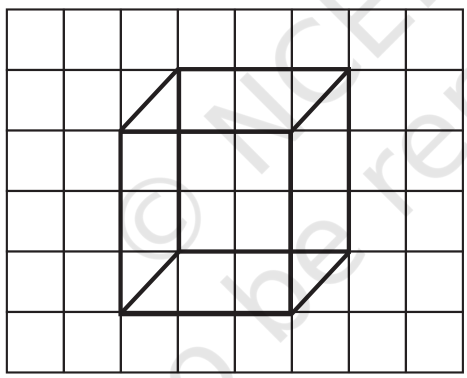
Step 1

Draw the **front** face.



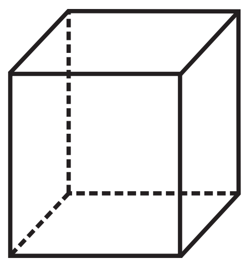
Step 2

Draw the **opposite** face. Sizes of the faces have to be same, but the sketch is somewhat off-set from step 1.



Step 3

Join the corresponding corners



Step 4

Redraw using dotted lines for hidden edges. (It is a convention) The sketch is ready now.

Fig 13.12

In the oblique sketch above, did you note the following?

(i) The sizes of the front faces and its opposite are same; and

(ii) The edges, which are all equal in a cube, appear so in the sketch, though the actual measures of edges are not taken so.

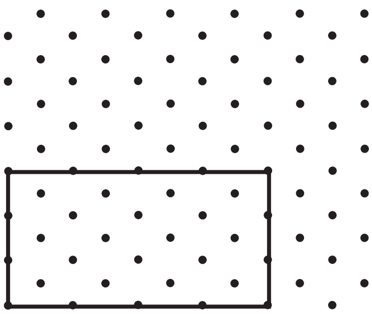
You could now try to make an oblique sketch of a cuboid (remember the faces in this case are rectangles)

Note: You can draw sketches in which measurements also agree with those of a given solid. To do this we need what is known as an **isometric sheet**. Let us try to make a cuboid with dimensions 4 cm length, 3 cm breadth and 3 cm height on given isometric sheet.

13.4.2 Isometric Sketches

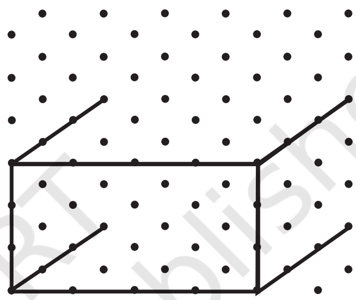
Have you seen an isometric dot sheet? (A sample is given at the end of the book). Such a sheet divides the paper into small equilateral triangles made up of dots or lines. *To draw sketches in which measurements also agree with those of the solid,* we can use isometric dot sheets. [Given on inside of the back cover (3rd cover page).]

Let us attempt to draw an isometric sketch of a cuboid of dimensions $4 \times 3 \times 3$ (which means the edges forming length, breadth and height are 4, 3, 3 units respectively) (Fig 13.13).



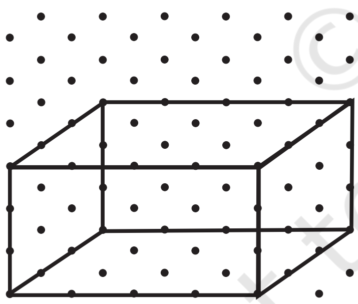
Step 1

Draw a rectangle to show the front face.



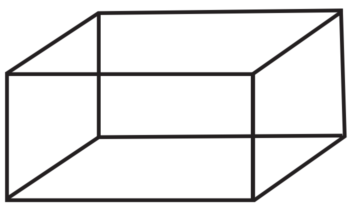
Step 2

Draw four parallel line segments of length 3 starting from the four corners of the rectangle.



Step 3

Connect the matching corners with appropriate line segments.



Step 4

This is an isometric sketch of the cuboid.

Fig 13.13

Note that the measurements are of exact size in an isometric sketch; this is not so in the case of an oblique sketch.

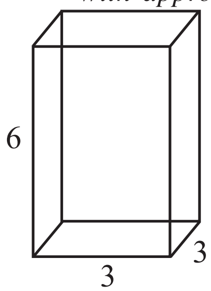


Fig 13.14 (i)

Example 1 Here is an oblique sketch of a cuboid [Fig 13.14(i)]. Draw an isometric sketch that matches this drawing.

Solution Here is the solution [Fig 13.14(ii)]. Note how the measurements are taken care of.

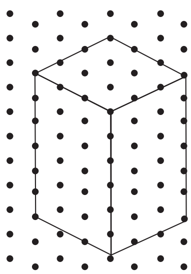
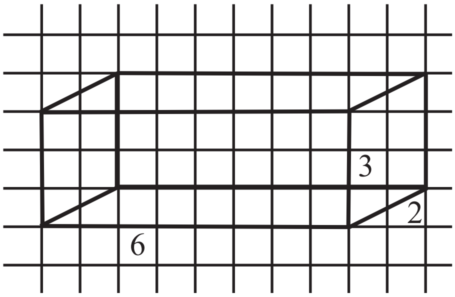


Fig 13.14 (ii)

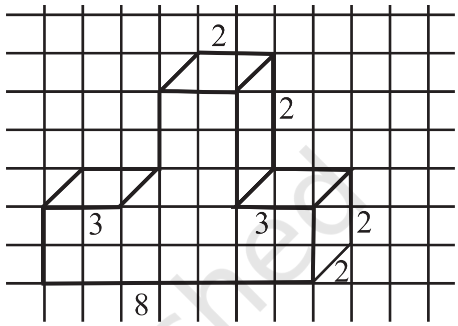
How many units have you taken along (i) 'length'? (ii) 'breadth'? (iii) 'height'? Do they match with the units mentioned in the oblique sketch?

EXERCISE 13.2

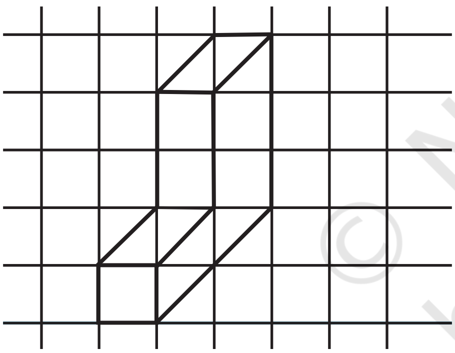
1. Use isometric dot paper and make an isometric sketch for each one of the given shapes:



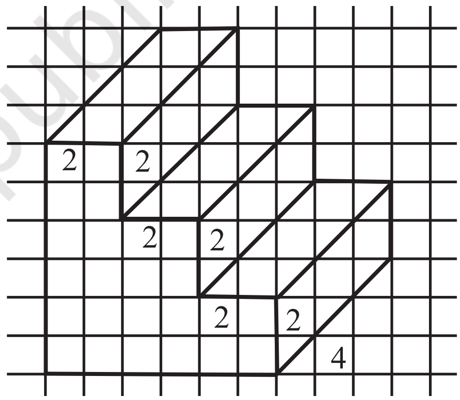
(i)



(ii)



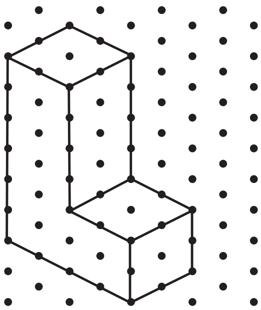
(iii)

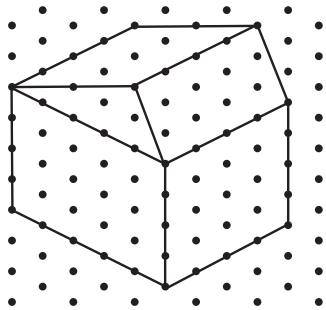


(iv)

Fig 13.15

2. The dimensions of a cuboid are 5 cm, 3 cm and 2 cm. Draw three different isometric sketches of this cuboid.
3. Three cubes each with 2 cm edge are placed side by side to form a cuboid. Sketch an oblique or isometric sketch of this cuboid.
4. Make an oblique sketch for each one of the given isometric shapes:





5. Give (i) an oblique sketch and (ii) an isometric sketch for each of the following:

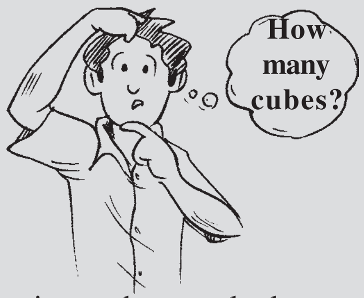
(a) A cuboid of dimensions 5 cm, 3 cm and 2 cm. (Is your sketch unique?)

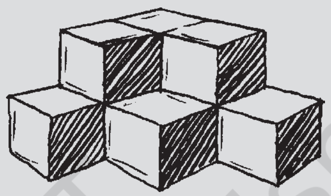
(b) A cube with an edge 4 cm long.

An isometric sheet is attached at the end of the book. You could try to make on it some cubes or cuboids of dimensions specified by your friend.

13.4.3 Visualising Solid Objects

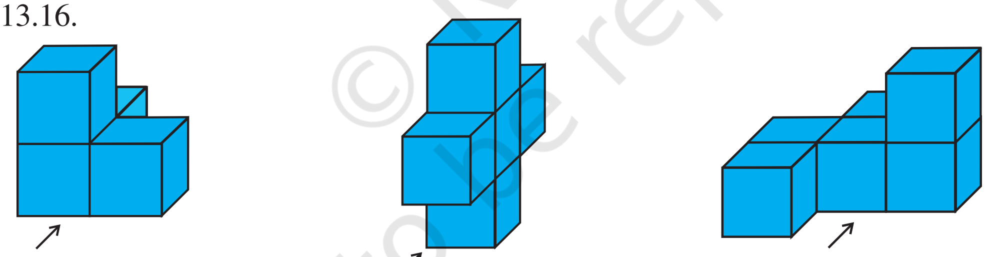
Do This





Sometimes when you look at combined shapes, some of them may be hidden from your view.

Here are some activities you could try in your free time to help you visualise some solid objects and how they look. Take some cubes and arrange them as shown in Fig 13.16.



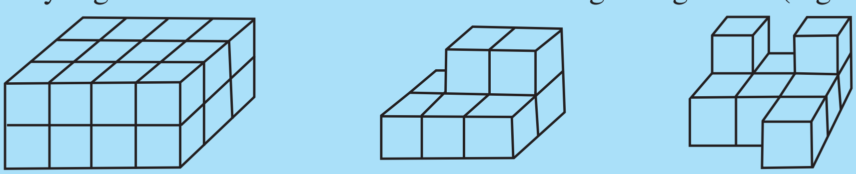
(i) (ii) (iii)

Fig 13.16

Now ask your friend to guess how many cubes there are when observed from the view shown by the arrow mark.

Try These

Try to guess the number of cubes in the following arrangements (Fig 13.17).



(i) (ii) (iii)

Fig 13.17

Such visualisation is very helpful. Suppose you form a cuboid by joining such cubes. You will be able to guess what the length, breadth and height of the cuboid would be.

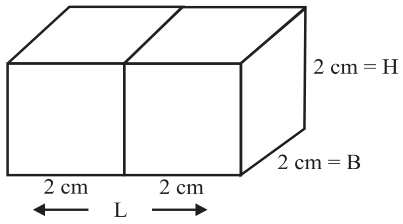


Fig 13.18

Example 2 If two cubes of dimensions 2 cm by 2cm by 2cm are placed side by side, what would the dimensions of the resulting cuboid be?

Solution As you can see (Fig 13.18) when kept side by side, the length is the only measurement which increases, it be comes 2 + 2 = 4 cm.

The breadth = 2 cm and the height = 2 cm.

Try These

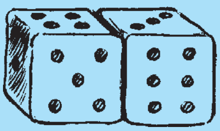


Fig 13.19

1. Two dice are placed side by side as shown: Can you say what the total would be on the face opposite to
   (a) 5 + 6 (b) 4 + 3
   (Remember that in a die sum of numbers on opposite faces is 7)
2. Three cubes each with 2 cm edge are placed side by side to form a cuboid. Try to make an oblique sketch and say what could be its length, breadth and height.

13.5 Viewing Different Sections of a Solid

Now let us see how an object which is in 3-D can be viewed in different ways.

13.5.1 One Way to View an Object is by Cutting or Slicing

Slicing game

Here is a loaf of bread (Fig 13.20). It is like a cuboid with a square face. You 'slice' it with a knife.

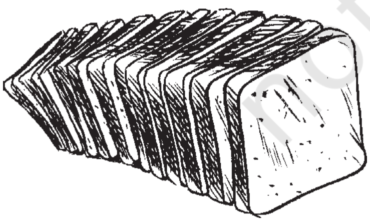


Fig 13.20

When you give a 'vertical' cut, you get several pieces, as shown in the Figure 13.20. Each face of the piece is a square! We call this face a **'cross-section'** of the whole bread. The cross section is nearly a square in this case.

Beware! If your cut is not 'vertical' you may get a different cross section! Think about it. The boundary of the cross-section you obtain is a plane curve. Do you notice it?

A kitchen play

Have you noticed cross-sections of some vegetables when they are cut for the purposes of cooking in the kitchen? Observe the various slices and get aware of the shapes that result as cross-sections.

Play this

Make clay (or plasticine) models of the following solids and make vertical or horizontal cuts. Draw rough sketches of the cross-sections you obtain. Name them wherever you can.

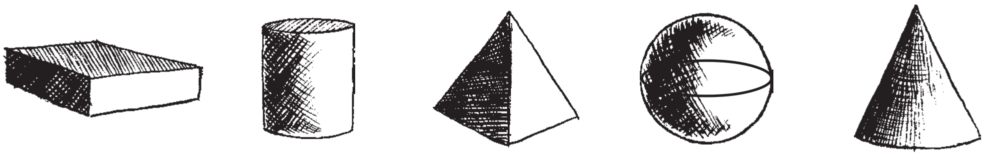


Fig 13.21

EXERCISE 13.3

1. What cross-sections do you get when you give a
   (i) vertical cut (ii) horizontal cut
   to the following solids?
   (a) A brick (b) A round apple (c) A die
   (d) A circular pipe (e) An ice cream cone

13.5.2 Another Way is by Shadow Play

A shadow play

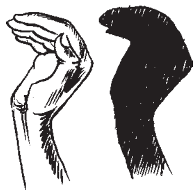


Fig 13.22

Shadows are a good way to illustrate how three-dimensional objects can be viewed in two dimensions. Have you seen a **shadow play**? It is a form of entertainment using solid articulated figures in front of an illuminated back-drop to create the illusion of moving images. It makes some indirect use of ideas in Mathematics.

You will need a source of light and a few solid shapes for this activity. (If you have an overhead projector, place the solid under the lamp and do these investigations.)

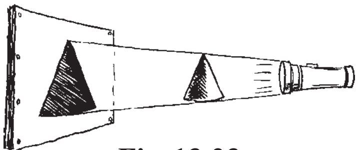


Fig 13.23

Keep a torchlight, *right in front of* a Cone. What type of shadow does it cast on the screen? (Fig 13.23)

The solid is three-dimensional; what is the dimension of the shadow?

If, instead of a cone, you place a cube in the above game, what type of shadow will you get?

Experiment with different positions of the source of light and with different positions of the solid object. Study their effects on the shapes and sizes of the shadows you get.

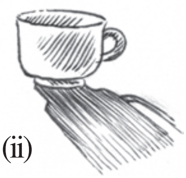
Here is another funny experiment that you might have tried already: Place a circular plate in the open when the Sun at the noon time is just *right above* it as shown in Fig 13.24 (i). What is the shadow that you obtain?

(i)

Will it be same during

(a) forenoons?

(b) evenings?



(ii)

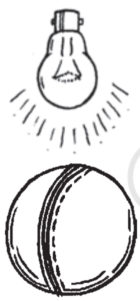
(iii)

Fig 13.24 (i) - (iii)

Study the shadows in relation to the position of the Sun and the time of observation.

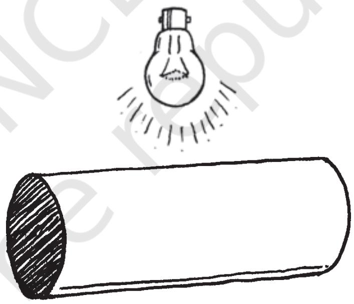
EXERCISE 13.4

1. A bulb is kept burning just right above the following solids. Name the shape of the shadows obtained in each case. Attempt to give a rough sketch of the shadow. (You may try to experiment first and then answer these questions).



A ball

(i)



A cylindrical pipe

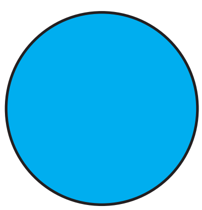
(ii)

A book

(iii)

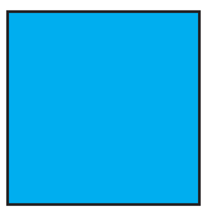
2. Here are the shadows of some 3-D objects, when seen under the lamp of an overhead projector. Identify the solid(s) that match each shadow. (There may be multiple answers for these!)

A circle



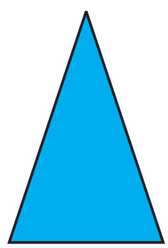
(i)

A square



(ii)

A triangle



(iii)

A rectangle

(iv)

3. Examine if the following are true statements:

(i) The cube can cast a shadow in the shape of a rectangle.

(ii) The cube can cast a shadow in the shape of a hexagon.

13.5.3 A Third Way is by Looking at it from Certain Angles to Get Different Views

One can look at an object standing in front of it or by the side of it or from above. Each time one will get a different view (Fig 13.25).

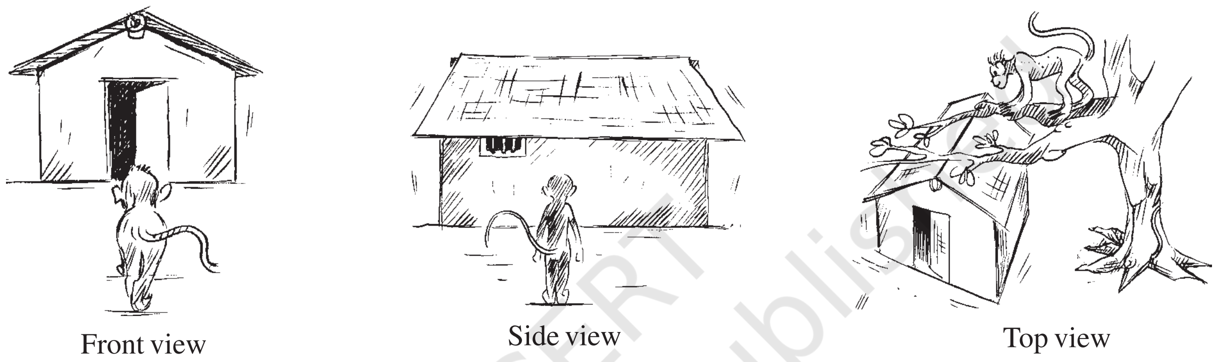


Fig 13.25

Here is an example of how one gets different views of a given building. (Fig 13.26)

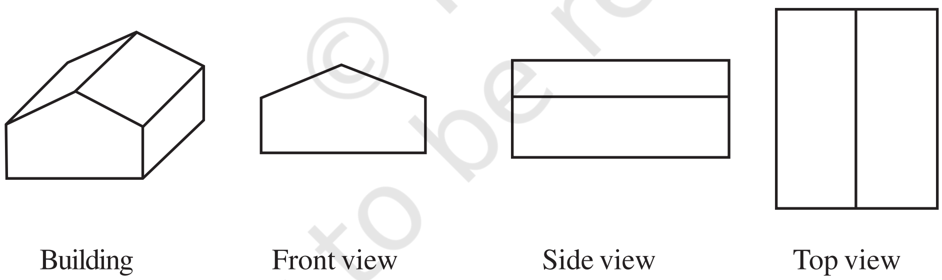


Fig 13.26

You could do this for figures made by joining cubes.

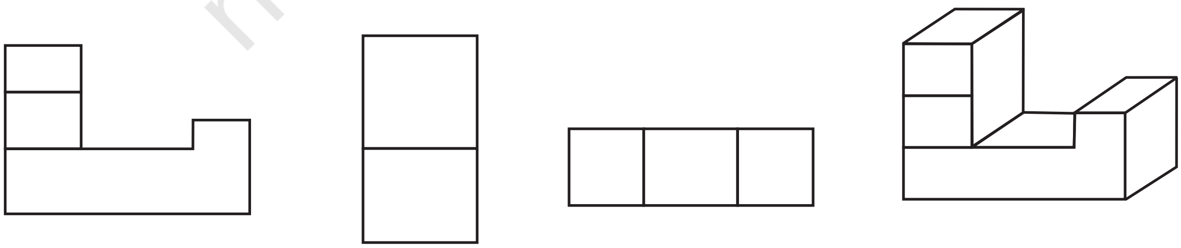


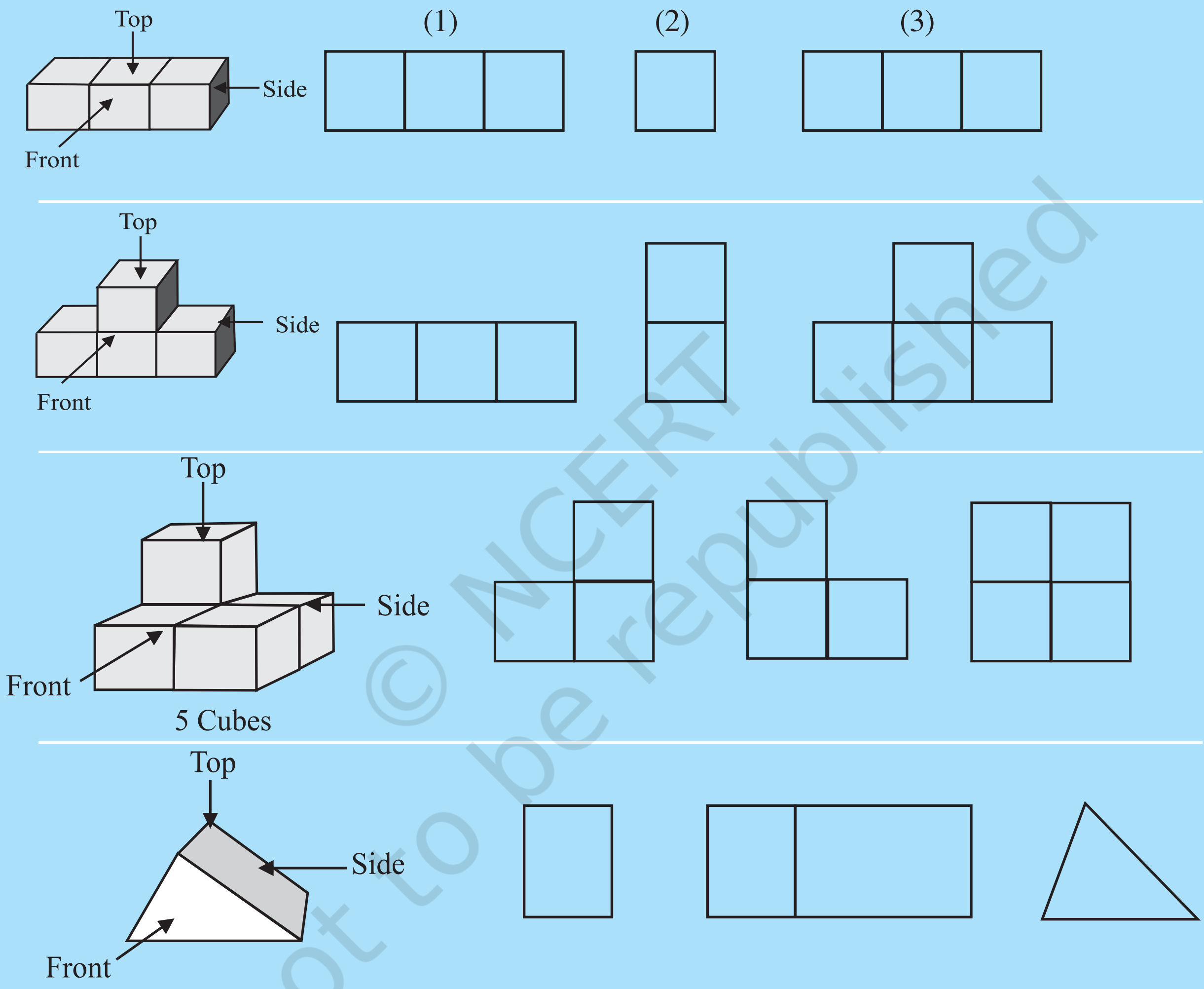
Fig 13.27

Try putting cubes together and then making such sketches from different sides.

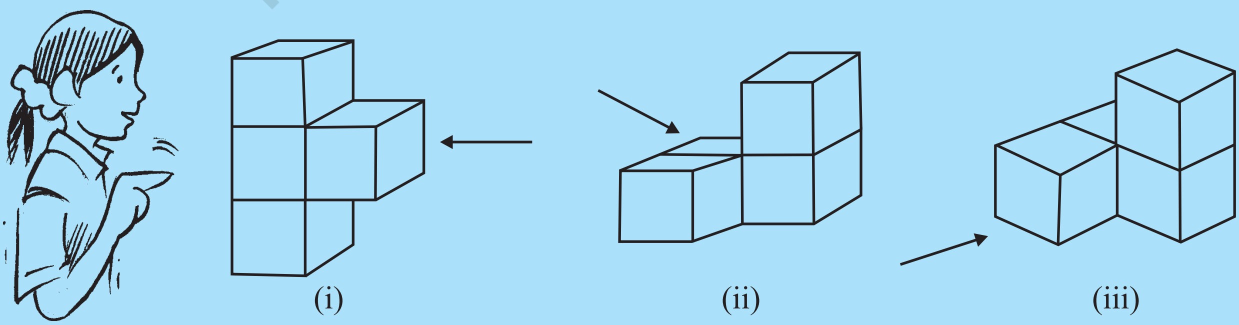
Try These

1. For each solid, the three views (1), (2), (3) are given. Identify for each solid the corresponding top, front and side views.

Solid **Its views**



2. Draw a view of each solid as seen from the direction indicated by the arrow.



What have We Discussed?

1. The circle, the square, the rectangle, the quadrilateral and the triangle are examples of **plane figures**; the cube, the cuboid, the sphere, the cylinder, the cone and the pyramid are examples of **solid shapes**.
2. Plane figures are of **two-dimensions** (**2-D**) and the solid shapes are of three-dimensions (**3-D**).
3. The corners of a solid shape are called its **vertices**; the line segments of its skeleton are its **edges**; and its flat surfaces are its **faces**.
4. A **net** is a skeleton-outline of a solid that can be folded to make it. The same solid can have several types of nets.
5. Solid shapes can be drawn on a flat surface (like paper) realistically. We call this **2-D representation of a 3-D solid**.
6. Two types of sketches of a solid are possible:
   (a) An **oblique sketch** does not have proportional lengths. Still it conveys all important aspects of the appearance of the solid.
   (b) An **isometric sketch** is drawn on an isometric dot paper, a sample of which is given at the end of this book. In an isometric sketch of the solid the measurements kept proportional.
7. **Visualising solid shapes** is a very useful skill. You should be able to see 'hidden' parts of the solid shape.
8. Different sections of a solid can be viewed in many ways:
   (a) One way is to view by cutting or **slicing** the shape, which would result in the cross-section of the solid.
   (b) Another way is by observing a 2-D **shadow** of a 3-D shape.
   (c) A third way is to look at the shape from different angles; the **front-view**, the **side-view** and the **top-view** can provide a lot of information about the shape observed.