



Informed Search Strategies

Vijay Kumar Meena
Assistant Professor
KIIT University

Acknowledgement

- These slides are taken (as it is or with some modifications) from various resources including -
 - Artificial Intelligence: A Modern Approach (AIMA) by Russell and Norvig.
 - Slides of Prof. Mausam (IITD)
 - Slides of Prof. Rohan Paul (IITD)
 - Slides of Prof. Brian Yu (Harvard University)

So Far...

- Definition of AI
- Brief history of AI
- Definition of Intelligent Agent
- Characteristics of Intelligent Agent
- Types of Intelligent Agents
- Modeling an AI problem to a search problem
- Uninformed Search Strategies
 - BFS
 - DFS
 - DLS
 - IDS
 - Bi-Directional Search
 - UCS

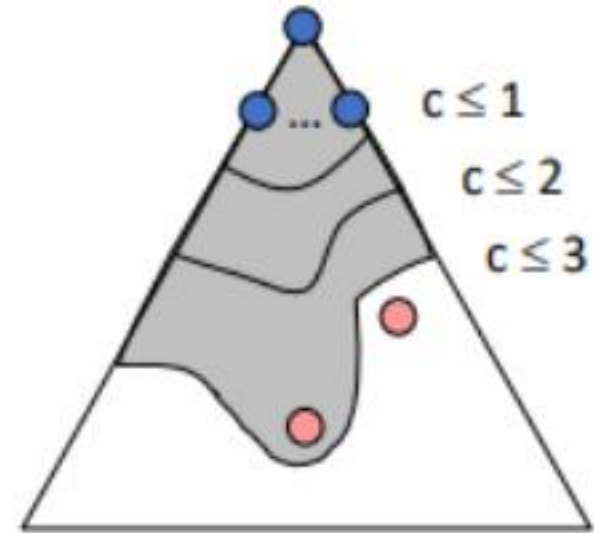
General search strategies

```
function TREE-SEARCH(problem) returns a solution, or failure
  initialize the frontier using the initial state of problem
  loop do
    if the frontier is empty then return failure
    choose a leaf node and remove it from the frontier
    if the node contains a goal state then return the corresponding solution
    expand the chosen node, adding the resulting nodes to the frontier
```

```
function GRAPH-SEARCH(problem) returns a solution, or failure
  initialize the frontier using the initial state of problem
  initialize the explored set to be empty
  loop do
    if the frontier is empty then return failure
    choose a leaf node and remove it from the frontier
    if the node contains a goal state then return the corresponding solution
    add the node to the explored set
    expand the chosen node, adding the resulting nodes to the frontier
    only if not in the frontier or explored set
```

Uninformed Search

- Uniform Cost Search
 - Expand the lowest cost path
 - Complete
 - Optimal
- Problem
 - Explores options in every “direction”
 - No information about goal location
- Informed Search
 - Use problem-specific knowledge beyond the definition of the problem to guide the search towards the goal.



Informed search

- Search algorithms like DFS, BFS, etc. are called blind search algorithms or uninformed search algorithms. They make no use of knowledge regarding problem itself.
- Informed search algorithms uses problem specific knowledge beyond the definition of the problem itself to find solutions more efficiently than uninformed search algorithms.
- **Heuristic Information** – Information about the problem (the nature of the states, the cost of transforming from one state to another, the promise of taking a certain path, and the characteristics of the goals) can sometimes be used to help guide the search more efficiently.
- Heuristic Information is usually stored in the form of an **evaluation function $f(n)$** which is a function of **any node n** .
- Using heuristic function can help to reduce the number of alternatives from an exponential number to a polynomial number and, thereby, obtain a solution in a tolerable amount of time.

Evaluation function

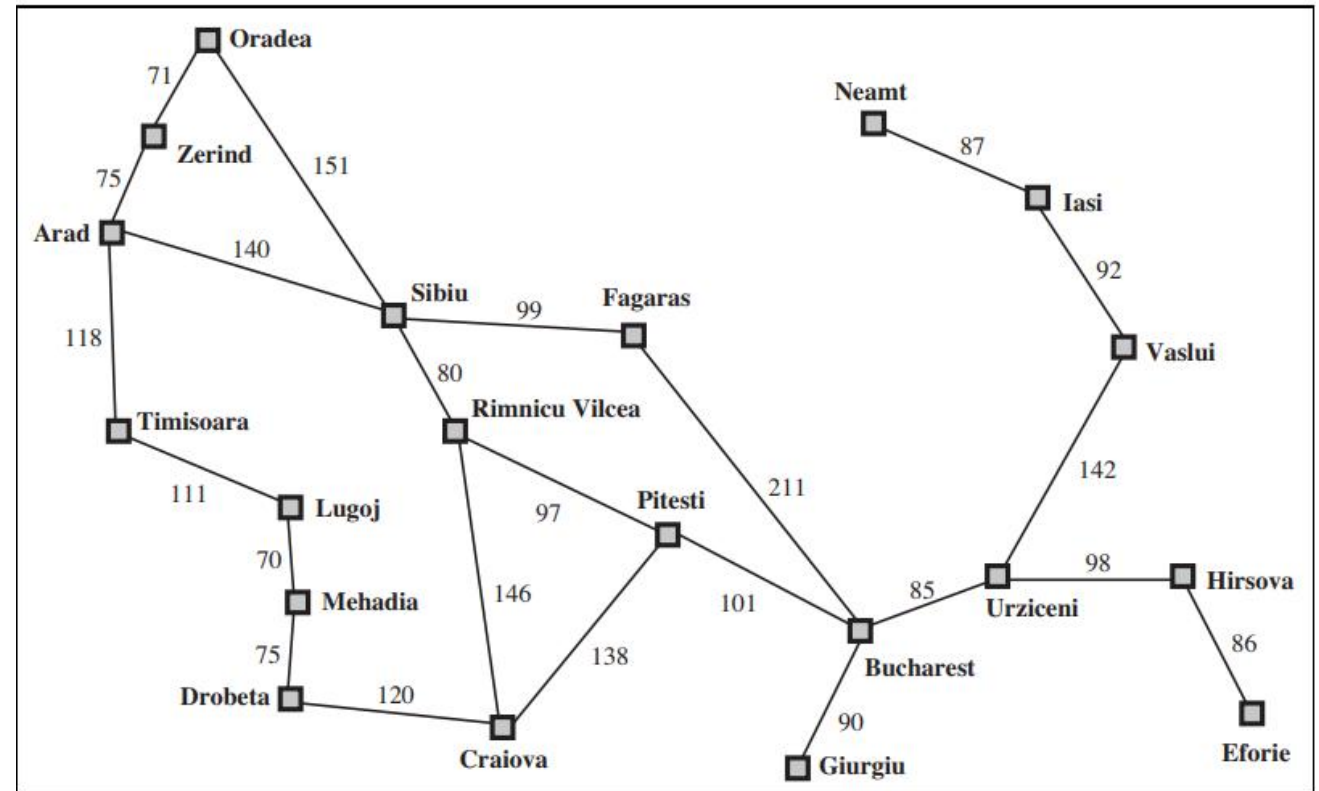
- The evaluation function **$f(n)$** is constructed as a cost estimation for any node **n** .
- This helps in choosing which node to expand next. Usually the node with **lower value of $f(n)$** .
- For any node evaluation function consists of two parts –
 - Path cost of the **node from start state**, represented as **$g(n)$**
 - Estimated cost of **goal from the current node**, represented as **heuristic function $h(n)$**
- **$f(n) = g(n) + h(n)$**
- Different algorithms uses different parts of evaluation function to select nodes for expansion.
- For example – Uniform Cost Search selects node with least path cost $g(n)$ for expansion first.

Heuristic function

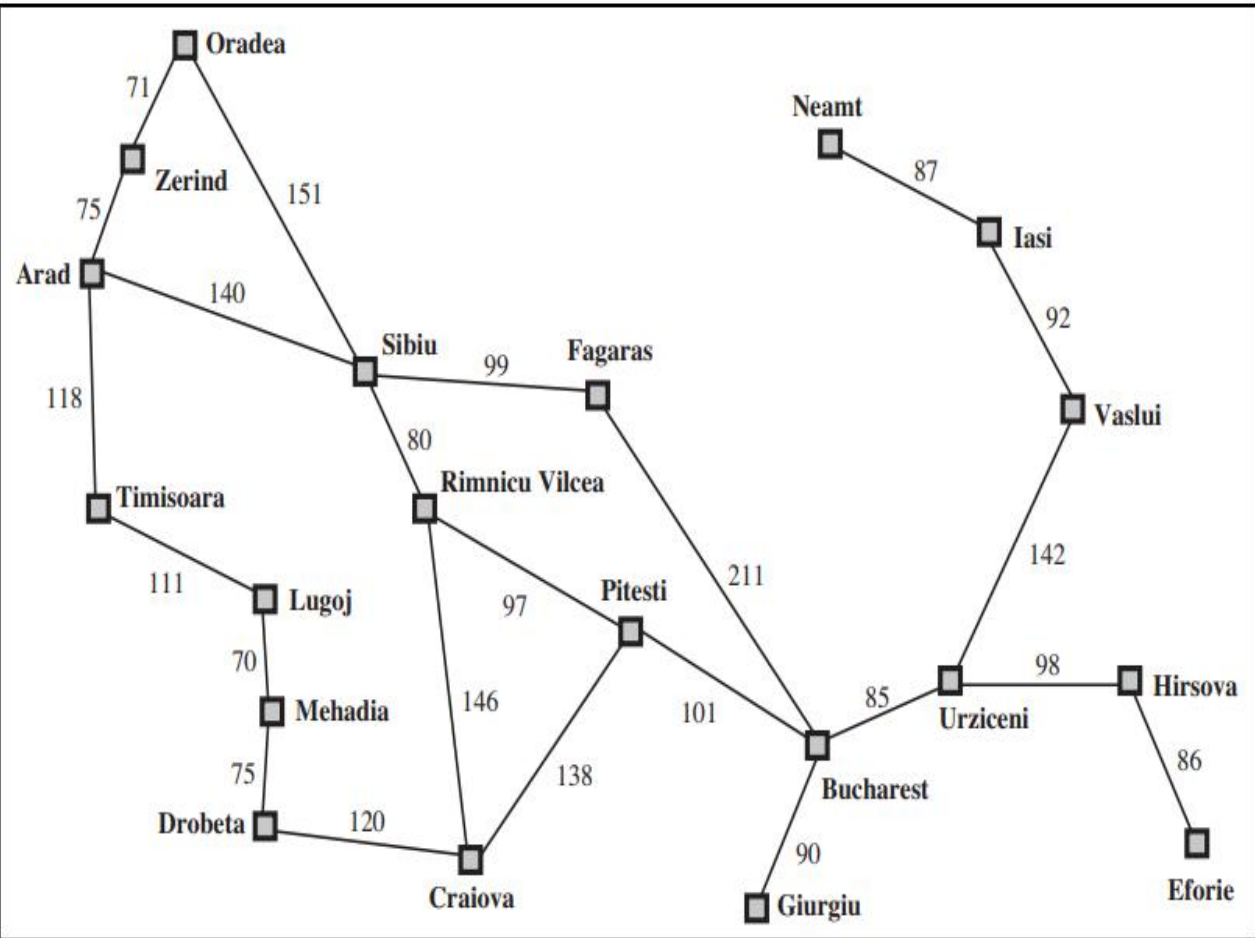
- Heuristic function $h(n)$ is an estimation of how far goal node is from the current node n .
- A heuristic is a rule of thumb for deciding which choice might be best.
- There is no general theory for finding heuristics, because every problem is different.
- Choice of heuristics depends on knowledge of the problem space.
- An informed guess of the next step to be taken in solving a problem would prune the search space.
- A heuristic may find a sub-optimal solution or fail to find a solution since it uses limited information.
- **Heuristics can be arbitrary, non-negative, problem-specific functions given the constraint that $h(n) = 0$ if n is goal node.**

Heuristic function: path finding

- Consider the problem of find path from city A to city B.
- What might be a good heuristic for this problem?
 - The straight-line distance from one city to another.
 - Can not be computed from directly from problem description. We need more information about problem to compute heuristic function.
- Is it always right?
 - Certainly not, Roads are rarely straight



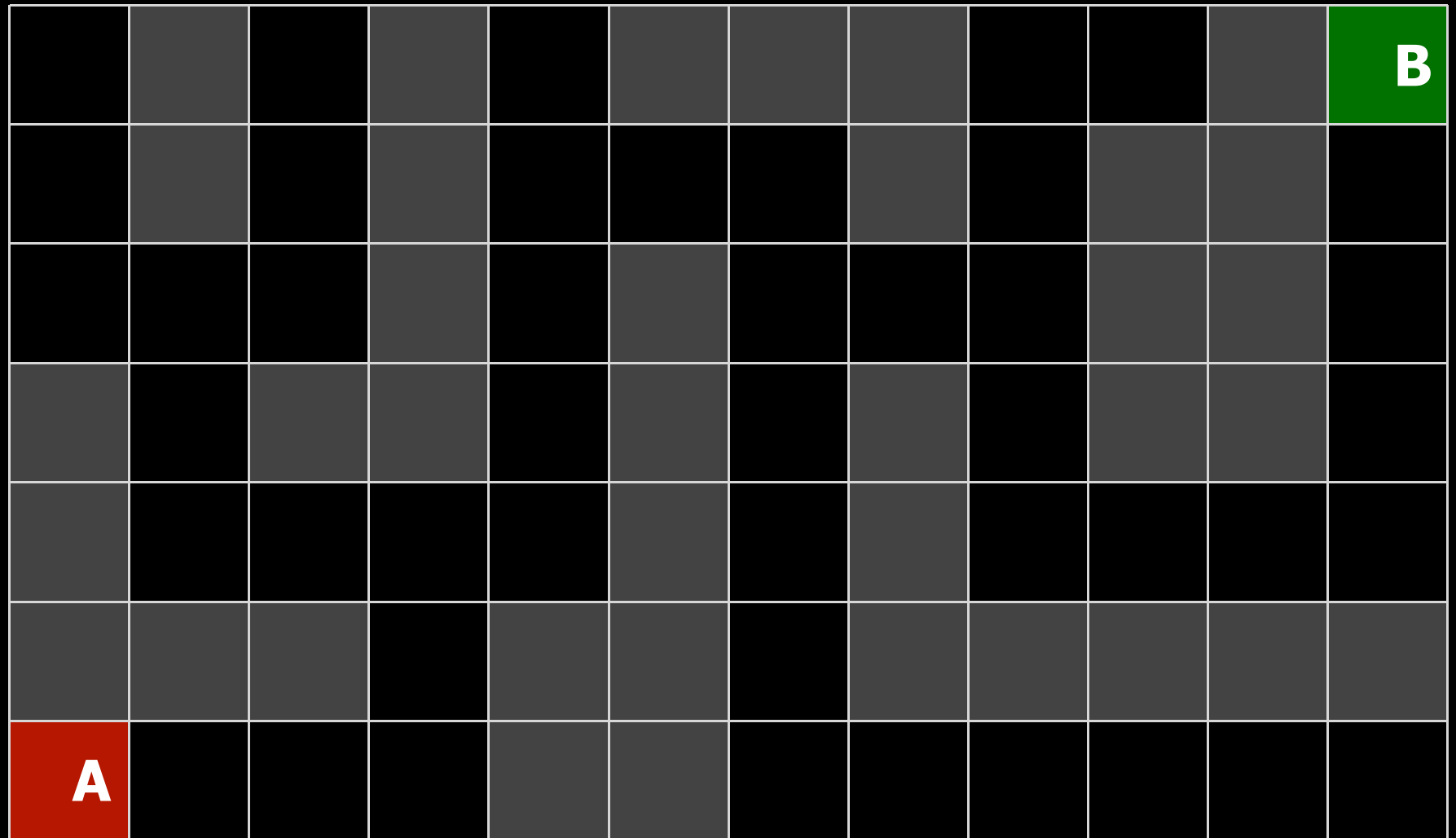
Heuristic function: Path Finding



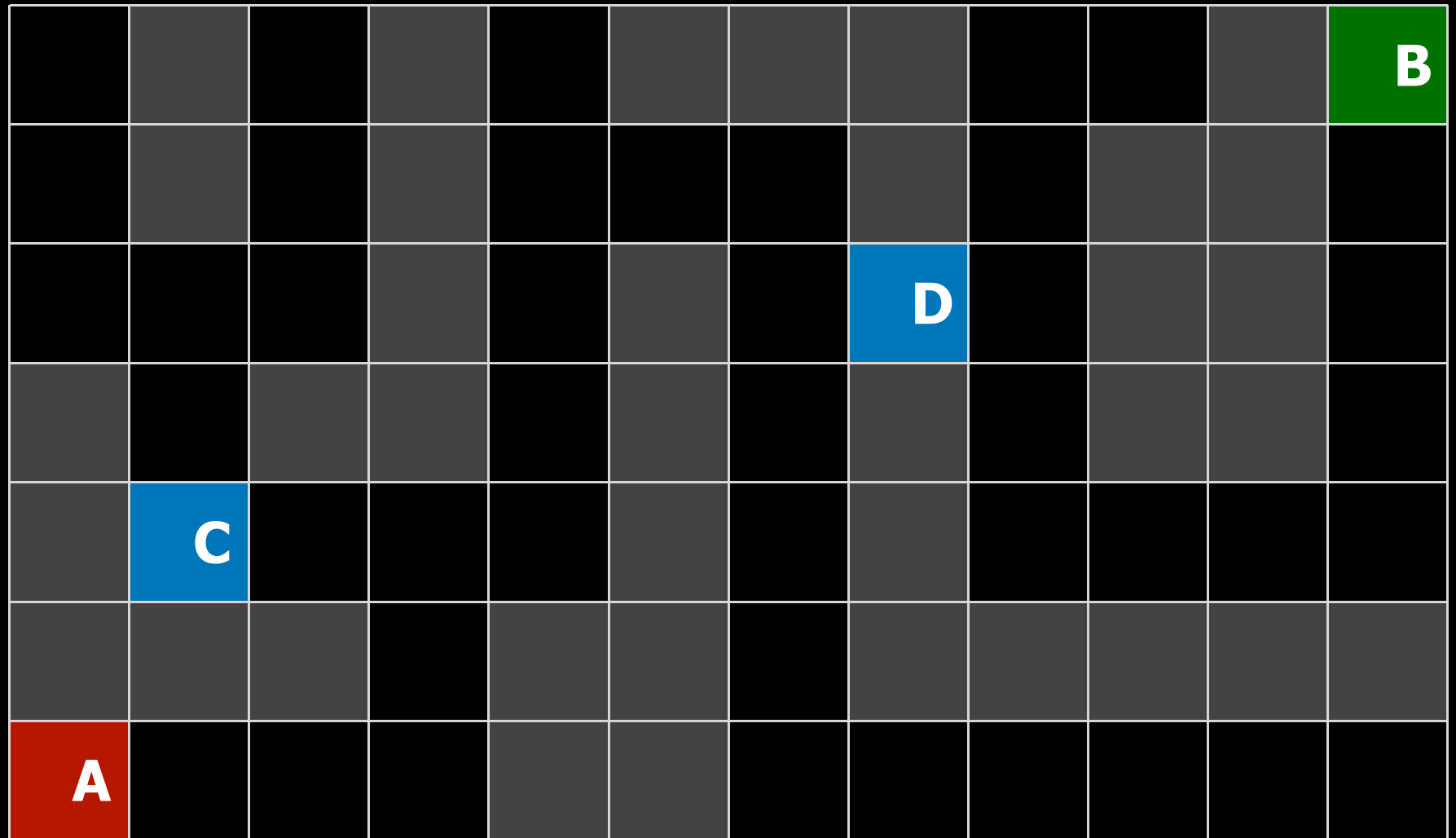
Arad	366	Mehadia	241
Bucharest	0	Neamt	234
Craiova	160	Oradea	380
Drobeta	242	Pitesti	100
Eforie	161	Rimnicu Vilcea	193
Fagaras	176	Sibiu	253
Giurgiu	77	Timisoara	329
Hirsova	151	Urziceni	80
Iasi	226	Vaslui	199
Lugoj	244	Zerind	374

Figure 3.22 Values of h_{SLD} —straight-line distances to Bucharest.

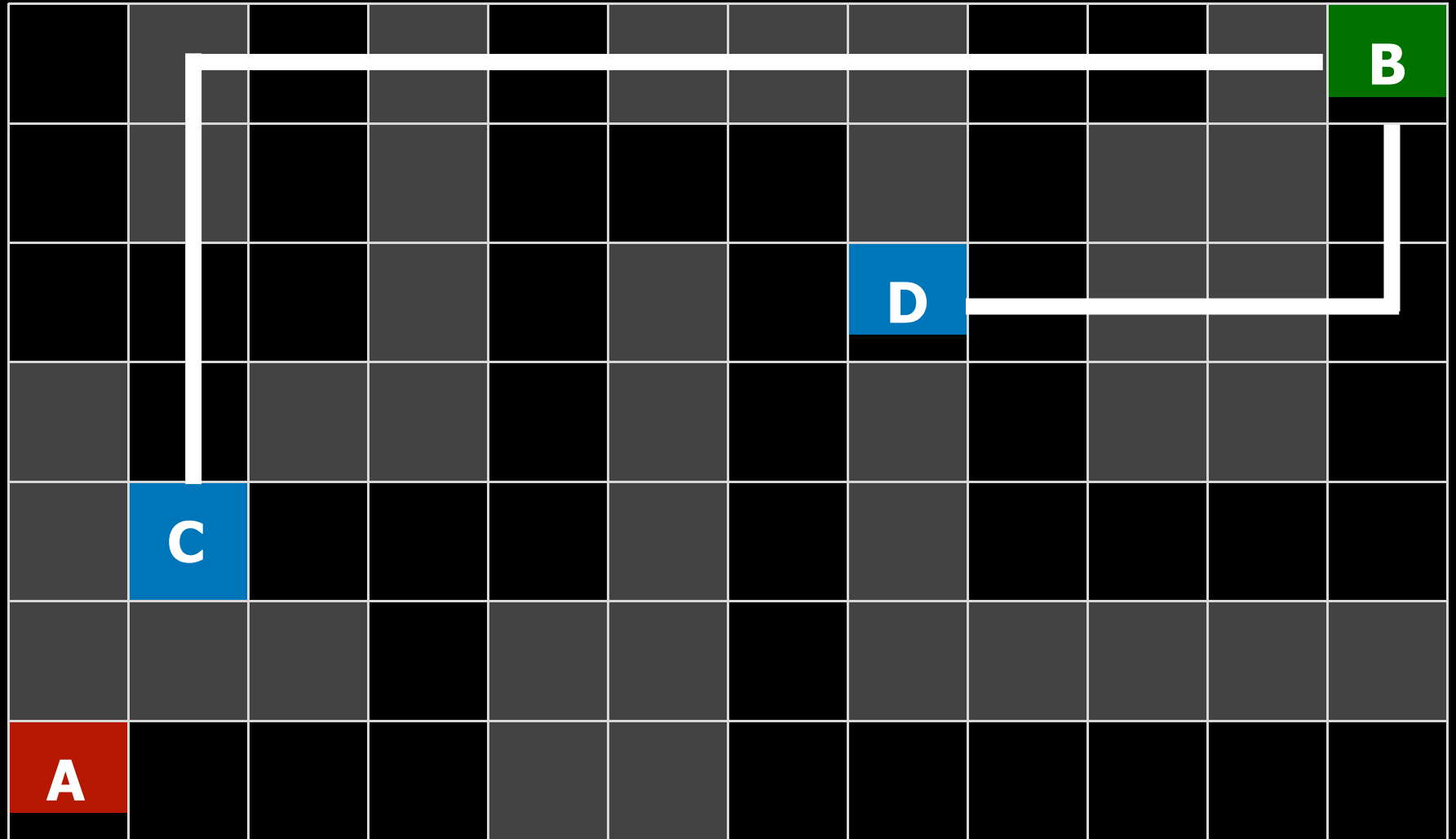
Heuristic function?



Heuristic function?



Heuristic function? Manhattan distance.



Best first search

- Best first Search is a strategy which uses evaluation function $f(n)$ to select which node to expand next.
- Best First Search uses priority queue as a frontier to select node with lowest evaluation function $f(n)$ value.
- There are two popular Best First Strategies –
 - Greedy Best First Search
 - A* Search

Greedy best first search

- Greedy Best First Search tries to expand the node that is closest to the goal, on the grounds that this is likely to lead to a solution quickly.
- Greedy Best First Search uses only heuristic function **$h(n)$** part of evaluation function for choosing which node to expand first i.e. for Greedy Best First Search **$f(n) = h(n)$** .

Greedy Best-First Search

11		9		7				3	2		B
12		10		8	7	6		4			1
13	12	11		9		7	6	5			2
	13			10		8		6			3
	14	13	12	11		9		7	6	5	4
			13			10					
A	16	15	14			11	10	9	8	7	6

Greedy Best-First Search

11		9		7				3	2		B
12		10		8	7	6		4			1
13	12	11		9		7	6	5			2
	13			10		8		6			3
	14	13	12	11		9		7	6	5	4
			13			10					
A	16	15	14			11	10	9	8	7	6

Greedy Best-First Search

11		9		7				3	2		B
12		10		8	7	6		4			1
13	12	11		9		7	6	5			2
	13			10		8		6			3
	14	13	12	11		9		7	6	5	4
			13			10					
A	16	15	14			11	10	9	8	7	6

Greedy Best-First Search

11		9		7				3	2		B
12		10		8	7	6		4			1
13	12	11		9		7	6	5			2
	13			10		8		6			3
	14	13	12	11		9		7	6	5	4
			13			10					
A	16	15	14			11	10	9	8	7	6

Greedy Best-First Search

11		9		7				3	2		B
12		10		8	7	6		4			1
13	12	11		9		7	6	5			2
	13			10		8		6			3
	14	13	12	11		9		7	6	5	4
			13			10					
A	16	15	14			11	10	9	8	7	6

Greedy Best-First Search

11		9		7				3	2		B
12		10		8	7	6		4			1
13	12	11		9		7	6	5			2
	13			10		8		6			3
	14	13	12	11		9		7	6	5	4
			13			10					
A	16	15	14			11	10	9	8	7	6

Greedy Best-First Search

11		9		7				3	2		B
12		10		8	7	6		4			1
13	12	11		9		7	6	5			2
	13			10		8		6			3
	14	13	12	11		9		7	6	5	4
			13			10					
A	16	15	14			11	10	9	8	7	6

Greedy Best-First Search

11		9		7				3	2		B
12		10		8	7	6		4			1
13	12	11		9		7	6	5			2
	13			10		8		6			3
	14	13	12	11		9		7	6	5	4
			13			10					
A	16	15	14			11	10	9	8	7	6

Greedy Best-First Search

11		9		7				3	2		B
12		10		8	7	6		4			1
13	12	11		9		7	6	5			2
	13			10		8		6			3
	14	13	12	11		9		7	6	5	4
			13			10					
A	16	15	14			11	10	9	8	7	6

Greedy Best-First Search

11		9		7				3	2		B
12		10		8	7	6		4			1
13	12	11		9		7	6	5			2
	13			10		8		6			3
	14	13	12	11		9		7	6	5	4
			13			10					
A	16	15	14			11	10	9	8	7	6

Greedy Best-First Search

11		9		7				3	2		B
12		10		8	7	6		4			1
13	12	11		9		7	6	5			2
	13			10		8		6			3
	14	13	12	11		9		7	6	5	4
			13			10					
A	16	15	14			11	10	9	8	7	6

Greedy Best-First Search

11		9		7				3	2		B
12		10		8	7	6		4			1
13	12	11		9		7	6	5			2
	13			10		8		6			3
	14	13	12	11		9		7	6	5	4
			13			10					
A	16	15	14			11	10	9	8	7	6

Greedy Best-First Search

11		9		7				3	2		B
12		10		8	7	6		4			1
13	12	11		9		7	6	5			2
	13			10		8		6			3
	14	13	12	11		9		7	6	5	4
			13			10					
A	16	15	14			11	10	9	8	7	6

Greedy Best-First Search

11		9		7				3	2		B
12		10		8	7	6		4			1
13	12	11		9		7	6	5			2
	13			10		8		6			3
	14	13	12	11		9		7	6	5	4
			13			10					
A	16	15	14			11	10	9	8	7	6

Greedy Best-First Search

11		9		7				3	2		B
12		10		8	7	6		4			1
13	12	11		9		7	6	5			2
	13			10		8		6			3
	14	13	12	11		9		7	6	5	4
			13			10					
A	16	15	14			11	10	9	8	7	6

Greedy Best-First Search

11		9		7				3	2		B
12		10		8	7	6		4			1
13	12	11		9		7	6	5			2
	13			10		8		6			3
	14	13	12	11		9		7	6	5	4
			13			10					
A	16	15	14			11	10	9	8	7	6

Greedy Best-First Search

11		9		7				3	2		B
12		10		8	7	6		4			1
13	12	11		9		7	6	5			2
	13			10		8		6			3
	14	13	12	11		9		7	6	5	4
			13			10					
A	16	15	14			11	10	9	8	7	6

Greedy Best-First Search

11		9		7				3	2		B
12		10		8	7	6		4			1
13	12	11		9		7	6	5			2
	13			10		8		6			3
	14	13	12	11		9		7	6	5	4
			13			10					
A	16	15	14			11	10	9	8	7	6

Greedy Best-First Search

11		9		7				3	2		B
12		10		8	7	6		4			1
13	12	11		9		7	6	5			2
	13			10		8		6			3
	14	13	12	11		9		7	6	5	4
			13			10					
A	16	15	14			11	10	9	8	7	6

Greedy Best-First Search

11		9		7				3	2		B
12		10		8	7	6		4			1
13	12	11		9		7	6	5			2
	13			10		8		6			3
	14	13	12	11		9		7	6	5	4
			13			10					
A	16	15	14			11	10	9	8	7	6

Greedy Best-First Search

11		9		7				3	2		B
12		10		8	7	6		4			1
13	12	11		9		7	6	5			2
	13			10		8		6			3
	14	13	12	11		9		7	6	5	4
			13			10					
A	16	15	14			11	10	9	8	7	6

Greedy Best-First Search

11		9		7				3	2		B
12		10		8	7	6		4			1
13	12	11		9		7	6	5			2
	13			10		8		6			3
	14	13	12	11		9		7	6	5	4
			13			10					
A	16	15	14			11	10	9	8	7	6

Greedy Best-First Search

11		9		7				3	2		B
12		10		8	7	6		4			1
13	12	11		9		7	6	5			2
	13			10		8		6			3
	14	13	12	11		9		7	6	5	4
			13			10					
A	16	15	14			11	10	9	8	7	6

Greedy Best-First Search

11		9		7				3	2		B
12		10		8	7	6		4			1
13	12	11		9		7	6	5			2
	13			10		8		6			3
	14	13	12	11		9		7	6	5	4
			13			10					
A	16	15	14			11	10	9	8	7	6

Greedy Best-First Search

11		9		7				3	2		B
12		10		8	7	6		4			1
13	12	11		9		7	6	5			2
	13			10		8		6			3
	14	13	12	11		9		7	6	5	4
			13			10					
A	16	15	14			11	10	9	8	7	6

Greedy Best-First Search

11		9		7				3	2		B
12		10		8	7	6		4			1
13	12	11		9		7	6	5			2
	13			10		8		6			3
	14	13	12	11		9		7	6	5	4
			13			10					
A	16	15	14			11	10	9	8	7	6

Greedy Best-First Search

11		9		7				3	2		B
12		10		8	7	6		4			1
13	12	11		9		7	6	5			2
	13			10		8		6			3
	14	13	12	11		9		7	6	5	4
			13			10					
A	16	15	14			11	10	9	8	7	6

Greedy Best-First Search

	10	9	8	7	6	5	4	3	2	1	B
	11										1
	12		10	9	8	7	6	5	4		2
	13		11						5		3
	14	13	12		10	9	8	7	6		4
			13		11						5
A	16	15	14		12	11	10	9	8	7	6

Greedy Best-First Search

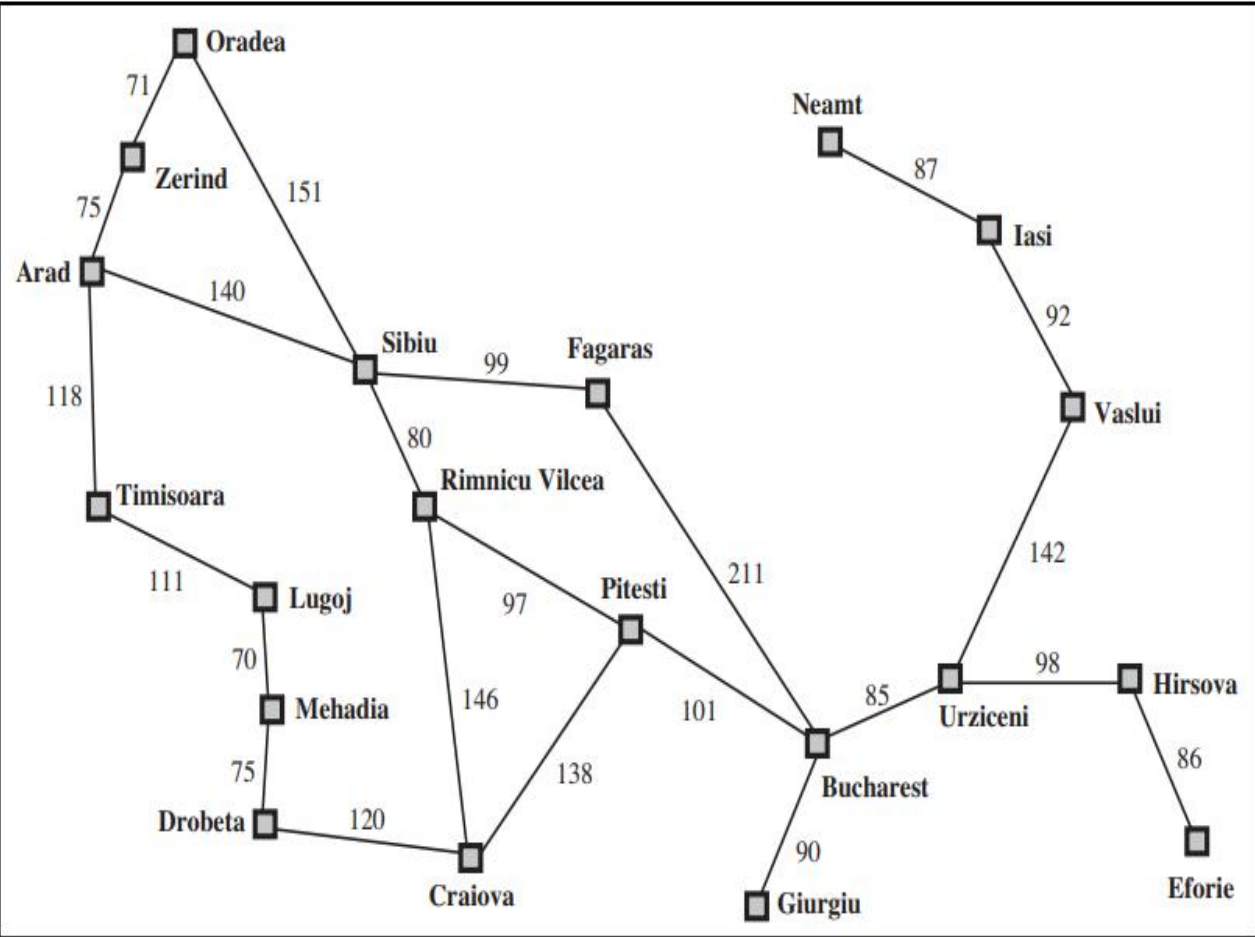
	10	9	8	7	6	5	4	3	2	1	B
	11										1
	12		10	9	8	7	6	5	4		2
	13		11						5		3
	14	13	12		10	9	8	7	6		4
			13		11						5
A	16	15	14		12	11	10	9	8	7	6

Greedy Best-First Search

	10	9	8	7	6	5	4	3	2	1	B
	11										1
	12		10	9	8	7	6	5	4		2
	13		11						5		3
	14	13	12		10	9	8	7	6		4
			13		11						5
A	16	15	14		12	11	10	9	8	7	6

Exercise

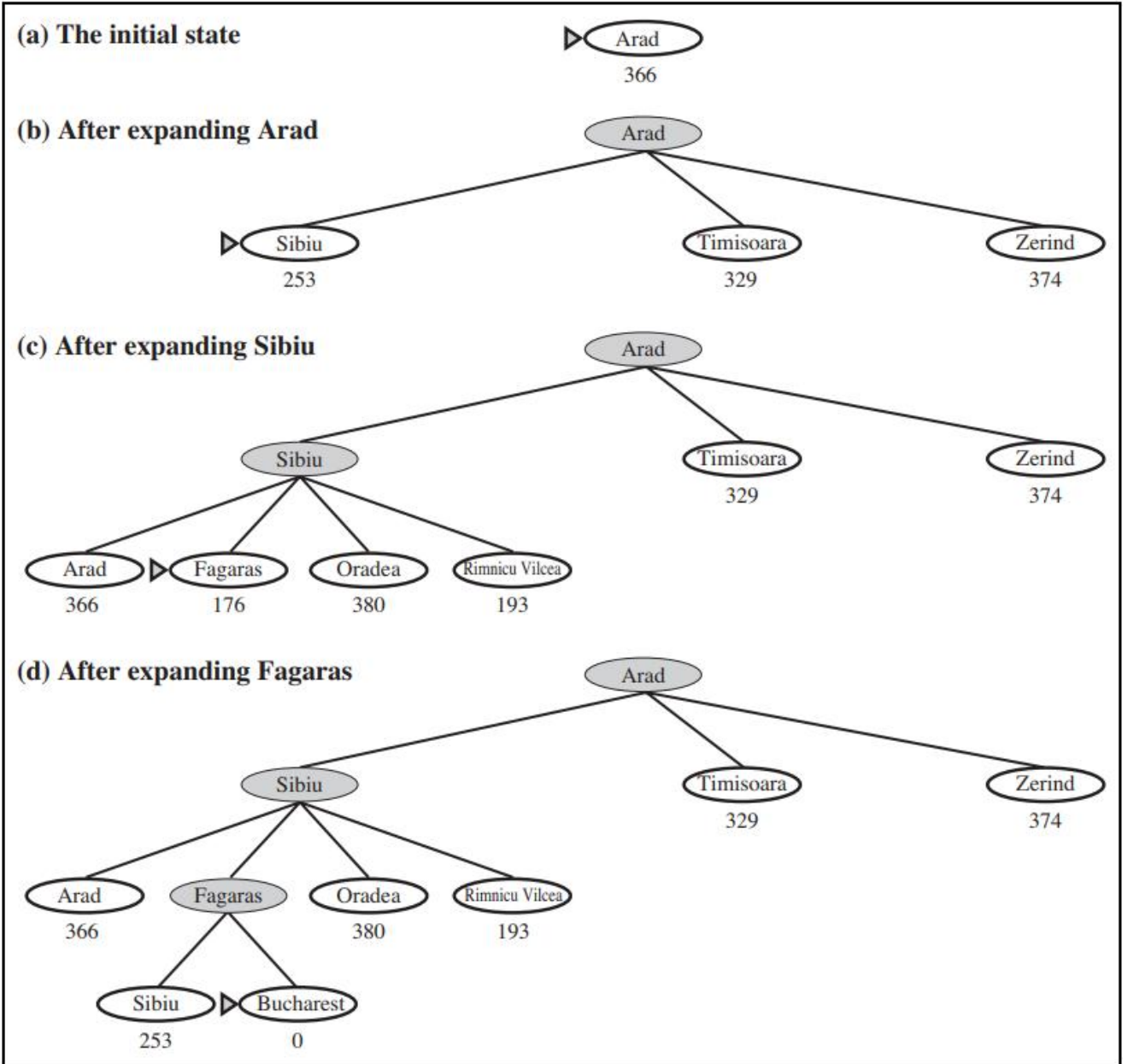
Run Greedy Best First Search to find path from **Arad** to **Bucharest**



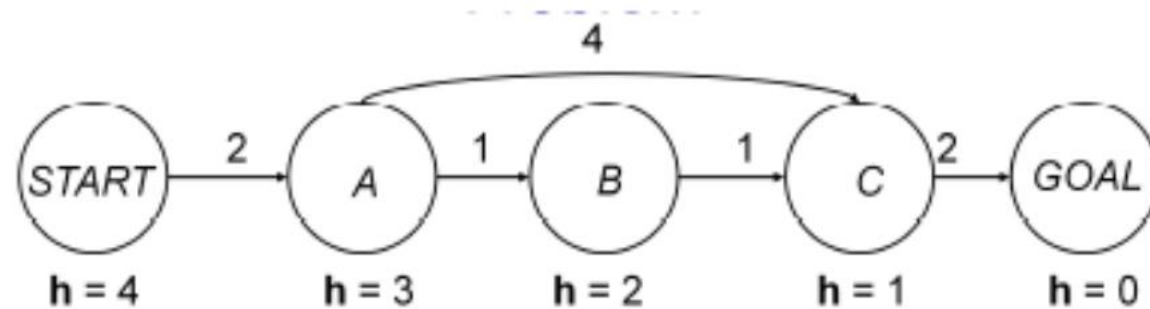
Arad	366	Mehadia	241
Bucharest	0	Neamt	234
Craiova	160	Oradea	380
Drobeta	242	Pitesti	100
Eforie	161	Rimnicu Vilcea	193
Fagaras	176	Sibiu	253
Giurgiu	77	Timisoara	329
Hirsova	151	Urziceni	80
Iasi	226	Vaslui	199
Lugoj	244	Zerind	374

Figure 3.22 Values of h_{SLD} —straight-line distances to Bucharest.

Greedy best first search



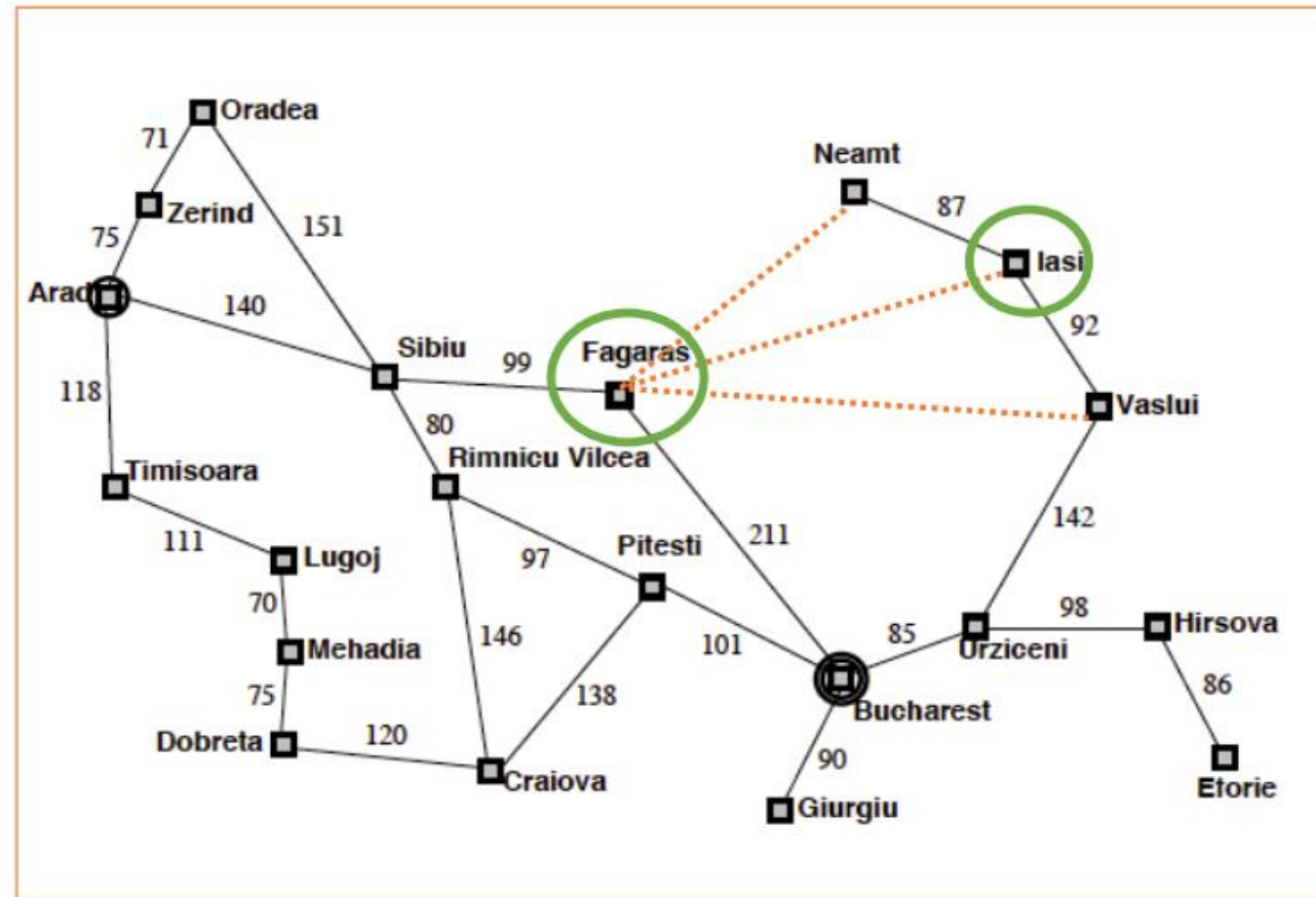
Is greedy best-first search optimal?



- At node A, we choose to go to node C, because it has a better (lower) heuristic value, instead of node B, which is optimal.
- Greedy Best-First Search is not optimal.

Is greedy best-first search complete?

- Greedy Best First Search is only complete in finite state space but not in infinite state space.
- Greedy Best First Search might end-up stuck on states which looks closer to goal but never leads to the goal.
- For example – Finding path from Iasi to Fagaras will cause Neamt to be expanded first but it will never lead to goal



Greedy best first search properties

- Greedy Best First Search has time and space complexity of $O(b^m)$ since it might have to explore all the nodes which look closer to goal state but never lead to the goal state.
- A good heuristic can reduce the complexity substantially.
- Greedy methods maximize short-term advantage without worrying about long-term consequences.

Similarity with Uninformed Search Strategies

➤ **Uniform Cost Search**

- Roughly “opposite” of Greedy best first search
- Uses $f(n) = g(n)$ where $g(n)$ is the sum of edge costs from start to node n .

➤ **Breadth First Search**

- No heuristic cost. A cost insensitive estimate of “cost” from start to node n .
- Uses $f(n) = g(n) = \text{depth}(n)$, where $\text{depth}(n)$ is the number of edges from start to node n .
- Both are best-first methods, they do not have a heuristic estimates and have different ways to measure cost from start to a node.

A* search algorithm

- A* is most widely known best first search algorithm.
- Main Idea behind A* is to minimize total path costs.
- Avoid expanding paths which are already expensive.
- Combine the greedy search (the estimated cost to go) with the uniform-search strategy (the cost incurred so far).

A* search algorithm

- A* expands nodes with lower evaluation function $f(n) = g(n) + h(n)$ where $g(n)$ is path cost function from start node to current node and $h(n)$ is heuristic function which is estimated cost from current node to goal node.
- A* is both complete and optimal if heuristic function $h(n)$ satisfies the following properties –
 - $h(n)$ should be admissible.
 - $h(n)$ should be consistent.

A* Algorithm

	10	9	8	7	6	5	4	3	2	1	B
	11										1
	12		10	9	8	7	6	5	4		2
	13		11						5		3
	14	13	12		10	9	8	7	6		4
			13		11						5
A	16	15	14		12	11	10	9	8	7	6

A* Algorithm

	10	9	8	7	6	5	4	3	2	1	B
	11										1
	12		10	9	8	7	6	5	4		2
	13		11						5		3
	14	13	12		10	9	8	7	6		4
			13		11						5
A	1+16	15	14		12	11	10	9	8	7	6

A* Algorithm

	10	9	8	7	6	5	4	3	2	1	B
	11										1
	12		10	9	8	7	6	5	4		2
	13		11						5		3
	14	13	12		10	9	8	7	6		4
			13		11						5
A	1+16	2+15	14		12	11	10	9	8	7	6

A* Algorithm

	10	9	8	7	6	5	4	3	2	1	B
	11										1
	12		10	9	8	7	6	5	4		2
	13		11						5		3
	14	13	12		10	9	8	7	6		4
			13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Algorithm

	10	9	8	7	6	5	4	3	2	1	B
	11										1
	12		10	9	8	7	6	5	4		2
	13		11						5		3
	14	13	12		10	9	8	7	6		4
			4+13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Algorithm

	10	9	8	7	6	5	4	3	2	1	B
	11										1
	12		10	9	8	7	6	5	4		2
	13		11						5		3
	14	13	5+12		10	9	8	7	6		4
			4+13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Algorithm

	10	9	8	7	6	5	4	3	2	1	B
	11										1
	12		10	9	8	7	6	5	4		2
	13		6+11						5		3
	14	13	5+12		10	9	8	7	6		4
			4+13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Algorithm

	10	9	8	7	6	5	4	3	2	1	B
	11										1
	12		7+10	9	8	7	6	5	4		2
	13		6+11						5		3
	14	13	5+12		10	9	8	7	6		4
			4+13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Algorithm

	10	9	8	7	6	5	4	3	2	1	B
	11										1
	12		7+10	8+9	8	7	6	5	4		2
	13		6+11						5		3
	14	13	5+12		10	9	8	7	6		4
			4+13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Algorithm

	10	9	8	7	6	5	4	3	2	1	B
	11										1
	12		7+10	8+9	9+8	7	6	5	4		2
	13		6+11						5		3
	14	13	5+12		10	9	8	7	6		4
			4+13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Algorithm

	10	9	8	7	6	5	4	3	2	1	B
	11										1
	12		7+10	8+9	9+8	10+7	6	5	4		2
	13		6+11						5		3
	14	13	5+12		10	9	8	7	6		4
			4+13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Algorithm

	10	9	8	7	6	5	4	3	2	1	B
	11										1
	12		7+10	8+9	9+8	10+7	11+6	5	4		2
	13		6+11						5		3
	14	13	5+12		10	9	8	7	6		4
			4+13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Algorithm

	10	9	8	7	6	5	4	3	2	1	B
	11										1
	12		7+10	8+9	9+8	10+7	11+6	12+5	4		2
	13		6+11						5		3
	14	13	5+12		10	9	8	7	6		4
			4+13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Algorithm

	10	9	8	7	6	5	4	3	2	1	B
	11										1
	12		7+10	8+9	9+8	10+7	11+6	12+5	13+4		2
	13		6+11						5		3
	14	13	5+12		10	9	8	7	6		4
			4+13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Algorithm

	10	9	8	7	6	5	4	3	2	1	B
	11										1
	12		7+10	8+9	9+8	10+7	11+6	12+5	13+4		2
	13		6+11						14+5		3
	14	13	5+12		10	9	8	7	6		4
			4+13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Algorithm

	10	9	8	7	6	5	4	3	2	1	B
	11										1
	12		7+10	8+9	9+8	10+7	11+6	12+5	13+4		2
	13		6+11						14+5		3
	14	6+13	5+12		10	9	8	7	6		4
			4+13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Algorithm

	10	9	8	7	6	5	4	3	2	1	B
	11										1
	12		7+10	8+9	9+8	10+7	11+6	12+5	13+4		2
	13		6+11						14+5		3
	14	6+13	5+12		10	9	8	7	15+6		4
			4+13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Algorithm

	10	9	8	7	6	5	4	3	2	1	B
	11										1
	12		7+10	8+9	9+8	10+7	11+6	12+5	13+4		2
	13		6+11						14+5		3
	7+14	6+13	5+12		10	9	8	7	15+6		4
			4+13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Algorithm

	10	9	8	7	6	5	4	3	2	1	B
	11										1
	12		7+10	8+9	9+8	10+7	11+6	12+5	13+4		2
	8+13		6+11						14+5		3
	7+14	6+13	5+12		10	9	8	7	15+6		4
			4+13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Algorithm

	10	9	8	7	6	5	4	3	2	1	B
	11										1
	9+12		7+10	8+9	9+8	10+7	11+6	12+5	13+4		2
	8+13		6+11						14+5		3
	7+14	6+13	5+12		10	9	8	7	15+6		4
			4+13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Algorithm

	10	9	8	7	6	5	4	3	2	1	B
	10+11										1
	9+12		7+10	8+9	9+8	10+7	11+6	12+5	13+4		2
	8+13		6+11						14+5		3
	7+14	6+13	5+12		10	9	8	7	15+6		4
			4+13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Algorithm

	11+10	9	8	7	6	5	4	3	2	1	B
	10+11										1
	9+12		7+10	8+9	9+8	10+7	11+6	12+5	13+4		2
	8+13		6+11						14+5		3
	7+14	6+13	5+12		10	9	8	7	15+6		4
			4+13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Algorithm

	11+10	12+9	8	7	6	5	4	3	2	1	B
	10+11										1
	9+12		7+10	8+9	9+8	10+7	11+6	12+5	13+4		2
	8+13		6+11						14+5		3
	7+14	6+13	5+12		10	9	8	7	15+6		4
			4+13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Algorithm

	11+10	12+9	13+8	7	6	5	4	3	2	1	B
	10+11										1
	9+12		7+10	8+9	9+8	10+7	11+6	12+5	13+4		2
	8+13		6+11						14+5		3
	7+14	6+13	5+12		10	9	8	7	15+6		4
			4+13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Algorithm

	11+10	12+9	13+8	14+7	6	5	4	3	2	1	B
	10+11										1
	9+12		7+10	8+9	9+8	10+7	11+6	12+5	13+4		2
	8+13		6+11						14+5		3
	7+14	6+13	5+12		10	9	8	7	15+6		4
			4+13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Algorithm

	11+10	12+9	13+8	14+7	15+6	5	4	3	2	1	B
	10+11										1
	9+12		7+10	8+9	9+8	10+7	11+6	12+5	13+4		2
	8+13		6+11						14+5		3
	7+14	6+13	5+12		10	9	8	7	15+6		4
			4+13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Algorithm

	11+10	12+9	13+8	14+7	15+6	16+5	4	3	2	1	B
	10+11										1
	9+12		7+10	8+9	9+8	10+7	11+6	12+5	13+4		2
	8+13		6+11						14+5		3
	7+14	6+13	5+12		10	9	8	7	15+6		4
			4+13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Algorithm

	11+10	12+9	13+8	14+7	15+6	16+5	17+4	3	2	1	B
	10+11										1
	9+12		7+10	8+9	9+8	10+7	11+6	12+5	13+4		2
	8+13		6+11						14+5		3
	7+14	6+13	5+12		10	9	8	7	15+6		4
			4+13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Algorithm

	11+10	12+9	13+8	14+7	15+6	16+5	17+4	18+3	2	1	B
	10+11										1
	9+12		7+10	8+9	9+8	10+7	11+6	12+5	13+4		2
	8+13		6+11						14+5		3
	7+14	6+13	5+12		10	9	8	7	15+6		4
			4+13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Algorithm

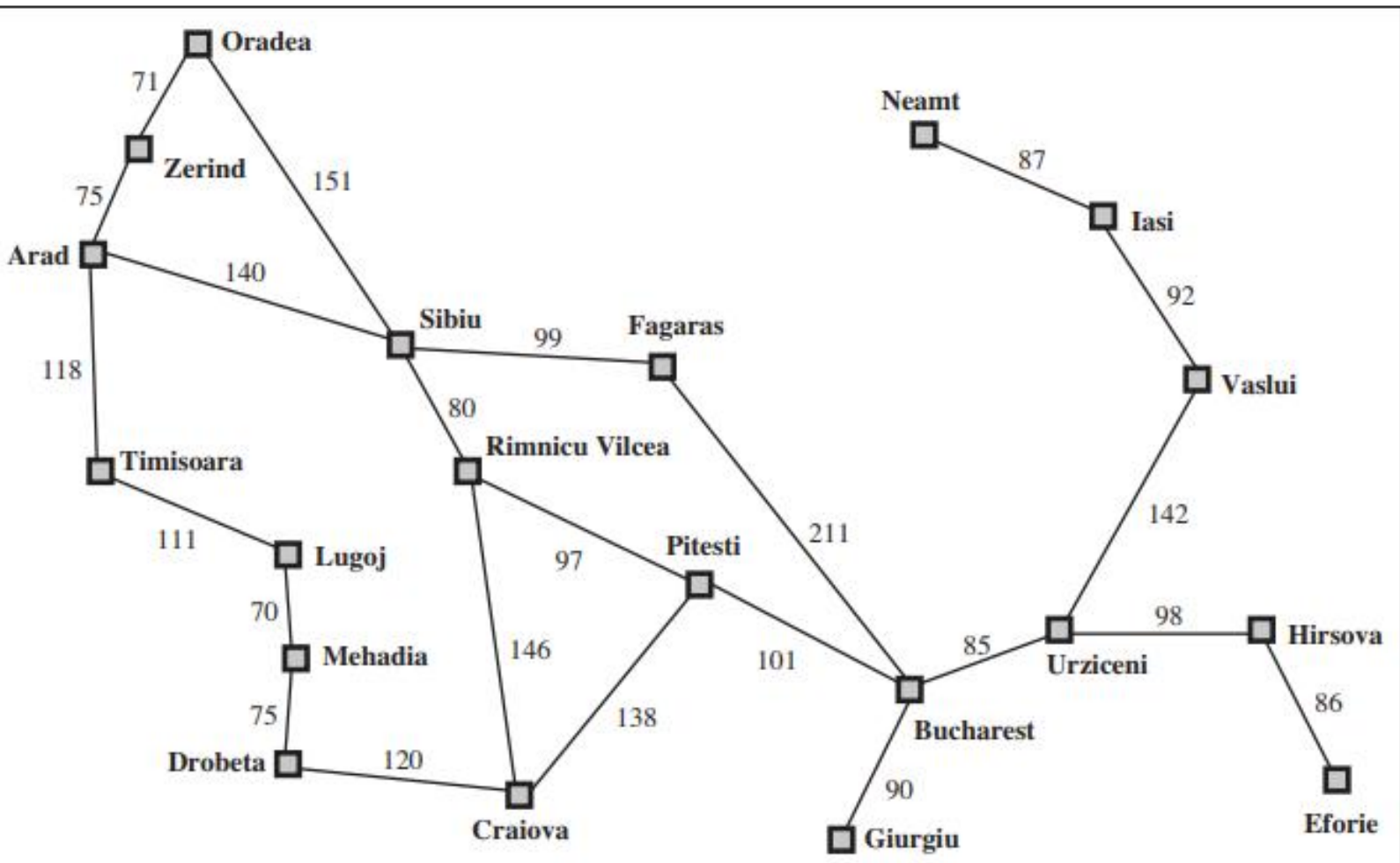
	11+10	12+9	13+8	14+7	15+6	16+5	17+4	18+3	19+2	1	B
	10+11										1
	9+12		7+10	8+9	9+8	10+7	11+6	12+5	13+4		2
	8+13		6+11						14+5		3
	7+14	6+13	5+12		10	9	8	7	15+6		4
			4+13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Algorithm

	11+10	12+9	13+8	14+7	15+6	16+5	17+4	18+3	19+2	20+1	B
	10+11										1
	9+12		7+10	8+9	9+8	10+7	11+6	12+5	13+4		2
	8+13		6+11						14+5		3
	7+14	6+13	5+12		10	9	8	7	15+6		4
			4+13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Algorithm: Exercise

➤ Use the A* search algorithm to find the path from Arad to Bucharest

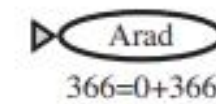


Arad	366
Bucharest	0
Craiova	160
Drobeta	242
Eforie	161
Fagaras	176
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244

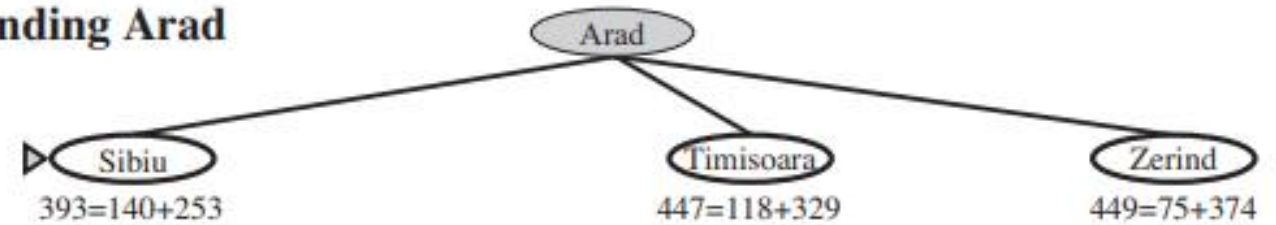
Mehadia	241
Neamt	234
Oradea	380
Pitesti	100
Rimnicu_Vilcea[Rimnicu Vilcea]	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

A* Algorithm: Exercise

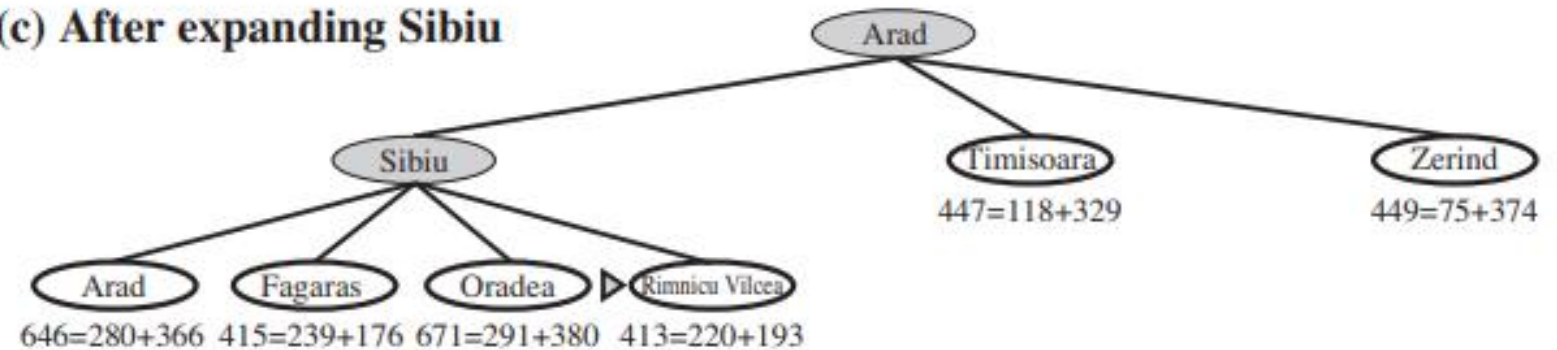
(a) The initial state



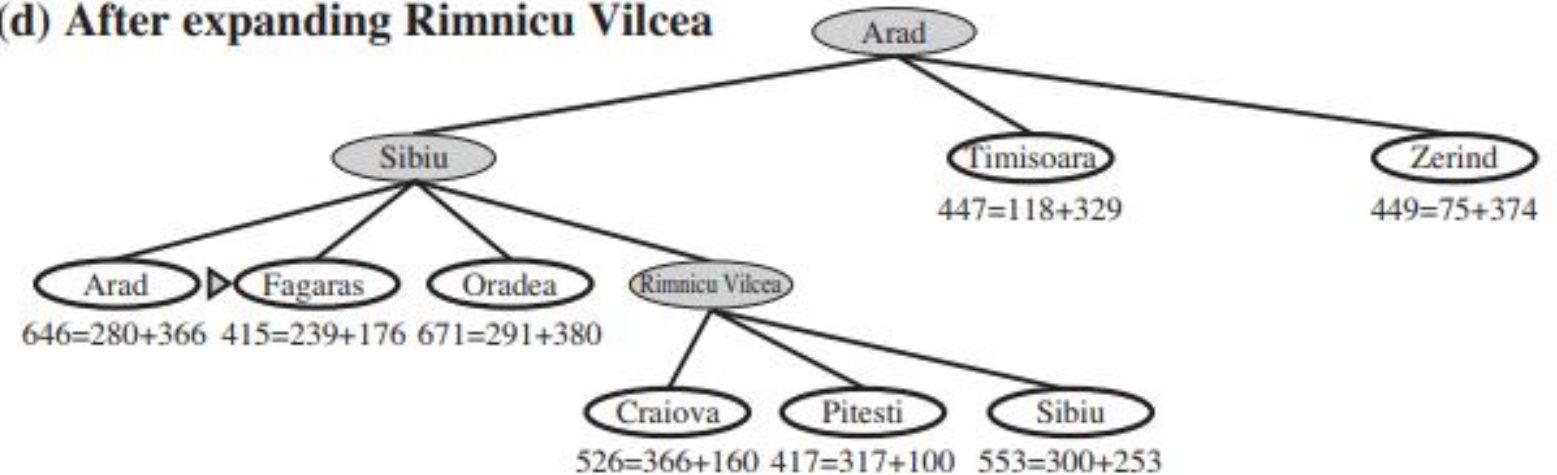
(b) After expanding Arad



(c) After expanding Sibiu

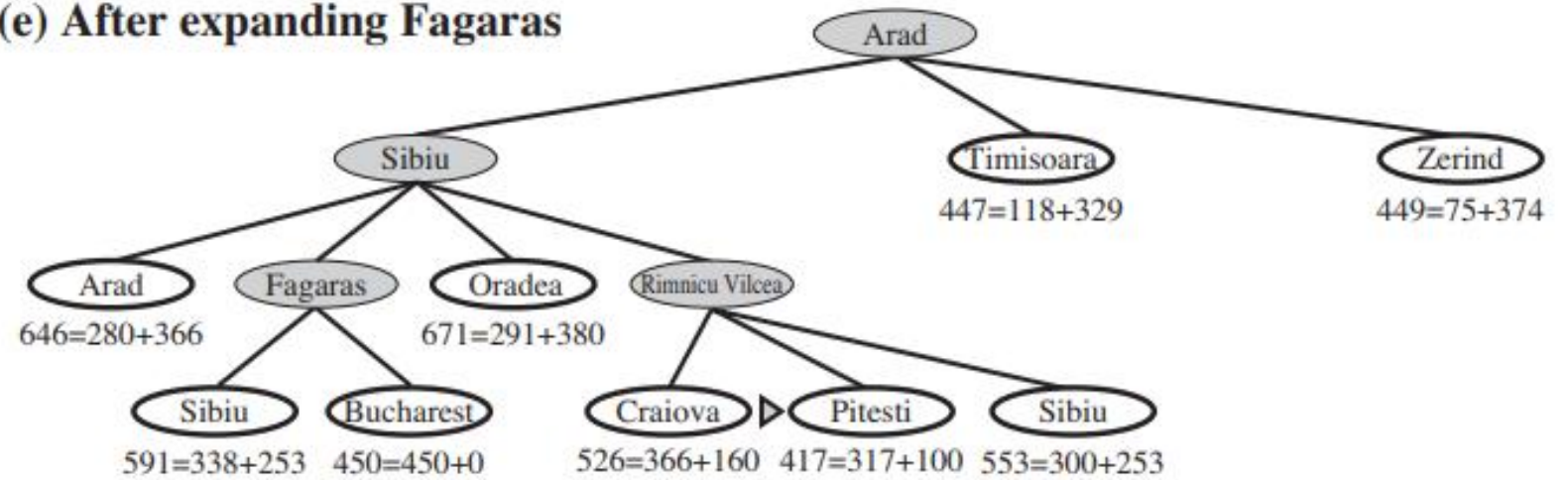


(d) After expanding Rimnicu Vilcea

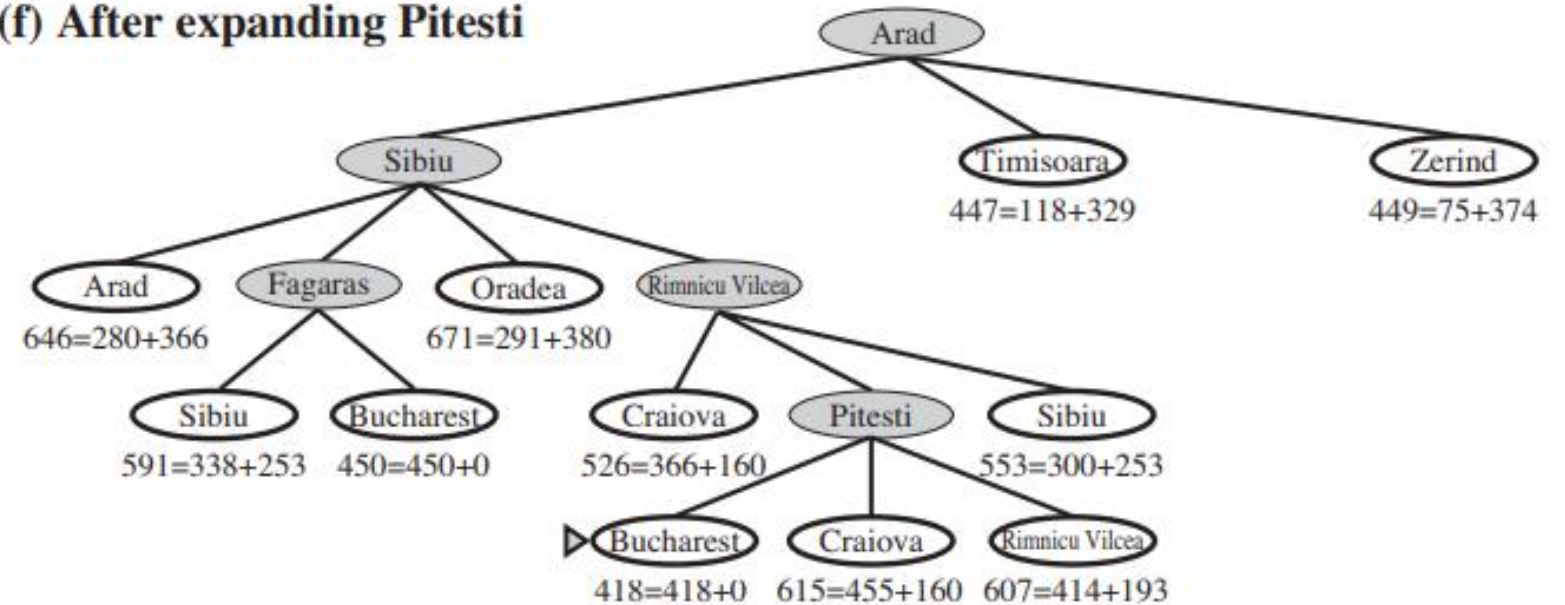


A* Algorithm: Exercise

(e) After expanding Fagaras



(f) After expanding Pitesti



Admissible heuristic

- The first condition required for optimality of A^* is that heuristic function must be admissible.
- An admissible heuristic is one that never overestimates the cost to reach the goal.
- Since $g(n)$ is the actual cost to reach node n from start state along current path and $f(n) = g(n) + h(n)$ therefore we can say that $f(n)$ never overestimates the true cost of a solution along the current path through n .
- Admissible heuristics are by nature optimistic because they think the cost of solving the problem is less than it actually is.
- An obvious example of an admissible heuristic is the straight-line distance that we used in the path finding problem between two cities.
- Straight-line distance is admissible because the shortest path between any two points is a straight line, so the straight line cannot be an overestimate.

Consistent heuristic

- Another condition required for optimality of A^* is that heuristic must be consistent or monotonic. This is called consistency or monotonicity of heuristic.
- A heuristic $h(\mathbf{n})$ is consistent if, for every node \mathbf{n} and every successor \mathbf{n}' of \mathbf{n} generated by any action \mathbf{a} , the estimated cost of reaching the goal from \mathbf{n} is no greater than the step cost of getting to \mathbf{n}' plus the estimated cost of reaching the goal from \mathbf{n} i.e. –

$$h(n) \leq c(n, a, n') + h(n')$$

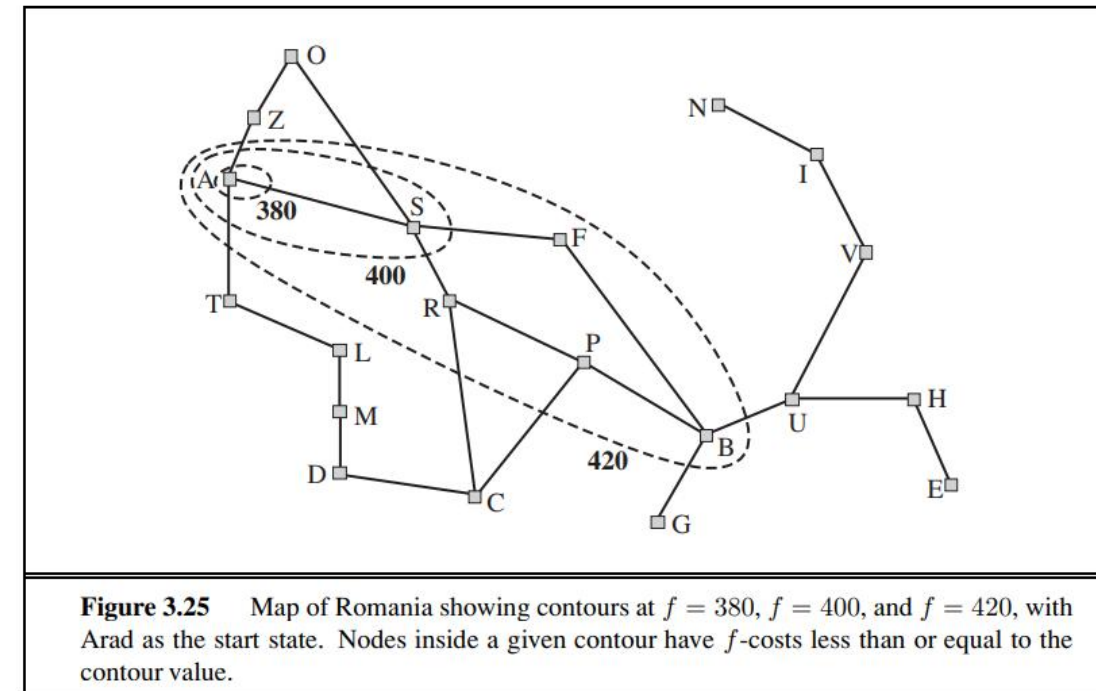
Consistent heuristic

- Consistency is a form of triangle inequality where triangle is formed by node **n**, successor of **n**, node **n'** and the goal node **G**.
- **Triangle inequality** says that length of any side of triangle can't be larger than sum of length of other two sides.
- For an admissible heuristic, the inequality makes perfect sense: if there were a route from **n** to **G** via **n'** that was cheaper than **h(n)**, that would violate the property that **h(n)** is a lower bound on the cost of to reach **G**.
- Every consistent heuristic is also admissible. Consistency is therefore a stricter requirement than admissibility.
- Straight-line distance heuristic is also consistent.

Optimality of A*

- As we mentioned earlier that, A* is optimal if heuristic function satisfies two properties -
 - $h(n)$ is admissible
 - $h(n)$ is consistent which is more stricter requirement.
- **Claim 1:** *If $h(n)$ is consistent, then the values of $f(n)$ along any path are nondecreasing.*
 - **Proof:** The proof follows directly from definition of consistency. Suppose n' is a successor of n ; then $g(n') = g(n) + c(n, a, n')$ for some action a , and we have $f(n') = g(n') + h(n') = g(n) + c(n, a, n') + h(n') \geq g(n) + h(n) = f(n)$.
- **Claim 2:** *Whenever A* selects a node n for expansion, the optimal path to that node has been found.*
 - **Proof:** If this wasn't the case, then there would have to be another frontier node n' on the optimal path from start node to n and that n' would have been selected first for the expansion.
- From above two claims, we can say that the sequence of nodes expanded by A* is in nondecreasing order of $f(n)$. Hence, the first goal node selected for expansion must be an optimal solution because f is the true cost for goal nodes (which have $h = 0$) and all later goal nodes will be at least as expensive.

Properties of A^*



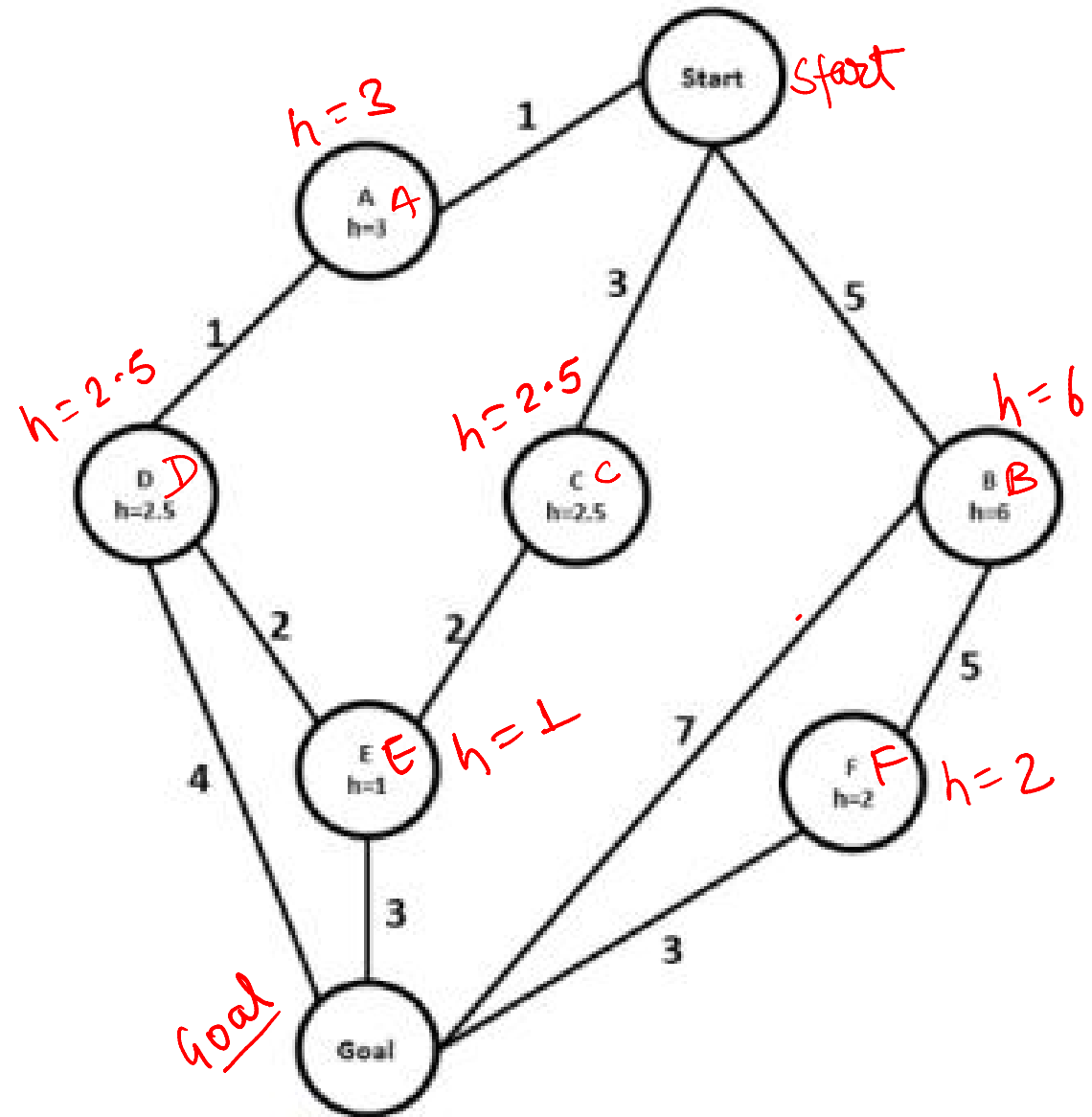
Properties of A*

- If C^* is the cost of the optimal solution path, then we can say the following -
 - A* expands all nodes with $f(n) < C^*$.
 - A* might then expand some of the nodes right on the “goal contour” (where $f(n) = C^*$) before selecting a goal node.
- Completeness requires that there be only finitely many nodes with cost less than or equal to C^* , a condition that is true if all step costs exceed some finite **eps** and if **b** is finite.
- Time and Space Complexity - $O(b^{\epsilon d}) = O((b^{\epsilon})^d)$
- A* keeps all the generated nodes in the memory or frontier. Therefore A* usually runs out of space long before it runs out of time. For this reason, A* is not practical for many large-scale problems.

Practice questions

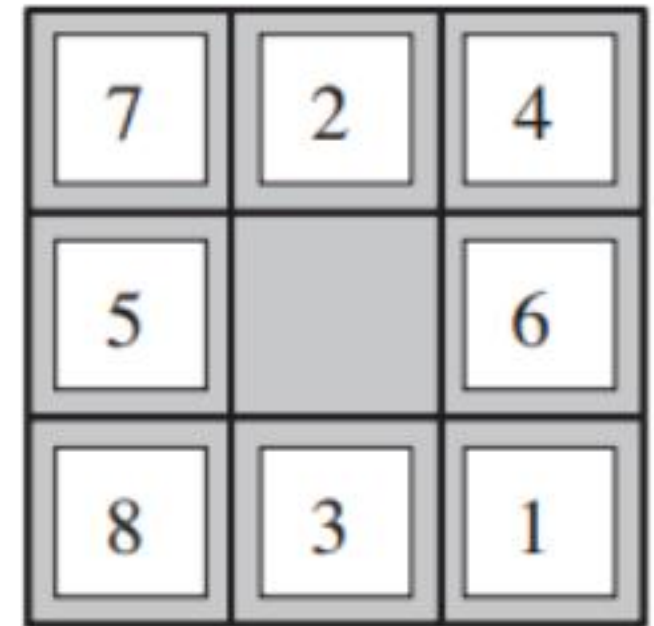
➤ Write down the order in which states are being explored by each of following search algorithms?

- Depth First Search
- Breadth First Search
- Iterative Deepening Search
- Uniform Cost Search
- Greedy Best First Search
- A* Search



Problem formulation: 8-puzzle

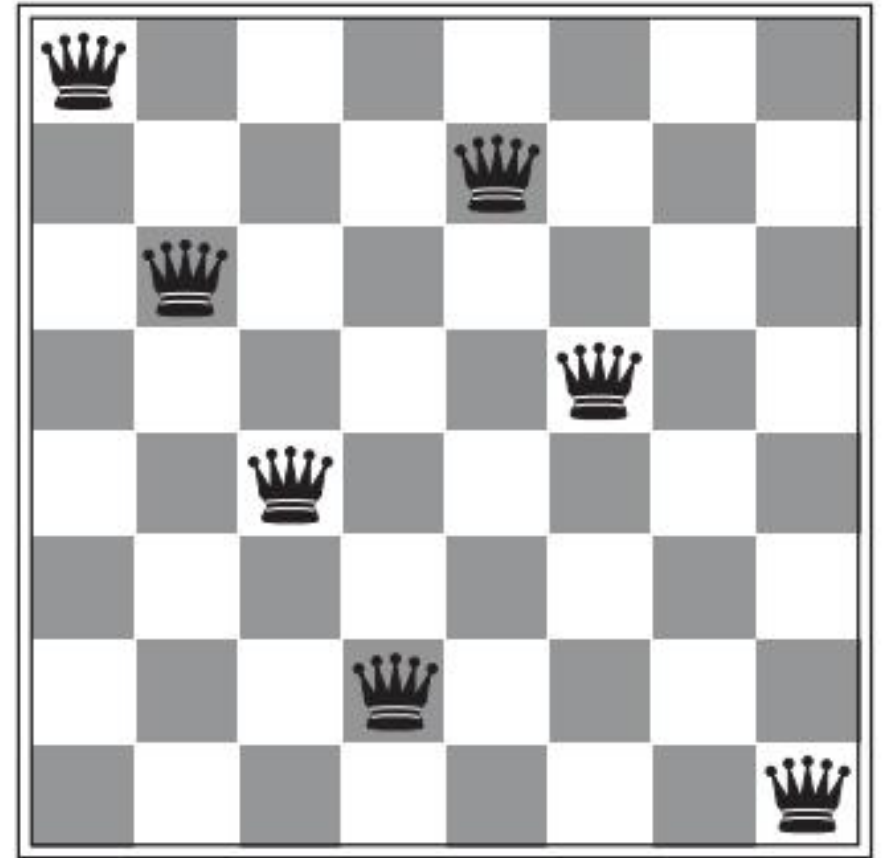
- The 8-puzzle, an instance of which is shown in the figure consists of a 3×3 board with eight numbered tiles and a blank space. A tile adjacent to the blank space can slide into the space. The object is to reach a specified goal state, such as the one shown on the right of the figure.
- Formulate 8-Puzzle as an AI problem and write down the following –
 - State Space
 - Start State
 - Goal State
 - Actions
 - Transition Model
 - Path Cost
 - Goal Test



7	2	4
5		6
8	3	1

Problem formulation: 8-queens

- In 8-queens problem, our goal is to put 8-queens on 8×8 chess-board such that no queens attack each other.
- Formulate 8-queens as an AI problem and write down the following –
 - State Space
 - Start State
 - Goal State
 - Actions
 - Transition Model
 - Path Cost
 - Goal Test



Practice questions

➤ Write down the order in which states are being explored by each of following search algorithms?

- Depth First Search
- Breadth First Search
- Iterative Deepening Search
- Uniform Cost Search
- Greedy Best First Search
- A* Search

