Finite State Machine with output

① Moore Machine:

A moore machine is a finite state machine whose output values are determined only by its current state. ~~of the machine~~

Formal definition:

A moore machine can be represented by 6-tuples. They are –

$$M = (Q, \Sigma, \Delta, \delta, \lambda, q_0\}$$

Where,

Q = set of all states

Σ = Set of input Alphabets.

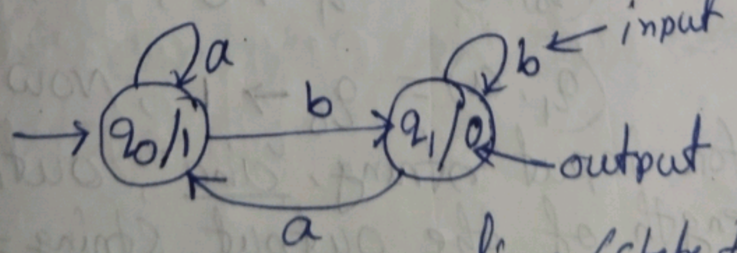
Δ = Set of output Alphabets.

δ = Transition function/Mapping
$(\delta : Q \times \Sigma \rightarrow Q)$

λ = output functions
$(\lambda : Q \rightarrow \Delta)$

q_0 = Initial state $(q_0 \in Q)$

Example:



Moore Machine (State transition Diagram)

$Q = \{q_0, q_1\}$

$\Sigma = \{a, b\}$

$\Delta = \{0, 1\}$

$\lambda : Q \rightarrow \Delta$

$q_0 \rightarrow 1$

$q_1 \rightarrow 0$

$q_0 = \{q_0\}$

Transition Table

$\delta : Q \times \Sigma \rightarrow Q$

Present state	Next state a	Next state b	$\lambda : Q \rightarrow \Delta$ Output
$\rightarrow q_0$	q_0	q_1	1
q_1	q_0	q_1	0

Question: Find the output of input string 'aba' in above machine.

Ans: In moore machine, the initial state always accept ε (Empty string).

Thus, $(q_0, \varepsilon) = q_0 \Rightarrow 1$, now, we are in q_0

$\xrightarrow{\varepsilon} \underset{1}{q_0} \xrightarrow{a} \underset{1}{q_0} \xrightarrow{b} \underset{0}{q_1} \xrightarrow{a} \underset{1}{q_0}$

$(q_0, a) = q_0 \rightarrow 1$, now, we are in q_0

$(q_0, b) = q_1 \rightarrow 0$, now, we are in q_1

$(q_1, a) = q_0 \rightarrow 1$, now we are in q_0

So, for input string, "aba", output is $= \underline{1101}$

ε

$\therefore$ The length of the output string = Length of input string $+ 1 = 3 + 1 = 4$.

Question: Find the output of input string 'abbbab'.

Ans: $(q_0, \varepsilon) = q_0 \rightarrow 1$

$(q_0, a) = q_0 \rightarrow 1$

$(q_0, b) = q_1 \rightarrow 0$

$(q_1, b) = q_1 \rightarrow 0$

$(q_1, b) = q_1 \rightarrow 0$

$(q_1, a) = q_0 \rightarrow 1$

$(q_0, b) = q_1 \rightarrow 0$

Hence, output = 1100010.

* A moore machine is very similar to DFA with a few key differences-

(i) It has not final states.

(ii) It does not accept or reject input instead it generates output from input.

(iii) It can-not have non-deterministic states.

Question 1: Construct a Moore Machine that take a set of all strings over alphabet $\{a, b\}$ as input and print 1 as output for every occurrence of 'aab' as substring.

OR

Construct a Moore Machine that takes a set of strings over $\{a, b\}$ and count number of occurrences of substring 'aab'.

Ans:

$L = \{ \underline{aab},\ a\underline{aab},\ aa\underline{aab},\ bb\underline{aab},\ a\underline{aab}ba,\ aa\underline{aab}bbb, \ldots \}$

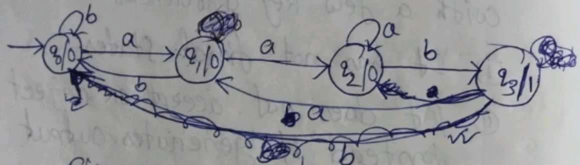


Fig: Moore Machine (Transition Diagram) (Same as DFA, without Final state)

$Q = \{q_0, q_1, q_2, q_3\}$

$\Sigma = \{a, b\}$

$\Delta = \{0, 1\}$

$\lambda : Q \to \Delta$

$q_0 \to 0$

$q_1 \to 0$

$q_2 \to 0$

$q_3 \to 1$

$q_0 = \{q_0\}$

Transition table

$\delta : Q \times \Sigma \to Q$

	a	b	output $\lambda : Q \to \Delta$
$\to q_0$	q_1	q_0	0
q_1	q_2	q_0	0
q_2	q_2	q_3	0
q_3	q_3	q_3	1

e.g.

Let, input string: $\underline{aab}\ \underline{aab}\ abb$

$(q_0, \varepsilon) \Rightarrow q_0 \to 0$

$\left.\begin{array}{l}(q_0, a) = q_1 \to 0 \\ (q_1, a) = q_2 \to 0 \\ (q_2, b) = q_3 \to 1\end{array}\right\} aab$

$\left.\begin{array}{l}(q_3, a) = q_2 \to 0 \\ (q_2, a) = q_2 \to 0 \\ (q_2, b) = q_3 \to 1\end{array}\right\} aab$

$\left.\begin{array}{l}(q_3, a) = q_1 \to 0 \\ (q_1, b) = q_0 \to 0 \\ (q_0, b) = q_0 \to 0\end{array}\right\} abb$

Hence, output string

= 0

$= \underset{\varepsilon}{\underline{0}}\ \underset{aab}{\underline{001}}\ \underset{aab}{\underline{001}}\ \underset{abb}{\underline{000}}$

(However we can omit the first $\underline{\underline{0}}$)

Question 2: Construct a Moore Machine that prints 'a' whenever the sequence '01' is encountered in any binary string.

Ans:

$\Sigma = \{0,1\}$

$\Delta = \{a, b\}$

$L = \{01, 001, 101, 00100, 00101, \ldots\}$

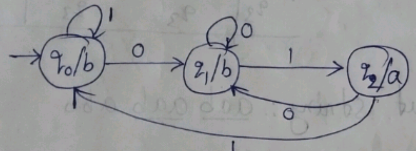


Fig: Transition Diagram (Moore Machine)

$Q = \{q_0, q_1, q_2\}$

Transition Table

$\lambda : Q \to \Delta$

$q_0 \to b$

$q_1 \to b$

$q_2 \to a$

$q_0 = \{q_0\}$

	0	1	output
$\to q_0$	q_1	q_0	b
q_1	q_1	q_2	b
q_2	q_1	q_0	a

Question 3: Construct a Moore Machine that take a set of all strings over {0,1} as input and produce 'x' as output if input end with 10 or produce 'y' as output if input end with 11, otherwise produce 'z'.

Ans:

$\Sigma = \{0,1\}$

$\Delta = \{x, y, z\}$

$L = \{10, 11, 010, 110, 110, 101, \ldots\}$

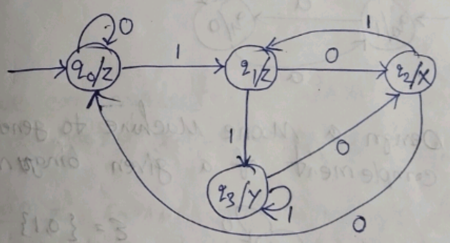


Fig: Moore Machin (Transition Diagram)

$Q = \{q_0, q_1, q_2, q_3\}$

Transition table

$\lambda : Q \to \Delta$

$q_0 \to z$

$q_1 \to z$

$q_2 \to x$

$q_3 \to y$

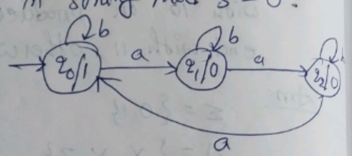
$q_0 = \{q_0\}$

	0	1	output
$\to q_0$	q_0	q_1	z
q_1	q_2	q_3	z
q_2	q_0	q_1	x
q_3	q_2	q_3	y

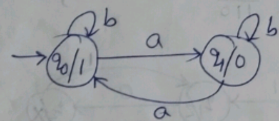
Question 4: Construct a Moore Machine that takes a set of all strings over alphabet $\{a, b\}$ as input and produce '1' as output if number of a's in string mod 3 = 0. (Divisible by 3)

Ans: $\Sigma = \{a, b\}$

$\Delta = \{0, 1\}$

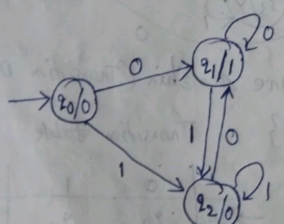


Question 5: Divisible by 2 or Mod by 2



Question 6: Design a Moore Machine to generate 1's complement of a given binary no.

Ans:



$\Sigma = \{0, 1\}$

$\Delta = \{0, 1\}$

$Q = \{q_0, q_1, q_2\}$

$\lambda : Q \to \Delta$

$q_0 \to 0$

$q_1 \to 1$

$q_2 \to 0$

	0	1	output
$\to q_0$	q_1	q_2	0
q_1	q_1	q_2	1
q_2	q_1	q_2	0

eg Let input String (binary): 1011

$(q_0, \varepsilon) = q_0 \to 0$

$(q_0, 1) = q_2 \to 0$

$(q_2, 0) = q_1 \to 1$

$(q_1, 1) = q_2 \to 0$

$(q_2, 1) = q_2 \to 0$

Hence output string (1's complement):

$\underset{\varepsilon}{0}$ 0100

We can neglect the initial 0 and thus the output which we get is 0100.

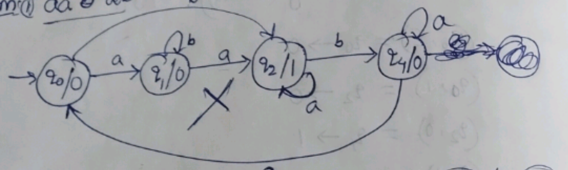
Question 7: Design the Moore M/c from the transition table given below, for the alphabet $\Sigma = \{a, b\}$ and the output alphabet is $\{0, 1\}$. For the following input sequence, find the respective output:

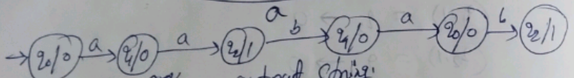
(i) aabab (ii) abbb (iii) ababb

	a	b	output
$\to q_0$	q_1	q_2	0
q_1	q_2	q_3	0
q_2	q_3	q_4	1
q_3	q_4	q_4	0
q_4	q_0	q_0	0

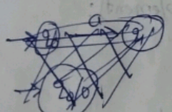
using STT

Ans(i) aabab

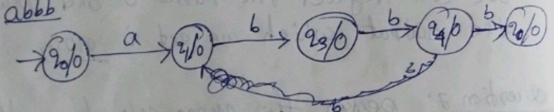




output String: $\frac{0}{\varepsilon}$ $\frac{01001}{aabab}$

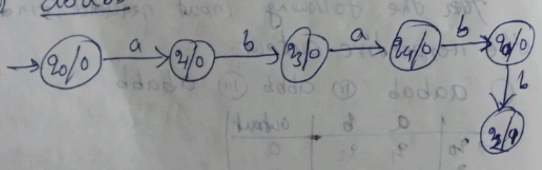


(ii) abbb



output string: $\frac{0}{\varepsilon}$ $\frac{0000}{abbb}$

(iii) ababb



output string: $\frac{0}{\varepsilon}$ $\frac{00001}{ababb}$

~~Mach~~

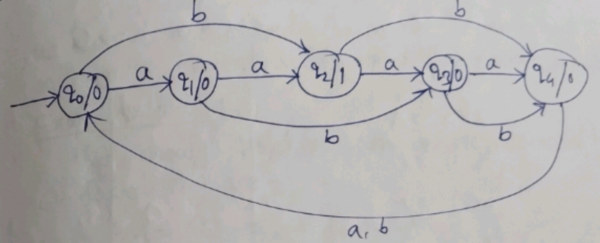
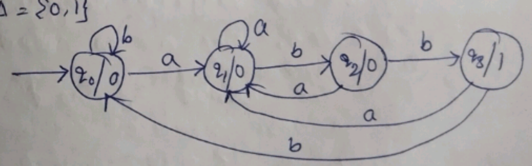


Fig: Moore machine (Transition Diagram) for the given transition table

Question 8: Count the occurrences of the sequences 'abb' in any input string over {a, b}

Ans: $\Sigma = \{a, b\}$

$\Delta = \{0, 1\}$



(ii) Mealy Machine:

A Mealy Machine is a finite state machine whose output values are determined both by its current state and the current inputs.

Formal definition:

A Mealy Machine can be represented by 6-tuples. They are–

$$M = (Q, \Sigma, \Delta, \delta, \lambda, q_0\}$$

where,

Q = set of all states

Σ = set of input Alphabets

Δ = set of output Alphabets

δ = Transition Function/Mapping

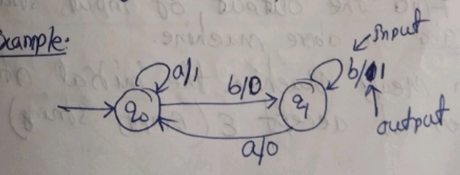
($\delta: Q \times \Sigma \rightarrow Q$)

λ = output functions

($\lambda: Q \times \Sigma \rightarrow \Delta$)

q_0 = initial states ($q_0 \in Q$)

Example:



$Q = \{q_0, q_1\}$

$\Sigma = \{a, b\}$

$\Delta = \{0, 1\}$

$\lambda : Q \times \Sigma \rightarrow \Delta$

$(q_0, a) \rightarrow 1$

$(q_0, b) \rightarrow 0$

$(q_1, a) \rightarrow 0$

$(q_1, b) \rightarrow 1$

$q_0 = \{q_0\}$

Transition Table

$\delta : Q \times \Sigma \rightarrow Q$

	a	output	b	output
$\rightarrow q_0$	q_0	1	q_1	0
q_1	q_0	0	q_1	1

Note: There is no final state in Moore Machine and Mealy Machine, because they are not language recognize as DFA & NFA, they are an output producer.

Question: Find the output of input string 'aba' in above machine.

Ans [In Mealy Machine, the initial state does not accept ε (Empty string).

Thus,

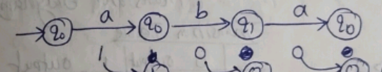
$(q_0, a) = q_0 \rightarrow 1$

$(q_0, b) = q_1 \rightarrow 0$

$(q_1, a) = q_0 \rightarrow 0$

So, for input string 'aba', output string is = 100

The length of the output string = Length of input string.



(we can write here also)

Question 1: From the following transition table design the Mealy Machine—

Present State	Next state			
	Input = 0		input = 1	
	state	output	state	output
$\rightarrow a$	b	x_1	c	x_1
b	b	x_2	d	x_3
c	d	x_3	c	x_1
d	d	x_3	d	x_2

Ans

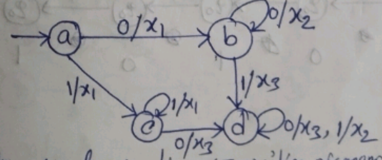


Fig Mealy Machine (Transition Diagram)

Here, $\Sigma = \{0, 1\}$

$\Delta = \{x_1, x_2, x_3\}$

Question 2: Construct a Mealy Machine that prints the 1's complement of an input bit (binary digit) string. (Binary no.)

Ans

$\Sigma = \{0, 1\}$

$\Delta = \{0, 1\}$

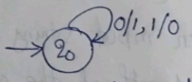


Fig: Mealy Machine (transition Diagram)

$Q = \{q_0\}$

$\lambda : Q \times \Sigma \to \Delta$

$(q_0, 0) \to 1$

$(q_0, 1) \to 0$

$q_0 = \{q_0\}$

Transition Table

	0	output	1	output
$\to q_0$	q_0	1	q_0	0

eg. Let, input string (binary): 1011

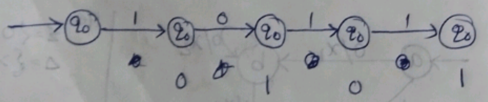
$(q_0, 1) = q_0 \to 0$

$(q_0, 0) = q_0 \to 1$

$(q_0, 1) = q_0 \to 0$

$(q_0, 1) = q_0 \to 0$

Hence output string (1's complement) = 0100



Question 3: Construct a Mealy Machine that take a set of all strings over alphabet $\{0,1\}$ and make 2's complement of an input string in binary number.

Ans:

[2's complement = 1's complement + 1]

$\Sigma = \{0, 1\}$

$\Delta = \{0, 1\}$

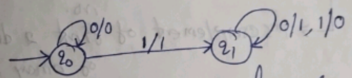


Fig: Mealy Machine (Transition Diagram)

$Q = \{q_0, q_1\}$

$\lambda : Q \times \Sigma \to \Delta$

$(q_0, 0) \Rightarrow q_0 \to 0$

$(q_0, 1) = q_1 \to 1$

$(q_1, 0) = q_1 \to 1$

$(q_1, 1) = q_1 \to 0$

$q_0 = \{q_0\}$

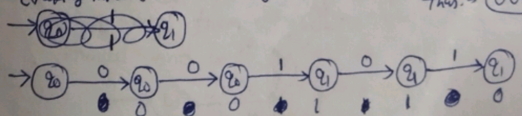
Transition table

	0	output	1	output
$\to q_0$	q_0	0	q_1	1
q_1	q_1	1	q_1	0

MSB LSB

eg, Let, input string (Binary): 10100 (taking input from Least Significant Bit (LSB) i.e. RHS→LHS

Thus → 00101



Hence, output string (2's complement): 01100 (Reverse order)

Procedure Example:

① 10100 (Binary)

1's comp: 01011

2's comp: +1

01100

↳ same as last 3 digit of binary no.

↳ 1's complement of first 2 digit of binary no.

② 1101 (Binary)

1's comp: 0010

2's comp: +1

0011

↳ same as last 1 digit of binary no.

↳ 1's complement of first 3 digit of binary no.

Question 4: Construct a Mealy M/c that take a set of all strings over $\Sigma = \{0,1\}$ as input and produce 'E' as output if 1's in string mod 2 = 0 (even no. of 1's) And produce 'F' as output if number of 1's in string mod $2 \neq 0$ (odd no. of 1's).

Ans: $\Sigma = \{0,1\}$

$\Delta = \{E,F\}$

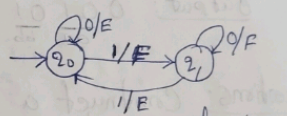
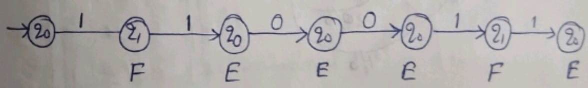


Fig: Mealy Machine (Transition Diagram)

	0	O/P	1	O/P
→q_0	q_0	E	q_1	F
q_1	q_1	F	q_0	E

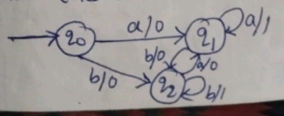
Eg. Input string: 110011



Hence, output string: FEEEFE

Examples: construct a Mealy M/c accepting lang. consisting of strings from Σ^* where $\Sigma = \{a,b\}$ & the string should end with either 'aa' or 'bb'.

Ans

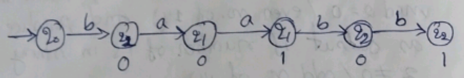


$\Sigma = \{a,b\}$

$\Delta = \{0,1\}$

e.g.

(i) input: baabb

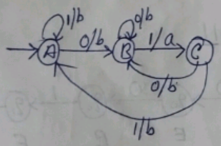


output: $0\,\underbrace{0\,1}_{aa}\,\underbrace{0\,1}_{bb}$

Question 6: Construct a Mealy M/c that prints 'a' whenever the sequence '01' is encountered in any i/p binary string.

Ans: $\Sigma = \{0,1\}$
$\Delta = \{a,b\}$

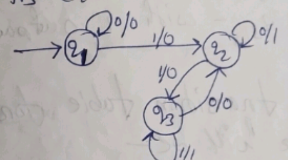
e.g. $\overset{b}{\underset{a}{\underline{0\,1}}}\ \overset{b}{\underset{a}{\underline{0\,1}}}\ \overset{b\,b}{\underset{a}{0\,\underline{0\,1}}}$



e.g.

Conversion of Mealy M/c to Moore M/c

Example 1: Convert the following Mealy M/c to its equivalent Moore Machine.



Ans: $\Sigma = \{0,1\}$ $\quad Q = \{q_1, q_2, q_3\}$
$\Delta = \{0,1\}$ $\quad \lambda : Q \times \Sigma \to \Delta$
$q_0 = \{q_1\}$

Transition table

	0	o/p	1	o/p
$\to q_1$	q_1	0	q_2	0
q_2	q_2	1	q_3	0
q_3	q_2	0	q_3	1

In above transition table,— ~~State~~

(i) state q_1 has only one output i.e. 0

(ii) State q_2 has two outputs 0 and 1

So, we create 2 states for q_2

q_{20} — with 0 output

q_{21} — with 1 output.

If a state has same output value then no need to convert it.

(ii) State q_3 has two outputs 0 and 1 so we create two states for q_3

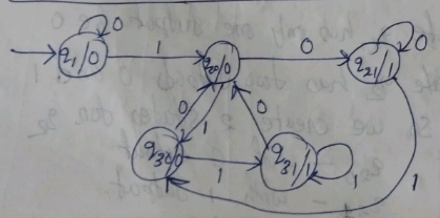
q_{30} - with ~~output~~ 0 output

q_{31} - with 1 output

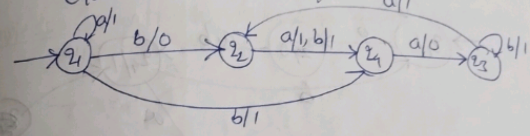
Hence the transition table for Moore Machine will be—

	0	1	output
$\rightarrow q_1$	q_1	q_{20}	0
q_{20}	q_{21}	q_{30}	0
q_{21}	q_{21}	q_{30}	1
q_{30}	q_{20}	q_{31}	0
q_{31}	q_{20}	q_{31}	1

Transition Diagram for Moore M/c:



Example 2: Convert the following Mealy M/c to equivalent Moore M/c



Ans: STT:

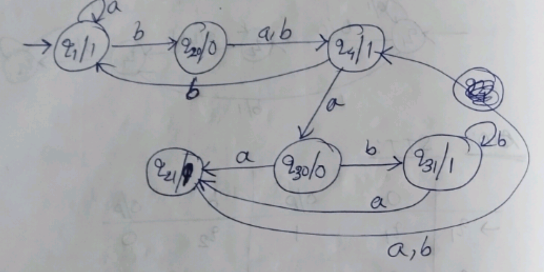
	a	o/p	b	o/p
$\rightarrow q_1$	q_1	1	q_2	0
q_2	q_4	1	q_4	1
q_3	q_2	1	q_3	1
q_4	q_3	0	q_1	1

STT for Moore Machine:

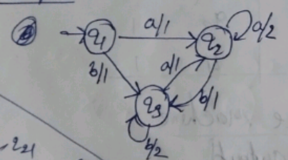
	a	b	output
$\rightarrow q_1$	q_1	q_{20}	1
q_{20}	q_4	q_4	0
q_{21}	q_4	q_4	1
q_{30}	q_{21}	q_{31}	0
q_{31}	q_{21}	q_{31}	1
q_4	q_{30}	q_1	1

[Note As states q_1 has same output value, hence we not convert it two parts. Same for state q_4.]

Transition Diagram for Moore Machine:



Example 3:



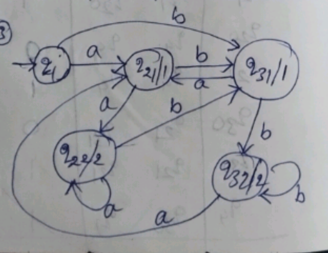
①	a	o/p	b	o/p
$\rightarrow q_1$	q_2	1	q_3	1
q_2	q_2	2	q_3	1
q_3	q_2	1	q_3	2

$q_2 < \begin{matrix} q_{21} \\ q_{22} \end{matrix}$ $q_3 < \begin{matrix} q_{31} \\ q_{32} \end{matrix}$

②

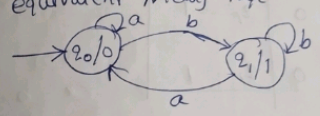
	a	b	output
$\rightarrow q_1$	q_{21}	q_{31}	—
q_{21}	q_{22}	q_{31}	1
q_{22}	q_{22}	q_{31}	2
q_{31}	q_{21}	q_{32}	1
q_{32}	q_{21}	q_{32}	2

③



Conversion of Moore M/c to Mealy M/c

Example 1: Convert the following Moore M/c to its equivalent Mealy M/c.



Ans: Transition Table of the Moore M/c

	a	b	output
$\rightarrow q_0$	q_0	q_1	0
q_1	q_0	q_1	1

The equivalent Mealy M/c can be obtained as –

(i) for state q_0

$$\lambda'(q_0, a) = \lambda(\delta(q_0, a)) = \lambda(q_0) = 0$$

$$\underbrace{\lambda'(q_0, b)}_{\text{Moore M/c}} = \underbrace{\lambda(\delta(q_0, b))}_{\text{Mealy M/c}} = \lambda(q_1) = 1$$

(ii) for state q_1

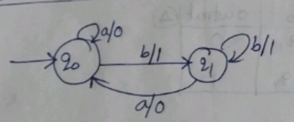
$$\lambda'(q_1, a) = \lambda(\delta(q_1, a)) = \lambda(q_0) = 0$$

$$\lambda'(q_1, b) = \lambda(\delta(q_1, b)) = \lambda(q_1) = 1$$

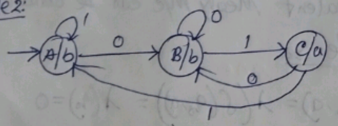
The transition table for the Mealy M/c

	a	o/p	b	o/p
$\rightarrow q_0$	q_0	0	q_1	1
q_1	q_0	0	q_1	1

Transition Diagram for Mealy M/c



Example2:



Ans:

Transition table of the Moore M/c

	0	1	output (λ)
$\rightarrow A$	B	A	b
B	B	C	b
C	B	A	a

Direct Method (by seeing Moore transition diagram)

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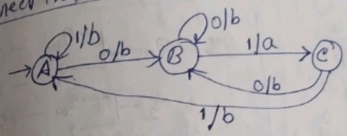
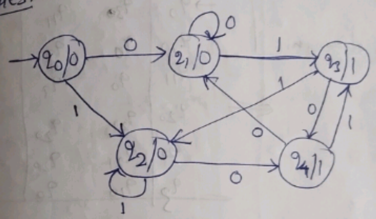
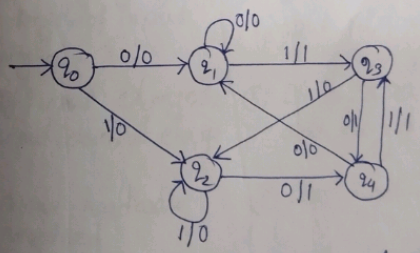


Examples:



Ans: (Directly)



Draw the STT for Mealy method

Q Convert the given mealy M/c to Moore M/c

state	a	b
$\rightarrow q_0$	$q_3, 0$	$q_1, 1$
q_1	$q_0, 1$	$q_3, 0$
q_2	$q_2, 1$	$q_2, 0$
q_3	$q_1, 0$	$q_0, 1$

Ans

q_1 — q_{10} — 0, q_{11} — 1

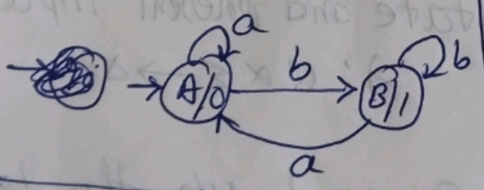
q_2 — q_{20} — 0, q_{21} — 1

state	a	b	output
$\rightarrow q_0$	q_3	q_{11}	1
q_{10}	q_0	q_3	0
q_{11}	q_0	q_3	1
q_{20}	q_{21}	q_{20}	0
q_{21}	q_{21}	q_{20}	1
q_3	q_{10}	q_0	0

Draw the diagram

Moore M/c vs Mealy M/c

Moore M/c	Mealy M/c
1. Defn. (normal)	1. Defn. (normal)
2. In moore machine, output is associated with every state	2. In mealy machine, output is given along the transition edge with input symbol.
3. In Moore M/c, output depends ~~on~~ upon the present state. $\lambda : Q \rightarrow \Delta$	3. In Mealy M/c, output depends upon the present state and present input. $\lambda : Q \times \Sigma \rightarrow \Delta$
4. In Moore M/c the length of ~~input~~ output string is equal to length of input string +1.	4. In Mealy M/c, the length of output string is equal to the length of input string
5. If input changes, output does not change.	5. If input changes, output also changes.
6. More hardware required.	6. Less hardware required.
7. Easy to design	7. Difficult to design.

moone M/c	mealy M/c
(8) output is placed on states	(8) output is placed on transitions.
(9) Generally, it has more no. of states than mealy M/c.	(9) Generally, it has lesser no. of states than moone M/c.
(10) moone M/c react slower to input.	(10) Mealy M/c react faster to input.
(11) Example: 	(11) Example: