Fluid Dynamics :- dynamics of liquid, gases & plasmas

$\boxed{\text{Macroscopic Phenomena}}$

Fluid :- CONTINUOUS medium with well-defined macroscopic quantities (density ρ, pressure p ...)

$\Rightarrow$ even at microscopic level, the fluid is composed of particles.

In astrophysical systems :-

Density → high (e.g. compact objects)
→ low (e.g. ISM, IGM)

Temperature → high (e.g accretion disk, jets)
→ low (e.g ISM)

Examples of fluids in the Universe :-

- Star Interiors, Compact objects
  (White Dwarf, Neutron stars)
- ISM, IGM, ICM
- Accretion disks, Jets, Stellar winds

Fluid element :- a region of fluid that is;

(a) small enough that there are no significant variations of any variable q

$$l_{region} << l_{scale} \sim \frac{q}{|\nabla q|}$$

(b) large enough to contain sufficient particles (to satisfy continuum limit)

$$n\, l_{region}^3 >> 1$$

(n: no. density of particles)

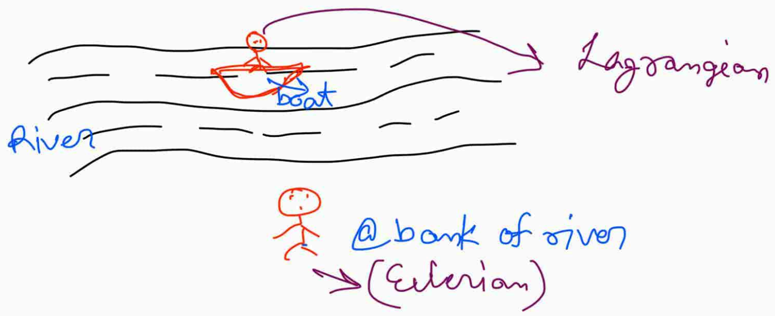
* Fluids are collection of molecules, widely spaced for a gas, closely spaced for a liquid.

* Fluids continuously deform under shear stress, while solids don't.

Eulerian vs Lagrangian descriptions :

(a) Eulerian :- considers the properties of fluid measured in a frame of reference, which is fixed in space.

(b) Lagrangian :- considers the properties of fluid measured in CO-MOVING frame of reference.



$$\frac{Df}{Dt} = \frac{d}{dt} f[(x(t), y(t), z(t), t]$$

$$= \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt} + \frac{\partial f}{\partial z}\frac{dz}{dt} + \frac{\partial f}{\partial t}$$

$$\boxed{\frac{Df}{Dt} = \frac{\partial f}{\partial t} + (u \cdot \nabla) f}$$

Lagrangian derivative   Eulerian derivative   Advection term

Fluid Dynamics equations

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{u}) = 0$$ continuity eqn $\Rightarrow$ Conservation of mass

$$\rho \frac{\partial \vec{u}}{\partial t} + \rho (\vec{u} \cdot \vec{\nabla}) \vec{u} = -\vec{\nabla} p + \rho \vec{g}$$ euler eqn $\Rightarrow$ mom. cons.

$$\frac{dQ}{dt} = \frac{dE}{dt} + \frac{dW}{dt}$$ $\Rightarrow$ energy cons.

Conservative form?

$$\frac{\partial U}{\partial t} + \vec{\nabla} \cdot \vec{F} = S \qquad U = \begin{bmatrix} \rho \\ \rho u \\ E \end{bmatrix} \quad \& \quad F = \begin{bmatrix} \rho \vec{u} \\ \rho \vec{u} \times \vec{u} + p \underline{\underline{I}} \\ (E + p) \vec{u} \end{bmatrix}$$

where $E = \underbrace{\frac{1}{2} \rho u^2}_{\text{K.E. density}} + \underbrace{e}_{\text{Internal energy density}}$

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{u}) = 0$$

$$\frac{\partial}{\partial t} (\rho \vec{u}) + \vec{\nabla} \cdot \left[\rho \vec{u} \times \vec{u} + p \underline{\underline{I}} \right] = \rho \vec{g}$$

$$\frac{\partial E}{\partial t} + \vec{\nabla} \cdot (E + p) \vec{u} = \rho \vec{u} \cdot \vec{g}$$

3 equations & 4 unknowns $\Rightarrow$ Need a closure

Equation of state

Ideal gas EoS : $p = \rho k_B T$

(other rel. correct EoS)

Why Numerical simulation?

complicated & non-linear nature of equations!

⇒ conservative form of fluid equations are solved on a grid using Numerical techniques!

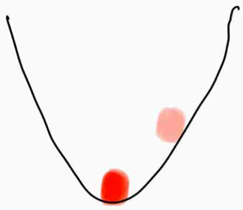
PLUTO code

- It is a finite volume/finite difference, shock-capturing code, which solves the system of conservation laws. → (PDEs)
- Equations are discretized and solved on a structured mesh.
- Written in C, C++ & fortran.

Instabilities?

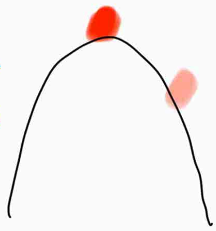
small perturbation in any system →

decay with time or oscillates around equilibrium point



Stable

Grow in amplitude with time & never returns to initial state.



Unstable

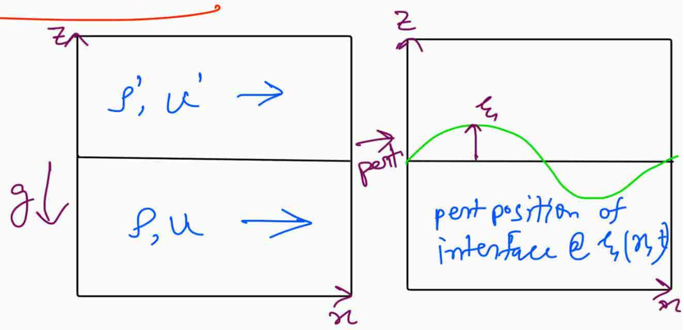
Instabilities at Two fluid interface

- constant gravity

Assume,

$$\xi = A \exp\{i(kx-\omega t)\}$$

perturbed position



⇓

Linear Stability Analysis

$$\boxed{\frac{\omega}{k} = \frac{\rho u + \rho' u'}{\rho + \rho'} \pm \sqrt{\frac{g}{k}\frac{(\rho - \rho')}{(\rho + \rho')} - \frac{\rho\rho'(u-u')^2}{(\rho+\rho')^2}}}$$

$$\frac{\omega}{k} = \underset{(\alpha)}{\text{Real}(\omega)} + \underset{(i\beta)}{\text{Imag}(\omega)}$$

$\xi_1 \sim A' e^{-i\alpha t}$ (osillatory)

$\xi_1 \sim A'' e^{\beta t}$ (exponential growth)

⇒ Imaginary part of freq. is related to unstable modes.

Case (I) :- Static fluid with denser fluid at top

$(\rho < \rho'$ & $u = u' = 0)$

$$\frac{\omega}{k} = \sqrt{\frac{g}{k}\frac{(\rho-\rho')}{(\rho+\rho')}} = \text{Imaginary part}$$

⇒ Rayleigh - Taylor Instability

(Supernova Explosions.)

Case (II) :- Moving fluid with denser fluid at bottom (RT stable)

$(\rho > \rho'$ & $u \neq 0$ & $u' \neq 0)$

unstable modes are: $k > \dfrac{(\rho^2 - \rho'^2)\, g}{\rho\rho'(u-u')^2}$

⇓

Kelvin - Helmholtz Instability

(Astrophysical Jets)