Deconstructing Multi-Heuristic A*: The Role of Heuristic Diversity in Cost-Aware Mars Rover Navigation

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***Abstract*—graph-search algorithms are a foundational component of autonomous robot navigation, particularly in large environments with different traversal cost. though optimal planners such as A* guarantee solution quality, their computational cost can be restricting in complex terrains. Bounded-suboptimal planners, like Weighted A* (WA*), does improve some efficiency by inflating heuristic estimations, but they remain vulnerable to misleading heuristics and trap-like geometrics. The Multi-Heuristic A* (MHA*) addresses this limitation by simultaneously using multiple heuristics by a multi-queue search architecture. Although MHA* has shown a strong performance, the extent of how much of it comes from heuristic diversity versus the underlying multi-queue mechanism remains unclear. This work brings a controlled ablation study which evaluates the role of heuristic diversity in MHA*. Experiments are conducted on synthetic Mars-like grid maps with terrain-dependent energy costs, that compares WA*, MHA* with diverse heuristics, and MHA* with homogeneous heuristics. All across 500 planning scenarios, the results show that MHA* with diverse heuristics achieves orders-of-magnitude reductions in node expansions and planning time on complex maps, with statistically significant improvements over WA*, but it had higher path cost. In contrast, homogeneous MHA* exhibits similar search behaviour to WA* but has a higher runtime because of the multi-queue overhead. These findings show that heuristic diversity, rather than multi-queue structure alone, is the main reason of MHA*'s performance and advantages in complex terrain planning.**

***Index Terms*—Path Planning, Heuristic Search, Multi-Heuristic A*, Robotics, Mars Rover**

I. Introduction

Autonomous mobile robots operating in large-scale, unstructured environments must repeatedly solve path planning problems that too under strict computational constraints. Planetary exploration rovers , in particular can face terrains with different traversal cost, with limited onboard computing resources, and the need for reliable real-time decision making.in such a setting , the rovers ability to quickly generate feasible, low-cost path is often more critical than guaranteeing strict optimality.

Graph based search algorithms still remains a dominant approach for addressing these challenges due to their strong theoretical foundations and predictable behaviour, the classical A* guarantees the completeness and optimality when it is guided by an admissible heuristic, which makes it a standard baseline in robotic planning. But as the environments get more complex and bigger in size, A*'s exhaustive search behaviour can lead to prohibitive planning times, espically in the high-resolution grids commonly used for terrain-aware navigation.

To face these limitations , bounded-suboptimal planners were proposed that trade optimality fro computational efficiency. Weighted A*(WA*) makes the search faster by inflating the heuristic estimate, biasing exploring toward the goal. While this approach often show substantial speeds, it can still be mislead by in accurate heuristics or complex obstacle placements, which can result in excessive node expansions before converging to a solution.

Multi-heuristic A*(MHA*) further extends on this idea by managing multiple concurrent searches that are driven by different types of heuristics. By combining an admissible anchor heuristic with other, mostly inadmissible heuristics, MHA* enables more aggressive exploration of the state space while keeping the bounded-suboptimality. Prior work has shown that this strategy can dramatically improve planning efficiency in challenging environments.

Despite its demonstrated success, most of the existing evaluations of MHA* primarly compare t against other planners such as, anytime varriants or sampling-based methods. As a result, it is still unclear that whether the observed performance gains comes from the multi-queue architechure or from the use of diverse heuristic guidance. This distinction is important for understanding when the additional complexity of MAH* is justified. This paper addresses this gap through a focused ablation study that isolates the role of heuristic diversity in MHA*. Using synthetic mars-like terrain maps with heterogeneous traversal costs, we compare WA*, MHA* with diverse heurestics, and MHA* with homogeneous heuristics in identical conditions. By analysing the planning time, nodes expamded, path cost, and path length across hundreds of maps, this work clarifies the mechanism supporting MHA*'s performance and provides practical insights for designing efficient heuristic planners in complex terrain environments.

II. Related Work

Graph based search methods have been central to robot motion planning in discretised state spaces, where the objective is to compute a minimum-cost path under a defined transition cost model. The classical A* algrothim, introduced by Hart, Nilsson and Raphael, expands states in order of evaluation function ($f(n) = g(n) + h(n)$), where ($g(n)$) is the accumulated cost from the start and ($h(n)$) is the heuristic estimate to the goal. When (h) is admissible, A* is both complete and optimal, which makes it a standard baseline for navigation and planning research [1]. But the computational load of optimal search can increase quickly with map size and environment complexity, which motivates approach that trade optimality for speed.

A big class of such approaches is bounded-suboptimal heuristic search. Pohl proposed weighted A* (WA*), that accelerated the planning by inflating the heuristic term, which uses $f(n) = g(n) + \epsilon \cdot h(n)$ with $\epsilon > 1$ [2]. This increased goal bias often reduces node expansion substantially, but WA* can still be slowed bby misleading heuristics and trap like geometries (e.g. concave obstacles), where the search must explore many states before finding an escape route. These cases can diminish the practical speed benefits of a purely single-heuristic, greedier strategy.

Multi-Heuristic A* (MHA*), given by Anie et al., addresses such limitations by using multiple heuristic within a multi-queue search architecture [3]. MHA* has an admissible "anchor" search to preserve bounded-suboptimality guarantees, while additional searches use arbitrary (potentially inadmissible) heuristics to encourage more aggressive exploration in different directions of the state space. This design enables planners to incorporate domain knowledge- such as terrain choices or goal-directed behaviours without sacrificing the therotical assurances provided by the anchor.

Beyond individual planners, standard references in motion planning provide broader context for algrothmic tradeoffs, environment representations, and practical considerations in robotics navigation [4], [5]. While MHA*'s empirical effectiveness is well established, published evaluations typically focus on comparing MHA* against alternative planning families (e.g. anytime search or sampling-based methods). In contrast, this work performs an ablation study aimed specifically at revealing the contribution of heuristic diversity from the contribution of the multi-queue mechanism itself. By comparing MHA* using diverse heuristic against an intentionally homogeneous-heuristic control, the study isolates whether the performance gains are intrinsic to parallel heuristic guidance or primarily driven by heuristic complementarity.

III. Problem formulation and environment

This work considers 2D grid-based path planning for a rover that is navigating a discretized terrain map. The environment is represented as a fixed-size occupancy-and-cost grid G, where each cell corresponds to a terrain type with an associated traversal cost. A planning problem is defined by a start state x_{start} and a goal state x_{goal}, and the objective is to compute a collision-free path that minimizes cumulative traversal cost.

A. State space and motion model

each state corresponds to a grid cell (r, c). The action set uses 4- connected motion (von Neumann neighbourhood), allowing moves in the cardinal directions:

$$A = \{(\pm 1, 0), (0, \pm 1)\} \tag{1}$$

Subjects to bounds $0 \leq r < H$, $0 \leq c < W$. Cells labelled as obstacle are non-traversable and are excluded from successor generation.

B. Terrain cost model

Traversable terrain cells are assigned positive integer costs that serve as a simplified proxy for energy expenditure. The terrain encoding used in this project is:

- Obstacle: cost ∞ (blocked),
- Bedrock: cost 1,

- Gravel: cost 5,
- Sand: cost 10.

Given a path π from x_{start} to x_{goal}, the total path cost is defined as:

$$J(\pi) = \sum_{(r,c)\in\pi} Cost(r, c) \tag{2}$$

where $Cost(r, c)$ is the terrain cost of the entered cell. Under this formulation, a lower-cost terrain is preferred, and sand regions impose a strong penalty that encourages the planner to route around them when possible.

C. Map Generation and Scenario Construction

To enable a controlled and repetable evaluation, synthetic "Mars-like" terrain maps are generated at a resolution of 100×100. Two environment classes are produced:

- **simple maps:** these maps were dominated by low-cost bedrock with relatively sparse obstacles and limited sand regions.
- **complex maps:** these maps had a higher obstacle density and substantially increased sand coverage, creating more challenging, high-cost structures that can include misleading local gradients for heuristic-guided search.

A total of 10 maps is created in total (5 simple and 5 complex). For each map, 50 planning scenarios are generated by sampling a valid start cell from the left boundary and a valid goal cell from the right boundary, making sure that both lie on traversable terrain. This way we get a total of 500 start-goal problems across the dataset. The use of fixed map sizes, consistent scenario counts per map, and controlled terrain distributions allows direct, fair comparisions between planners that are under identical task distributions.

IV. Planning algorithms

In this section we describe the planning algorithms evaluated in this work, which includes the baseline bounded-suboptimal planner and the multi-heuristic variants. All planners operate on the same grid representation and cost model descrbed in section III, which ensures that performance arise from search strategy and heuristic guidance.

A. Weighted A*

The Weighted A* (WA*) extends classical A* by biasing the search much more aggressively towards the goal through heuristic inflation. Each state (n) is elevated using

$$f(n) = g(n) + \epsilon \cdot h(n) \tag{3}$$

where $g(n)$ is the cost-to-come, $h(n)$ is a heuristic estimate of the cost-to-go, and $\epsilon \geq 1$ is a user-defined weight. When $\epsilon = 1$, WA* reduces to standard A*. Larger values of ϵ prioritize heuristic guidance, usually reducing node expansions and planning time at the expense of solution optimality.

In this work, WA* serves as the primary baseline that is bounded suboptimal planner. A Euclidean distance heuristic is used as the admissible heuristic h_{anchor}, and a fixed weight $\epsilon = 1.5$ is applied accorss all the experiments. This choice shows a common trade off between the speed and solution quality while maintaining comparability with anchor search used the MHA*

B. Multi-Heuristic A*

The Multi-Heuristic A* (MHA*) generalizes the bounded-suboptimal search by managing multiple priority queues, each of which is guided by a different heuristic. One of which is designated to the anchor search and used admissible heuristic which preserves the bounded-suboptimality promise. While other queues use inadmissible heuristics that may be more informative or goal-directed in specific regions. At every expansion step, the nodes are selected from the queue according to a scheduling policy. In this implementation, a simple round-robin strategy is used to select between queues, while all searches share a common cost-to-come map and closed set. When the goal is reached by any of the queue, the algorithm is terminated and returns to corresponding solution path.

C. Heuristic design

3 heuristic functions are applied

- **Anchor heuristic:** Euclidean distance to the goal, that is admissible under the grid motion model.
- **Scaled Manhattan heuristic:** A Manhattan-distance heuristic is multiplied by a constant factor to bias against the unnecessary turns and encourage axis-aligned progress.
- **Aggressive heuristic:** A heavily scaled Euclidean distance heuristic that strongly prioritizes direct motion toward the goal and is intentionally inadmissible.

All of the heuristic are computed solely from geometric information and do not explicitly encode terrain cost, ensuring

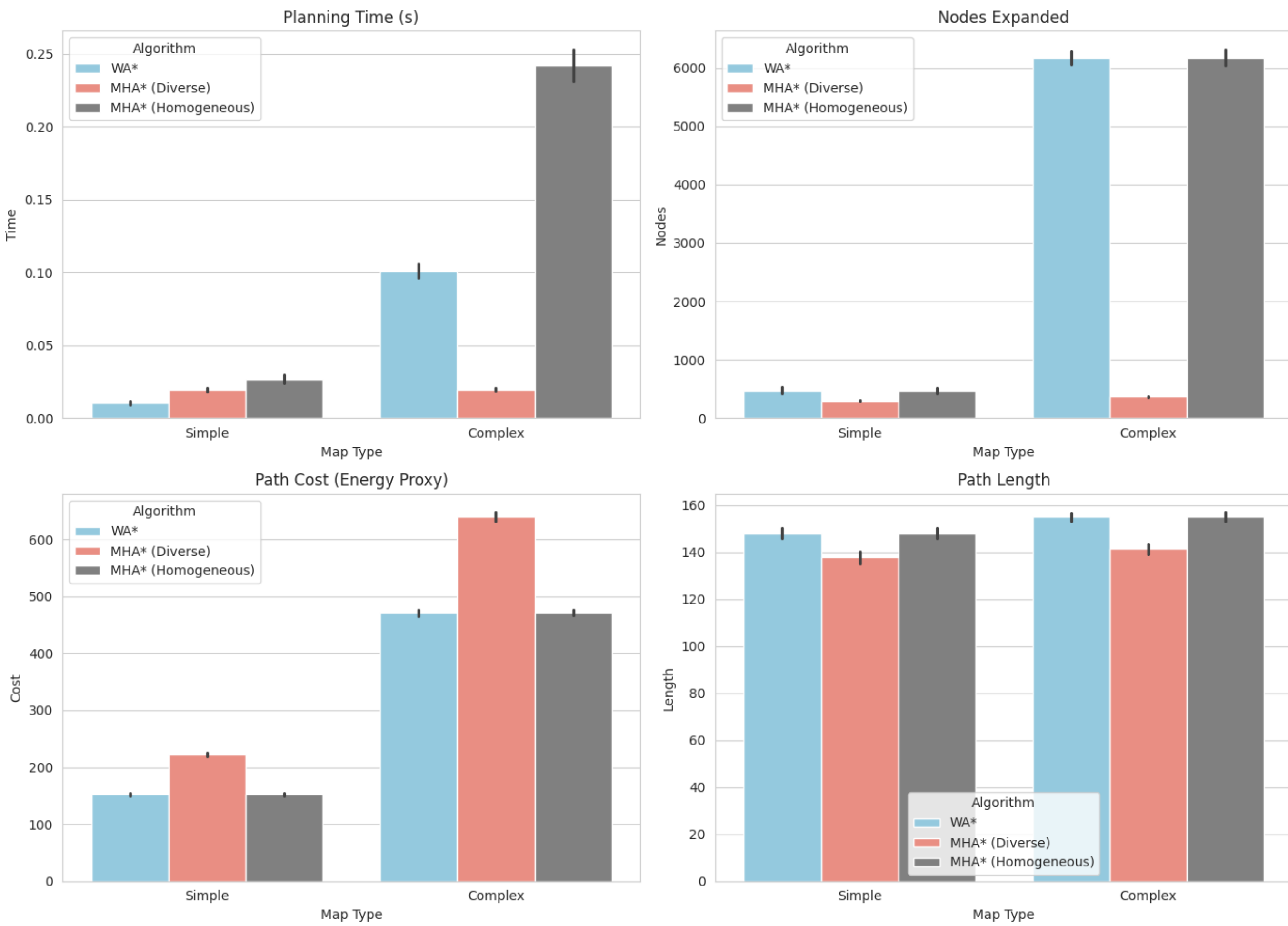


Fig. 1. Aggregate Bar Plots showing Planning Time, Nodes Expanded, Path Cost, and Path Length for Simple and Complex maps.

that differences in the behaviour arise from search guidance rather than the emedded cost cost knowledge.

D. Heuristic-diversity ablation setup

To isolate the effect of heuristic diversity, three planners are evalauated:

- WA* using admissible heuristic.
- MHA*(diverse) using anchor plus 2 distinct inadmissible heuristic.
- MHA*(homogeneous) using 3 identical copies of anchor heuristic.

This homogenous configuration helps as a negative control: it preserves the multi-queue architecture of MHA* while also removing heuristic diversity. Any performance difference between the diverse and homogenous variants can be attributed to the presence or absence of complementary heuristic guidance, rather than the multi-queue mechanisn.

All planners use identical weight parameters for heuristic inflation, ensuring a fair comparison of search behaviour and performance.

V. Experimental Methodology

This section explains the experimental protocol used to evaluate the planners performance and to test whether heuristic diversity is the primary driver of MHA*'s empirical gains.

A. Experimental setup

All of the experiments are conducted on the synthetic mars-like terrain dataset described in section III. This dataset contains of 10 maps (5 simple and 5 complex), each of them is associated with 50 start-goal pairs, which results in total of 500 planning problems in total. For each scenario all 3 planners are executed o the same rid and same start- goal pair. To ensure that the competition is fair the planners share the same motion model(4 connected grid) and terrain cost definition.

A trial will only be considered successful if the planner returns a valid collision free path from start to goal. To avoid bias the results are summarized over scenarios for which all 3 planners produce valid paths. In total 487 out of 500 scenarios meet this criteria.

B. Evaluation metrics

Planner performance is evaluated using 4 metrics computed per trial:

1) **Planning time (s):** wall-clock time runtime required to find a solution path.
2) **Nodes expanded:** the numbers of states removed from an open list and expanded(proxy for search effort).
3) **Path cost:** total cumulative terrain cost along the returned path, treated as an energy proxy.
4) **Path length:** number of states in the returned path(step count).

All these metrics together capture the computational efficiency (time, expansions) and solution quality (cost, length). Because bounded-suboptimal planners may trade cost for speed, so reporting both categories is necessary for a meaningful comparison.

C. Statistical analysis

All the results are summarized separately for both simple and complex map classes. For each metric and planner the, descriptive statistics include the mean, standard devation, and median across trials. In addition, a 95% confidence interval (CI) for the mean is also computed.

Now to assess whether observed performance differences are statically significant, welch's two-sample t-testis is applid for comparing planning time between WA* and MHA*(diverse) for each map class. statistical significance is reported using the corresponding p-values. Practical signifince is reported via cohen's d effect size, computed using pooled standard devation.

D. Visualization

1) **Aggregate bar plots:** these compare average planning time, node expanded, path cost and path length across map types and planners, with 95% confidence interval error bars.
2) **Qualitative path overlays:** this represents a complex map instance, displaying the returned paths for all 3 planners along with a summary table for all above statics for that current run.

Together, these visualizations connect the quantitative results to intuitive planner behaviors in structured and high-cost regions.

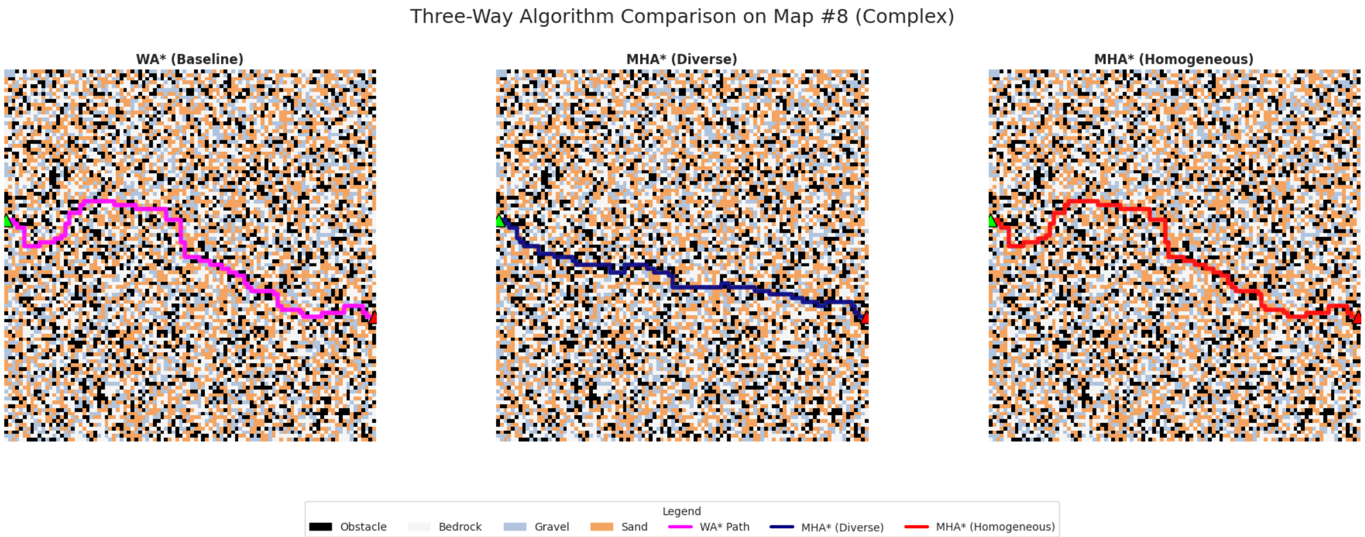
VI. RESULTS

The experimental evaluation reveals a clear and consistent difference in how the planners perform across simple and complex map environments. On simple maps, WA* demonstrates the fastest average planning time, which out performs both variants of MHA*. The stastical analysis conforms that the difference between WA* and MHA*(diverse) is significant, with a moderate to large effect size. This suggests that in relatively uniform environments where the heuristic guidance is already effective, the additionaloverhed of managing multiple heuristic queue in MHA* outweighs any potential benefits.

A much different trend emerges on complex maps, in the more challenging environments the MHA*(diverse) significantly outperforms both WA* and MHA*(Homogenous), acheaving much lower planning times and dramatically reducing the number of expanded nodes. Welche's t-tests confirms that these improvements are highly statistically significant, with a very large effect size. Although the MHA*(homogenous) expands a similar number of nodes as WA*, it consistently incurs higher runtime due to the overhead of maintaining multiple queues, which results in inferior overall performance. The analysis of path costs further shows that MHA*(diverse) tends to produce higher-cost solutions, reflecting a stronger goal bias and illustrating the expected tradeoff between computational efficiency and solution quality in bounded-suboptimal search.

VII. DISCUSSION

The results suggest that the performance advantage associated with the MHA* comes primarily from heuristic diversity rather than from the multi-queue framework itself. When the heuristics provide complementary perspective on the



Metric	WA* (Baseline)	MHA* (Diverse)	MHA* (Homogeneous)
Planning Time	0.0979 sec	0.0160 sec	0.2251 sec
Total Path Cost	481.0	678.0	481.0
Nodes Expanded	6808	349	6808
Path Length	156	134	156

Fig. 2. Three-Way Algorithm Comparison on a Complex Map showing paths taken by WA*, MHA* (Diverse), and MHA* (Homogeneous).

TABLE I
SUMMARY STATISTICS BY MAP TYPE AND ALGORITHM

Map	Algo	Time(s)	Nodes	Cost	Len
Complex	MHA* (Div)	0.020	380	656	144
	MHA* (Homo)	0.245	6285	487	157
	WA*	0.100	6285	487	157
Simple	MHA* (Div)	0.022	292	221	136
	MHA* (Homo)	0.034	536	152	147
	WA*	0.014	536	152	147

search space, the planner is better able to avoid misleading regions and reduce unnecessary exploration, particularly in environments with complex terrain and obstacle structures. This benigit is mostly visible in complex maps, where reliance on a single heuristic makes planners more likely to be trapped in locally suboptimal regions.

In contrast, when all heuristics are identical, the multi-queue framework offers little to no practical advantage. The homogenoys MHA* variant exhibits search behaviour that closely resembls with WA* but suffers from additional runtime due to queue management. This shows that simply increasing the architectural complexity does not guarantee improvement performance. Instead, these findings highlights the importance of careful heuristic design and reinforce the idea that multi-heuristic planning is most effective in environments where no single heuristic can reliability guide the search on its own.

VIII. Conclusion and Future Work

This work presented a controlled ablation study examining the role of heuristic diversity in Multi-Heuristic A* for terrain-aware path planning. By systematically compaing WA*, and MHA* with diverse heuristic and homogenous heuristics on a synthetic mars-like environment, the study shows that heuristic diversity is the key driver behindMHA*'s performance gains in complex terrains. While the homogenous variant has the same multi-queue architecture, it fails to deliver similar benefits, which confirms that diverse heuristic guidance not parallel queues alone is responsible for improved efficiency.

Future work could extend this analysis by incorporating terrain-aware or learned heuristics that explicitly model traversal cost, enabling a better balance between planning speed and path quality. Additional directions include evaluating on more realistic terrain dataset, integration of rover kinematics and dynamic constrains, and investigation of adaptive or anytime heuristic weighting strategies to dynamically manage the tradeoff between computational efficiency and solution optimality.

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