STBC , STLC Algorithm

2021/05/28 김상은

BPS LAB

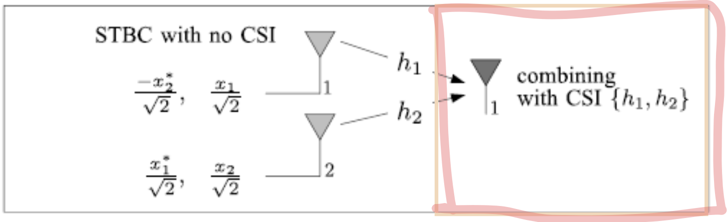
Space Time Code

- Spatial Diversity : Multiple Channel Gain
- Time Diversity : Consecutive Time Transmission

CSI(Channel State/Status Information)
: 통신 채널에 대한 상태 정보

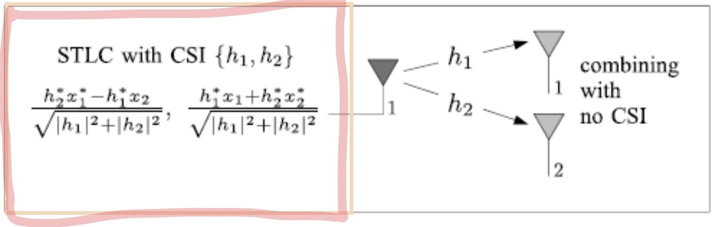
Pilot으로 Channel estimation

TX : RX = 2 : 1



STBC(Space Time Block Code)

TX : RX = 1 : 2



STLC(Space Time Line Code)

Space Time Code

- Spatial Diversity parameter

안테나 개수

TX : M 개 RX : N 개

MN spatial channel

$\boxed{\alpha}$ = MN 으로 정의

채널을 표시할 때,

$$h_{(m-1)(N+n)}$$

TX m번째 안테나 RX n번째 안테나

- α spatial channel gain의 합

$$\boxed{\gamma_\alpha} = \sum_{m=1}^{M}\sum_{n=1}^{N}\left|h_{(m-1)(N+n)}\right|^2$$

SNR ← $\boxed{\gamma_2}$

→ STBC $\sum_{m=1}^{2}\sum_{n=1}^{1}|\quad|^2$

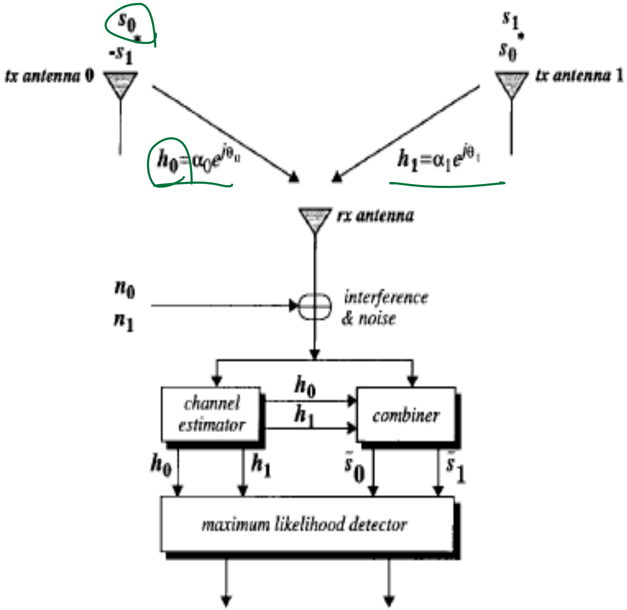
→ STLC $\sum_{m=1}^{1}\sum_{n=1}^{2}|\quad|^2$

STBC. TX RX
2 : 1

STLC TX : RX
1 : 2

MN = 2

Space Time Block Code(STBC)

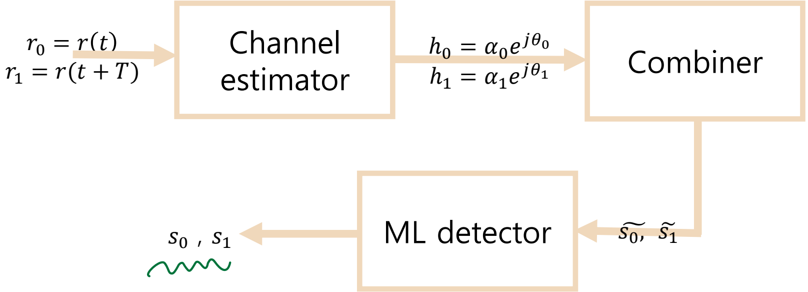


1. TX Signal

	antenna 0	antenna 1
time t	s_0	s_1
time $t+T$	$-s_1^*$	s_0^*

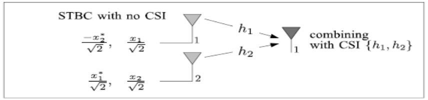
2. RX Signal

- RX Signal 처리 순서



Space Time Block Code(STBC)

T = symbol duration



STBC(Space Time Block Code)

$r_0 = r(t)$
$r_1 = r(t+T)$

Channel estimator

가정

$h_0(t) = h_0(t+T) = \alpha_0 e^{j\theta_0}$

$h_1(t) = h_1(t+T) = \alpha_1 e^{j\theta_1}$

⇒ 시간 delay (여기서 T) 후에 보내도 채널 환경이 일정한 상태

RX

t 시간에 받은 신호 ⇒ $r_0(t) = h_0 S_0 + h_1 S_1 + n_0$

t+T 시간에 받은 신호 ⇒ $r_1(t+T) = -h_0 S_1^* + h_1 S_0^* + n_1$

> 노이즈+간섭 AWGN채널.

$$r = \begin{bmatrix} r_{1,1} \\ r_{1,2}^* \end{bmatrix} = \frac{1}{\sqrt{2}} \underbrace{\begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix}}_{H_{(1,2)}} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} z_{1,1} \\ z_{1,2}^* \end{bmatrix} \triangleq \frac{1}{\sqrt{2}} H_{(1,2)} x_0 + z$$

이때 이 $H_{(1,2)}$의 특징 ⇒ orthogonal
$H^H H = I$

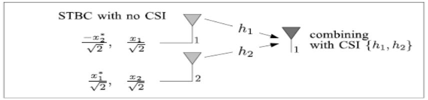
$$H^H_{(1,2)} r = \begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix} \begin{bmatrix} r_{1,1} \\ r_{1,2}^* \end{bmatrix} \triangleq \frac{\gamma_2}{\sqrt{2}} I_2 x + \underbrace{H^H_{(1,2)} z}_{= z'}$$

⇒ z'는 가우시안 벡터 평균 : 0

covariance (공분산) : $E[z'(z')^H] = \gamma_2 \sigma_z^2 I_2.$

Space Time Block Code(STBC)

여기서 우리가 찾아내야하는것 ⇒ s_0, s_1



STBC(Space Time Block Code)

Combiner → $\tilde{s_0}, \tilde{s_1}$

$$\tilde{s}_0 = h_0^* r_0 + h_1 r_1^*$$

$$= h_0^*(h_0 s_0 + \cancel{h_1 s_1} + n_0) + h_1(-\cancel{h_0^* s_1} + h_1^* s_0 + n_1^*)$$

$$= \underbrace{(h_0 h_0^* + h_1 h_1^*)}_{\alpha_0^2+\alpha_1^2} s_0 + h_0^* n_0 + h_1 n_1^*$$

$$\tilde{s}_1 = h_1^* r_0 - h_0 r_1^*$$

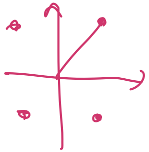
$$= h_1^*(\cancel{h_0 s_0} + h_1 s_1 + n_0) - h_0(-h_0^* s_1 + \cancel{h_1^* s_0} + n_1^*)$$

$$= \underbrace{(h_0 h_0^* + h_1 h_1^*)}_{\alpha_0^2+\alpha_1^2} s_1 + h_1^* n_0 - h_0 n_1^*$$

⇓

$\tilde{s}_0, \tilde{s}_1$

QPSK



ML detector → s_0, s_1

$$d^2(x,y) \Rightarrow squared\ Euclidean\ distance$$
$$\Rightarrow (x-y)(x^*-y^*)$$

$$\boxed{\cancel{(\alpha_0{}^2+\alpha_1{}^2-1)|s_i|^2}} + d^2(\tilde{s_0}, s_i) \le \boxed{\cancel{(\alpha_0{}^2+\alpha_1{}^2-1)|s_k|^2}} + d^2(\tilde{s_0}, s_k)$$

$$d^2(\tilde{s_0}, s_i) \le d^2(\tilde{s_0}, s_k)$$

QPSK symbol의 경우

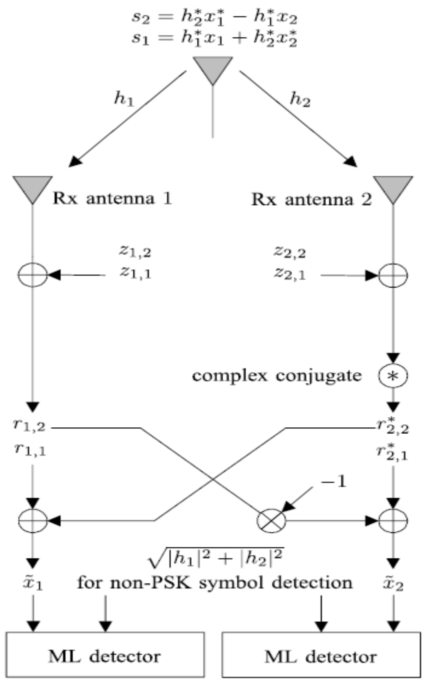
$\tilde{s_0} \Rightarrow s_0$
$\tilde{s_1} \Rightarrow s_1$

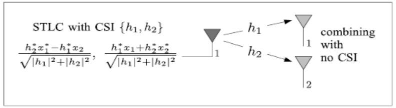
signal 에너지

$$\mathrm{SNR}_{\mathrm{STBC}}(\gamma_2) = \frac{\gamma_2 \sigma_x^2}{2\sigma_z^2}.$$

노이즈 에너지

Space Time Line Code(STLC)

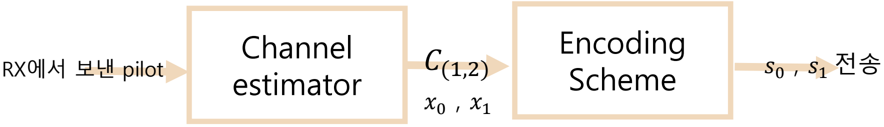




STLC(Space Time Line Code)

1. TX Signal

- TX Signal 처리 순서



$$\begin{bmatrix} s_1^* \\ s_2 \end{bmatrix} = \boldsymbol{C}_{(1,2)} \begin{bmatrix} x_1^* \\ x_2 \end{bmatrix} = \begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix} \begin{bmatrix} x_1^* \\ x_2 \end{bmatrix}$$

	Tx time $t = 1$	Tx time $t = 2$
Tx antenna 1	$s_1 = h_1^* x_1 + h_2^* x_2^*$	$s_2 = h_2^* x_1^* - h_1^* x_2$

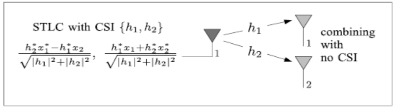
2. RX Signal

	Rx time $t = 1$	Rx time $t = 2$
Rx antenna 1	$r_{1,1}$	$r_{1,2}$
Rx antenna 2	$r_{2,1}$	$r_{2,2}$

- RX Signal 처리 순서

Space Time Line Code(STLC)

$C_{(1,2)} = H_{(1,2)}$ 와 동일하게 디자인됨.



STLC(Space Time Line Code)

Encoding Scheme → s_0 , s_1 전송

STLC encoding matrix

$$\begin{bmatrix} s_1^* \\ s_2 \end{bmatrix} = \boxed{C_{(1,2)}} \begin{bmatrix} x_1^* \\ x_2 \end{bmatrix} = \underbrace{\begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix}}_{= H_{(1,2)}} \begin{bmatrix} x_1^* \\ x_2 \end{bmatrix}$$

전송 신호

$$\begin{cases} s_1^* = h_1 x_1^* + h_2 x_2 \Rightarrow s_1 = h_1^* x_1 + h_2^* x_2^* \\ s_2 = h_2^* x_1^* - h_1^* x_2 \end{cases}$$

RX Signal => Time 2번, Antenna 2개
=> 총 4개의 신호
=> $r_{1,1} r_{1,2}, r_{2,1} r_{2,2}$

$$\begin{bmatrix} r_{1,1} & r_{1,2} \\ r_{2,1} & r_{2,2} \end{bmatrix} = \frac{1}{\sqrt{\gamma_2}} \underset{2\times 1}{\begin{bmatrix} h_1 \\ h_2 \end{bmatrix}} \underset{1\times 2}{\begin{bmatrix} s_1 & s_2 \end{bmatrix}} + \begin{bmatrix} z_{1,1} & z_{1,2} \\ z_{2,1} & z_{2,2} \end{bmatrix}$$

$$= \frac{1}{\sqrt{\gamma_2}} \begin{bmatrix} s_1 h_1 & s_2 h_1 \\ s_1 h_2 & s_2 h_2 \end{bmatrix}$$

$$r_{1,1} = \frac{1}{\sqrt{\gamma_2}} h_1 s_1 + z_{1,1} = \frac{1}{\sqrt{\gamma_2}} h_1 h_1^* x_1 + h_2^* x_2^* + z_{1,1}$$

$$r_{1,2} = \frac{1}{\sqrt{\gamma_2}} h_1 s_2 + z_{1,2} = \frac{1}{\sqrt{\gamma_2}} h_1 h_2^* x_1^* - h_1^* x_2 + z_{1,2}$$

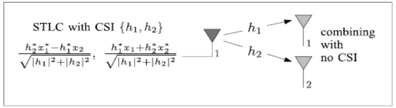
$$r_{2,1} = \frac{1}{\sqrt{\gamma_2}} h_2 s_1 + z_{2,1} = \frac{1}{\sqrt{\gamma_2}} h_2 h_1^* x_1 + h_2^* x_2^* + z_{2,1}$$

$$r_{2,2} = \frac{1}{\sqrt{\gamma_2}} h_2 s_2 + z_{2,2} = \frac{1}{\sqrt{\gamma_2}} h_2 h_2^* x_1^* - h_1^* x_2 + z_{2,2}$$

알아내고 싶은것 ⇒ x_1 , x_2

Space Time Line Code(STLC)

" blind coding technique "



STLC(Space Time Line Code)

Decoding Scheme $\widetilde{x_0}, \widetilde{x_1}$

$$\boxed{\tilde{x}_0 = r_{1,1} + r_{2,2}^*}$$

$$= \frac{1}{\sqrt{\gamma_2}}\left[h_1h_1^* x_1 + \cancel{h_1 h_2^* x_2} + h_2h_2^* x_1 - \cancel{h_2^* h_1 x_2^*}\right] + z_{1,1} + z_{2,2}^*$$

$$= \frac{1}{\sqrt{\gamma_2}}(h_1h_1^* + h_2h_2^*)x_1 + z_{1,1} + z_{2,2}^*$$

$$\boxed{\tilde{x}_1 = r_{2,1}^* - r_{1,2}}$$

$$= \frac{1}{\sqrt{\gamma_2}}\left[\cancel{h_2 h_1^* x_1^*} + h_2^* h_2 x_2 - \cancel{h_1 h_2^* x_1^*} + h_1 h_1^* x_2\right] + z_{2,1}^* - z_{1,2}$$

$$= \frac{1}{\sqrt{\gamma_2}}(h_1h_1^* + h_2h_2^*)x_2 + z_{2,1}^* - z_{1,2}$$

ML detector 할때 필요함.
⇒ QPSK symbol에서는 안해도 됨.

STLC 와 STBC 비교했을때 STLC 장점.

장 h 계산 X ⇒ 2 operation 이 줄어듦.

받은 신호를 그대로 더하고 빼주기만 하면됨 →

$\boxed{\tilde{s}_0 = h_0^* r_0 + h_1 r_1^*}$ $\boxed{\tilde{s}_1 = h_1^* r_0 - h_0 r_1^*}$

STBC: 곱셈 4번, 덧셈뺄셈 2번, 채널추정 2번 — 8번 계산

STLC: 덧셈뺄셈 2번

6개 연산차이 ⇒ 연산량 75%. $\frac{6}{8} = \frac{3}{4}$

ML detector x_0, x_1

$$d^2(x,y) \Rightarrow squared\ Euclidean\ distance$$
$$\Rightarrow (x-y)(x^*-y^*)$$

$$\boxed{\begin{array}{l}\widetilde{x_0} \Rightarrow x_0 \\ \widetilde{x_1} \Rightarrow x_1\end{array}}$$

$$({\alpha_0}^2 + {\alpha_1}^2 - 1)|s_i|^2 + d^2(\widetilde{s_0}, s_i) \le ({\alpha_0}^2 + {\alpha_1}^2 - 1)|s_k|^2 + d^2(\widetilde{s_0}, s_k)$$

$$d^2(\widetilde{s_0}, s_i) \le d^2(\widetilde{s_0}, s_k)$$

$$\mathrm{SNR}_{\mathrm{STLC}}(\gamma_2) = \frac{\gamma_\alpha \sigma_x^2}{2\sigma_z^2} = \frac{\gamma_2 \sigma_x^2}{2\sigma_z^2}$$

effective channel gain

Space Time Line Code(STLC)

Table

	STLC encoding matrices	Decoding function: $f(a,b)=a+b$ for $\tilde{x}_1$	for $\tilde{x}_2$
$\boldsymbol{C}_{(1,2)}$	$\boldsymbol{C}^a_{(1,2)} = \begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix}$	$f(r^*_{1,1}, r_{2,2})$	$f(r^*_{2,1}, -r_{1,2})$
	$\boldsymbol{C}^b_{(1,2)} = \begin{bmatrix} h_1 & h_2 \\ -h_2^* & h_1^* \end{bmatrix}$	$f(r^*_{1,1}, -r_{2,2})$	$f(r^*_{2,1}, r_{1,2})$
	$\boldsymbol{C}^c_{(1,2)} = \begin{bmatrix} h_1 & -h_2 \\ h_2^* & h_1^* \end{bmatrix}$	$f(r^*_{1,1}, r_{2,2})$	$f(-r^*_{2,1}, r_{1,2})$
	$\boldsymbol{C}^d_{(1,2)} = \begin{bmatrix} -h_1 & h_2 \\ h_2^* & h_1^* \end{bmatrix}$	$f(-r^*_{1,1}, r_{2,2})$	$f(r^*_{2,1}, r_{1,2})$
Type	Encoding s_1 and s_2	Decoding for $\tilde{x}_1$	Decoding for $\tilde{x}_2$
1	$\begin{bmatrix} s_1^* \\ s_2 \end{bmatrix} = \boldsymbol{C}_{(1,2)} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$	$f(\cdot,\cdot)$	$f(\cdot,\cdot)$
2	$\begin{bmatrix} s_1^* \\ s_2 \end{bmatrix} = \boldsymbol{C}_{(1,2)} \begin{bmatrix} x_1^* \\ x_2 \end{bmatrix}$	$f^*(\cdot,\cdot)$	$f(\cdot,\cdot)$
3	$\begin{bmatrix} s_1^* \\ s_2 \end{bmatrix} = \boldsymbol{C}_{(1,2)} \begin{bmatrix} x_1 \\ x_2^* \end{bmatrix}$	$f(\cdot,\cdot)$	$f^*(\cdot,\cdot)$
4	$\begin{bmatrix} s_1^* \\ s_2 \end{bmatrix} = \boldsymbol{C}_{(1,2)} \begin{bmatrix} x_1^* \\ x_2^* \end{bmatrix}$	$f^*(\cdot,\cdot)$	$f^*(\cdot,\cdot)$
5	$\begin{bmatrix} s_1 \\ s_2^* \end{bmatrix} = \boldsymbol{C}^*_{(1,2)} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$	$f^*(\cdot,\cdot)$	$f^*(\cdot,\cdot)$
6	$\begin{bmatrix} s_1 \\ s_2^* \end{bmatrix} = \boldsymbol{C}^*_{(1,2)} \begin{bmatrix} x_1^* \\ x_2 \end{bmatrix}$	$f(\cdot,\cdot)$	$f^*(\cdot,\cdot)$
7	$\begin{bmatrix} s_1 \\ s_2^* \end{bmatrix} = \boldsymbol{C}^*_{(1,2)} \begin{bmatrix} x_1 \\ x_2^* \end{bmatrix}$	$f^*(\cdot,\cdot)$	$f(\cdot,\cdot)$
8	$\begin{bmatrix} s_1 \\ s_2^* \end{bmatrix} = \boldsymbol{C}^*_{(1,2)} \begin{bmatrix} x_1^* \\ x_2^* \end{bmatrix}$	$f(\cdot,\cdot)$	$f(\cdot,\cdot)$
9	$\begin{bmatrix} s_1^* \\ s_2 \end{bmatrix} = -\boldsymbol{C}_{(1,2)} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$	$-f(\cdot,\cdot)$	$-f(\cdot,\cdot)$
10	$\begin{bmatrix} s_1^* \\ s_2 \end{bmatrix} = -\boldsymbol{C}_{(1,2)} \begin{bmatrix} x_1^* \\ x_2 \end{bmatrix}$	$-f^*(\cdot,\cdot)$	$-f(\cdot,\cdot)$
11	$\begin{bmatrix} s_1^* \\ s_2 \end{bmatrix} = -\boldsymbol{C}_{(1,2)} \begin{bmatrix} x_1 \\ x_2^* \end{bmatrix}$	$-f(\cdot,\cdot)$	$-f^*(\cdot,\cdot)$
12	$\begin{bmatrix} s_1^* \\ s_2 \end{bmatrix} = -\boldsymbol{C}_{(1,2)} \begin{bmatrix} x_1^* \\ x_2^* \end{bmatrix}$	$-f^*(\cdot,\cdot)$	$-f^*(\cdot,\cdot)$
13	$\begin{bmatrix} s_1 \\ s_2^* \end{bmatrix} = -\boldsymbol{C}^*_{(1,2)} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$	$-f^*(\cdot,\cdot)$	$-f(\cdot,\cdot)^*$
14	$\begin{bmatrix} s_1 \\ s_2^* \end{bmatrix} = -\boldsymbol{C}^*_{(1,2)} \begin{bmatrix} x_1^* \\ x_2 \end{bmatrix}$	$-f(\cdot,\cdot)$	$-f^*(\cdot,\cdot)$
15	$\begin{bmatrix} s_1 \\ s_2^* \end{bmatrix} = -\boldsymbol{C}^*_{(1,2)} \begin{bmatrix} x_1 \\ x_2^* \end{bmatrix}$	$-f^*(\cdot,\cdot)$	$-f(\cdot,\cdot)$
16	$\begin{bmatrix} s_1 \\ s_2^* \end{bmatrix} = -\boldsymbol{C}^*_{(1,2)} \begin{bmatrix} x_1^* \\ x_2^* \end{bmatrix}$	$-f(\cdot,\cdot)$	$-f(\cdot,\cdot)$

16가지!

s_1 s_1^* ⇒ 2가지
s_2 s_2^* ⇒ 2가지 } 4가지

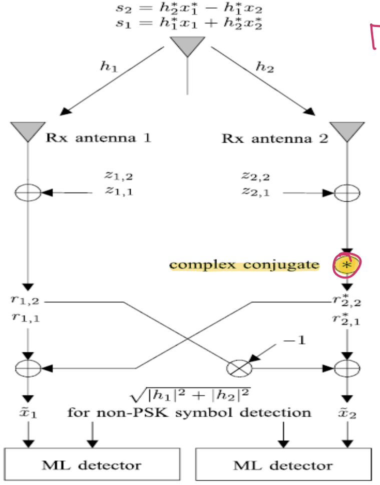
x_1 x_1^* ⇒ 2가지
x_2 x_2^* ⇒ 2가지 → 4가지

⇒ 동일한 성능

⇒ 구현이 다름.

RX 용력에 따라서 어떤 matrix를 사용할지 정할수있음.

	STLC encoding matrices	Decoding function: $f(a,b)=a+b$ for $\tilde{x}_1$	for $\tilde{x}_2$
$\boldsymbol{C}_{(1,2)}$	$\boldsymbol{C}^a_{(1,2)} = \begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix}$	$f(r^*_{1,1}, r_{2,2})$	$f(r^*_{2,1}, -r_{1,2})$
	$\boldsymbol{C}^b_{(1,2)} = \begin{bmatrix} h_1 & h_2 \\ -h_2^* & h_1^* \end{bmatrix}$	$f(r^*_{1,1}, -r_{2,2})$	$f(r^*_{2,1}, r_{1,2})$
	$\boldsymbol{C}^c_{(1,2)} = \begin{bmatrix} h_1 & -h_2 \\ h_2^* & h_1^* \end{bmatrix}$	$f(r^*_{1,1}, r_{2,2})$	$f(-r^*_{2,1}, r_{1,2})$
	$\boldsymbol{C}^d_{(1,2)} = \begin{bmatrix} -h_1 & h_2 \\ h_2^* & h_1^* \end{bmatrix}$	$f(-r^*_{1,1}, r_{2,2})$	$f(r^*_{2,1}, r_{1,2})$
Type	Encoding s_1 and s_2	Decoding for $\tilde{x}_1$	Decoding for $\tilde{x}_2$
1	$\begin{bmatrix} s_1^* \\ s_2 \end{bmatrix} = \boldsymbol{C}_{(1,2)} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$	$f(\cdot,\cdot)$	$f(\cdot,\cdot)$
2	$\begin{bmatrix} s_1^* \\ s_2 \end{bmatrix} = \boldsymbol{C}_{(1,2)} \begin{bmatrix} x_1^* \\ x_2 \end{bmatrix}$	$f^*(\cdot,\cdot)$	$f(\cdot,\cdot)$



→ 안테나 2에서 들어온 신호 conjugate

2 ⇒ $r_{1,1} + r^*_{2,2}$
⇒ $r^*_{2,1} - r_{1,2}$

1 ⇒ $r^*_{1,1} + r_{2,2}$
⇒ $r^*_{2,1} - r_{1,2}$

↳ time = 1 일때 들어온 신호 conjugate.

Space Time Block Code(STBC)

STBC, STLC는
diversity gain은 같고
성능도 동일함.

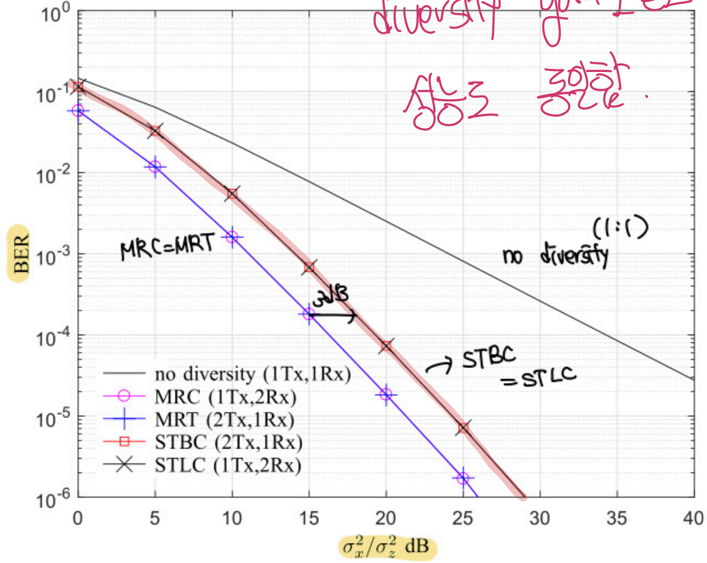


FIGURE 4. **BER performance comparison of coherent BPSK with 1 × 2 MRC, 2 × 1 MRT, 2 × 1 STBC, and 1 × 2 STLC during Rayleigh fading.**

THANK-YOU

For watching my presentation

BPS