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Group 8

(1) (1.1) $u = a\hat{i} + b\hat{j} + c\hat{k} \rightarrow \mathbb{R}^3$ ($\hat{i}$, $\hat{j}$ & $\hat{k}$ are the unit vectors to denote the basis)

$v = x\hat{i} + y\hat{j} + z\hat{k}$

$w = d\hat{i} + e\hat{j} + f\hat{k}$

(a) $u \cdot u = |u||u| \cos\theta \quad (\theta = 0)$

$= |u|^2 =$ Square of length.

The given statement is true.

(b) $(u \cdot v)^2 + (u \times v)^2 = |u|^2 |v|^2$

LHS

$|u|^2 \cdot |v|^2 \cos^2\theta + (|u| \cdot |v| \sin\theta \, \hat{n})^2$ $\quad (\because \hat{n} \to \text{unit vector})$

$= (|u| \cdot |v|)^2 \cos^2\theta + (|u|^2 |v|^2) \sin^2\theta \, (\hat{n} \cdot \hat{n})$

$= (|u|^2 |v|^2)(\cos^2\theta + \sin^2\theta) = |u|^2 |v|^2 = \text{RHS}$

(given eq^n is ~~false~~ true)

(c) $|u+v|^2 = |u|^2 + |v|^2 + 2(u \cdot v)$

LHS

$|u+v|^2 = (u+v) \cdot (u+v) = u \cdot u + v \cdot v + u \cdot v + v \cdot u$

$= |u|^2 + |v|^2 + 2(u \cdot v)$

$= \text{RHS}$ (given eq^n is true)

(d) $(u \cdot v)^2 = u^2 \cdot v^2$

LHS: $(u \cdot v)^2 = (|u| \, |v| \cos\theta)^2 = (|u|^2 |v|^2 \cos^2\theta)$

$\cancel{\neq (|u^2| \, |v^2| \cos\theta) \cos\theta}$

$= (u^2 \cdot v^2) \cos^2\theta$

$\neq \text{RHS}$.

$\text{LHS} \neq \text{RHS}$

The given equation is ~~true~~ false

(e) $u \times (v \times w) = (u \cdot w)v - (u \cdot v)w$

~~case let a = i + j + k~~

our, $u = a\hat{i} + b\hat{j} + c\hat{k}$; $v = x\hat{i} + y\hat{j} + z\hat{k}$

$w = d\hat{i} + e\hat{j} + f\hat{k}$.

($\hat{i}, \hat{j}, \hat{k} \rightarrow$ unit vectors ~~along~~ bases ~~along direction~~)

<u>LHS</u>

$u \times (v \times w) = u \times m$ (where $m = v \times w$)

$$m = \begin{vmatrix} \hat{i} & \hat{j} & k \\ x & y & z \\ d & e & f \end{vmatrix} = (yf - ze)\hat{i} - (xf - dz)\hat{j} + (xe - yd)\hat{k}$$

$$u \times m = \begin{vmatrix} \hat{i} & \hat{j} & k \\ a & b & c \\ (yf-ze) & -(xf-dz) & (xe-yd) \end{vmatrix} = [b(xe-yd) + c(xf-dz)]\hat{i} - [a(xe-yd) - c(yf-ze)]\hat{j} + [-a(xf-dz) - b(yf-ze)]\hat{k}$$

$$= (bxe - byd + cxf - cdz)\hat{i} + (cyf - cze - axe + ayd)\hat{j} + (adz - axf - byf + bze)\hat{k} \quad \text{———} (1)$$

<u>RHS</u>

$(u \cdot w)v - (u \cdot v)w$

$$= (ad + be + cf)(x\hat{i} + y\hat{j} + z\hat{k}) - (ax + by + cz)(d\hat{i} + e\hat{j} + f\hat{k})$$

$$= (adx + bex + cfx)\hat{i} + (ady + bey + cfy)\hat{j} + (adz + bez + cfz)\hat{k}$$

$$- [(adx + byd + czd)\hat{i} + (axe + bye + cze)\hat{j} + (axf + byf + czf)\hat{k}]$$

$$= (bxe + cfx - byd - cdz)\hat{i} + (cyf + ayd - cze - axe)\hat{j}$$

$$+ (adz + bze - axf - byf)\hat{k} \quad \text{——— } (2)$$

So, from 1 & 2, LHS = RHS.
The given statement is true.

(f) $(u \cdot v) \cdot w = u \cdot (v \cdot w)$

LHS

$(u \cdot v) \cdot w = (ax + by + cz) \cdot (d\hat{i} + e\hat{j} + f\hat{k})$

RHS

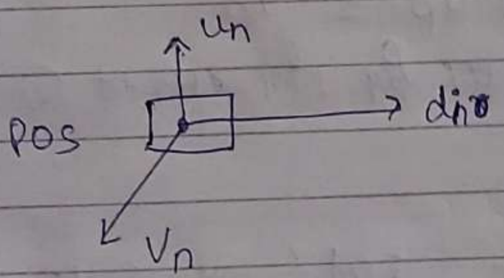
$u \cdot (v \cdot w) = (a\hat{i} + b\hat{j} + c\hat{k}) \cdot (dx + ey + fz)$

so, LHS $\neq$ RHS.
The given equat^n is false

(1.2) ⓐ <u>In air</u>

There will be no gravity. The given up vector may not be perpendicular to the direct^n (forward) ~~dir~~ vector. Hence first we need to find the correct basis vectors (unit vectors) for our camera.

ⓐ



Here, pos represents the pos^n of the camera.

un: normalized up vector.

dn: normalized direct^n vector

vn: normalized basis orthogonal to un & dn.

$$dn = \frac{\vec{dir}}{|\vec{dir}|} \qquad vn = \frac{dn \times up}{|dn \times up|} \qquad Un = vn \times dn$$

ⓑ

∵ Here $\vec{dir}$ & up vectors are given forward & up vectors. 'x' → cross product.

① Rotate left by an angle 'α'

$$\begin{bmatrix} V_n' \\ d_n' \\ U_n' \\ pos' \end{bmatrix} = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 & 0 \\ \sin\alpha & \cos\alpha & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} V_n \\ d_n \\ U_n \\ pos \end{bmatrix}$$

d_n' = new direct^n vector.

$= V_n \sin\alpha + d_n \cos\alpha$

U_n': new up vector $= U_n$

pos': new posit^n vector $= pos$.

② Look up by an angle 'β'

$$\begin{bmatrix} V_n' \\ d_n' \\ U_n' \\ pos' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\beta & -\sin\beta & 0 \\ 0 & \sin\beta & \cos\beta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} V_n \\ d_n \\ U_n \\ Pos \end{bmatrix}$$

$d_n' = d_n \cos\beta - U_n \sin\beta$

$U_n' = d_n \sin\beta + U_n \cos\beta$

$pos' = pos$

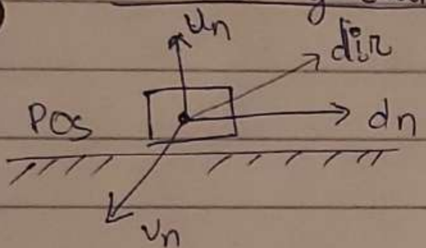
③ Move forward by a distance 'd'

$d_n' = d_n$

$U_n' = U_n$

$pos' = pos + d\,(d_n)$

(b) on the ground:



on the ground our 'up' vector is fixed but our forward vector is not perpendicular to 'up' vector. So we need to find the forward vector.

u_n, d_n, v_n are the orthonormal basis which represent the up, forward & side vector.

$$u_n = \frac{\vec{Up}}{|\vec{Up}|} \qquad v_n = -\frac{\vec{u_n} \times \vec{dir}}{|\vec{u_n} \times \vec{dir}|} \qquad d_n = u_n \times v_n$$

(1) Rotate (yaw) by an angle 'α' (left)

d_n' = new directⁿ (forward)

$= v_n \sin\alpha + d_n \cos\alpha$

$u_n' = u_n$ (U_n' = new up vector)

$pos' = pos$ (pos' = new position vector)

(2) look up by an angle β

$$dn' = dn \cos\beta - un \sin\beta$$
$$un' = un$$
$$pos' = pos$$

③ Move forward by a distance d.

$$dn' = dn$$
$$un' = un$$
$$pos' = pos + d\,(dn)$$

NB: The notation dn', un', pos' are the direct" vector for forward, up & position respectively.