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Group 8

5.1 (a) $f(w_i, w_o) = K_d + K_s \cos^n(w_r, w_o)$ (Phong BRDF)

$K_s = 0$

so, $f(w_i, w_o) = K_d = \rho_d / c_d$ (given)

Total hemispherical reflection: $\rho(w_i, w_o) = \int_\Omega f(w_i, w_o) \cos w_o \, dw.$

$\rho(w_i, w_o) = \int_\Omega f(w_i, w_o) \cos\theta \sin\theta \, d\theta \, d\phi$ $\{ dw = \sin\theta \, d\theta \, d\phi$

$= \int_0^{2\pi} \int_0^{\pi/2} K_d \cos\theta \sin\theta \, d\theta \, d\phi$

$= K_d \pi$

Energy conservat^n: $\rho \leq 1$

$\Rightarrow K_d \pi \leq 1 \Rightarrow \boxed{c_d \geq \rho_d \pi}$

(b) $K_d = 0$

$f(w_i, w_o) = K_s \cos^n(\theta) = \frac{\rho_s}{c_s} \cos^n\theta$

$\rho(w_o) = \int f(w_i, w_o) \cos\theta \sin\theta \, d\theta \, d\phi$

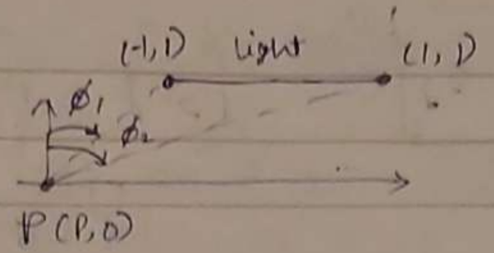
$= \int_0^{2\pi} \int_0^{\pi/2} K_s \cos^{n+1}\theta \sin\theta \, d\theta \, d\phi$

$= \int_0^{2\pi} \frac{K_s}{n+2} \, d\phi$ (using integrat^n by substitut^n)

$= \frac{2\pi K_s}{n+2}$

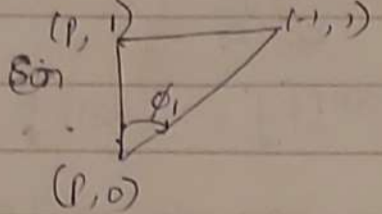
Energy conservat^n: $\rho \leq 1 \Rightarrow \boxed{c_s \geq \frac{\rho_s 2\pi}{n+2}}$

5·2 (a)

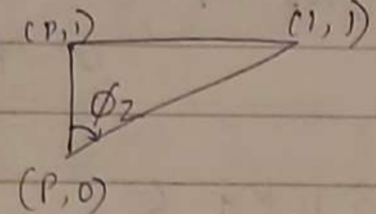


$L(x,\phi) = 1 \quad \{\phi_1 < \phi < \phi_2\}$

$f_r(w, x, w_0) = \frac{1}{\pi}$



$\sin\phi_1 = \dfrac{-(P+1)}{\sqrt{1+(P+1)^2}}$ (anti clockwise angle)



$\sin\phi_2 = \dfrac{-(P-1)}{\sqrt{1+(P-1)^2}}$

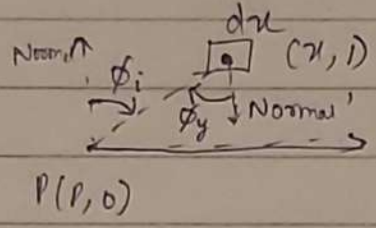
$$L(x,w_0) = L_e(x,w_0) + \int_0^{\pi} f_r(w,x,w_0) \cdot L(x,w) \cdot \cos\phi \, dw$$

$$= 0 + \int_{\phi_1}^{\phi_2} (1) \cos\phi \, d\phi$$

$$= \frac{1}{\pi}(\sin\phi_2 - \sin\phi_1) = \frac{1}{\pi}\left(\frac{-(P-1)}{\sqrt{1+(P-1)^2}} + \frac{(P+1)}{\sqrt{1+(P+1)^2}} \right)$$

$$= \frac{1}{\pi}\left(\frac{1+P}{\sqrt{1+(P+1)^2}} - \frac{(P-1)}{\sqrt{1+(P-1)^2}} \right)$$

(b) we have to consider an infintesimally small element on the light source of length dx. To get the whole light source we need to integrate it from -1 to 1



$\phi_i = \phi_y$

ϕ_i = angle subtended b/w the line from point to light source element & the normal.

ϕ_y = angle subtended b/w the line from point to light source element & the normal'.

$$L(x, w_0) = le(x, w_0) + \int_{y \in S} f_r(w_i, x, w_0) \cdot L(y, -w_i(x,y)) \cdot \frac{\cos\phi_i \cos\phi_y}{|x-y|} \cdot dA_y$$

$$= 0 + \int_{-1}^{1} \frac{1}{\pi} (1) \frac{\cos\phi_i \cos\phi_y}{|x-y|} \, dx$$

$$= \frac{1}{\pi} \int_{-1}^{1} \frac{\cos^2\phi_i}{\sqrt{1+(P-x)^2}} \, dx.$$

$$= \frac{1}{\pi} \int_{-1}^{1} \frac{1}{\cancel{}} (1+(P-x)^2)^{-3/2} \, dx \qquad \left\{ \cos\phi_i = \frac{1}{\sqrt{1+(P-x)^2}} \right.$$

Applying integrat^n by substitution.

$$= \frac{-1}{\pi} \left[\frac{\cancel{x}(P-x)}{\sqrt{1+(P-x)^2}} \right|_{-1}^{1} = \frac{-1}{\pi} \left\{ \frac{(P-1)}{\sqrt{1+(P-1)^2}} - \frac{P+1}{\sqrt{1+(P+1)^2}} \right\}$$

Proof of integratⁿ step.

$$\int \frac{dx}{(1+(p-x)^2)^{3/2}}$$

$p - x = t$.

$dx = -dt$.

$$= \int \frac{-dt}{(1+t^2)^{3/2}}$$

$t = \tan\theta$.

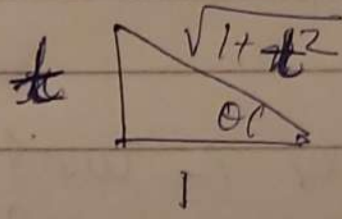
$dt = \sec^2\theta \, d\theta$

$$= -\int \frac{\sec^2\theta}{\sec^3\theta} d\theta$$

$(1+\tan^2\theta = \sec^2\theta)$

$$= -\int \cos\theta \, d\theta$$

$$= -\sin\theta = \frac{-t}{\sqrt{1+t^2}}$$



$$= -\frac{x}{1+\cancel{x^2}}$$

$$= -\frac{(p-x)}{\sqrt{1+(p-x)^2}}$$