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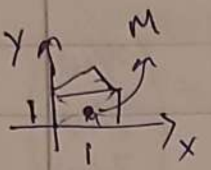
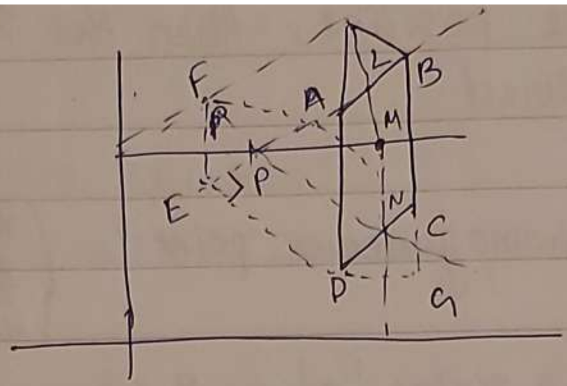
Group 8

(1)





M is the mid pt. of LN as it is evident from the small 1x2 structure.

So, $\triangle PML$ & $\triangle PMN$ are similar (SAS)

So, $PL = PN$

And $\angle LPN = 90^\circ$ (PN is parallel to ED & $\angle PED = 90^\circ$)

So, As $PL = PN$, $\angle PNL = PLN = 45^\circ$ ($\angle LPN = 90^\circ$)

So, $\angle ADC = 45^\circ$. & $\angle CDG = 45^\circ$

$DC = 4$ (given) so, $DG = 4 \times \cos 45^\circ = 4/\sqrt{2} = 2\sqrt{2}$

This structure has been sheared along Y-axis by 45°.

Also, AE || DC.

$\angle AED = \angle EDC = 90^\circ$. So, $\angle ADE = 45^\circ$.

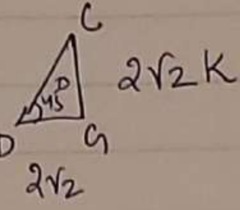
$AD = \frac{5}{\cos 45^\circ} = 5\sqrt{2}$.

(1) <u>Step-1</u> (Scaling by $S_x = 2\sqrt{2}$, $S_y = 5\sqrt{2}$. (because of DG & AD)

$$\begin{pmatrix} S_x & 0 \\ 0 & S_y \end{pmatrix} = \begin{pmatrix} 2\sqrt{2} & 0 \\ 0 & 5\sqrt{2} \end{pmatrix}$$

In 3D, $\begin{pmatrix} 2\sqrt{2} & 0 & 0 \\ 0 & 5\sqrt{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$

Shearing along Y axis



Step-II) From the figure, we will shear along Y-axis by the amount (shear factor) K.

$$\tan 45^\circ = \frac{2\sqrt{2}K}{2\sqrt{2}} \Rightarrow \boxed{K = 1}$$

$$\text{shear Matrix} = \begin{pmatrix} 1 & 0 & 0 \\ SH_y & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Step-III) Translate to reach the desired point by $(4-\sqrt{2},\ 5-3.5\sqrt{2})$

$$\text{Translation Matrix} = \begin{pmatrix} 1 & 0 & 4-\sqrt{2} \\ 0 & 1 & 5-\frac{7}{2}\sqrt{2} \\ 0 & 0 & 1 \end{pmatrix}$$

The overall transformation matrix. (T)(Sh)(Sc)

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 4-\sqrt{2} \\ 0 & 1 & 5-3.5\sqrt{2} \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 2\sqrt{2} & 0 & 0 \\ 0 & 5\sqrt{2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

To prove the correctness, we have initial point M at $(0.5, 0.5, 1)$ and after transformation $(4, 5, 1)$

After transformat^n, scaling $\rightarrow \begin{pmatrix} \sqrt{2} \\ 2.5\sqrt{2} \\ 1 \end{pmatrix}$

After shearing along y-axis $\rightarrow \begin{pmatrix} \sqrt{2} \\ 3.5\sqrt{2} \\ 1 \end{pmatrix}$

After translation $\rightarrow \begin{pmatrix} 4 \\ 5 \\ 1 \end{pmatrix} \rightarrow$ (Transformed point)

(4.2) (a) Let P be an homogeneous point $= \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \begin{pmatrix} x/w \\ y/w \\ z/w \\ 1 \end{pmatrix}$

Multiplying by a scalar 'a' $= P \times a$

$$= \begin{pmatrix} ax \\ ay \\ az \\ aw \end{pmatrix} = \begin{pmatrix} x/w \\ y/w \\ z/w \\ 1 \end{pmatrix}$$

In, ~~so~~ this we get an equivalent homogeneous point.

(b) we have 3 homogeneous point.

$P_0 = (a_0, b_0, c_0, 1)$; $P_1 = (a_1, b_1, c_1, 1)$; $P_2 = (a_2, b_2, c_2, 1)$

~~$P_3 = (a_3, b_3, c_3, 1)$~~

Summing them component wise $= P_0 + P_1 + P_2$

$$= (a_0 + a_1 + a_2,\ b_0 + b_1 + b_2,\ c_0 + c_1 + c_2,\ 3)$$

$$= \left(\frac{a_0 + a_1 + a_2}{3},\ \frac{b_0 + b_1 + b_2}{3},\ \frac{c_0 + c_1 + c_2}{3},\ 1 \right)$$

$\therefore$ This is the centre b/w that points.

(4.3) If we have a homogeneous line 'L' passing b/w two homogeneous points. P & Q. Then the points will satisfy the equation

$$L^T P = 0$$
$$\& \quad L^T Q = 0$$

where, $L = \begin{pmatrix} x_L \\ y_L \\ z_L \\ 1 \end{pmatrix}$

$P = \begin{pmatrix} x_P \\ y_P \\ z_P \\ 1 \end{pmatrix}$ $Q = \begin{pmatrix} x_Q \\ y_Q \\ z_Q \\ 1 \end{pmatrix}$

But this equat^n will also hold if $L = P \times Q$. as $P \times Q$ will be perpendicular to P.

So, $(P \times Q)^T P = 0$. So, $L = P \times Q$

Hence, we can say that cross product b/w 2 homogeneous points gives a homogeneous coordinates of ~~the~~ the line.