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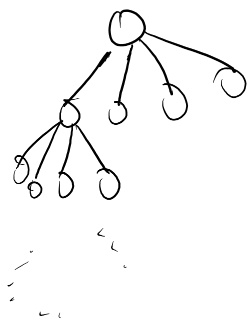
Group 8

CG assignment 3

Friday, December 3, 2021 5:22 PM

3-1) given a tree with arity A, h is the depth of the tree



D	Inner Nodes	Leaves
0	1	0
1	$A - l_1$	l_1
2	$A^2 - l_1 A - l_2$	l_2
$\vdots$		
$h-1$	$A^{h-2} - l_1 A^{h-3} \cdots - l_{h-1}$	l_{h-1}
h	0	$A^{h-1} - l_1 A^{h-2} - l_2 A^{h-3} \cdots l_{h-1} A$

Here $l_1, l_2, l_3 \cdots l_{h-1}$ are the No. of Nodes which turned into leaves at deapth $1, 2, \cdots h-1$,

$$I = \Sigma \text{ inner nodes } = 1 + A - l_1 + A^2 - l_1 A - l_2 + \cdots$$

$$= 1 + A + A^2 + \cdots A^{h-2}$$
$$- l_1(1 + A + A^2 + \cdots A^{h-3})$$
$$- l_2(1 + A + A^2 + \cdots A^{h-4})$$

$$- l_{h-1}$$

$$= \frac{A^{h-1} - 1}{A-1} - l_1 \frac{(A^{h-2} - 1)}{A-1} - \cdots l_{h-1} \frac{(A-1)}{(A-1)}$$

$$I(A-1) = (A^{h-1} - 1) - l_1(A^{h-2} - 1) - \cdots l_{h-1}(A-1)$$

$$I(A-1) + 1 = A^{h-1} - l_1(A^{h-2} - 1) - \cdots l_{h-1}(A-1) \quad 1$$

$$L = \Sigma \text{ leaf nodes} = A^{h-1} - l_1(A^{h-2}) - l_2 A^{h-3} - \cdots - l_{h-1}A$$

$$+ l_1 + l_2 + \cdots l_{h-1}$$

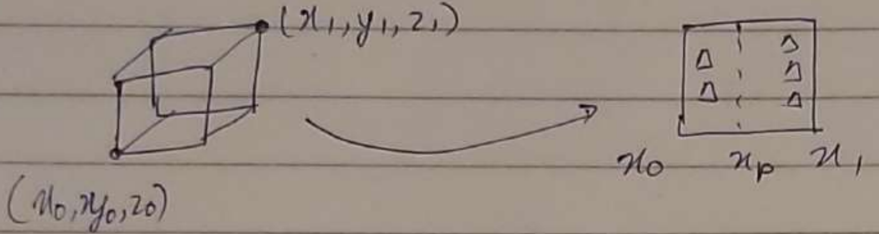
$$= A^{h-1} - l_1(A^{h-2} - 1) - l_2(A^{h-3} - 1) \cdots - l_{h-1}(A-1).$$

$$I(A-1) + 1 = L$$

$$\boxed{I = \frac{L-1}{A-1}}$$

3.2 $\text{cost} = C_{trav} + \text{Prob}(\text{hit } L)^{*}\, \text{Cost}(L) + \text{Prob}(\text{hit } R)^{*}\, \text{cost}(R)$

$$= C_{trav} + \frac{1}{SA(C)} \left\{ SA(L)^{*}\, \text{triangle count}(L) + SA(R)^{*}\, \text{triangle count}(R) \right\}$$



Let's take the case ~~of a~~ ~~sing~~ ~~simple square~~.

$$\text{cost} = C_{trav} + \frac{1}{SA(C)} \Big\{ \cancel{SA} \left[(x_p - x_0)\, y_i + y_i z_i + z_i (x_p - x_0)\right]^{*} 2 \times \text{tri.count}(L)$$

$$+ \left[(x_p - x_p)\, y_i + y_i z_i + z_i\, (x_1 - x_p)\right]^{*} 2 \times \text{tri.count}(R) \Big\}$$

On solving and arranging the terms.

$$\text{cost} = m\, x_p + C$$

where:
$y_i = |y_1 - y_0|$, $SA(C)$ = Surface Area of Parent BOX
$z_i = |z_1 - z_0|$,
m = slope ; C = intercept.

To minimize the cost we need to find the split plane x_p. But we need to keep in mind that the triangle count in Left & Right is fixed. Only surface Area changes from left to right.
Hence the cost funct^n becomes a linear function in terms of x_p. To minimize the cost we need to find the point which is close to the corners of the bounding box. (Here x_1 or x_0) depending on the slope m.