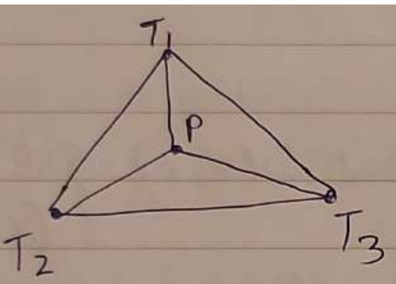
Devikalyan Das 7007352
Pranay Raj Kamuni 7015552
Group 8

2 1 a



P : centre of mass = centroid.

Area of triangle T = A

$P(\alpha, \beta, \gamma) = \alpha T_1 + \beta T_2 + \gamma T_3$

where α, β, γ are Barycentric coordinates.

$$\alpha = \frac{\text{Signed area } (\Delta\, T_2 T_3 P)}{\text{Signed area } (\Delta\, T_1 T_2 T_3)} = \frac{A/3}{A} = 1/3$$

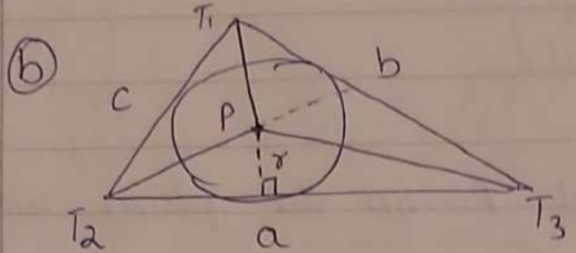
($\because$ centroid divides Δ into 3 equal areas)

$$\beta = \frac{\text{Signed area } (\Delta\, T_3 T_1 P)}{\text{Signed area } (\Delta\, T_1 T_2 T_3)} = 1/3$$

$$\gamma = \frac{\text{signed area } (\Delta T_1 T_2 P)}{\text{signed area } (\Delta T_1 T_2 T_3)} = 1/3$$

$$P(\alpha, \beta, \gamma) = (1/3, 1/3, 1/3)$$

(b)



P : centre of the inscribed circle (incentre)
In incentre, the angle bisectors meet.

Area of $\Delta T_1 T_2 T_3 = s\gamma$ $\begin{cases} \gamma : \text{inradius} \\ s : \text{semiperimeter.} \end{cases}$

$s = \frac{a+b+c}{2}$ $\{$ a, b, c are the lengths of the sides of the triangle

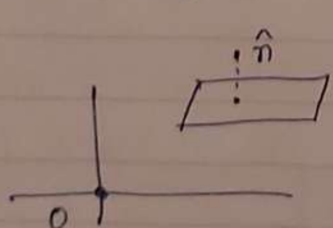
$P(\alpha, \beta, \gamma)$: (Incentre with Barycentric coordinates α, β, γ.

$$\gamma = \frac{\text{signed area } (\Delta T_1 T_2 P)}{\text{signed area } (\Delta T_1 T_2 T_3)} = \frac{\frac{1}{2} c \gamma}{s\gamma} = \frac{c}{a+b+c}$$

$$\beta = \frac{\text{signed area } (\Delta P T_3 T_1)}{\text{signed area } (\Delta T_1 T_2 T_3)} = \frac{\frac{1}{2} b \gamma}{s\gamma} = b/a+b+c$$

$$\alpha = \frac{\text{signed area } (\Delta P T_2 T_3)}{\text{signed area } (\Delta T_1 T_2 T_3)} = \frac{\frac{1}{2} a \gamma}{s\gamma} = a/a+b+c$$

(2.2) (a)



a = arbitrary pt. on plane
$= (x, y, z)$

$\hat{n}$ = unit normal vector of the plane
$= \frac{n}{\|n\|} = (\hat{n}_1, \hat{n}_2, \hat{n}_3)$

n: normal vector of the plane.
$= (n_1, n_2, n_3)$

$$\hat{n} = \left(\frac{n_1}{\|n\|}, \frac{n_2}{\|n\|}, \frac{n_3}{\|n\|}\right)$$

Suppose we have a point P_0 on the plane with coordinate (x_0, y_0, z_0).

So, equat^n of the plane $\Rightarrow (a - P_0) \cdot \hat{n} = 0$

$$(x - x_0) n_1 + (y - y_0) n_2 + (z - z_0) n_3 = 0.$$

$$\Rightarrow n_1 x + n_2 y + n_3 z - (n_1 x_0 + n_2 y_0 + n_3 z_0) = 0 \quad (1)$$

Comparing with the eq^n given $Ax + By + Cz + D = 0$

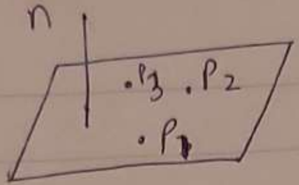
$n_1 = A, \; n_2 = B, \; n_3 = C$
$D = -(n_1 x_0 + n_2 y_0 + n_3 z_0)$
$= -(A x_0 + B y_0 + C z_0)$

So, our $a = (x, y, z)$, $n = (A, B, C)$

The point 'a' satisfies the equat^n if it is (x_0, y_0, z_0)

we can write equation (1) as
$a \cdot n + D = 0$
$\Rightarrow a = \left(\frac{-D}{A}, \frac{-D}{B}, \frac{-D}{C}\right)$

(b)



$\overrightarrow{P_2P_3} \in P_2 \in P_3$

$\overrightarrow{P_3P_2} = P_2 - P_3$

$\overrightarrow{P_3P_1} = P_1 - P_3$

$$n = \overrightarrow{P_3P_2} \times \overrightarrow{P_3P_1} = (P_2 - P_3) \times (P_1 - P_3)$$

$\hat{n}$ = unit normal vector.

$$= \frac{(P_2 - P_3) \times (P_1 - P_3)}{\|(P_2 - P_3) \times (P_1 - P_3)\|}$$

$\{ \because$ '$\times$' $\rightarrow$ cross product$\}$

a = arbitary point on the plane.

P_0 = suppose any pt on the plane.

Eq^n of plane : $(P_0 - a) \cdot \hat{n} = 0$

where $a = P_1 / P_2 / P_3$

(2.3) (a) $R(t) = O + td$

$(R_x, R_y, R_z) = (O_x, O_y, O_z) + t(d_x, d_y, d_z)$

Quadric Equation : $ax^2 + by^2 + cz^2 + dxy + exz + fyz + gx + hy + iz + j = 0$

Substituting the ray eq^n in quadric equation.

$$a(O_x + td_x)^2 + b(O_y + td_y)^2 + c(O_z + td_z)^2 + e(O_x + td_x)(O_z + td_z) + f(O_y + td_y)(O_z + td_z) + g(O_x + td_x) + d(O_x + td_x)(O_y + td_y)$$
$$+ h(O_y + td_y) + i(O_z + td_z) + j = 0$$

Arranging the terms,

$$\cancel{t^2}\, a(O_x^2 + t^2 d_x^2 + 2t\, O_x d_x) + b(O_y^2 + t\, d_y^2 + 2t\, O_y d_y) +$$

$$c(O_z^2 + t^2 d_z^2 + 2t\, O_z d_z) + \cancel{d(O_x O_y + t(O_x d_y + O_y d_x)}$$

$$d(O_x O_y + t(O_x d_y + O_y d_x) + t^2(d_x d_y)) +$$

$$e(O_x O_z + t(O_z d_x + O_x d_z) + t^2(d_x d_z)) +$$

$$f(O_y O_z + t(O_z d_y + O_y d_z) + t^2 d_z d_y) +$$

$$g(O_x + t d_x) + h(O_y + t d_y) + i(O_z + t d_z) + j = 0$$

$$\Rightarrow t^2(a d_x^2 + b d_y^2 + c d_z^2 + d\, d_x d_y + e\, d_x d_z + f\, d_z d_y) +$$

$$t(2a\, O_x d_x + 2b\, O_y d_y + 2c\, O_z d_z + d\, O_x d_y + d\, O_y d_x + e(O_z d_x + O_x d_z) + f(O_z d_y + O_y d_z) + g\, d_x + h\, d_y + i\, d_z) +$$

$$(a O_x^2 + b O_y^2 + c O_z^2 + d O_x O_y + e O_x O_z + f O_y O_z + g O_x + h O_y + i O_z + j) = 0$$

$$\Rightarrow At^2 + Bt + C = 0 \quad \text{———} \quad (2)$$

$$A = a d_x^2 + b d_y^2 + c d_z^2 + d\, d_x d_y + e\, d_x d_z + f\, d_y d_z)$$

$$B = 2a O_x d_x + 2b O_y d_y + 2c O_z d_z + d O_x d_y + d O_y d_x + e(O_z d_x + O_x d_z) + f(O_z d_y + O_y d_z) + g d_x + h d_y$$

$+ i d_z$

$$C = a\,o_x^2 + b\,o_y^2 + c\,o_z^2 + d\,o_x o_y + e\,o_x o_z + f\,o_y o_z + g\,o_x + h\,o_y + i\,o_z + j$$

The roots of the eqn. (2).

$$t = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$ ~~(2 roots)~~

$$= \frac{-B \pm \sqrt{Det}}{2A}$$ $\{ Det = B^2 - 4AC$

<u>Constraints</u>: ~~Det > 0~~ Conditions for the ray to intersect successfully:

(1) $Det \geq 0$

(2) The two roots t_0, t_1 are the distance. Hence ~~if~~ $t_0, t_1 > 0$.

~~If to~~ If both roots are positive, we will take the smallest t.

(3) If $A = 0$; then $t = -C/B$.

(b) Sphere equation general form with centre (c_1, c_2, c_3) and radius r is

$$(x-c_1)^2 + (y-c_2)^2 + (z-c_3)^2 = r^2$$

Opening the equation:

$$x^2+y^2+z^2-2c_1x-2c_2y-2c_3z-r^2=0 \quad \text{—} (1)$$

Quadric eqn: $ax^2+by^2+cz^2+dxy+exz+fyz+gx+hy+iz+j=0$ —— (2)

comparing both the eqn (1) & (2).

for sphere we need:

$$a=b=c=1 \quad \& \quad d=e=f=0$$

So, ~~general~~ from the eqn (2) of the previous question

$$A = d_x^2+d_y^2+d_z^2$$

$$B = 2O_xd_x+2O_yd_y+2O_zd_z+gd_x+hd_y+id_z$$

$$C = O_x^2+O_y^2+O_z^2+gO_x+hO_y+iO_z+j$$

$$At^2+Bt+C=0.$$

$$t = \frac{-B \pm \sqrt{B^2-4AC}}{2A} = \frac{-B\pm\sqrt{Det}}{2A}.$$

for successfull intersectn.

1. Det > 0

2. The roots t_0, t_1 must be positive.

   $t_0, t_1 > 0$

   If both are positive, then the smallest distance will be considered.