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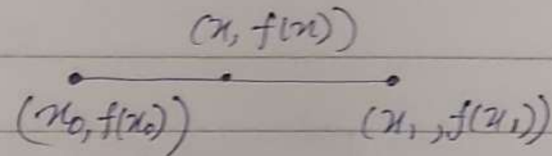
Group 8

(1) (a) $x \in [x_0, x_1]$



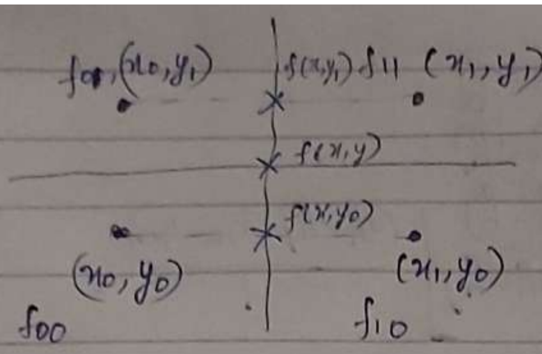
Line equation

$$f(x) - f(x_0) = \frac{f(x_1) - f(x_0)}{(x_1 - x_0)} (x - x_0)$$

$(y = mx : \text{form})$

$$\Rightarrow f(x) = f(x_0) + (x - x_0) \frac{(f(x_1) - f(x_0))}{(x_1 - x_0)}$$

(b)



Bilinear interpolation is about applying linear interpolation along two basis direction.

$f_{00}, f_{10}, f_{01}, f_{11}$ → are the values of the func^n at the four points $(x_0,y_0), (x_1,y_0), (x_0,y_1), (x_1,y_1)$ respectively.

Along x direction,

$$f(x, y_0) = f_{00} + \frac{(x - x_0)}{(x_1 - x_0)}(f_{10} - f_{00})$$

$$= \left(\frac{1}{x_1 - x_0}\right)\left((x - x_0) f_{10} + (x_1 - x) f_{00}\right)$$

$$f(x, y_1) = \left(\frac{1}{x_1 - x_0}\right)\left((x_1 - x) f_{01} + (x - x_0) f_{11}\right)$$

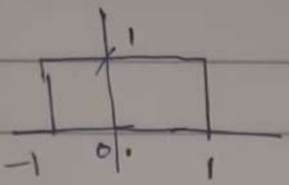
Along y-direc^n;

$f(x,y_1)$
$f(x,y)$
$f(x,y_0)$

$$f(x,y) = \frac{(y - y_0)(f(x,y_1)) + (y_1 - y)(f(x,y_0))}{(y_1 - y_0)}$$

$$= \frac{1}{(y_1 - y_0)(x_1 - x_0)}\Big[(y - y_0)(x_1 - x) f_{01} + (y - y_0)(x - x_0) f_{11} + (y_1 - y)(x - x_0) f_{10} + (y_1 - y)(x_1 - x) f_{00}\Big]$$

(6.2) $B(x) = \begin{cases} 0 & \text{for } x \leq -1 \\ 1 & \text{for } -1 < x < 1 \\ 0 & \text{for } 1 \leq x. \end{cases}$

B(x) 

$$F(f) = \int_{-\infty}^{\infty} B(x) \cdot e^{-2\pi i k x} \, dx.$$ [Fourier Transform of Box function]

$$= \int_{-1}^{1} e^{-2\pi i k x} \, dx$$

$$= \frac{1}{2\pi i k} \left[e^{2\pi i k} - e^{-2\pi i k} \right] = \frac{1}{\pi k} \sin(2\pi k)$$

$$= 2 \frac{\sin(2\pi k)}{2\pi k} = 2 \operatorname{sinc}(2k)$$ (sinc function)

(Given $\operatorname{sinc}(x) = \left(\frac{\sin(\pi x)}{\pi x}\right)$)

(6.3) (a) Yes exact reconstruction is possible because the sampling rate is more than the nyquist rate.

$f_s = \frac{1}{T}$ $\quad$ $N_q = \frac{1}{2T}.$

for Reconstruction of signal, we need filter with infinite support & sampling rate > Nyquist rate.

(b) In fourier space, we need multiplication with the box function & then do inverse fourier transformation to reconstruct the signal.

(c) In image space, we need convolution with the sinc function to reconstruct the signal. (sinc should be infinitely supported)