Lecture 9 - Communication Networks

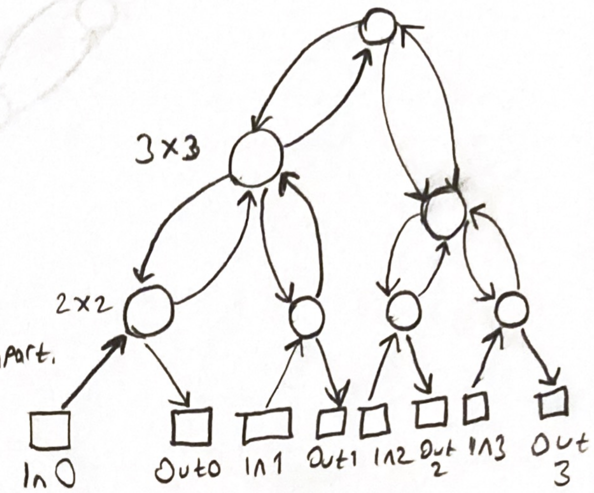
Complete Binary Tree

○ = switch: Direct Packets through Netw

□ = Terminal: Source & destination of data

Latency: The time required for a packet to travel from an input to an output

Diameter: of a network is the length of the shortest path between the input and output that are furthest apart.



Diameter = 6
(from In 0 to Out 3)
Switch count: 7

A permutation is a function

$$\pi : \{0 \dots N-1\} \longrightarrow \{0 \dots N-1\}$$

Such that no two numbers are mapped to same value.
$(\pi(i) = \pi(j)$ iff $i = j)$

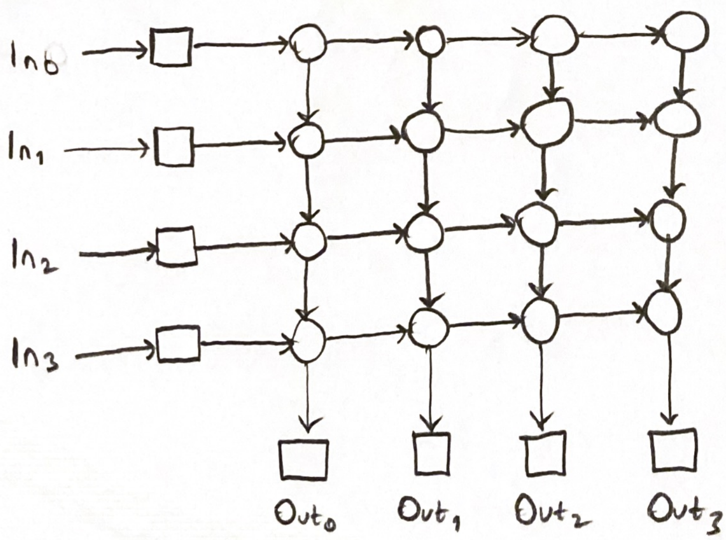
Ex\ $\pi(i) = N-1-i \rightarrow$ congestion $= 4$
$\pi(i) = i \longrightarrow$ congestion $= 1$

Permutation Routing Problem for π:
For each i direct the packet at In_i to $Out_{\pi(i)}$: Path taken is denoted by $P_{i,\pi(i)}$

The congestion of paths $P_{0,\pi(0)}, \dots, P_{N-1,\pi(N-1)}$
Congestion is equal to the largest number of paths that pass through a single switch

Max. congestion: $\max\limits_{\pi}$ min. congestion $P_{0,\pi(0)} \dots P_{N-1,\pi(N-1)}$
solutions
$P_{0,\pi(0)} \dots P_{N-1,\pi(N-1)}$

2D - Array



Theorem\ The congestion of an N-input array is 2.

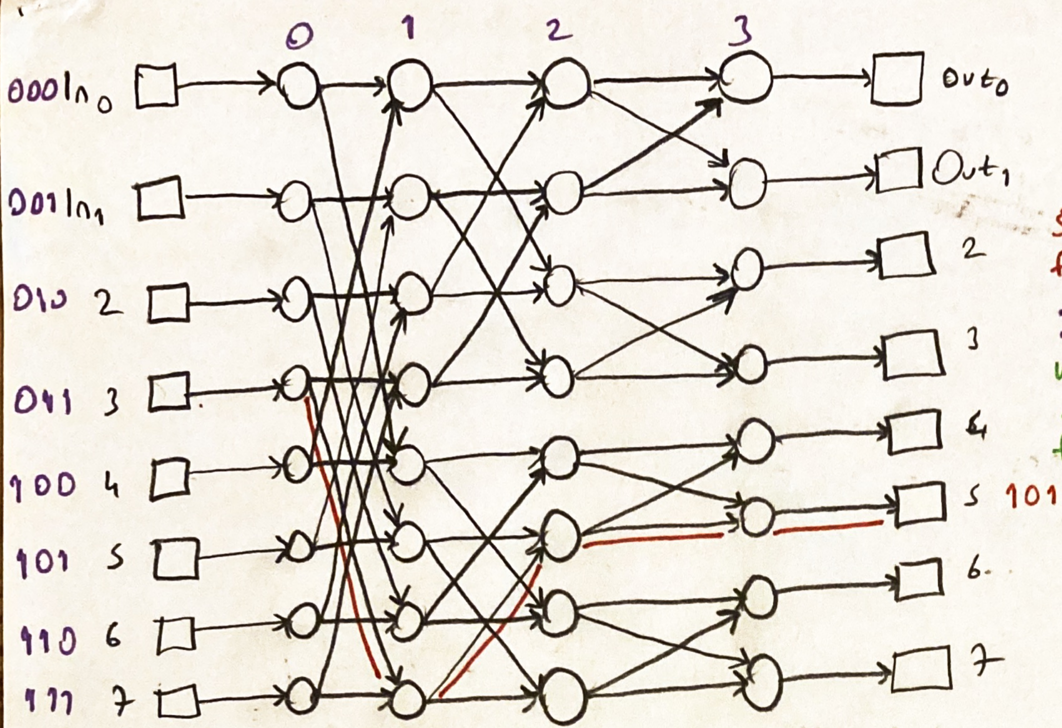
Pf\ Let π be a perm

$P_{i,\pi(i)}$ = Path from In_i rightward to column $\pi(i)$ and downward to $Out_{\pi(i)}$

Switch in row i and column $\pi(i)$ transmits ≤ 2 packets

If $\pi(0) = 0$, $\pi(N-1) = N-1 \Rightarrow$ congestion 2

■



Butterfly

Suppose we want to go from In 3 to Out 5

3: 011 and 5: 101

What will do is for every different bit we cross, for every same bit we go straight

0 cross 1
1 cross 0
1 straight 1

So a switch is uniquely identified by its row and column.

$$(b_1, \ldots, b_{\log N}, l) \quad l \to \text{level}$$

$$(b_1, \ldots, b_{l+1}, \ldots, b_{\log N}, l+1) \qquad (b_1, \ldots, \overline{b_{l+1}}, \ldots, b_{\log N}, l+1)$$

$$(x_1, \ldots, x_{\log N}, 0) \longrightarrow (y_1, x_2, x_3, \ldots, x_{\log N}, 1)$$

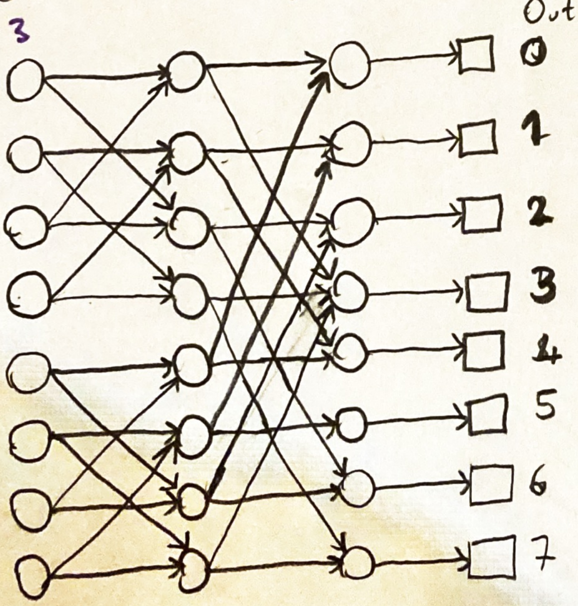
$$\downarrow$$

$$(y_1, y_2, x_3, \ldots, x_{\log N}, 2)$$

$$(y_1, \ldots, y_{\log N}, \log N)$$

Benes Network

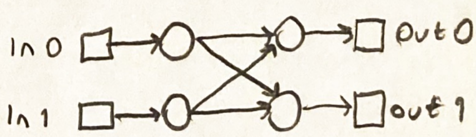
Consider the butterfly that we draw, Now, we will expand it;



Thm The congestion of N-input Benes network is 1, when $N = 2^a$ for some $a \geq 1$

Pf By induction on a: $P(a)$ = 'The theorem is true for a'

Base Case: $N = 2^1$



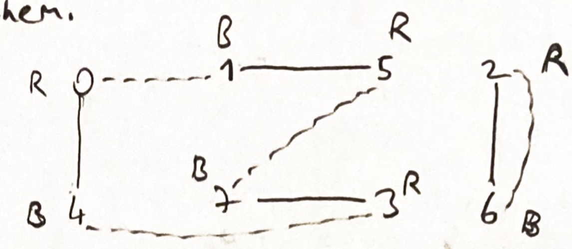
$\pi(0) = 0$	$\pi(0) = 1$
$\pi(1) = 1$	$\pi(1) = 0$

In both cases, max. congestion is 1.

Inductive step: Assume P(a)

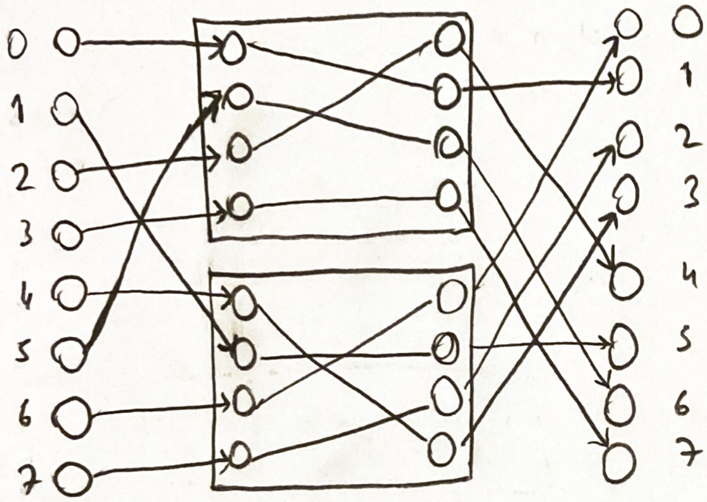
Ex) $\pi(0)=1$ $\quad$ $\pi(4)=3$
$\pi(1)=5$ $\quad$ $\pi(5)=6$
$\pi(2)=4$ $\quad$ $\pi(6)=0$
$\pi(3)=7$ $\quad$ $\pi(7)=2$

Constraint graph:
If two packets must pass through different subnetworks, then there is an edge between them.



Ex) The packet destined for Out_0 ($\pi(6)=0$) and the packet for Out_4 ($\pi(2)=4$) can't pass through the same subnetwork.

Key Insight: A 2-coloring of the constraint graph



Remaining part of the proof can be found in recitation notes and readings

N x N Network	Diameter	Switch size	# Switches	Congestion
Binary Tree	$2*(1+\log N)$	3x3	$2*N-1$	N
2D Array	$2N$	2x2	N^2	2
Butterfly	$2+\log N$	2x2	$N*(1+\log N)$	$\sqrt{N}$ or $\sqrt{N/2}$
Benes	$1+2*\log N$	2x2	$2*N*\log N$	1