HW 5

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```
library(purrr)
library(tidyr)
library(knitr)
library(invgamma)
```

2.3

```
set.seed(6)

b<- seq(0,1, by = .0001)

a_ <- 0
b_ <- 0

#checking a range of values to get the
#appropiate a and b that meets conditions
for(i in 1:length(b)){
  a = (100*b[i]^2)
  med = qgamma(.5,a,b[i])
  if(med%>%round(0) == 75){
      a_= a;b_ = b[i]
      break
  }
}

#condition 1, var = 100

var<- a_/b_^2
var

## [1] 100

#condition 2 median = 75

med_<- rgamma(10000, a_, b_)%>%median()
med_%>%round()

## [1] 75
```

The prior that staify both conditions is `Gamma`(56.175025, 0.7495)

2.5

```r
#information given in exercise
c= c(1,1,2,2)
a<- c(.1,1,.1,1)

#matrix to store values
data<- matrix(.0,nrow = 4, ncol = 9)

data[,1]<- c
data[,2]<- a
data[,3]<- a

#function to build determine the sse
sse <- function(){
  x2=15;x=-2;n= 20
  ((1/(n-1))* x2- (x/n)^2)%>%return()
}

#function for the mean
mean<- function(a, b){
  n = 20
  ((sse() + b)/(n + 2*a - 2))%>%return()
}



#calcuating the std
for(i in 1:nrow(data)){
  #post A
  data[i,4]= (20/2) + data[i,3]
  #post B
  data[i,5] = (data[i,3]  + sse())
  #P(sigma>c)
  data[i,6] = 1- pinvgamma(data[i,1], data[i,4], data[i,5])
  # post mean
  data[i,7] = mean(data[i,3], data[i,3])
  # post std
  data[i,8] = (sse()/(20-1))^.5
  #adding random a and b to see what effect the prior had on the posterior
  data[i,9] = mean(10, 10)

}

df2<- as.data.frame(data)

names(df2)<- c("c", "a", "b", "A", "B",  "postProb", "postMean", "postStd",
"priorTest")
```

```r
df2%>%kable(caption = "Results Of Posterior")
```

Results Of Posterior

c	a	b	A	B	postProb	postMean	postStd	priorTest
1	0.1	0.1	10.1	0.8794737	0.0e+00	0.0483227	0.2025461	0.2836704
1	1.0	1.0	11.0	1.7794737	2.8e-06	0.0889737	0.2025461	0.2836704
2	0.1	0.1	10.1	0.8794737	0.0e+00	0.0483227	0.2025461	0.2836704
2	1.0	1.0	11.0	1.7794737	0.0e+00	0.0889737	0.2025461	0.2836704

The results for `P(`θ`> c)` are in column `postProb`.

```r
#P(sigma>c | c= 1, a=b=0.1) / P(sigma>c | c=1, a=b=1)

r1 <- data[1,6]/data[2,6]

#P(sigma>c | c= 2, a=b=0.1) / P(sigma>c | c=2, a=b=1)

r2 <- data[3,6]/data[4,6]
```

The ratio when `c = 1` the ratio is 0.0095544 and when `c = 2` the ratio is 0.0118331. Also, the posterior is very sensative to the prior. This is shown in column `priorTest` where the prior mean chnages greatly when I plug in any value for rate and shape.

Problem 3 and 4 in chapter 2 of Reich and Ghosh. In addition to the problems in the book, find the conjugate prior for p and derive its posterior assuming

$Y|p \sim \text{Binomial}(n,p)$

$Z|p \sim \text{Binomial}(m,p)$

$$P(p|X) \propto \overbrace{P(Y|p)\cdot P(Z|p)}^{P(X|p)} \cdot P(p)$$

$$\propto \binom{n}{y} p^{y}(1-p)^{n-y} \cdot \binom{m}{z} p^{z}(1-p)^{m-z} \cdot \frac{\Gamma(a+b)}{\Gamma(a)+\Gamma(b)} p^{a-1}(1-p)^{b-1}$$

$$\propto p^{y}(1-p)^{n-y} p^{z}(1-p)^{m-z} \cdot p^{a-1}(1-p)^{b-1}$$

$$\propto p^{(y+z+a)-1}(1-p)^{(n-y+m-z+b)-1}$$

Using a Beta prior our posterior is a Beta. Therefore

$\text{Beta}(y+z+a,\ n-y+m-z+b)$