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Hw 3

Question # 8

$X_1 \sim \text{Bernoulli}(\frac{1}{2})$ $X_2 \sim \text{Bernoulli}(\frac{1}{3})$. \perp

$Y_1 = 1$ if both $X_1 = 1$ and $X_2 = 1$. & $Y_1 = 0$ otherwise

$Y_2 = 1$ if either $X_1 = 1$ or $X_2 = 1$. & $Y_2 = 0$ otherwise

There are 4 possible outcomes

$$P(X_1=1, X_2=1) = \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$$

$$P(X_1=0, X_2=0) = \frac{1}{2} \times \frac{2}{3} = \frac{2}{6}$$

$$P(X_1=1, X_2=0) = \frac{1}{2} \times \frac{2}{3} = \frac{2}{6}$$

$$P(X_1=0, X_2=1) = \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$$

$$\textcircled{a} P(X_1=1 \text{ and } X_2=1 | Y_1=1) = \frac{P(X_1=1, X_2=1)}{P(Y_1=1)} = \frac{P(X_1=1, X_2=1)}{P(X_1=1, X_2=1)} = \frac{\frac{1}{6}}{\frac{1}{6}} = 1$$

$$\textcircled{b} P(X_1=1 \text{ and } X_2=1 | Y_2=1) = \frac{P(X_1=1, X_2=1)}{P(X_1=1, X_2=1) + P(X_1=1, X_2=0) + P(X_1=0, X_2=1)}$$

$$= \frac{\frac{1}{6}}{\frac{1}{6} + \frac{2}{6} + \frac{1}{6}} = \frac{1}{4}$$

$$\textcircled{c} P(X_1=1 | Y_1=1) = \frac{P(X_1=1, X_2=1)}{P(X_1=1, X_2=1)} = \frac{\frac{1}{6}}{\frac{1}{6}} = 1$$

$$\textcircled{d} P(X_1=1 | Y_2=1) = \frac{P(X_1=1, X_2=1) + P(X_1=1, X_2=0)}{P(X_1=1, X_2=1) + P(X_1=1, X_2=0) + P(X_1=0, X_2=1)}$$

$$= \frac{\frac{1}{6} + \frac{2}{6}}{\frac{1}{6} + \frac{2}{6} + \frac{1}{6}}$$

$$= \frac{3}{4}$$

Question #9

- a) $\frac{51}{75}$ prior is a preferred prior when the prior is important and needs to be included. Here as with equal probability prior, I think the data is good and the prior don't play a major role.

$$\begin{aligned} \text{b) } P(H|Y) &= \frac{P(Y|H) \cdot P(H)}{P(Y|H) \cdot P(H) + P(Y|M) \cdot P(M)} \\ &= \frac{\binom{1000}{3} \left(\frac{3.24}{1000}\right)^3 \left(1 - \frac{3.24}{1000}\right)^{1000-3} \cdot .5}{\binom{1000}{3} \left(\frac{3.24}{1000}\right)^3 \left(1 - \frac{3.24}{1000}\right)^{1000-3} \cdot .5 + \binom{1000}{3} \left(\frac{.23}{1000}\right)^3 \left(1 - \frac{.23}{1000}\right)^{1000-3} \cdot .5} \\ &= .9928 \end{aligned}$$

- c) I think the assumption we are making in b is highly unlikely because for us to have a 50% prior means that half of the words would contain "upon". This not possible in valid sentences where meaning is conveyed.
- d) Changing to 1 would decrease the posterior probability. Because 1 get us much closer to Madison expected usage.
- e) I think the posterior would still be high as in B, in favour of Hamilton. However, the value is a little higher than b because of the use of more information.