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Question 4

$Y_i | \delta^2 \sim Normal(0, \delta_i^2)$

$\delta_i^2 \sim InvGamma(a, b)$

$b \sim Gamma(1,1)$

$$P(\delta_i^2 | Y_i, b) \propto \prod_{i=1}^{n} \left[P(Y_i | \delta_i^2) \cdot P(\delta_i^2) \right] \cdot P(b)$$

$$\propto \prod_{i=1}^{n} \left[\frac{1}{(2\pi\delta^2)^{\frac{1}{2}}} \cdot e^{-\frac{1}{2}\frac{y_i^2}{\delta_i^2}} \cdot \frac{b^a}{\Gamma(a)} (\delta_i^2)^{a-1} \cdot e^{-\frac{b}{\delta_i^2}} \right] \cdot b^{1-1} \cdot e^{-b}$$

$$\propto \prod_{i=1}^{n} (\delta_i^2)^{-\frac{1}{2}} \cdot e^{-\sum_{i=1}^{n} \frac{y_i^2}{2\delta^2}} \cdot \prod_{i=1}^{n} (\delta_i^2)^{-a-1} \cdot e^{-\sum_{i=1}^{n} \frac{b}{\delta_i^2}}$$

$$\propto \prod_{i=1}^{n} (\delta_i^2)^{-\frac{1}{2}-a-1} \cdot e^{-\sum_{i=1}^{n} \frac{y_i^2}{2\delta_i^2} - \sum_{i=1}^{n} \frac{b}{\delta_i^2}}$$

$$\propto \prod_{i=1}^{n} (\delta_i^2)^{-(\frac{1}{2}+a)-1} \cdot e^{-\sum_{i=1}^{n} \frac{\frac{y_i^2+2b}{2}}{\delta_i^2}}$$

$$\propto InGamma\left(\frac{1}{2}+a, \frac{y_i^2+2b}{2}\right)$$

$$P(b | \delta_i^2, Y_i) \propto \prod_{i=1}^{n} P(Y_i | \delta_i^2) \cdot P(\delta_i^2 | b) \cdot P(b)$$

$$\propto \prod_{i=1}^{n} \left[b^a \cdot e^{-\frac{b}{\delta_i^2}} \right] \cdot b^{1-1} \cdot e^{-b}$$

$$\propto b^{na} \cdot e^{-b\sum_{i=1}^{n} \frac{1}{\delta_i^2}} \cdot b^{1-1} \cdot e^{-b}$$

$$\propto b^{na+1-1} \cdot e^{-b\left(\sum_{i=1}^{n} \frac{1}{\delta_i^2}+1\right)}$$

$$\propto Gamma\left(na+1, \sum_{i=1}^{n} \frac{1}{\delta_i^2}+1\right)$$

Part a

$$P(\delta_{i=1}^2 | Y_{i=1}, b) \propto InGamma\left(\frac{1}{2}+a, \frac{y_{i=1}^2+2b}{2}\right)$$

$$P(b | \delta_{i=1}^2, Y_{i=1}) \propto Gamma\left(a+1, \frac{1}{\delta_{i=1}^2}+1\right)$$

Part B

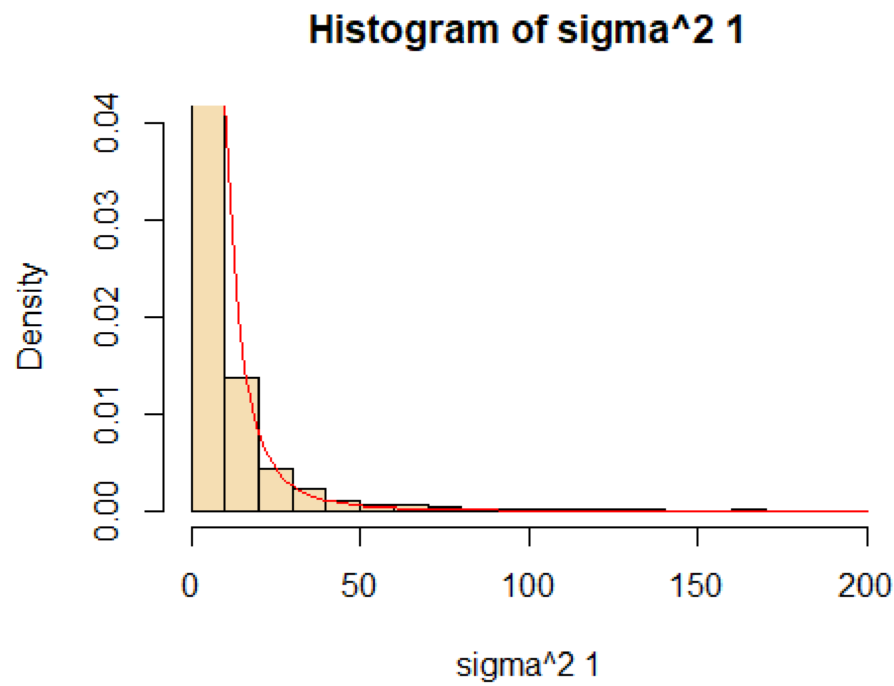
My steps for Gibbs Sampler would be:

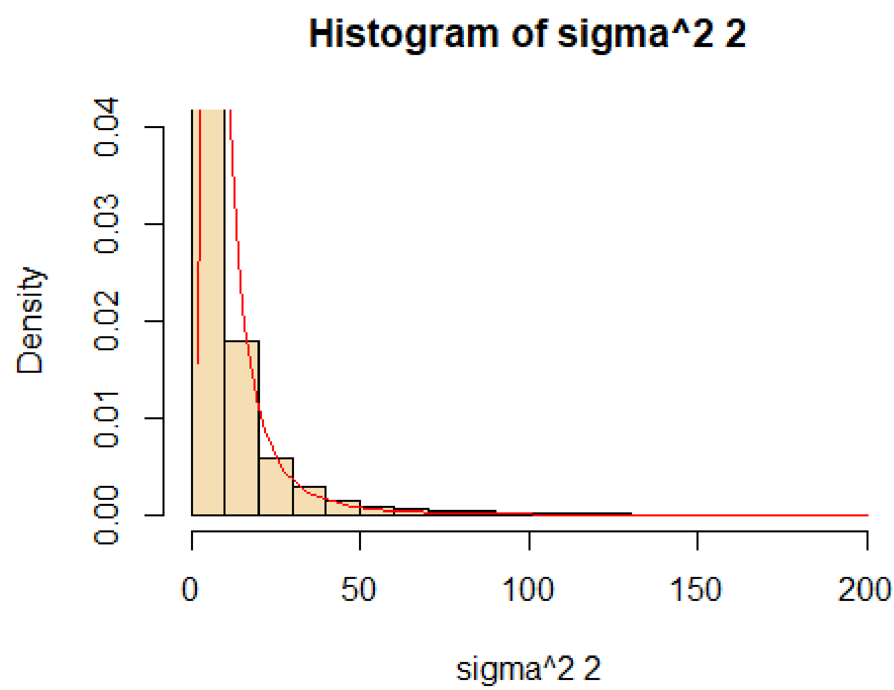
- First import the data (Yi), then assign some variable to the data (Y).
- Get the count of the data and assign a variable (n).
- Assign the number of simulations to a variable. (S)
- Create a matrix(samples) or list to hold my various P(sigmai^2|Yi, b) and P(b|sigma^2, yi)
- Assign initial values for sigma^2(sigma) and b (B), to any values which eventually converge.
- Assign a and b from prior.
- Loop from 1 to S with inner loop from 1 to n over P(sigmai^2|Yi, b) for each yi.
- Also within the loop from 1 to S, pass values generated from each iteration in previous bullet point into P(b|sigma^2, yi)
- Store these values generates from each iteration into matrix samples.

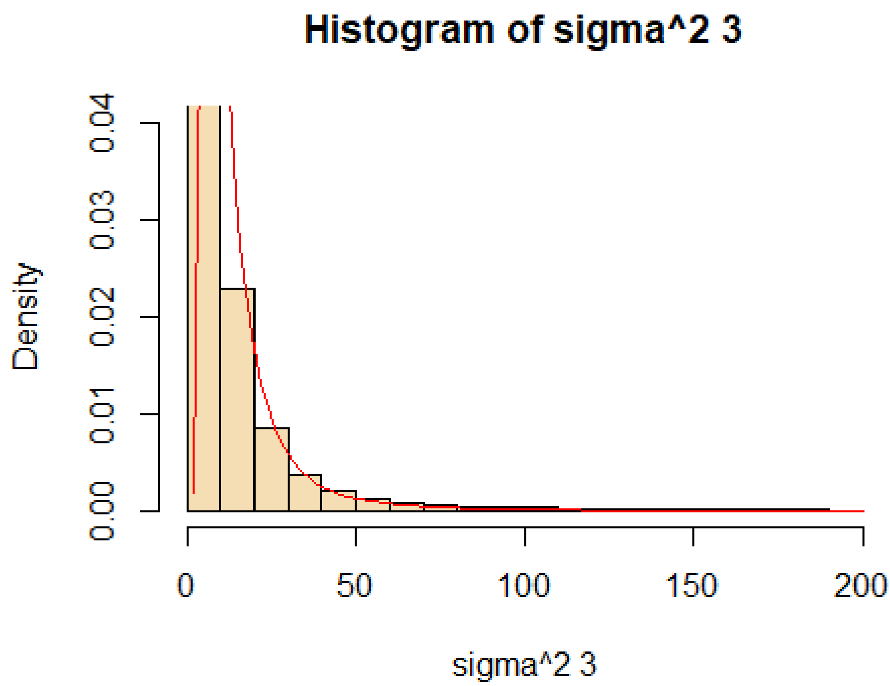
Part C

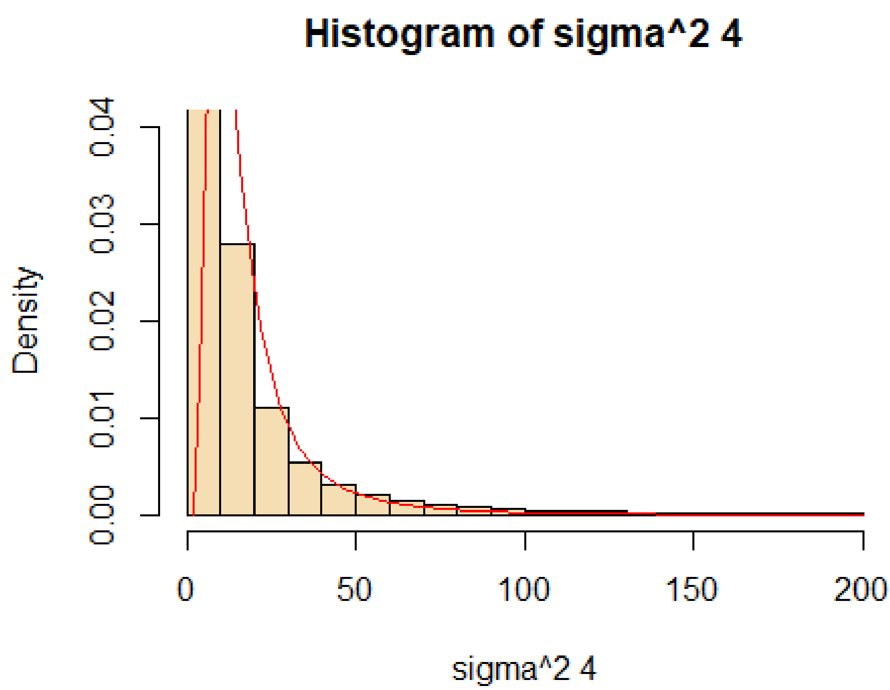
```r
library(invgamma)

#yi
Y<- c(1:10)

#n
n<- length(Y)


# of simulations
S<- 25000

samples<- matrix(NA, nrow = S, ncol = 11)

#names to assign to matrix
colnames(samples)<- c("s1", "s2", "s3", "s4", "s5", "s6", "s7", "s8", "s9",
```

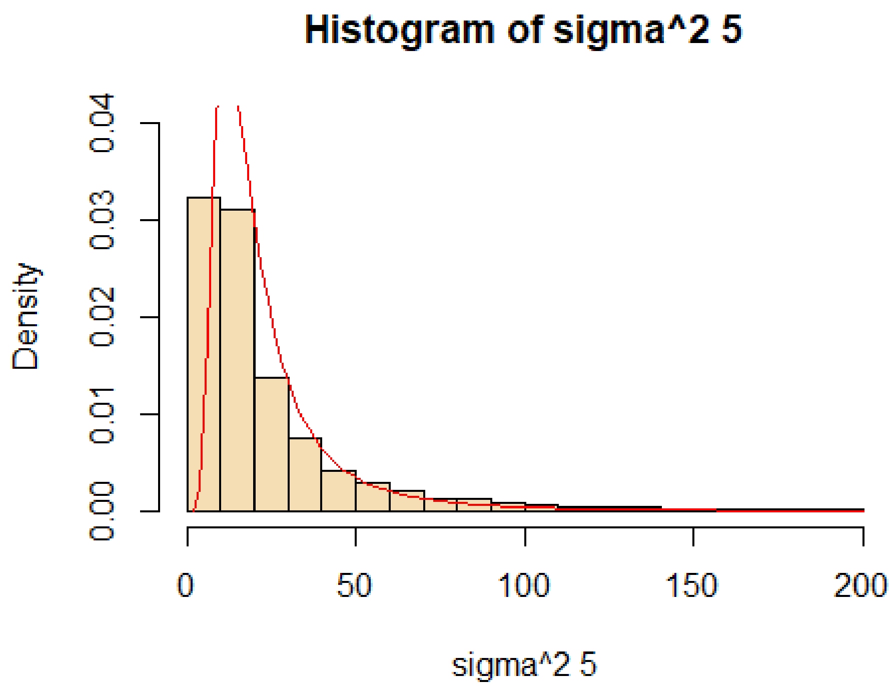
```
"s10", "B")


#inital values
sigma <- 1
b<- .1

#assign by Dr Reich
a<- 1




#Gibbs sampler
for(s in 1:S){
  for(i in 1:n){
    #sigma
    sigma[i] <- 1/(rgamma(1,.5 + a, (Y[i]^2 + 2* b)/2))
  }
  #b
  b <- rgamma(1,n*a + 1, 1+sum(1/sigma))
  samples[s,] <- c(sigma, b)
}


# getting the inverse gamma rate and shape
para<- function(x){
  m = mean(x)
  v = var(x)
  a = (m^2/v) + 2
  b = m * (a- 1)
  return(c(a, b))
}

#Plot of sigma
# op <- par(pty="m", mfrow=c(5, 2), mar=c(4.2, 4.2, 1, 1))
for(i in 1:10){
  temp=samples[,i]
  #selecting only the values less than 200 for hist
  values= temp[temp<200]
  hist(values, freq = F, col = "Wheat", xlab =paste("sigma^2", i), main =
paste("Histogram of sigma^2", i), ylim = c(0, .04))
  x= seq(0, 200, length = length(values))

  #gamma distrubtion overlay
  curve(dinvgamma(x,para(values)[1],para(values)[2]), add = TRUE, col =
"red");
}
```

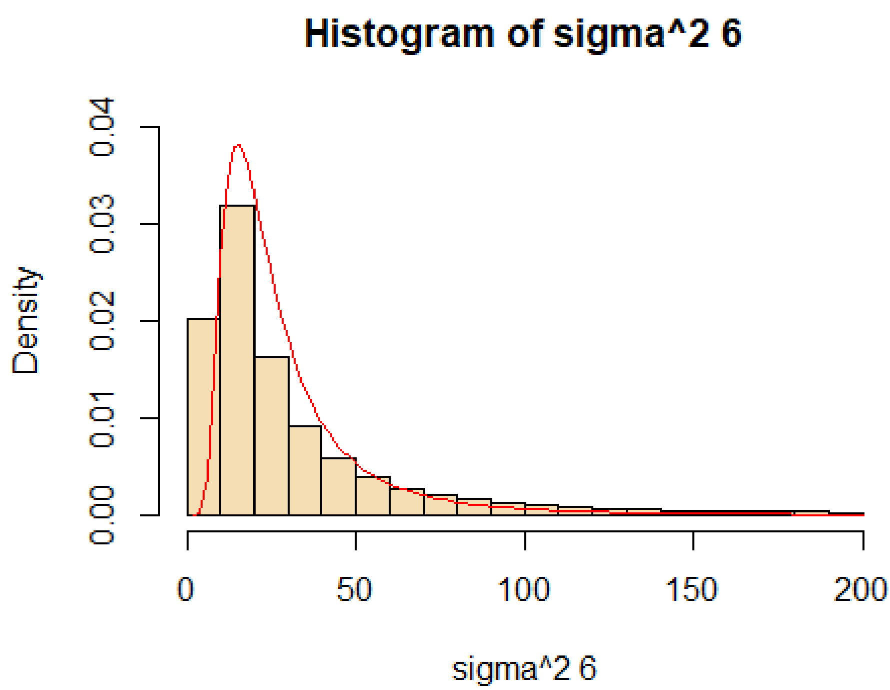

Histogram of sigma^2 1
Density
0.00
0.01
0.02
0.03
0.04
0
50
100
150
200
sigma^2 1

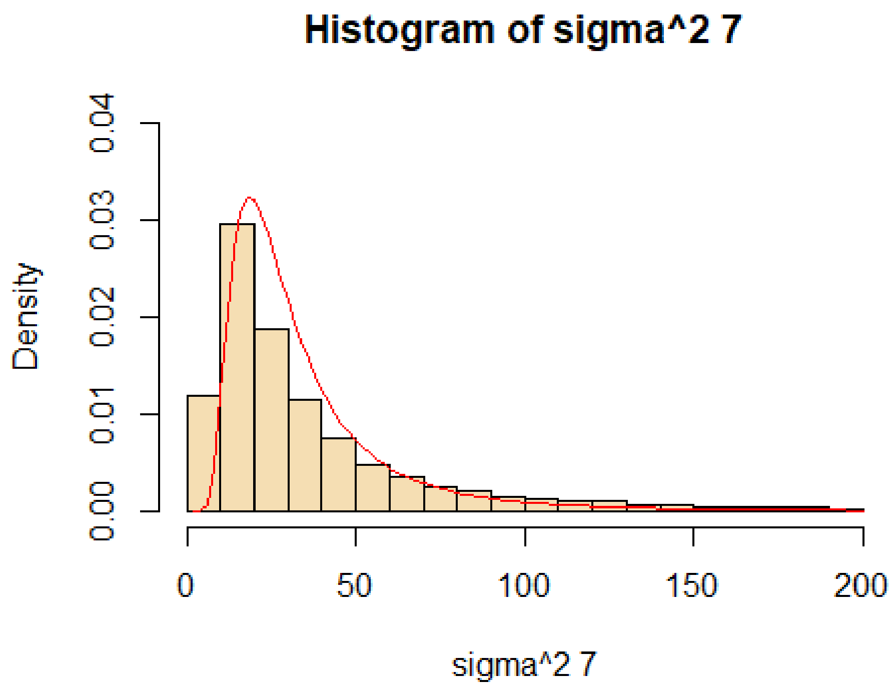

Histogram of sigma^2 2
Density
0.00
0.01
0.02
0.03
0.04
0
50
100
150
200
sigma^2 2

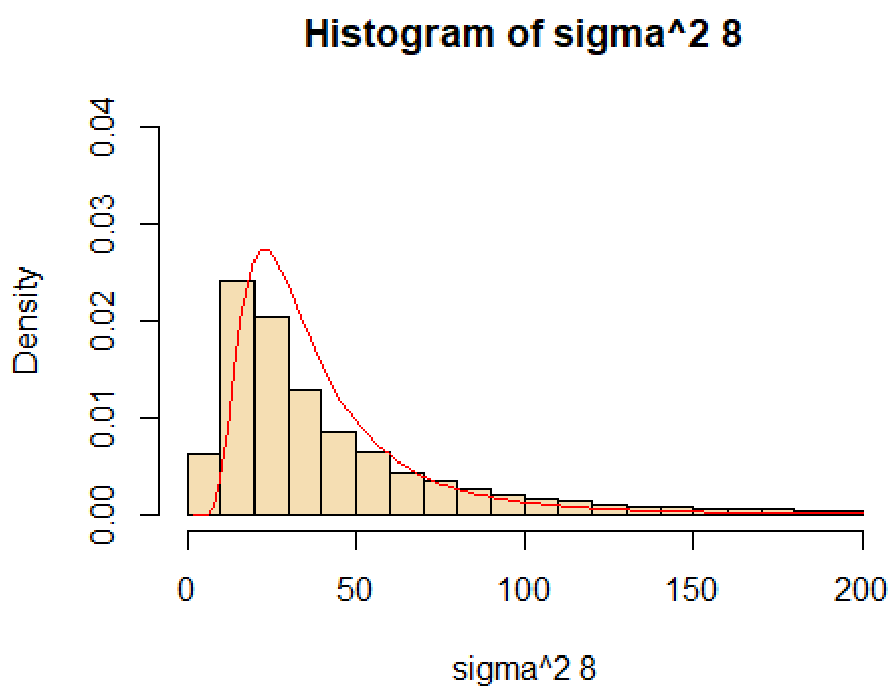

Histogram of sigma^2 3
Density
0.00
0.01
0.02
0.03
0.04
0
50
100
150
200
sigma^2 3

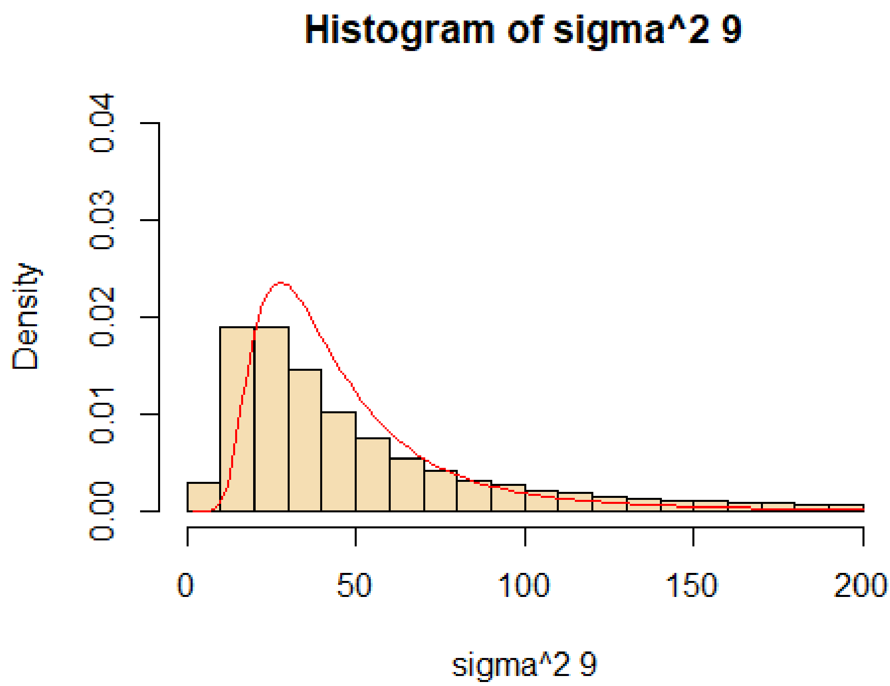

Histogram of sigma^2 4
Density
0.00
0.01
0.02
0.03
0.04
0
50
100
150
200
sigma^2 4

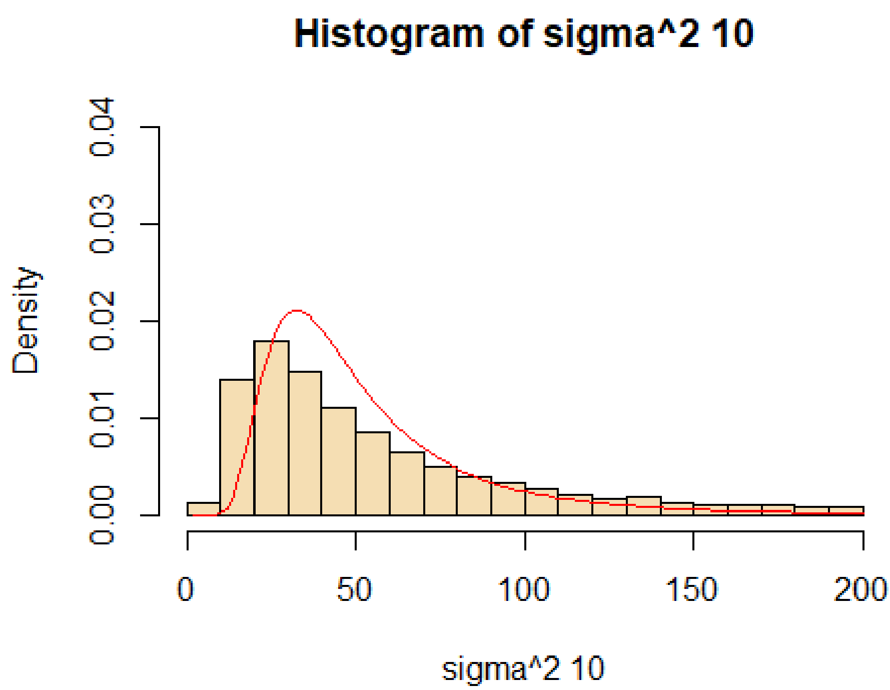

Histogram of sigma^2 5
Density
0.00
0.01
0.02
0.03
0.04
0
50
100
150
200
sigma^2 5


Histogram of sigma^2 6
Density
0.00
0.01
0.02
0.03
0.04
0
50
100
150
200
sigma^2 6


Histogram of sigma^2 7
Density
0.00
0.01
0.02
0.03
0.04
0
50
100
150
200
sigma^2 7


Histogram of sigma^2 8
Density
0.00
0.01
0.02
0.03
0.04
0
50
100
150
200
sigma^2 8


Histogram of sigma^2 9
Density
0.00
0.01
0.02
0.03
0.04
0
50
100
150
200
sigma^2 9


Histogram of sigma^2 10
Density
0.00
0.01
0.02
0.03
0.04
0
50
100
150
200
sigma^2 10

```
# par(op)
```

```r
#histogram of B
hist(samples[,11], freq = F, col = "wheat", main = "Histogram of b")
```