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Brief Paper

On the Use of Adaptive Updating Rules for Actuator and Sensor Fault Diagnosis*

HONG WANG,† ZHEN J. HUANG‡ and STEVE DALEY§

Key Words—Fault detection and diagnosis; adaptive updating rules; linear systems; observers; actuators, sensors.

Abstract—A novel approach is presented for the fault detection and diagnosis (FDD) of faults in actuators and sensors via the use of adaptive updating rules. The system considered is linear time-invariant and is subjected to an unknown input that represents either model uncertainty or unmeasurable disturbances. First, fault detection and diagnosis for linear actuators and sensors is considered, where a fixed observer is used to detect the fault whilst an adaptive diagnositic observer is constructed to diagnose the fault. Using the augmented error technique from model reference adaptive control, an observation error model is formulated and used to establish an adaptive diagnostic algorithm that produces an estimate of the gains of actuator and the sensor. An extension to the fault detection and diagnosis to cover nonlinear actuators is also made, where a similar augmented error model to that used for linear actuators and sensors is obtained. As a result, a convergent adaptive diagnostic algorithm for estimating the parameters in the nonlinear actuators is developed. Two simulated numerical examples are included to demonstrate the use of the proposed approaches. © 1997 Elsevier Science Ltd. All rights reserved.

1. *Introduction*

Owing to the increasing demand for high reliability for many industrial processes, fault detection and diagnosis (FDD) algorithms and their application to a wide range of industrial and commercial processes have been the subjects of intensive investigation over the past two decades. Many methods have been developed, and examples include the robust-redundancy-relation approaches (Chow et al., 1986; Basseville, 1988; Frank, 1994), the observer-based approaches (Patton *et al.*, 1989; Patton and Chen, 1991; Viswanadham and Srichander, 1987; Ge and Fang, 1988; Park and Rizzoni, 1994), identification-based methods (Isermann, 1984; Freyermuth, 1991), and the neural-network-based methods (Zhang and Roberts, 1992; Patton *et al.*, 1994; Wang *et al.*, 1994). These methods have proven to be capable of successfully detecting and diagnosing certain types of system faults, which are generally assumed to be unexpected changes of some parameters in the system (Daley and Wang, 1993).

In observer-based FDD approaches, observers are constructed based upon a knowledge of the parameter matrices in the state-space representation of the healthy system. This enables fault detection to be performed by checking the observation error in relation to a pre-specified threshold (Frank, 1994). As for diagnosis, either a single observer (Massoumnia, 1986; White and Speyer, 1987) or a number of observers (Ge and Fang, 1988) are connected in parallel to the system, whereupon residual signals are generated, with each being sensitive to one particular fault but insensitive to others. Since the parameter matrices of the system are assumed to be exactly known, difficulties will most certainly arise when applying these methods to practical systems. As a result, researchers have sought to enhance the robustness of observer-based FDD such that the generated residual signals are sensitive to the fault but insensitive to model uncertainties and external disturbances. This has lead to the use of unknown input observers (Viswanadham and Srichander, 1987; Phatak and Viswanadham, 1988; Wuennenberg and Frank, 1988; Chen and Zhang, 1991; Hou and Muller, 1994), where the model uncertainties and slowly varying parameters are treated as unknown inputs to the system. However, these methods are still limited to systems whose dominant parts are linear and known (Ding and Frank, 1993).

Since in general the residual signals are functions of faults, model uncertainties and unknown parameters of the system, adaptive observers have been proposed (Sidar, 1983; Li and Zhang, 1993; Frank *et al.*, 1991; Ding and Frank, 1993) to further enhance the robustness. In these approaches, adaptive observers are used to track the slow variations of unknown system parameters, and the estimated parameters are then used to evaluate the residual signals in order to reduce the number of false alarms. As a result, the sensitivity of residual signals to both model uncertainties and slowly varying parameters is decreased. In fact, adaptive observer-based FDD combines the observer-based FDD methods with with parameter-estimation FDD approaches, and is capable of simultaneously estimating unknown parameters, slowly varying faults and the state of the system (Sidar, 1983; Li and Zhang, 1993; Ding and Frank, 1993). Since fault detection and diagnosis for abrupt faults is performed through the evaluation of adaptive residual signals, the implementation of the overall structures are somewhat complicated. This is true especially when a number of adaptive observers are connected to the system in order to diagnose different fault types. An added limitation is that the conditions for convergence and stability are very complex, and a priori knowledge of the norms of the state and inputs is required (Sidar, 1983).

As a result, simplified approaches for fault diagnosis are still in need of further development. This forms the main purpose of this paper, where a simple adaptive diagnostic algorithm, together with the use of an observer-based FDD framework, is developed for the diagnosis of sensor and actuator faults, which can be either abrupt or incipient.

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† Department of Paper Science, UMIST, P.O. Box 88, Manchester M60 1QD, U.K.

‡ Radio Technology Department, The South East University, Nanjing, P.R. China.

§ Mechanical Engineering Centre, European Gas Turbines Ltd, GEC Alsthom, Leicester LE8 3LH, U.K.

2. *Detection observer*

Assume that the system to be considered can be expressed by

$$\dot{x}(t) = Ax(t) + Bf(t)u(t) + v(t), \tag{1}$$

$$y(t) = g(t)[Cx(t) + Df(t)u(t)], \tag{2}$$

where $x(t) \in \mathbb{R}^n$ is the unmeasurable state vector, $u(t) \in \mathbb{R}^m$ is the measurable input vector and $y(t) \in \mathbb{R}^r$ is the measurable output vector; $v(t)$ is a unmeasurable term representing model uncertainties and the input noise of the system. It also represents the output noises through the dynamic equivalent principles. A, B, C and D are known parameter matrices of dimensions $n \times n$, $n \times m$, $r \times n$ and $r \times m$, respectively; $f(t)$ and $g(t)$ are $m \times m$ and $r \times r$ time-varying matrices representing the gains of actuator and sensor connected to $u(t)$ and $y(t)$, respectively. Both sensor and actuator faults are considered. It is assumed that the pair (A, C) is observable; (ii) $g(t)$ is always a non-singular matrix; (iii) $v(t)$ is uniformly bounded (i.e. there is a known positive number δ_0 such that $\|v(t)\| \le \delta_0$), and (iv) when no fault occurs, we have

$$g(t) = g_H, \quad f(t) = f_H, \tag{3}$$

where g_H and f_H are known matrices and the subscript H stands for healthy sensors and actuators.

The purpose of FDD is therefore to generate an alarm signal when a fault occurs and to produce an accurate estimate of the matrices f and g that define the faulty behaviour of the actuator and sensor. For this purpose, the following observer is constructed:

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bf_H u(t) + L_d[\hat{y}(t) - y(t)], \tag{4}$$

$$\hat{y}(t) = g_H[C\hat{x}(t) + Df_H u(t)], \tag{5}$$

where $\hat{x}(t) \in \mathbb{R}^n$ is the state vector of the observer and $L_d \in \mathbb{R}^{n\times r}$ is the gain matrix such that $A + L_d g_H C$ is a stable matrix. Define

$$e(t) = \hat{x}(t) - x(t), \tag{6a}$$

$$e_0(t) = \hat{y}(t) - y(t), \tag{6b}$$

$$\tilde{f}(t) = f_H - f(t), \quad \tilde{g}(t) = g_H - g(t). \tag{6c}$$

Then it can be shown that the equality

$$\begin{aligned} e_0 = \hat{y}(t) - y(t) &= g_H C\hat{x}(t) + g_H Df_H u(t) \\ &\quad - g(t)Cx(t) - g(t)Df(t)u(t) \\ &= g_H[Ce(t) + D\tilde{f}(t)u(t)] + \tilde{g}(t)g^{-1}(t)y(t) \end{aligned} \tag{7}$$

holds. As a result, the observation error equation

$$\begin{aligned} \dot{e}(t) &= (A + L_d g_H C)e(t) + (B + L_d g_H D)\tilde{f}(t)u(t) \\ &\quad + L_d\tilde{g}(t)g^{-1}y(t) - v(t) \end{aligned} \tag{8a}$$

can be obtained. If no fault occurs, (3) holds and $\tilde{f} = \tilde{g} = 0$. As a result, the second and third terms in (7) and (8a) are zero. This means that the observation error satisfies

$$\dot{e}(t) = (A + L_d g_H C)e(t) - v(t), \tag{8b}$$

$$e_0(t) = g_H Ce(t). \tag{8c}$$

Since $\| v(t) \le \delta_0$, it can be further obtained that

$$\|e_0(t)\| \le \max_{\omega \ge 0} \|g_H C(\mathrm{j}\omega I - A - L_d g_H C)^{-1}\| \, \delta_0 = \lambda, \tag{8d}$$

which is calculable because all the matrices involved in (8d) are known. Therefore the fault detection can be readily carried out as follows:

$$\|e_0(t)\| = \|\hat{y}(t) - y(t)\| \quad \begin{cases} \le \lambda & \text{if no fault occurs,} \\ > \lambda & \text{if fault has occurred,} \end{cases} \tag{9}$$

where λ is the threshold defined by (8d).

This is the well known observer based fault-detection method (Patton *et al.* 1989; Frank, 1994), where the gain matrices L_d, g_H and f_H in (4) and (5) are fixed. Therefore the observer (4), (5) is also referred to as a detection observer for the system (1), (2).

3. *An adaptive diagnostic observer and its error equation*

To diagnose the fault after the alarm (9) has been generated, the adaptive diagnostic observer

$$\dot{x}_m(t) = Ax_m(t) + B\hat{f}(t)u(t) + L\varepsilon(t), \tag{10}$$

$$\varepsilon(t) = \hat{g}(t)^{-1}[y_m(t) - y(t)], \tag{11}$$

$$y_m(t) = \hat{g}(t)[Cx_m(t) + D\hat{f}(t)u(t)] \tag{12}$$

is constructed, where $x_m(t) \in \mathbb{R}^n$ is the observer state vector, $L \in \mathbb{R}^{n\times r}$ is a pre-specified observer gain matrix that makes $A + LC$ stable, and $\hat{f}(t)$ and $\hat{g}(t)$ are the estimates of $f(t)$ and $g(t)$ respectively. The initial values of $\hat{f}(t)$ and $\hat{g}(t)$ are set to f_H and g_H respectively. It is assumed that after $t > t_f$, a fault occurs such that

$$f(t) = f = \text{constant} \ne f_H, \tag{13}$$

$$g(t) = g = \text{constant} \ne g_H, \tag{14}$$

and that g is nonsingular. Moreover, the faults may occur either simultaneously or at different times. Denote

$$e_m(t) = x_m(t) - x(t). \tag{15}$$

Then it can be obtained from (1) and (10) that

$$\dot{e}_m(t) = Ae_m(t) + B[\hat{f}(t) - f(t)]u(t) + L\varepsilon(t) - v(t), \tag{16}$$

$$\begin{aligned} y_m(t) - y(t) &= \hat{g}(t)Cx_m(t) + \hat{g}(t)D\hat{f}(t)u(t) \\ &\quad - gCx(t) - gDfu(t), \\ &= \hat{g}(t)[Ce_m(t) + D\tilde{f}(t)]u(t) + \tilde{g}(t)g^{-1}y(t), \end{aligned} \tag{17}$$

$$\tilde{f}(t) = \hat{f}(t) - f, \quad \tilde{g}(t) = \hat{g}(t) - g. \tag{18}$$

From (11) and (17), $\varepsilon(t)$ can be further expressed as

$$\varepsilon(t) = Ce_m(t) + D\tilde{f}(t)u(t) + [g^{-1} - \hat{g}^{-1}(t)]y(t). \tag{19}$$

Substituting (19) into (16), it can be further obtained that

$$\begin{aligned} \dot{e}_m(t) &= (A + LC)e_m(t) + (B + LD)\tilde{f}(t)u(t) \\ &\quad + L[g^{-1} - \hat{g}^{-1}(t)]y(t) - v(t). \end{aligned} \tag{20}$$

As a result, the combination of (19) and (20) gives the relationship between the estimation errors $(\tilde{f}(t), \tilde{g}(t))$ and the observation error $\varepsilon(t)$. The purpose of fault diagnosis is to use available $\varepsilon(t)$, $u(t)$ and $y(t)$ to construct a diagnostic algorithm for $\hat{g}(t)$ and $\hat{f}(t)$ such that

$$\lim_{t\to\infty} \hat{g}(t) = g, \quad \lim_{t\to\infty} \hat{f}(t) = f \tag{21}$$

Compared with the detection observer in Section 2, the adaptive diagnostic observer uses $\varepsilon(t)$ instead of the output error. Otherwise, the matrix $A + LC$ in (20) will be of the form $A + L\hat{g}(t)C$, which may not be regarded as a stable matrix. Since (19) and (20) are in a similar form to the error equation between the reference model and the adjustable part of the system found in model reference adaptive control (Landau, 1979; Narendra and Annaswamy, 1989; Åström and Wittenmark, 1989; Sun and Ioannou, 1992), in the following section an adaptive updating rule, which has been widely used in adaptive control, will be constructed and applied to tune $\hat{g}(t)$ and $\hat{f}(t)$.

4. *Single-input and single-output systems*

Denote

$$A_0 = A + LC, \quad B_0 = B + LD, \quad h = g^{-1}, \tag{22a}$$

$$\hat{h}(t) = \hat{g}^{-1}(t), \quad \tilde{h}(t) = h - \hat{h}(t). \tag{22b}$$

Then (20) can be expressed as

$$\dot{e}_m(t) = A_0 e_m(t) + B_0\tilde{f}(t)u(t) + L\tilde{h}(t)y(t) - v(t). \tag{23}$$

When the system (1), (2) is single-input and single-output, three transfer functions

$$W_1(s) = C(sI - A_0)^{-1}B_0 + D, \tag{24}$$

$$W_2(s) = C(sI - A_0)^{-1}L + 1, \tag{25}$$

$$W_3(s) = -C(sI - A_0)^{-1}. \tag{26}$$

can be readily defined, which lead to the following input–output expression of the error equations (19) and (20) in Laplace-transform form:

$$\varepsilon(t) = W_1(s)\tilde{f}(t)u(t) + W_2(s)\tilde{h}(t)y(t) + W_3(s)v(t). \tag{27}$$

Since under fault conditions $g(t)$ and $f(t)$ take constant values different from g_H and f_H, it can be shown, using the augmented error technique (Narendra and Annaswamy, 1989), that

$$\begin{aligned}\varepsilon(t) = \theta^T(t)\zeta(t) &+ [W_1(s)\hat{f}(t) - \hat{f}(t)W_1(s)]u(t)\\ &+ [W_2(s)\hat{h}(t) - \hat{h}(t)W_2(s)]y(t)\\ &+ \delta(t) + W_3(s)v(t),\end{aligned} \tag{28}$$

$$\theta^T(t) = [\tilde{f}(t) \quad \tilde{h}(t)], \tag{29}$$

$$\zeta = \begin{bmatrix} W_1(s)u(t) \\ W_2(s)y(t) \end{bmatrix}, \tag{30}$$

where $\delta(t)$ decays exponentially (Narendra and Annaswamy, 1989) and can be neglected. Since the second and third terms of (28) are all measurable, we can define

$$\begin{aligned}\varepsilon_1(t) = \varepsilon(t) &- [W_1(s)\hat{f}(t) - \hat{f}(t)W_1(s)]u(t)\\ &+ [W_2(s)\hat{h}(t) - \hat{h}(t)W_2(s)]y(t)\end{aligned} \tag{31}$$

as the augmented error signal for the system. As a result, it can be seen that

$$\varepsilon_1(t) = \theta^T(t)\zeta(t) + v_0, \tag{32}$$

$$v_0 = W_3(s)v(t), \tag{33}$$

$$\|v_0\| \le \max_{\omega \ge 0} \|W_3(j\omega)\|\, \delta_0 = \lambda_1. \tag{34}$$

Since $W_3(t)$ is known after L is selected, λ_1 is a known number, which can be used to construct a dead zone for an adaptive adjusting algorithm. Equation (32) is in the standard format of the error equations widely used in adaptive control (Narendra and Annaswamy, 1989, equation (8.24) of Chapter 8). The adaptive updating laws used in adaptive control can therefore be directly applied to tune $\theta(t)$. This is summarised by the following theorem.

Theorem 1. With the error model (32) and the fact that $W_i(s)$ $(i = 1, 2, 3)$ are stable, and that $\delta(t)$ decays exponentially and can be neglected, the tuning rules

$$\dot{\theta}(t) = \begin{cases} -\Gamma \dfrac{\varepsilon_1(t)\zeta(t)}{1 + \zeta^T(t)\zeta(t)} & (\|\varepsilon_1(t)\| > \lambda_1), \\ 0 & (\|\varepsilon_1(t)\| \le \lambda_1) \end{cases} \tag{35}$$

realizes a bounded $\theta(t)$ and $\dot{\theta}(t) \in L^2$.

The proof of this theorem is omitted here, since the same formulations presented by Narendra and Annaswamy (1989) can be applied. In (35), $\Gamma = \Gamma^T > 0$ is a pre-specified gain matrix that defines the tuning rate.

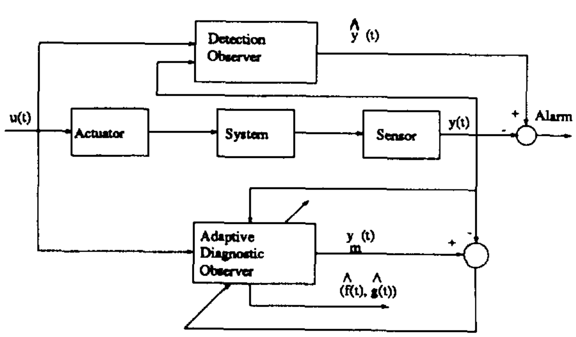

Fig. 1. Structure of the proposed method.

The tuning rule (35) is referred to as the adaptive diagnostic algorithm for the actuator and sensor fault diagnosis of the system (1), (2). Letting $\Gamma = \mathrm{diag}\,(\gamma_1, \gamma_2)$ and using the adaptive diagnostic algorithm (35) and the definition of the vectors $\theta(t)$ and $\zeta(t)$ in (29) and (30), it can be further shown that

$$\frac{d\hat{f}(t)}{dt} = -\gamma_1 \frac{\varepsilon_1(t)W_1(s)u(t)}{1 + \zeta(t)^T\zeta(t)}, \tag{36}$$

$$\frac{d\hat{g}(t)}{dt} = -\gamma_2 \frac{\varepsilon_1(t)\hat{g}^2(t)W_2(s)y(t)}{1 + \zeta(t)^T\zeta(t)} \quad (\|\varepsilon_1(t)\| > \lambda_1). \tag{37}$$

It is well known in adaptive control that a persistently exciting signal $u(t)$ can improve the accuracy of the estimation. This signal may therefore be required in practice for an improved diagnosis. However, the application of such signals can cause problems for practical systems in which case an exponentially decaying persistent signal should be used to improve the accuracy of adaptive diagnostic algorithm (36), (37). The overall fault detection and diagnosis procedure is shown in Fig. 1.

Although the term $\delta(t)$ in (28) tends to zero exponentially, the speed of this decay depends on the zeros of the denominators of transfer functions $W_1(s)$ and $W_2(s)$, which are part of the eigenvalues of matrix $A + LC$. Therefore, to ensure the validity of neglecting $\delta(t)$ and to provide a good convergence of the adaptive diagnostic algorithm (35), the fixed gain matrix L should be selected such that the eigenvalues of the matrix $A + LC$ are not only placed in the left-hand side of the complex plane, but are also far from the imaginary axis.

5. *Multiple-input and multiple-output systems*

When n or $r > 1$, the original system (1), (2) is multiple-input and multiple-output. In this case, the transfer functions $W_1(s)$ and $W_2(s)$ are $r \times m$ and $r \times r$ matrices with $w_1^{ij}(s)$ and $w_2^{ij}(s)$ being their ijth elements respectively. However, owing to the non-commutative property of matrix multiplications, a compact matrix form such as (28) cannot be obtained directly. In this section, we shall use the component form to derive the required error equation (32) for MIMO systems.

Since L can be selected such that $A + LC$ is stable, it can be seen that all the $w_1^{ij}(s)$ and $w_2^{ij}(s)$ are stable transfer functions. Denote $\tilde{f}_{ij}(t)$ and $\tilde{h}_{ij}(t)$ as the ijth elements of the matrices $\tilde{f}(t)$ and $\tilde{h}(t)$ respectively. Then the MIMO version of the error equation (27) becomes

$$\begin{aligned}\varepsilon_k(t) = &\sum_{i=1}^{m}\sum_{j=1}^{m} w_1^{ki}(s)\tilde{f}_{ij}(t)u_j(t) + \sum_{i=1}^{r}\sum_{j=1}^{r} w_2^{ki}(s)\tilde{h}_{ij}(t)y_j(t)\\ &+ v_{0,k} \quad (k = 1, 2, \ldots, r),\end{aligned} \tag{38}$$

where $\varepsilon_k(t)$ is the kth component of the vector $\varepsilon(t)$, $v_{0,k}(t)$ is the kth component of vector $v_0(t)$, $u_j(t)$ and $y_j(t)$ are the jth components of the vectors $u(t)$ and $y(t)$, respectively. Using the same technique as in Section 4, it can be seen that

$$\begin{aligned} w_1^{ki}(s)\tilde{f}_{ij}(t)u_j(t) = &\tilde{f}_{ij}(t)w_1^{ki}(s)u_j(t)\\ &+ [w_1^{ki}(s)\hat{f}_{ij}(t) - \hat{f}_{ij}(t)w_1^{ki}(s)]u_j(t) + \delta_k(t),\end{aligned} \tag{39}$$

$$\begin{aligned} w_2^{ki}(s)\tilde{h}_{ij}(t)y_j(t) = &\tilde{h}_{ij}(t)w_2^{ki}(s)y_j(t)\\ &+ [w_2^{ki}(s)\hat{h}_{ij}(t) - \hat{h}_{ij}(t)w_2^{ki}(s)]y_j(t) + \delta_k(t),\end{aligned} \tag{40}$$

where $\delta_k(t)$, again, decays exponentially (Narendra and Annaswamy, 1989). Let

$$\begin{aligned}\xi_k(t) = &\sum_{i=1}^{m}\sum_{j=1}^{m} \{[w_1^{ki}(s)\hat{f}_{ij}(t) - \hat{f}_{ij}(t)w_1^{ki}(s)]u_j(t)\}\\ &+ \sum_{i=1}^{r}\sum_{j=1}^{r} \{[w_2^{ki}(s)\hat{h}_{ij}(t) - \hat{h}_{ij}(t)w_2^{ki}(s)]y_j(t)\}\\ &+ 2\delta_k(t).\end{aligned} \tag{41}$$

Then it can be obtained from (38) that

$$\varepsilon_k(t)=\sum_{i=1}^{m}\sum_{j=1}^{m}\tilde f_{ij}(t)w_1^{ki}(s)u_j(t)+\sum_{i=1}^{r}\sum_{j=1}^{r}\tilde h_{ij}(t)w_2^{ki}(s)y_j(t)+\xi_k(t)+v_{0,k}. \quad (42)$$

Define

$$\theta=[\tilde f_{11},\tilde f_{12},\ldots,\tilde f_{1m},\tilde f_{21},\ldots,\tilde f_{2m},\ldots,\tilde f_{m1},\ldots,\tilde f_{m2},\ldots,\tilde f_{mm},\ldots \tilde h_{11},\tilde h_{12},\ldots,\tilde h_{1r},\ldots,\tilde h_{r1},\tilde h_{r2},\ldots,\tilde h_{rr}]^{\mathrm T}\in\mathbf R^{mm+rr}, \quad (43)$$

$$\zeta_k(t)=[w_1^{k1}(s)u_1(t),\ w_1^{k1}(s)u_2(t)\ \ldots,\ w_1^{k1}(s)u_m(t)\ w_1^{k2}(s)u_1(t),\ \ldots,\ w_1^{k2}(s)u_m(t),\ \ldots,\ w_1^{km}(s)u_1(t),\ \ldots,\ w_1^{km}(s)u_m(t)\ w_2^{k1}(s)y_2(t),\ w_2^{k1}(s)y_2(t),\ \ldots,\ w_2^{k1}(s)y_r(t),\ \ldots,\ w_2^{kr}(s)y_1(t),\ \ldots,\ w_2^{kr}(s)y_r(t)]^{\mathrm T}\in\mathbf R^{mm+rr},$$
$$k=1,2,\ldots,r. \quad (44)$$

Then (42) can be expressed as

$$\varepsilon_k(t)=\theta^{\mathrm T}\zeta_k+\xi_k+v_{0,k}. \quad (45)$$

As a result, a measurable augmented error $\sigma_k(t)$ can be defined as

$$\sigma_k(t)=\varepsilon_k(t)-\xi_k(t),\quad (k=1,2,\ldots,r). \quad (46a)$$

Using (45), it can be seen that

$$\sigma_k(t)=\theta^{\mathrm T}\zeta_k(t)+v_{0,k}\quad (k=1,2,\ldots,r). \quad (46b)$$

Similarly to Section 4, the adaptive diagnostic algorithm for matrices $\hat f(t)$ and $\hat h(t)$ should be such that

$$\lim_{t\to\infty}\varepsilon_k(t)=0. \quad (47)$$

However, since all the equations in (46b) $(k=1,2,\ldots,r)$ are subjected to a single parameter estimation error vector θ and the existence of term v_0, a compact form such as (35) for tuning $\hat f(t)$ and $\hat h(t)$ cannot be obtained. As a result, an adaptive tuning algorithm for $\hat f(t)$ and $\hat h(t)$ should be selected to minimize the magnitude of $\varepsilon_k(t)$. This can be achieved by choosing the following performance index function:

$$J=\frac12\sum_{k=1}^{r}[\sigma_k(t)-v_0]^2=\theta^{\mathrm T}\left[\sum_{k=1}^{r}\zeta_k(t)\zeta_k^{\mathrm T}(t)\right]\theta. \quad (48)$$

Using the well-known gradient descent rule in the continuous-time domain and the dead zones in Section 4, the adaptive diagnostic algorithm for $\hat f(t)$ and $\hat h(t)$ can be formulated to give

$$\frac{d\hat f_{ij}(t)}{dt}=\begin{cases}-\lambda_{ij}\dfrac{\partial J}{\partial\hat f_{ij}(t)} & (\|\sigma_k(t)\|>\lambda_1),\\ 0 & (\|\sigma_k(t)\|\le\lambda_1),\end{cases} \quad (49)$$

$$\frac{d\hat h_{ij}(t)}{dt}=\begin{cases}-\mu_{ij}\dfrac{\partial J}{\partial\hat h_{ij}(t)}, & (\|\sigma_k(t)\|>\lambda_1),\\ 0 & (\|\sigma_k(t)\|\le\lambda_1),\end{cases} \quad (50)$$

where λ_{ij} and μ_{ij} are positive numbers representing the learning rates. From (46b) and (48), we have

$$\frac{\partial J}{\partial\hat f_{ij}(t)}=\sum_{k=1}^{m}\sigma_k(t)w_1^{ki}(s)u_j(t), \quad (51)$$

$$\frac{\partial J}{\partial\hat h_{ij}(t)}=\sum_{k=1}^{r}\sigma_k(t)w_2^{ki}(s)y_j(t). \quad (52)$$

The adaptive diagnostic algorithm (49)–(52) constitutes an estimate of the matrices $\hat f(t)$ and $\hat h(t)$, which can be further expressed as

$$\frac{d\hat f_{ij}(t)}{dt}=\begin{cases}-\lambda_{ij}\displaystyle\sum_{k=1}^{m}\sigma_k(t)w_1^{ki}(s)u_j(t) & (\|\sigma_k(t)\|>\lambda_1),\\ 0 & \|\sigma_k(t)\|\le\lambda_1),\end{cases} \quad (53a)$$

$$\frac{d\hat h_{ij}(t)}{dt}=\begin{cases}-\mu_{ij}\displaystyle\sum_{k=1}\sum_{k=1}^{m}\sigma_k(t)w_2^{ki}(s)y_j(t) & (\|\sigma_k(t)\|>\lambda_1),\\ 0 & (\|\sigma_k(t)\|\le\lambda_1).\end{cases} \quad (53b)$$

Because the signals $\sigma_k(t)$, $u_j(t)$ and $y_j(t)$ are all measurable, and $W_1(s)$ and $W_2(s)$ are known after L is selected, the adaptive diagnostic algorithm (53) is realizable in practice. Using $\hat h(t)$ in (53b), $\hat g(t)$ can be readily obtained via the inverse

$$\hat g(t)=\hat h^{-1}(t). \quad (54)$$

Since the adaptive diagnostic algorithm (53) is obtained via the use of gradient descent rule and the perfomance function J is quadratic with respect to θ, a global minimum of the performance function J can be guaranteed if $\sum_{k=1}^{r}\zeta_k(t)\zeta_k^{\mathrm T}(t)$ is uniformly positive-definite (i.e. there is a positive number α such that $\sum_{k=1}^{r}\zeta_k(t)\zeta_k^{\mathrm T}(t)>\alpha I$). However, this is a difficult pre-condition for many systems. Further studies are necessary in order to tune $\hat f(t)$ and $\hat g(t)$ to ensure convergence and realize (21). However, in the case when the original system is multiple-input and single-output, $\hat g(t)$ and hence $\hat h(t)$) becomes a scalar. This means that the vector $\sigma(t)$ consists of only one element. As a result, the output error equation (46) becomes

$$\sigma_1(t)=\theta^{\mathrm T}\zeta_1(t)+v_0, \quad (55)$$

which is in the same form as (32) of Section 4 without $\delta(t)$. In this case, the compact form such as the adaptive diagnostic algorithm (35) can still be used, which can guarantee the convergence of the diagnostic algorithm when $u(t)$ is persistently exciting.

Another issue is the selection of threshold λ, dead zone λ_1, and adaptive learning rates γ_1, γ_2, λ_{ij} and μ_{ij}. As for the threshold, (8d) is normally used for fault detection purpose. This means that the threshold is selected as the maximum allowable magnitude of the output error $e_0(t)$ when no fault occurs. Since δ_0 is assumed known, this threshold is completely calculable from (8d). The dead zone λ_1 is in fact clearly defined by (34), since both the transfer function (matrix for MIMO cases) $W_3(s)$ and the upper bound on the input noise δ_0 are known. Therefore the most flexible parts are the adaptive learning rates, which determine the speed of the fault diagnosis for the actuators and sensors. In general, high learning rates will lead to fast convergence, whilst at the same time resulting in adaptive diagnostic algorithms too sensitive to noises. As a result, care should be taken in selecting these parameters, and a good compromise is needed. An optimal selection of these learning rates can be found in Wang (1987).

6. *Extension to nonlinear actuators*

In the above sections, we have assumed that the actuators and sensors are all linear. However, there are many practical systems where the characteristics of actuators and sensors are nonlinear. For example, the saturation shown in Fig. 2 is exhibited by many of the hydraulic valves and electric actuators that are used widely in a number of industries.

Therefore it is necessary to extend the fault detection and diagnosis algorithms to cope with nonlinear actuators and sensors in dynamic systems. This forms the purpose of this section, where the extension to nonlinear actuators will be particularly addressed.

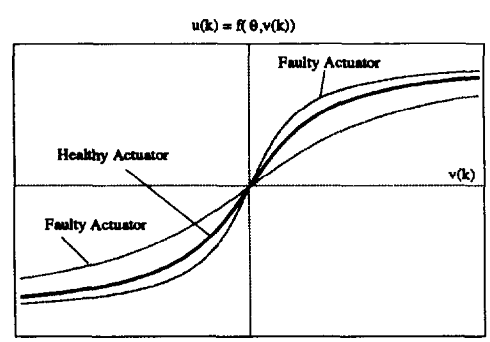

Fig. 2. Saturation in actuators and sensors.

System representation. To simplify the presentation, the system considered is expressed by

$$\dot{x}(t) = Ax(t) + Bf(\theta(t), u(t)) + v(t), \tag{56a}$$

$$y(t) = Cx(t), \tag{56b}$$

where $f(\theta, u(t))$ ($f: \mathbb{R}^{p+m} \mapsto \mathbb{R}^m$) is an $m \times 1$ known vector mapping representing the characteristics of the nonlinear actuator and $\theta \in \mathbb{R}^p$ is a parameter vector, which is assumed to change unexpectedly when faults occur in the actuator. Moreover, it is still assumed that the pair (A, C) is observable and that, when no fault occurs, the equality

$$\theta(t) = \theta_H \tag{57}$$

holds, where θ_H is a known vector and the subscript H stands for the healthy actuator. The purpose of fault detection and diagnosis is therefore to generate an alarm signal when a fault occurs and produce an accurate estimate of the vector θ that defines the faulty behaviour of the actuator.

Fault detection. Similarly to the detection observer in Section 2, the following nonlinear observer is constructed:

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bf(\theta_H, u(t)) + L_d[\hat{y}(t) - y(t)], \tag{58a}$$

$$\hat{y}(t) = C\hat{x}(t), \tag{58b}$$

where $\hat{x}(t) \in \mathbb{R}^n$ is again the state vector of the observer and $\hat{y}(t) \in \mathbb{R}^m$ is the output vector of the observer. Since it has been assumed that the pair (A, C) is observable, $L_d \in \mathbb{R}^{n \times m}$ can be selected such that $A + L_dC$ is a stable matrix. Define

$$e(t) = \hat{x}(t) - x(t), \tag{59a}$$

$$e_0(t) = \hat{y}(t) - y(t) = Ce(t). \tag{59b}$$

Then the observation error becomes

$$\dot{e}(t) = (A + L_dC)e(t) + B[f(\theta_H, u(t)) - f(\theta(t), u(t))] - v(t). \tag{60}$$

By taking $\|e_0(t)\|$ as a detection signal, the fault detection can be readily carried out in a similar way as has been discussed in Section 2.

Fault diagnosis. To diagnosis the fault, the observer

$$\dot{x}_m(t) = Ax_m(t) + Bf(\hat{\theta}(t), u(t)) + L[Cx_m(t) - y(t)] \tag{61}$$

is constructed, where $x_m(t) \in \mathbb{R}^n$ is the observer state vector and $\hat{\theta}(t)$ is an estimate of $\theta(t)$. The value of $\hat{\theta}(t)$ is set to θ_H until a fault is detected. It is assumed that after $t \geq t_f$ (or $\|e_0(t)\| > \lambda$ in (9)) a fault occurs such that

$$\theta(t) = \theta = \text{constant} \neq \theta_H. \tag{62}$$

Denote

$$e_m(t) = x_m(t) - x(t), \tag{63}$$

$$\varepsilon(t) = Cx_m(t) - y(t) = Ce_m(t). \tag{64}$$

Then it can again be obtained that

$$\dot{e}_m(t) = (A + LC)e_m(t) + B[f(\hat{\theta}(t), u(t)) - f(\theta, u(t))] - v(t) \quad (t > t_f). \tag{65}$$

As a result, the purpose of fault diagnosis is to find a diagnostic algorithm for $\hat{\theta}(t)$ such that

$$\lim_{t\to\infty} \varepsilon(t) = 0, \quad \lim_{t\to\infty} \hat{\theta}(t) = \theta \tag{66}$$

Again, (64) and (65) are in a similar form to the error equation between the reference model and the adjustable part of the system found in model reference adaptive control (Narendra and Annaswamy, 1989). As a result, the following theorem can be obtained, which produces a convergent adaptive diagnostic algorithm for estimating the fault vector θ.

Theorem 2. Suppose that

(i) the gain matrix L for the adaptive observer (61) can be selected such that the following conditions are satisfied:

$$(A + LC)^T P + P(A + LC) = -Q, \tag{67}$$

$$C = B^T P, \tag{68}$$

where P and Q are two positive-definite matrices;

(ii) $f(\theta, u(t))$ is continuous and first-order differentiable;

(iii) there is a known positive function $K_1(u(t)) > 0$ and a positive known number K_2 such that for any θ_1, $\theta_2 \in \mathbb{R}^p$ the inequalities

$$\left\| f(\theta_1, u(t)) - f(\theta_2, u(t)) - \left.\frac{\partial f(\theta, u(t))}{\partial \theta}\right|_{\theta=\theta_1} (\theta_1 - \theta_2) \right\| \leq K_1(u(t)), \tag{69}$$

$$\|\theta\| \leq K_2 \tag{70}$$

hold, where $\partial f(\theta, u(t))/\partial\theta$ is the Jacobian matrix of $f(\theta, u(t))$ with respect to θ.

Then the adaptive diagnostic algorithm

$$\frac{d\hat{\theta}(t)}{dt} = \begin{cases} -\sigma\hat{\theta}(t) - \left[\dfrac{\partial f(\hat{\theta}(t), u(t))}{\partial \hat{\theta}(t)}\right]^T \varepsilon(t) & (\varepsilon(t), \hat{\theta}(t) \in D_R), \\ -\left[\dfrac{\partial f(\hat{\theta}(t), u(t))}{\partial \hat{\theta}(t)}\right]^T \varepsilon(t) & (\varepsilon(t), \hat{\theta}(t) \in \bar{D}_R) \end{cases} \tag{71}$$

guarantees that, within a finite time period, the variables $(\varepsilon(t), \hat{\theta}(t))$ converge to the set $\bar{D}_R$ exponentially at a rate greater than $e^{-\delta t}$, where

$$\delta = \min\left\{\frac{\lambda_{\min}(Q)}{2\lambda_{\max}(P)}, \sigma\right\}, \tag{72}$$

$$D_R = \left\{(\varepsilon(t), \hat{\theta}(t)) \;\middle|\; \frac{\lambda_{\min}(P)}{\|C\|^2}\|\varepsilon(t)\|^2 + \frac{1}{2}\|\hat{\theta}(t)\|^2 > K_2^2 + \frac{1}{\delta}[\lambda_{\min}(Q)\,\|Q^{-1}P\|^2 \times [\|B\|^2K_1^2(u(t)) + \delta_0^2] + \sigma K_2^2]\right\}, \tag{73}$$

$$\bar{D}_R = \left\{(\varepsilon(t), \hat{\theta}(t) \;\middle|\; \frac{\lambda_{\min}(P)}{\|C\|^2}\|\varepsilon(t)\|^2 + \frac{1}{2}\|\hat{\theta}(t)\|^2 \leq K_2^2 + \frac{1}{\delta}[\lambda_{\min}(Q)\,\|Q^{-1}P\|^2 \times [\|B\|\,K_1^2(u(t)) + \delta_0^2] + \sigma K_2^2]\right\}, \tag{74}$$

and $\lambda_{\min}(Z)$ and $\lambda_{\min}(Z)$ are for the minimum and maximum eigenvalues of the positive-definite symmetric matrix Z.

Proof. See the Appendix.

7. *Simulation examples*

The system to be considered in the simulation is second order and is given by

$$\dot{x}(t) = \begin{bmatrix} 0.65 & -2.45 \\ 0.3 & -0.9 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} f(t)u(t) + \begin{bmatrix} 0.01\sin(23.6t) \\ -0.02 \end{bmatrix}, \tag{75}$$

$$y(t) = g(t)[-3.5 \quad 5.5]x(t) + g(t)f(t)u(t). \tag{76}$$

It can be seen that in this case $\delta_0 = 0.0224$ and the system is observable. It is assumed that the healthy actuator and sensor have the gains $f_H = 1$ and $g_H = 1$. For this system, the observer gains are selected as

$$L_d = L = -\begin{bmatrix}1\\2\end{bmatrix}, \tag{77}$$

which makes $(A + LC)$ stable, with two real eigenvalues, -1.35 and -6.39. As a result, the following transfer functions can be obtained:

$$W_1(s) = \frac{s^2 + 5.75s + 5.15}{s^2 + 7.75s + 8.65}, \tag{78}$$

$$W_2(s) = \frac{s^2 + 0.25s + 0.15}{s^2 + 7.75s + 8.65}, \tag{79}$$

$$W_3(s) = \left[-\frac{3.5s + 1.5}{s^2 + 7.75s + 8.65} \quad \frac{5.5s + 5}{s^2 + 7.75s + 8.65} \right]. \tag{80}$$

Using these transfer functions, it can be shown that the threshold and the size of dead zones are

$$\lambda = \lambda_1 = 0.0194. \tag{81}$$

To illustrate the use of the algorithm, faults are created as follows:

$$f(t) = \begin{cases} 1 & (t < 7\,\mathrm{s}), \\ 0.4 & (t \geq 7\,\mathrm{s}), \end{cases} \tag{82}$$

$$g(t) = \begin{cases} 1 & (t < 20\,\mathrm{s}), \\ 1.08 & (t \geq 20\,\mathrm{s}). \end{cases} \tag{83}$$

With $\Gamma = \mathrm{diag}\,(30, 2.5)$ and the sampling period being 0.01, the simulation results are shown in Figs 3–6, where the system is subjected to a step input at $t = 0$. It can be seen that both $\hat{f}(t)$ and $\hat{g}(t)$ can track the behaviour of the actuator and sensor faults in a desired manner.

Although many actuator and sensor faults lead to an abrupt change in gain, in practice the faults can also cause the gain to drift. Such faults are extremely difficult to detect immediately from a simple visual inspection of the output signals. To simulate this phenomena, it is assumed that the faulty actuator and sensor for the system (75), (76) are given by

$$f(t) = \begin{cases} f_H = 1 & (t \leq 7\,\mathrm{s}), \\ 1 - 0.05(t - 7) & (t > 7\,\mathrm{s}), \\ 0.375 + 0.05(t - 20) & (t > 20\,\mathrm{s}), \end{cases} \tag{84}$$

$$g(t) = \begin{cases} 1 & (t \leq 7\,\mathrm{s}), \\ 1 - 0.01(t - 7) & (t > 7\,\mathrm{s}), \\ 0.87 + 0.01(t - 20) & (t \geq 20\,\mathrm{s}). \end{cases} \tag{85}$$

Because the adaptive diagnostic algorithm (35) can only follow constant or slowly time-varying faults, the driftings (84) and (85) are made very slow in order to maintain our assumptions (13) and (14). However, to increase the tracking ability of the adaptive diagnostic algorithm (35), the learning rate for the tuning of $\hat{g}(t)$ is modified to $\gamma_2 = 10$. Figures 7–10 show the responses for both detection and diagnosis for the system (75), (76) subjected to (84) and (85). The drifting gain faults in both the actuator and sensors can again be diagnosed, and $(\hat{f}(t), \hat{g}(t))$ follows $(f(t), g(t))$ with the desired accuracy.

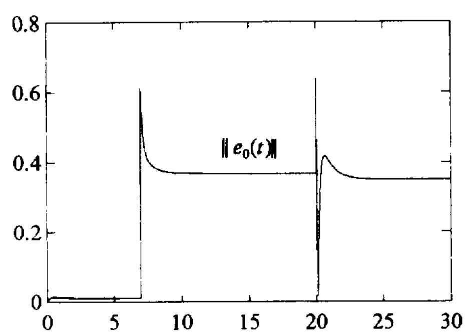

Fig. 3. Detection signal.

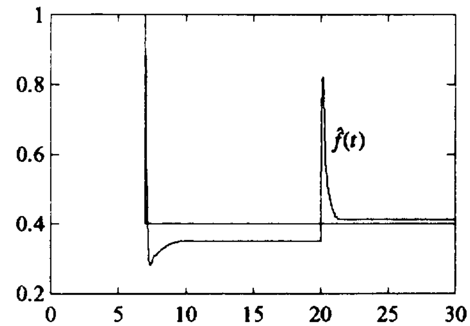

Fig. 4. Fault diagnosis for $f(t)$.

As for nonlinear actuators, we reconsider the system with

$$\dot{x}(t) = \begin{bmatrix} 0.65 & -2.45 \\ 0.3 & -0.9 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \tan^{-1}[\theta(t)u(t)] + \begin{bmatrix} 0.01\sin(23.6t) \\ -0.02 \end{bmatrix}, \tag{86}$$

$$y(t) = [-3.5 \quad 5.5]x(t), \tag{87}$$

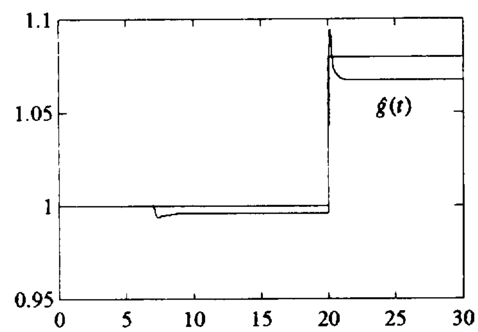

Fig. 5. Fault diagnosis for $g(t)$.

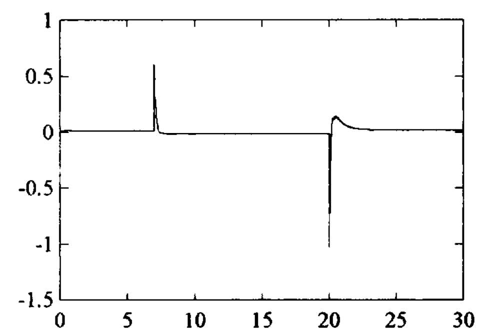

Fig. 6. The augmented error $\varepsilon_1(t)$.

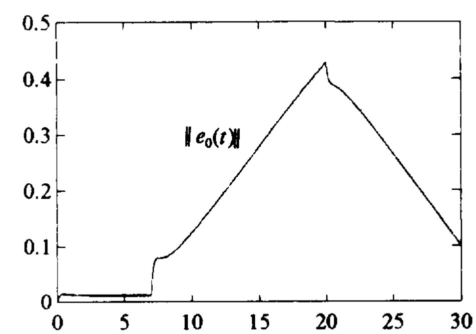

Fig. 7. Detection signal.

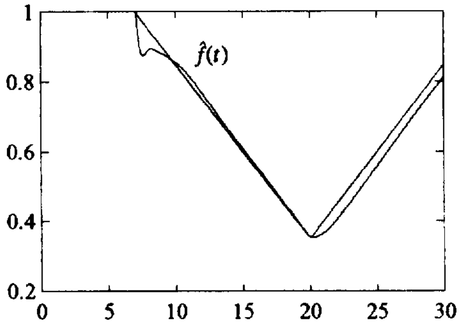

Fig. 8. Fault diagnosis for $f(t)$.

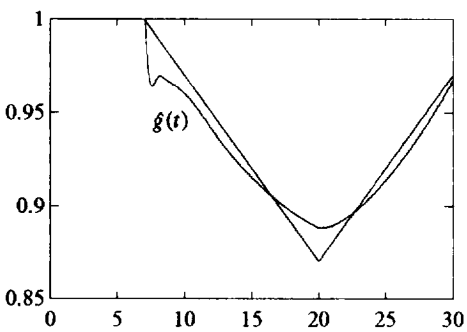

Fig. 9. Fault diagnosis for $g(t)$.

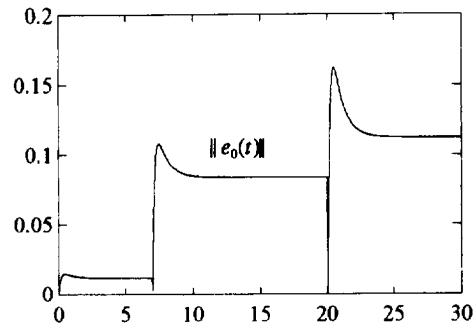

Fig. 11. Detection signal.

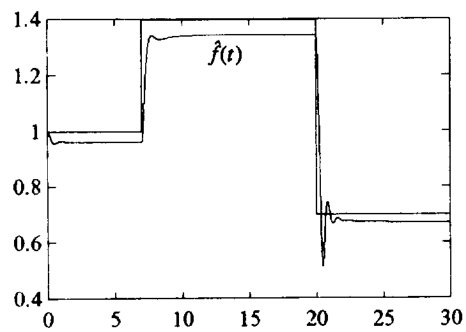

Fig. 12. Fault diagnosis.

where the nonlinear actuator is characterized by $f(\theta(t), u(t)) = \tan^{-1}[\theta(t)u(t)]$. It can be seen that the gain matrices in (77) makes the triple $(A + LC, B, C)$ strictly positive real, with

$$P = \begin{bmatrix} 3 & -3.5 \\ -3.5 & 5.5 \end{bmatrix}, \quad Q = \begin{bmatrix} 26.2 & -43.4250 \\ -43.425 & 75.25 \end{bmatrix}. \tag{88}$$

Moreover, with the choice of the nonlinear actuator in (86), it is assumed that $0.5 \le \|\theta\| \le 2$. Using the well-known Cauchy theorem, it can be shown that

$$f(u_1, u(t)) - f(\theta_2, u(t)) = \left.\frac{\partial f(\theta, u(t))}{\partial \theta}\right|_{\theta=\bar{\theta}} (\theta_1 - \theta_2), \tag{89}$$

where θ_1 and θ_2 are two arbitrary numbers in the closed interval $[0.5, 2]$, $\bar{\theta}$ is related to θ_1 and θ_2 and is also in $[0.5, 2]$. As a result, the following inequality can be formulated:

$$\begin{aligned} &\left| \tan^{-1}[\theta_1 u(t)] - \tan^{-1}[\theta_2 u(t)] - \left.\frac{\partial \tan^{-1}[\theta u(t)]}{\partial \theta}\right|_{\theta=\theta_1} (\theta_1 - \theta_2) \right| \\ &= \left| \frac{u(t)}{1 + [\bar{\theta} u(t)]^2} - \frac{u(t)}{1 + [\theta_1 u(t)]^2} \right| |\theta_1 - \theta_2| \\ &\le \frac{2|u(t)|(|\theta_1| + |\theta_2|)}{1 + \min(|\theta_1|, |\theta_2|)^2 u(t)^2} \le \frac{4|u(t)|}{1 + 0.25u(t)^2} \le \frac{16}{|u(t)|} = K_1(u(t)). \end{aligned} \tag{90}$$

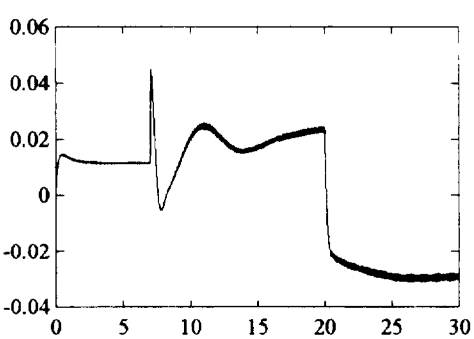

Fig. 10. The augmented error $\varepsilon_1(t)$.

In the simulation, the healthy actuator has $\theta_H = 1$, and the fault is created as follows:

$$\theta(t) = \begin{cases} 1 & (t > 7\,\text{s}), \\ 1.4 & (t \ge 7\,\text{s}), \\ 0.7 & (t > 20\,\text{s}). \end{cases} \tag{91}$$

Based on the principles outlined in Section 6, $K_2 = 2$ and $\sigma = 1$. The simulation results are shown in Figs 11 and 12, where the threshold is again equal to 0.0194. It can be seen that $\hat{\theta}(t)$ tracks the faulty actuator behaviour in a desired manner.

8. *Discussion and conclusions*

In this paper we have developed an adaptive algorithm for fault detection and diagnosis of actuators and sensors in deterministic dynamic systems. Two observers—a fixed detection observer and an adaptive diagnostic observer—were constructed separately to detect and diagnose the fault. A new form of observation error equation for the adaptive diagnostic observer was formulated and used to construct an adaptive diagnostic algorithm, which can be obtained directly via the use of the augmented error signal technique from model reference adaptive control. Single-input and single-output systems were first considered, and it was shown that a compact form of adaptive diagnostic algorithm with dead zones can be obtained. Extension to arbitrary multi-input and multi-output systems was then considered, and a gradient descent rule in the continuous-time domain was used to construct an adaptive diagnostic algorithm. A further extension to nonlinear actuators was performed via the use of the same fault detection and diagnosis for linear actuators and sensors. Two simulated examples have been included to demonstrate the applicability of the proposed methods and encouraging results have been obtained.

A bounded single term $v(t)$ has been used to represent model uncertainties and input and output noises through the dynamic equivalent principle. However, the model uncertainties in matrices (A, B, C, D) will lead to the following equations for the system:

$$\begin{aligned} \dot{x}(t) &= (A + \Delta A)x(t) + (B + \Delta B)f(t)u(t) + v(t), \\ y(t) &= g(t)[(C + \Delta C)x(t) + (D + \Delta D)f(t)u(t)]. \end{aligned}$$

In this case, similar error equations to (19) and (20) can still be obtained, but the unmeasurable term will include (ΔAx, $\Delta B\bar{f}u$, $\bar{g}\Delta Cx$, $\bar{g}\Delta D\bar{f}u$) as part of its component. As a result, the assumption on the boundness of the unmeasurable term no longer holds. Two alternative approaches may therefore be used to treat this case. The first is to use normalized signals in combination with dead zones (Narendra and Annaswamy, 1989) in the design of an adaptive diagnostic algorithm. The other approach is to choose the observer gain matrix L such that error equation is made insensitive to these model uncertainties. This can be achieved via the use of parametric design in robust fault diagnosis observers (Wang *et al.*, 1993).

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Appendix—Proof of Theorem 2

Since in the nonlinear adaptive observer (61), $\hat{\theta}(t) = \theta_H$ until a fault is detected (see (62)), the adaptive diagnostic algorithm (71) is only utilised after $t > t_f$, where, based upon the assumption (62), $\theta(t) = \theta$. Therefore, when $t > t_f$, the following Lyapunov function can be chosen for the system (64), (65) and (71):

$$V(e_m(t), \hat{\theta}(t)) = e_m^T P e_m + \|\hat{\theta}(t) - \theta\|^2. \tag{A.1}$$

It can be shown that

$$\begin{aligned}\dot{V}(e_m(t), \hat{f}(t)) &= -e_m(t)^T Q e_m(t) + 2e_m(t)^T PB[f(\hat{\theta}(t), u(t)) \\ &\quad - f(\theta, u(t))] + 2[\hat{\theta}(t) - \theta]^T \frac{d\hat{\theta}(t)}{dt} + 2e_m(t)^T P v(t). \end{aligned}\tag{A.2}$$

Using assumptions (ii) and (iii) of the theorem, it can be seen that

$$f(\hat{\theta}(t), u(t)) - f(\theta, u(t)) = \left.\frac{\partial f(\theta, u(t))}{\partial \theta}\right|_{\theta = \hat{\theta}(t)} [\hat{\theta}(t) - \theta] + \Delta, \tag{A.3}$$

where the term Δ satisfies $\|\Delta\| \le K_1(u(t))$. Using (71), it can be obtained that

$$\begin{aligned}\dot{V}(e_m(t), \hat{\theta}(t)) &= -e_m(t)^T Q e_m(t) \\ &\quad + 2e_m(t)^T PB\left[\frac{\partial f(\hat{\theta}(t), u(t))}{\partial \hat{\theta}(t)} [\hat{\theta}(t) - \theta] + \Delta\right] \\ &\quad + 2[\theta(t) - \theta]^T \frac{d\hat{\theta}(t)}{dt} + 2e_m(t)^T P v(t) = -e_m(t)^T Q e_m(t) \\ &\quad + 2e_m(t)^T P[B\Delta + v(t)] - 2\sigma[\hat{\theta}(t) - \theta]^T \hat{\theta}(t). \end{aligned}\tag{A.4}$$

Using assumption (iii) of the theorem, it can be shown that

$$\begin{aligned}\dot{V}(e_m(t), \hat{\theta}(t)) &= -[e_m(t) + Q^{-1}P(B\Delta + v)]^T Q[e_m(t) \\ &\quad + Q^{-1}P(B\Delta + v)] + (B\Delta + v)^T P Q^{-1} P(B\Delta + v) \\ &\quad - 2\sigma[\hat{\theta}(t) - \theta]^T[\hat{\theta}(t) - \theta] - 2\sigma[\theta(t) - \theta]^T \theta \\ &\le -\delta V(e_m(t), \hat{\theta}(t)) \\ &\quad + \lambda_{\min}(Q) \|Q^{-1}P\|^2[\|B\|^2 K_1^2(u(t)) + \delta_0^2] + \sigma K_2^2 \end{aligned}\tag{A.5}$$

On the other hand, it can be seen that the following equality holds when $(\varepsilon(t), \hat{\theta}(t)) \in D_R$:

$$V(e_m(t), \hat{\theta}(t)) \geq \lambda_{min}(P) \|e_m(t)\|^2 + \frac{1}{2}\|\hat{\theta}(t)\|^2 - \|\theta\|^2 \geq \frac{\lambda_{min}(P)}{\|C\|^2}\|\varepsilon(t)\|^2 + \frac{1}{2}\|\hat{\theta}(t)\|^2 - K_2^2 \geq \frac{1}{\delta}\{\lambda_{min}(Q)\|Q^{-1}P\|^2[\|B\|^2 K_1^2(u(t)) + \delta_0^2] + \sigma K_2^2\} \quad (A.6)$$

Using the inequality (A.5) and the definition of D_R, it can be seen that $\dot{V}(e_m(t), \hat{\theta}(t)) < 0$ $(\varepsilon(t), \hat{\theta}(t)) \in D_R)$. As a result, the dynamic system represented by (64), (65) and (71) is stable. Since the inequality (A.5) holds for all $(\varepsilon(t), \hat{\theta}(t)) \in D_R$, the pair $(e_m(t), \hat{\theta}(t))$ are therefore uniformly bounded and converge to D_R exponentially at a rate greater than $e^{-\delta t}$.

Denote by $m_L\{t \mid (\varepsilon(t), \hat{\theta}(t)) \in D_R\}$ the Lebesgue measure of the time period when the pair $(\varepsilon(t), \hat{\theta}(t))$ are inside D_R. Then it can be shown that

$$m_L\{t \mid (\varepsilon(t), \hat{\theta}(t)) \in D_R\} = \int_{t_f}^{t} \frac{dV(e_m(t), \hat{\theta}(t))}{\dot{V}(e_m(t), \hat{\theta}(t))} \leq \int_{t_f}^{t} \frac{|dV(e_m(t), \hat{\theta}(t))|}{|\dot{V}(e_m(t), \hat{\theta}(t)|} \leq \int_{f}^{t} \frac{|dV(e_m(t), \hat{\theta}(t))|}{\delta\,|V(e_m(t)\hat{\theta}(t)) - E_0^2|} = -\frac{1}{\delta}\log\left|\frac{\delta V(e_m(t), \hat{\theta}(t)) - E_0^2}{\delta V(e_m(t_f), \hat{\theta}(t_f)) - E_0^2}\right| < +\infty \quad (A.8)$$

when $(\varepsilon(t), \hat{\theta}(t)) \in D_R$, where

$$E_0^2 = \lambda_{min}(Q)\|Q^{-1}P\|^2[\|B\|^2 K_1^2(u(t)) + \delta_0^2] + \sigma K_2^2 \quad (A.9)$$

□