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PCA-based detection of damage in time-varying systems

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ABSTRACT

When performing Structural Health Monitoring, it is well known that the natural frequencies do not depend only on the damage but also on environmental conditions, such as temperature and humidity. The Principal Component Analysis is used to take this problem into account, because it allows eliminating the effect of external factors. The purpose of the present work is to show that this technique can be successfully used not only for time-invariant systems, but also for time-varying ones. Referring to the latter, one of the most studied systems which shows these characteristics is the bridge with crossing loads, such as the case of the railway bridge studied in present paper; in this case, the mass and the velocity of the train can be considered as "environmental" factors.This paper, after a brief description of the PCA method and one example of its application on time-invariant systems, presents the great potentialities of the methodology when applied to time-varying systems. The results show that this method is able to better detect the presence of damage and also to properly distinguish among different levels of crack depths.

1. Introduction

In the last years, the detection of damage in structures has been one of the most important issues in civil and mechanical engineering.

It is well known how changes in environmental conditions can affect structural vibration properties. Some field tests have found that the variations of structural vibration properties due to the changing environmental conditions (e.g. temperature, humidity, boundary conditions) are very significant, and are often more important with respect to those due to rather severe structural damage. Moreover, modal parameters are influenced in different ways by the temperature: whilst it influences quite largely the natural frequencies, it has a very small effect on mode shapes and on damping factors [1].

Sohn et al. [2] studied the effect of the temperature on the Alamosa Canyon Bridge, and they presented an adaptive filter that accommodates the changes in temperature to the damage detection system of a large-scale bridge. They also noted that the variation of the frequencies grows up to 6.6% in a period of 24 h.

In [3] the influence of temperature on natural frequencies is studied by defining the probability of damage existence, which is useful to give a confidence level of damage occurrence. In [4], instead, both the excitation sources and the temperature effect are studied as uncertainties added to the system, and successively an ARX model is formulated to write a dependence of natural frequencies upon the temperature and to fix confidence intervals for the damage evaluation.

The Principal Component Analysis (PCA) technique, contrarily to the method just cited, does not require to measure environmental parameters because they are taken into account as embedded variables [5]. The main idea of the method is

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to eliminate the contribution due to environmental conditions by removing linear correlations among the data. This fact implies the compression of the data by reducing the number of dimensions: they are represented in terms of a minimum of variables while preserving most of the information present in the data set.

The PCA is nowadays applied in a number of fields, i.e. the image processing, the modelling of turbulence and the reduced order models. As far as the structural damage is concerned, the PCA was firstly applied to normal modes, examining the relationship between them and the proper orthogonal modes [6]. Successively, the PCA has been extended to be applied to different system characteristics, usually called "features" [5].

Since the PCA is basically a linear tool, it is not able to treat nonlinear cases, i.e. cases where the relationship between couples of features is not linear. For this reason, some modifications of the algorithm, as for example the local PCA, have been proposed [7,8]. The main idea of these works is the splitting of the features in different clusters [9], and then the application of the standard PCA to each ensemble.

The PCA method is usually coupled with the discipline of Novelty Detection, developed during the last years, where the main issue is to identify, from measured data, whether or not a machine or a structure has deviated from a normal condition, i.e. if the data is novel [11,12]. This technique is based on the calculation of a distance between data coming from a healthy reference case and data coming from a new acquisition. The most used distances are the Euclidean norm and the Mahalanobis norm. The second one should be preferred because it takes into account the correlation between all the data [10].

The purpose of the present paper is to show the great potential of the PCA method applied to Linear Time-Invariant (LTI) systems and to Linear Time-Varying (LTV) systems. In time-varying systems, the conventional definition of modal parameters is no more valid [13,14] because they are changing instant by instant.

The main idea of the paper is to show that the PCA method can be successfully applied, without any alteration, to LTV systems. This is possible because the method is not sensitive to the type of features considered; it only compares a new set of data with the reference one.

The application presented concerns the transit of a moving mass over a beam, which simulates a train crossing a railway bridge. In this case the natural frequencies of the system are influenced both by the mass weight and its speed, and these effects can be seen as environmental factors. This issue will be discussed deeply in the article because, for the application of the PCA method, it is necessary that the correlation among the features is linear or almost linear. The choice of the features is another fundamental point in this analysis; since the modal parameters are identified in many time instants, only one mode can be considered sufficient to build up the features.

The paper starts with a mathematical presentation of the PCA method. Then, an experimental example on LTI systems and a numerical one on LTV systems are proposed. The aim of the paper is to show the versatility and the reliability of the method, even in very different situations. Just two limitations are envisaged for the present method: the correct applicability restricted to linear and weakly nonlinear cases and its capability of detecting but not localizing the damage.

2. Principal Component Analysis

The Principal Component Analysis is also known as proper orthogonal decomposition [5]. Consider a matrix $Y \in \Re^{n \times N}$, called features matrix, containing the n features of N simulations. The method allows writing a matrix $X \in \Re^{m \times N}$, called scores matrix, with $m < n$, representing the reduced order matrix of Y. The transformation can be written as

$$X = TY \tag{1}$$

where the matrix $T \in \Re^{m \times n}$ is called loading matrix. The linear mapping of Eq. (1) is representative of the reduction of the problem order from n to m: this value corresponds physically to the number of environmental conditions that affect considerably the structure. It is important to notice that the number of selected features must be always greater than the number of environmental conditions.

The matrix T is calculated by the application of the singular value decomposition (SVD), computed from the covariance matrix of the features:

$$YY^T = USU^T \tag{2}$$

where U is an orthogonal matrix, i.e. $UU^T = I$ and the matrix S contains the singular values. The loading matrix T is built using the first m columns of U. The number m can be either chosen a priori or selected after the SVD, by inspecting the singular value plot.

The definition

$$\hat{Y} = T^T X = T^T T Y \tag{3}$$

allows firstly to project the original data in the m-dimensional space, and then to re-map the data back to the original space. The loss of information due to this procedure can be evaluated by calculating the residual error matrix $E = Y - \hat{Y}$.

Two types of novelty index can be now introduced: the Euclidean norm and the Mahalanobis norm. The second must be preferred because it takes into account the inverse of the correlation matrix of the features $R=(1/N)YY^T$:

$$NI_k^M=\sqrt{E_k^T R^{-1} E_k} \tag{4}$$

where the subscript k means that the value has been obtained at time t_k.

Since these norms are separately calculated for all the available data, it is possible to perform a statistical analysis. In particular, the following properties can be defined:

$$\text{Mean: } M_{NI}=\overline{NI} \tag{5}$$

$$\text{Standard deviation: } \sigma_{NI}=std(NI) \tag{6}$$

$$\text{Threshold value: } V=\overline{NI}+\alpha\sigma_{NI} \tag{7}$$

The coefficient α is usually taken equal to 3, corresponding to a confidence interval of 99.7%, with the assumption of a normal distribution.

The threshold value is useful to distinguish the damaged from the undamaged case. In fact, the value is calculated using the novelty index coming from the data of the healthy structure. Then, the novelty indices referred to the data coming from a new measurement on the structure are obtained and compared with the threshold. If the values are above it damage has occurred, otherwise the structure is still undamaged.

The percentage of novelty index values that exceeds the threshold can be used as an indicator of the damage detection. The optimal situation is when this percentage is equal to zero in the undamaged case (p_U=0) and when it is equal to one in the damaged case (p_D=1).

3. Experimental example

The experimental application was conducted in the Laboratory of the Mechanical Department, where a clamped-free beam was mounted (see Fig. 1) in a controlled temperature chamber. The characteristics of the beam are summarised in Table 1.

The measurements were conducted at twelve different temperatures, as listed in Table 2.

Each temperature has been maintained constant for 75 min to allow the test rig stabilizing its temperature, except for the $-35\,^{\circ}$C case when 90 min were fixed.

The force has been applied to the beam by a contactless magnetic transducer Brüel & Kjær MM0004 (55 mm from the clamping) and the velocity has been measured by a similar transducer (150 mm away from the clamped edge). To limit the mechanical noise on the measured signal, the environmental chamber was set in stand-by mode during all the signal acquisitions. The excitation was a linear sine sweep from 0.5 to 550 Hz and lasted 4.5 min, while the sampling frequency

Fig. 1. The beam considered.

Table 1
Experimental application: beam properties.

Length	L=0.260 m
Width	h=0.001 m
Depth	b=0.030 m
Mass	M=0.060 kg

Table 2
List of temperatures.

Temperature (°C)											
−35	−20	0	20	40	60	80	100	120	140	160	175

Table 3
Cases analyzed for the experimental application.

File	Scenario	Added mass position (m)
1	Healthy	–
2	Healthy	–
3	Added mass, m=0.8 g	0.26
4	Added mass, m=0.8 g	0.10
5	Added mass, m=0.67 g	0.26
6	Added mass, m=0.67 g	0.13
7	Added mass, m=0.67 g	0.10
8	Reduction of the clamping stiffness	

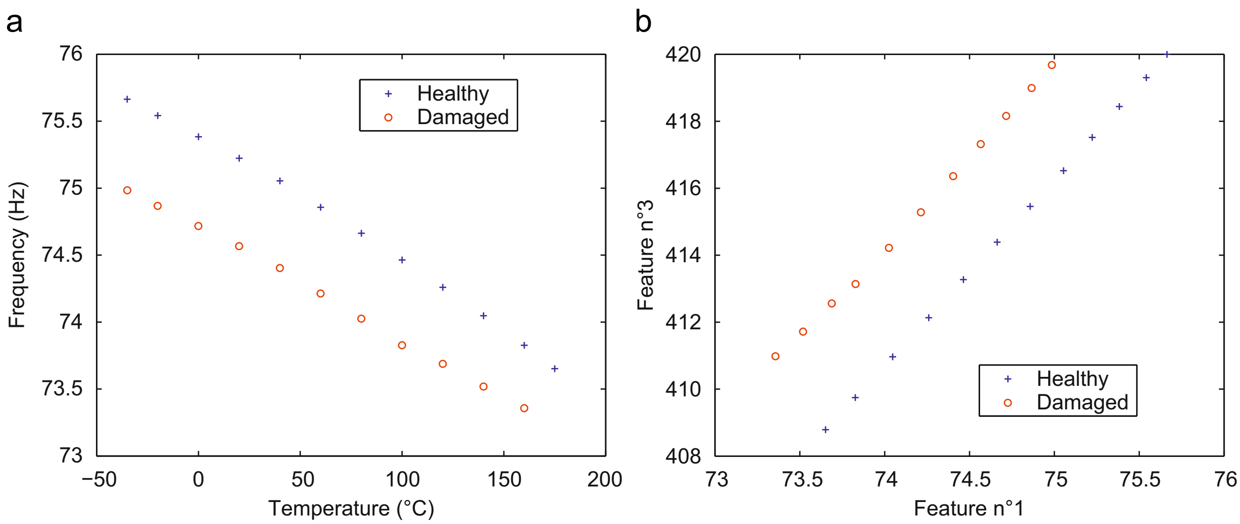

Fig. 2. Second frequency as function of the temperature (a) and first feature against the third one (b), case 7.

was fixed to 4096 Hz. The natural frequencies are extracted in the frequency domain by a least square fitting of the auto-spectrum of the output signal. The features considered are the second, the third and the fourth natural frequency, because, due to the adopted frequency resolution and the low damping level, the first mode resonance was not well defined.

In order to reproduce a structural modification, a little mass has been added (cases 3–7) or the stiffness of the clamped edge has been modified (case 8) by reducing the screwing torque (see Fig. 1). The cases analyzed are presented in Table 3.

The second natural frequency as function of the temperature is depicted in Fig. 2a, referred to case 7; in Fig. 2b, the relation between the first and the third feature is shown, for the same case.

The ranges of variation of the second frequencies are very similar in the healthy and in the damaged case. The linearity among the features indicates that the beam is composed by a homogeneous material. When this supposition holds, the ratio of two different natural frequencies of the system is always constant, even if the dependence of the natural frequencies on the temperature is not linear. The PCA method, therefore, is used when the correlation between two different frequencies is linear or almost linear, while a different approach such as the local PCA must be adopted in all the other cases [7].

The results obtained by applying PCA to all cases are visualized in Fig. 3. In the figure, as usually in all the images representing novelty detection results, different lines are present: the mean M_{NI} and the standard deviation σ_{NI} (both over and under the mean value) of each ensemble, and the threshold value.

Not all the files have the same number of measures because in some cases the acquisition failed. The results are very clear and they allow to distinguish the different levels of “damage”, which are dependent upon the position and the weight of the added mass. In particular, the largest values of the Mahalanobis norm are reached when the mass is located at the free end of the beam.

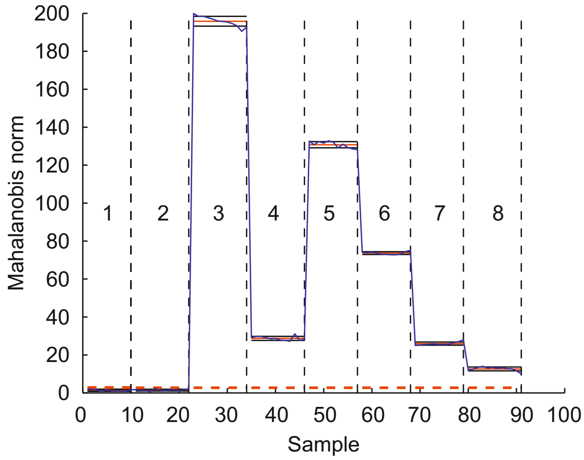

Fig. 3. Experimental application: novelty index for the different cases.

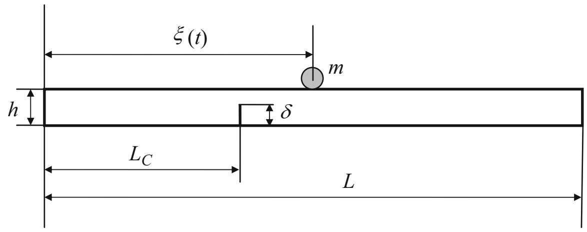

Fig. 4. Damaged beam subjected to a moving mass.

4. Application on a LTV system

The LTV systems have been the object of many studies and approaches in recent years. The main difference with respect to the LTI systems is the variation of the modal parameters at each time instant. The time-varying frequencies, indeed, are identified in many time instants and therefore much information about the system behaviour is available. When using the PCA method, the use of a large number of features is advisable in order to obtain better results.

A non-negligible time-varying phenomenon can be observed during the passage of a train on a railway bridge: the train transit gives origin to some parameter modifications, due to its mass which is somehow comparable with that of the bridge [13].

In this section, a damaged beam model, with a load passing over it, is used to test the capabilities of the PCA methodology in finding the presence of damage.

4.1. The model of cracked beam

Let us consider a simply supported Euler–Bernoulli beam of length L, travelled by a mass m with constant velocity v (Fig. 4). The crack is located at $x=L_c$, with relative depth $\gamma=\delta/h$, where δ is the depth of the crack.

The equation of motion is

$$\rho A\frac{\partial^2 w}{\partial t^2}(x,t)+EI\frac{\partial^4 w}{\partial x^4}(x,t)=-m(a_v+g)X(t)\delta(x-\xi(t)) \tag{8}$$

where $w(x, t)$ is the vertical deflection of the beam; ρ, E, A and I are the mass density, Young's modulus of the material, the cross-sectional area and the area moment of inertia, respectively, all supposed constant on the whole length of the beam; a_v is the vertical acceleration of the moving mass. Moreover, $\xi(t)=vt$ is the instantaneous position of the mass along the beam, $\delta(x)$ is the Dirac delta function and $X(t)$ is a window function, defined by unit step functions u in the following manner:

$$X(t)=u(t)-u\left(t-\frac{l}{v}\right) \tag{9}$$

Assuming the stiffness of the moving system infinite and neglecting the possibility that the mass may separate from the beam, after some mathematical manipulations [15], the equation of motion can be written in the following form:

$$[M(t)]\{\ddot{q}(t)\}+[C(t)]\{\dot{q}(t)\}+[K(t)]\{q(t)\}=g\{\tau(t)\} \tag{10}$$

$$w(x,t)=\sum_{i=1}^{n}\phi_i(x)q_i(t) \tag{11}$$

$$[M(t)]=[I]_{n\times n}+mX(t)\{\boldsymbol{\Phi}(\xi)\}\{\boldsymbol{\Phi}(\xi)\}^T \tag{12}$$

$$[C(t)]=2mvX(t)\{\boldsymbol{\Phi}(\xi)\}\{\boldsymbol{\Phi}^I(\xi)\}^T \tag{13}$$

$$[K(t)]=[\omega^2]+mv^2X(t)\{\boldsymbol{\Phi}(\xi)\}\{\boldsymbol{\Phi}^{II}(\xi)\}^T \tag{14}$$

$$\{\tau(t)\}=-mX(t)\{\boldsymbol{\Phi}(\xi)\} \tag{15}$$

Where $[\omega^2]$ is the diagonal matrix containing the square of the first n angular frequencies of the beam, $\{\Phi(\xi)\}$ the mass-normalized mode shapes ϕ_i of the beam, $\{q(t)\}$ the modal coordinates and g the gravity acceleration.

The crack is modelled as a rotational spring with stiffness K_t, linked to γ by the following relation [16]:

$$K_t=\frac{EI}{6h\pi\gamma^2 f(\gamma)} \tag{16}$$

where $f(\gamma)=0.638-1.035\gamma+3.720\gamma^2-5.173\gamma^3+7.553\gamma^4-7.332\gamma^5+2.491\gamma^6$.

The time histories are generated by integrating Eq. (10) and the output-only identification is performed with the Short Time Stochastic Subspace Identification (ST-SSI) method [13]. It is based on the frozen technique, consisting of the classical LTI identification proposed by Van Overschee and De Moor [17], but performed in different windows, in which the whole time period is divided.

4.2. Effect of the environmental conditions

In the LTI example, the temperature was the unique environmental condition considered. In the case of a train crossing a railway bridge, the mass and the velocity of the moving load can be considered as "environmental" conditions, because, when analyzing real data, usually the measurements of the train mass and velocity are not performed: their effect on the natural frequencies is relevant but it is unknown [18]. The PCA method is useful to eliminate the dependence on these factors. The temperature, for simplicity, is not taken into account in this example. The characteristics of the beam considered are shown in Table 4.

4.2.1. Analytical approach

The sensitivity of the frequencies with respect to mass and velocity is studied in this subsection by using an analytical approach based on the first modal approximation of the undamaged beam. By using the first mode, mass and stiffness matrices, from Eqs. (12) and (14), reduce to scalars:

$$M=1+m\left(\frac{\sin((\pi/L)\xi)}{\sqrt{(1/2)\mu L}}\right)^2 \tag{17}$$

$$K=\omega_{1,I}^2-\frac{\pi^2}{L^2}mv^2\left(\frac{\sin((\pi/L)\xi)}{\sqrt{(1/2)\boldsymbol{\mu}L}}\right)^2 \tag{18}$$

where $\mu=\rho A$, $\omega_{1,I}$ is the first angular frequency of the beam and ξ is the position in which the modal shape is evaluated. After some simplifications, these scalars become

$$M_{L_1}=1+\frac{2m}{\mu L}\sin\left(\frac{\pi}{L}\xi\right)^2 \tag{19}$$

Table 4
LTV system: characteristics of the beam considered.

Young's modulus	E	70 GPa
Mass density	ρ	2982 kg/m^3
Beam length	L	1.86 m
Beam width	b	0.15 m
Beam depth	h	0.015 m
Crack location	L_c	0.73 m

$$K_{L_1} = 4\pi^2 \left(f_{1,I}^2 - \frac{1}{4L^2} \frac{2m}{\mu L} v^2 \sin\left(\frac{\pi}{L}\xi\right)^2 \right) \tag{20}$$

The first natural frequency in L_1 is written as

$$f_{L_1} = \frac{1}{2\pi}\sqrt{\frac{K_{L_1}}{M_{L_1}}} = \sqrt{\frac{f_{1,I}^2 - (1/4L^2)(2m/\mu L)v^2 \sin((\pi/L)L_1)^2}{1+(2m/\mu L)\sin((\pi/L)L_1)^2}} \tag{21}$$

The effect of the velocity on natural frequencies can be neglected if

$$v \ll \frac{2Lf_{1,I}}{\sqrt{2m/\mu L}} \frac{1}{\sin((\pi/L)L_1)} \tag{22}$$

as usually occurs, since $\mu L > 2m$ (the mass of the bridge is greater than twice the mass of the train).

4.2.2. Mass effect

The effect of the mass is very clear: it can be easily confused with the presence of damage, since it reduces the natural frequencies.

In Fig. 5, four features ($L_1 = L/4$, $L_2 = L/2$, $L_3 = L/3$, $L_4 = 3L/4$) extracted from a numerical simulation, and built with the use of four modes but no added noise, are depicted. The range of variation of the mass is equal to [1.5, 2.5] kg, while the velocity is fixed at $v = 0.1$ m/s.

It has been verified that, for this case, the analytical method provides results very close to the numerical one (the relative error is about 0.02%).

Within the chosen range, all the features are almost linearly dependent on the first one, and the regression lines highlight this behaviour, as shown in Fig. 5.

4.2.3. Velocity effect

In Fig. 6, four features ($L_1 = L/4$, $L_2 = L/2$, $L_3 = L/3$, $L_4 = 3L/4$) extracted from a numerical simulation, and built with the use of four modes but no added noise, are depicted. The mass is equal to $m = 2$ kg, while the velocity v has been chosen in the interval [0.1, 10] m/s, that corresponds to $t_{cross} = 18.60$ and $t_{cross} = 0.186$ s, respectively.

The results of the analytical method are in good agreement with those of the numerical method: the relative error is about 0.01%.

The choice of a large range of velocities shows that this environmental condition does not affect very much the features: the effect induced for realistic velocities is practically negligible (see Eq. (22)).

4.3. Choice of the features

The feature choice is a fundamental issue in the damage detection, especially when using the PCA method. The main idea is to select some points along the first natural frequency evolution graph. Actually this approach could fail when analyzing data originated by trains travelling at different velocities: since the sampling frequency is usually constant, then the number of points in the extracted first frequencies varies with the train speed. Moreover, there could be little identification errors, due to the output-only analysis. In order to solve these two problems, a least square regression is

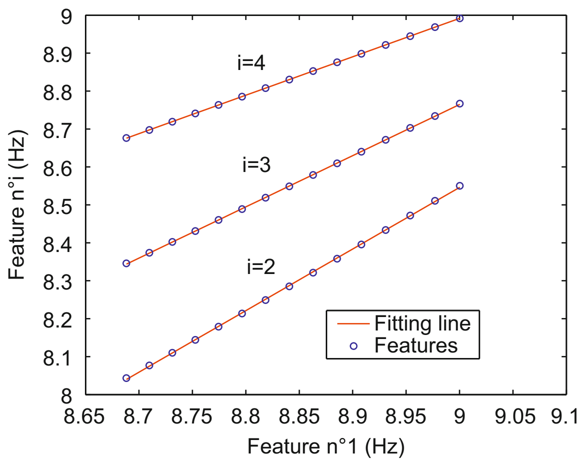

Fig. 5. Second feature as function of the first one, $L_1 = L/4$, $L_2 = L/2$, $L_3 = L/3$, $L_4 = 3L/4$, m in [1.5, 2.5] kg, $v = 0.1$ m/s.

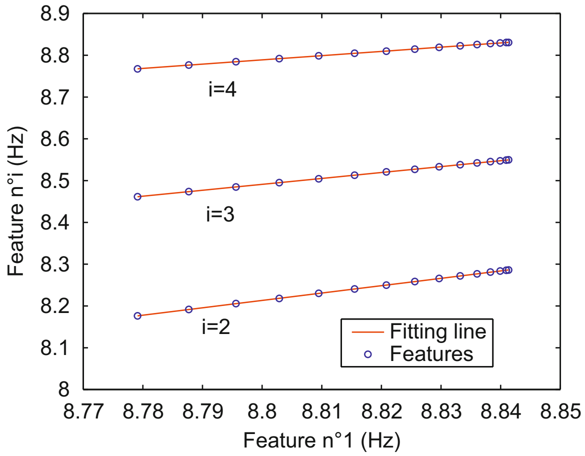

Fig. 6. Second feature as function of the first one, $L_1=L/4$, $L_2=L/2$, $L_3=L/3$, $L_4=3L/4$, $m=2$ kg, v in [0.1, 10] m/s.

Table 5
List of the cases analyzed.

	Range of mass (kg)	Range of velocity (m/s)	L_c (m)	γ
Healthy case 1 (He)	From 1.5 to 2.5	From 0.15 to 0.3	–	0
Damaged case (D1)	From 1.5 to 2.5	From 0.15 to 0.3	0.73	0.7
Damaged case (D2)	From 1.5 to 2.5	From 0.15 to 0.3	0.73	0.5
Damaged case (D3)	From 1.5 to 2.5	From 0.15 to 0.3	0.73	0.3

applied, with a high-order polynomial, and the first and the last part of the frequency are eliminated to reduce the boundary effects.

The main advantage with respect to the LTI case is the possibility of choosing a higher number of features, which are different values of a certain eigenfrequency. Usually the first frequency is the most suitable choice because, when analyzing real train-bridge systems, it is the most excited mode and can be identified quite easily. If higher frequencies can be extracted from the identification process, new sets of features can be chosen and consequently other damage detections may be carried on. As a consequence, this procedure is useful to compare the different results and to reduce false alarms and missed alarms.

The choice of the natural frequencies as basis for the feature selection is explained by the fact that damping factors and mode shapes are more difficult to estimate with high precision.

The features are inserted in the PCA method, no matter if they are coming from LTI or a LTV system. The most important aspect is the linear correlation among the features, which is not trivial for an LTV system, where the mass is considered as environmental condition. It has been previously shown that when the range of the mass travelling on the beam is not too large, the correlation between couples of features is almost linear and consequently the PCA can be applied. The threshold value, in order to distinguish damaged and undamaged cases, is chosen analogously to the LTI case, so by taking the coefficient α equal to 3.

4.4. Application of the method

The PCA algorithm is applied to cases listed in Table 5, both with 0% and 3% of Gaussian noise added to the accelerations. The masses and the velocities are chosen within a uniform random distribution in the specified range.

In Fig. 7 the results of the Mahalanobis norm are depicted with two principal components and ten features along the first time-varying frequency.

The novelty index increases as the relative crack depth grows, as expected. In particular, when $\gamma=0.3$ the method fails with 3% noise, while it is quite satisfying without noise. The main reasons are the use of the output-only identification (no external measured force is applied) and the presence of two environmental conditions. When $\gamma=0.5$, the damage is clearly found even with the largest noise. The case in which $\gamma=0.7$ is presented to put in evidence that the PCA is not only able to detect the damage but also to give an indication about its severity, producing an increasing value of the mean index when the crack depth increases.

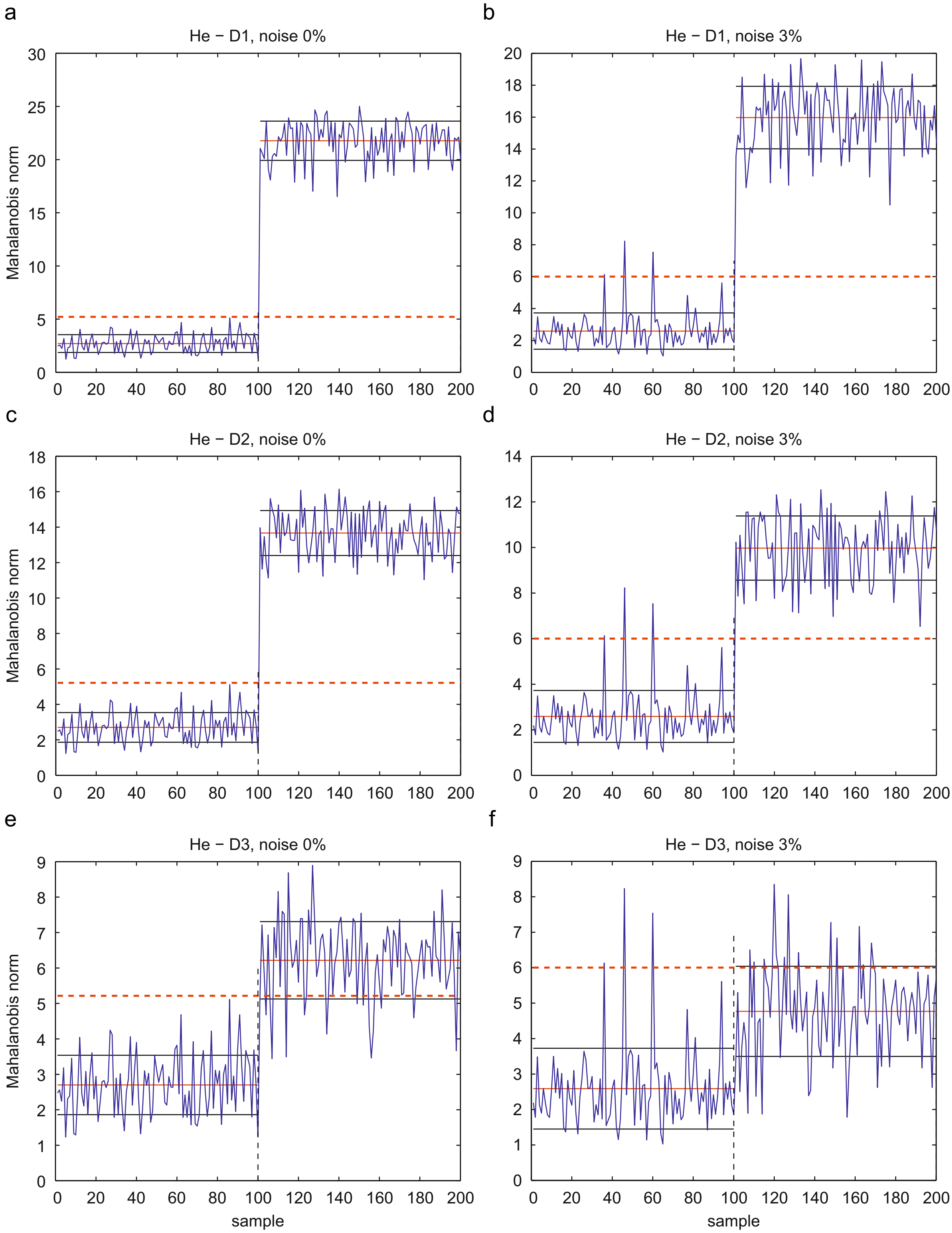

Fig. 7. LTV system: novelty index for the different analyses.

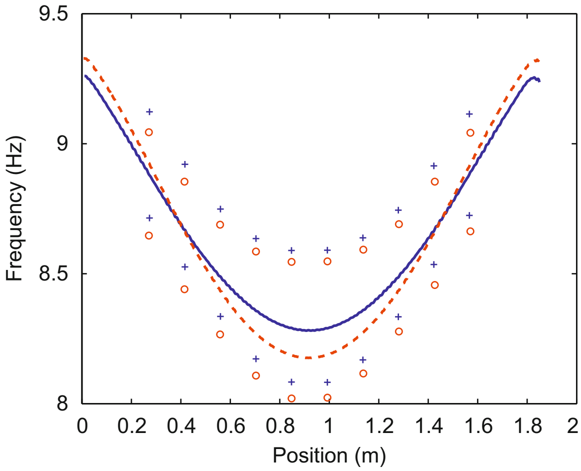

Fig. 8. LTV system: range of variation of the features of the healthy case ("+") and of the damaged case ("o") and two examples of natural frequencies: healthy (solid line) and damaged (dashed line).

The three anomalous values of the novelty index in the healthy case with 3% noise are due to irregularities in the first natural frequency, depending on the output-only identification method.

Fig. 8, for the healthy case (He) and the last damaged case (D3), shows the trends of the first natural frequencies extracted from the simulations that produce the novelty index closest to the mean of the respective ensemble. Moreover, Fig. 8 shows the variation ranges of the features, which are not so different for the two cases analyzed. The presence of the crack does not modify too much the features, but the PCA method is able to detect the damage because the important characteristic is not the frequency lowering (that could also be due to the mass variation) but the loss of its symmetry.

On the basis of these considerations, it is possible to understand that the frequency is influenced by four characteristics:

1. the load mass,
2. the load crossing speed,
3. the position of the crack,
4. the depth of the crack.

The third element is responsible for the variation of the symmetry, because it causes a time shift of the minimum of the frequency [13].

5. Conclusion

The main purpose of the PCA method is the elimination of the effect produced by the environmental conditions that influence the structure examined. Usually, the most relevant factor is the temperature. When analyzing particular types of Time-Varying systems, as for example the railway bridge crossed by a train mentioned in the paper, there exist other elements to be considered as environmental conditions, in particular the mass and the velocity, since they are usually not measured. The natural frequencies, which change with time, depend on the mass and the velocity of the load, but also on the position and on the depth of the crack. The PCA method is demonstrated to be useful in the removal of the first two effects, making it possible to detect the damage. In particular, the identified natural frequencies have been subjected to a regression process, in order to obtain a smoother trend to be used for the feature selection.

For the damage selection process, there are many advantages in analyzing the Time-Varying systems, with respect to the Time-Invariant ones:

- The modal parameters contain much information about the systems, because there is a different value for each time instant.
- Only one mode is necessary to build the feature matrix: this is a fundamental benefit, since usually in practical problems the first mode is the most excited and consequently correctly identified when using an output-only procedure.
- If it is possible, the identification of other frequencies is useful to improve the results, bearing in mind that higher modes are more affected by the presence of damage [13].

As the analysis demonstrates, there are some limits in the detection of the crack, since a possible wide mass range could alter the linear correlation among the features, while realistic velocities do not have a great influence on the features. Moreover, the method is able to distinguish the different levels of damage.

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