Spike-Timing Dependent Plasticity

What are they? How do they work?

Birla Institute of Technology and Science
K.K. Birla Goa Campus
Department of Computer Science & Information Systems

Param Biyani

Supervised by
Prof. Basabdatta Sen Bhattacharya

Hebbian Learning

Spike-Timing Dependent Plasticity (STDP) is an unsupervised biological learning method for SNNs, based on **Hebbian Learning** rules [1]. Hebb's Rule -

When an axon of cell A is near enough to excite cell B and repeatedly or persistently takes part in firing it, some growth process or metabolic change takes place in one or both cells such that A's efficiency, as one of the cells firing B, is increased.

The idea is that if A's firing is responsible for B's firing, then the ability of A's firing to trigger B's firing will increase. The synapse is characterized by a parameter w_{ij} , called the **synaptic weight**, or the *synaptic efficacy*. In the case where a presynaptic spike is followed closely by a postsynaptic spike, then it is presumed that the presynaptic neuron caused the spike in the postsynaptic neuron, and so the weight of the synapse between the neurons is increased. This is known as potentiation.

If a postsynaptic spike is emitted shortly before a presynaptic spike is emitted, then the presynaptic spike cannot have caused the postsynaptic spike, and so the weight of the synapse between the neurons is reduced. This is known as depression.

Appropriate stimulation paradigms can induce changes of the postsynaptic response that last for hours or days. If the stimulation paradigm leads to a persistent increase of the synaptic efficacy, the effect is called long-term potentiation (LTP) of synapses. If the result is a decrease of the synaptic efficacy, it is called long-term depression (LTD).

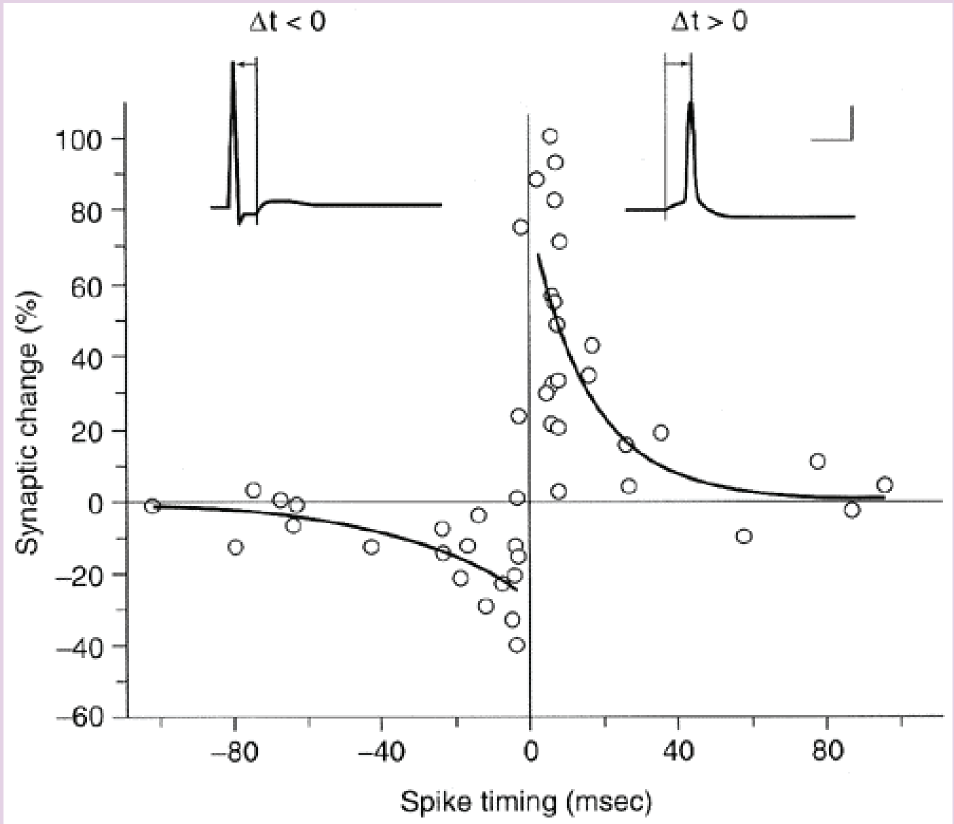
The idea is that modifications of the synaptic weights are driven by correlations in the firing activity of pre- and postsynaptic neurons. In order to find a mathematically formulated learning rule based on Hebb's postulate we focus on a single synapse with weight w_{ij} that transmits signals from a presynaptic neuron *j* to a postsynaptic neuron *i*.

$$\frac{\mathrm{d}}{\mathrm{d}\ t} w_{ij} = F\left(w_{ij}; \nu_i, \nu_j\right)$$

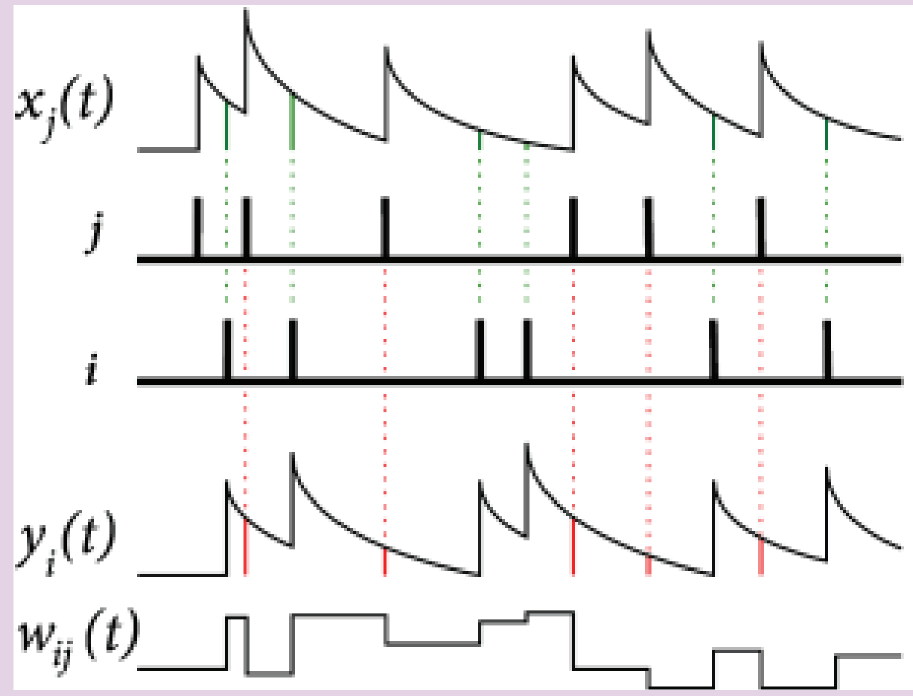
The general formula for change in synaptic efficacy. Here *F* is the undetermined function. The activity of the presynaptic neuron is denoted by v_j and that of the postsynaptic neuron by v_i.

The resulting change in the synaptic efficacy Δw_{ij} after several repetitions of the experiment turns out to be a function of the difference $t_j(f) - t_i(f)$ between the firing times of pre- and post-synaptic neuron. This observation has given rise to the term *'Spike-Timing-Dependent Plasticity' (STDP)*. Bi and Poo measured the changes in synaptic efficacy induced in the synapses of hippocampal neurons by pairs of pre and postsynaptic spikes with different relative timings. The relationship between the magnitude of these changes and the relative timing of the pre and postsynaptic spikes is known as STDP.

Pair-Based Models of STDP



The timing requirements between pre- and postsynaptic spikes. Experimentally measured weight changes (circles) as a function of $t_j(f) - t_i(f)$ in milliseconds overlayed on a schematic two-phase learning window (solid line). Observe how the shape of the two curves aren't similar, STDPs are temporally assymetrical.



Implementation of pair-based plasticity by local variables: The presynaptic spikes leave a trace $x_j(t)$, postsynaptic spikes a trace $y_i(t)$. The weight increases at the moment of a postsynaptic spike proportional to the momentary value of the trace $x_j(t)$ left by previous presynaptic spike arrivals. Analogously we get depression for post-before-pre pairings at the moment of a presynaptic spike (vertical dashed lines highlight moments of spike firing) [1].

The data recorded by Bi and Poo, 1998 [2] suggests that the magnitude of the changes in synaptic efficacy (Δw_{ij}) is related to the relative spike timings with the following exponential functions:

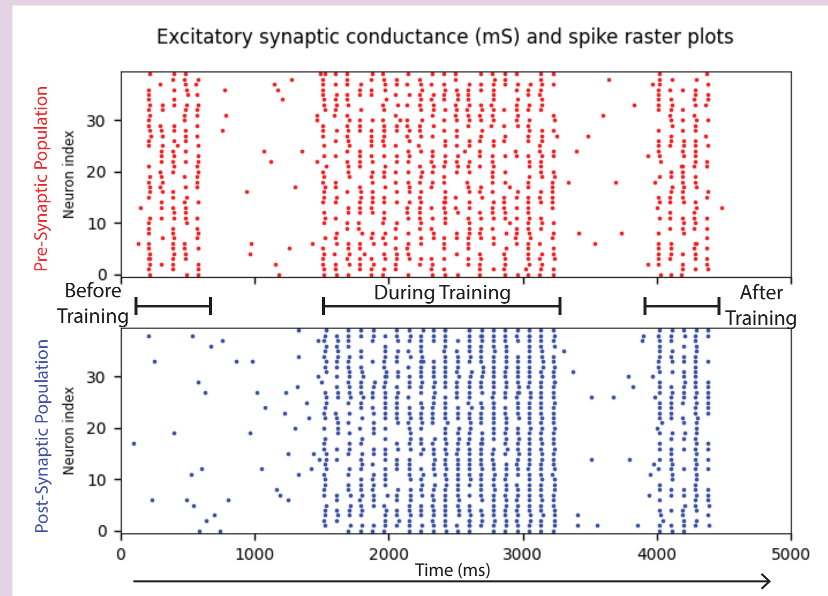
$$\Delta w_+ = A_+(w) \cdot \exp(-|\Delta t|/\tau_+) \quad \text{at} \quad t_{\text{post}} \quad \text{for} \quad t_{\text{pre}} < t_{\text{post}}$$
$$\Delta w_- = A_-(w) \cdot \exp(-|\Delta t|/\tau_-) \quad \text{at} \quad t_{\text{pre}} \quad \text{for} \quad t_{\text{pre}} > t_{\text{post}}$$

Where $\Delta t = t_j - t_i$ represents the relative timing of pre and post-synaptic spikes, $\tau_\pm$ defines the time constant of the exponentials and the $A_\pm$ functions define how the magnitude of the change in weight depends on the current synaptic efficacy [3]. We consider two update rules here:

1. Additive weight dependence, where $A_\pm$ are constant values, independent of w. Seperate values for A_+ and A_- denotes a possible asymmetry between increasing and decreasing the synaptic efficacy.
2. Multiplicative weight dependence, where

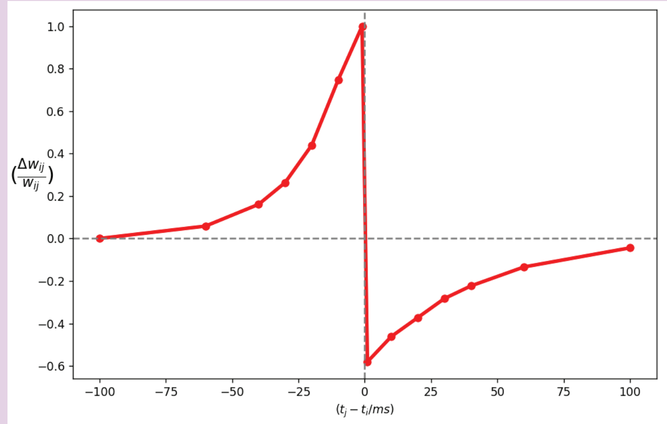
$$A_+ = \lambda_+(1-w),\ A_- = \lambda\_w$$

STDP spike raster on SpiNNaker



Spike raster plot showing the various phases of STDP training and the correlation established between a pre and postsynaptic neuron populations. At the beginning, the stimulation to pre-synaptic neurons does not trigger a response in post-synaptic neurons. Then during the training, both the populations are given a stimulus with a time difference, $t_{post} - t_{pre}$ of 10 ms. After that, only the pre-synaptic population is given a stimulus which now generates a response in the postsynaptic population implying successful training by STDP [5].

STDP Curves on SpiNNaker



$\tau_+ = 20$
$\tau_- = 40$
$A_+ = 0.01$
$A_- = 0.005$
$w_{max} = 1$
$w_{min} = 0$

The STDP characteristic curve

Both additive and multiplicative weight change rules are implemented on SpiNNaker. Figure above shows the STDP curves generated using additive weight dependence rules, with the given parameters. While the time window is smaller, the change in weight is larger. $|\Delta w_{ij}|$ keeps decreasing as $|t_i - t_j|$ increases. Also, the potentiation side of the graph has higher change in weight for the same time difference for the depression side, however, it has a much smaller window of change.

Unsupervised Learning

Here, it is also important to note that Hebbian learning is *unsupervised*, because there is no notion of 'good' or 'bad' changes of a synapse. Synaptic changes happen whenever there is joint activity of pre- and postsynaptic neurons, i.e., they are driven by the neuronal firing patterns. These patterns may reflect sensory stimulation as well as ongoing brain activity, but there is no feedback signal from a 'supervisor' or from the environment.

In artificial neural networks some, or even all, neurons receive input from external sources as well as from other neurons in the network. Inputs from external sources are typically described as a statistical ensemble of potential stimuli. Unsupervised learning in the field of artificial neural networks refers to changes of synaptic connections which are driven by the statistics of the input stimuli – in contrast to supervised learning or reward-based learning where the network parameters are optimized to achieve, for each stimulus, an optimal behavior. Hebbian learning rules, as introduced in the previous section, are the prime example of unsupervised learning in artificial neural networks.

SpiNNaker

SpiNNaker, short for 'Spiking Neural Network Architecture', is a many core, massively parallel neuromorphic computer used for simulating large-scale SNNs in wall-clock time at 1 ms resolution [7]. It is designed by the Advanced Processor Technologies Research Group (APT) at the Department of Computer Science, University of Manchester, and consists of over 1 million cores, where each core is a very-low-power ARM968 processor. SpiNNaker's low-power consumption and scalability as compared to conventional supercomputers and GPUs makes it a promising platform for computing biologically realistic large-scale SNNs. For running simulations on the platform, one needs to write code via the SpiNNaker toolchain, called the sPyNNaker API, and is implemented using the PyNN library. The Ebrains Project Portal provides an efficient access to the million-core SpiNNaker server at Manchester.

References

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[3] Song S, Miller KD, Abbott LF. Competitive Hebbian learning through spike-timing-dependent synaptic plasticity. Nat Neurosci. 2000 Sep;3(9):919-26. doi: 10.1038/78829. PMID: 10966623.

[4] James Knight (2017). Plasticity in large-scale neuromorphic models of the neocortex. PhD Thesis, University of Manchester.

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