Chapter 2: Fuzzy Logic

2.1 Fuzzy Sets

Fuzzy sets are a fundamental concept in fuzzy logic, a mathematical framework for dealing with uncertainty and imprecision in decision-making. Unlike classical sets, where an element either belongs to the set (membership degree = 1) or does not belong to the set (membership degree = 0), fuzzy sets allow for partial membership degrees between 0 and 1, representing the degree of belongingness of an element to the set.

Fuzzy sets provide a flexible and intuitive way to represent and reason with vague or imprecise concepts, such as "tallness" or "hotness," in a wide range of applications, including control systems, decision support systems, and pattern recognition. By allowing for partial membership degrees, fuzzy sets capture the inherent uncertainty and subjectivity present in many real-world scenarios, making fuzzy logic a valuable tool for dealing with complex and uncertain information.

Components of Fuzzy Sets:

- Membership Function: A function that maps each element of the universe of discourse to a membership degree between 0 and 1, indicating the extent to which the element belongs to the fuzzy set.
- Universe of Discourse: The set of all possible values or elements that the fuzzy set can take.

Example of Fuzzy Sets:

Let's consider the example of a fuzzy set representing the concept of "tallness" in humans. In classical set theory, a person is either tall or not tall, with a clear boundary between the two categories. However, in reality, the distinction between tall and not tall is subjective and depends on individual perceptions and cultural norms.

- Universe of Discourse: The universe of discourse for the fuzzy set "tallness" could be the set of heights of human beings, ranging from very short to very tall.
- Membership Function: The membership function for the fuzzy set "tallness" assigns membership degrees to each height value in the universe of discourse, indicating the degree of tallness of a person.

Example Membership Function:

- Very Short: μ(Height) = 0 for heights less than 150 cm.

- Short: μ(Height) = 0.2 for heights between 150 cm and 160 cm.

- Medium: μ(Height) = 0.5 for heights between 160 cm and 170 cm.

- Tall: μ(Height) = 0.8 for heights between 170 cm and 180 cm.

- Very Tall: μ(Height) = 1 for heights greater than 180 cm.

Interpretation:

- If a person's height is 165 cm, their membership degree in the fuzzy set "tallness" would be 0.5, indicating that they are moderately tall.

- If a person's height is 185 cm, their membership degree in the fuzzy set "tallness" would be 1, indicating that they are very tall.

1.1 Operations on Fuzzy Sets

Fuzzy sets support various operations similar to classical sets, but with a different interpretation due to their fuzzy nature. There are some basic operations on fuzzy sets, each providing a different way to combine or manipulate fuzzy information. These operations enable fuzzy logic to handle uncertainty and vagueness in a wide range of applications, including control systems, decision-making, and pattern recognition. Here are some common operations on fuzzy sets:

- Union (Max): The union of two fuzzy sets A and B is another fuzzy set where the membership degree of each element is the maximum of the corresponding membership degrees in sets A and B. Let's consider two fuzzy sets representing the concepts of "tall" and "young" in a group of people.

  - Fuzzy set A (Tall):

  - $\mu(A) = \{0.1/150, 0.5/160, 0.8/170, 0.6/180, 0.3/190\}$

  - Fuzzy set B (Young):

  - $\mu(B) = \{0.9/20, 0.7/25, 0.4/30, 0.1/35, 0.0/40\}$

  The union of A and B ($A \cup B$) would be:

  - $\mu(A \cup B) = \{\max(0.1, 0.9)/150, \max(0.5, 0.7)/160, \max(0.8, 0.4)/170, \max(0.6, 0.1)/180, \max(0.3, 0.0)/190\}$

- $\mu(A \cup B) = \{0.9/150, 0.7/160, 0.8/170, 0.6/180, 0.3/190\}$

- Intersection (Min): The intersection of two fuzzy sets A and B is another fuzzy set where the membership degree of each element is the minimum of the corresponding membership degrees in sets A and B. Continuing from the previous example:

  The intersection of A and B ($A \cap B$) would be:

  - $\mu(A \cap B) = \{\min(0.1, 0.9)/150, \min(0.5, 0.7)/160, \min(0.8, 0.4)/170, \min(0.6, 0.1)/180, \min(0.3, 0.0)/190\}$

- $\mu(A \cap B) = \{0.1/150, 0.5/160, 0.4/170, 0.1/180, 0.0/190\}$

- Complement: The complement of a fuzzy set A is another fuzzy set where the membership degree of each element is 1 minus the membership degree of the corresponding element in set A. Let's use the fuzzy set A (Tall) from the previous example:

  The complement of A (A') would be:

  - $\mu(A') = \{1 - 0.1/150, 1 - 0.5/160, 1 - 0.8/170, 1 - 0.6/180, 1 - 0.3/190\}$

- $\mu(A') = \{0.9/150, 0.5/160, 0.2/170, 0.4/180, 0.7/190\}$

- Difference: The difference between two fuzzy sets A and B is another fuzzy set where the membership degree of each element is the minimum of the membership degree of the corresponding element in set A and the complement of the membership degree of the corresponding element in set B. Continuing from the previous example:

  The difference between A and B (A - B) would be:

  - $\mu(A - B) = \{\min(0.1, 0.1')/150, \min(0.5, 0.5')/160, \min(0.8, 0.6')/170, \min(0.6, 0.9')/180, \min(0.3, 1')/190\}$

  - $\mu(A - B) = \{0.0/150, 0.0/160, 0.2/170, 0.0/180, 0.0/190\}$

2.2 Fuzzy Relations

Fuzzy relations extend the concept of crisp (or classical) relations to handle fuzzy sets, allowing for the representation of imprecise or uncertain relationships between elements of two or more sets. Fuzzy relations are widely used in fields such as control systems, decision-making, and pattern recognition.

Fuzzy relations provide a flexible framework for representing and reasoning about imprecise or uncertain relationships between elements of sets. By allowing for partial membership degrees, fuzzy relations capture the inherent complexity and ambiguity present in many real-world scenarios, making them a powerful tool for modeling and analyzing fuzzy information in various applications. Here are some common types of fuzzy relations along with simple examples:

- Fuzzy Equivalence Relations: A fuzzy equivalence relation on a set A is reflexive, symmetric, and transitive. Let's consider a fuzzy relation "similarity" between movies based on their genre. The membership degree in the "similarity" relation between two movies represents how similar their genres are. For instance, if Movie A and Movie B both belong to the genres "Action" and "Adventure," they might have a high membership degree in the "similarity" relation.

- Fuzzy Partial Ordering Relations: A fuzzy partial ordering relation on a set A is reflexive, antisymmetric, and transitive. Suppose we have a set of fruits and a fuzzy relation "sweetness" between fruits. The membership degree in the "sweetness" relation indicates how sweet a fruit is. If Fruit A has a higher sweetness degree than Fruit B, and Fruit B has a higher sweetness degree than Fruit C, then there exists a fuzzy partial ordering relation between these fruits based on their sweetness levels.
- Fuzzy Similarity Relations: A fuzzy similarity relation on a set A measures the degree of similarity between elements of A. Consider a set of colors represented as fuzzy sets. A fuzzy similarity relation "color similarity" between two colors measures the degree of similarity between them based on attributes like hue, saturation, and brightness. For instance, two shades of blue may have a high degree of similarity, while a blue and a red may have a lower degree of similarity.
- Fuzzy Preference Relations: A fuzzy preference relation on a set A captures the degree to which one element of A is preferred over another. Suppose we have a set of restaurants and a fuzzy relation "taste preference" between restaurants based on customer reviews. The membership degree in the "taste preference" relation represents how much a customer prefers one restaurant's food over another. A restaurant with higher membership degrees in the "taste preference" relation is preferred over others.
- Fuzzy Compatibility Relations: A fuzzy compatibility relation on a set A measures the degree to which elements of A are compatible with each other. Consider a set of ingredients and a fuzzy relation "ingredient compatibility" between ingredients in a recipe. The membership degree in the "ingredient compatibility" relation indicates how well certain ingredients go together in a dish. For example, tomatoes and basil may have a high membership degree in the "ingredient compatibility" relation for making a tomato basil pasta sauce.

2.3 Membership Functions

Membership functions are a core component of fuzzy logic, representing the degree of membership or truth for each element in a fuzzy set. These functions map input values to membership degrees between 0 and 1, indicating the degree to which the input belongs to the fuzzy set.

Membership functions play a crucial role in fuzzy logic, enabling the representation of fuzzy concepts and the modeling of uncertain or imprecise information. By selecting appropriate membership functions and tuning their parameters, fuzzy systems can accurately capture the complex relationships between variables in various real-world applications. Here are some common types of membership functions along with examples:

- Triangular Membership Function: The triangular membership function is characterized by a triangular shape, with a peak at the center and linear decrease towards the edges. Let's consider a fuzzy set "Temperature" with a triangular membership function representing the concept of "Moderate Temperature." The membership function might have a peak at 25 degrees Celsius, indicating the most moderate temperature, with linear decrease towards cooler and warmer temperatures.

- Trapezoidal Membership Function: The trapezoidal membership function is similar to the triangular function but with flat edges, resulting in a trapezoidal shape. Suppose we have a fuzzy set "Speed" with a trapezoidal membership function representing the concept of "Medium Speed." The membership function might have flat edges between 40 and 60 km/h, indicating a range of speeds considered medium, with linear decrease outside this range.
- Gaussian Membership Function: The Gaussian membership function follows the shape of a Gaussian (bell) curve, with a peak at the center and symmetric decrease towards the edges. Consider a fuzzy set "Distance" with a Gaussian membership function representing the concept of "Near." The membership function might have a peak at 0 meters, indicating maximum nearness, with symmetric decrease as distance increases.
- Sigmoidal (S-Shaped) Membership Function: The sigmoidal membership function has an S-shaped curve, with gradual increase or decrease towards the center and steeper slopes towards the edges. Let's say we have a fuzzy set "Brightness" with a sigmoidal membership function representing the concept of "Medium Brightness." The membership function might have a gradual increase from 0 to 50% brightness, representing a transition from dark to medium brightness, followed by a gradual decrease from 50 to 100% brightness.
- Piecewise Linear Membership Function: The piecewise linear membership function consists of multiple linear segments joined together, allowing for more flexibility in modeling complex relationships. Suppose we have a fuzzy set "Age" with a piecewise linear membership function representing the concept of "Young Adult." The membership function might have a gradual increase from 18 to 25 years old, followed by a gradual decrease from 25 to 35 years old, capturing the fuzzy boundary between young and adult ages.

2.3.1 Features of Membership Functions:

Membership functions in fuzzy logic serve as a bridge between linguistic terms (e.g., "tall," "hot," "fast") and numerical values. They define how membership degrees are assigned to input values, indicating the degree to which an element belongs to a fuzzy set.

Membership functions are a crucial component of fuzzy logic, enabling the representation of imprecise or uncertain information in a linguistic form. By understanding the features of membership functions and selecting appropriate shapes, ranges, centers, spreads, and smoothness, fuzzy systems can accurately model and reason with complex real-world phenomena. Here are some key features of membership functions illustrated through examples:

- Shape and Range: Membership functions can have different shapes (e.g., triangular, trapezoidal, Gaussian) and ranges depending on the characteristics of the fuzzy set being represented. Consider the fuzzy set "Temperature" with a triangular membership function representing "Warm." The membership function ranges from 20 to 30 degrees Celsius, with a peak at 25 degrees Celsius.

- Center and Spread: The center of a membership function represents the peak or centroid of the fuzzy set, while the spread determines the width or variability of the membership function. For the fuzzy set "Speed" representing "Moderate Speed," the center might be at 50 km/h with a spread of 10 km/h, indicating that speeds between 45 and 55 km/h are considered moderate.
- Degree of Membership: Membership functions assign membership degrees between 0 and 1 to input values, indicating the degree to which each value belongs to the fuzzy set. In the fuzzy set "Brightness" representing "Low," an input value of 30% brightness might have a membership degree of 0.8, indicating a high degree of belongingness to the "Low" brightness category.
- Monotonicity: Membership functions can be monotonic (increasing or decreasing) or non-monotonic, depending on the nature of the linguistic term being represented. A triangular membership function representing "Young" might be monotonically increasing from 18 to 25 years old, while a trapezoidal function representing "Medium" might have a plateau in the middle.
- Smoothness: Smooth membership functions have continuous derivatives and can provide more accurate representations of fuzzy concepts. A Gaussian membership function representing "Near" might have a smooth curve, ensuring a gradual transition from maximum to minimum membership degrees as distance increases.
- Interpretability: Membership functions should be interpretable and meaningful to users, reflecting their linguistic understanding of the fuzzy concept being represented. In a fuzzy controller for room temperature, the membership functions for "Cold," "Comfortable," and "Hot" should correspond to temperature ranges that people intuitively perceive as such.

2.3.2 Methods of Membership Value Assignments

In fuzzy logic, membership value assignments determine the degree of membership of an element in a fuzzy set. These assignments can be based on various methods, each capturing different characteristics of the relationship between the input value and the fuzzy set.

Methods of membership value assignments in fuzzy logic provide flexibility and adaptability in representing fuzzy concepts and relationships between variables. By selecting appropriate methods and techniques, fuzzy systems can accurately model complex and uncertain information in various real-world applications, enabling effective decision-making and reasoning in fuzzy environments. Here are some common methods of membership value assignments along with examples:

- Linguistic Terms: Linguistic terms are descriptive labels used to represent fuzzy sets and their membership values. These terms are often defined by experts or through linguistic rules. Consider a fuzzy set "Temperature" with linguistic terms "Cold," "Warm," and "Hot." The membership value assignment for the input temperature of 25°C might be 0.8

for "Warm" and 0.2 for "Cold," indicating a high degree of membership in the "Warm" category and a lower degree of membership in the "Cold" category.

- Numerical Ranges: Numerical ranges define fuzzy sets based on specific intervals or ranges of input values, with membership values assigned according to the degree of overlap between the input value and the range. Suppose we have a fuzzy set "Speed" with numerical ranges representing "Low," "Medium," and "High" speeds. For an input speed of 55 km/h, the membership value assignment might be 0.5 for "Medium," indicating that the speed falls halfway between the "Low" and "High" ranges.
- Mathematical Functions: Mathematical functions, such as linear, exponential, or sigmoidal functions, can be used to assign membership values based on the input value's relationship with specific parameters or characteristics. Consider a fuzzy set "Brightness" with a sigmoidal membership function representing the concept of "Medium Brightness." The membership value assignment for an input brightness level of 75% might be calculated using the sigmoidal function, resulting in a membership value close to 1.
- Data-driven Approaches: Data-driven approaches use statistical techniques or machine learning algorithms to learn membership value assignments from observed data, such as training examples or historical records. In a customer segmentation task, membership value assignments for demographic attributes (e.g., age, income) might be learned from a dataset of customer profiles using clustering algorithms like k-means or hierarchical clustering.
- Expert Knowledge: Expert knowledge involves eliciting membership value assignments from domain experts or stakeholders who possess expertise in the problem domain. In medical diagnosis, a team of doctors might define membership value assignments for symptoms and diagnostic tests based on their clinical experience and knowledge of disease patterns.

2.4 Fuzzy Rules and Fuzzy Reasoning

Fuzzy rules and fuzzy reasoning are fundamental concepts in fuzzy logic, allowing systems to make decisions or draw conclusions based on fuzzy sets and fuzzy logic principles.

Fuzzy rules and reasoning provide a flexible and intuitive framework for decision-making in systems where inputs and outputs are imprecise or uncertain. By defining linguistic rules and applying fuzzy reasoning techniques, fuzzy logic systems can effectively model complex relationships and make decisions based on fuzzy input data, enabling applications in diverse domains such as control systems, pattern recognition, and decision support systems. Here's an overview of fuzzy rules and reasoning along with examples:

- Fuzzy Rules: Fuzzy rules express relationships between input variables and output variables in linguistic terms, using conditional statements that relate fuzzy sets to each other. These rules are typically of the form: "If (input is A) then (output is B)." Example is as under:

  - Rule 1: If Temperature is Cold, then Heater Setting is High.

- Rule 2: If Temperature is Warm, then Heater Setting is Medium.

- Rule 3: If Temperature is Hot, then Heater Setting is Low.

➢ Fuzzy Reasoning: Fuzzy reasoning involves applying fuzzy rules to input data to determine the appropriate output. It involves three main steps: fuzzification, rule evaluation, and defuzzification.

a. Fuzzification: Convert crisp input values into fuzzy sets using membership functions. For a temperature of 20°C, fuzzification assigns membership degrees to the fuzzy sets "Cold," "Warm," and "Hot" based on their respective membership functions.

b. Rule Evaluation: Apply fuzzy rules to the fuzzy input values to determine the degree to which each rule is satisfied. Given the input temperature of 20°C, Rule 1 (If Temperature is Cold) is satisfied to a degree of 0.6, Rule 2 (If Temperature is Warm) is satisfied to a degree of 0.8, and Rule 3 (If Temperature is Hot) is satisfied to a degree of 0.

c. Aggregation and Inference: Combine the outputs of individual rules to generate a single fuzzy output. Aggregating the outputs of the fuzzy rules yields a fuzzy output indicating the appropriate heater setting, with membership degrees assigned to fuzzy sets like "High," "Medium," and "Low."

d. Defuzzification: Convert the fuzzy output back into a crisp value using techniques such as centroid or max membership. Defuzzification determines the crisp heater setting based on the aggregated fuzzy output, resulting in a specific heater setting value (e.g., "High," "Medium," or "Low").

2.5 Fuzzy Inference System

A Fuzzy Inference System (FIS) is a computational framework that applies fuzzy logic principles to make decisions or draw conclusions based on imprecise or uncertain input data. It consists of four main components: fuzzifier, rule base, inference engine, and defuzzifier.

Fuzzy Inference Systems provide a powerful framework for decision-making in systems where inputs and outputs are imprecise or uncertain. By combining fuzzy logic principles with a rule-based approach, FIS can effectively model complex relationships and make decisions based on fuzzy input data, enabling applications in various domains such as control systems, pattern recognition, and decision support systems. Let's delve into each component and the overall process of a Fuzzy Inference System, along with examples:

➢ Fuzzifier: The fuzzifier converts crisp input data into fuzzy sets using membership functions, assigning membership degrees to each fuzzy set based on the input values. Suppose we have an input variable "Temperature" with the crisp value of 25°C. The fuzzifier assigns membership degrees to fuzzy sets like "Cold," "Warm," and "Hot" based on their respective membership functions.

- ➢ Rule Base: The rule base contains a set of fuzzy rules that express relationships between input variables and output variables in linguistic terms, using conditional statements. The example is as under:

  - Rule 1: If Temperature is Cold, then Heater Setting is High.

  - Rule 2: If Temperature is Warm, then Heater Setting is Medium.

  - Rule 3: If Temperature is Hot, then Heater Setting is Low.

- ➢ Inference Engine: The inference engine applies fuzzy rules to the fuzzy input data, determining the degree to which each rule is satisfied and combining the outputs of individual rules to generate a single fuzzy output. Given the input temperature of 25°C and the fuzzy rules, the inference engine evaluates each rule's satisfaction degree and aggregates the outputs to determine the appropriate heater setting.
- ➢ Defuzzifier: The defuzzifier converts the aggregated fuzzy output back into a crisp value, providing a specific decision or action based on the fuzzy inference. Defuzzification determines the crisp heater setting based on the aggregated fuzzy output, resulting in a specific value (e.g., "High," "Medium," or "Low") for the heater setting.

2.5.1 Overall Process of Fuzzy Inference System

- ➢ Fuzzification: Convert crisp input data into fuzzy sets using membership functions.
- ➢ Rule Evaluation: Apply fuzzy rules to the fuzzy input data to determine the degree to which each rule is satisfied
- ➢ Aggregation and Inference: Combine the outputs of individual rules to generate a single fuzzy output.
- ➢ Defuzzification: Convert the fuzzy output back into a crisp value using techniques such as centroid or max membership.

  Example Application: Consider an automatic climate control system in a car. The Fuzzy Inference System takes inputs such as outside temperature, desired temperature set by the user, and current cabin temperature. Based on fuzzy rules and input data, it determines the appropriate settings for air conditioning, fan speed, and heating, providing optimal comfort for passengers.

2.6 Fuzzy Expert System

A Fuzzy Expert System (FES) is an extension of a traditional expert system that incorporates fuzzy logic principles to handle uncertainty and imprecision in expert knowledge and decision-making. It combines domain expertise with fuzzy logic techniques to provide reasoning capabilities similar to those of human experts.

Fuzzy Expert Systems leverage fuzzy logic principles to handle uncertainty and imprecision in expert knowledge and decision-making. By combining domain expertise with fuzzy reasoning techniques, FES can provide intelligent recommendations and decisions in various applications, ranging from environmental control systems to financial forecasting and medical diagnosis. Let's explore the components and functionality of a Fuzzy Expert System, along with examples:

- Components of a Fuzzy Expert System:

  Knowledge Base: The knowledge base contains expert knowledge represented in the form of fuzzy rules, membership functions, and linguistic variables.

  Inference Engine: The inference engine applies fuzzy reasoning techniques to the knowledge base, making inferences and drawing conclusions based on fuzzy logic principles.

  User Interface: The user interface allows users to interact with the Fuzzy Expert System, providing input data and receiving output results in a user-friendly manner.

- Functionality of a Fuzzy Expert System:

  Fuzzification: Convert crisp input data into fuzzy sets using appropriate membership functions, representing linguistic variables and their degrees of membership.

  Fuzzy Inference: Apply fuzzy rules from the knowledge base to the fuzzy input data, determining the degree to which each rule is satisfied and aggregating the outputs to generate a single fuzzy output.

  Defuzzification: Convert the aggregated fuzzy output back into a crisp value or decision using defuzzification techniques, providing a specific recommendation or action based on the fuzzy inference.

- Example of a Fuzzy Expert System:

  Objective: Design a Fuzzy Expert System to control air quality in a smart building based on environmental factors such as temperature, humidity, and pollution levels.

  Components:

    Knowledge Base:

      - Fuzzy rules: IF (Temperature is High) AND (Humidity is High) THEN (Air Conditioner Setting is High)

      - Membership functions: Triangular or trapezoidal membership functions representing linguistic variables such as "High," "Medium," and "Low" for temperature, humidity, and air conditioner setting.

    Inference Engine:

- Apply fuzzy reasoning techniques to evaluate fuzzy rules based on input data.

- Determine the appropriate air conditioner setting based on environmental conditions and expert knowledge encoded in the fuzzy rules.

User Interface:

- Allow users to input environmental data such as temperature, humidity, and pollution levels.

- Display recommendations or actions generated by the Fuzzy Expert System, such as adjusting air conditioner settings or activating air purifiers.

Example Interaction:

User: "Current temperature is 30°C, humidity is 80%, and pollution level is moderate."

Fuzzy Expert System:

- Fuzzification: Convert crisp input data into fuzzy sets using appropriate membership functions.

- Fuzzy Inference: Apply fuzzy rules to determine the appropriate air conditioner setting.

- Defuzzification: Convert the aggregated fuzzy output back into a crisp value, recommending a specific air conditioner setting (e.g., "High").

User Interface: Display the recommendation to the user, indicating the recommended air conditioner setting based on the input environmental conditions.

2.7 Fuzzy Decision Making

Fuzzy decision-making is a process that allows systems to make decisions based on imprecise or uncertain input data using fuzzy logic principles. It involves defining fuzzy rules, evaluating fuzzy inputs, and applying fuzzy reasoning techniques to determine the best course of action.

Fuzzy decision-making provides a flexible and robust framework for making decisions in complex and uncertain environments. By incorporating fuzzy logic principles, systems can effectively model imprecise or uncertain input data and derive optimal decisions based on linguistic rules and fuzzy reasoning techniques. Examples of applications include autonomous vehicles, industrial control systems, and financial forecasting, where decision-making processes must account for uncertainty and variability in input data. Here's a detailed overview of fuzzy decision-making along with examples:

- Problem Formulation: Identify the decision problem and define the input variables, output variables, and fuzzy rules that govern the decision-making process. Consider a decision problem of determining the optimal speed of a self-driving car based on road conditions and traffic flow. The input variables could include "Road Condition" and "Traffic Flow," while the output variable could be "Speed." Fuzzy rules could describe how road conditions and traffic flow affect the optimal speed.
- Fuzzification: Convert crisp input values into fuzzy sets using appropriate membership functions. Fuzzification assigns membership degrees to fuzzy sets representing road conditions (e.g., "Dry," "Wet," "Icy") and traffic flow (e.g., "Light," "Moderate," "Heavy") based on their respective membership functions.
- Rule Evaluation: Apply fuzzy rules to the fuzzy input values to determine the degree to which each rule is satisfied. Evaluate fuzzy rules such as "If Road Condition is Dry and Traffic Flow is Light, then Speed is High" based on the degree of satisfaction of the input variables.
- Aggregation and Inference: Combine the outputs of individual rules to generate a single fuzzy output representing the optimal decision. Aggregating the outputs of fuzzy rules yields a fuzzy output indicating the optimal speed for the self-driving car, with membership degrees assigned to fuzzy sets like "High," "Medium," and "Low."
- Defuzzification: Convert the fuzzy output back into a crisp decision using defuzzification techniques. Defuzzification determines the crisp optimal speed for the self-driving car based on the aggregated fuzzy output, resulting in a specific speed value (e.g., 60 km/h).
- Decision Implementation: Implement the decision by taking the determined optimal action based on the defuzzified output. Set the speed of the self-driving car to the determined optimal value (e.g., 60 km/h) based on the fuzzy decision-making process.