DS 320 Algorithms for Data Science | **Discussion #1**
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1 Asymptotics

For each of the following pairs f and g, determine their asymptotic relationship by writing *True* or *False* in the blanks.

$f(n)$	$g(n)$	$f = O(g)$	$f = \Omega(g)$	$f = \Theta(g)$
$5n^2 - 1000$	$10n^2$			
n^3	$3n^7 - 2n$			
$\sqrt{n}$	$\ln n$			
n^{100}	100^n			
n^n	$n!$			
$\binom{n}{2}$	$\binom{n}{3}$			

2 Induction Practice

Induction can be used to prove classic formulas such as the formula for the sum of the first n natural numbers.

a. Prove that for all $n \geq 1$, $\sum_{j=1}^{n} j = \frac{n(n+1)}{2}$.

In class, we discussed Big-O notation and its uses. While it is generally easier to use limits to prove upper bounds, we can also use induction. Let's try a few examples.

b. Prove that for all $n \geq 4$, $2^n \geq n^2$.

c. The Fibonacci numbers may be defined by the recurrence relation

$$F_1 = 1, \quad F_2 = 1, \qquad F_n = F_{n-1} + F_{n-2} \quad \text{for} \quad n \geq 3.$$

For example, the first six Fibonacci numbers $F_1, F_2, F_3, F_4, F_5, F_6$ are 1, 1, 2, 3, 5, 8.

Prove that $F_n \leq 2^n$ for all $n \in \mathbb{N}$, using mathematical induction.