

1 Design Incomparable Functions

Design a pair of functions f and g from \mathbb{N} to \mathbb{N} such that neither $f(n) = O(g(n))$ nor $g(n) = O(f(n))$.

Hint : The functions we have looked at so far either converge to infinity or towards a constant. Think about functions that may not converge the same way.

2 Graph Concepts

In class we saw definitions for graphs $G = (V, E)$, paths of edges e_1, e_2, \dots, e_k , and adjacency matrix representations A . Another important concept you will encounter is a special type of path called a cycle. A cycle is any path which starts and ends at the same vertex, and contains unique vertices in between. Let v_1, v_2, \dots, v_{k+1} be the sequence of vertices seen along a path e_1, e_2, \dots, e_k . That is, $e_1 = (v_1, v_2), e_2 = (v_2, v_3), \dots, e_k = (v_k, v_{k+1})$.

Definition 1. A cycle is a path with at least $k = 3$ edges for which $v_1 = v_{k+1}$ and $v_i \neq v_j$ for any other $i, j \neq 1, k + 1$ in between.

Definition 2. A triangle is a cycle consisting of exactly $k = 3$ edges, therefore containing 3 unique vertices.

Exercises

- a) Draw an undirected graph with $G(V, E)$ with $n = 4$ vertices, where there exists an edge (i, j) between vertices i and j for every possible i, j pair. How many edges are in this graph, i.e. what is $|E|$? How would you represent this graph as an adjacency matrix A ? How many cycles of length n (i.e. cycles that visit every vertex) are in this graph? How many triangles?
- b) Draw an undirected graph with $G(V, E)$ with $n = 4$ vertices and only $m = 3$ edges. Write its adjacency matrix representation. Can you still ensure that there exists a path between any pair of vertices? Are there any cycles or triangles??

3 Count Edges

Given an undirected graph $G = (V, E)$ and its adjacency matrix representation A . Design an algorithm that uses A to count the number of edges m in G . What is its run-time?

4 Count Triangles

Given an undirected graph $G = (V, E)$ and its adjacency matrix representation A . Design an algorithm that uses A to count the number of triangles in G . What is its run-time?