E0-270: Assignment 1
Principal Component Analysis and Support Vector Machines Implementation on MNIST Dataset

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1 Introduction

Principal Component Analysis (PCA) and Support Vector Machine (SVM) are widely used techniques in machine learning for dimensionality reduction and classification, respectively. In this report presented the methodology, results for SVM and PCA, and analysis of implementing PCA and SVM algorithms on the MNIST dataset.

2 Methodology

2.1 PCA

PCA is a technique for representing the data in reduced dimensionality while preserving most of its variance. Given a dataset X of n data points each with m features, PCA finds a new set of k orthogonal basis vectors, called principal components, that best represent the data. The principal components are found by computing the eigenvectors of the covariance matrix of the centered data, where the centering involves subtracting the mean of each feature across all data points. The first principal component corresponds to the eigenvector with the largest eigenvalue, and each subsequent principal component corresponds to the next largest eigenvalue.

To implement PCA, first, compute the mean of the data and subtract it from each data point. Then, computed the covariance matrix of the centered data using numpy.cov() function in python which uses the given formula to compute covariance where the input of numpy.cov() function is a centered data matrix.

$$\Sigma = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})(x_i - \bar{x})^T \tag{1}$$

where x_i is the i-th data point, $\bar{x}$ is the mean of the data, and T denotes the transpose operation. then found the eigenvectors and eigenvalues of the covariance matrix using the numpy.linalg.eig() function in Python. sorted the eigenvalues in descending order and selected the top k eigenvectors corresponding to the k largest eigenvalues. Finally, the data were projected onto the selected eigenvectors to obtain the reduced dimensional data.

2.2 SVM

SVM is a binary classification algorithm that finds the hyperplane that separates the given data into two classes with minimum classification error. Given a dataset X of n data points each with m features and their corresponding labels $y \in -1, 1$, SVM finds the weight vector w and bias term b that define the hyperplane $w^T x + b = 0$. The optimization problem for SVM is given by:

$$\min_{w,b} \frac{1}{2}||w||^2 + C\sum_{i=1}^{n} \max\{0, 1 - y_i(w^T x_i + b)\} \tag{2}$$

where C is a hyperparameter that controls the trade-off between maximizing the margin and minimizing the classification error, and the hinge loss function $\max\{0, 1-y_i(w^T x_i+b)\}$ penalizes points that are misclassified or lie within the margin.

To implement SVM, Used the stochastic gradient descent algorithm to optimize the objective function.initialized the weight vector and bias term to zeros and updated them using the gradient of the objective function concerning the weight vector and bias term at each step using below updation rule for w and b.

$$w = w - \alpha * (w - C * y_i * x_i) \tag{3}$$

$$b = b + \alpha * (C * y_i) \tag{4}$$

In the above equations (3) and (4) α is a learning rate which is the hyperparameter.
In implementation, Used various learning rates, C parameters, and iterations to train the model.

2.3 Experimental Setup

To implement PCA and SVM used, the MNIST dataset given with the assignment; dataset consists of 60,000 training images and 10,000 test images. Each image is a 28 $\times$28 grayscale image of a handwritten digit, and the task is to classify what digit it is. So, it is a 10-class classification problem.

Every pixel of the image is represented by a single integer between 0 and 255, so each image is a 784-dimensional vector.

The dataset is already split into training and test sets. Each image is a row in the given CSV files. The first column is the label, and the remaining 784 columns are flattened images.

For implementation of PCA and SVM algorithms.Trained SVM on the original data and on the reduced dimensional data obtained from PCA, which is normalized between -1 and 1.

The implementation of the MultiClassSVM used ten binary SVM classifiers in the context of 1-vs-All to extract the multiclass functionality from binary class functionality

Evaluation of the performance of the models on the test set using four metrics: accuracy, precision, recall, and F1 score is done by taking the average performance of all SVM binary classifier models. Implementation calculated these metrics using the functions given in the boilerplate code structure of the assignment.

3 Results and Analysis

3.1 PCA

First analyzed is the effect of the number of principal components used on the variance captured by the reduced dimensional data. Figure 1 shows that increasing the number of principal components used leads to a higher percentage of variance explained by the reduced dimensional data. For example, using 100 principal components captures more than 85% of the variance in the data. Afterward, the variance captured almost converges as the principal components exceed 200.

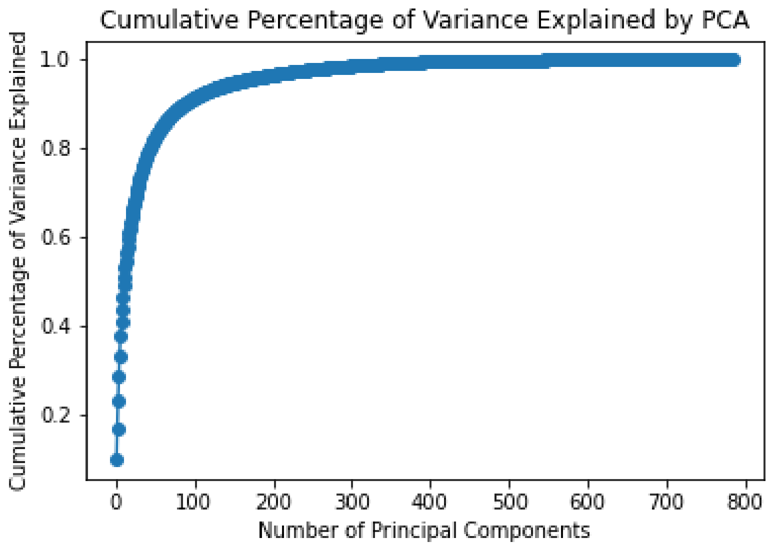
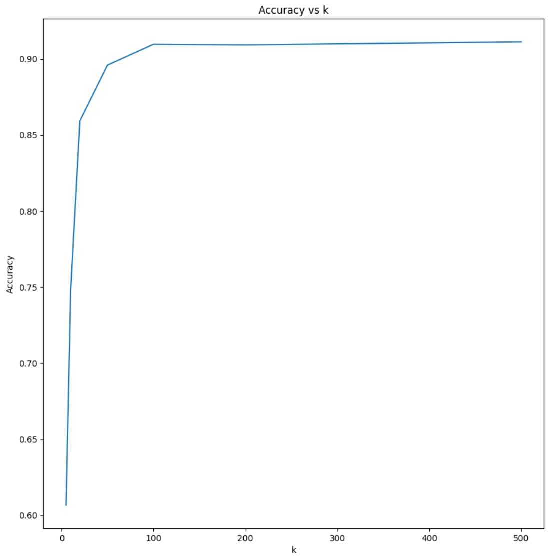


Figure 1: Percentage of variance explained by the reduced dimensional data with respect to the number of principal components used for PCA

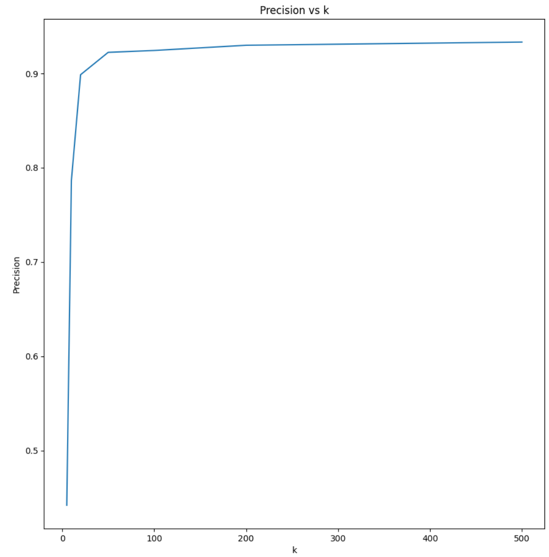
3.2 SVM

Next, In the implementation, trained SVM on the original data and the reduced dimensional data obtained from PCA, normalized between -1 and 1 using different numbers of principal components. Figure 2 shows the performance metrics of SVM on the test set with respect to the number of principal components used.

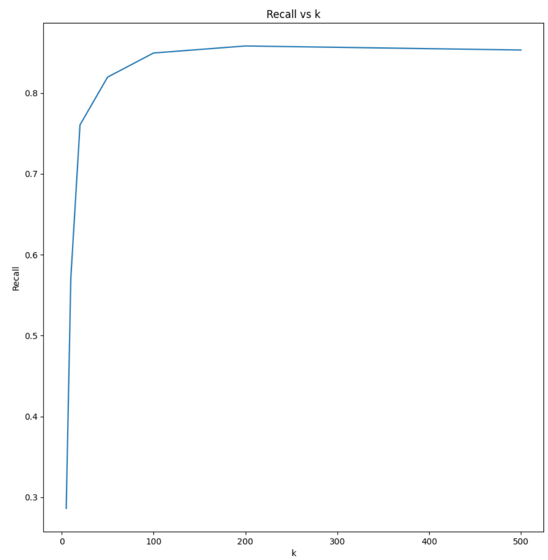
We observe that using PCA to reduce the dimensionality of the data leads to a significant improvement in the performance of SVM in terms of training efforts. From the graphs in Figure 2a we can easily observe that using 50 principal components leads to almost the same percentage of accuracy, precision, recall, and F1 score compared to using the original data performance given in Figure 3.



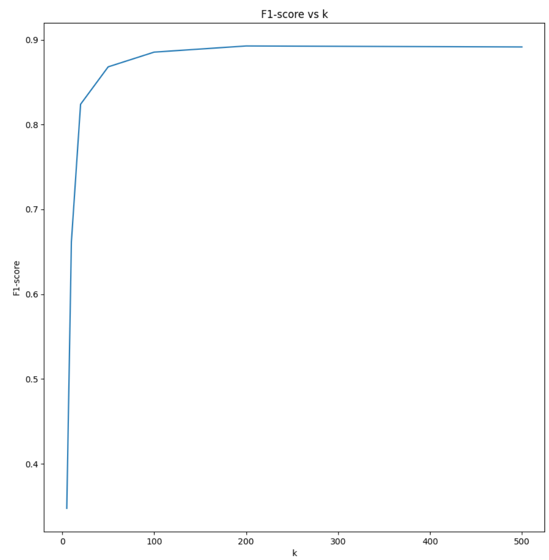
(a) Accuracy



(b) Precision



(c) Recall



(d) F1 Score

Figure 2: Performance metrics of SVM on the test set with respect to the number of principal components used for PCA

By training SVM on the original data without using PCA. Figure 3 shows the performance metrics of SVM on the test set. Figure 3 observed that SVM achieves an accuracy of about 92%, which is almost the same as the accuracy achieved by SVM with PCA. This indicates that reducing the dimensionality of the data using PCA can help improve the performance of SVM in training efforts with almost the same accuracy.

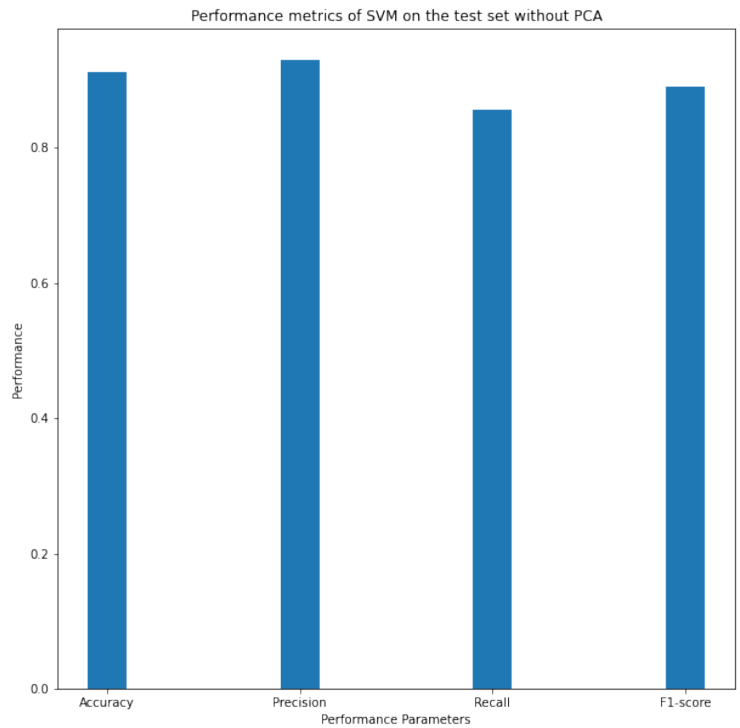


Figure 3: Performance metrics of SVM on the test set without PCA

4 Conclusion

In this report, the implementation of PCA and SVM algorithms evaluated their performance on the MNIST dataset. Observed that using PCA to reduce the dimensionality of the data can help improve the performance of SVM. Specifically, using 200 principal components led to almost the same accuracy, precision, recall, and F1 score compared to the original data.

Findings suggest that PCA can be a helpful technique for dimensionality reduction in machine learning tasks. Furthermore, implementing SVM can serve as a basic template for building more complex and powerful Multi-Class SVM models.

In conclusion, Report demonstrated the efficacy of PCA and SVM algorithms for classification tasks and highlighted the importance of dimensionality reduction in improving the performance of machine learning models. The code provided can be used to build more complex hyperplane-based Multi-Class classification models.