Digital Design & Computer Architecture Sarah Harris & David Harris

Chapter 1:

From Zero to One

Chapter 1 :: Topics

- The Art of Managing Complexity
- Number Systems
 - Binary Numbers
 - Hexadecimal Numbers
 - Bits, Bytes, Nibbles
 - Addition
 - Signed Numbers
 - Extension
- Logic Gates
- Logic Levels
- CMOS Transistors
- Transistor-Level Gate Design
- Power Consumption

Chapter 1: From Zero to One

The Art of Managing Complexity

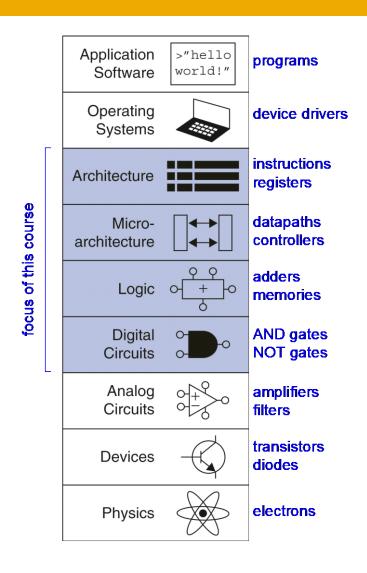
The Art of Managing Complexity

 How do we design things that are too big to fit in one person's head at once?

- Abstraction
- Discipline
- The Three –y's
 - Hierarchy
 - Modularity
 - Regularity

Abstraction

Hiding details when they aren't important



Discipline

- Intentionally restrict design choices
- Example: Digital discipline
 - Discrete voltages instead of continuous
 - Simpler to design than analog circuits can build more sophisticated systems
 - Digital systems replacing analog predecessors:
 i.e., digital cameras, digital television, cell phones, CDs

The Three -y's

- Hierarchy
 - _
- Modularity
 - _
- Regularity
 - _

Example: Model T Ford

- Famous early (1908) example of interchangeable parts.
 - Most previous cars were hand-crafted by skilled tradesmen.
 - Mass production on moving assembly lines greatly reduced cost.



en.wikipedia.org/wiki/Ford_Model_T#/media/File:1925_Ford_Model_T_touring.jpg

Henry Ford:

I will build a motor car for the great multitude. It will be large enough for the family, but small enough for the individual to run and care for. It will be constructed of the best materials, by the best men to be hired, after the simplest designs that modern engineering can devise. But it will be so low in price that no man making a good salary will be unable to own one — and enjoy with his family the blessing of hours of pleasure in God's great open spaces.

Example: Model T Ford

Hierarchy

- Car has chassis, wheels, seats, engine.
- Engine has cylinders, spark plugs, exhaust, carburetor.
- Carburator has air intake,
 inlet needle, feed pipe,
 coupling nut.

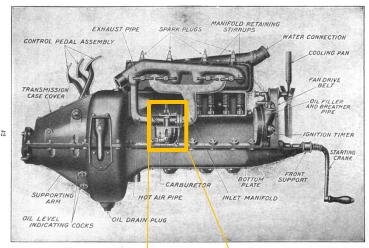


Fig. 8.—Valve Side of the Ford Model T Unit Power Plant Showing Manifolds, Carburetor and Interior of One of the Valve Spring Chambers.

https://en.wikipedia.org/wiki/Ford_Model_T_engine#/media/File:Pagé 1917 Model T Ford Car Figure 08.png

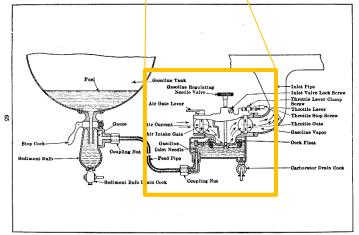


Fig. 14.—The Ford Model T Fuel Supply and Gas Making System.

https://en.wikipedia.org/wiki/Ford_Model_T_engine#/media/File:Pagé_1917_Model_T_Ford_Car_Figure_14.png

Example: Model T Ford

Modularity

- Function of coupling nut:
 - Hold fuel line to intake elbow
 - Prevent leaks
 - Easily removable
- Interface of coupling nut:
 - Standardized diameter, thread pitch, torque

Regularity

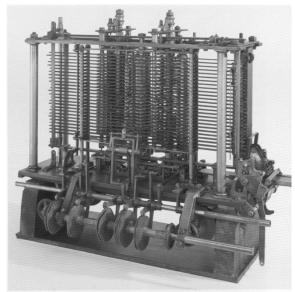
- Interchangeable parts
 - Standard nut could be purchased from many suppliers
- "Any customer can have a car painted any color that he wants so long as it is black." - Henry Ford

The Digital Abstraction

- Most physical variables are continuous
 - Voltage on a wire
 - Frequency of an oscillation
 - Position of a mass
- Digital abstraction considers discrete subset of values

The Analytical Engine

- Designed by Charles
 Babbage from 1834 1871
- Considered to be the first digital computer
- Built from mechanical gears, where each gear represented a discrete value (0-9)
- Babbage died before finishing it





Digital Discipline: Binary Values

Two discrete values:

- 1's and 0's
- 1, TRUE, HIGH
- 0, FALSE, LOW
- 1 and 0: voltage levels, rotating gears, fluid levels, etc.
- Digital circuits use voltage levels
 - 0: low voltage (GND)
 - 1: high voltage (V_{DD})
- Bit: Binary digit

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Number Systems: Binary Numbers

Number Systems

Decimal numbers

1's column 10's column 100's column 1000's column Decimal numbers in digital systems mean any base 10 numbers, not just those with a decimal point.

$$5374_{10} = 5 \times 10^3 + 3 \times 10^2 + 7 \times 10^1 + 4 \times 10^0$$
five three seven four thousands hundreds tens ones

Binary numbers

Counting in Binary

Binary

• • •

Decimal

Powers of Two

•
$$2^0 =$$

•
$$2^1 =$$

•
$$2^2 =$$

•
$$2^3 =$$

•
$$2^4 =$$

•
$$2^5 =$$

•
$$2^6 =$$

•
$$2^7 =$$

•
$$2^8 =$$

•
$$2^9 =$$

•
$$2^{10} =$$

•
$$2^{11} =$$

•
$$2^{12} =$$

•
$$2^{13} =$$

•
$$2^{14} =$$

•
$$2^{15} =$$

Handy to memorize

Number Conversion

- Binary to decimal conversion:
 - Convert 10011, to decimal

_

- Decimal to binary conversion:
 - Convert 47₁₀ to binary

_

Decimal to Binary Conversion

Two methods:

- Method 1: Find the largest power of 2 that fits, subtract and repeat
- Method 2: Repeatedly divide by 2, remainder goes in next most significant bit

Decimal to Binary Conversion

53₁₀

Method 1: Find the largest power of 2 that fits, subtract and repeat

Method 2: Repeatedly divide by 2, remainder goes in next most significant bit

Binary Values and Range

N-digit decimal number

- How many values?
- Range?
- Example: 3-digit decimal number:

N-bit binary number

- How many values?
- Range:
- Example: 3-digit binary number:

•

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Number Systems: Hexadecimal Numbers

Hexadecimal Numbers

Hex Digit	Decimal Equivalent	Binary Equivalent
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
А	10	1010
В	11	1011
С	12	1100
D	13	1101
E	14	1110
F	15	1111

Hexadecimal Numbers

- Base 16
- Shorthand for binary

Hexadecimal to Binary Conversion

- Hexadecimal to binary conversion:
 - Convert 4AF₁₆ (also written 0x4AF) to binary

__

- Hexadecimal to decimal conversion:
 - Convert 4AF₁₆ to decimal

_

Hexadecimal and Binary Prefixes

- Hard to write subscripts in text files
- Some programming languages uses prefixes
 - Hex: 0x
 - $0x23AB = 23AB_{16}$
 - Binary: 0b
 - $0b1101 = 1101_2$

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Number Systems: Bytes, Nibbles, & All That Jazz

Bits, Bytes, Nibbles...

- Byte: 8 bits
 - Represents one of _____ values
 - **–** [__, ___]
- Nibble: 4 bits
 - Represents one of _____ values
 - **–** [__, ___]

One binary digit is ___ bit
One hex digit is ___ bits or ___ nibble
Two hex digits make ___ byte

Most significant on left Least significant on right 10010110

most least significant bit bit bit

byte

10010110

nibble

CEBF9AD7

most least significant byte byte

Large Powers of Two

•
$$2^{10} = 1$$
 kilo

•
$$2^{20} = 1 \text{ mega}$$

•
$$2^{30} = 1$$
 giga

•
$$2^{40} = 1 \text{ tera}$$

•
$$2^{50} = 1$$
 peta

•
$$2^{60} = 1 \text{ exa}$$

$$\approx 10^3 (1024)$$

$$\approx 10^6 (1,048,576)$$

$$\approx 10^9 (1,073,741,824)$$

$$\approx 10^{12}$$

$$\approx 10^{15}$$

$$\approx 10^{18}$$

Estimating Powers of Two

• What is the value of 2^{24} ?

 How large of a value can a 32-bit integer variable represent?

Chapter 1: From Zero to One

Number Systems: Addition

Addition

Decimal

Binary

Binary Addition Examples

 Add the following 4-bit binary numbers

Add the following
 4-bit binary
 numbers

Overflow

- Digital systems operate on a fixed number of bits
- Overflow: when result is too big to fit in the available number of bits
- See previous example of 11 + 6

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Number Systems: Signed Numbers

Signed Binary Numbers

- Sign/Magnitude Numbers
- Two's Complement Numbers

Sign/Magnitude Numbers

- 1 sign bit, N-1 magnitude bits
- Sign bit is the most significant (left-most) bit
 - Positive number: sign bit = 0

$$A: \{a_{N-1}, a_{N-2}, \dots a_2, a_1, a_0\}$$

Negative number: sign bit = 1

$$A = (-1)^{a_{N-1}} \sum_{i=0}^{N-2} a_i \, 2^i$$

• Example, 4-bit sign/mag representations of ± 6:

```
+6 =
```

Range of an N-bit sign/magnitude number:

Sign/Magnitude Numbers

Problems:

Addition doesn't work, for example -6 + 6:

```
1110
+ 0110
10100 (wrong!)
```

Two representations of 0 (± 0):

```
1000
```

0000

Two's Complement Numbers

- Don't have same problems as sign/magnitude numbers:
 - Addition works
 - Single representation for 0

Two's Complement Numbers

msb has weight of -2^{N-1}

$$A = a_{N-1}(-2^{N-1}) + \sum_{i=0}^{N-2} a_i 2^i$$

- Most positive 4-bit number:
- Most negative 4-bit number:
- The most significant bit still indicates the sign (1 = negative, 0 = positive)
- Range of an N-bit two's complement number:

Reversing the Sign

- Reverse the sign of a two's complement number
- Method:
 - 1. Invert the bits
 - 2. Add 1
- Example: Reverse the sign of $3_{10} = 0011_2$
 - 1.
 - 2.

Historically, this reversing the sign method has been called: "Taking the Two's complement". But this terminology can be confusing, so we instead we call it "reversing the sign".

Two's Complement Examples

- Reverse the sign of $6_{10} = 0110_2$
 - 1.
 - 2.

- What is the decimal value of the two's complement number 1001₂?
 - 1.
 - 2.

Two's Complement Addition

Add 6 + (-6) using two's complement numbers

Add -2 + 3 using two's complement numbers

Subtraction

- Subtract a 2's complement number by reversing the sign and adding.
- Reverse sign by taking 2's complement

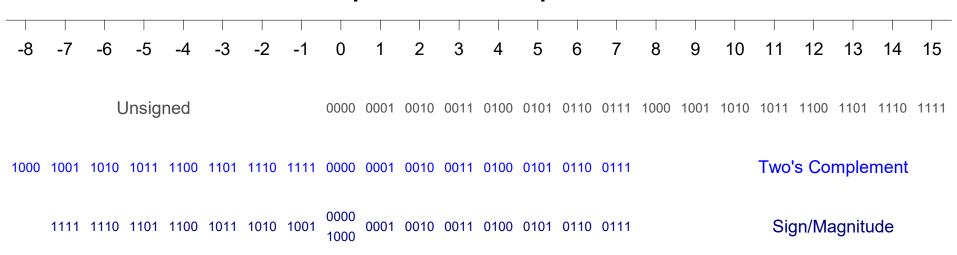
• Ex:
$$3 - 5 = 3 + (-5)$$

$$\begin{array}{r}
0011 & 3 \\
+ & 1011 \\
\hline
1110 & -2
\end{array}$$

Number System Comparison

Number System	Range
Unsigned	$[0, 2^N-1]$
Sign/Magnitude	$[-(2^{N-1}-1), 2^{N-1}-1]$
Two's Complement	$[-2^{N-1}, 2^{N-1}-1]$

For example, 4-bit representation:



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Number Systems: Extension

Increasing Bit Width

Extend number from N to M bits (M > N):

- Sign-extension for 2's complement numbers
- Zero-extension for unsigned numbers

Sign-Extension

- Sign bit copied to msb's
- Number value is same

Example 1:

- 4-bit representation of 3 = 0011
- 8-bit sign-extended value: 00000011

Example 2:

- 4-bit representation of -5 = 1011
- 8-bit sign-extended value: 11111011

Zero-Extension

- Zeros copied to msb's
- Value changes for negative numbers

Example 1:

– 4-bit value =

$$0011 = 3_{10}$$

- 8-bit zero-extended value: $00000011 = 3_{10}$

Example 2:

– 4-bit value =

$$1011 = -5_{10}$$

- 8-bit zero-extended value: $00001011 = 11_{10}$

Chapter 1: From Zero to One

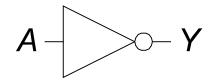
Logic Gates

Logic Gates

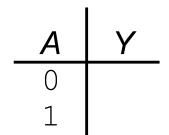
- Perform logic functions:
 - inversion (NOT), AND, OR, NAND, NOR, etc.
- Single-input:
 - NOT gate, buffer
- Two-input:
 - AND, OR, XOR, NAND, NOR, XNOR
- Multiple-input

Single-Input Logic Gates

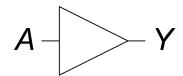
NOT



$$Y = \overline{A}$$



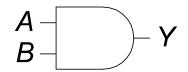
BUF



$$Y = A$$

Two-Input Logic Gates

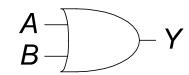
AND



$$Y = AB$$

Α	В	Y
0	0	
0	1	
1	0	
1	1	

OR

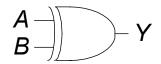


$$Y = A + B$$

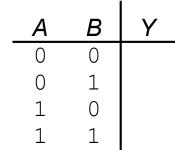
A	В	Y
0	0	
0	1	
1	0	
1	1	

More Two-Input Logic Gates

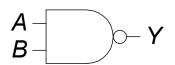
XOR



$$Y = A \oplus B$$



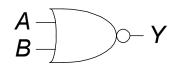
NAND



$$Y = \overline{AB}$$

Α	В	Y
0	0	
0	1	
1	0	
1	1	

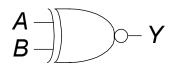
NOR



$$Y = \overline{A + B}$$

A	В	Y
0	0	
0	1	
1	0	
1	1	

XNOR

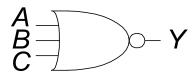


$$Y = \overline{A + B}$$

	Α	В	Y
•	0	0	
	0	1	
	1	0	
	1	1	

Multiple-Input Logic Gates

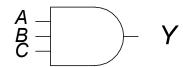
NOR₃



$$Y = \overline{A + B + C}$$

A	В	С	Y
0	0	0	1
0	0	1	
Ο	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

AND3



$$Y = ABC$$

Α	В	С	Υ
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

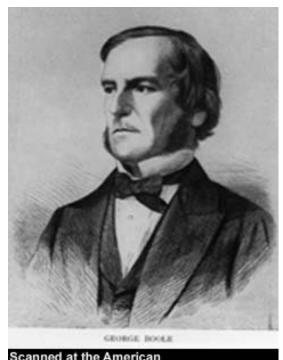
Truth table rows are listed in binary order.

Multi-input XOR: Odd parity

SystemVerilog Description

George Boole, 1815-1864

- Born to working class parents
- Taught himself mathematics and joined the faculty of Queen's College in Ireland
- Wrote An Investigation of the Laws of Thought (1854)
- Introduced binary variables
- Introduced the three fundamental logic operations: AND, OR, and NOT



Scanned at the American Institute of Physics

Chapter 1: From Zero to One

Logic Levels

Logic Levels

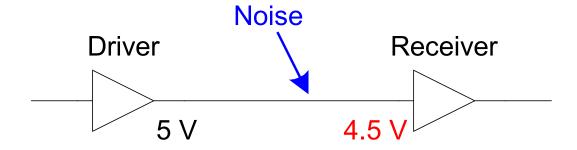
- Discrete voltages represent 1 and 0
- For example:
 - -0 = ground (GND) or 0 volts
 - $-1 = V_{DD}$ or 5 volts
- What about 4.99 volts? Is that a 0 or a 1?
- What about 3.2 volts?

Logic Levels

- Range of voltages for 1 and 0
- Different ranges for inputs and outputs to allow for noise

What is Noise?

- Anything that degrades the signal
 - E.g., resistance, power supply noise, coupling to neighboring wires, etc.
- Example: a gate (driver) outputs 5 V but, because of resistance in a long wire, receiver gets 4.5 V

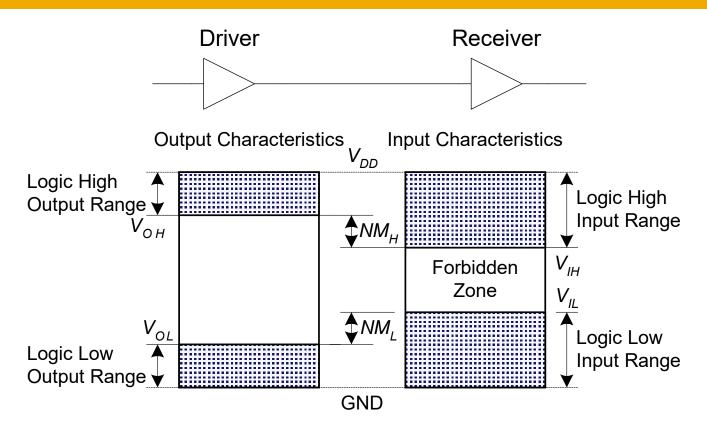


The Static Discipline

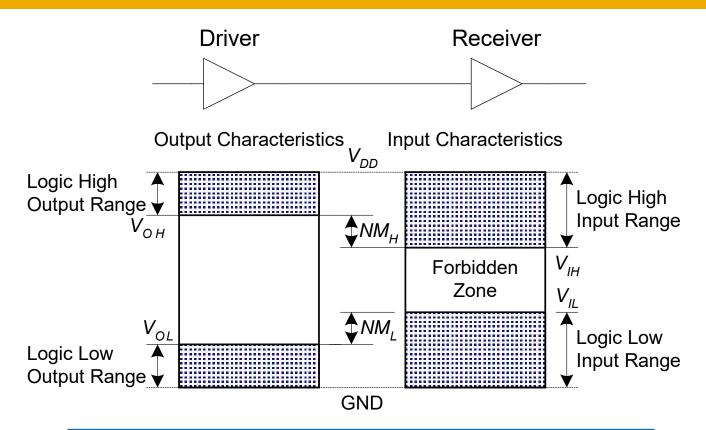
 With logically valid inputs, every circuit element must produce logically valid outputs

 Use limited ranges of voltages to represent discrete values

Noise Margins



Noise Margins



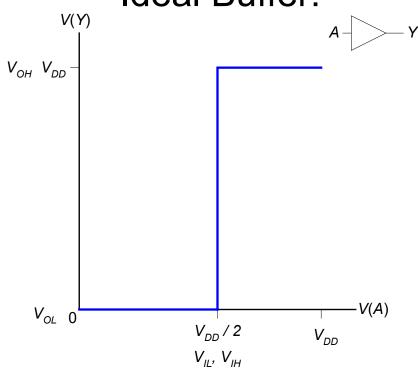
High Noise Margin: NM_H =

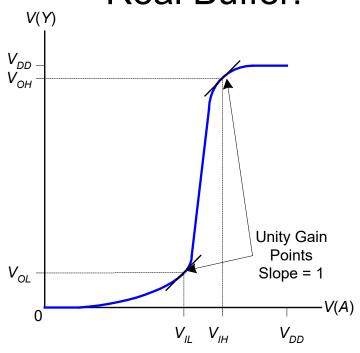
Low Noise Margin: $NM_L =$

DC Transfer Characteristics



Real Buffer:

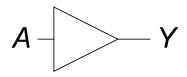


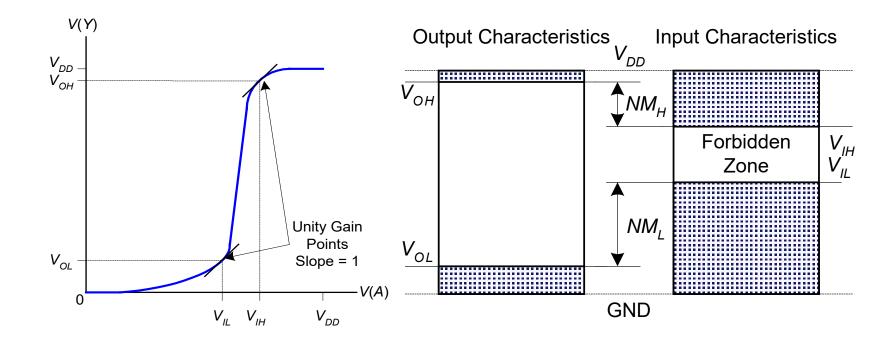


$$NM_H = NM_L = V_{DD}/2$$

$$NM_H$$
, $NM_L < V_{DD}/2$

DC Transfer Characteristics





V_{DD} Scaling

- In 1970's and 1980's, $V_{DD} = 5 \text{ V}$
- V_{DD} has dropped
 - Avoid frying tiny transistors
 - Save power
- 3.3 V, 2.5 V, 1.8 V, 1.5 V, 1.2 V, 1.0 V, ...
 - Be careful connecting chips with different supply voltages

V_{DD} Scaling

- In 1970's and 1980's, $V_{DD} = 5 \text{ V}$
- V_{DD} has dropped
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- 3.3 V, 2.5 V, 1.8 V, 1.5 V, 1.2 V, 1.0 V, ...
 - Be careful connecting chips with different supply voltages

Chips operate because they contain magic smoke

Proof: if the magic smoke is let out, the chip stops working

Logic Family Examples

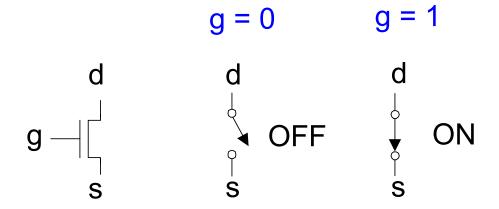
Logic Family	V _{DD}	V _{IL}	V _{IH}	V _{OL}	V _{OH}
TTL	5 (4.75 - 5.25)	0.8	2.0	0.4	2.4
CMOS	5 (4.5 - 6)	1.35	3.15	0.33	3.84
LVTTL	3.3 (3 - 3.6)	0.8	2.0	0.4	2.4
LVCMOS	3.3 (3 - 3.6)	0.9	1.8	0.36	2.7

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CMOS Transistors

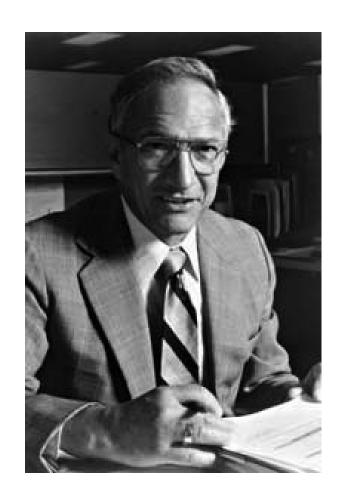
Transistors

- Logic gates built from transistors
- 3-ported voltage-controlled switch
 - 2 ports connected depending on voltage of 3rd
 - d and s are connected (ON) when g is 1



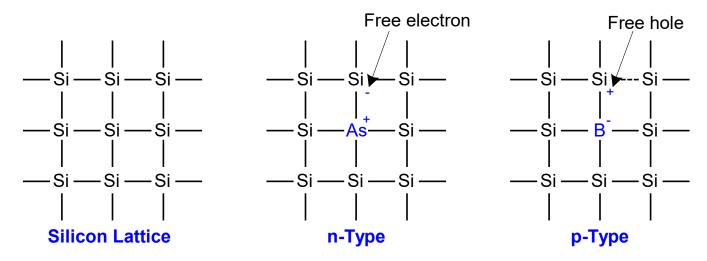
Robert Noyce, 1927-1990

- Nicknamed "Mayor of Silicon Valley"
- Cofounded Fairchild
 Semiconductor in 1957
- Cofounded Intel in 1968
- Co-invented the integrated circuit



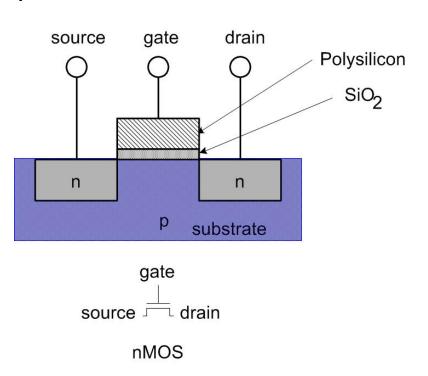
Silicon

- Transistors built from silicon, a semiconductor
- Pure silicon is a poor conductor (no free charges)
- Doped silicon is a good conductor (free charges)
 - n-type (free negative charges, electrons)
 - p-type (free positive charges, holes)



MOS Transistors

- Metal oxide silicon (MOS) transistors:
 - Polysilicon (used to be metal) gate
 - Oxide (silicon dioxide) insulator
 - Doped silicon



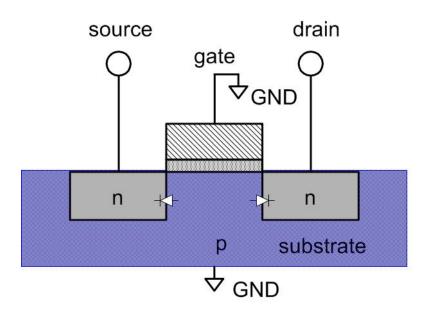
Transistors: nMOS

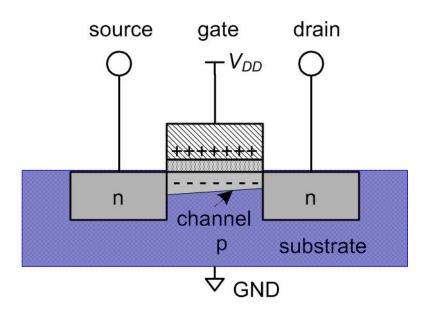
Gate = 0

OFF (no connection between source and drain)

Gate = 1

ON (channel between source and drain)

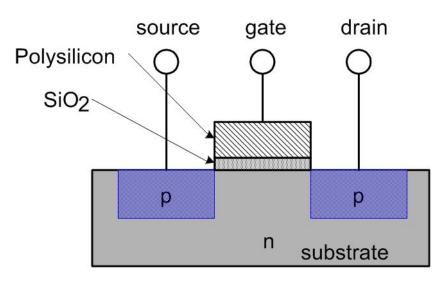


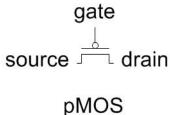


Transistors: pMOS

pMOS transistor is opposite

- **ON** when **Gate** = **0**
- OFF when Gate = 1

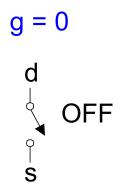


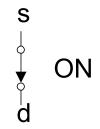


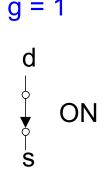
Transistor Function

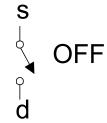
nMOS g — s

pMOS g⊸|



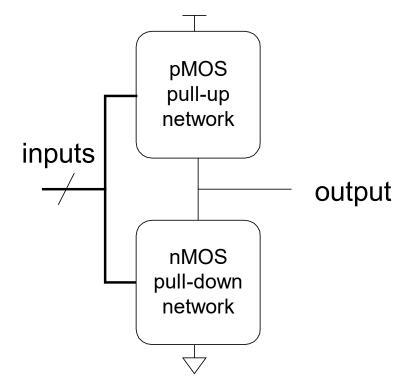






Transistor Function

- nMOS: pass good 0's, so connect source to GND
- pMOS: pass good 1's, so connect source to V_{DD}

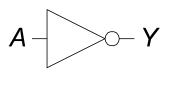


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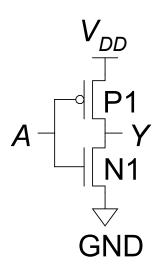
Gates from Transistors

CMOS Gates: NOT Gate





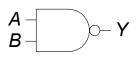
$$Y = \overline{A}$$



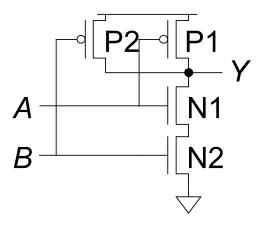
\boldsymbol{A}	P1	N1	Y
0			
1			

CMOS Gates: NAND Gate

NAND

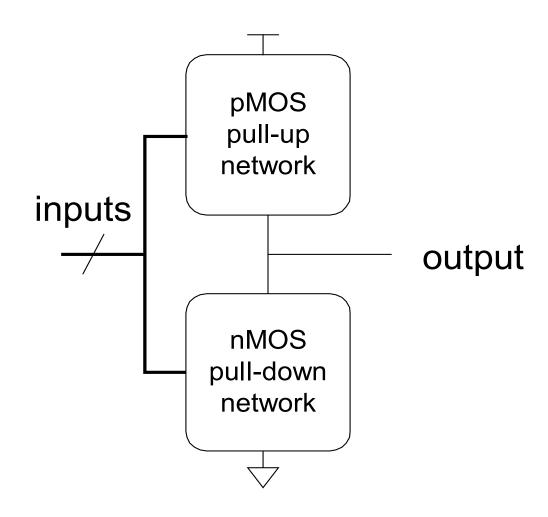


$$Y = \overline{AB}$$



\boldsymbol{A}	B	P1	P2	N1	N2	Y
0	0					
0	1					
1	0					
1	1					

CMOS Gate Structure



NOR3 Gate

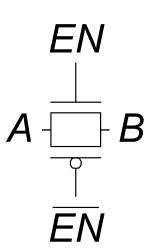
How do you build a three-input NOR gate?

AND2 Gate

How do you build a two-input AND gate?

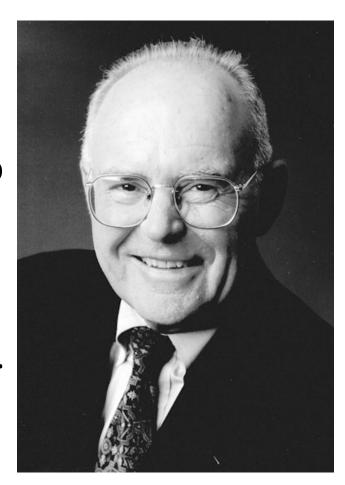
Transmission Gates

- nMOS pass 1's poorly
- pMOS pass 0's poorly
- Transmission gate is a better switch
 - passes both 0 and 1 well
- When *EN* = 1, the switch is ON:
 - -EN = 0 and A is connected to B
- When *EN* = 0, the switch is OFF:
 - A is not connected to B

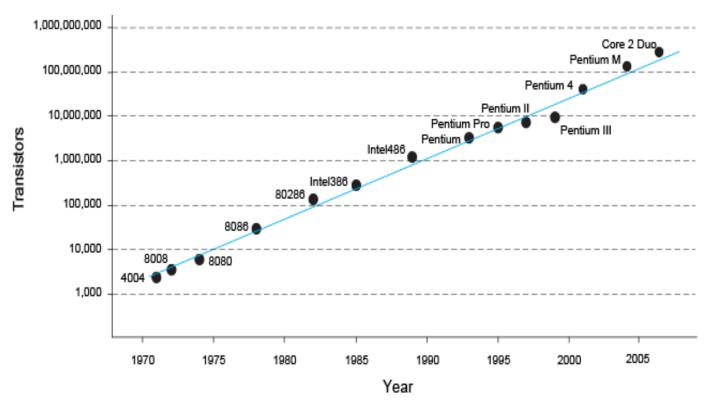


Gordon Moore, 1929-

- Cofounded Intel in 1968 with Robert Noyce.
- Moore's Law: number of transistors on a computer chip doubles every year (observed in 1965)
- Since 1975, transistor counts have doubled every two years.
- Corollaries: transistors get faster and lower power



Moore's Law



"If the automobile had followed the same development cycle as the computer, a Rolls-Royce would today cost \$100, get one million miles to the gallon, and explode once a year . . ." (Robert Cringely, Infoworld)

Robert Cringley

Chapter 1: From Zero to One

Power Consumption

Power Consumption

Power = Energy consumed per unit time

- Dynamic power consumption
- Static power consumption

Dynamic Power Consumption

Power to charge transistor gate capacitances

- Energy required to charge a capacitance, C, to V_{DD} is $CV_{DD}^{\ 2}$
- Circuit running at frequency f (f cycles per second)
- Capacitor is charged α times per cycle (discharging from 1 to 0 is free)
- Dynamic power consumption:

$$P_{dynamic} = \alpha C V_{DD}^2 f$$

Activty Factor α

• α is the fraction of cycles that cap is charged

Clock

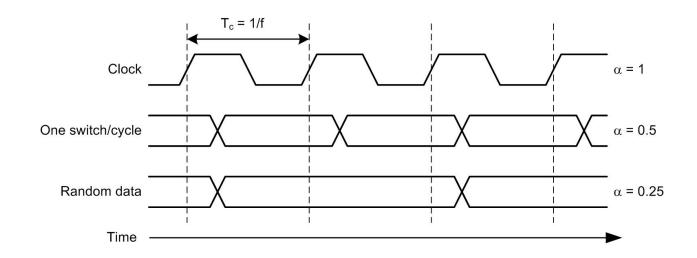
- $\alpha = 1$
- Data switching once per cycle
- α = 0.5

Random data

 $\alpha = 0.25$

Typical data

 α = 0.1



Static Power Consumption

- Power consumed when no gates are switching
- Caused by the quiescent supply current, I_{DD}
 (also called the leakage current)
- Static power consumption:

$$P_{static} = I_{DD}V_{DD}$$

Units

 Chips have tiny capacitances, small currents, and high frequencies.

Unit	Prefix	Value
Tera	Т	1012
Giga	G	109
Mega	М	10 ⁶
Kilo	K	10 ³
Mili	m	10 ⁻³
Micro	μ	10 ⁻⁶
Nano	n	10 ⁻⁹
Pico	р	10 ⁻¹²
Femto	f	10 ⁻¹⁵

Power Consumption Example

 Estimate the power consumption of a mobile phone running Angry Birds

```
-V_{DD} = 0.8 \text{ V}
 - C = 5 \text{ nF}
 -f=2 GHz
 -\alpha = 0.1
 -I_{DD} = 100 \text{ mA}
P = \alpha C V_{DD}^2 f + I_{DD} V_{DD}
       = (0.1)(5 \text{ nF})(0.8 \text{ V})^2(2 \text{ GHz}) + (100 \text{ mA})(0.8 \text{ V})
       = (0.64 + 0.08) \text{ W} \approx 0.72 \text{ W}
```

Power Consumption Example

 If the phone has a 8 W-hr battery, estimate its battery life sitting idle in your pocket.

```
-V_{DD} = 0.8 \text{ V}
-I_{DD} = 100 \text{ mA}
```

$$P_{static} = I_{DD}V_{DD} = 0.08 \text{ W}$$

```
Battery life = Capacity / Consumption
= (8 W-hr) / (0.08 W) = 100 hr (4 days)
```

About these Notes

Digital Design and Computer Architecture Lecture Notes

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