Introduction to Googology

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Abstract.

Contents

CHAPTER 1

The Beginning of Googology

1.1. The Simplest Unary Operation

1.1.1. The Positive Integers. The set of positive integers, $\mathbb{N}^*$, is defined as satisfying the following conditions:

(1) For any arbitrary positive integer a, its successor, a^+ is a positive integer as well.
(2) There exists a positive integer such that no positive integer's successor is that integer. Notate this as 1.
(3) Except for 1, every positive integer is the successor of a positive integer.
(4) If $1 \in S$, and whenever $n \in S$, $n^+ \in S$ as well; then $S = \mathbb{N}^*$.

But problems arise. Until now, we might have wondered how to express things like "1's successor," "1's successor's successor," and so on. Could they be expressed as 1^+, 1^{++}, etc?

For a long time, people defined [TODO] as follows: $1^+ = 2, 2^+ = 3, 3^+ = 4, 4^+ = 5, 5^+ = 6, 6^+ = 7, 7^+ = 8, 8^+ = 9, 9^+ = 10$. We use two digits in "10" to refer to the successor of the number 9.

Then, $10^+ = 11, 11^+ = 12, \ldots, 19^+ = 20, \ldots, 99^+ = 100, \ldots$ Thus, we now have a way to theoretically express any positive integer.

However, this simple operation by itself is practically useless. The really useful stuff is the following...

1.1.2. Addition.