Contents

1. The Beginning of Googology

1.1. The Simplest Operaion

1.1.1. The Positive Integers and The Successor Operation

The set of positive integers N^* are defined to have the following properties:
1. If a is a positive integer, then so too is its successor a^+.
2. There exists a positive integer a such that no positive integer's successor is a. Notate this as 1.
3. Except for 1, every positive integer is the successor of a positive integer.
4. If $1 \in S$, and whenever $a \in S$, $a^+ \in S$ as well; then $S = N^*$.

We could use the phrases "1's successor," "1's succeessor's successor," etc. But that gets unweildy very quickly, so we use expressions like 1^+, $1^+ +$, etc. instead.

For a long time people used to define things as $1^+ = 2$,$2^+ = 3$,$3^+ = 4$,$4^+ = 5$,$5^+ = 6$,$6^+ = 7$,$7^+ = 8$,$8^+ = 9$,$9^+ = 10$. We use two digits in "10" to refer to the successor of 9.

Similarly, $10^+ = 11, 11^+ = 12, ..., 19^+ = 20, ..., 99^+ = 100, ...$ Hence, following this procedure, we can notate any positive integer in theory.

However, this unary operation is only used for counting, and is practically useless. A more useful operation is the following:

1.1.2. Addition

The addition operator is defined as follows, using the addition symbol "+."
1. $a + 1 = a^+$.
2. $a + b^+ = (a + b)^+$.

Very simple! The only problem is that it is too abstract. We can derive a simpler expression: a + b = a$^{+++ \cdots ++}$ (with b many +es.) This new definition is very clear.

However, sometimes this is not useful. So, we need...

1.1.3. Multiplication

Multiplication is defined using the multiplication symbol $\times$.
1. $a \times 1 = a$.
2. $a \times b^+ = a \times b + a$.

This is already a very simple definition. A more accessible definition is $a \times b = a + a + a + ...a$ (with b as.)

We can now discover that $10 \times 10 = 100$,$100 \times 10 = 1000$, 100…0 (with n 0s) $\times$ 10 = 100…0 (with $n + 1$ 0s.)

1.1.4. Exponentiation

TODO