# Project Euler - Problem 103

Proloy Mishra

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## 1 Definition

**Definition 1.1** (The Problem)**.** *Let $S(A)$ be the sum of a set $A$. Then the problem, given a natural number $n$, is to find a set $A$ which minimizes $S(A)$ s.t. $|A| = n$, and for any 2 disjoint subsets $X$ and $Y$ of it:*
*1. $S(X) \neq S(Y)$, and*
*2. $|X| > |Y| \implies S(X) > S(Y)$.*

## 2 Getting a more generalized (and useful) requirement

**Theorem 2.1.** *A satisfying 1.1 is equivalent to a similar set of requirements which don't require that $X$ and $Y$ be disjoint, merely distinct.*

*Proof.* Let $X \neq Y$, where $X, Y \subseteq A$. Separate them out into pairwise disjoint sets $X - Y$, $Y - X$, and $X \cap Y$. By the requirement of $A$, $S(X - Y) \neq S(Y - X)$. Hence,

$$\begin{aligned}
&S(X - Y) + S(X \cap Y) \neq S(Y - X) + S(X \cap Y) \\
&\implies S((X - Y) \cup (X \cap Y)) \neq S((Y - X) \cup (X \cap Y)) \\
&\implies S(X) \neq S(Y).
\end{aligned}$$

The converse is ez.
Similarly, requirement #2 can be generalized too. □

Note since $A$ satisfying #2 means that $|X| \neq |Y| \implies S(X) \neq S(Y)$, requirement #1's generality is actually redundant. Its only contribution is with same-length sets. so, now, a better definition is:

**Definition 2.1** (The Problem - Definition 2)**.** *The problem, given a natural number $n$, is to find a set $A$ which minimizes $S(A)$ s.t. $|A| = n$, and for any 2 distinct subsets $X$ and $Y$ of it:*
*1. $|X| = |Y| \implies S(X) \neq S(Y)$, and*
*2. $|X| > |Y| \implies S(X) > S(Y)$.*