A Simple Yet Effective and Efficient Collaborative Filtering Based Recommendation System

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Abstract

In this paper, we describe a simple yet effective way to recommend items to a user and predict the rating of an item given by a user with high accuracy.

1 Introduction

A common problem (insert relevant papers) with typical filtering based recommendation systems they usually consider all users similar to a user. To mitigate performance problems, (insert relevant papers) show that we can simply consider only a handful of neighbors for a particular user.

2 Algorithm

Let us first describe the assumption problem that we want to solve. Let $\mathcal{M}$ be a set of movies and $\mathcal{U}$ be a set of users. We are given a set of triplets $(u, m, r) \in \mathcal{S}$ such that $u \in \mathcal{U}, m \in \mathcal{U}$ and $1 \leq r \leq 5$ which denotes the rating r of movie m given by user u. We are also given a set of pairs $(u, m) \in \mathcal{T}$ such that $u \in \mathcal{U}, m \in \mathcal{M}$ and we want to predict the rating r given by user m to the movie m. Let r_{um} be the rating of movie m given by user u and C_{uv} be the set of items that both u and v rated. We use the *Pearson Correlation Coefficient* (see Freedman, Pisani, and Purves [1]) as the measure of correlation between user u and v. However, we will use a normalized version of it for practical consideration as follows:

$$S_{uv} = \frac{\sum_{m \in C_{uv}} (r_{um} - \bar{r_u})(r_{vm} - \bar{r_v})}{\sqrt{\sum_{m \in C_{uv}} (r_{um} - \bar{r_u})^2} \sqrt{\sum_{m \in C_{uv}} (r_{vm} - \bar{r_v})^2}}$$

$$S_{uv} \leftarrow \frac{S_{uv} + 1}{2}$$

This normalization is done using the fact that Pearson correlation coefficient p_{uv} is always in the range $[-1, 1]$. We call two users u and v similar if $S_{uv} \geq s$ for some positive real number s such that $0 \leq s \leq 1$. Typically, we want s in the range $[.5, 1]$. For this paper, we will consider $s \in \{.7, .8, .9\}$. For a movie m, let $\mathcal{U}_m$ be the set of users who rated m and M_u be the set of movies rated by user u. For a set or tuple A, let $\bar{A}$ denote the average of the numbers in A. For a tuple of weights $\mathbf{w} = (w_1, \dots, w_n)$ such that $0 \leq w_i \leq 1$ and $\sum_{i=1}^{n} w_i = 1$ and a tuple of positive real numbers $\mathbf{a} = (a_1, \dots, a_n)$, the *weighted harmonic mean* of $\mathbf{a}$ is defined as

$$\mathfrak{H}(\mathbf{a}, \mathbf{w}) = \frac{\sum_{i=1}^{n} w_i}{\sum_{i=1}^{n} \frac{w_i}{a_i}}$$

Usually, *weighted arithmetic mean* is used to predict the ratings in a recommendation system. But in this paper, we have investigated the results using harmonic mean. Next, we describe the rating prediction algorithm for a pair (u, m).

Algorithm 1: Algorithm to predict rating

Input: Test data in the format (u, m), Threshold t, similarity s, T to take first T neighbors for a user u
Output: A single integer in the range $[1, 5]$ denoting the predicted rating
Data: Train data in the format (u, m, r)

$W \leftarrow []$
$X \leftarrow []$
$tot \leftarrow 0$
$R \leftarrow U_m$
res $= 0$
for $v \in R$ **do**
    if $|C_{uv}| < t$ **then**
        continue
    end
    if $S_{uv} < s$ **then**
        continue
    end
    if $S_{uv} \geq s$ **then**
        $W \leftarrow [W, s]$
        $X \leftarrow [X, r_{vm}]$
        $tot \leftarrow tot + 1$
        if $tot > T$ **then**
            break
        end
    end
end
if $tot > 0$ **then**
    res $= \mathfrak{H}(X, W)$
end
else
    if $|M_u| > 0$ **then**
        res $= \bar{M_u}$
    end
    else
        res $= \bar{R_m}$
    end
end
res = res+.5
res = floor(res)
return res

2.1 Algorithm Principles

Our algorithm is based on the following principles.

First principle *Two users u and v are more relevant for each other if it is ensured that $|C_{uv}| \geq t$ for some large enough positive integer t.* It is obvious how this principle helps us establish better correlation between users since two users u and v can have very high correlation with very low number of common items between them.

Second principle *If first principle is established, then it is possible to get an accurate estimation of predicted rating with a lower number of neighbors instead of using a very large number of neighbors.* This helps us predict a rating a lot more efficiently with better accuracy. It is also possible to get a mathematical sense of why this works in practice. Let m and n be positive integers such that $m > n$. Let u be a user and $\mathcal{N}_m = \{v : |C_{uv}| \geq t\}$ and $\mathcal{N}_n = \{u : |C_{uv}| \geq t\}$ be two sets of neighbors of a user u such that $|\mathcal{C}_m| = m$ and $|\mathcal{N}_n| = n$. Using the condition $|C_{uv}| \geq t$, it can be assumed that if $s < t$ for a positive integer s, then

$$P(\sigma^2(A) \leq \sigma^2(B)) \tag{1}$$

should be very high where $A = \{r_{vm} : |C_{uv}| \geq t\}$, $B = \{r_{vm} : |C_{uv}| \geq s\}$ and $P(x)$ denotes the probability of the random variable x. Since higher value of the threshold t ensures better similarity between two users, we can say in a non-rigorous way that the *Pigeonhole principle* ensures (1) is high enough in practice more often than not.

Third principle *If two sets of ratings A and B have similar variance and consist ratings given by similar users of u only, then $P(|\bar{A} - \bar{B}| < \epsilon)$ is very high for some positive real number ϵ which is considerably smaller than* 1. We can show that a special case holds very often in practice. Since A and B has ratings from similar users only, assume that both A and B consist of 4 and 5 only (our data set rating range is $[1, 5]$). If $|A| = m$ and $|B| = n$ and a is the number of 4 in A whereas b is the number of 4 in B, then

$$\begin{aligned}
\bar{A} &= \frac{4a + 5(m - a)}{m} \\
&= \frac{5m - a}{m} \\
&= 5 - \frac{a}{m} \\
\bar{B} &= \frac{4b + 5(n - b)}{n} \\
&= \frac{5n - b}{n} \\
&= 5 - \frac{b}{n}
\end{aligned}$$

As we can see, it does not matter if $m \gg n$ or $n \gg m$ since the difference $|\bar{A} - \bar{B}|$ or the ratio $\frac{\bar{A}}{\bar{B}}$ only depends on the ratio $\frac{a}{m}$ and $\frac{b}{n}$. So as long as these ratios are similar, $\bar{A}$ and $\bar{B}$ are similar as well.

3 Results

We show the results of our algorithm on the Movielens 1 million data set and backup our claims with empirical evidence below. The quality of performance is measured using the $F1$ score. The $F1$ score is the harmonic mean of precision and recall.

$$P = \frac{TP}{TP + FP}$$
$$R = \frac{TP}{TP + FN}$$
$$F1 = \frac{2PR}{R + P}$$

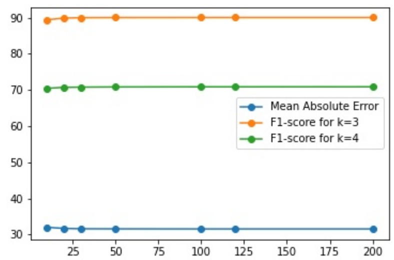
We have chosen $F1$ as an accuracy metric for couple of reasons.

1. It is well known that the harmonic mean of n positive real numbers $a_1, \dots, a_n$ is less than $\min\{a_1, \dots, a_n\}$ for $n > 1$. So an $F1$ value of x indicates that the model has at least x precision and x recall.

2. $F1$ score can penalize bad precision or bad recall scenarios properly where arithmetic or geometric means may fail. For example, if we consider a recommendation where we do not predict anything, technically, we do not have any errors. So the precision would be 100 percent whereas recall would be 0 percent. An arithmetic average would give us an accuracy of 50 percent which is clearly wrong. $F1$ score penalizes all such scenarios accordingly. For this particular example, we would still have an $F1$ score of 0 percent.

For this paper, we call a test data point *true positive* if a certain movie m is to be recommended to user u based on the rating. If the rating is over k for some fixed k, then the movie is to be recommended, otherwise it is to be discarded. We have showed results for both $k \geq 3$ and $k \geq 4$ in this paper.

We have also used *mean absolute percentage error* as a metric in place of the usual *mean absolute error*. It is easy to see that mean absolute error puts equal weights on all ratings whereas mean absolute percentage errors mitigates that problem to some extent. So we felt it important to also show the results in terms of mean absolute percentage errors.



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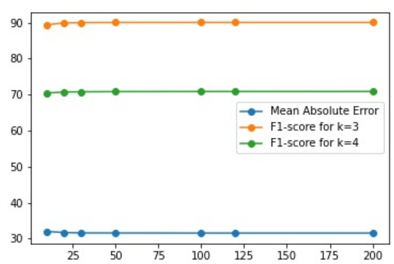

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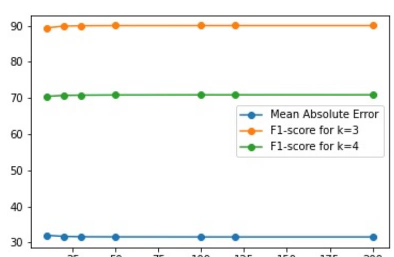

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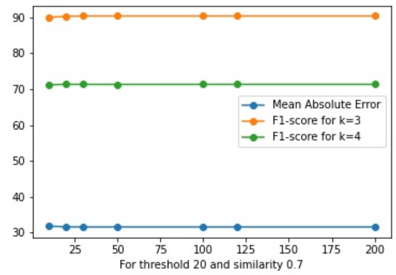
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Mean Absolute Error
F1-score for k=3
F1-score for k=4
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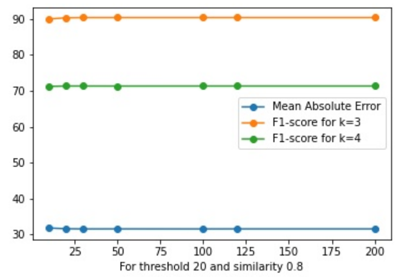
For threshold 20 and similarity 0.7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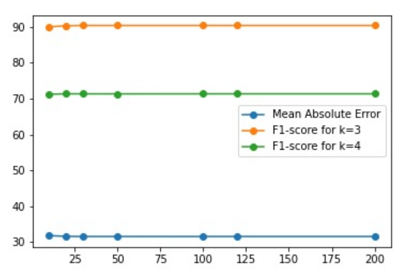
For threshold 20 and similarity 0.8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Mean Absolute Error
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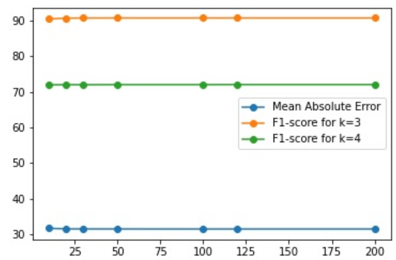
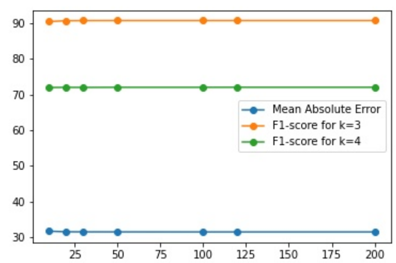
For threshold 20 and similarity 0.9



For threshold 30 and similarity 0.7



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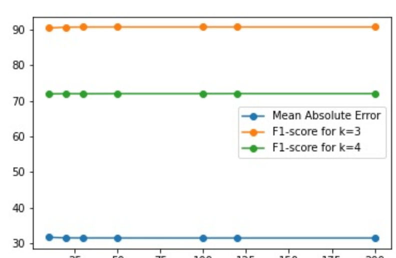

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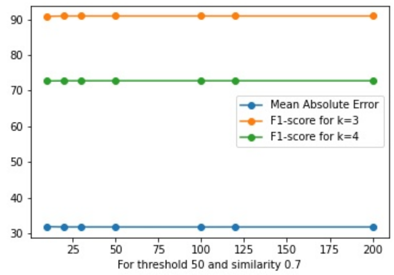
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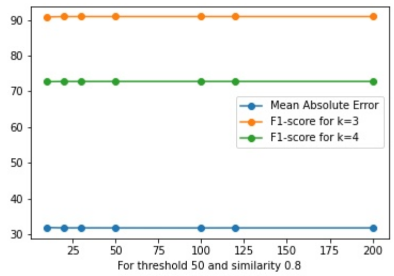
Mean Absolute Error
F1-score for k=3
F1-score for k=4
For threshold 50 and similarity 0.7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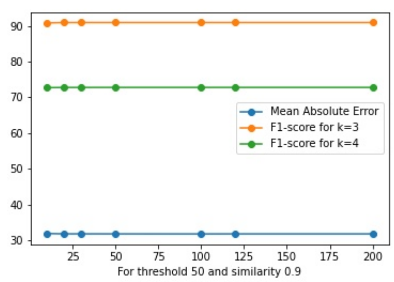
Mean Absolute Error
F1-score for k=3
F1-score for k=4
For threshold 50 and similarity 0.8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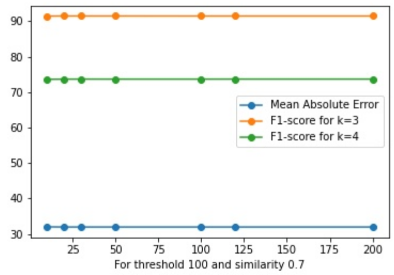
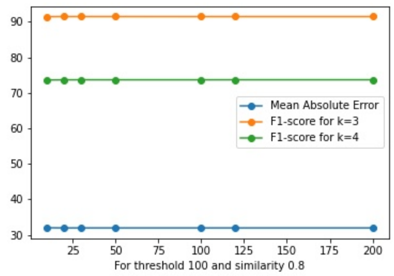
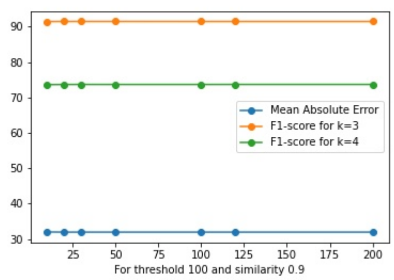
Mean Absolute Error
F1-score for k=3
F1-score for k=4
For threshold 50 and similarity 0.9


Mean Absolute Error
F1-score for k=3
F1-score for k=4
For threshold 100 and similarity 0.7


Mean Absolute Error
F1-score for k=3
F1-score for k=4
For threshold 100 and similarity 0.8


Mean Absolute Error
F1-score for k=3
F1-score for k=4
For threshold 100 and similarity 0.9

References

[1] David Freedman, Robert Pisani, and Roger Purves. "Statistics (international student edition)". In: *Pisani, R. Purves, 4th edn. WW Norton & Company, New York* (2007).