A Simple Yet Effective and Efficient Collaborative Filtering Based Recommendation System

Masum Billal Mahmudul Hasan Munna

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Abstract

In this paper, we describe a simple yet effective way to recommend items to a user and predict the rating of an item given by a user with high accuracy.

1 Introduction

A common problem (insert relevant papers) with typical filtering based recommendation systems they usually consider all users similar to a user. To mitigate performance problems, (insert relevant papers) show that we can simply consider only a handful of neighbors for a particular user.

2 Algorithm

Let us first describe the assumption problem that we want to solve. Let $\mathcal{M}$ be a set of movies and $\mathcal{U}$ be a set of users. We are given a set of triplets $(u, m, r) \in \mathcal{S}$ such that $u \in \mathcal{U}, m \in \mathcal{U}$ and $1 \leq r \leq 5$ which denotes the rating r of movie m given by user u. We are also given a set of pairs $(u, m) \in \mathcal{T}$ such that $u \in \mathcal{U}, m \in \mathcal{M}$ and we want to predict the rating r given by user m to the movie m. Let r_{um} be the rating of movie m given by user u and C_{uv} be the set of items that both u and v rated. We use the *Pearson Correlation Coefficient* (see Freedman, Pisani, and Purves [1]) as the measure of correlation between user u and v. However, we will use a normalized version of it for practical consideration as follows:

$$S_{uv} = \frac{\sum_{m \in C_{uv}} (r_{um} - \bar{r_u})(r_{vm} - \bar{r_v})}{\sqrt{\sum_{m \in C_{uv}} (r_{um} - \bar{r_u})^2} \sqrt{\sum_{m \in C_{uv}} (r_{vm} - \bar{r_v})^2}}$$

$$S_{uv} \leftarrow \frac{S_{uv} + 1}{2}$$

This normalization is done using the fact that Pearson correlation coefficient p_{uv} is always in the range $[-1, 1]$. We call two users u and v similar if $S_{uv} \geq s$ for some positive real number s such that $0 \leq s \leq 1$. Typically, we want s in the range $[.5, 1]$. For this paper, we will consider $s \in \{.7, .8, .9\}$. For a movie m, let $\mathcal{U}_m$ be the set of users who rated m and M_u be the set of movies rated by user u. For a set or tuple A, let $\bar{A}$ denote the average of the numbers in A. For a tuple of weights $\mathbf{w} = (w_1, \ldots, w_n)$ such that $0 \leq w_i \leq 1$ and $\sum_{i=1}^{n} w_i = 1$ and a tuple of positive real numbers $\mathbf{a} = (a_1, \ldots, a_n)$, the *weighted harmonic mean* of $\mathbf{a}$ is defined as

$$\mathfrak{H}(\mathbf{a}, \mathbf{w}) = \frac{\sum_{i=1}^{n} w_i}{\sum_{i=1}^{n} \frac{w_i}{a_i}}$$

Usually, *weighted arithmetic mean* is used to predict the ratings in a recommendation system. But in this paper, we have investigated the results using harmonic mean. Next, we describe the rating prediction algorithm for a pair (u, m).

Algorithm 1: Algorithm to predict rating

Input: Test data in the format (u, m), Threshold t, similarity s, T to take first T neighbors for a user u

Output: A single integer in the range $[1, 5]$ denoting the predicted rating

Data: Train data in the format (u, m, r)

$W = []$
$X = []$
$tot \leftarrow 0$
$R \leftarrow U_m$
res $= 0$
for $v \in R$ **do**
  if $|C_{uv}| < t$ **then**
    continue
  end
  if $S_{uv} < s$ **then**
    continue
  end
  if $S_{uv} \geq s$ **then**
    $W \leftarrow [W, s]$
    $X \leftarrow [X, r_{vm}]$
    $tot \leftarrow tot + 1$
    if $tot > T$ **then**
      break
    end
  end
end
if $tot > 0$ **then**
  res $= \mathfrak{H}(X, W)$
end
else
  if $|M_u| > 0$ **then**
    res $= \bar{M_u}$
  end
  else
    res $= \bar{R_m}$
  end
end
res = res+.5
res = floor(res)
return res

2.1 Algorithm Principles

Our algorithm is based on the following principles.

First principle *Two users u and v are more relevant for each other if it is ensured that $|C_{uv}| \geq t$ for some large enough positive integer t.* It is obvious how this principle helps us establish better correlation between users since two users u and v can have very high correlation with very low number of common items between them.

Second principle *If first principle is established, then it is possible to get an accurate estimation of predicted rating with a lower number of neighbors instead of using a very large number of neighbors.* This helps us predict a rating a lot more efficiently with better accuracy. It is also possible to get a mathematical sense of why this works in practice. Let m and n be positive integers such that $m > n$. Let u be a user and $\mathcal{N}_m = \{v : |C_{uv}| \geq t\}$ and $\mathcal{N}_n = \{u : |C_{uv}| \geq t\}$ be two sets of neighbors of a user u such that $|\mathcal{C}_m| = m$ and $|\mathcal{N}_n| = n$. Using the condition $|C_{uv}| \geq t$, it can be assumed that if $s < t$ for a positive integer s, then

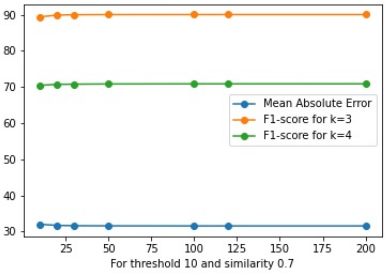
$$P(\sigma^2(A) \leq \sigma^2(B)) \tag{1}$$

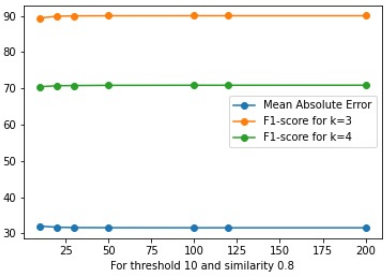
should be very high where $A = \{r_{vm} : |C_{uv}| \geq t\}$, $B = \{r_{vm} : |C_{uv}| \geq s\}$ and $P(x)$ denotes the probability of the random variable x. Since higher value of the threshold t ensures better similarity between two users, we can say in a non-rigorous way that the *Pigeonhole principle* ensures (1) is high enough in practice more often than not.

Third principle *If two sets of ratings A and B have similar variance and consist ratings given by similar users of u only, then $P(|\bar{A} - \bar{B}| < \epsilon)$ is very high for some positive real number ϵ which is considerably smaller than* 1. We can show that a special case holds very often in practice. Since A and B has ratings from similar users only, assume that both A and B consist of 4 and 5 only (our data set rating range is $[1, 5]$). If $|A| = m$ and $|B| = n$ and a is the number of 4 in A whereas b is the number of 4 in B, then

$$\begin{aligned}
\bar{A} &= \frac{4a + 5(m - a)}{m} \\
&= \frac{5m - a}{m} \\
&= 5 - \frac{a}{m} \\
\bar{B} &= \frac{4b + 5(n - b)}{n} \\
&= \frac{5n - b}{n} \\
&= 5 - \frac{b}{n}
\end{aligned}$$

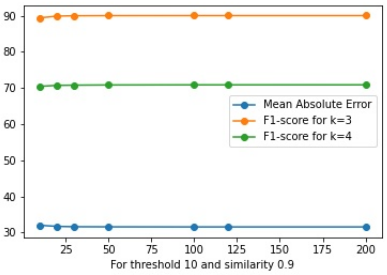
As we can see, it does not matter if $m \gg n$ or $n \gg m$ since the difference $|\bar{A} - \bar{B}|$ or the ratio $\frac{\bar{A}}{\bar{B}}$ only depends on the ratio $\frac{a}{m}$ and $\frac{b}{n}$. So as long as these ratios are similar, $\bar{A}$ and $\bar{B}$ are similar as well.

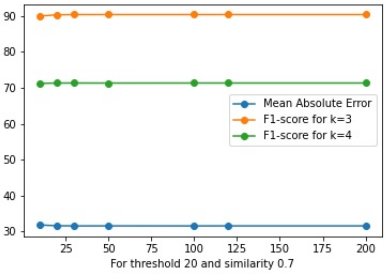


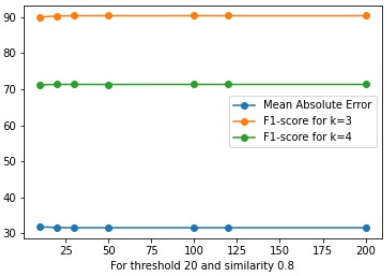


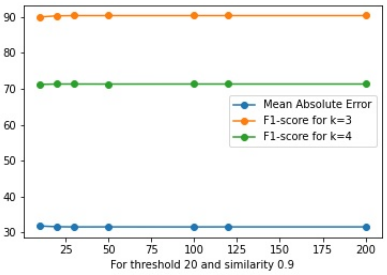
3 Results

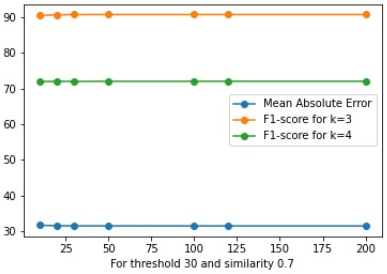
We show the results of our algorithm on the Movielens 1 million data set and backup our claims with empirical evidence below.

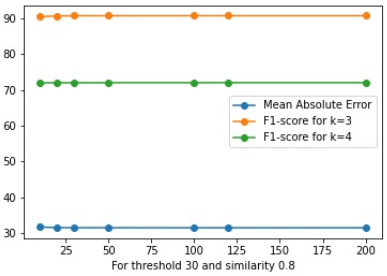


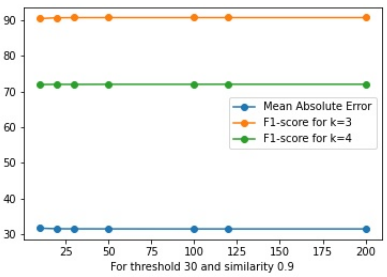

Mean Absolute Error
F1-score for k=3
F1-score for k=4
For threshold 20 and similarity 0.7

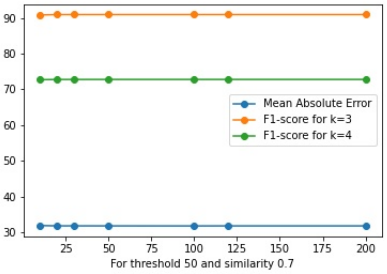

Mean Absolute Error
F1-score for k=3
F1-score for k=4
For threshold 20 and similarity 0.8

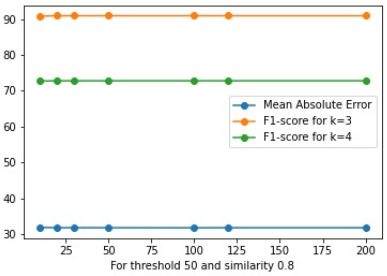

Mean Absolute Error
F1-score for k=3
F1-score for k=4
For threshold 20 and similarity 0.9

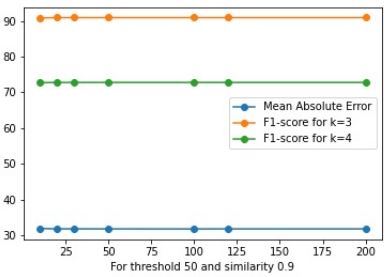

Mean Absolute Error
F1-score for k=3
F1-score for k=4
For threshold 30 and similarity 0.7

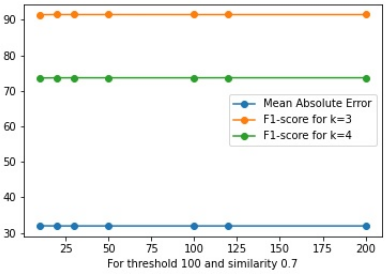

Mean Absolute Error
F1-score for k=3
F1-score for k=4
For threshold 30 and similarity 0.8


Mean Absolute Error
F1-score for k=3
F1-score for k=4
For threshold 30 and similarity 0.9

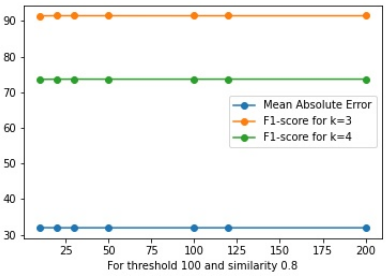

Mean Absolute Error
F1-score for k=3
F1-score for k=4
For threshold 50 and similarity 0.7


Mean Absolute Error
F1-score for k=3
F1-score for k=4
For threshold 50 and similarity 0.8

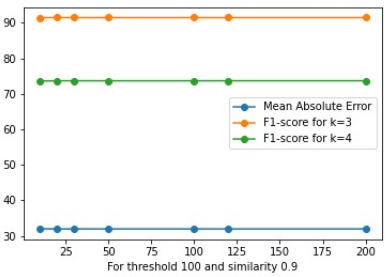

Mean Absolute Error
F1-score for k=3
F1-score for k=4
For threshold 50 and similarity 0.9



For threshold 100 and similarity 0.7



For threshold 100 and similarity 0.8



For threshold 100 and similarity 0.9

References

[1] David Freedman, Robert Pisani, and Roger Purves. "Statistics (international student edition)". In: *Pisani, R. Purves, 4th edn. WW Norton & Company, New York* (2007).