ELEMENTARY ANALYTIC NUMBER THEORY

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Elementary Analytic Number Theory

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Contents

Notations

$\gcd(a, b)$ Greatest common divisor of a and b

$\operatorname{lcm}(a, b)$ Least common multiple of a and b

$\varphi(n)$ Euler's totient function of n

$\tau(n)$ Number of divisors of n

$\sigma(n)$ Sum of divisors of n

$\omega(n)$ Number of distinct prime divisors of n

$\Omega(n)$ Number of total prime divisors of n

$\lambda(n)$ Liouville function of n

$\mu(n)$ Mőbius function of n

$\vartheta(x)$ Tchebycheff function of the first kind

$\psi(x)$ Tchebycheff function of the second kind

$\zeta(s)$ Zeta function of the complex number s

Chapter 1

Introduction

Chapter 2

Arithmetic Functions

Arithmetic function. A real or complex valued function $f : \mathbb{N} \to \mathbb{C}$ is an *arithmetic function*. In this book, we will be concerned with $f : \mathbb{N} \to \mathbb{N}$ primarily.

2.1 Dirichlet Convolution and Generalization

Definition. For two arithmetic functions f and g, the *Dirichlet product* or *Dirichlet convolution* of f and g is defined as

$$f * g = \sum_{d|n} f(d) g\left(\frac{n}{d}\right)$$

We will discuss here how this came to be. Consider the zeta function.

$$\zeta(s) = \frac{1}{1^s} + \frac{1}{2^s} + \dots$$