§4 Proof of Prime Number Theorems

§4 ARTICLES

- ABEL, NIELS HENRIK, "Untersuchungen über die Reihe: $1 + (m/1)x + m \cdot (m-1)/(1 \cdot 2) \cdots x^2 + m \cdot (m-1) \cdot (m-2)/(1 \cdot 2 \cdot 3) \cdots x^3 + ...$ ", Journal für Math., vol. I (1826), pp. 311-339, DOI: 10.1515/9783112347386-030.
- Ayoub, Raymond, "Euler and the zeta function", The American Mathematical Monthly, vol. LXXXI, no. 10 (1974), pp. 1067-1086, doi: 10.2307/2319041.
- Beumer, Martin, "The arithmetical function $\tau_k(n)$ ", The American Mathematical Monthly, vol. LXIX, no. 8 (1962), pp. 777-781, doi: 10.2307/2310778.
- Cahen, Eugène, "Sur la fonction $\xi(s)$ de Riemann et sur des fonctions analogues", fr, *Annales scientifiques de l'École Normale Supérieure*, vol. 3e série, 11 (1894), pp. 75-164, doi: 10.24033/asens.401.
- Euler, Leonhard, "De Progressionibus Harmonicis Observationes", Commentarii academiae scientiarum Petropolitanae, vol. vii (1740), pp. 150-161.
- "De summatione innumerabilium progressionum", Commentarii academiae scientiarum Petropolitanae, vol. v (1738), pp. 91-105.
- "E-20: De summatione innumerabilium Progressionum", *Spectrum* (2020), pp. 52-64, doi: 10.1090/spec/098/10.
- "E-43: De Progressionibus Harmonicis Observationes", *Spectrum* (2020), pp. 133-141, doi: 10. 1090/spec/098/23.
- "E-72: Variae Observationes circa series Infinitas", *Spectrum* (2020), pp. 249-260, poi: 10.109 0/spec/098/41.
- "Variae Observationes circa series infinitas", Commentarii academiae scientiarum Petropolitanae, vol. IX (1744), pp. 160-188.
- HARDY, GODFREY HAROLD and SRINIVASA RAMANUJAN AIYANGAR, "The normal number of prime factors of a number n", Quarterly Journal of Mathematics, vol. XLVIII (1917), pp. 76-92.
- Haukkanen, Pentti, "Expressions for the Dirichlet Inverse of an Arithmetical Function", Notes on Number Theory and Discrete Mathematics, ISSN 1310-5132 Volume 6, 2000, Number 4, Pages 118—124, vol. vi, no. 4 (2000), pp. 118-124, doi: https://nntdm.net/volume-06-2000/number-4/118-124/.
- JENSEN, JOHAN LUDWIG WILLIAM VALDEMAR, "OM RÆKKERS KONVERGENS", Tidsskrift for mathematik, 5th ser., vol. II (1884), pp. 63-72, ISSN: 09092528, 24460737, http://www.jstor.org/stable/24540057.
- "Sur une généralisation d'un théorème de Cauchy", Comptes Rendus (Mar. 1888).
- Landau, Edmund, "Handbuch der Lehre von der Verteilung der Primzahlen", Monatshefte für Mathematik und Physik, vol. xxII, no. 1 (Dec. 1911), doi: 10.1007/bf01742852.

§4. Articles Masum Billal

Landau, Edmund, "Über die Anzahl der Gitterpunkte in geweissen Bereichen", Nachrichten von der Gesellschaft der Wissenschaften zu Göttingen, Mathematisch-Physikalische Klasse, vol. XIX (1912), pp. 687-772.

- "Über eine idealtheoretische funktion", Transactions of the American Mathematical Society, vol. XIII, no. 1 (1912), pp. 1-21, poi: 10.1090/s0002-9947-1912-1500901-6.
- LEHMER, DERRICK NORMAN, "Asymptotic evaluation of certain Totient Sums", American Journal of Mathematics, vol. xxII, no. 4 (1900), pp. 293-335, doi: 10.2307/2369728.
- MacMillan, Kieren and Jonathan Sondow, "Proofs of power sum and binomial coefficient congruences via Pascal's identity", *The American Mathematical Monthly*, vol. cxviii, no. 6 (2011), pp. 549-551, doi: 10.4169/amer.math.monthly.118.06.549.
- MÖBIUS, AUGUST FERDINAND, "Über eine besondere art von Umkehrung der Reihen." Journal für die reine und angewandte Mathematik (Crelles Journal), vol. IX (1832), pp. 105-123, DOI: 10.1515/crll.1832.9.105.
- Pascal, Blaise, "Sommation des puissances numériques", Oeuvres complètes, Jean Mesnard, ed., Desclée-Brouwer, Paris, vol. III (1964), pp. 341-367.
- RIEMANN, BERNHARD, "Ueber die anzahl der primzahlen unter einer gegebenen grösse", *Monatsberichte der Berliner Akademie* (Nov. 1859), pp. 136-144, doi: 10.1017/cbo9781139568050.008.
- TCHEBYCHEFF, "Mémoire sur les nombres premiers." fre, Journal de Mathématiques Pures et Appliquées (1852), pp. 366-390, http://eudml.org/doc/234762.

§4 Books

- AIYANGAR, SRINIVASA RAMANUJAN, "Highly Composite Numbers", in, Collected papers of Srinivasa Ramanujan, ed. by Godfrey Harold Hardy, Peruvemba Venkatesvara Seshu Aiyar, and Bertram Martin Wilson, Cambridge University Press, 1927, pp. 78-128.
- BACHMANN, PAUL GUSTAV HEINRICH, Analytische zahlentheorie, 1894, vol. II.
- Cojocaru, Alina Carmen and Maruti Ram Pedaprolu Murty, An introduction to sieve methods and their applications, Cambridge University Press, 2006.
- DIRICHLET, JOHANN PETER GUSTAV LEJEUNE, "Über Die Bestimmung Der Mittleren Werthe", in, G. Lejeune Dirichlet's Werke, ed. by Leopold Kronecker and László Fuchs, Druck Und Verlag Von Georg Reimer., 1897, vol. II, pp. 49-66.
- Vorlesungen Über Zahlentheorie, ed. by R. Dedekind, Cambridge University Press, 1879.
- HARDY, GODFREY HAROLD, Orders of Infinity: The 'Infinitärcalcül' of Paul Du Bois-Reymond, Cambridge Tracts in Mathematics, Cambridge University Press, 1910.
- HARDY, GODFREY HAROLD and MARCEL RIESZ, The general theory of Dirichlet's series, Cambridge University Press, 1915.
- Jordan, Camille, Traitée des substitutions et des équations algébriques, Gauthier-Villars, Paris, 1870.
- Knoebel, Arthur, Reinhard Laubenbacher, Jerry Lodder, and David Pengelley, "Sums of numerical powers", in, *Mathematical Masterpieces: Further chronicles by the explorers*, Springer-Verlag, 2007, pp. 32-37.
- Landau, Edmund, Handbuch der Lehre von der Verteilung der Primzahlen, 1909, vol. II.
- Mascheroni, Lorenzo, Adnotationes ad calculum Integralem Euleri, Galeatii, 1790.

§4. Books Masum Billal

$\S 4$ Glossary

Erdős He is Erdős

§4 INDEX

Abel partial summation formula, 6 Average order, 2

Dirichlet convolution, 16
Dirichlet hyperbola method, 18
Dirichlet inverse, 17
Dirichlet series, 15
Divisor closed set, 17
Dual convolution, 17

Euler's summation formula, 7 Euler-Mascheroni constant, 9

general convolution, 19 General Dirichlet series, 15 Generalized number of divisor, 2 Generalized sum of divisor, 3

Identity function, 17

Jordan function, 13

Logarithmic integral, 21

Möbius function, 16

Normal order, 2

Pascal identity, 5

Summatory function, 1

Tchebycheff's psi function, 21 Tchebycheff's theta function, 21

Zeta function, 12