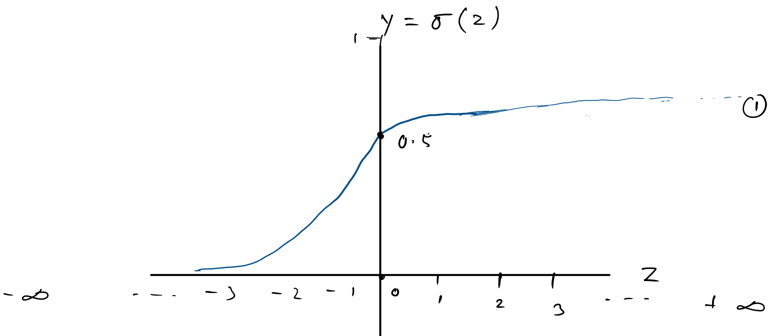
Sigmoid Function

23 August 2025 13:45



① Sigmoid function

Mathematical function

$-\infty$, $+\infty$

⇓

Sigmoid

⇓

[0 − 1]

$$\boxed{\sigma(z) = \frac{1}{1+e^{-z}}}$$

$z = 0$, $y = 0.5$

z = big , y tends to 1

z = small , y tends to 0

$$\left[\begin{array}{c} -\infty < z < +\infty \\ \Downarrow \\ 0 < y < 1 \end{array}\right]$$

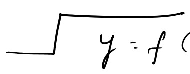
value $(-\infty$ to $+\infty)$

$Z = \boxed{W_0 + W_1 * 8.2 + W_2 * 9}$ [Model]

⇓   ⇓

Z (o/p)   o/p

⇓

step function  $y = f(z)$

$z \geq 0 \Rightarrow +ve \Rightarrow$ class 1

$z < 0 \Rightarrow -ve \Rightarrow$ class 2

Current Implementation

$Z = W_0 + W_1 * 8.2 + W_2 * 9$ → [+1000, −999, 0]   100000, −99999

⇓ z

 $y = \sigma(z) = \frac{1}{1+e^{-z}}$

0 > ~~z~~ > 1

1

⇓

$y = \sigma(z) \quad \frac{1}{1+e^{-z}}$

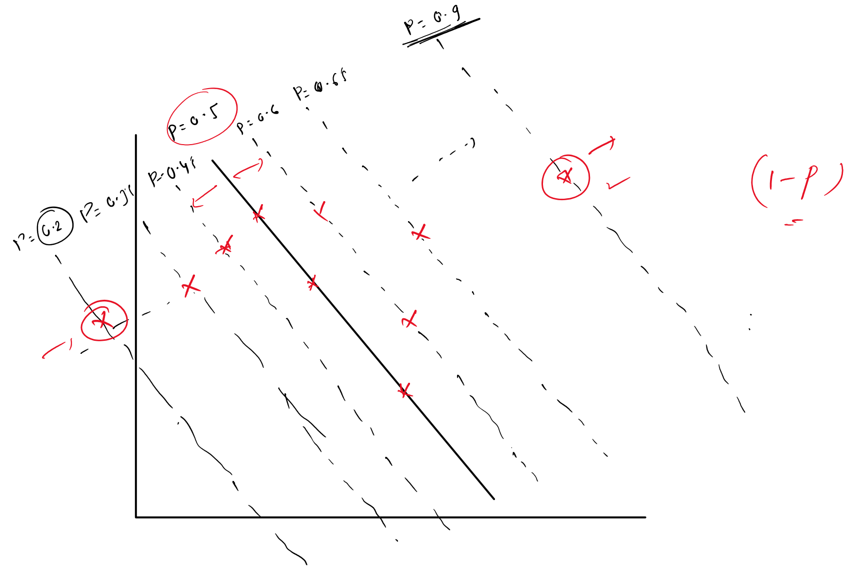
$\Downarrow$

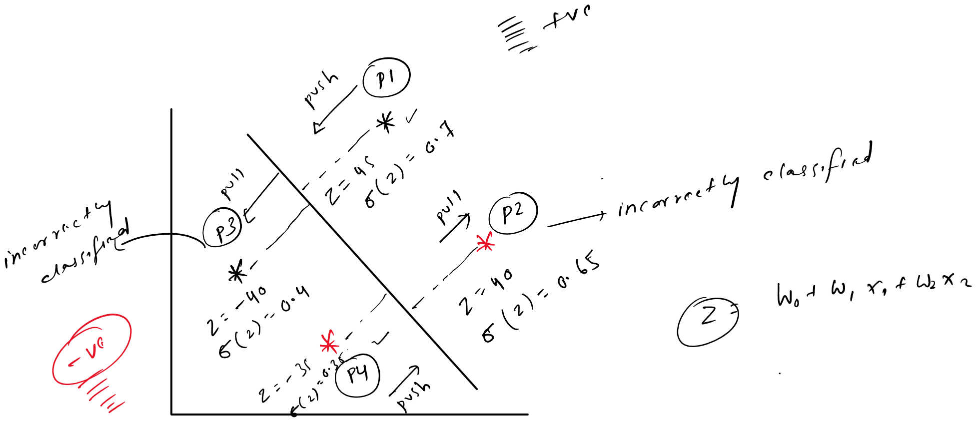
$y = [0 \text{ to } 1]$

$$\begin{cases} y \geq 0.5 \Rightarrow +ve \Rightarrow \text{class } 1 \\ y < 0.5 \Rightarrow -ve \Rightarrow \text{class } 2 \end{cases}$$

$0 > \quad \infty \quad > 1$

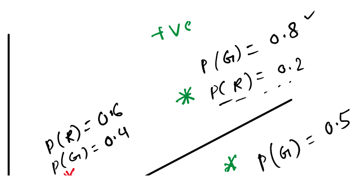
0.5 } $-ve$ } $+ve$

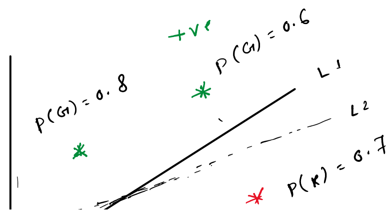


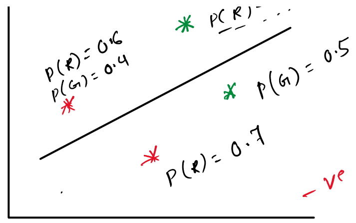


$z = w_0 + w_1 x_1 + w_2 x_2$

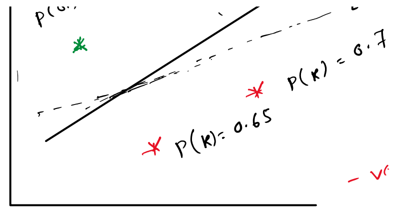
	y_i (Data)	$\hat{y}_i$ (Model o/p)	$y_i - \hat{y}_i$
P1	1	0.7	0.3
P2	0	0.65	−0.65
P3	1	0.4	0.6
P4	0	0.35	−0.35







Model 1



Model 2

Maximum Likelihood

$0.6 * 0.8 * 0.7 * 0.5$

$= 0.168$

$0.8 * 0.6 * 0.65 * 0.7$

$= 0.2184.$

(MLE)

0.2184666611112222 3

0.2184666 1 111 2222 4

Applying the log. transformation.

$\boxed{\log(ab) = \log a + \log b}$

$\log(0.6 * 0.8 * 0.7 * 0.5) = \log(0.6) + \log(0.8) + \log(0.7) + \log(0.5)$ → Maximize

$= -ve \quad -ve \quad -ve \quad -ve \quad = -ve$

Intent

$\boxed{\log([0-1]) = -ve}$

⇓ Transformation.

consider $-ve$ logs.

$-\log(0.6)$

$-(-)$

$+$

$\underbrace{-\log(0.6)}_{+ve} \underbrace{-\log(0.8)}_{+ve} \underbrace{-\log(0.7)}_{+ve} \underbrace{-\log(0.5)}_{+ve} = +ve.$

Cross entrophy.

Minimize

Intent

M1

$\log(0.6)$

-0.221848

$\boxed{0.221}$

M2

$\log(0.8)$

-0.0969

$\boxed{0.096}$

y	$\hat{y}$
1	.
0	.
1	.
0	

Loss Function

$$= \boxed{-y_i \log(\hat{y}_i) - (1-y_i)\log(1-\hat{y}_i)}$$

y_i	$\hat{y}_i$
1	0.8
1	0.6

$-\log(0.8) - (1-1)\log(1-\hat{y}_i) = -\log(0.8).$

$-\log(0.6) - (1-)\log(1-\hat{y}_i) = -\log(0.6)$

1	0.6	$-\log(0.6) - (1-)\log(1-\hat{y}_i) = -\log(0.6)$
0	0.7	$-0(\log 0.7) - (1-0)\log(1-0.7) = -\log(0.3)$
0	0.5	$-0(\log 0.5) - (1-0)\log(1-0.5) = -\log(0.5)$

$$Loss = \sum_{i=1}^{n} -y_i \log(\hat{y}_i) - (1-y_i)\log(1-\hat{y}_i)$$

$$\boxed{Loss = -\frac{1}{n}\sum_{i=1}^{n} y_i \log(\hat{y}_i) + (1-y_i)\log(1-\hat{y}_i)}$$

- Log loss error.
- Binary cross entrophy.