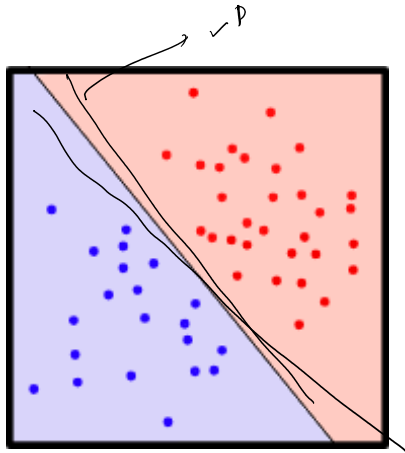


Logistic Regression

20 July 2025 08:04



Perceptron Algorithm

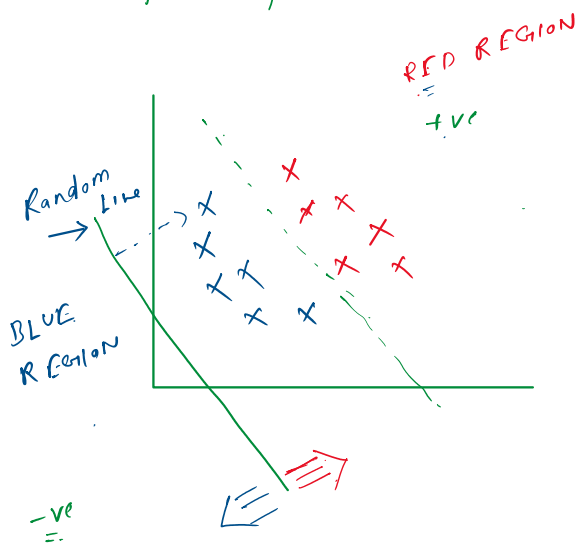
① Logistic Regression

- classification technique
- Binary / Multiclass classification.

Pre-requisites

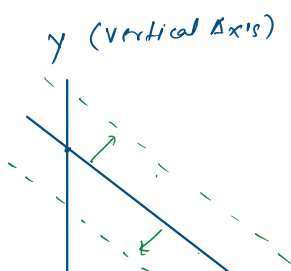
≡ Data must be linearly separable.

~

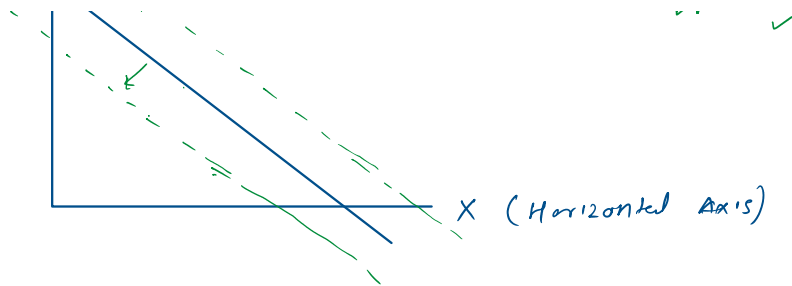


Algorithm

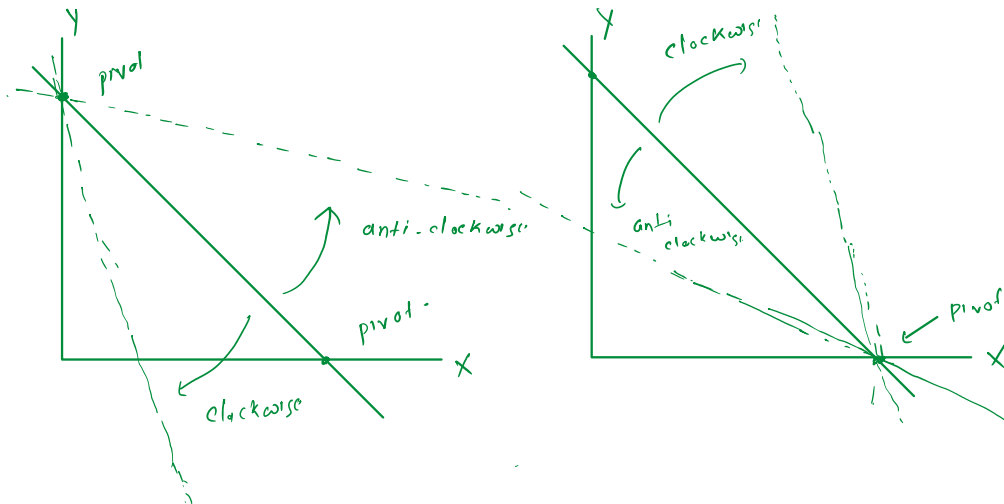
- ① Start with a random line.
- ② Pick an epoch value, 1000 (iteration)
- ③ Repeat the step 1000 times.
 - pick a random point
 - check,
 - If the point is correctly classified:
Ignore the point.
 - If the point is incorrectly classified:
- Move the line towards the point
- ④ Line is at proper place separating the data points.



$$\textcircled{A}x + \textcircled{B}y + \textcircled{C} = 0$$



① changing the intercept



$$y = mx + c \rightarrow \text{Standard equation.}$$

$$w_0x + w_1y + b = 0 \rightarrow \text{General eqn.}$$

① $2x + 3y = 10$

\uparrow increase 15
 \downarrow decrease
 5

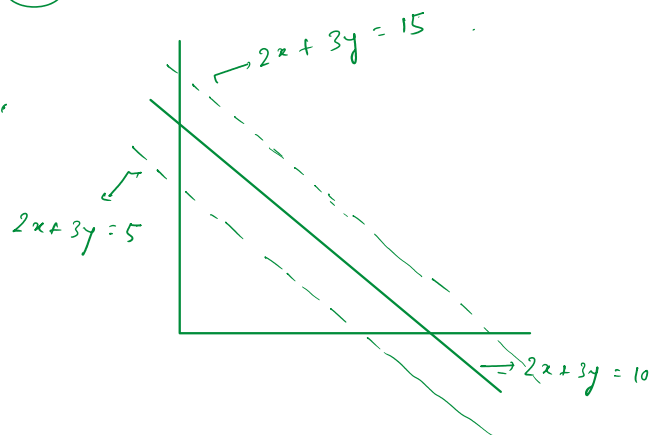
-5

\uparrow

$$2x + 3y + (-10) = 0$$

\downarrow

-15

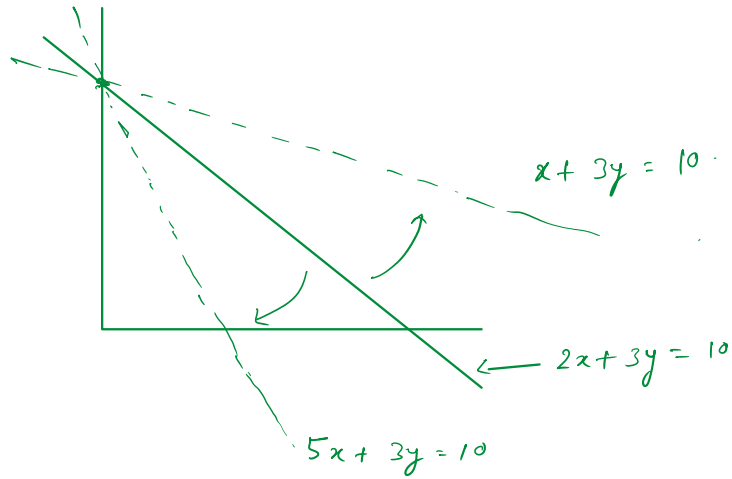


② $\overset{5}{\uparrow}$
 $\boxed{2}x + 3y + (-10) = 0$ [coefficient of x]

2

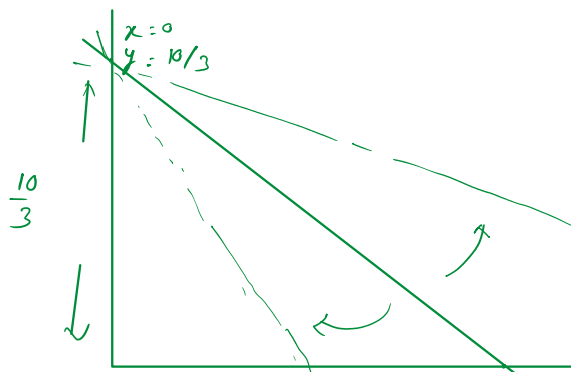
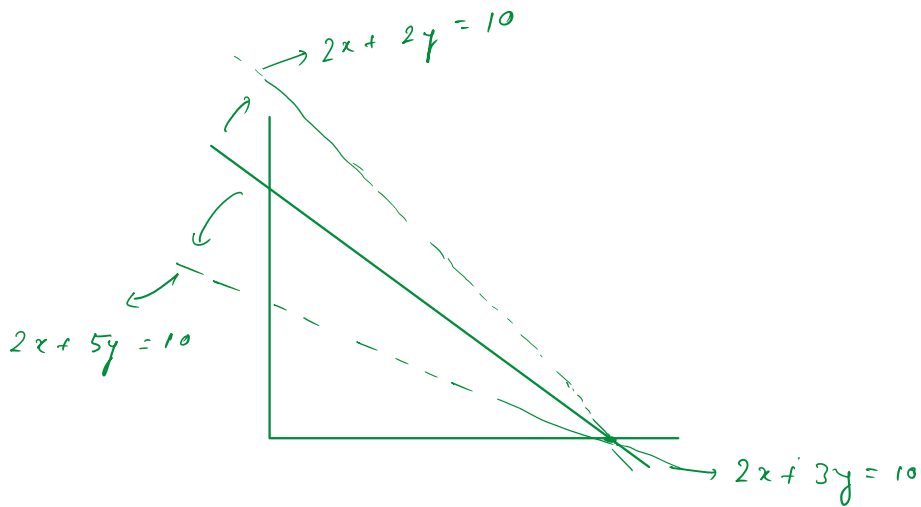
$$\boxed{2}x + 3y + (-10) = 0 \quad [\text{coefficient of } x]$$

↓
↓



$$2x + \boxed{3}y + (-10) = 0$$

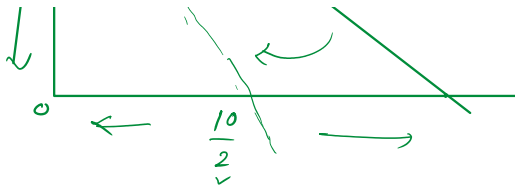
↑
5
↓
2



$$2x + 3y + (-10) = 0$$

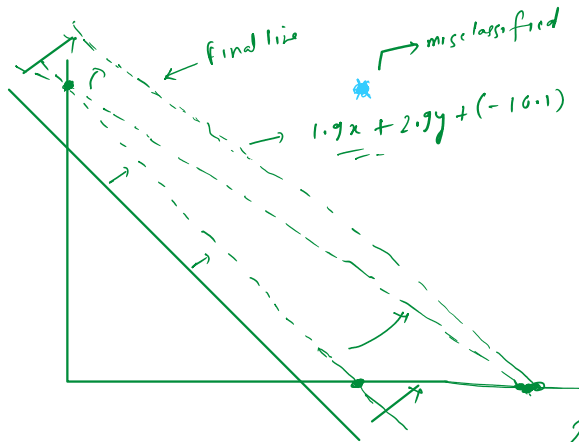
$$x = 0, y = \frac{10}{3}$$

$$y = 0, x = \frac{10}{2} = \boxed{5}$$



$$\frac{10}{5} = 2$$

$$2x + 3y + (-10) = 0$$



+ve

Learning Rate
0.1

$$2x + 3y + (-10) = 0$$

$\downarrow \quad \quad \downarrow \quad \quad \downarrow$
 $-0.1 \quad -0.1 \quad -0.1$

$$1.9x + 2.9y + (-10.1) = 0$$

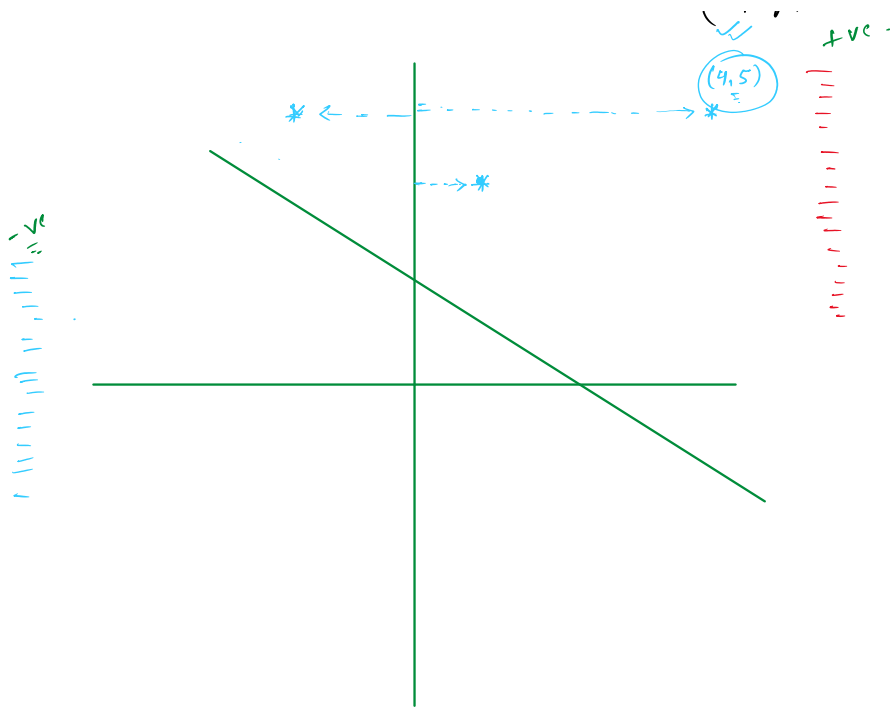
Algo

1. Start with random line $ax + by + c = 0$
2. Pick the epoch value, 1000
3. Pick a learning rate value, 0.1 \rightarrow 0.01 \leftarrow 0.001
4. [Repeat 1000 times]
 - Pick a random point
 - check if correctly classified.
Ignore it
 - check if incorrectly classified.
 - Add ± 0.1 to a
 - Add ± 0.1 to b
 - Add ± 0.1 to c

(x_i, y_i)

$(4, 5)$

+ve



$$\begin{array}{rcl}
 2x + 3y - 10 = 0 \\
 \downarrow \quad \downarrow \quad \downarrow \\
 0.1 \quad 0.1 \quad 0.1
 \end{array}$$

$\eta \rightarrow$ Learning Rate
 0.1

$$\eta x_i \rightarrow 0.1 \times 4 \rightarrow 0.4 \checkmark$$

$$\eta y_i \rightarrow 0.1 \times 5 \rightarrow 0.5$$

$$\eta = 0.1 \\
 (x_i, y_i) = (4, 5)$$

$$\begin{array}{rcl}
 2x + 3y - 10 = 0 \\
 \downarrow \quad \downarrow \quad \downarrow \\
 \eta x_i \quad \eta y_i \quad \eta \\
 \downarrow \quad \downarrow \quad \downarrow \\
 0.1 \times 4 \quad 0.1 \times 5 \quad 0.1 \\
 \downarrow \\
 0.4 \quad 0.5 \quad 0.1
 \end{array}$$

$$1.6x + 2.5y - 10.1 = 0$$

$$\left. \begin{array}{l} \eta = 0.1 \\ (5, 6) \end{array} \right\}$$

$$\begin{array}{rcl}
 2x + 3y - 10 = 0 \\
 \downarrow \quad \downarrow \quad \downarrow \\
 0.5 \quad 0.6 \quad 0.1
 \end{array}$$

$$1.5x + 2.4y - 10.1 = 0$$

$$\left. \begin{array}{l} \eta = 0.1 \\ (25, 20) \end{array} \right\}$$

$$\begin{array}{rcl}
 2x + 3y - 10 = 0 \\
 \downarrow \quad \downarrow \quad \downarrow \\
 -2.5 \quad -2 \quad -0.1
 \end{array}$$

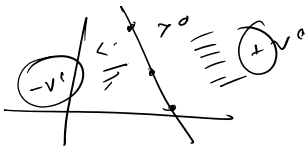
$$-0.5x + y - 10.1 = 0$$

x_0	x_1 income	x_2 age	y loan	\hat{y}
1	55k	22	1	1
1	45k	30	0	1
1	30k	29	1	0
1	50k	24	1	1

$$w_0 x_0 + w_1 x_1 + w_2 x_2 = 0$$

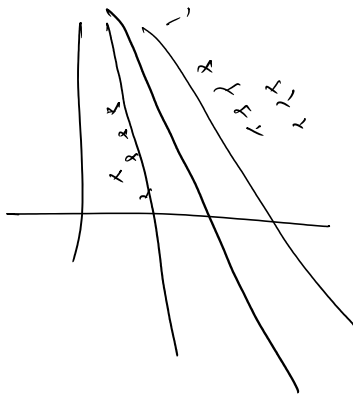
Model

$$\sum_{i=0}^2 w_i x_i = 0$$



$$\sum_{i=0}^2 w_i x_i \geq 0 \quad \text{loan app.}$$

$$\sum_{i=0}^2 w_i x_i < 0 \quad \text{loan rejected}$$



① Select a random point (p)

② if $p \in N\text{-Region}$ & $\sum_{i=0}^2 w_i x_i \geq 0$

$$w_{\text{new}} = w_{\text{old}} - \eta p$$

if $p \in P\text{-Region}$ & $\sum_{i=0}^2 w_i x_i < 0$

$$w_{\text{new}} = w_{\text{old}} + \eta p$$

Learning

datapoint

Incorrectly

$$w_n = w_0 + \eta (\check{y}_i - \hat{y}_i) x_i$$

$$w_n = w_0 + \eta (1 - 1) x_i = w_0$$

$$w_n = w_0 + \eta (1 - 0) x_i$$