2a) Verify that $(H_R - E\cdot I + i\Delta(E))\, G_R(E) = I$ can be solved for each column of $G_R(E)$ individually, reducing every calculation to the form $\hat{A}\vec{x} = \vec{b}$.

$\Rightarrow$ $G_R(E) = (H_R - E\cdot I + i\cdot\Delta(E))^{-1}$

$\Rightarrow$ Let $(H_R - E\cdot I + i\cdot\Delta(E)) = A$, which implies that $G_R = A^{-1}$

$\Rightarrow$ With $A = (a_{ij})$ and $A^{-1}(\hat{a}_{ij})$ we get

$$\begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \\ a_{n1} & \cdots & a_{nn} \end{pmatrix} \cdot \begin{pmatrix} \hat{a}_{11} & \cdots & \hat{a}_{1n} \\ \vdots & & \\ \hat{a}_{n1} & \cdots & \hat{a}_{nn} \end{pmatrix} = \begin{pmatrix} 1 & & 0 \\ 0 & \ddots & \\ \vdots & & 1 \end{pmatrix}$$

$\Rightarrow$ The j-th column of the inverse $\hat{a}_j = \begin{pmatrix} \hat{a}_{1j} \\ \vdots \\ \hat{a}_{nj} \end{pmatrix}$ is therefore the solution

of the system of linear equations: $\underline{\underline{A\cdot\hat{a}_j = \vec{e}_j}}$

where $\vec{e}_j$ is the j-th unit vector!

$\Rightarrow$ $\underline{\underline{A^{-1} = G_R(E) = (\hat{a}_1 \mid \hat{a}_2 \mid \ldots \mid \hat{a}_n)}}$
