Defining "Simulation" of Infinite Constructs and Complexity Limits

Defining "Simulation" of Infinite Constructs Within the Finite Framework

Conceptual Idea

In classical mathematics, infinite constructs (e.g., an infinite sequence converging to a limit) can be defined exactly, but not always approximated finitely without invoking limits. Our finite framework avoids actual infinities but can still "simulate" or approximate infinite constructs to any desired accuracy by increasing capacity.

Definition (Simulation of Infinite Constructs)

Consider a classical infinite process $\{x_n\}$ converging to a limit L. To "simulate" this process in the finite framework:

1. **Representation at Finite Capacity:** At any fixed capacity a, we can represent only a finite set of increments. Define a function that maps each classical number x_n to a finite increment-based representation $\widehat{x_n}^{(a)}$. Initially, $\widehat{x_n}^{(a)}$ may approximate x_n coarsely.

2. **Refinement to Achieve Arbitrary Precision:** Given $\varepsilon > 0$, we want $|\widehat{x_n}^{(a)} - L| < \varepsilon$. Because we can refine capacity at will, there exists a capacity state $a(\varepsilon, n)$ large enough that increments are so fine the represented value $\widehat{x_n}^{(a(\varepsilon,n))}$ lies within ε-distance of L.

3. **No Actual Limit Reached, Only Approximations:** We never reach the "final" infinite stage. Instead, we choose increasingly large capacities for increasingly large n to get closer and closer to L. Thus, for every $\varepsilon > 0$, we can find some n and a sufficiently high capacity a such that $|\widehat{x_n}^{(a)} - L| < \varepsilon$.

Proposition (Finite Simulation of Infinite Processes)

For any convergent sequence $\{x_n\}$ with limit L in classical mathematics, and any $\varepsilon > 0$, there exists a capacity state a and an index n in the finite framework such that the finite representation $\widehat{x_n}^{(a)}$ differs from L by less than ε. As $\varepsilon \to 0$, we increase a and n together to approximate L arbitrarily closely.

This proposition shows how infinite constructs (like limits) are never truly realized but can be "emulated" to any desired precision within the finite framework.

Attempting to Impose a Finite Complexity Limit on an Infinite Framework

Overview

We now consider the opposite direction: starting from a classically infinite framework (like standard set theory or real analysis with full infinite sets), we attempt to impose a hard finite complexity limit Ω. This means we try to say: "No entity with complexity greater than Ω shall exist." But classical infinite frameworks inherently allow the construction of arbitrarily complex entities.

What is Complexity in an Infinite Framework?

Complexity could be defined as the size of sets, the nesting of definitions, or the length of infinite expansions. In classical set theory, for any finite bound Ω, we can construct sets or numbers more complex than that bound. For example, consider a simple diagonalization argument:

- Suppose we adopt a complexity measure C that assigns a finite integer complexity measure to each construct (e.g., complexity might count the number of elements in a set, or the Turing complexity of describing a number).
- Assume there is a universal finite limit Ω beyond which no objects can be considered.

Lemma (Contradiction from Imposing a Finite Limit Ω on an Infinite Framework)

Let the classical infinite framework be standard set theory (ZFC) or real analysis. Suppose we impose a finite complexity limit Ω such that no definable entity may exceed complexity Ω. Then, we reach a contradiction.

Proof Sketch

1. **Classical Infinity Assumption:** Classical infinite frameworks assert the existence of infinite sets (e.g., the natural numbers $\mathbb{N}$). From $\mathbb{N}$, we can construct increasingly complex entities: larger finite sets, sequences, and so forth. Complexity can be roughly correlated with size or nesting depth.

2. **Diagonal Construction:** Given any finite complexity limit Ω, consider all entities definable within that limit. Because the framework is infinite, we can form a new entity (like a set that differs from each entity at complexity level $\leq \Omega$ at some point) that must have complexity greater than Ω. This is analogous to Cantor's diagonal argument, where attempting to list all sets or all infinite expansions leads to the construction of a new entity outside the listed set.

3. **Contradiction:** We find an entity that must exist (because infinite frameworks are not capacity-limited) yet is not allowed by the finite limit Ω. This contradicts the assumption that we could impose a hard finite complexity limit on an inherently infinite system.

Conclusion

In an infinite framework, any finite complexity bound Ω is illusory. The infinite nature of the framework ensures that one can always produce objects beyond any predetermined finite complexity limit.

Interpreting the Asymmetry

- In the finite framework (our PG-based approach), simulating infinite constructs is a matter of approximation. We never violate our finite nature; we just choose larger capacities to get finer approximations.
- In a classical infinite framework, trying to force a finite complexity limit contradicts the very essence of infinitude. Infinite sets and objects mean you can surpass any finite constraint.

Proposition (Asymmetry of Simulation)

The finite capacity framework can simulate infinite constructs within any arbitrary ε-error by increasing capacity, while the classical infinite framework cannot incorporate a hard finite complexity limit Ω without contradiction.

This proposition highlights the one-way nature of the approximation and the impossibility of imposing finite constraints on inherently infinite structures.

Conclusion

We have:

1. A proposition stating how infinite constructs (like limits) can be simulated arbitrarily closely in the finite framework by refining capacity.
2. A lemma showing that imposing a finite complexity limit on a classical infinite framework leads to contradiction.

Together, these results clarify the asymmetry: the finite framework approximates infinite math as closely as desired, while the infinite framework cannot forcibly reduce itself to a strictly finite complexity system.