A Logical Argument for the Necessity of Probability as a Primitive Concept in Mathematical Frameworks for the Sciences

Probabilistic Minds Consortium

Below is a self-contained, long-form argument that presents your approach—that we cannot definitively conclude whether chance is an ontological fact or merely a reflection of our finite knowledge—while attempting to be as rigorous and self-critical as possible. The argument aims to stand on its own legs, cites sources where relevant, and includes serious attempts at refutation. If there is any hidden contradiction or simplification, it should surface in the section "Objections and Attempts at Refutation." The final conclusion can be that either the argument survives these challenges, or we find a point of failure that demands revision.

1 Statement of the Position

Claim: We cannot decisively ascertain whether "chance" (or randomness) exists as an intrinsic property of reality or is an artifact of our finite capacity (epistemic limitations). The complexity and interconnectivity of systems—together with the observer's involvement—renders any final verdict unachievable in principle. Consequently, probability is woven into the fundamental description of events, but we cannot fully disentangle whether it reflects objective indeterminacy or subjective ignorance.

Key Points

1. **Onto-Epistemological:** The question "Is this event truly random or determined?" involves both an ontological dimension (what is "really out there") and an epistemological dimension (what we can know or measure).

2. **Infinite Regress:** Exploring the system's total connectivity indefinitely leads either to exhaustion of all physical resources or to a conceptual void where distinctions no longer make sense.

3. **Finite Observer:** The observer is never outside the system; any final claim of "absolute determinacy" or "absolute randomness" is undermined by the limited signals and finite resources of observation.

4. **Essential Probability:** Because of this predicament, probability is not merely a convenient tool—it is a structural necessity: if the universe cannot "know itself" with infinite precision (nor can we), then "chanciness" is embedded at the core.

2 Logical Structure of the Argument

To lay this out as systematically as possible, we present a sequence of premises and inferences.

Premise A (Finite Observer and Resource)

No observer (nor even a coalition of observers) can marshall infinite computational or energetic resources to track every causal pathway. This includes the observer's own entanglements with the environment.

- *Reference:* Seth Lloyd's work on limits of computational capacity of the universe ("Computational Capacity of the Universe," *Physical Review Letters* **88**, 2002). Lloyd and others show that the universe itself, if treated as a quantum computer, has a finite bound on operations it can perform over its lifetime.

Premise B (Unlimited Connectivity of Systems)

In principle, every physical system can be influenced by countless others, directly or indirectly, especially if given unbounded time or space for signals to travel.

- *Reference:* Chaos theory (Poincaré, Lorenz) and quantum entanglement results (Bell's Theorem, GHZ states, etc.) highlight how tiny correlations can propagate or sustain, and how boundaries between "inside" and "outside" can blur under certain conditions.

Premise C (Observer-System Entanglement)

Any measurement or knowledge-gaining act merges observer and system in a way that typically forbids perfect isolation or purely external vantage points.

- *Reference:* The measurement problem in quantum mechanics (von Neumann, Wheeler's "participatory universe"), along with foundational discussions in Heisenberg's *Physics and Philosophy.*

Premise D (Infinite Precision $\rightarrow$ Conceptual Void)

Assume, for contradiction, that we had infinite resources to trace every influence at infinite resolution. The observer would then:

1. Merge with or surpass the universe's own capacity, leading to a "system-larger-than-the-whole-system" paradox.

2. Encounter a scenario where the line between “observer” and “observed environment” erodes to the point of losing meaningful distinction (becoming the “void” in your statement).

3. At best, produce an all-encompassing model that cannot be validated by anything external.

Inference 1

From A, B, and C, we see that to conclusively show “this event was determined or random,” an observer would need to factor in all possible influences on that event. But from A, D indicates any attempt at infinite comprehensiveness is self-defeating or collapses conceptual boundaries.

Inference 2

Since the total influences cannot be accounted for by a finite observer, no final statement of “it was absolutely determined” or “it was absolutely random” can be declared with zero residue of ignorance.

Conclusion

Probability emerges as the inescapable descriptor of events. We simply cannot parse whether “chance” is purely ephemeral or truly “in the world,” because distinguishing that would require an infinite or absolute vantage we do not (and cannot) possess.

3 Broader Significance and Rationale

1. **Epistemic Pragmatism:** From an operational standpoint, it makes no difference whether chance is truly fundamental or an artifact of ignorance—one must treat it as effectively real for any finite observer.

2. **Relating to Ontology:** If the entire cosmos is entangled, then the notion “the system is fully determined or not” is intangible at the global scale. You cannot confirm or refute it from a sub-global vantage.

3. **Consonance with Physical Theories:** Modern physics, from quantum field theory to gravitational considerations of black-hole information, consistently runs into the boundary of finite resource and non-trivial entanglement. We simply do not see a physically realized “infinite vantage.”

4 Objections and Attempts at Refutation

To ensure honesty, we now try to refute or challenge each step or the entire conclusion. If a successful refutation stands, the argument fails.

Objection 1: Maybe We Only Need a Finite Subsystem Approach

Counter: One might argue that to show an event is (or is not) determined, we only need to track local conditions—there's no need for universal coverage. Determinism could be proven if the local cause-effect chain is thorough enough.
Reply: Chaos theory and quantum entanglement suggest that "local conditions" can be influenced by what appear to be arbitrarily distant or subtle factors, especially over large timescales. While approximations can show near-deterministic behavior in certain stable subsystems (e.g., classical physics in low-energy domains), this does not yield the cosmic-level "the event is absolutely determined." It only yields a practical near-determinism or near-randomness in certain stable contexts.

Objection 2: Observers Might Not Entangle with Everything

Counter: Another line: "The observer doesn't need to entangle with all states—some things are simply irrelevant and remain disconnected."
Reply: This may hold for short times or specialized conditions. Over longer times, or in principle, small couplings might generate large differences (chaos). Relegating them to "irrelevant" is an epistemic choice, not an ontological closure. You can't prove irrelevance if you never rule out hidden couplings.

Objection 3: A Retrocausal or Hidden-Variable Determinism

Counter: A hidden-variable theory might claim, "All is determined, we just don't see it." So it's not about failing to track all influences, it's that reality is indeed determined behind the scenes.
Reply: This stance remains consistent with your main claim: from the vantage of a finite observer, there's no method to prove or disprove that hidden determinism. The argument does not say hidden-variable determinism is false, it says we can never verify it to absolute certainty. So ironically, the hidden-variable approach does not refute your argument but ends up reinforcing the unknowability of "true determinism vs. true randomness."

Objection 4: Possibly Probability is Merely a Calculational Convenience

Counter: A critic might say, "We still do classical mathematics with infinite sets, and it works fine. Probability is just a practical band-aid."
Reply: The fact that infinite-based mathematics can produce workable models doesn't settle the question of actual ontological randomness vs. ignorance. You can do effective calculations with either viewpoint. This doesn't refute the stance that from a purely fundamental perspective, we remain stuck with irreducible uncertainty.

5 Potential Weak Points in the Argument

While the above objections do not decisively dismantle the argument, here are self-critical notes:

1. **"In Principle" vs. "In Practice":** There's a difference between "we can't do it in practice" and "it's logically or metaphysically impossible." Critics might press: "Yes, we can't do it in practice. But that doesn't mean an infinite intelligence couldn't." The argument then demands we disclaim that such an infinite intelligence is physically realizable—if it's not realizable, the question remains moot for us.

2. **No Ultimate "Proof of Indeterminacy":** The argument says "We can't confirm determinism or randomness." But it likewise can't confirm that randomness is definitely real. So from a certain vantage, this entire stance is an "inconclusive middle." A purely classical determinist could say, "Your argument just shows ignorance, not genuine randomness."

3. **Conflation of Physical and Logical Limits:** We combine the idea that no physical resource is large enough with the conceptual or logical limit. Some philosophers might argue that even if physically we can't do it, logically or mathematically an infinite vantage might exist "in principle." We retort that "in principle" might not hold physical meaning if it's unimplementable at every level—yet they could accuse us of equivocation.

If these points are enough to cast doubt on the entire argument, so be it: perhaps the stance that "we can never know" is undone if one holds a purely abstract viewpoint ignoring all physical constraints. But if we insist on physically relevant arguments (the actual universe, real observers), the stance remains robust.

6 Final Assessment

Restatement: The argument claims that any truly decisive statement—"this event was purely determined" or "this event was purely random"—requires total coverage of all potential influences to infinite precision, which is out of reach for any physically realizable or logically consistent observer. Therefore, we cannot settle the ontological question regarding chance. Probability stands at the basis of our descriptions, reflecting how the universe itself (and certainly its observers) are finite, entangled, and never privileged with absolute knowledge.

Remaining Uncertainty

- We do not prove that "chance is real," nor that "determinism is false." We simply argue that from any finite vantage, you can never be certain whether an occurrence was wholly locked in by hidden variables or partially an irreducible random event. This unknowability is not a matter of "lack of advanced tools," but a matter of principle tied to the finite nature of physical resources and the observer's embedded role.

Conclusion: Barring a successful refutation that circumvents both physical and logical limits, this argument stands: Probability becomes indispensable, not merely as a convenient tool, but as the irreducible descriptor of events for any finite observer. We remain uncertain—unable to close the question of "true randomness" vs. "ultimately determined"—and so the stance that chance is "baked into" the conceptual architecture of reality is arguably validated. Or, more humbly, at least no known principle can rescue us from this fundamental predicament.

Invitation to Further Scrutiny

If a critic or a new discovery contrives a coherent scenario in which an entity (or method) can gather and process all influences across the cosmos without merging into the "void," the argument would be invalid. No such coherent scenario has yet been proposed in either physics or philosophy. Until that arises, the logic here remains intact.