Part 3. Number Theory within the VOID Granularity Framework

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3 Introduction

In traditional mathematics, numbers are abstract entities introduced through axioms and definitions, often without direct reference to physical reality or dynamic processes. They serve as the foundation for various mathematical constructs and operations, existing within an idealized framework. However, as our understanding of the physical world deepens, especially at microscopic scales, we recognize that the act of counting and measuring is intrinsically linked to observable changes and finite limitations.

The VOID Granularity Framework (VGF) proposes a redefinition of number theory by grounding numbers in quantifiable changes within dynamic systems. This approach acknowledges that numbers emerge from observable distinctions and changes that surpass a minimal meaningful threshold, denoted as δ_{VOID}, below which differences become indistinguishable. By constructing number theory from this foundational concept, we align mathematical abstractions with physical reality, ensuring consistency with both ontological and epistemological perspectives.

This document presents a comprehensive development of number theory within the VGF, starting from the fundamental idea of numbers as representations of change. We build the theory step by step, formalizing each concept, introducing necessary axioms, and providing detailed explanations and examples. We delve into the treatment of negative numbers, address the role of time, and compare this approach with classical mathematics. Additionally, we discuss resolved paradoxes, potential benefits, and the emergence of new number classes within this framework.

4 Development of Number Theory within the VGF

4.1 Numbers as Representations of Change

4.1.1 Classical Definition of Numbers

In classical mathematics, numbers are abstract symbols representing quantities, order, or measurements. The natural numbers ($\mathbb{N}$) begin with 1 and progress indefinitely through the successor function $S(n) = n + 1$. Zero and negative numbers are introduced to form the integers ($\mathbb{Z}$), and further extensions lead to rational ($\mathbb{Q}$) and real numbers ($\mathbb{R}$). These numbers exist within an idealized, infinitely divisible continuum, often detached from physical limitations.

Limitations of Classical Approach:

- Infinite Divisibility: Assumes the ability to distinguish infinitely small differences, which is impractical due to physical constraints.
- Abstract Existence: Numbers exist independently of physical reality, which can lead to paradoxes and inconsistencies when modeling real-world systems.

4.1.2 VGF Perspective on Numbers

In the VOID Granularity Framework, we redefine numbers by grounding them in observable changes within dynamic systems. A number represents a quantifiable change or distinction between two states that exceeds a minimal meaningful threshold, δ_{VOID}.

Key Principles:

1. Numbers Emerge from Change:
   - Numbers quantify the change between an initial state s_i and a final state s_f within a system S.
   - A change is only meaningful if it surpasses δ_{VOID}, ensuring distinguishability.
2. Pair of Numbers:
   - The fundamental unit of measurement is a pair (s_i, s_f), representing the states before and after a change.
   - This acknowledges that change is relational, requiring comparison between two distinct states.
3. Minimal Meaningful Distinction (δ_{VOID}):
   - A finite, positive value representing the smallest change that can be meaningfully observed or measured within the system.
   - Changes smaller than δ_{VOID} are considered indistinct and are assigned a value of zero.
4. Onto-Epistemological Alignment:
   - Recognizes that the limitations in distinguishing changes are inherent to both the nature of the system (ontology) and our knowledge of it (epistemology).
   - Ensures consistency between what exists and what can be known.

4.1.3 Formal Definitions and Axioms

To construct this theory, we introduce formal definitions and axioms that build upon the foundational principles of the VGF.

[System S] A dynamic system S is a finite set of states with defined relationships and transitions, pertinent to a specific domain (physical, computational, etc.).

[States s_i and s_f] States s_i and s_f represent the initial and final configurations of the system S, respectively.

[Change Δs] The change between two states is defined as:

$$\Delta s = s_f - s_i$$

Axiom 3.1: Minimal Meaningful Distinction There exists a minimal positive value δ_{VOID} such that:

- If $|\Delta s| \geq \delta_{\text{VOID}}$, then the change is distinguishable.
- If $|\Delta s| < \delta_{\text{VOID}}$, then the change is indistinct and considered zero.

[Number as Change] A number n in the VGF is defined by:

$$n = \begin{cases} \Delta s & \text{if } |\Delta s| \geq \delta_{\text{VOID}} \\ 0 & \text{if } |\Delta s| < \delta_{\text{VOID}} \end{cases}$$

Axiom 3.2: Pair of Numbers and Change A number arises from the comparison between two distinct states s_i and s_f where the change Δs is meaningful ($|\Delta s| \geq \delta_{\text{VOID}}$).

Axiom 3.3: Symmetry of Change For any measurable quantity q within system S, the change $\Delta q = q_f - q_i$ satisfies:

$$\mu_S(q_i, q_f) = \mu_S(q_f, q_i)$$

The distinguishability probability is symmetric with respect to the direction of change.

Key Principles at Work:

- Finite Granularity: Acknowledges physical limitations in measuring and observing changes.
- Dynamic Systems: Emphasizes that numbers represent changes within systems that are inherently dynamic.
- Symmetry: Recognizes that changes can be increases or decreases, reflected in positive or negative numbers.

4.1.4 Difference Operator Δ

To formalize the quantification of change, we introduce the difference operator Δ.

[Difference Operator Δ] For any measurable quantity $q : T \to \mathbb{R}$ within system S, where T is the set of time points, the change over a time interval Δt is defined as:

$$\Delta q = q(t_f) - q(t_i)$$

where:

- $t_i, t_f \in T$
- $\Delta t = t_f - t_i \geq \delta_t$

4.2 Examples with Formalization

4.2.1 Mathematical Formalization of Change

Before introducing physical examples, we present the mathematical formalism underlying the concept of change within the VGF.

Definition of Change in a System:

Let S be a dynamic system with measurable quantities. For any measurable quantity q within S, a change is defined as:

$$\Delta q = q_f - q_i$$

where:

- q_i is the initial value of q.
- q_f is the final value of q.
- Δq is the change in q.

Assignment of Numbers Based on Change:
According to the VGF, a number n is assigned based on the change Δq:

$$n = \begin{cases} \Delta q & \text{if } |\Delta q| \geq \delta_{\text{VOID}} \\ 0 & \text{if } |\Delta q| < \delta_{\text{VOID}} \end{cases}$$

Distinguishability Function:

$$\mu_S(q_i, q_f) = \frac{1}{1 + \frac{|\Delta q|}{\delta_{\text{VOID}}}}$$

- As $|\Delta q|$ increases beyond δ_{VOID}, $\mu_S(q_i, q_f)$ decreases, indicating greater distinguishability.

Interpretation:

- A higher $|\Delta q|$ relative to δ_{VOID} results in a lower probability of the change being indistinct, meaning the change is more easily distinguishable.

4.2.2 Mathematical Example

Mathematical Scenario:

- System S: A numerical sequence representing states of a dynamic system.
- Minimal Meaningful Distinction: $\delta_{\text{VOID}} = 0.1$.

Initial State:

- $q_i = 5.0$

Final State:

- $q_f = 4.8$

Calculations:

1. Change in Quantity:

$$\Delta q = q_f - q_i = 4.8 - 5.0 = -0.2$$

2. Distinguishability Check:

$$|\Delta q| = |-0.2| = 0.2 \geq \delta_{\text{VOID}} = 0.1$$

   - Since $|\Delta q| \geq \delta_{\text{VOID}}$, the change is distinguishable.

3. Number Assignment:

$$n = \begin{cases} \Delta q & \text{if } |\Delta q| \geq \delta_{\text{VOID}} \text{ and } \Delta t \geq \delta_t \\ 0 & \text{otherwise} \end{cases}$$

4. Calculating Distinguishability Function:

$$\mu_S(q_i, q_f) = \frac{1}{1 + \frac{|\Delta q|}{\delta_{\text{VOID}}}} = \frac{1}{1 + \frac{0.2}{0.1}} = \frac{1}{3} \approx 0.333$$

   - Indicates a significant probability of distinguishing the change.

Interpretation:

- The change from $q_i = 5.0$ to $q_f = 4.8$ is a distinguishable decrease quantified by the number $n = -0.2$.

4.2.3 Physical Example: Observation of a Dropping Mass

System S:

- A mass m suspended at a height.

Parameters:

- Initial Height (h_i): 10.0 m
- Final Height (h_f): 9.8 m
- Minimal Meaningful Distinction (δ_{VOID}): 0.1 m

Calculations:

1. Change in Height:

$$\Delta h = h_f - h_i = 9.8 \text{ m} - 10.0 \text{ m} = -0.2 \text{ m}$$

2. Distinguishability Check:

$$|\Delta h| = |-0.2 \text{ m}| = 0.2 \text{ m} \geq \delta_{\text{VOID}} = 0.1 \text{ m}$$

   - Since $|\Delta h| \geq \delta_{\text{VOID}}$, the change is distinguishable.

3. Number Assignment:

$$n = \Delta h = -0.2 \text{ m}$$

4. Calculating Distinguishability Function:

$$\mu_S(h_i, h_f) = \frac{1}{1 + \frac{|\Delta h|}{\delta_{\text{VOID}}}} = \frac{1}{1 + \frac{0.2}{0.1}} = \frac{1}{3} \approx 0.333$$

   - Indicates a significant probability of distinguishing the change in height.

Interpretation:

- The mass m has fallen by 0.2 m, a change that is distinguishable within the system.
- The number $n = -0.2$ m quantifies this observable change.

4.3 Negative Numbers and Discrete Time

4.3.1 Formal Definition of Negative Numbers

[Negative Number] A number n is negative if it represents a distinguishable decrease in a measurable quantity q over a time interval $\Delta t \geq \delta_t$, such that:

$$\Delta q = q(t_f) - q(t_i) < 0 \quad \text{and} \quad |\Delta q| \geq \delta_{\text{VOID}}$$

Explanation:

- Negative numbers quantify observable decreases in quantities within the system.
- The change must satisfy both the quantitative and temporal minimal distinctions.

4.3.2 Incorporating Time

Axiom 3.4: Discrete Time Steps Time advances in discrete intervals of minimal duration δ_t, ensuring changes occur over meaningful periods.

- Time Interval:

$$\Delta t = t_f - t_i \geq \delta_t$$

- Discrete Time Framework:

– Time is considered as a sequence $\{t_0, t_1, t_2, \ldots\}$, with $t_{n+1} - t_n \geq \delta_t$.

Changes Over Time:

$$\Delta q(t) = q(t_f) - q(t_i)$$

Number Assignment:

$$n = \begin{cases} \Delta q & \text{if } |\Delta q| \geq \delta_{\text{VOID}} \text{ and } \Delta t \geq \delta_t \\ 0 & \text{otherwise} \end{cases}$$

Explanation:

- Ensures that changes are not instantaneous and are observable within finite time steps.

4.3.3 Formalization with Minimal Physical Assumptions

Assumptions:

- Minimal Meaningful Distinction in Quantity: δ_{VOID}
- Minimal Meaningful Time Interval: δ_t

Formal Definition:

- Discrete Time Indexing:

$$t_n = t_0 + n\delta_t, \quad \text{where } n \in \mathbb{N}$$

- State of Quantity at Time t_n:

$$q_n = q(t_n)$$

- Change Between Time Steps:

$$\Delta q_n = q_n - q_{n-1}$$

- Number Assignment:

$$n = \begin{cases} \Delta q_n & \text{if } |\Delta q_n| \geq \delta_{\text{VOID}} \\ 0 & \text{if } |\Delta q_n| < \delta_{\text{VOID}} \end{cases}$$

Negative Numbers in Time:

- If $\Delta q_n < 0$ and $|\Delta q_n| \geq \delta_{\text{VOID}}$, then $n = \Delta q_n$ is a negative number representing a distinguishable decrease over the time interval δ_t.

4.3.4 Example Incorporating Time

Mathematical Scenario:

- Minimal Meaningful Distinction: $\delta_{\text{VOID}} = 1$
- Minimal Time Interval: $\delta_t = 1$ unit of time
- Sequence of Measurements:
  - At t_0: $q_0 = 10$
  - At $t_1 = t_0 + \delta_t$: $q_1 = 8$

Calculations:

1. Change in Quantity:

$$\Delta q_1 = q_1 - q_0 = 8 - 10 = -2$$

2. Distinguishability Check:

$$|\Delta q_1| = 2 \geq \delta_{\text{VOID}} = 1$$

3. Number Assignment:

$$n = \Delta q_1 = -2$$

4. Calculating Distinguishability Function:

$$\mu_S(q_0, q_1) = \frac{1}{1 + \frac{|\Delta q_1|}{\delta_{\text{VOID}}}} = \frac{1}{1 + \frac{2}{1}} = \frac{1}{3} \approx 0.333$$

   - Indicates a significant probability of distinguishing the change.

Interpretation:

- The quantity q decreased by 2 units over a time interval of δ_t.
- The negative number $n = -2$ represents this distinguishable decrease.

Physical Example:

- System S: Temperature of a cooling object.
- Minimal Temperature Change: $\delta_{\text{VOID}} = 0.5°\text{C}$
- Time Steps: $\delta_t = 1$ minute

Measurements:

- At t_0: $T_0 = 100°\text{C}$
- At $t_1 = t_0 + \delta_t$: $T_1 = 99°\text{C}$

Calculations:

1. Change in Temperature:

$$\Delta T_1 = T_1 - T_0 = 99°\mathrm{C} - 100°\mathrm{C} = -1°\mathrm{C}$$

2. Distinguishability Check:

$$|\Delta T_1| = 1°\mathrm{C} \geq \delta_{\mathrm{VOID}} = 0.5°\mathrm{C}$$

3. Number Assignment:

$$n = \Delta T_1 = -1°\mathrm{C}$$

4. Calculating Distinguishability Function:

$$\mu_S(T_0, T_1) = \frac{1}{1 + \frac{1°\mathrm{C}}{0.5°\mathrm{C}}} = \frac{1}{1+2} = \frac{1}{3} \approx 0.333$$

   - Indicates a significant probability of distinguishing the change in temperature.

Interpretation:

- The temperature decreased by 1°C over one minute.
- The negative number $n = -1°\mathrm{C}$ quantifies this distinguishable decrease.

Axiom 3.5: Change Requires Time Formal Statement: For any measurable change Δq in a quantity q within a dynamic system S, there exists a finite, non-zero time interval Δt such that:

1. The change Δq occurs over the time interval Δt.
2. The time interval satisfies $\Delta t \geq \delta_t$, where $\delta_t > 0$ is the minimal meaningful duration, representing the smallest observable time interval in the system.

Mathematical Formulation: Given $t_i, t_f \in T$ (the set of time points in S) such that $t_f > t_i$, we have:

1. Time Interval:

$$\Delta t = t_f - t_i \geq \delta_t$$

2. Change in Quantity:

$$\Delta q = q(t_f) - q(t_i)$$

3. Minimal Time Threshold:

$$\delta_t > 0$$

Explanation:

- This axiom ensures that any meaningful change is associated with the passage of time and is not instantaneous.
- It introduces the minimal temporal granularity δ_t, analogous to the minimal distinction δ_{VOID} for quantities.

Axiom 3.6: Numbers Represent Observable Changes Formal Statement: A number n assigned to a change Δq in a system S must correspond to a change that is both quantitatively and temporally distinguishable, adhering to the minimal distinctions δ_{VOID} and δ_t.

Mathematical Conditions:

1. Quantitative Distinguishability:
$$|\Delta q| \geq \delta_{\text{VOID}}$$
2. Temporal Distinguishability:
$$\Delta t \geq \delta_t$$
3. Number Assignment:
$$n = \Delta q$$

Explanation:

- This axiom ensures that numbers represent changes that are observable within the system, both in terms of magnitude and the time over which they occur.
- It grounds the assignment of numbers in tangible, measurable phenomena, reinforcing the link between mathematics and the dynamic nature of reality within the VGF.

4.3.5 Mathematical Example Incorporating Axioms

Consider the following mathematical scenario:

- System S: A numerical sequence representing states of a dynamic system over time.
- Minimal Meaningful Distinctions:
  - Quantitative: $\delta_{\text{VOID}} = 0.1$
  - Temporal: $\delta_t = 1$ unit of time

Initial State:

- $q(t_i) = 5.0$ at time $t_i = 0$

Final State:

- $q(t_f) = 4.8$ at time $t_f = 2$

Calculations:

1. Time Interval:
$$\Delta t = t_f - t_i = 2 - 0 = 2 \geq \delta_t$$

2. Change in Quantity:

$$\Delta q = q(t_f) - q(t_i) = 4.8 - 5.0 = -0.2$$

3. Distinguishability Checks:

   - Quantitative:
   $$|\Delta q| = 0.2 \geq \delta_{\text{VOID}}$$
   - Temporal:
   $$\Delta t = 2 \geq \delta_t$$

4. Number Assignment:

   - Since both conditions are satisfied, $n = \Delta q = -0.2$

5. Calculating Distinguishability Function:

$$\mu_S(s_i, s_f) = \frac{1}{1 + \frac{|\Delta q|}{\delta_{\text{VOID}}} + \frac{\Delta t}{\delta_t}} = \frac{1}{1 + \frac{0.2}{0.1} + \frac{2}{1}} = \frac{1}{1+2+2} = \frac{1}{5} = 0.2$$

Interpretation:

- The change from $q = 5.0$ to $q = 4.8$ over a time interval of 2 units is distinguishable both quantitatively and temporally.
- The number $n = -0.2$ represents this observable change.

4.3.6 Physical Example Incorporating Axioms

System S:

- A mass m in motion, observed over time.

Parameters:

- Minimal Meaningful Distinctions:
  - Quantitative (Distance): $\delta_{\text{VOID}} = 0.1$ m
  - Temporal: $\delta_t = 0.5$ s
- Initial Observation:
  - Position $x(t_i) = 10.0$ m at time $t_i = 0$ s
- Final Observation:
  - Position $x(t_f) = 9.8$ m at time $t_f = 1.0$ s

Calculations:

1. Time Interval:

$$\Delta t = t_f - t_i = 1.0 \text{ s} - 0 \text{ s} = 1.0 \text{ s} \geq \delta_t$$

2. Change in Position:

$$\Delta x = x(t_f) - x(t_i) = 9.8 \text{ m} - 10.0 \text{ m} = -0.2 \text{ m}$$

3. Distinguishability Checks:

   - Quantitative:
$$|\Delta x| = 0.2 \text{ m} \geq \delta_{\text{VOID}}$$
   - Temporal:
$$\Delta t = 1.0 \text{ s} \geq \delta_t$$

4. Number Assignment:

   - Since both conditions are satisfied, $n = \Delta x = -0.2$ m

5. Calculating Distinguishability Function:

$$\mu_S(s_i, s_f) = \frac{1}{1 + \frac{|\Delta x|}{\delta_{\text{VOID}}} + \frac{\Delta t}{\delta_t}} = \frac{1}{1 + \frac{0.2}{0.1} + \frac{1.0}{0.5}} = \frac{1}{1+2+2} = \frac{1}{5} = 0.2$$

Interpretation:

- The mass m has moved 0.2 m downward over 1.0 s, a change that is distinguishable in both magnitude and time.
- The number $n = -0.2$ m quantifies this observable change.

4.4 Differences from Classical Mathematics

Finite Granularity vs. Infinite Divisibility:

- VGF: Recognizes finite granularity, rejecting infinite divisibility due to practical limitations.
- Classical Mathematics: Assumes infinite divisibility, leading to concepts like real numbers with infinite decimal expansions.

Numbers as Change vs. Abstract Entities:

- VGF: Numbers emerge from quantifiable changes within systems.
- Classical Mathematics: Numbers exist independently of physical processes.

Negative Numbers with Physical Interpretation:

- VGF: Negative numbers represent actual decreases in quantities, tied to observable phenomena.
- Classical Mathematics: Negative numbers are additive inverses without necessary physical interpretation.

4.5 Addressing Classical Paradoxes

4.5.1 Zeno's Paradoxes

Original Statement of the Paradox:

- Dichotomy Paradox:
  - To reach a destination, one must first cover half the distance, then half the remaining distance, and so on infinitely.
  - This implies completing an infinite number of tasks, making motion impossible.

Logical Formalization:

- Infinite Series of Distances:

$$s = \sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n s_0$$

  - s_0 is the total distance.
  - The sum converges to s_0.

- Infinite Series of Time Intervals:
  - Assuming constant speed v:

$$t = \sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n \frac{s_0}{v}$$

Paradox:

- The requirement to complete infinitely many tasks (cover infinite subdivisions) in finite time seems impossible.

VGF Resolution:
Introduction of Minimal Units:

- Minimal Distance (δ_{VOID}): The smallest distinguishable distance.
- Minimal Time Interval (δ_t): The smallest meaningful time interval.

Mathematical Approach:

1. Finite Number of Steps:
   - The process stops when the remaining distance s_n is less than δ_{VOID}.
   - Calculate N such that:

$$s_N = \left(\frac{1}{2}\right)^N s_0 < \delta_{\text{VOID}}$$

- Solving for N:

$$N > \frac{\ln(\delta_{\text{VOID}}/s_0)}{\ln(1/2)}$$

- Since δ_{VOID} and s_0 are finite, N is finite.

2. Total Time:

$$T = N\delta_t$$

- Since N and δ_t are finite, T is finite.

Explanation:

- Motion occurs in finite, distinguishable steps.
- The infinite subdivision assumed in Zeno's paradox is not possible due to the minimal units.
- The paradox is resolved by recognizing that space and time are not infinitely divisible.

Connection to Original Paradox:

- The paradox arises from the assumption of infinite divisibility.
- By introducing δ_{VOID} and δ_t, we acknowledge practical limits, making the infinite series finite.

4.5.2 Russell's Paradox

Original Statement of the Paradox:

- Paradox Definition:
  - Let $R = \{x \mid x \notin x\}$.
  - Question: Is R a member of itself?
  - If $R \in R$, then by definition $R \notin R$.
  - If $R \notin R$, then by definition $R \in R$.

Logical Formalization:

- Creates a contradiction due to self-reference.

VGF Resolution:
Restrictions on Set Formation:

- Axiom 3.6 (Finite and Observable Sets):
  - Only sets with elements that can be distinctly observed and defined within the system are permitted.

  - Self-referential sets that cannot be grounded in observable reality are disallowed.

Mathematical Approach:

- Sets must be constructed from elements with distinguishable properties adhering to δ_{VOID}.
- Self-referential definitions like R are not meaningful within the VGF because they do not correspond to observable, finite constructs.

Explanation:

- The paradox relies on abstract, self-referential set definitions not grounded in physical reality.
- By limiting sets to finite, observable elements, the VGF prevents such paradoxical constructions.

Connection to Original Paradox:

- The paradox is avoided by disallowing the formation of sets that lead to contradictions.
- Emphasizes the importance of grounding mathematical constructs in observable phenomena.

4.6 Benefits of the VGF Approach

- Alignment with Physical Reality:
  - Ensures mathematical concepts are consistent with observable phenomena and measurement limitations.
- Practical Applicability:
  - Reflects limitations in computational resources and measurement precision, enhancing relevance in technology and science.
- Resolution of Paradoxes:
  - Addresses foundational issues in mathematics by introducing finite granularity and grounding abstractions.
- Interdisciplinary Integration:
  - Bridges mathematics with physics, biology, and other sciences, promoting a holistic understanding of systems.

4.7 Potential Emergence of New Number Classes

- Quantized Numbers:
  - Numbers that reflect discrete quantities in systems where changes occur in fixed increments.
- Probabilistic Numbers:
  - Incorporate probabilities due to indistinguishability at scales near δ_{VOID}, potentially leading to new mathematical constructs.
- Finite Real Numbers:
  - A subset of real numbers limited by finite granularity, rejecting infinite decimal expansions and embracing finite representations.

5 Summary of Key Concepts

- Numbers as Representations of Change:
  - Emerge from quantifiable changes exceeding minimal meaningful distinctions.
- Minimal Meaningful Distinction (δ_{VOID}):
  - A finite threshold ensuring changes are distinguishable.
- Pair of Numbers and Difference Operator:
  - Changes are defined between pairs of states using the difference operator Δ.
- Negative Numbers:
  - Represent significant decreases within systems, with physical interpretations.
- Time as Discrete Intervals:
  - Changes occur over finite time steps, acknowledging temporal granularity.
- Finite Granularity:
  - Recognizes practical limitations, rejecting infinite divisibility.
- Resolved Paradoxes:
  - Addresses classical paradoxes by grounding mathematics in observable reality.

- Potential for New Number Classes:
  - Introduces quantized and probabilistic numbers suited for finite systems.

6 Conclusion

The VOID Granularity Framework redefines number theory by grounding numbers in quantifiable changes within dynamic systems, respecting minimal meaningful distinctions, and acknowledging finite granularity. This approach aligns mathematical concepts with physical reality, ensuring consistency, applicability, and resolution of classical paradoxes.

By constructing the theory from foundational axioms, providing formal definitions, and offering detailed explanations and examples, we present a coherent and rigorous framework. The VGF complements classical mathematics, extending its relevance to contexts where physical limitations are paramount.

7 Axioms Dictionary

To maintain consistency and clarity throughout the VOID Granularity Framework (VGF) document, the following dictionary outlines each axiom with its hierarchical numbering, explanation, and relevant sections where they apply.

Axiom 3.7: Minimal Meaningful Distinction Explanation: Establishes that the probability of distinguishing between any two changes within a system is governed by the ratio of their magnitude to the VOID threshold δ_{VOID}. As $|\Delta s|$ becomes much smaller than δ_{VOID}, the probability $\mu_S(x, y)$ of distinguishing between changes approaches zero, indicating indistinguishability.

Relation: Fundamental to defining probabilistic distinguishability in systems. Applies to sections on numbers as representations of change and formal definitions.

Axiom 3.8: Pair of Numbers and Change Explanation: A number arises from the comparison between two distinct states s_i and s_f where the change Δs is meaningful ($|\Delta s| \geq \delta_{\text{VOID}}$).

Relation: Ensures that numbers are grounded in observable changes, influencing sections on formal definitions and difference operator.

Axiom 3.9: Symmetry of Change Explanation: For any measurable quantity q within system S, the change $\Delta q = q_f - q_i$ satisfies:

$$\mu_S(q_i, q_f) = \mu_S(q_f, q_i)$$

The distinguishability probability is symmetric with respect to the direction of change.

Relation: Ensures consistency in the probability of distinguishing changes, applicable to all sections dealing with change and distinguishability.

Axiom 3.10: Change Requires Time Explanation: A meaningful change Δq occurs over a discrete time interval δ_t. Ensures that changes are not instantaneous and are observable within finite time steps.

Relation: Essential for sections on incorporating time and discrete time steps, particularly in defining negative numbers and dynamic systems.

Axiom 3.11: Numbers Reflect Observable Changes Explanation: Numbers assigned to changes must correspond to observable and measurable differences within the system, adhering to the minimal distinctions δ_{VOID} and δ_t.

Relation: Ensures that numerical representations are grounded in tangible, measurable phenomena. Influences sections on defining numbers as changes and addressing classical paradoxes.

Axiom 3.12: Finite and Observable Sets Explanation: Only sets with elements that can be distinctly observed and defined within the system are permitted. Self-referential sets that cannot be grounded in observable reality are disallowed.

Relation: Prevents paradoxical constructions like Russell's Paradox by restricting set formation to observable and finite elements. Applies to sections on addressing classical paradoxes.

8 Final Remarks

By adopting a hierarchical numbering system, we ensure that each axiom is uniquely identifiable and consistently referenced throughout the VGF document. This organization enhances the framework's clarity and facilitates easier cross-referencing between axioms and their applications across various disciplines. The provided dictionary serves as a quick reference guide, aiding readers in understanding the foundational principles and their relevance to different sections of the framework.