PRIOR GEOMETRY: EXTENSIVE FORMALIZATION PART 3

SECTION 10: ADDITIONAL FORMAL DEFINITIONS AND PROPERTIES

10.1 Minimal Increment Transitions

(10.1.1) Define a minimal increment transition between patterns x and y at (a, e) if $d_{(a,e)}(x, y) = 1$. That is, they differ by exactly one increment of probability distribution.

(10.1.2) Construct a graph $G_{(a,e)}$ with vertices as stable patterns and edges representing minimal increment transitions. This graph encodes geometric adjacency.

10.2 Polygonal Loops and Inside/Outside

(10.2.1) A polygonal loop $C = (x_1, \ldots, x_n)$ is a simple cycle in $G_{(a,e)}$.

(10.2.2) Removing edges of C from $G_{(a,e)}$ splits the graph into components. Designate one reference pattern x_{ref} known as "outside." Patterns not reachable from x_{ref} after removing C are "inside" the loop.

(10.2.3) Thus, polygonal loops define finite analogs of planar curves and inside/outside regions, purely combinational and finite.

10.3 Refinement of Numeric Scales

(10.3.1) Consider two numeric scales: $\{0, 1/L, \ldots, 1\}$ and $\{0, 1/M, \ldots, 1\}$. To combine them, find capacity a' with increments $1/R$ where $R \geq \text{lcm}(L, M)$. Both scales embed into the finer scale.

(10.3.2) This embedding shows how rational increments from different intervals can be reconciled into a common refined scale.

10.4 Negative Increments and Symmetry

(10.4.1) Define negative increments by choosing a chain in the opposite direction. For any increment k/L, a corresponding pattern chain in the reverse order defines $-k/L$.

(10.4.2) Thus, negative increments appear as reversible steps along stable pattern chains, ensuring that if we interpret increments as "positions," negative increments represent going backward.

10.5 Representing Integers and Basic Arithmetic

(10.5.1) Integers appear as repeated increments of a chosen unit increment $1/N(a)$. For example, $3 * (1/N(a))$ is just the third pattern after the start in a chain.

(10.5.2) Since all arithmetic is finite, representing large integers requires longer chains or higher capacity states.

10.6 Algebraic Closure and Finite Domains

(10.6.1) Within a given capacity a, the set of representable increments is finite. Addition and multiplication remain closed within this finite set of rational increments.

(10.6.2) If a desired rational number cannot be represented exactly at capacity a, refine to a' with larger $N(a')$ until representation is possible. If no capacity within Ω can represent it exactly, we approximate it by the closest available increment.

10.7 Actions on Patterns and Dynamic Loops

(10.7.1) Changing $(a, e) \to (a', e')$ modifies μ and p_x distributions, potentially altering which patterns are stable and the structure of lines and polygons.

(10.7.2) This dynamical aspect shows that geometry and arithmetic are not fixed but depend on the joint state and can evolve over time.

10.8 Comparison with Classical Infinite Concepts

(10.8.1) Classical number theory relies on infinite sets and limits. Here, no actual infinity is present. All sets are finite at each step.

(10.8.2) Approximations to classical concepts like real numbers, continuity, or measure theory are done by capacity refinement and selection of increasingly large $N(a)$.

(10.8.3) Without infinite sets, no perfect realization of π, e, or irrational numbers occurs. Instead, we get sequences of rational increments approximating them to arbitrary precision, constrained by Ω.

SECTION 11: PROOF SKETCHES AND CONSISTENCY ARGUMENTS

11.1 Consistency with Probability Axioms

(11.1.1) Probability measures $\mu^{(a,e)}(x)$ and distributions $p_x^{(a,e)}(g)$ are finite rational numbers.

(11.1.2) Normalization conditions are easily met by assigning uniform or arbitrary rational distributions that sum to 1, requiring no infinite processes.

11.2 Updating without Contradictions

(11.2.1) For each $(a, e) \to (a', e')$, define U_μ and U_p as simple piecewise functions or lookup tables. Since A and E are finite, and increments are finite rationals, no contradictions arise.

(11.2.2) Probability updates remain finite transformations, ensuring internal consistency at every step.

11.3 Constructing a Model

(11.3.1) Start with minimal sets: $A = \{a1\}$, $E = \{e1\}$, $X = \{x1\}$, $G_{x1} = \{g11, g12\}$, etc.

(11.3.2) Assign $\mu^{(a1,e1)}(x1) = 1$ and $p_{x1}^{(a1,e1)}(g11) = 1$, trivial stable pattern.

(11.3.3) Add more states, patterns, and features step by step, always ensuring finite rational increments. This yields at least one non-empty model, proving the system is consistent.

SECTION 12: APPLICATION TO INTEGER SCALES AND RATIONALS

12.1 Integer Approximations

(12.1.1) Fix capacity a. The smallest increment is $1/N(a)$. Counting k increments from a baseline pattern defines $k/N(a)$.

(12.1.2) If $N(a) = 10$, for example, increments are multiples of 0.1. Integers like 1,2,3 appear as $10/10, 20/10, 30/10$ but must remain within representable sets.

12.2 Rational Density under Refinement

(12.2.1) Increasing capacity state $a \to a'$ with $N(a') = N(a) * Q$ introduces finer increments of size $1/(N(a) * Q)$, making the rationals denser.

(12.2.2) Thus, by repeated refinements (bounded by Ω), we simulate rational density up to the complexity limit.

12.3 Operations with Minimal Error

(12.3.1) For any target rational number r, we can approximate r by $k/N(a')$ at a sufficiently high capacity a'.

(12.3.2) The error $|r - k/N(a')| < \epsilon$ if we choose a' so $N(a') > k/(\epsilon)$. This process requires no infinite limit, just a finite large $N(a')$.

SECTION 13: STRUCTURED COMPLEXES AND MAPPINGS

13.1 Complexes of Patterns

(13.1.1) Define a finite set of patterns arranged so that minimal increment edges form higher structures like cubes, tetrahedra, etc.

(13.1.2) Each complex remains finite, and each topological property (connectedness, cycles) is computed combinationally.

13.2 Maps Between Pattern Sets

(13.2.1) A map $F : X \to X$ between pattern sets can be defined increment-wise. If increments correspond to rational numbers, F can represent linear transformations over finite vector spaces.

(13.2.2) Changing capacity refines these maps, allowing finer approximations of linear maps and eigen decompositions.

SECTION 14: NO SOLIPSISM PRINCIPLE

14.1 Joint Dependence on (a,e)

(14.1.1) Probability structures $p_x^{(a,e)}$ depend on both observer state a and environment e. This prevents solipsism: distributions are not purely subjective but constrained by external signals.

(14.1.2) As $(a, e) \to (a', e')$, environment changes force recalculation of probabilities, ensuring external grounding.

14.2 Capacity and External Constraints

(14.2.1) Higher capacity states reveal finer distinctions in signals S_e. If S_e changes drastically, patterns compatible with S_e must also change.

(14.2.2) This coupling ensures that the finite increments are always tied to external conditions, not abstractly chosen.

SECTION 15: STABILITY AND EMERGENT REGULARITIES

15.1 Attractors as Stable Patterns

(15.1.1) If repeated environmental interactions lead to certain x always having high $\mu^{(a,e)}(x)$ and stable $p_x^{(a,e)}$, x is an emergent stable pattern.

(15.1.2) Such attractors represent geometric objects: points, lines, etc., formed by consistent probability assignments.

15.2 Parameter Variation

(15.2.1) Varying capacity systematically reveals more subtle patterns and increments.

(15.2.2) At low capacity, multiple stable patterns might appear merged as one pattern. At high capacity, they differentiate into several distinct stable patterns.

SECTION 16: SYNTHESIS OF THE FORMAL SYSTEM

16.1 Summary

- Finite sets A, E, X, G_x
- Probability increments quantized by $N(a)$
- Capacity $V(a)$ controls granularity
- Stability defines geometric-like entities
- Refined increments yield rational arithmetic
- Complex algebraic structures built from finite increments
- Approximation of classical concepts via capacity refinement
- Finite complexity Ω forbids true infinity

16.2 Concluding Formal Remarks

(16.2.1) The entire framework is self-contained, finite, and relies on no infinite sets, no limits, and no continuity arguments.

(16.2.2) By careful selection of increments, chains, and distributions, all classical-like structures (numbers, geometry, algebra) appear as finite approximations.

(16.2.3) This ensures a consistent, finite, rational-based mathematical universe where complexity and capacity guide the emergence of structured concepts.