PRIOR GEOMETRY: EXTENSIVE FORMALIZATION

SECTION 1: PRELIMINARIES

1.1 Finite Sets and Basic Objects

(1.1.1) Let A be a finite set. Elements of A are called *observer states.*

(1.1.2) Let E be a finite set. Elements of E are called *environment states.*

(1.1.3) Consider the Cartesian product $A \times E$. An element $(a, e) \in A \times E$ is called a *joint system state.*

1.2 Observer Capacity and Associated Parameters

(1.2.1) Define a function $V : A \to \mathbb{N}$, where $\mathbb{N}$ is the set of positive integers, that assigns to each observer state $a \in A$ a natural number $V(a)$.

(1.2.2) The value $V(a)$ is called the *capacity* of the observer state a. Capacity relates to how finely the observer can discriminate increments of probability.

(1.2.3) For each $a \in A$, define $N(a)$ as a positive integer determined by $V(a)$. The function $V \mapsto N(a)$ is non-decreasing: if $V(a') > V(a)$, then $N(a') \geq N(a)$.

(1.2.4) $N(a)$ determines the resolution of probability increments associated with the observer state a. Smaller increments correspond to larger $N(a)$.

1.3 Probability Assignments and Patterns

(1.3.1) Let X be a finite set of abstract entities called *patterns.*

(1.3.2) For each pattern $x \in X$, let G_x be a finite set of features associated with x. Each feature set G_x is finite.

(1.3.3) For each joint state $(a, e) \in A \times E$, define a probability measure $\mu^{(a,e)} : X \to \{0, \frac{1}{N(a)}, \frac{2}{N(a)}, \ldots, 1\}$ mapping each pattern x to a rational probability increment compatible with $N(a)$.

(1.3.4) For each pattern x and joint state (a, e), define a probability distribution $p_x^{(a,e)} : G_x \to \{0, \frac{1}{N(a)}, \ldots, 1\}$ such that $\sum_{g \in G_x} p_x^{(a,e)}(g) = 1$.

(1.3.5) The existence measure $\mu^{(a,e)}(x)$ indicates the "presence strength" or "existence level" of pattern x at state (a, e). The distribution $p_x^{(a,e)}$ defines how that pattern's probability mass is allocated among its features G_x.

1.4 Consistency Conditions for Probability

(1.4.1) For each (a, e), $\mu^{(a,e)}(x) \in \{0, \frac{1}{N(a)}, \dots, 1\}$ and $\sum_x \mu^{(a,e)}(x)$ can be less than or equal to 1 (the theory may allow some normalization conditions as needed, but we must remain finite and closed).

(1.4.2) Each $p_x^{(a,e)}(g) \in \{0, \dots, 1\}$ with increments of $1/N(a)$.

(1.4.3) If $V(a') > V(a)$, then $N(a') \geq N(a)$, ensuring that finer resolutions are possible at higher capacities.

1.5 Environment Signals and Constraints

(1.5.1) For each $e \in E$, define a finite set S_e called the *signal set* available in environment state e.

(1.5.2) Each pattern x and its features G_x must be compatible with the available signals S_e at environment state e. Compatibility means: for each $g \in G_x$, g must be interpretable as a feature derived from or not contradicting S_e.

(1.5.3) If state (a, e) changes to (a', e'), the available signals change from S_e to $S_{e'}$. Probability measures $\mu^{(a',e')}(x)$ and $p_x^{(a',e')}(g)$ must be redefined or updated accordingly to remain consistent.

1.6 Update Mechanisms

(1.6.1) Define update functions U_μ and U_p that handle transitions in the joint state space. For a state transition $(a, e) \to (a', e')$, we have:

$$\mu^{(a',e')}(x) = U_\mu(\mu^{(a,e)}(x), a, e, a', e'), \quad p_x^{(a',e')}(g) = U_p(p_x^{(a,e)}(g), a, e, a', e').$$

(1.6.2) U_μ and U_p must map rational increments consistent with $1/N(a)$ to rational increments consistent with $1/N(a')$.

(1.6.3) These updates ensure no infinite regress: each step is finite, well-defined, and does not require limits or infinite processes.

SECTION 2: STABILITY AND GEOMETRIC INTERPRETATIONS

2.1 Stability of Patterns

(2.1.1) A pattern x is stable over a region $R \subseteq A \times E$ if for all $(a, e) \in R$:

$$|\mu^{(a,e)}(x) - \mu^{(a_0,e_0)}(x)| \leq \epsilon_\mu$$

and

$$\sum_g |p_x^{(a,e)}(g) - p_x^{(a_0,e_0)}(g)| \le \epsilon_p$$

for some fixed reference $(a_0, e_0) \in R$ and fixed small ϵ_μ, ϵ_p.

(2.1.2) Stability ensures that the pattern x does not drastically change its existence probability or internal feature distribution in that region of states.

2.2 Emergence of Points as Stable Probability Configurations

(2.2.1) Consider a stable pattern x that remains recognizable as a single entity under slight modifications of (a, e). Such stable patterns act like "points" in a discrete probability configuration space.

(2.2.2) Each "point" corresponds to a probability distribution tuple $(\mu^{(a,e)}(x), p_x^{(a,e)})$ stable under some conditions.

(2.2.3) Points are finite probability structures chosen from rational increments defined by $N(a)$.

2.3 Constructing Distances and Geometric Relations

(2.3.1) Define a distance-like measure between patterns x and y at a fixed (a, e):

$$d_{(a,e)}(x, y) = N(a) \sum_g |p_x^{(a,e)}(g) - p_y^{(a,e)}(g)|.$$

Here $N(a)$ rescales the increments so we count how many minimal steps of size $1/N(a)$ differ.

(2.3.2) $d_{(a,e)}(x, y)$ is symmetric, non-negative, and satisfies the triangle inequality due to the L^1 nature of absolute differences.

(2.3.3) This defines a finite metric space on the set of stable patterns at (a, e). Changes in (a, e) modify the metric due to changes in $N(a)$ and distributions.

2.4 Lines and Higher Constructs

(2.4.1) A line-like construct arises from a finite sequence of stable patterns $(x_0, \dots, x_L)$ where each consecutive pair differs by a minimal increment ($d_{(a,e)}(x_i, x_{i+1}) = 1$).

(2.4.2) Such sequences define a "line" in this discrete probability space. By increasing capacity, we can insert more intermediate patterns to refine the line.

(2.4.3) Polygons are loops: sequences of patterns $(x_1, \dots, x_n)$ forming a cycle where x_n connects back to x_1. Removing this cycle from the pattern graph may disconnect the set into "inside" and "outside" sets, defining a topological notion in a finite combinational setting.

SECTION 3: REFINEMENT AND CAPACITY INCREASE

3.1 Capacity Increase and Finer Probability Increments

(3.1.1) If $V(a') > V(a)$, then $N(a') \geq N(a)$. We can define $N(a') = MN(a)$ for some integer $M \geq 1$ to ensure finer increments.

(3.1.2) Under capacity refinement, a probability increment $1/N(a)$ can be further subdivided into M increments of size $1/(MN(a))$.

(3.1.3) Refinement preserves consistency: previously defined distributions remain representable as coarser approximations within the finer grid.

3.2 Monotonic Refinement of Increments

(3.2.1) Consider two patterns x and y with $d_{(a)}(x, y) = L$ increments at capacity a. At higher capacity a' with finer resolution, the number of increments required $d_{(a')}(x, y) = L'$ satisfies $L' \geq L$, ensuring monotonic refinement of the scale.

(3.2.2) Larger capacity states can represent all increments from lower capacity states plus additional intermediate increments.

3.3 Existence of Models and Non-Contradiction

(3.3.1) The axioms define finite sets, finite distributions, and finite update functions. Constructive examples:

$$E = \{e1\}, A = \{a1\}, \text{ minimal: } X = \{x1, x2\}, G_{x1} = \{g11, g12\}, \ldots$$

Assign $\mu^{(a1,e1)}(x1) = 1/2$, $p_{x1}^{(a1,e1)}(g11) = 1$, etc. Everything is finite and rational.

(3.3.2) No contradictions arise since no infinite procedures are needed. We have closed finite rational sets at each step.

SECTION 4: EMERGENCE OF NUMBERS AND ARITHMETIC

4.1 Constructing Increment-Based Numbers

(4.1.1) Consider a pair of stable patterns (p_u, p_v) at capacity a such that $d_{(a)}(p_u, p_v) = L$.

(4.1.2) The increments between p_u and p_v define a finite "interval" of L steps. We label them as $0, 1/L, 2/L, \ldots, 1$.

(4.1.3) These fractions $0, \ldots, k/L, \ldots, 1$ represent rational numbers constructed directly from the finite increments of probability distributions.

4.2 Operations on Increment-Derived Numbers

(4.2.1) Given two fractions k/L and m/M from possibly different intervals, to add them:

(a) Find a capacity state a' with increments small enough to represent both $1/L$ and $1/M$ as $1/R$ where $R \geq \mathrm{lcm}(L, M)$.

(b) Represent k/L as k'/R and m/M as m'/R.

(c) Define $(k/L) + (m/M) = (k' + m')/R$.

(4.2.2) Similarly for subtraction, multiplication, and division (except division by zero), we refine capacity to find common denominators and perform operations with finite rational increments.

4.3 No Infinite Sets, No Irrationals

(4.3.1) All constructed "numbers" are finite rationals with denominators bounded by capacity refinement.

(4.3.2) As capacity increases, denominators grow, approximating finer rational values but never introducing true irrational numbers.

(4.3.3) There is no completed infinite set of rationals; at each state we only have a finite subset. However, by moving through capacity states, we can approach denser and denser rational sets.