PRIOR GEOMETRY: EXTENSIVE FORMALIZATION PART 2

SECTION 5: ALGEBRAIC STRUCTURES FROM FINITE INCREMENTS

5.1 VOID Groups

(5.1.1) Consider a set of increments $\{k/L \mid k = 0, \ldots, L\}$ representing a segment of rational increments. Include negative increments by considering a dual chain in the opposite direction.

(5.1.2) Under increment addition, we get a structure that approximates a group:

- Identity: 0 increment.
- Inverses: For each k/L, define $(-k)/L$ as the inverse increment.
- Associativity: Holds exactly for addition of rational numbers. In the discrete setting, differences from exact rational arithmetic can occur if we consider multiple distinct intervals, but capacity refinement can make these negligible.

(5.1.3) Thus, we form a "VOID Group," a finite structure that mimics a group at the given capacity.

5.2 VOID Rings and Fields

(5.2.1) Introduce multiplication by considering two intervals forming a grid of increments.

(5.2.2) Multiplying increments $(k/L) * (m/M)$ requires a common refinement. After refinement, define multiplication as counting combined increments.

(5.2.3) Distributivity, commutativity, identity elements (1 and 0), and existence of inverses (except for zero) give rise to field-like structures at sufficiently refined capacities.

(5.2.4) Such a "VOID Field" is finite, rational, and capacity-dependent. Increasing capacity allows finer approximations of inverses and a richer subset of rationals.

5.3 Vector Spaces Over a VOID Field

(5.3.1) Consider tuples of increments $(v_1, \ldots, v_n)$ with each v_i in the VOID Field. Define vector addition and scalar multiplication component-wise.

(5.3.2) With finite dimensions and finite increments, we obtain a finite structure approximating a vector space. All linear algebra operations remain finite and can be refined by increasing capacity.

5.4 Matrices and Linear Maps

(5.4.1) A matrix is a finite array of increments from the VOID Field.

(5.4.2) Matrix addition and multiplication are defined component-wise and row-column-wise using VOID Field operations.

(5.4.3) Increasing capacity reduces rounding error and makes matrix operations more closely resemble classical linear algebra.

SECTION 6: POLYGONS, POLYHEDRA, AND TOPOLOGICAL CONSTRUCTS

6.1 Polygonal Loops

(6.1.1) A polygonal loop is a cycle of stable patterns $(x_1, \ldots, x_n)$ where each consecutive pair differs by one increment and x_n connects back to x_1.

(6.1.2) Removing these edges from the pattern graph can partition the pattern set, defining inside and outside sets in a purely finite combinational manner.

6.2 Higher-Dimensional Constructs

(6.2.1) Chains of stable patterns can form 2D, 3D complexes if we arrange them in complex graphs, mimicking polygons and polyhedra.

(6.2.2) Each face, edge, vertex corresponds to stable patterns or sets of patterns. No continuity is needed; all are finite combinational structures.

SECTION 7: APPROXIMATION AND EPSILON-DELTA CONTROL

7.1 Error Measures

(7.1.1) For any algebraic property (associativity, distributivity) or geometric configuration, define an error measure as the difference in increments from the ideal classical value.

(7.1.2) Errors are multiples of $1/N(a)$. Reducing increments by increasing $N(a)$ reduces the error.

7.2 Epsilon-Delta Capacity Arguments

(7.2.1) Given $\epsilon > 0$, choose capacity a' with $N(a')$ large enough so that increments are below ϵ/K for some scaling factor K.

(7.2.2) Under capacity a', the error in representing operations (like matrix multiplication associativity) is less than ϵ.

(7.2.3) Thus, for any desired precision, we can refine capacity to reduce error, mimicking classical epsilon-delta arguments without infinite limits, using only finite steps.

SECTION 8: FINITE INFORMATION LIMIT

8.1 Complexity Bound Omega

(8.1.1) Assume there is a finite upper bound Ω on total representable complexity. Ω limits how large $N(a)$ or how big sets can become.

(8.1.2) As we refine capacity, we must remain under Ω. Beyond Ω, no further refinements are allowed.

8.2 Consequences of Finite Omega

(8.2.1) No true infinite processes can be completed.

(8.2.2) Classical concepts relying on infinite sets or limits (real numbers, uncountable sets, exact irrational numbers) cannot be fully realized.

(8.2.3) We only get finite approximations at any achievable capacity state.

SECTION 9: FORMAL AXIOMATIC SUMMARY

This section restates key axioms and definitions in a condensed, formal listing:

9.1 Sets and States

(Ax1) A and E are finite sets. Elements of A are observer states, elements of E are environment states.

(Ax2) There is a function $V : A \to \mathbb{N}$. For each $a \in A$, define $N(a) \in \mathbb{N}$ with the property that if $V(a') > V(a)$, then $N(a') \geq N(a)$.

(Ax3) The system state is $(a, e) \in A \times E$.

9.2 Probability Assignments

(Ax4) X is a finite set of patterns. For each $x \in X$, G_x is a finite set of features.

(Ax5) For each (a, e), $\mu^{(a,e)}(x) \in \{0, 1/N(a), \ldots, 1\}$ and $p_x^{(a,e)}(g) \in \{0, 1/N(a), \ldots, 1\}$ with $\sum_g p_x^{(a,e)}(g) = 1$.

(Ax6) If environment changes to e', signals S_e change. Patterns must remain compatible with S_e. Probability distributions must adjust accordingly.

9.3 Updates and Finite Steps

(Ax7) There exist update functions U_μ, U_p that define $\mu^{(a',e')}$ and $p_x^{(a',e')}$ from $\mu^{(a,e)}$ and $p_x^{(a,e)}$ given a state transition $(a,e) \to (a',e')$. U_μ and U_p preserve rational increments and yield no contradictions.

9.4 Capacity Refinement

(Ax8) Higher capacity states allow finer increments. For any N, we can find a' with $N(a') \geq N \cdot N(a)$.

9.5 Stability and Geometry

(Ax9) A pattern x stable over $R \subseteq A \times E$ if $\mu^{(a,e)}(x)$ and $p_x^{(a,e)}(g)$ vary by less than fixed small epsilons. Stable patterns represent geometric points.

(Ax10) Distances between patterns defined by L^1 differences in p_x distributions scaled by $N(a)$. Lines and polygons form by chains and loops of minimal-increment pattern transitions.

9.6 Arithmetic and Algebraic Structures

(Ax11) Increment-based rational numbers constructed from finite pattern chains. Addition, multiplication of these numbers defined by capacity refinement and common denominators.

(Ax12) By extending these numbers to tuples and grids, form finite analogs of groups, rings, fields, and vector spaces. Each structure is finite and capacity-dependent.

9.7 Approximation and Limits

(Ax13) For any $\epsilon > 0$, choose capacity a' large enough so increments $< \epsilon$. Then any deviation from classical algebraic or geometric properties $< \epsilon$.

9.8 Finite Complexity Omega

(Ax14) There is a finite upper bound Ω on complexity. No infinite sets or processes appear. All constructs remain finite at every stage.