Introduction to Causal Inference

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Nordic Probabilistic AI School
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Copenhagen, Denmark

Predictive vs Explainable, Trustworthy AI

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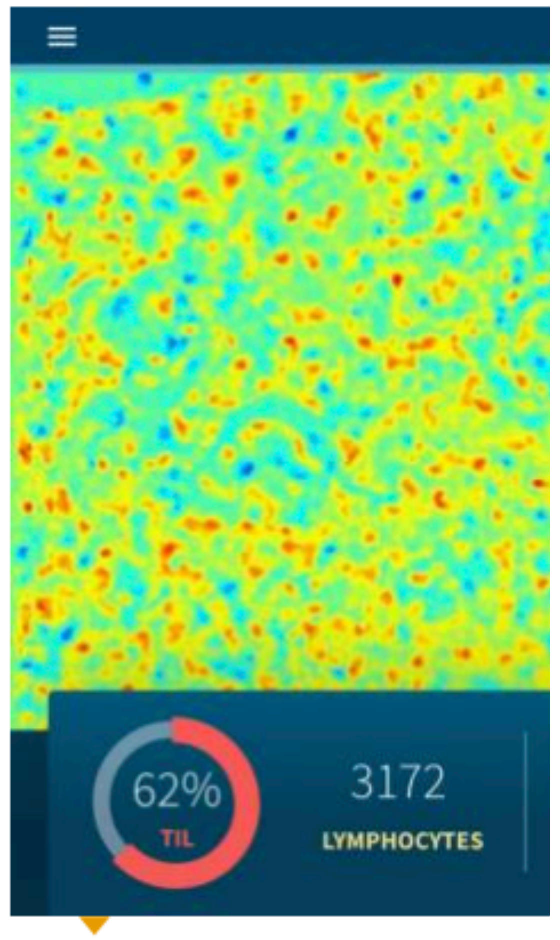
Making the Role of AI

Analysis system for the diagnos



Detection of tumor-infiltrating lym generate a heatmap showing TILs (of Klauschen/Charité

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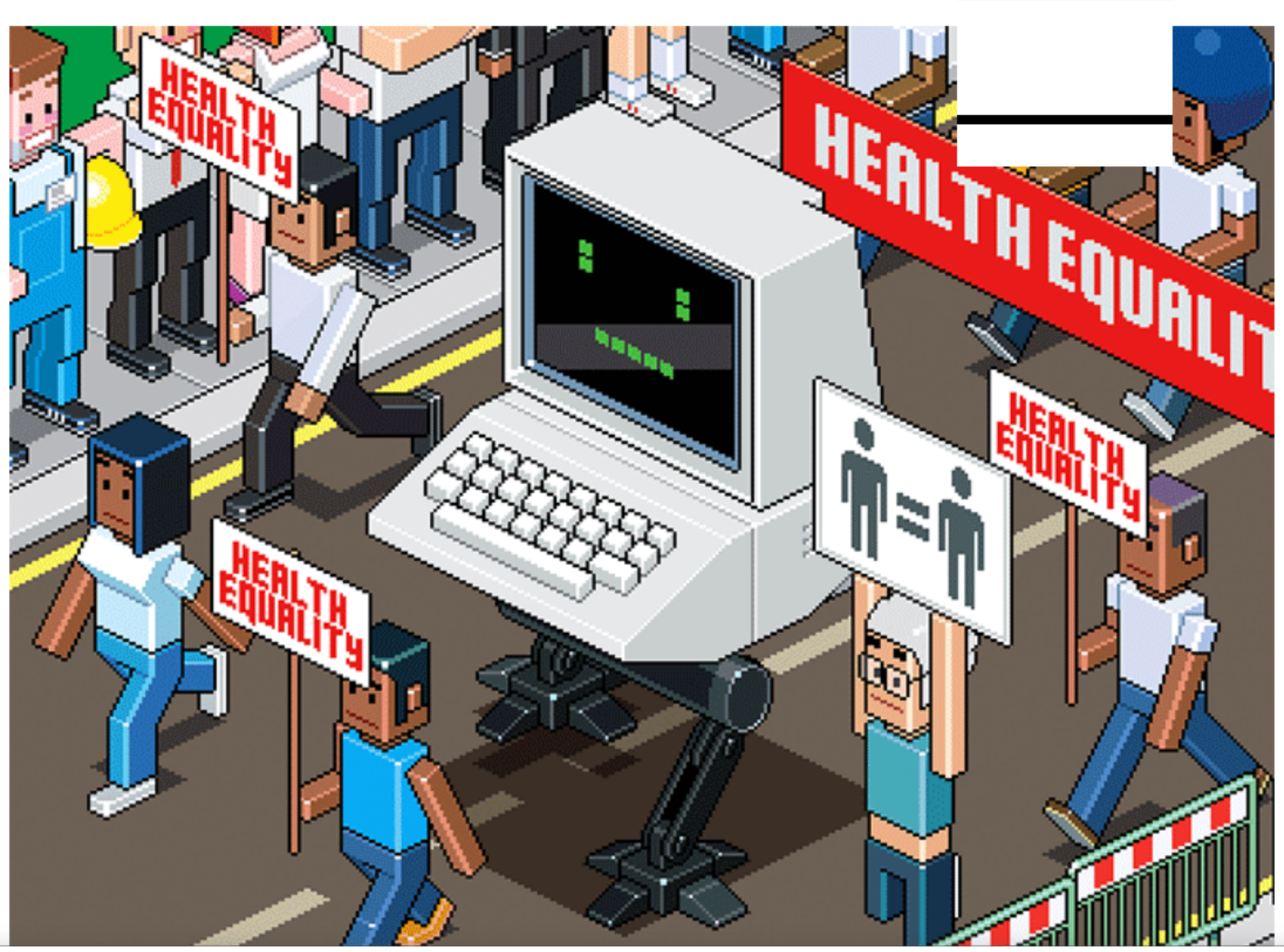
A fairer way forward for AI in health care

Without careful implementation, artificial intelligence could widen health-care inequality.

Linda Nordling



Predictive vs Explainable, Trustworthy AI

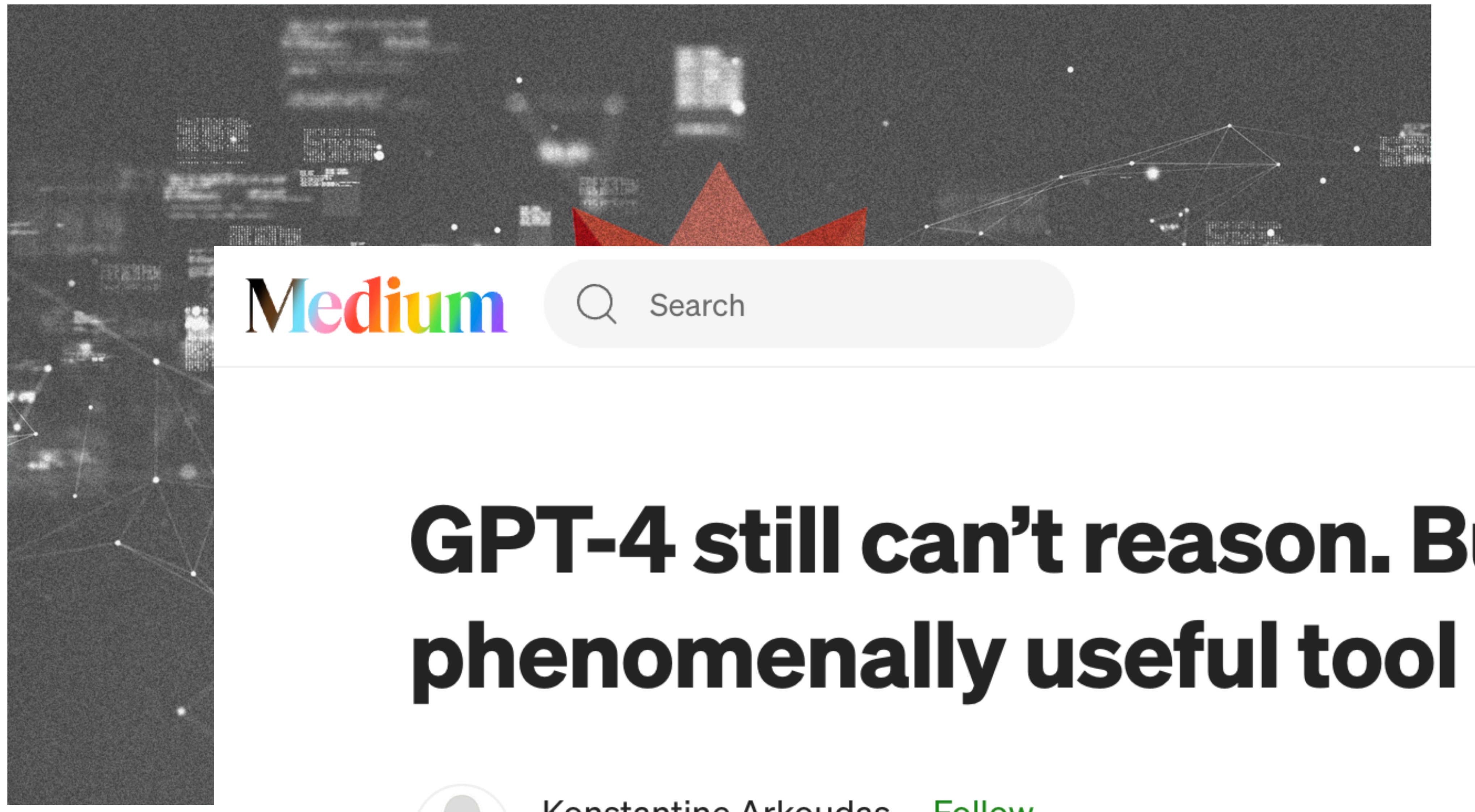
Chat GPT - Impressive Abilities:

- Debug code
- Play Games
- Translate text
- Explain complex topics
- Create AI art
- Grade essays
- Write a resume
- Brainstorm ideas
- Play games with ChatGPT
- Create chatbots
- Language tutor
- Create jokes
- Solve coding errors
- Write poetry

06-08-2024 | TECH

This classic answer engine still outsmarts AI chatbots

For questions involving hard data and math calculations, 15-year-old WolframAlpha is a fast, accurate alternative to inaccurate AI chatbots.



[Image: RICHARD JON

BY CHR

Medium Search

GPT-4 still can't reason. But it's a phenomenally useful tool anyway.

Konstantine Arkoudas · Follow

12 min read · Apr 11, 2024

Causality: A Missing Link to Reasoning in AI

The ability to understand cause-and-effect relationships is crucial for deeper understanding and decision-making processes.

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Why artificial intelligence needs to understand consequences

A machine with a grasp of cause and effect could learn more like a human, through imagination and regret.

Prediction vs Effect of Interventions

Statistical Association vs Causation

Predictive Tasks

Task: Can I guess how severe is a fire by **observing** the number of firefighters?

Yes!

X: **Number of firefighters in action**

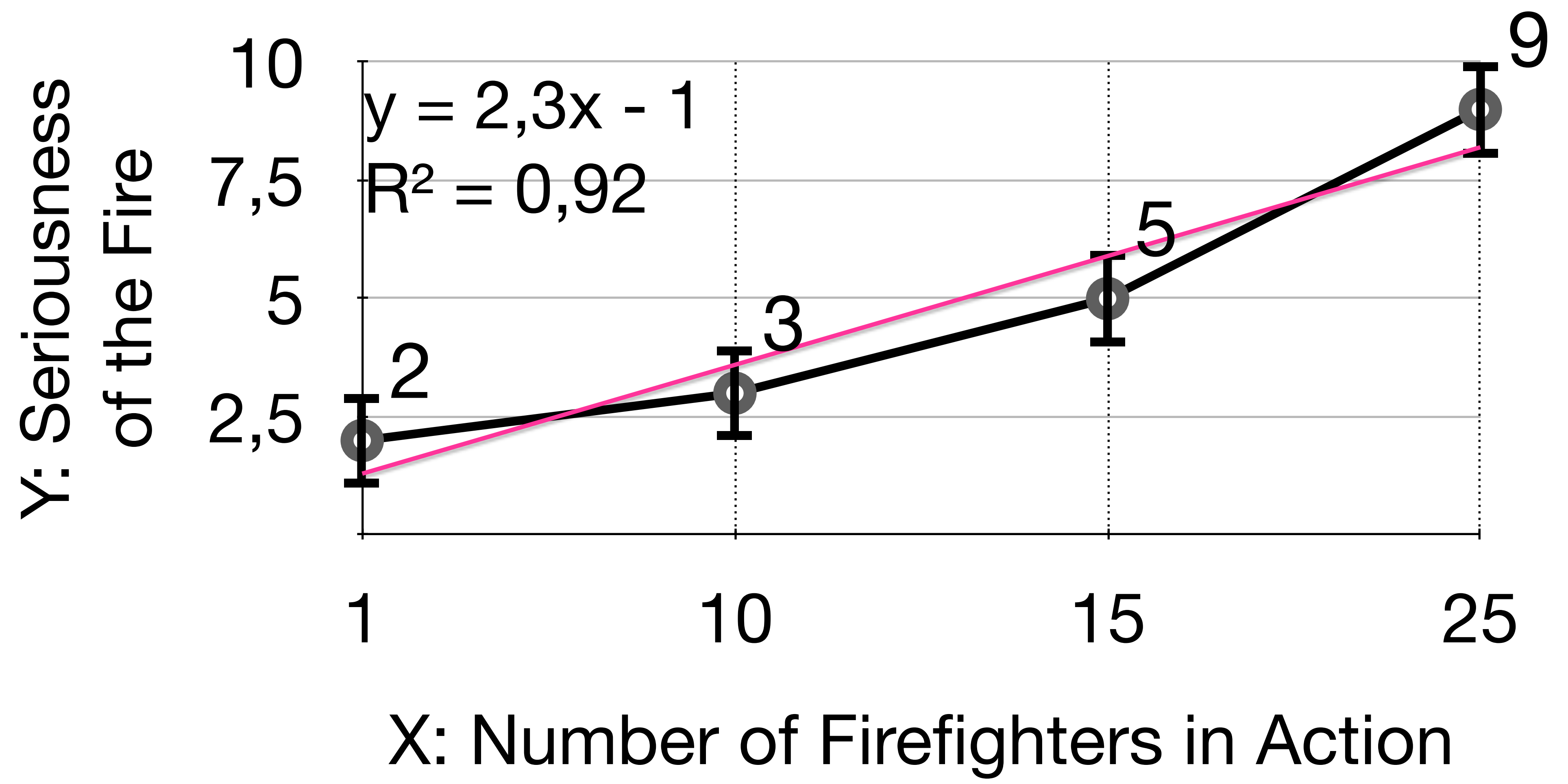
Y: **Severity of the (initial) fire**

$\rho_{XY} \neq 0 \implies X$ **is a good predictor of** Y

$$P(Y = y \mid X = x) \neq P(Y = y)$$

Observational Probability Distribution

Correlation between severity of fire and number of firefighters in action



Conclusion: The severity of the fire increases with the number of firefighters.

Prediction ⇒ Decision-Making?

Conclusion: The severity of the fire increases with the number of firefighters.

The fewer firefighters, the weaker the fire.

Should we decrease the number of firefighters to reduce the fire?

Causal Effect $\neq$ Statistical Association

The **causal effect** of X on Y is a quantity that tells us how much Y changes after an intervention $do(X = x)$, e.g., $E[Y \mid do(X = x)]$.

If a different number of firefighters were dispatched, $do(X = x)$, would this change the expected severity of the initial fire Y?

Causal Effect $\equiv$ Effect of an Intervention

This quantity is derived from the probability of Y obtained after making the intervention $do(X = x)$, which is denoted by $P(Y \mid do(X = x))$ and is commonly referred as ***Interventional Distribution***.

Causal Effect ≡ Effect of an Intervention

This question can also be answered if knowledge about the underlying reality is available!

X: **Number of firefighters in action**

Y: **(Initial) Severity of the fire**

$$\begin{cases} X = f_X(Y, U_X) \\ Y = f_Y(U_Y) \end{cases}$$

Underlying
Structural Causal Model (SCM)

Y is not a function of X

In other words, X **is not a cause of** Y

In this case, $\forall x, E[Y \mid do(X = x)] = E[Y]$.

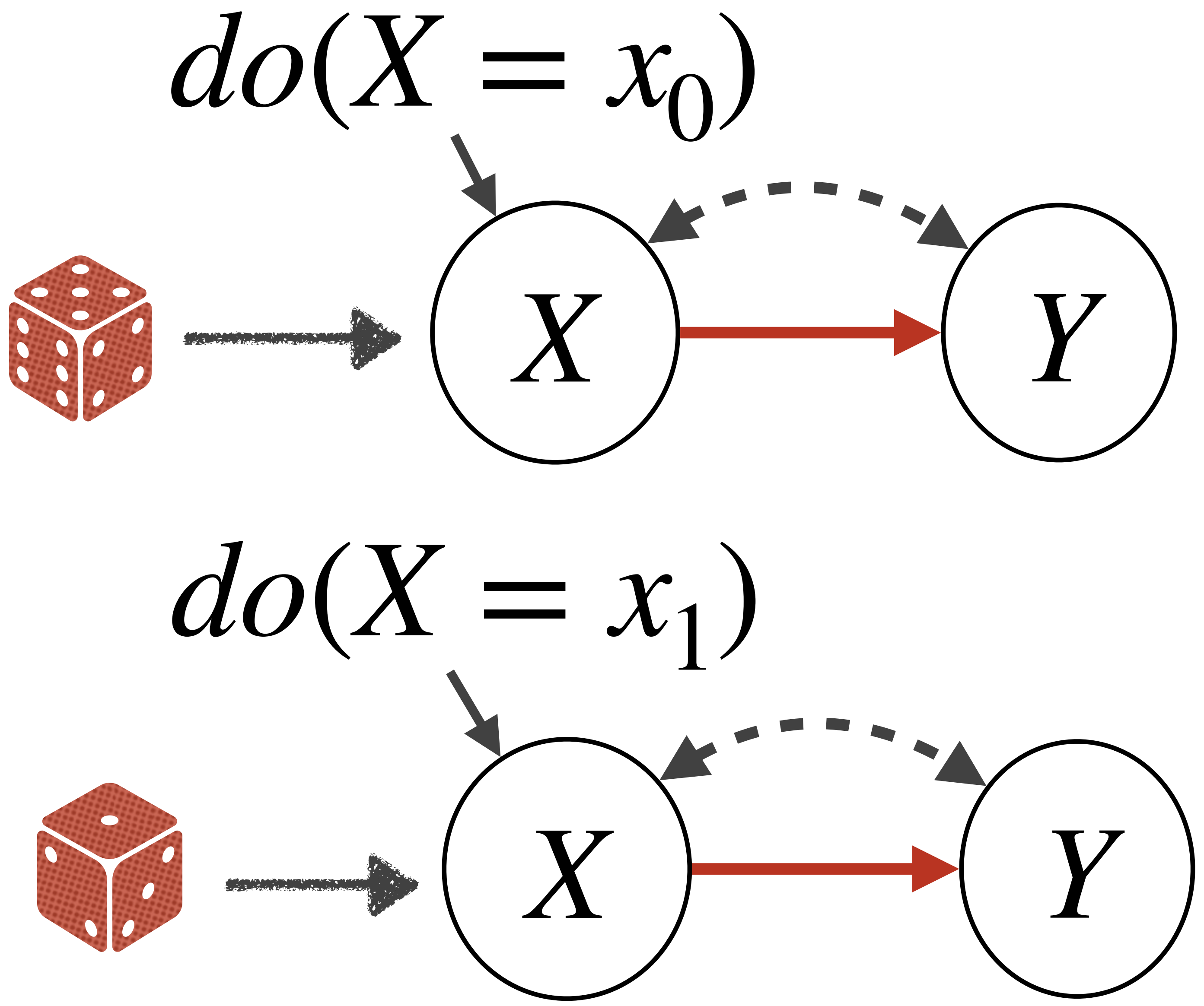
Changing the number of firefighters does not change the (initial) severity of the fire.

The action/intervention on X, $do(X = x)$ is independent of Y,
i.e., $P(Y = y \mid do(X = x)) = P(Y = y)$

Randomized Experiments

One way to access the interventional distribution $P(Y \mid \textcolor{red}{do(X = x)})$ is through a *perfectly realized* Randomized Experiments / Control Trials (e.g. RCT):

Randomization of the X's assignment



$\mathbb{E}[Y \mid do(X = x_0)]$

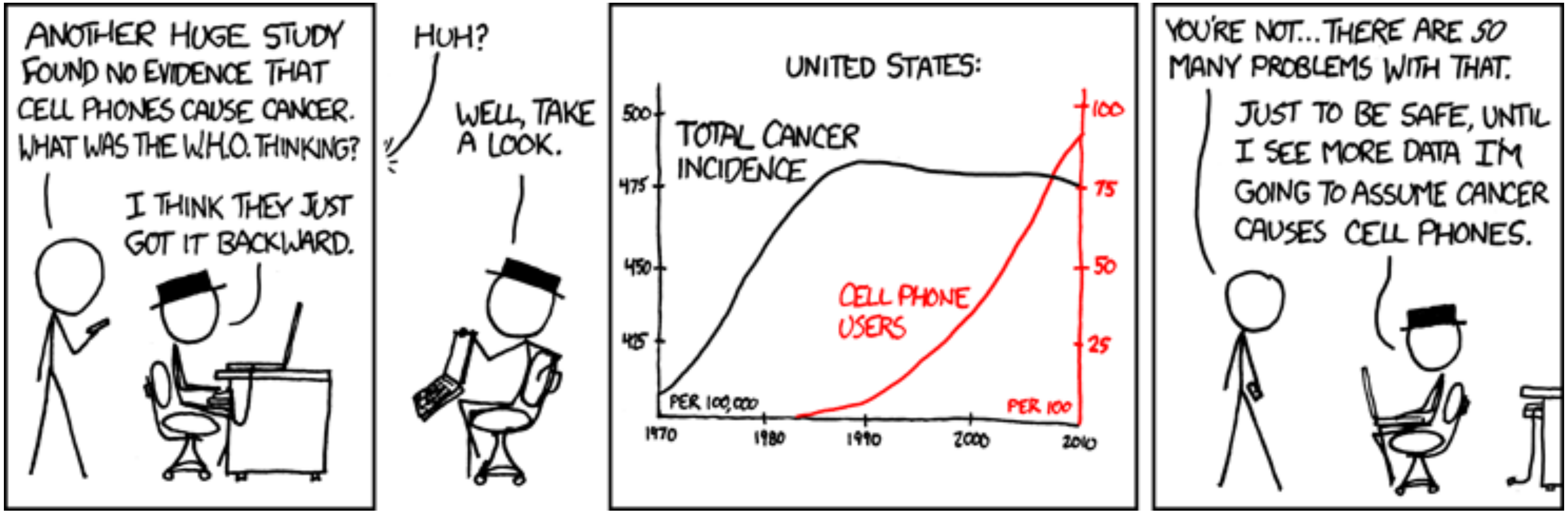
$\mathbb{E}[Y \mid do(X = x_1)]$

Average Causal Effect: $\mathbb{E}[Y \mid do(X = x_0)] - \mathbb{E}[Y \mid do(X = x_1)]$

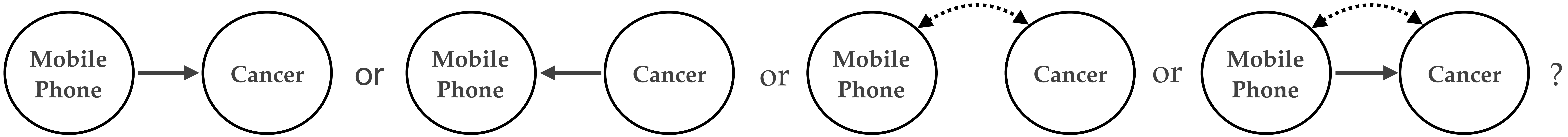
What is the causal effect of the number of firefighters X on the severity of the initial fire Y ?

Can we identify causal effects without conducting a randomized experiment?

Association vs Causation

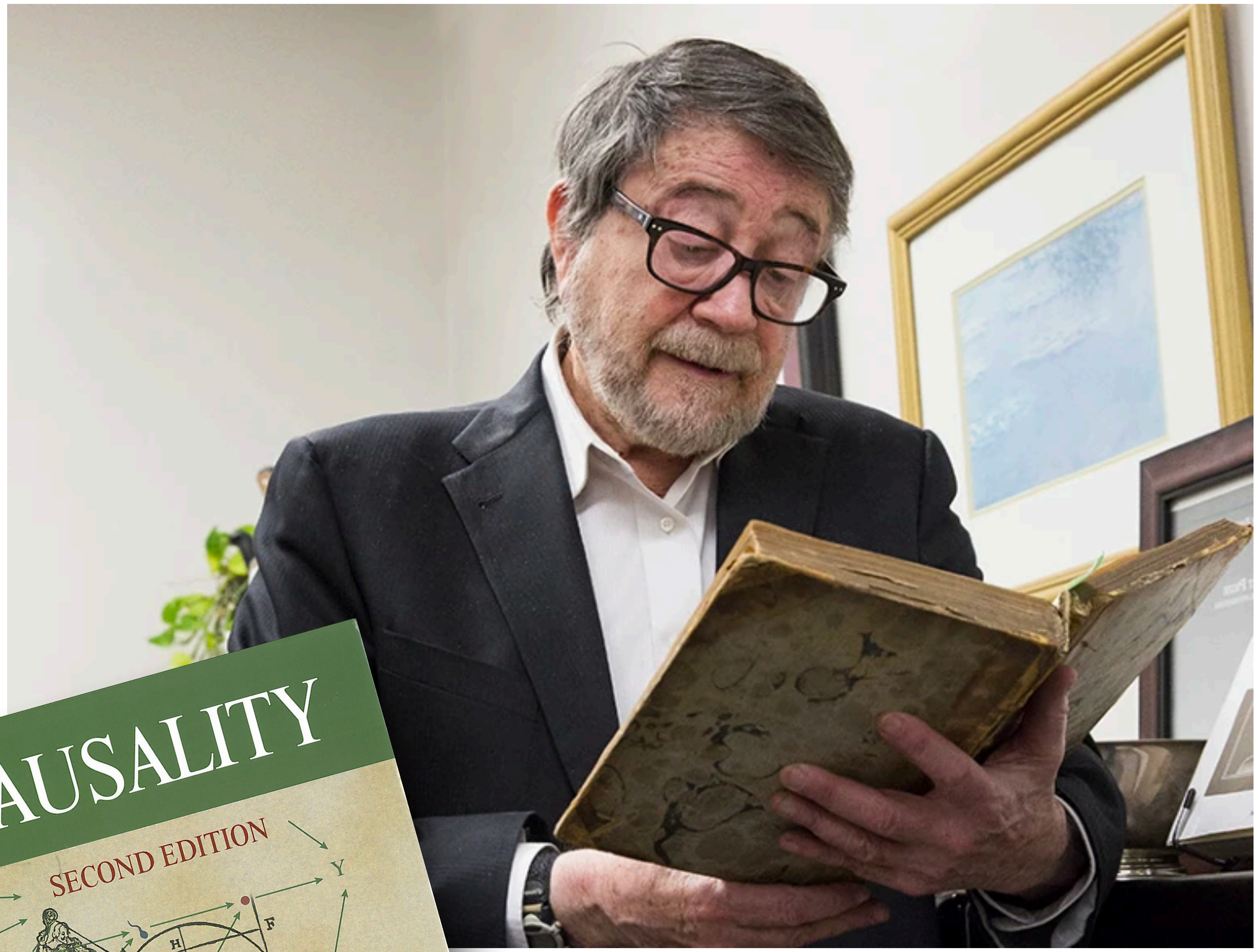


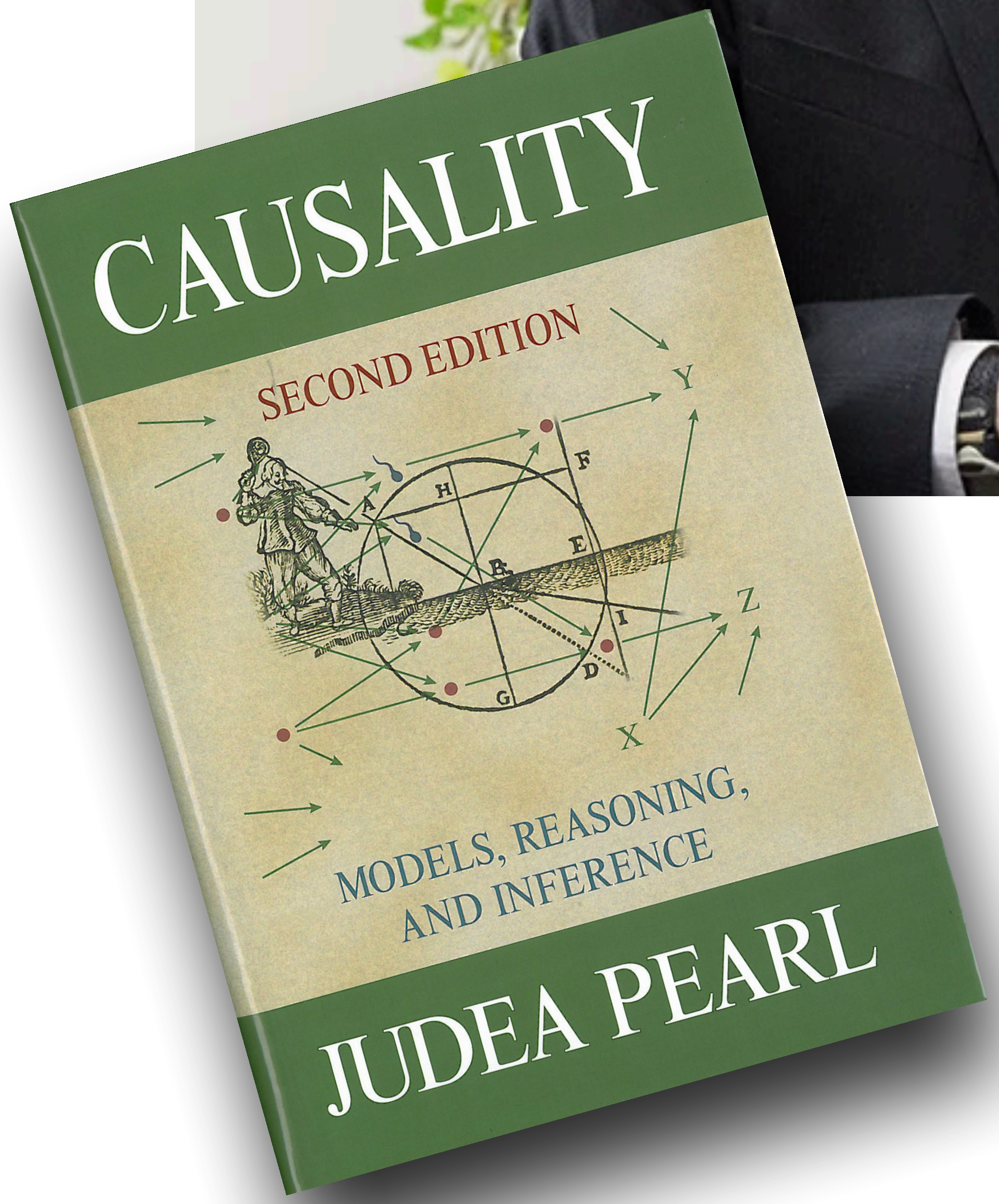
Will we be able to decide the true relationship just by “seeing” more data?



The Mathematical Framework of Causal Data Science

Judea Pearl — Causality





Director of the Cognitive Systems Laboratory at the University of California, Los Angeles.

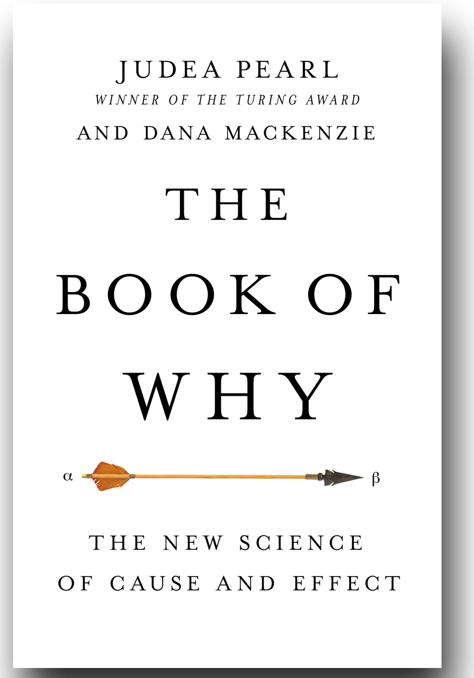
In 2011, he won the A. M. Turing Award (the highest distinction in computer science and a $250,000 prize)

"for fundamental contributions to artificial intelligence through the development of a calculus for probabilistic and causal reasoning."

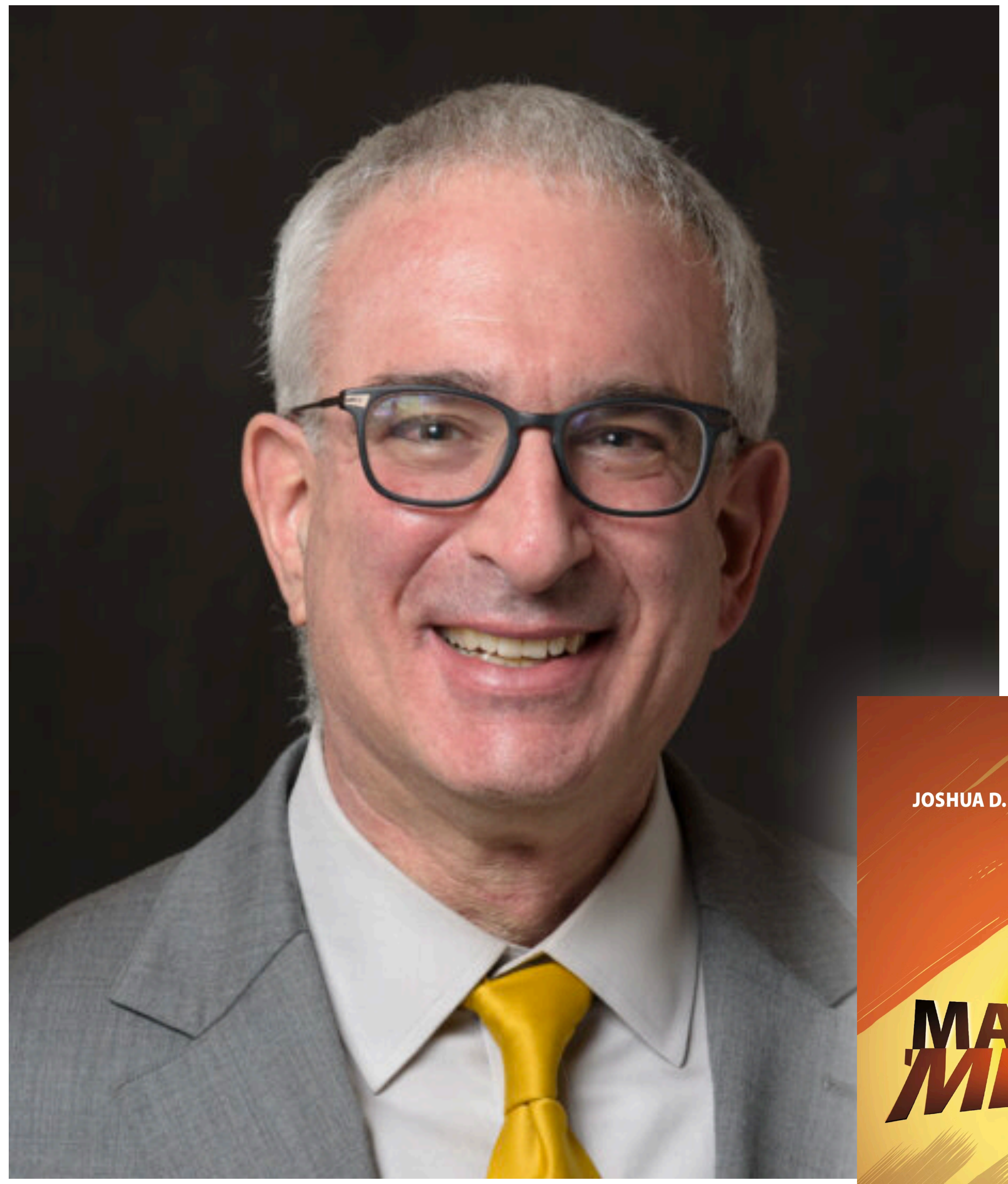
— Association for Computing Machinery (ACM)

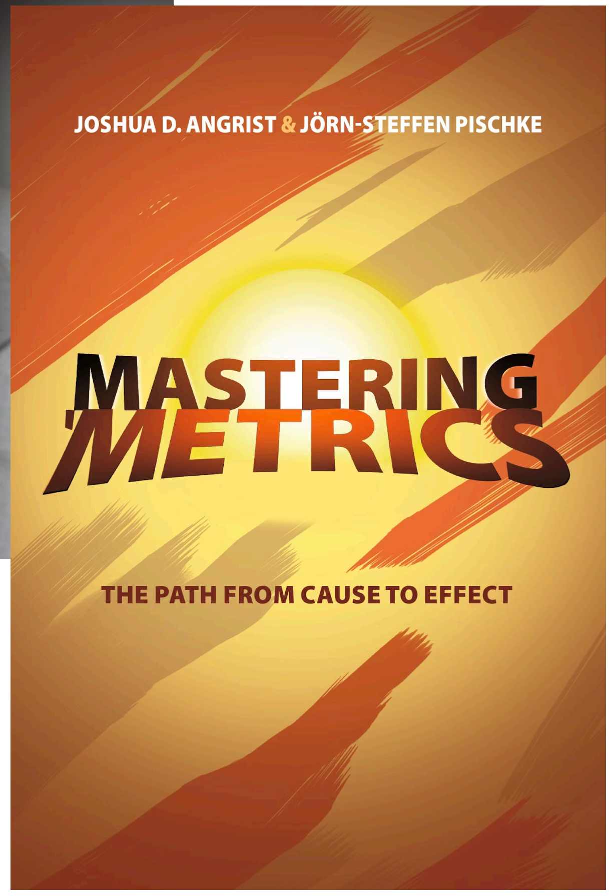
"Deep learning has instead given us machines with truly impressive abilities but no intelligence. The difference is profound and lies in the absence of a model of reality."

— The Book of Why: The New Science of Cause and Effect



Guido W. Imbens, Joshua D. Angrist & Donald B. Rubin



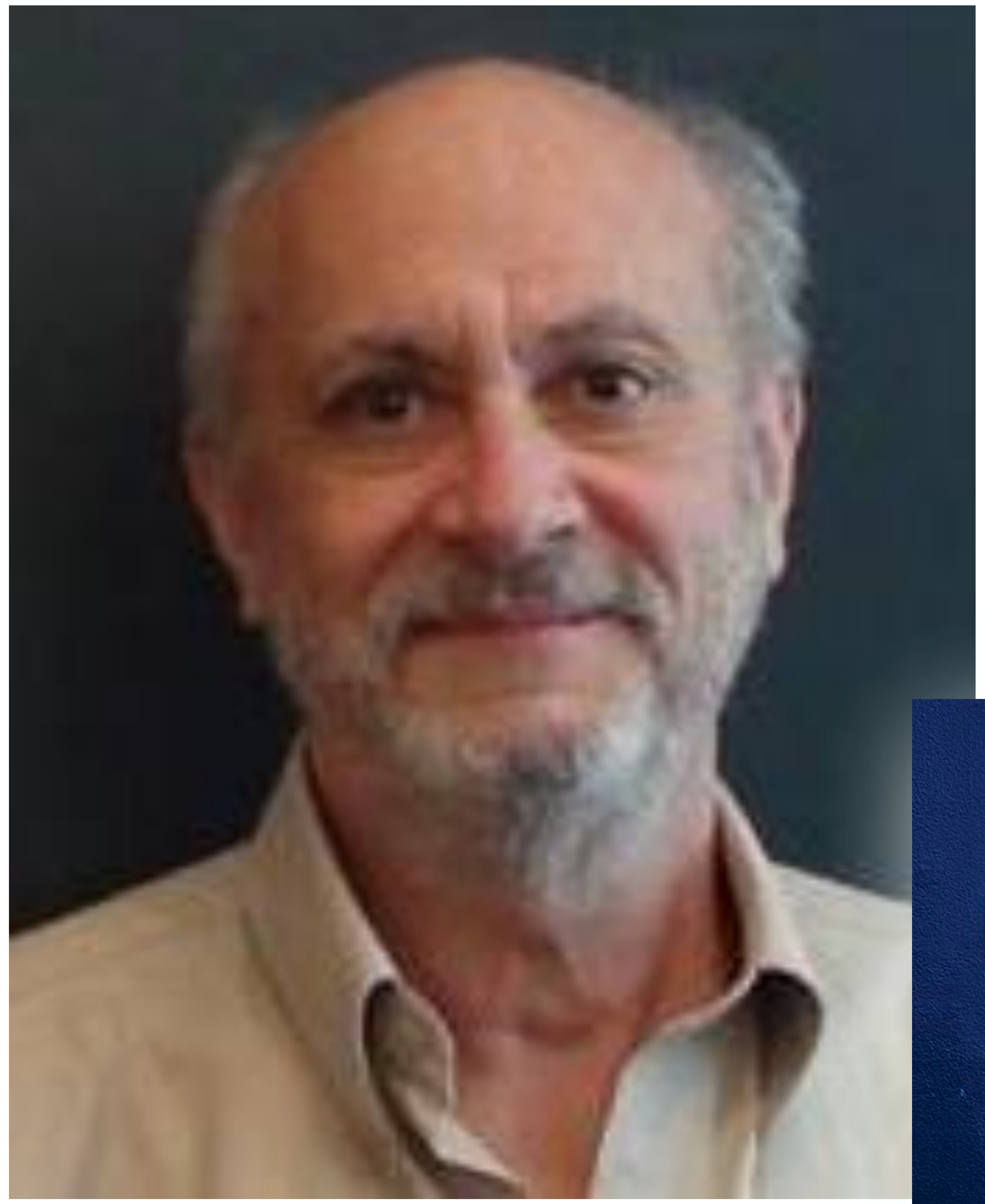


Joshua D. Angrist

Professor of
Economics at MIT

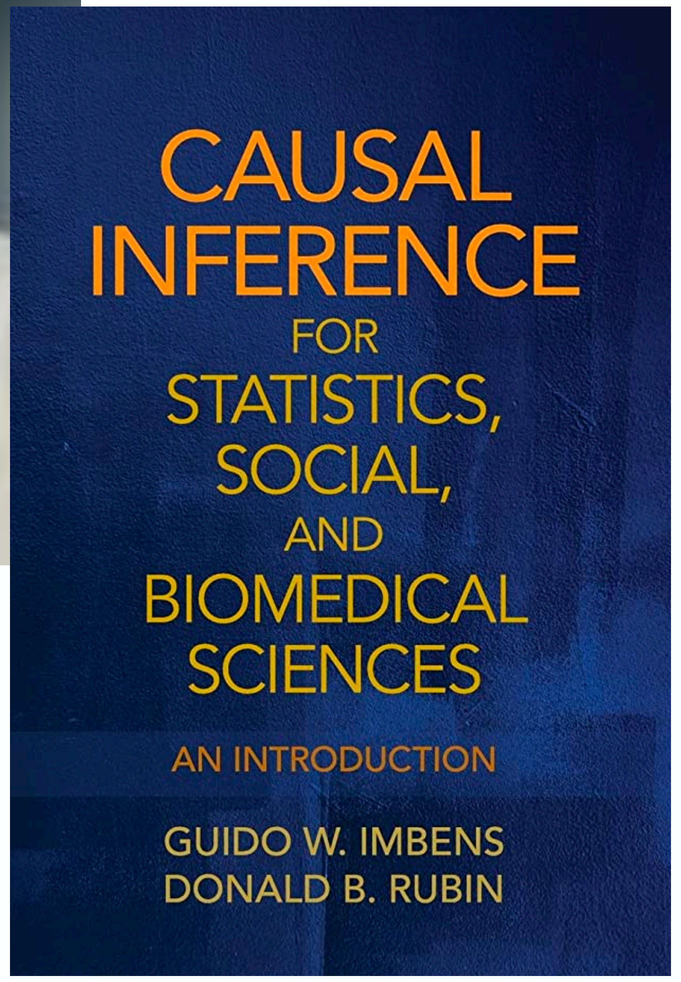
Guido W. Imbens

Professor of Applied
Econometrics at
Stanford University



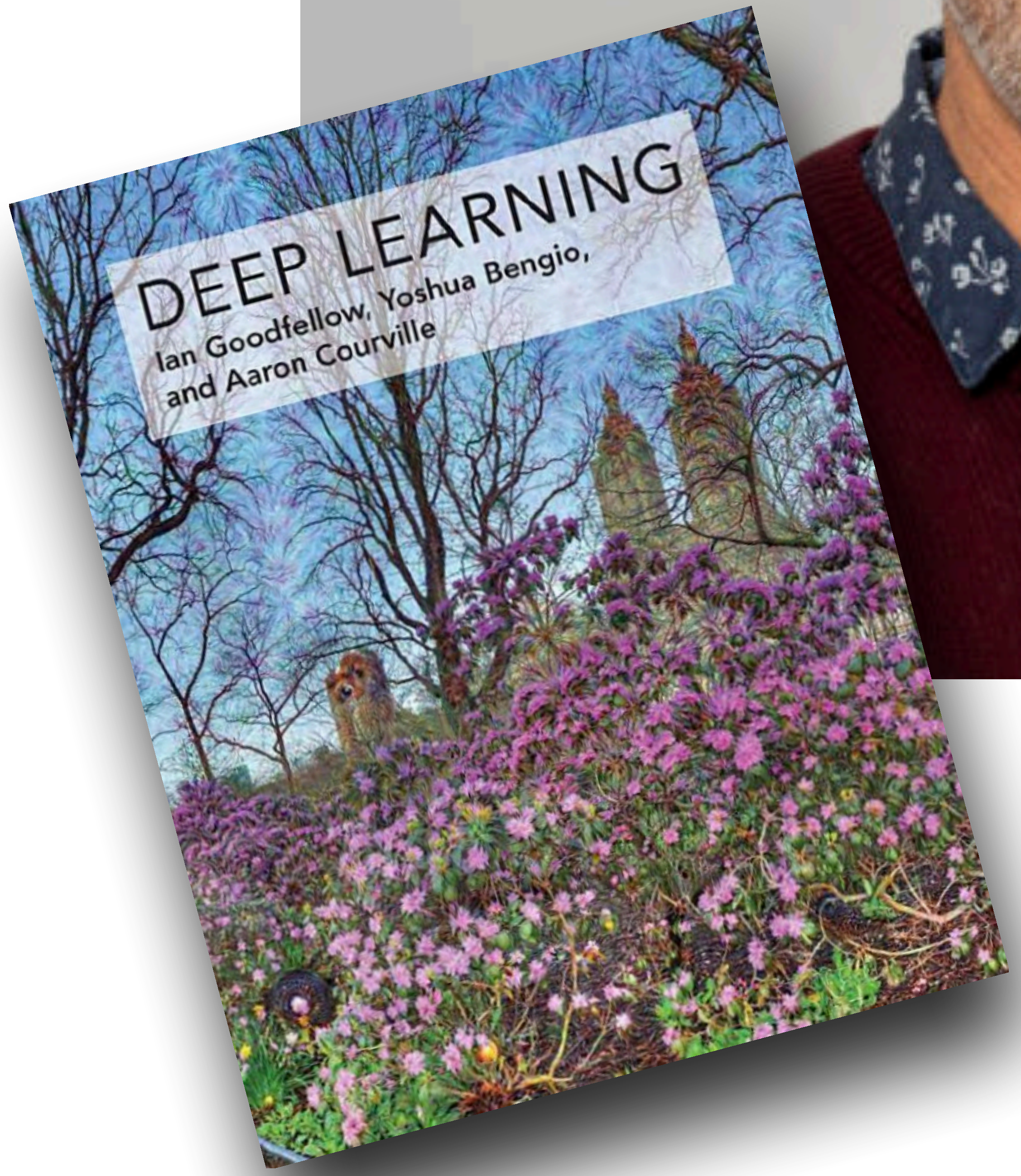
Donald B. Rubin

Professor of
Statistics at
Harvard University



In 2021, Angrist & Imbens won the Nobel Prize in Economics
“for their methodological contributions to the analysis of causal relationships”

Yoshua Bengio — Deep Learning



Professor at the University of Montreal, and the Founder and Scientific Director of Mila – Quebec AI Institute

In 2018, he won the A. M. Turing Award, with Geoffrey Hinton, and Yann LeCun

“for conceptual and engineering breakthroughs that have made deep neural networks a critical component of computing.”

— Association for Computing Machinery (ACM)

“Causality is very important for the next steps of progress of machine learning,” — interview with *IEEE Spectrum*.

Why causality is so important?

Causality allows important capabilities such as

Causal Effect: can determine the effect of *unrealized* interventions rather than just predicting an outcome (i.e., can distinguish between association and causation)

- **Causal Effect Identification and Estimation**

Explainability: provides a better understanding of the underlying mechanisms

- **Causal Discovery**

Fairness: captures and disentangles any mechanisms of discrimination that may be present, including direct, indirect-mediated, and indirect-confounded.

Generalizability: allows the transportability of causal effects across different domains.

Data Fusion: provides language and theory to cohesively combine prior knowledge and data from multiple and heterogeneous studies.

Causal Data Science

Goal is to develop language, criteria, and algorithms for:

- **Data-Fusion:** cohesively combining heterogenous datasets,
- **Causal Inference:** inferring the effects of interventions, and
- **Decision-Making:** making robust and generalizable decisions.

PNAS

Causal inference and the data-fusion problem

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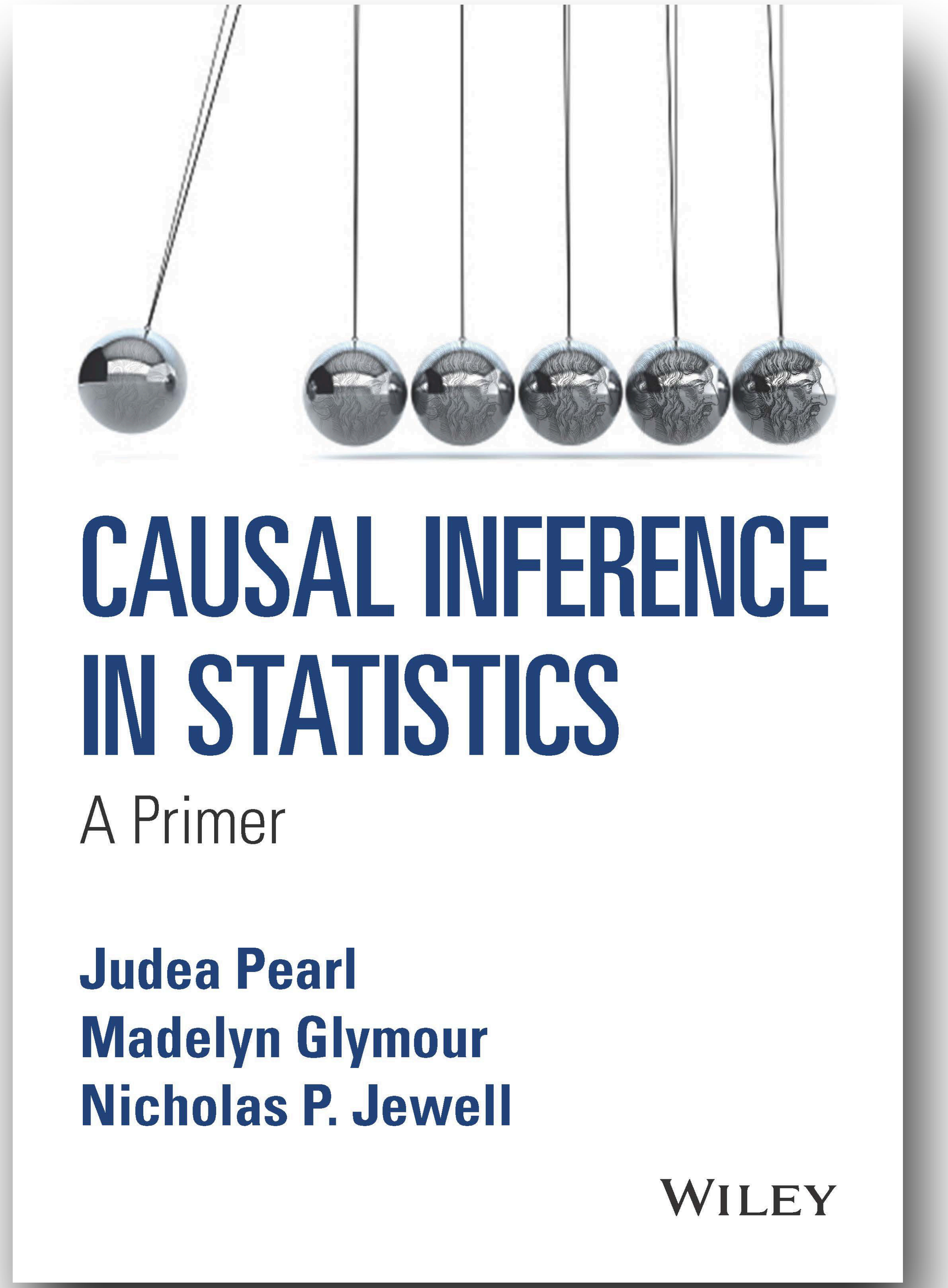
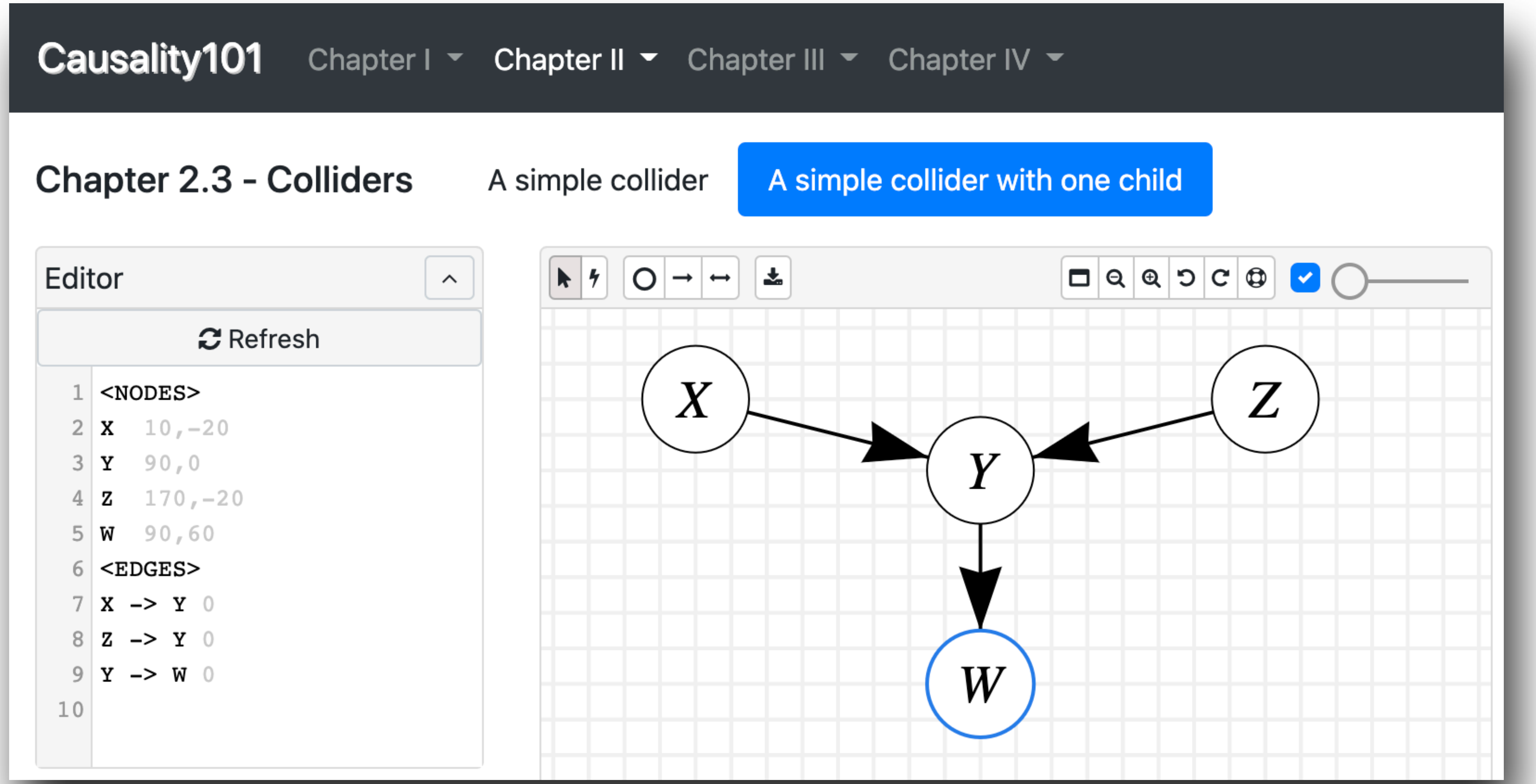
Edited by Richard M. Shiffrin, Indiana University, Bloomington, IN, and approved March 15, 2016 (received for review June 29, 2015)

http://causalfusion.net

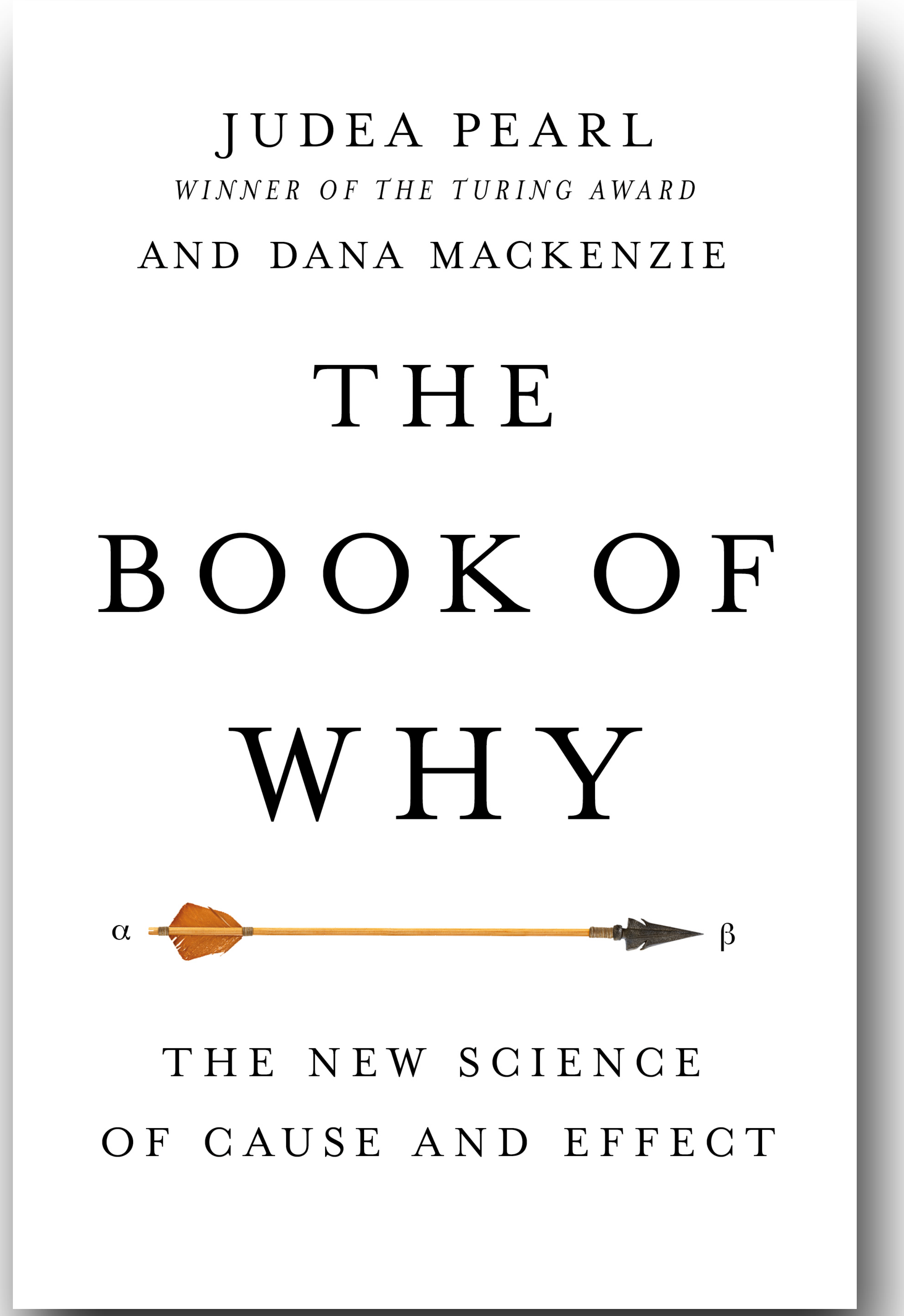
Causality Theory by Judea Pearl

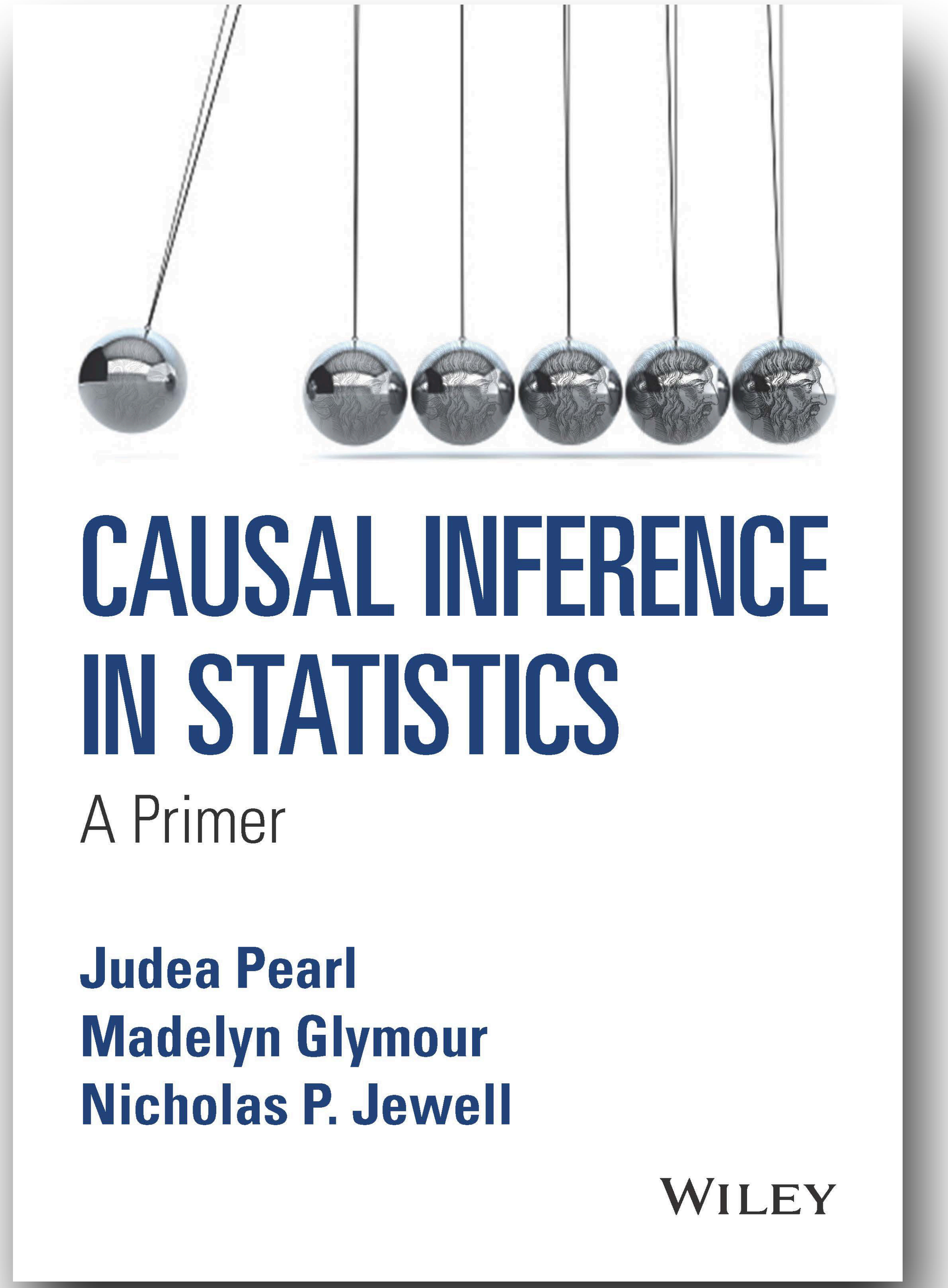


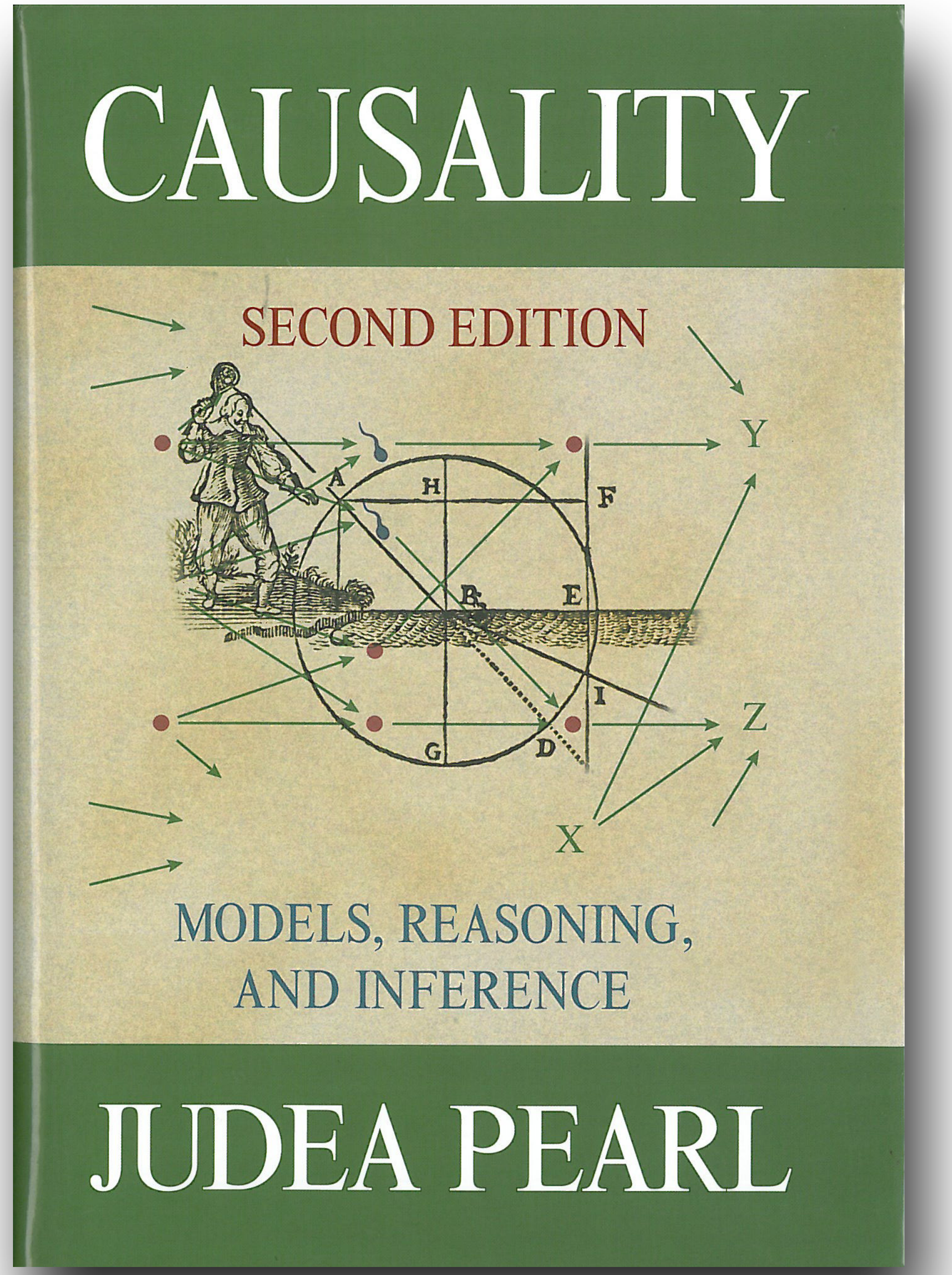
https://causality101.net/



Causality Theory by Judea Pearl







Structural Causal Model (SCM)

EXPLAINABILITY AND THE DATA GENERATING MODEL

Structural Causal Model (SCM)

Definition: A structural causal model $\mathcal{M}$ (or, data generating model) is a tuple $\langle \mathbf{V}, \mathbf{U}, \mathcal{F}, P(\mathbf{u}) \rangle$, where

- $\mathbf{V} = \{V_1, \ldots, V_n\}$: are endogenous variables
- $\mathbf{U} = \{U_1, \ldots, U_m\}$: are exogenous variables
- $\mathcal{F} = \{f_1, \ldots, f_n\}$: are functions determining $\mathbf{V}$, i.e., $v_i \leftarrow f_i(pa_i, u_i)$, where $Pa_i \subseteq \mathbf{V}$ are endogenous causes (parents) of V_i and $U_i \subseteq \mathbf{U}$ are exogenous causes of V_i.
- $P(\mathbf{U})$ is the probability distribution over $\mathbf{U}$.

Assumption: $\mathcal{M}$ is recursive, i.e., there are no feedback (cyclic) mechanisms.

Structural Equation Model (SEM)

$$\mathscr{M} = \begin{cases} \mathbf{V} = \{X, Y, Z\} \\ \mathbf{U} = \{\epsilon_X, \epsilon_Y, \epsilon_Z\} \\ \mathscr{F} = \begin{cases} Z = \beta_{Z0} + \epsilon_Z \\ X = \beta_{X0} + \beta_{XZ}Z + \epsilon_X \\ Y = \beta_{Y0} + \beta_{YZ}Z + \beta_{YX}X + \epsilon_Y) \end{cases} \\ \mathbf{U} \sim \mathscr{N}\left(\mathbf{0}, \Sigma = \begin{bmatrix} \sigma_X & 0 & 0 \\ 0 & \sigma_Y & 0 \\ 0 & 0 & \sigma_Z \end{bmatrix}\right) \end{cases}$$

- **Linear functions**
- **Normal distribution**
- **Markovianity / Causal Sufficiency**: Error terms in $\mathbf{U}$ are independent of each other (diagonal covariance matrix).

Full specification of an SCM <u>requires</u> parametric and distributional assumptions.

Estimation of such models usually requires strong assumptions (e.g., Markovianity).

Statistical Association vs Causation

Pre-Interventional/ Observational SCM

$$\mathscr{M} = \begin{cases} \mathbf{V} = \{X, Y\} \\ \mathbf{U} = \{U_{XY}, U_X, U_Y\} \\ \mathscr{F} = \begin{cases} X = f_X(U_X, U_{XY}) \\ Y = f_Y(X, U_Y, U_{XY}) \end{cases} \\ P(\mathbf{U}) \end{cases}$$

$do(X = x)$

Post-Interventional / Interventional SCM

$$\mathscr{M}_x = \begin{cases} \mathbf{V} = \{X, Y\} \\ \mathbf{U} = \{U_{XY}, U_X, U_Y\} \\ \mathscr{F} = \begin{cases} X = x \\ Y = f_Y(x, U_Y, U_{XY}) \end{cases} \\ P(\mathbf{U}) \end{cases}$$

Observational Distribution

$$P(\mathbf{V}) \doteq P_{\mathscr{M}}(\mathbf{V})$$

$$\neq$$

Interventional Distribution

$$P(\mathbf{V} \mid do(X = x)) \doteq P_{\mathscr{M}_x}(\mathbf{V})$$

Can we ***predict*** better the value of Y after ***observing*** that $X = x$?

$P(Y = y \mid X = x) \neq P(Y = y) \implies X$ is ***correlated*** to Y

Can we ***predict*** better the value of Y after ***making an intervention*** $do(X = x)$?

$\exists x$ s.t. $P_{\mathscr{M}_x}(Y = y) \neq P(Y = y) \implies X$ is ***a cause*** of Y

Statistical Association vs Causation

Pre-Interventional/ Observational SCM

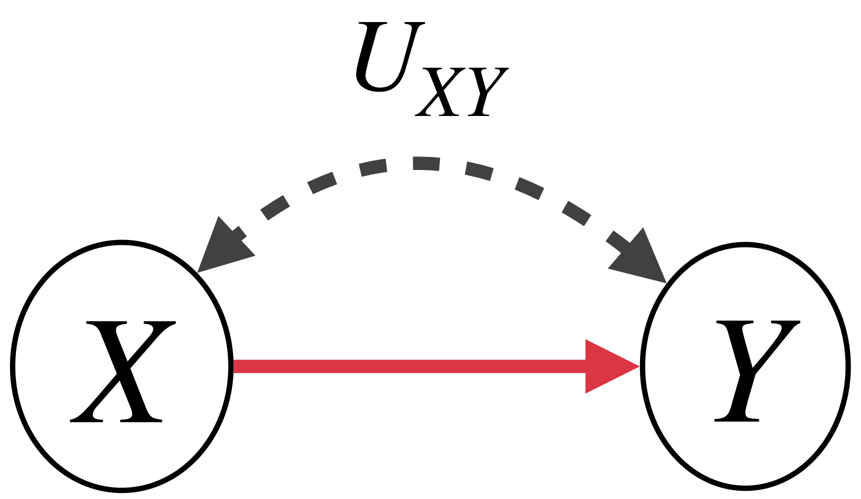
$$\mathcal{M} = \begin{cases} \mathbf{V} = \{X, Y\} \\ \mathbf{U} = \{U_{XY}, U_X, U_Y\} \\ \mathcal{F} = \begin{cases} X = f_X(U_X, U_{XY}) \\ Y = f_Y(X, U_Y, U_{XY}) \end{cases} \\ P(\mathbf{U}) \end{cases}$$

Loss of Information

Observational Data

$P_{\mathcal{M}}(\mathbf{V})$

Observational Distribution



Observational Causal Diagram

$do(X = x)$

Post-Interventional / Interventional SCM

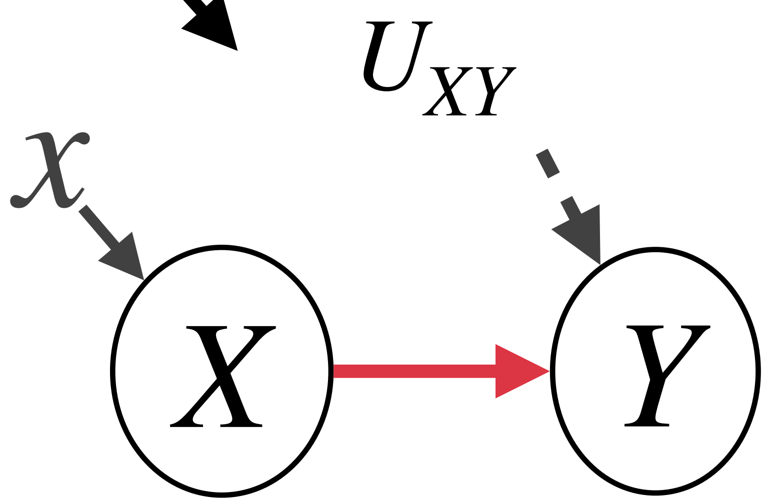
$$\mathcal{M}_x = \begin{cases} \mathbf{V} = \{X, Y\} \\ \mathbf{U} = \{U_{XY}, U_X, U_Y\} \\ \mathcal{F} = \begin{cases} X = x \\ Y = f_Y(x, U_Y, U_{XY}) \end{cases} \\ P(\mathbf{U}) \end{cases}$$

Loss of Information

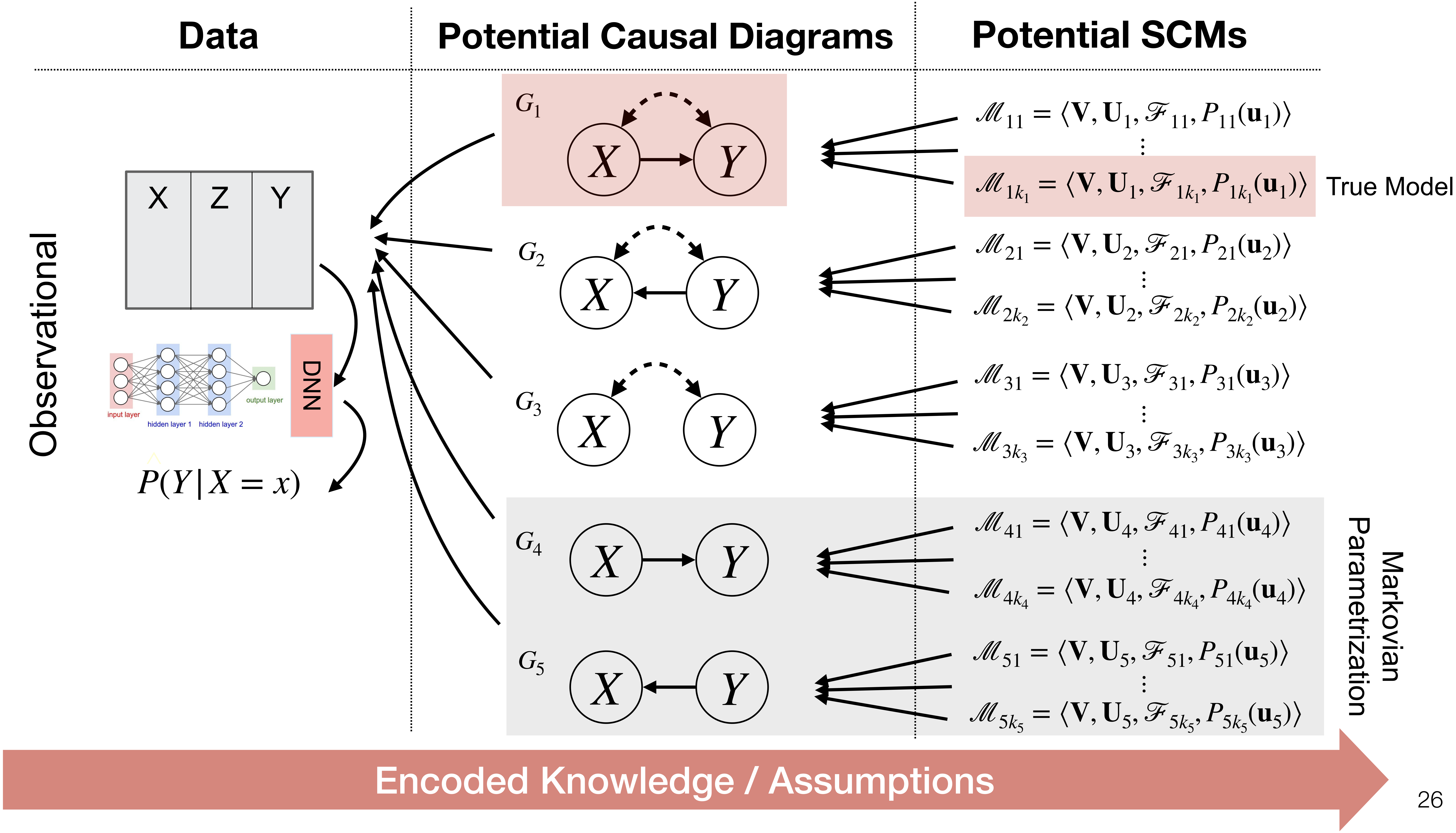
Interventional Data

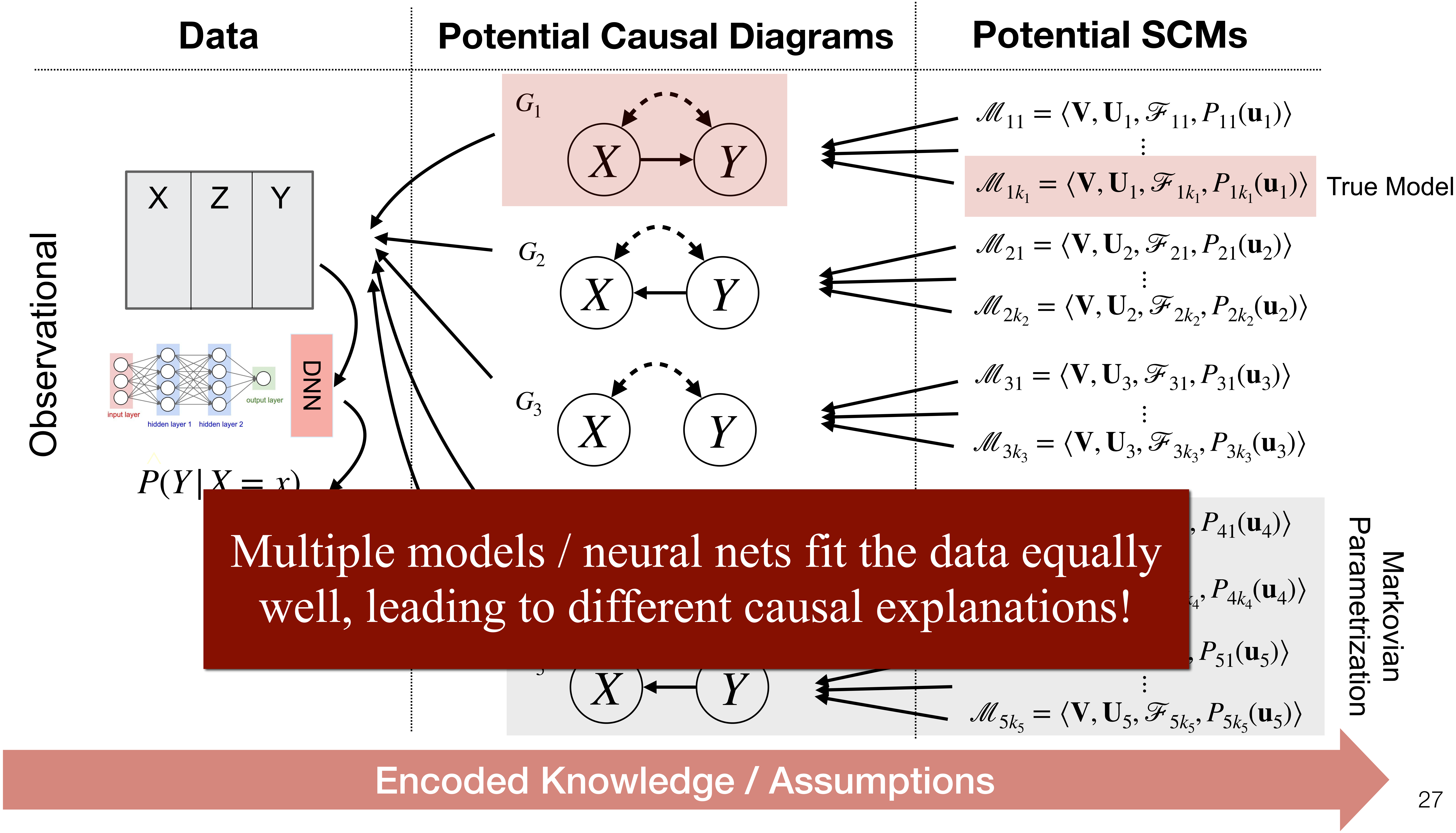
$P_{\mathcal{M}_x}(\mathbf{V}) \doteq P(\mathbf{V} \mid do(x))$

Interventional Distribution



Interventional Causal Diagram


Data
Potential Causal Diagrams
Potential SCMs
Observational
X
Z
Y
input layer
hidden layer 1
hidden layer 2
output layer
DNN
$\hat{P}(Y|X=x)$
G_1
G_2
G_3
G_4
G_5
$\mathcal{M}_{11} = \langle \mathbf{V}, \mathbf{U}_1, \mathcal{F}_{11}, P_{11}(\mathbf{u}_1) \rangle$
$\mathcal{M}_{1k_1} = \langle \mathbf{V}, \mathbf{U}_1, \mathcal{F}_{1k_1}, P_{1k_1}(\mathbf{u}_1) \rangle$
True Model
$\mathcal{M}_{21} = \langle \mathbf{V}, \mathbf{U}_2, \mathcal{F}_{21}, P_{21}(\mathbf{u}_2) \rangle$
$\mathcal{M}_{2k_2} = \langle \mathbf{V}, \mathbf{U}_2, \mathcal{F}_{2k_2}, P_{2k_2}(\mathbf{u}_2) \rangle$
$\mathcal{M}_{31} = \langle \mathbf{V}, \mathbf{U}_3, \mathcal{F}_{31}, P_{31}(\mathbf{u}_3) \rangle$
$\mathcal{M}_{3k_3} = \langle \mathbf{V}, \mathbf{U}_3, \mathcal{F}_{3k_3}, P_{3k_3}(\mathbf{u}_3) \rangle$
$\mathcal{M}_{41} = \langle \mathbf{V}, \mathbf{U}_4, \mathcal{F}_{41}, P_{41}(\mathbf{u}_4) \rangle$
$\mathcal{M}_{4k_4} = \langle \mathbf{V}, \mathbf{U}_4, \mathcal{F}_{4k_4}, P_{4k_4}(\mathbf{u}_4) \rangle$
$\mathcal{M}_{51} = \langle \mathbf{V}, \mathbf{U}_5, \mathcal{F}_{51}, P_{51}(\mathbf{u}_5) \rangle$
$\mathcal{M}_{5k_5} = \langle \mathbf{V}, \mathbf{U}_5, \mathcal{F}_{5k_5}, P_{5k_5}(\mathbf{u}_5) \rangle$
Markovian Parametrization
Encoded Knowledge / Assumptions


Data
Potential Causal Diagrams
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$\hat{P}(Y|X = x)$
G_1
G_2
G_3
X
Y
$\mathcal{M}_{11} = \langle \mathbf{V}, \mathbf{U}_1, \mathcal{F}_{11}, P_{11}(\mathbf{u}_1) \rangle$
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True Model
$\mathcal{M}_{21} = \langle \mathbf{V}, \mathbf{U}_2, \mathcal{F}_{21}, P_{21}(\mathbf{u}_2) \rangle$
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$\mathcal{M}_{31} = \langle \mathbf{V}, \mathbf{U}_3, \mathcal{F}_{31}, P_{31}(\mathbf{u}_3) \rangle$
$\mathcal{M}_{3k_3} = \langle \mathbf{V}, \mathbf{U}_3, \mathcal{F}_{3k_3}, P_{3k_3}(\mathbf{u}_3) \rangle$
$P_{41}(\mathbf{u}_4) \rangle$
$P_{4k_4}(\mathbf{u}_4) \rangle$
$P_{51}(\mathbf{u}_5) \rangle$
$\mathcal{M}_{5k_5} = \langle \mathbf{V}, \mathbf{U}_5, \mathcal{F}_{5k_5}, P_{5k_5}(\mathbf{u}_5) \rangle$
Markovian Parametrization
Multiple models / neural nets fit the data equally well, leading to different causal explanations!
Encoded Knowledge / Assumptions

Bayesian Network

A DAG, possibly with latent confounders (ADMG),
representing the **conditional independences**
implied by an SCM

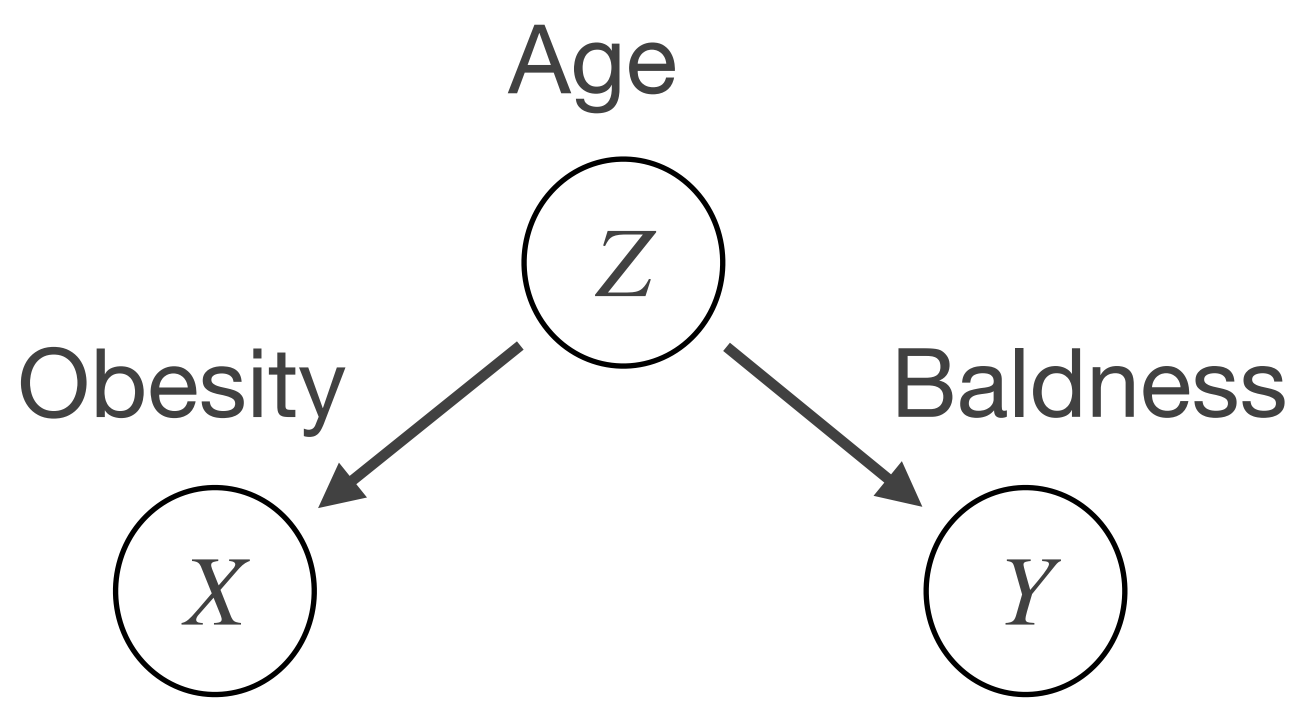
Directed
Acyclic Graph

Acyclic Directed
Mixed Graph

Conditional independencies implied by d-separation

Fork

Z **as a common cause**

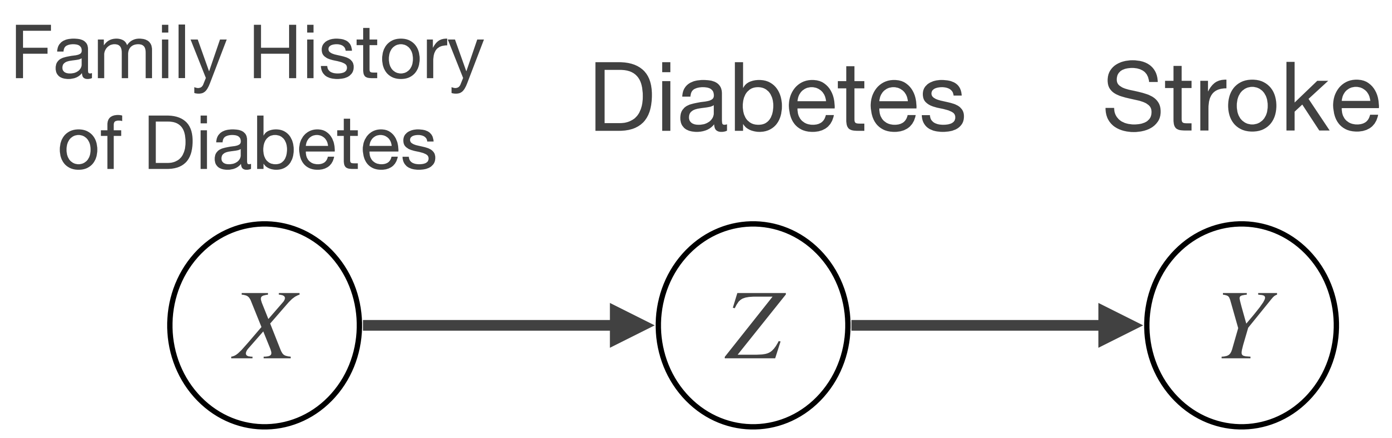


$$X \not\!\perp\!\!\!\perp Y$$

$$X \perp\!\!\!\perp Y | Z$$

Chain

Z **as a mediator**



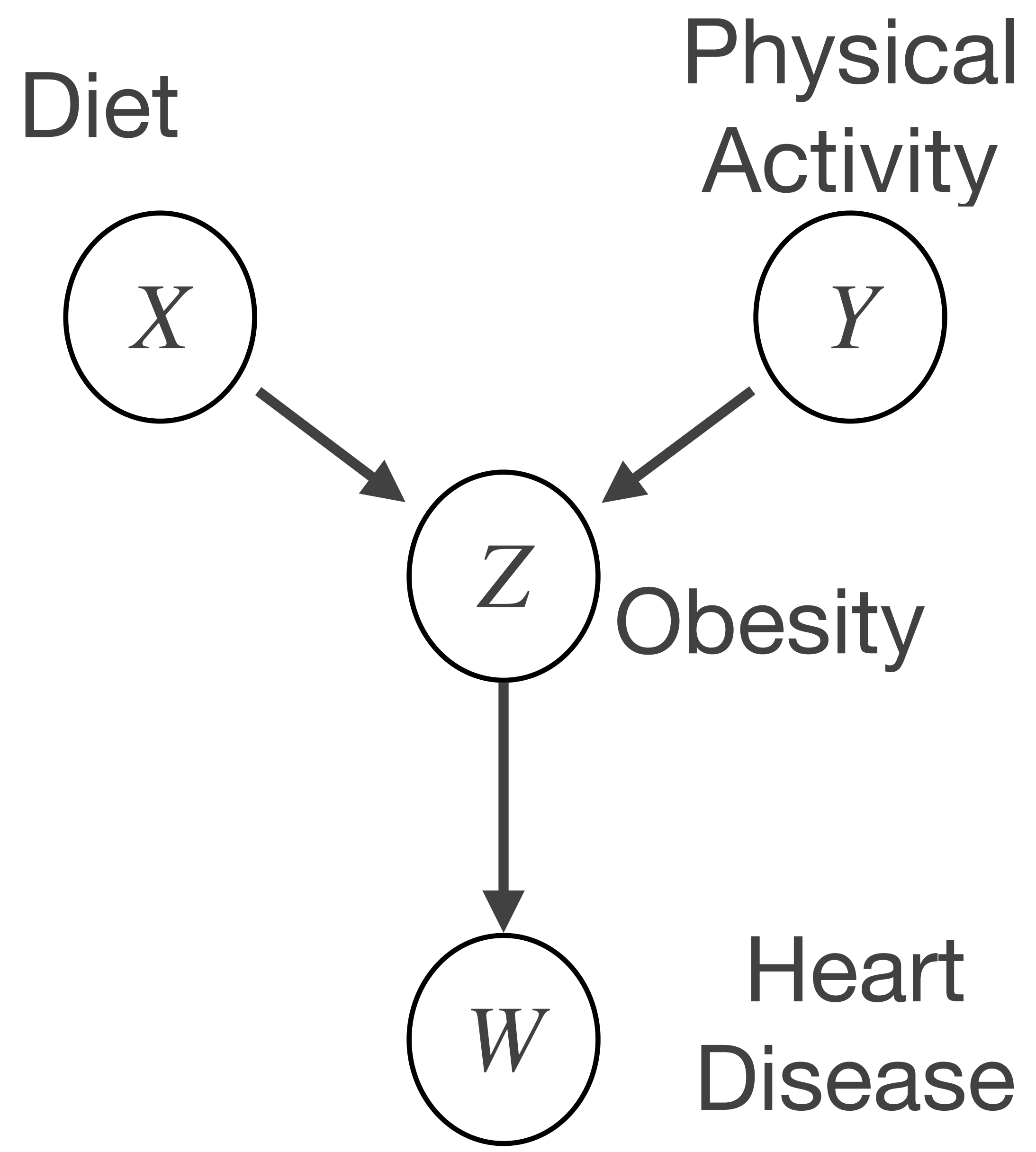
$$X \not\!\perp\!\!\!\perp Y$$

$$X \perp\!\!\!\perp Y | Z$$

In both cases, Z is a non-collider!

V-Structure

Z **as a collider or common effect**



$$X \perp\!\!\!\perp Y$$

$$X \not\!\perp\!\!\!\perp Y | Z$$

$$X \not\!\perp\!\!\!\perp Y | W$$

BN - Encoder of Conditional Independences

Bayesian Networks (BN) are **Minimal Independence Maps**:

$$(\mathbf{X} \perp\!\!\!\perp \mathbf{Y} \mid \mathbf{Z})_G \Rightarrow (\mathbf{X} \perp\!\!\!\perp \mathbf{Y} \mid \mathbf{Z})_P$$

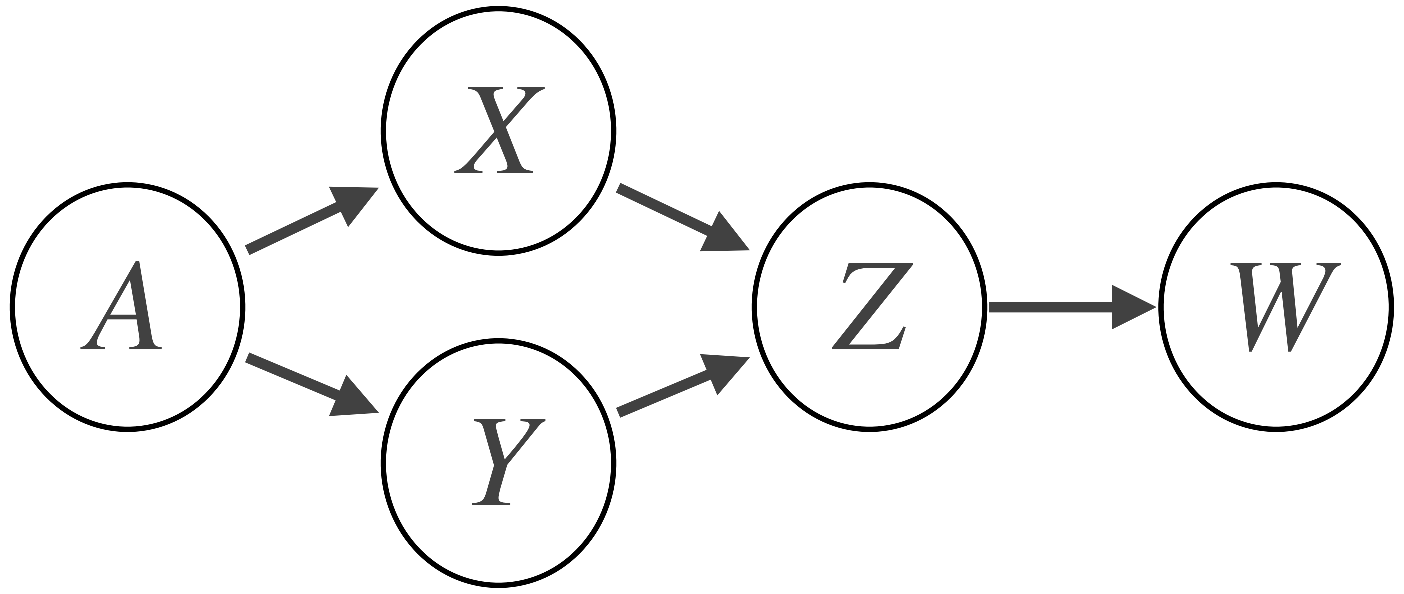
No edges of G can be removed without ceasing such a property.

Observational Distribution

$$P(\mathbf{V}) \doteq P_{\mathscr{M}}(\mathbf{V}) = \sum_{\mathbf{u}} \prod_{V_i \in \mathbf{V}} P(v_i \mid pa_i, u_i) P(\mathbf{u})$$

Factorization obtained by Chain Rule and conditional independencies implied by the SCM $\mathscr{M}$.

Edges have no causal semantics!



$$P(\mathbf{v}) = P(w \mid z, x, y, a)\ P(z \mid x, y, a)\ P(x \mid y, a)\ P(y \mid a)\ P(a)$$
$$= P(w \mid z)\ P(z \mid x, y)\ P(x \mid a)\ P(y \mid a)\ P(a)$$

$W \perp\!\!\!\perp X, Y, A \mid Z$ $\quad A \perp\!\!\!\perp Z \mid X, Y$ $\quad Y \perp\!\!\!\perp X \mid A$

Markov Equivalence Class

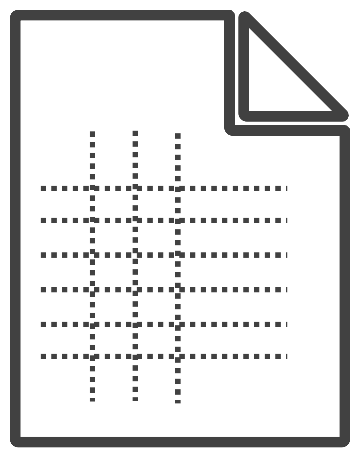
$$\mathcal{M}_1 = \begin{cases} \mathbf{V} = \{X, Y\} \\ \mathbf{U} = \{U_x, U_Y\} \\ \mathcal{F} = \begin{cases} f_X(U_X) \\ f_Y(X, U_Y) \end{cases} \\ P(\mathbf{U}) \end{cases}$$

$\vdots$

$$\mathcal{M}_{N-1} = \begin{cases} \mathbf{V} = \{X, Y\} \\ \mathbf{U} = \{U_x, U_Y, U_{X,Y}\} \\ \mathcal{F} = \begin{cases} f_X(Y, U_X, U_{X,Y}) \\ f_Y(U_Y, U_{X,Y}) \end{cases} \\ P(\mathbf{U}) \end{cases}$$

$$\mathcal{M}_N = \begin{cases} \mathbf{V} = \{X, Y\} \\ \mathbf{U} = \{U_x, U_Y\} \\ \mathcal{F} = \begin{cases} f_X(U_X) \\ f_Y(U_Y) \end{cases} \\ P(\mathbf{U}) \end{cases}$$

Data



Conditional (in)dependencies

$$\frac{P(\mathbf{v})}{X \not\perp\!\!\!\perp Y}$$

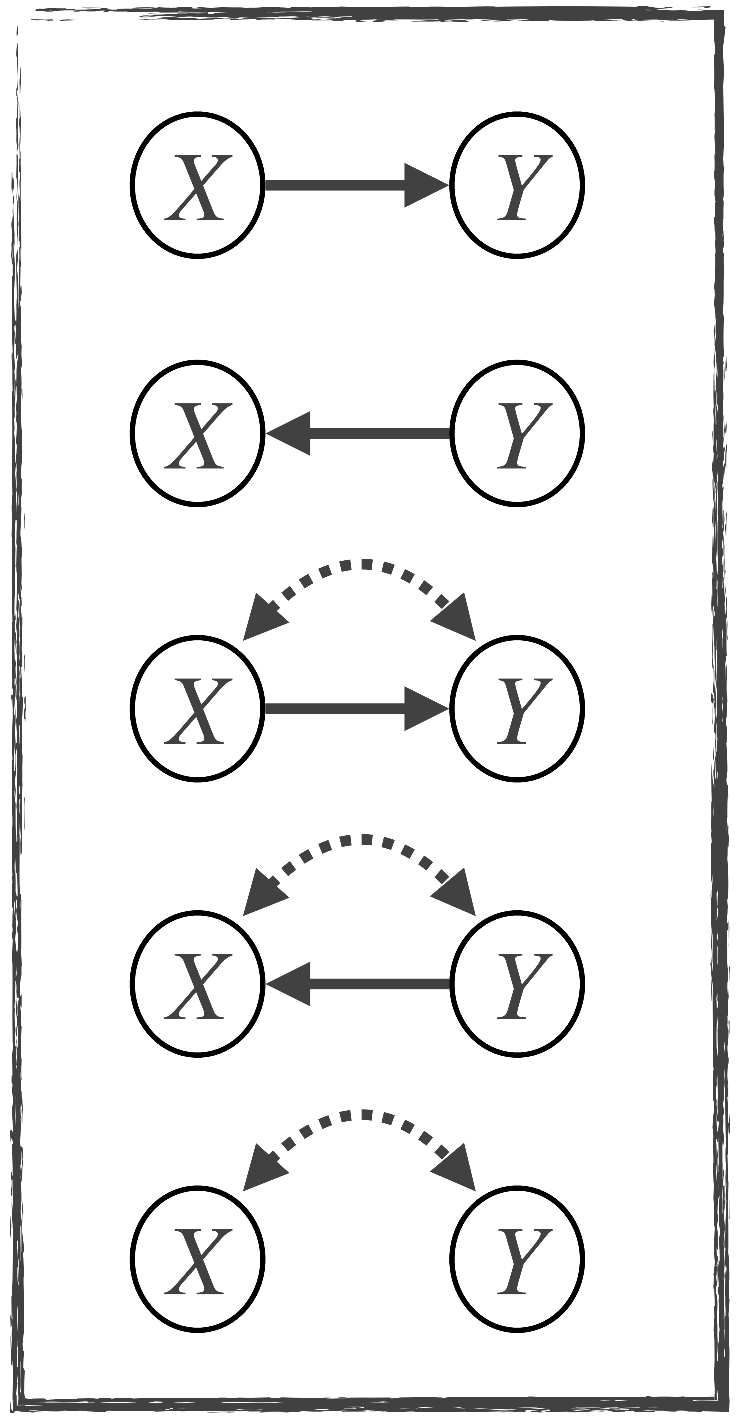
$$P(x, y) = \sum_{u_x, u_y} P(x \mid y) P(y) P(u_x, u_y)$$

$$P(x, y) = \sum_{u_x, u_y} P(y \mid x) P(x) P(u_x, u_y)$$

$\vdots$

Markov Equivalence Class

(class of models implying the same set of conditional independencies)

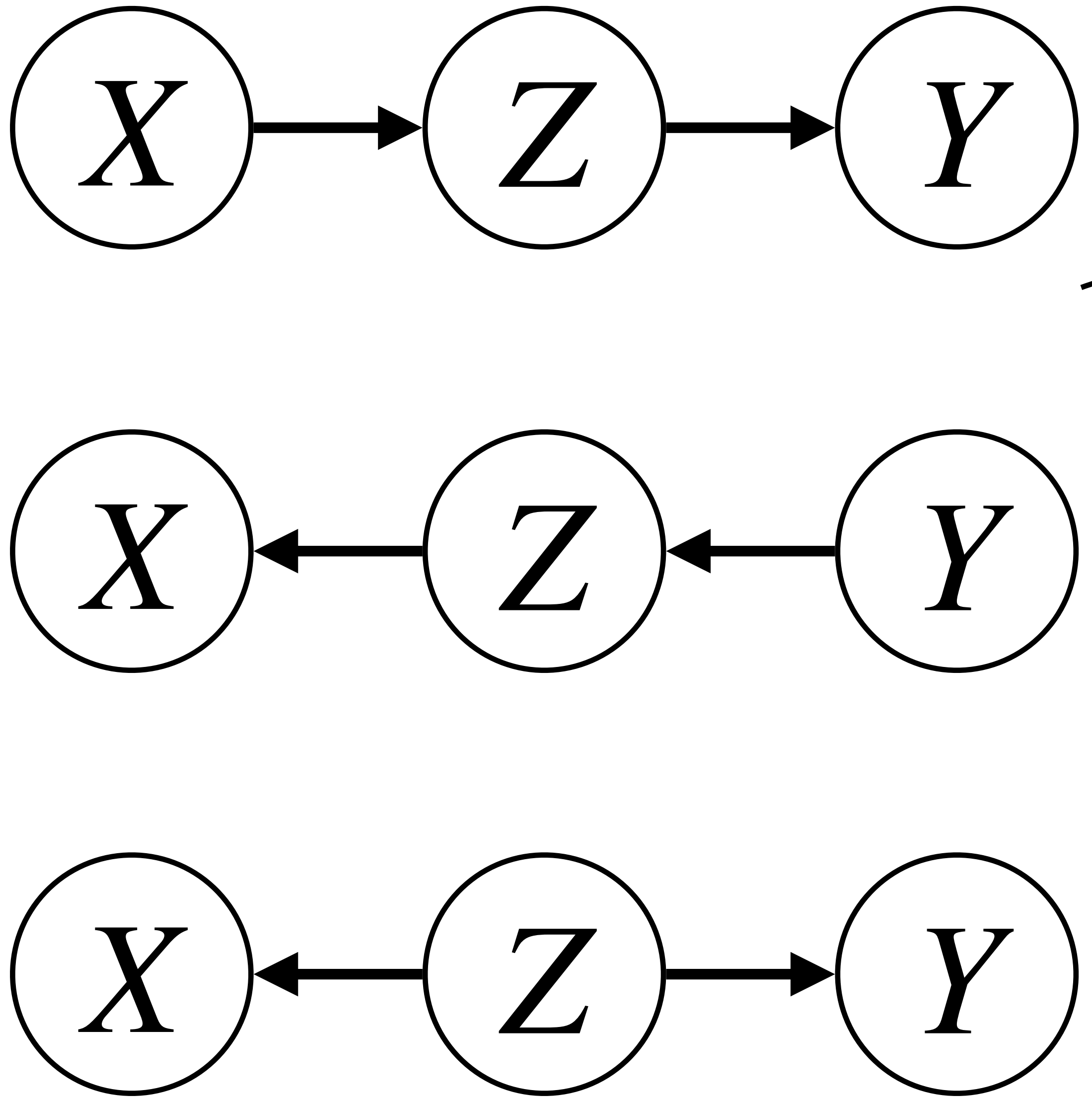


Correlation does not imply causation!

Equivalent Bayesian Networks

Distribution	Factorization	Bayesian Networks
$P(X, Y, Z)$ with $P(Y \mid X, Z) = P(Y \mid X)$ i.e., $X \perp\!\!\!\perp Y \mid Z$	$P(x, y, z) = {\color{red}P(y \mid x, z)}P(z \mid x)P(x)$ $= {\color{red}P(y \mid z)}P(z \mid x)P(x)$ $P(x, y, z) = {\color{red}P(x \mid y, z)}P(y \mid z)P(z)$ $= {\color{red}P(x \mid z)}P(z \mid y)P(y)$ $P(x, y, z) = {\color{red}P(y \mid x, z)}P(x \mid z)P(z)$ $= {\color{red}P(y \mid z)}P(x \mid z)P(z)$	

Observational Data



Markov Equivalent

Two models are considered ***Markov equivalent*** if they imply the same conditional independencies.

Equivalent Bayesian Networks

Distribution | **Factorization** | **Bayesian Networks**

$P(X, Y, Z)$

with $P(Y | X, Z) = P(Y | X)$

i.e., $X \perp\!\!\!\perp Y | Z$

$$P(x, y, z) = P(y | x, z)P(z | x)P(x) = P(y | z)P(z | x)P(x)$$

$$P(x, y, z) = P(x | y, z)P(y | z)P(z) = P(x | z)P(z | y)P(y)$$

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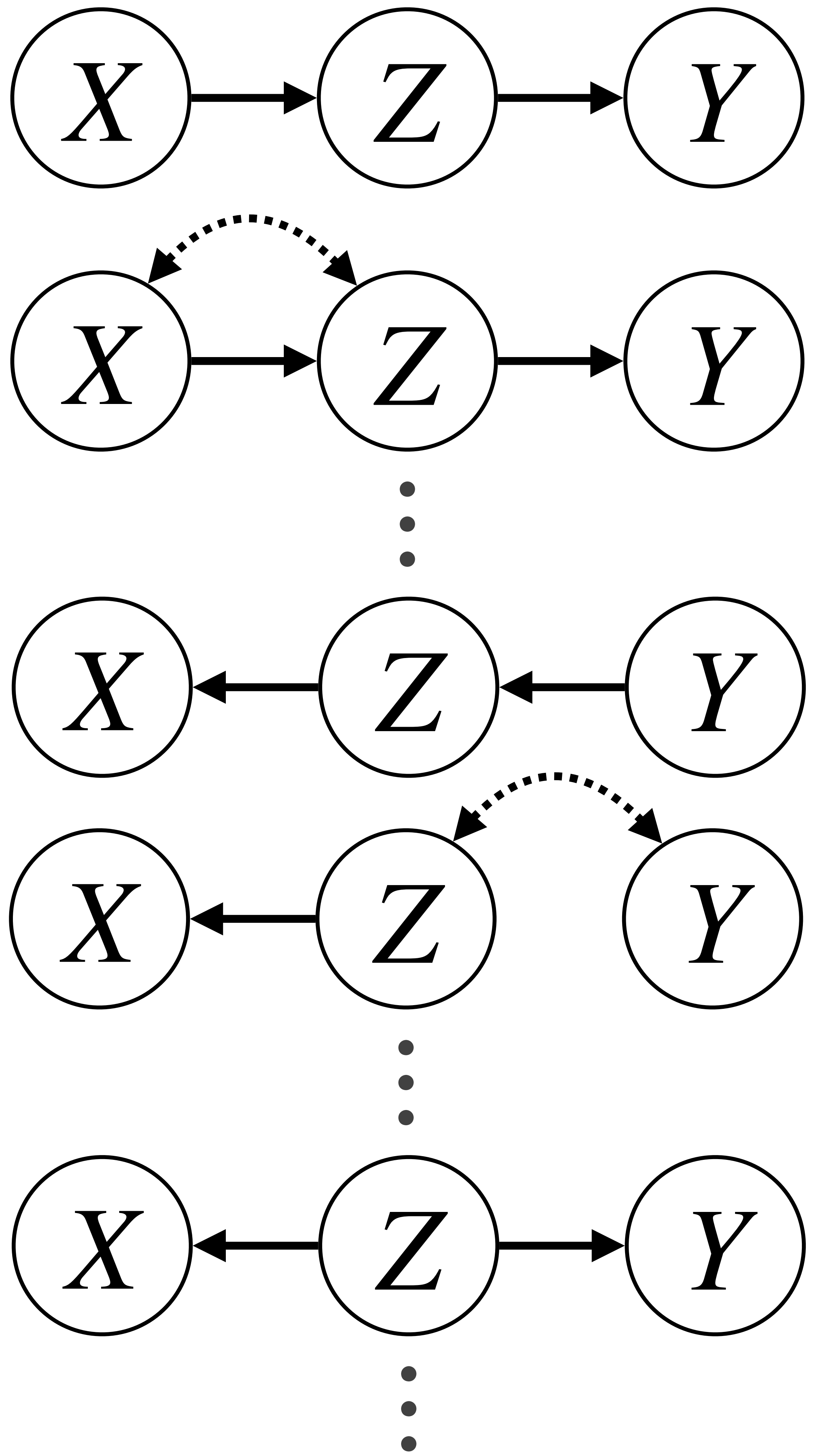
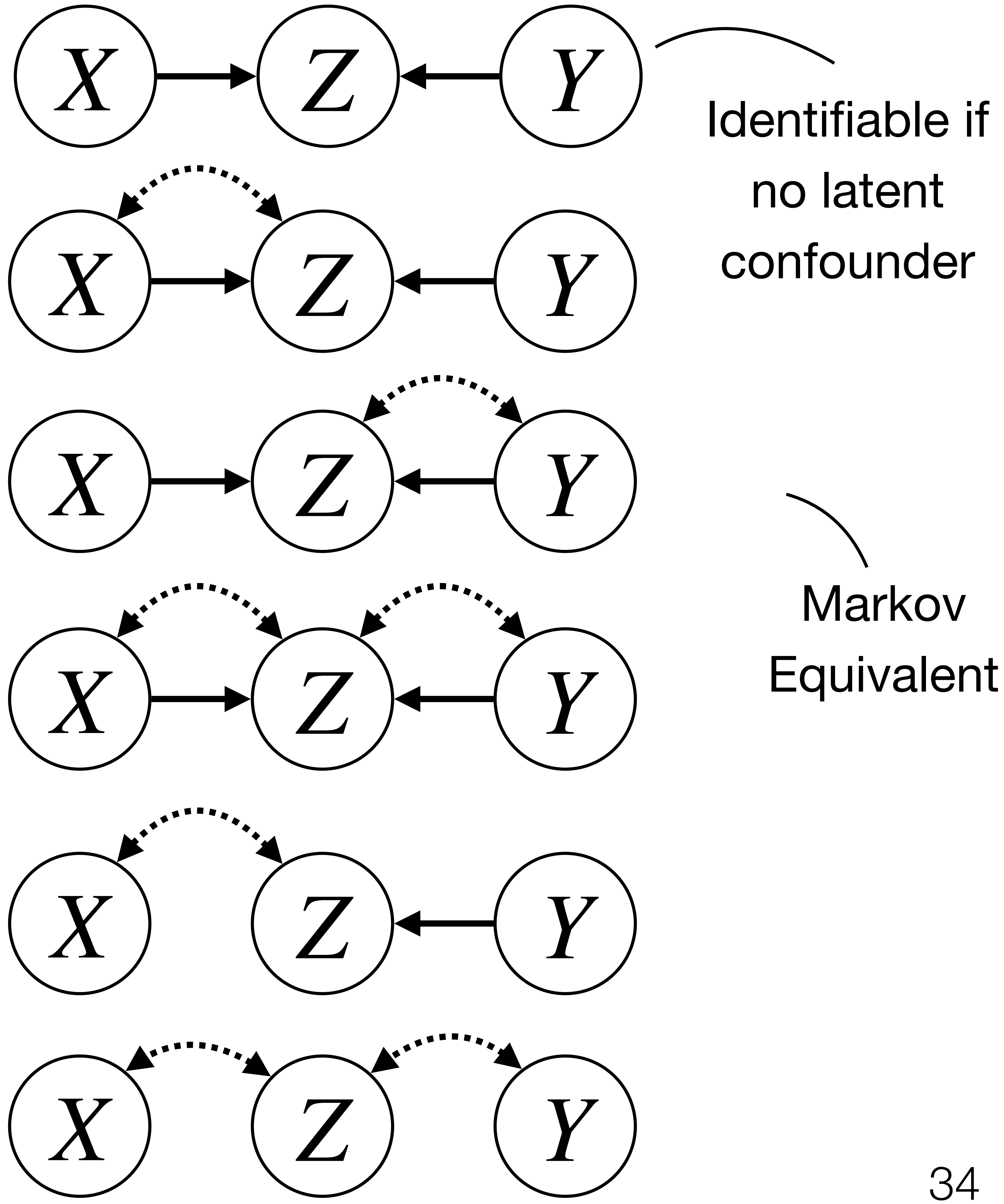


Markov Equivalent

Invariance:
Z is **never** a collider
(either ancestor of X and Y).

Equivalent Bayesian Networks

Distribution	Factorization	Bayesian Networks
$P(X, Y, Z)$ with $P(Y \mid X) = P(Y)$ i.e., $X \perp\!\!\!\perp Y$	$P(x, y, z) = P(z \mid x, y)P(x \mid y)P(y)$ $= P(z \mid x, y)P(x)P(y)$	

Equivalent Bayesian Networks

Distribution

$$P(X, Y, Z)$$

with $P(Y \mid X) = P(Y)$

i.e., $X \perp\!\!\!\perp Y$

Factorization

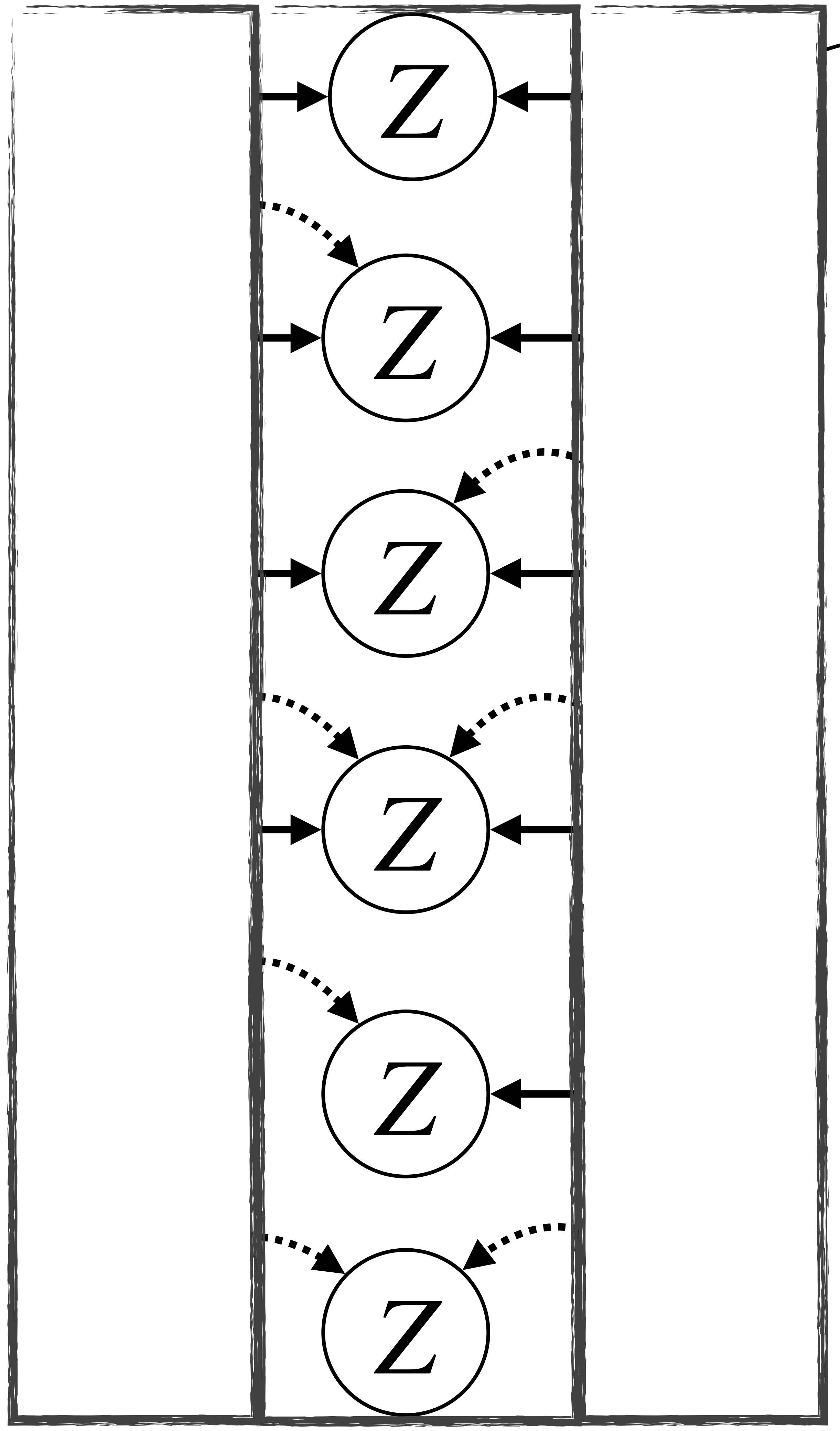
$$\begin{aligned} P(x, y, z) &= P(z \mid x, y){\color{red}P(x \mid y)}P(y) \\ &= P(z \mid x, y){\color{red}P(x)}P(y) \end{aligned}$$

Invariance:

Z is **always** a collider

(non-ancestor of X and Y).

Bayesian Networks



Identifiable if no latent confounder

Markov Equivalent

Causal Bayesian Network

A DAG, possibly with latent confounders (ADMG),
representing the **causal and confounding relationships**
implied by an SCM

CBN: Encoder of <u>Structural</u> <u>Causal</u> Knowledge

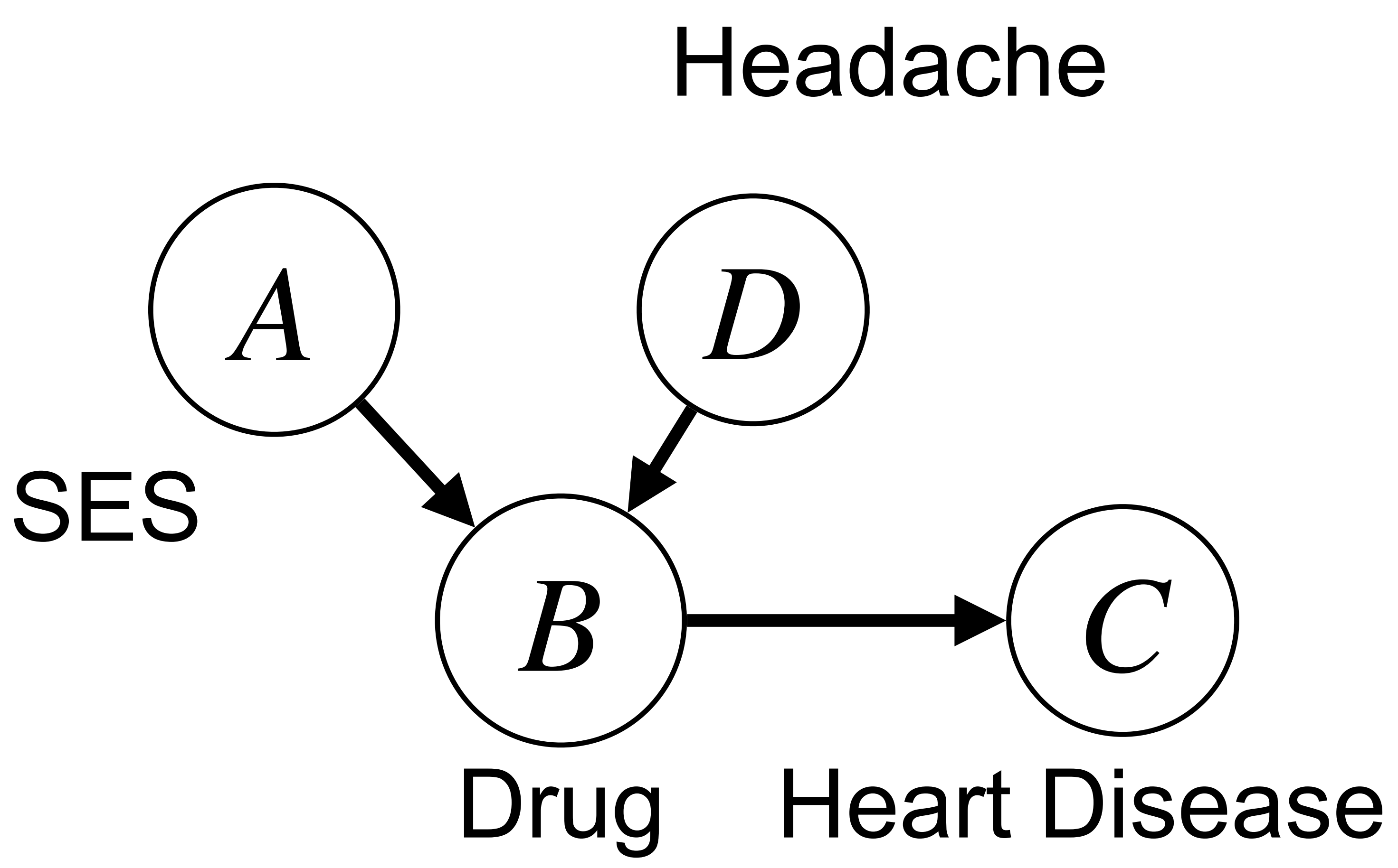
Structural Causal Model (SCM)

$$\mathcal{M} = \langle \mathbf{V}, \mathbf{U}, \mathcal{F}, P(\mathbf{u}) \rangle$$

$$\mathcal{M} = \begin{cases} \mathbf{V} = \{A, B, C, D\} \\ \mathbf{U} = \{U_A, U_B, U_C, U_D, U_{CD}\} \\ \mathcal{F} = \begin{cases} A \leftarrow f_A(U_A) \\ B \leftarrow f_B(A, D, U_B) \\ D \leftarrow f_Z(U_D, U_{CD}) \\ C \leftarrow f_X(B, U_C, U_{CD}) \end{cases} \\ P(\mathbf{U}) \end{cases}$$

Induced Causal Bayesian Network (CBN)

Causal Diagram



An SCM $\mathcal{M} = \langle \mathbf{V}, \mathbf{U}, \mathcal{F}, P(\mathbf{u}) \rangle$ induces a causal diagram such that, **for every** $V_i, V_j \in \mathbf{V}$:

$V_i \rightarrow V_j$, if V_i appears as argument of $f_j \in \mathcal{F}$.

CBN: Encoder of Structural Causal Knowledge

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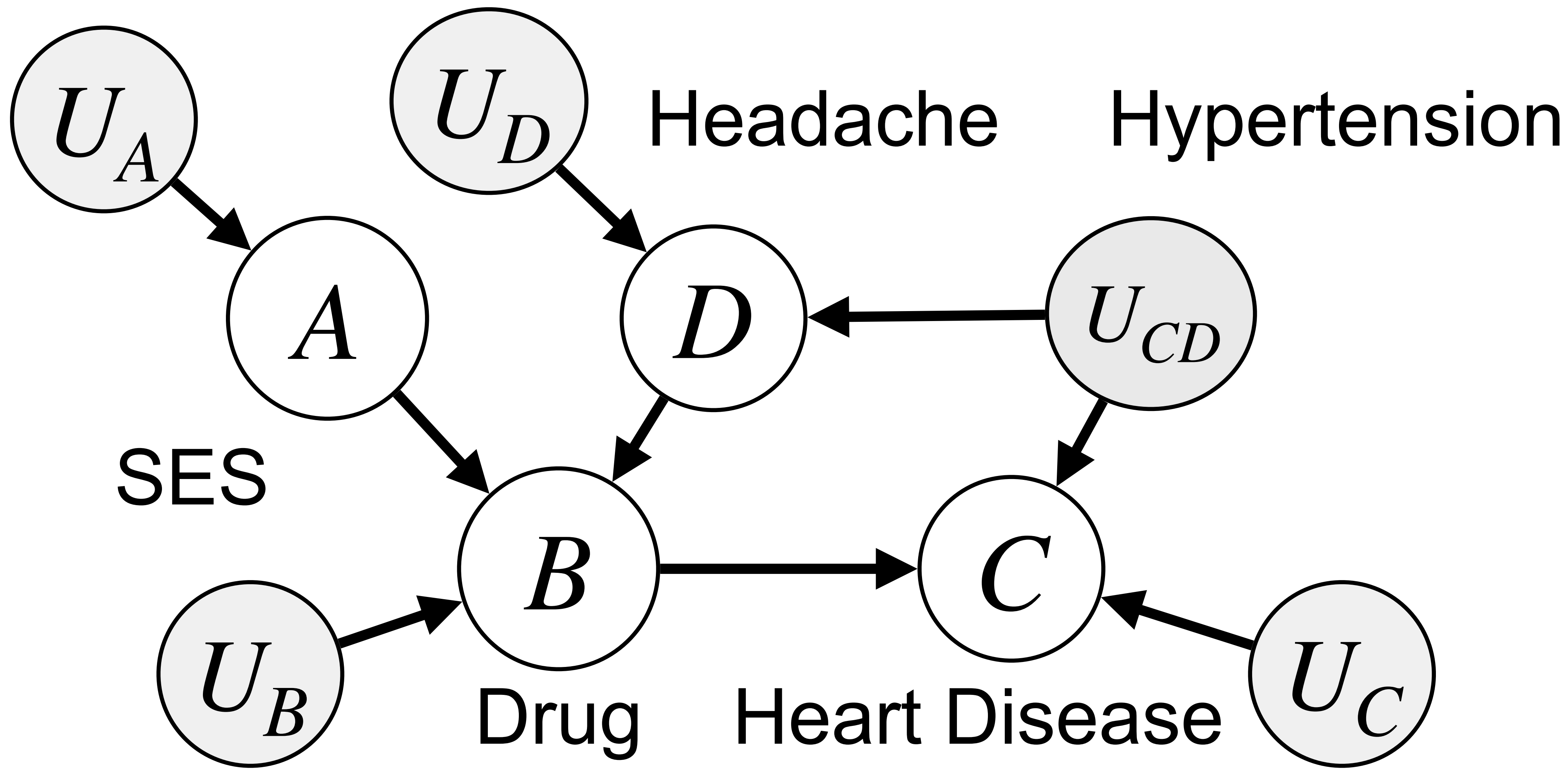
Induced Causal Bayesian Network (CBN)

Causal Diagram



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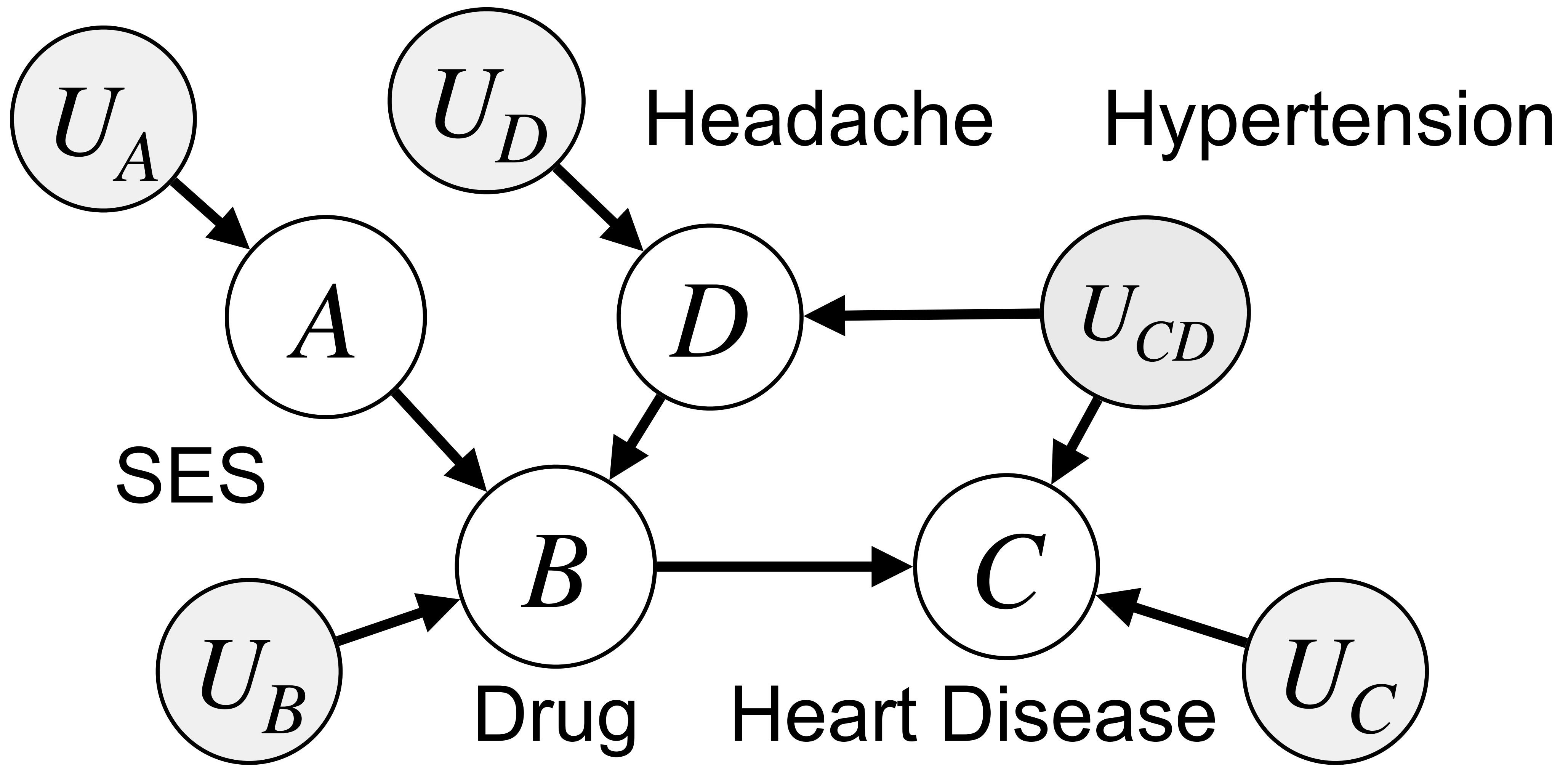
Induced Causal Bayesian Network (CBN)

Causal Diagram



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$V_i \rightarrow V_j$, if V_i appears as argument of $f_j \in \mathscr{F}$.

$V_i \leftarrow\!\!\!-\!\!\!\rightarrow V_j$ if the corresponding $U_i, U_j \in \mathbf{U}$ are correlated or f_i , f_j share some argument $U \in \mathbf{U}$.

CBN: Encoder of Structural Causal Knowledge

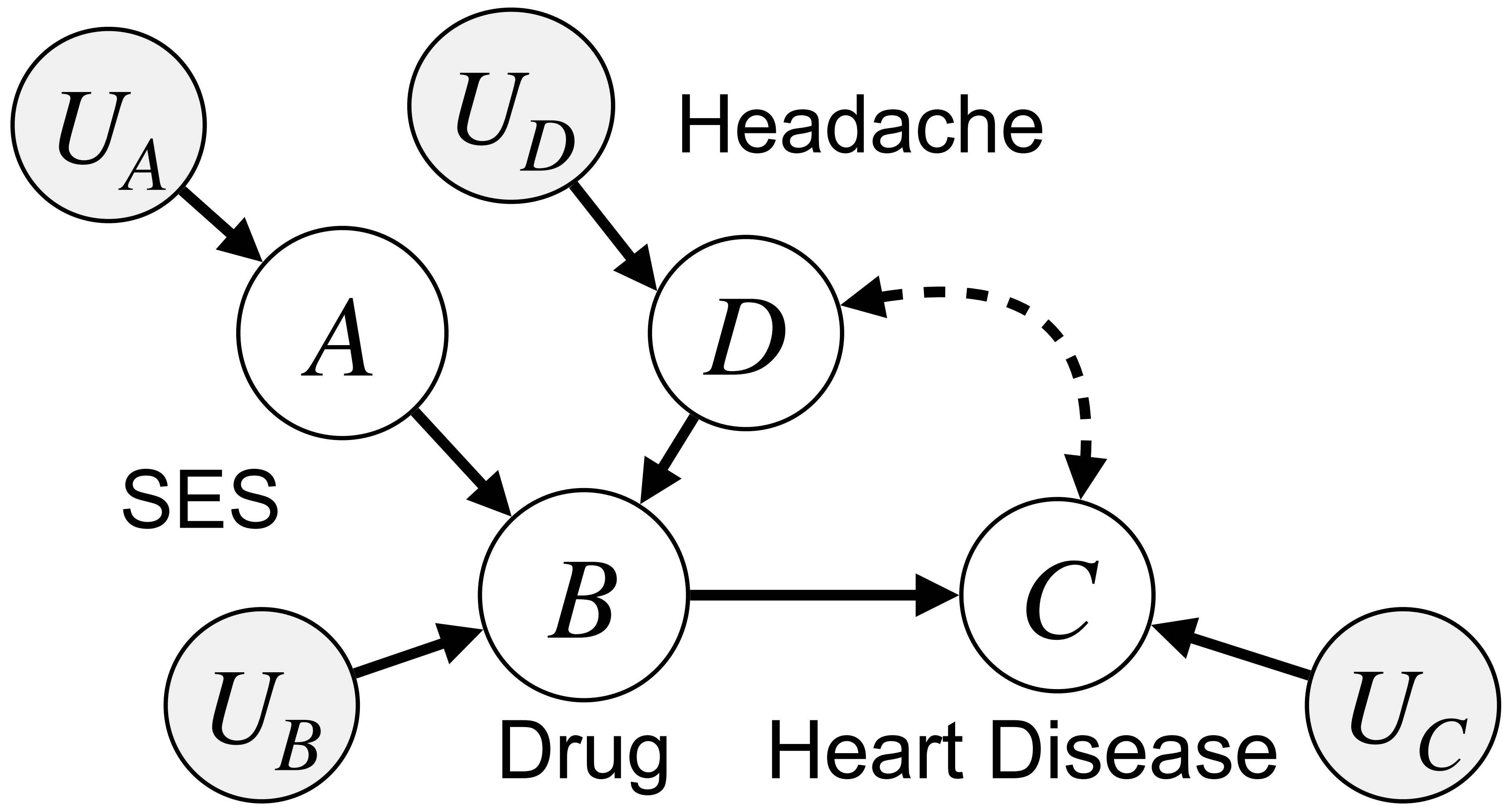
Structural Causal Model (SCM)

$\mathscr{M} = \langle \mathbf{V}, \mathbf{U}, \mathscr{F}, P(\mathbf{u}) \rangle$

$$
\mathscr{M} = \begin{cases} \mathbf{V} = \{A, B, C, D\} \\ \mathbf{U} = \{U_A, U_B, U_C, U_D, U_{CD}\} \\ \mathscr{F} = \begin{cases} A \leftarrow f_A(U_A) \\ B \leftarrow f_B(A, D, U_B) \\ D \leftarrow f_Z(U_D, U_{CD}) \\ C \leftarrow f_X(B, U_C, U_{CD}) \end{cases} \\ P(\mathbf{U}) \end{cases}
$$

Induced Causal Bayesian Network (CBN)

Causal Diagram



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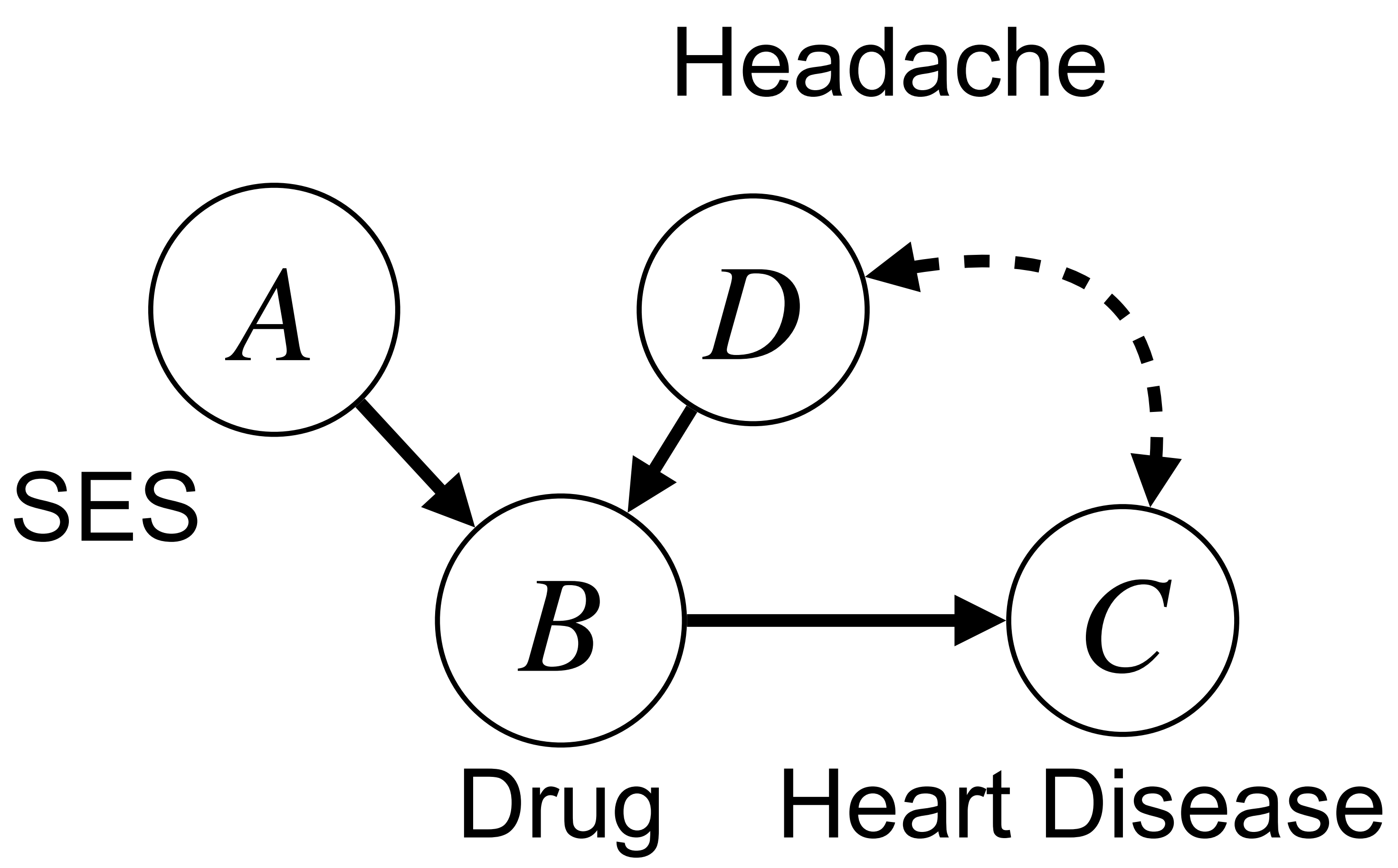
Induced Causal Bayesian Network (CBN)

Causal Diagram



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CBN: Encoder of Structural Causal Knowledge

Let $\mathbf{P}_*$ be the collection of all interventional distributions $P(\mathbf{V} \mid do(\mathbf{x}))$, $\mathbf{X} \subseteq \mathbf{V}$, including the null (observational) distribution $P(\mathbf{V})$.

An Acyclic Directed Mixed Graph (ADMG) G is a CBN for $\mathbf{P}_*$ if for every intervention $do(\mathbf{X} = \mathbf{x})$, $\mathbf{X} \subseteq \mathbf{V}$, if it hold:

Interventional Distribution

$$P(\mathbf{V} \mid do(X = x)) \doteq P_{\mathscr{M}_x}(\mathbf{V})$$

$$= \sum_{\mathbf{u}} \prod_{V_i \in \mathbf{V} \setminus \mathbf{X}} P(v_i \mid pa_i, u_i) P(\mathbf{u}) \Big|_{\mathbf{X}=\mathbf{x}}$$

Truncated factorization implied by the SCM $\mathscr{M}_x$.

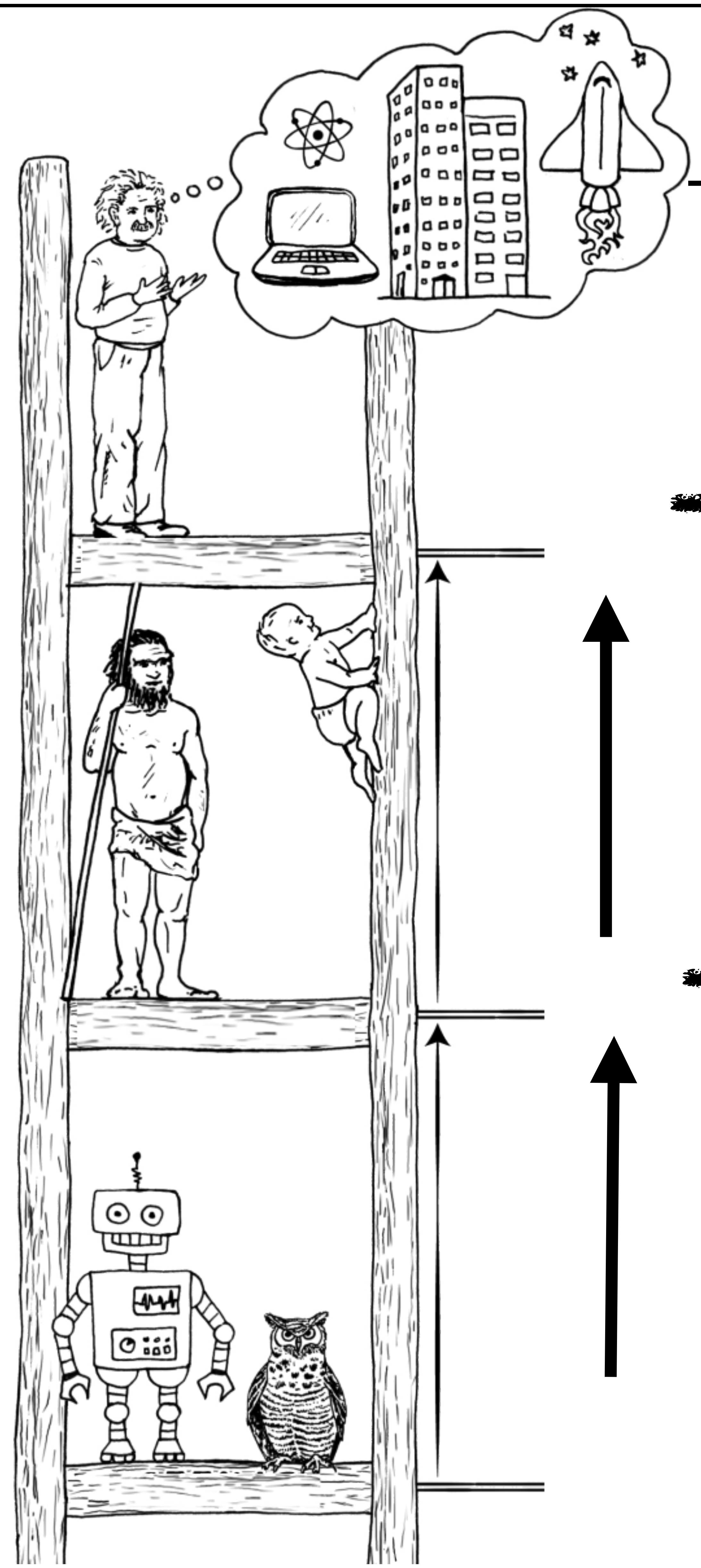
Semi-Markov relative to $G_{\overline{\mathbf{X}}}$

Pearl’s Causal Hierarchy (PCH)

The Three Inferential Layers

Ladder of Causation

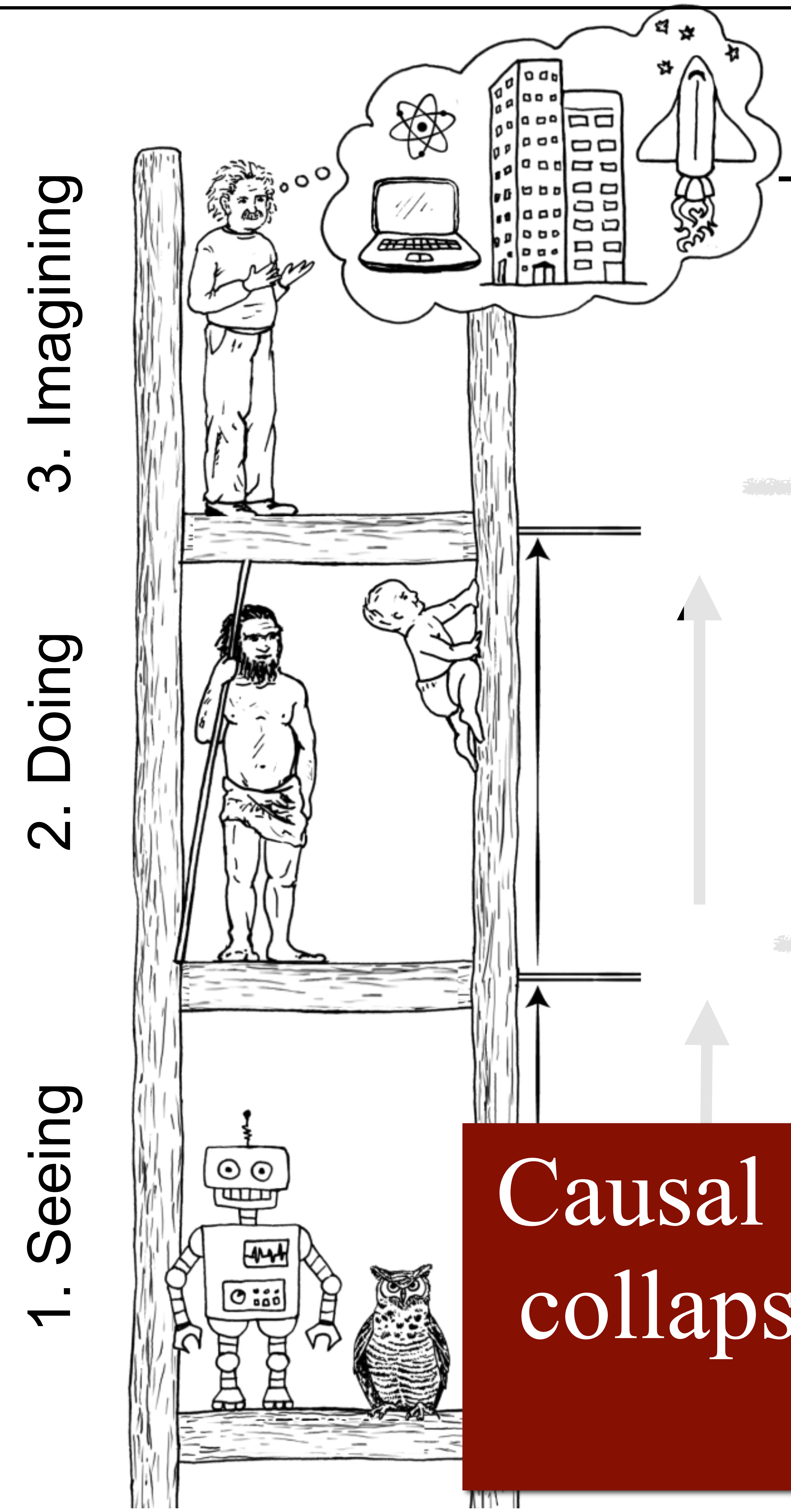
3. Imagining
2. Doing
1. Seeing



Layer	Task / Language	Typical Question	Examples
Counterfactual $P(y_x \mid x', y')$	Structural Causal Model	What if I had acted differently?	Was it the aspirin that stopped my headache?
Interventional $P(y \mid do(x), c)$	ML- Reinforcement (Causal Bayes Net)	What if I do X? What would Y be if I intervene on X?	Will my headache be cured if I take aspirin?
Associational $P(y \mid x)$	ML- (Un)Supervised (Bayesian Networks, Decision trees, Deep nets, …)	What if I see? How would seeing X change my belief in Y?	What does a symptom tell us about the disease?

* Book of Why & On Pearl's Hierarchy and the Foundations of Causal Inference, E. Bareinboim, J. Correa, D. Ibeling, T. Icard, in press. https://causalai.net/r60.pdf

Ladder of Causation



Cross-layer inferences:

Doing

most of the inferences are about causal effects (policies, treatments, decisions)

Seeing

most of the available data is observational, passively collected

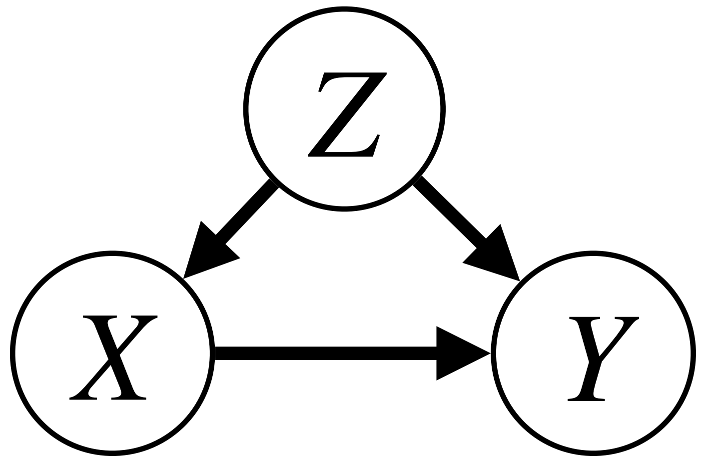
Causal Hierarchy Theorem : The ladder almost never collapses. That is, for almost any SCM, the rungs of the ladder remain distinct.

Causal Pipeline from a Causal Diagram

1 **Query**

$P(y \mid do(x))$

2 **Causal Contraints**



3 $P(x, m, y)$

Observational Distribution

Inference Engine

Solution

yes / no

$$P(y \mid do(x)) = \sum_z P(y \mid x, z) P(z)$$

$$\mathbb{E}(y \mid do(x)) = \sum_z \mathbb{E}(y \mid x, z) P(z) \rightarrow \hat{\mathbb{E}}(y \mid do(x)) = \sum_z \hat{\mathbb{E}}((y \mid x, z) \hat{P}(z)$$

Observational Data

Interventional Distribution ← Available Distributions

👍

Causal Modeling / Causal Discovery → Causal Effect Identification → Causal Effect Estimation

Causal Effect Identification from Causal Diagrams

Causal Effect

The **causal effect** of a (set of) treatment variable(s) $\mathbf{X}$ on a (set of) outcome variable(s) $\mathbf{Y}$ is a quantity derived from $P(\mathbf{Y} \mid do(\mathbf{X}))$ that tells us how much $\mathbf{Y}$ changes due to an intervention $do(\mathbf{X} = \mathbf{x})$.

Examples:

- *Average Treatment Effect* (ATE) for discrete treatments:

$$\mathbb{E}[\mathbf{Y} \mid do(\mathbf{X} = \mathbf{x}')] - \mathbb{E}[\mathbf{Y} \mid do(\mathbf{X} = \mathbf{x})],$$

where $\mathbb{E}[\mathbf{Y} \mid do(\mathbf{X} = \mathbf{x})] = \sum_{\mathbf{y} \in \Omega_{\mathbf{Y}}} \mathbf{y} P(\mathbf{y} \mid do(\mathbf{x}))$

defined for two treatment levels $\mathbf{x}'$ and $\mathbf{x}$ of $\mathbf{X}$.

- *Average Treatment Effect* (ATE) for continuous treatments,

$$\frac{\partial \mathbb{E}[Y_i \mid do(X_j = x_j)]}{\partial x_j}, \text{ for all } Y_i \in \mathbf{Y}, \text{ and } X_j \in \mathbf{X}.$$

Jacobian of $\mathbb{E}[\mathbf{Y} \mid do(\mathbf{X} = \mathbf{x})]$, where

$$\mathbb{E}[\mathbf{Y} \mid do(\mathbf{X} = \mathbf{x})] = \int_{\Omega_{\mathbf{y}}} \mathbf{y} P(\mathbf{y} \mid do(\mathbf{x})) d\mathbf{y},$$

and $\Omega_{\mathbf{Y}}$ is the space of all possible values that $\mathbf{Y}$ might take on

The derivative shows the rate of change of $\mathbf{Y}$ w.r.t. $do(\mathbf{X} = \mathbf{x})$

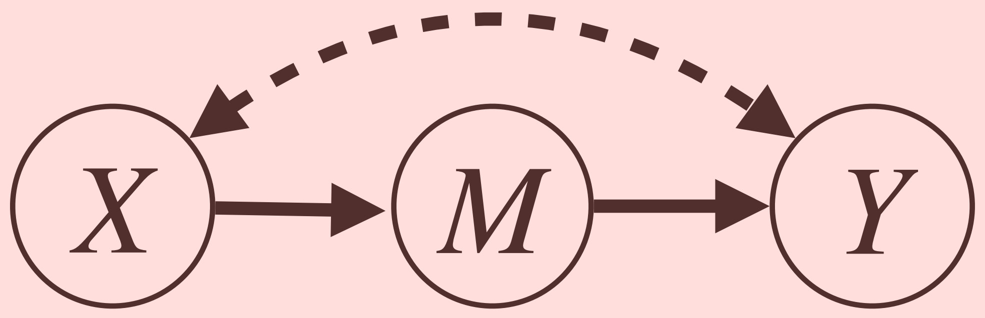
Classical Causal Effect Identification

Structural knowledge available

1 **Query**

$P(y \mid do(x))$

2 **Causal Contraints**



3 **Probability Distributions**

$P(x, m, y)$ **Observational Distribution**

Inference Engine

Solution

yes / no

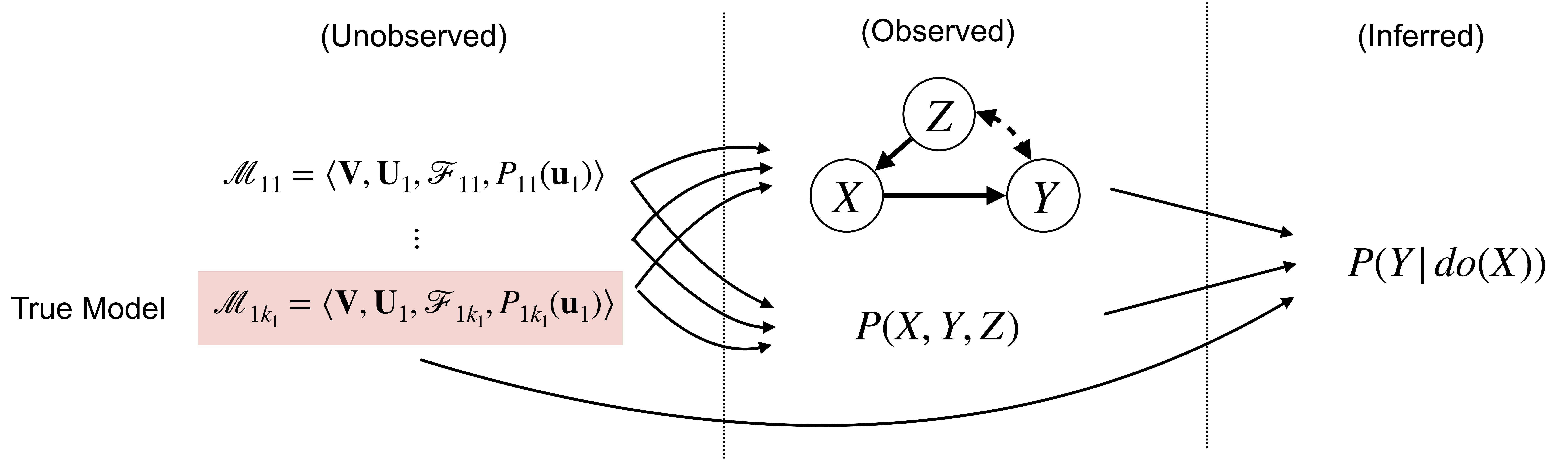
$$P(y \mid do(x)) = \sum_m P(m \mid x) \sum_{x'} P(y \mid m, x') P(x')$$

Interventional Distribution ← Available Distributions 👍

- Tian, J. and Pearl, J. (2002) A General Identification Condition for Causal Effects. In Proceedings of the Eighteenth National Conference on Artificial Intelligence (AAAI 2002), pp. 567–573, Menlo Park, CA, 2002. AAAI Press/MIT Press.

The Effect Identification Problem

Causal Effect Identifiability: The causal effect of a (set of) treatment variable(s) $\mathbf{X}$ on a (set of) outcome variable(s) $\mathbf{Y}$ is said to be identifiable from a causal diagram G and the probability of the observed variables $P(\mathbf{V})$ if the interventional distribution $P(\mathbf{Y} \mid do(\mathbf{X}))$ is *uniquely computable,* i.e., if for every pair of SCMs $\mathscr{M}_1$ and $\mathscr{M}_2$ that induce G and $P^{\mathscr{M}_1}(\mathbf{V}) = P^{\mathscr{M}_2}(\mathbf{V}) = P(\mathbf{V}) > 0$, $P^{\mathscr{M}_1}(\mathbf{Y} \mid do(\mathbf{X})) = P^{\mathscr{M}_2}(\mathbf{Y} \mid do(\mathbf{X})) = P(\mathbf{Y} \mid do(\mathbf{X}))$.



In words, causal effect identifiability means that, no matter the form of true SCM, for all models $\mathscr{M}$ agreeing with $\langle G, P(\mathbf{V}) \rangle$, they also agree in $P(\mathbf{y} \mid do(\mathbf{x}))$.

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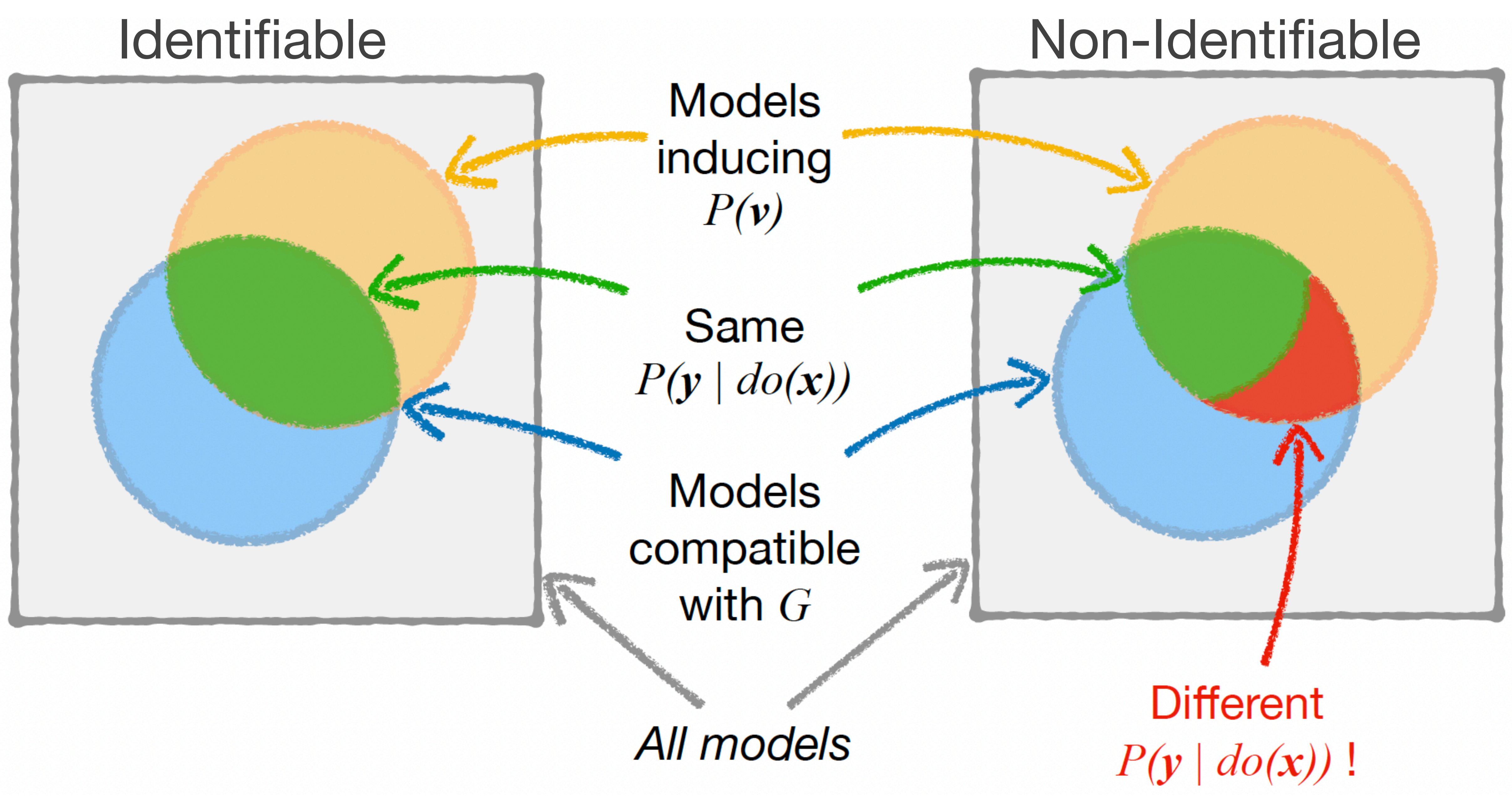


In words, causal effect identifiability means that, no matter the form of true SCM, for all models $\mathscr{M}$ agreeing with $\langle G, P(\mathbf{V})\rangle$, they also agree in $P(\mathbf{y} \mid do(\mathbf{x}))$.

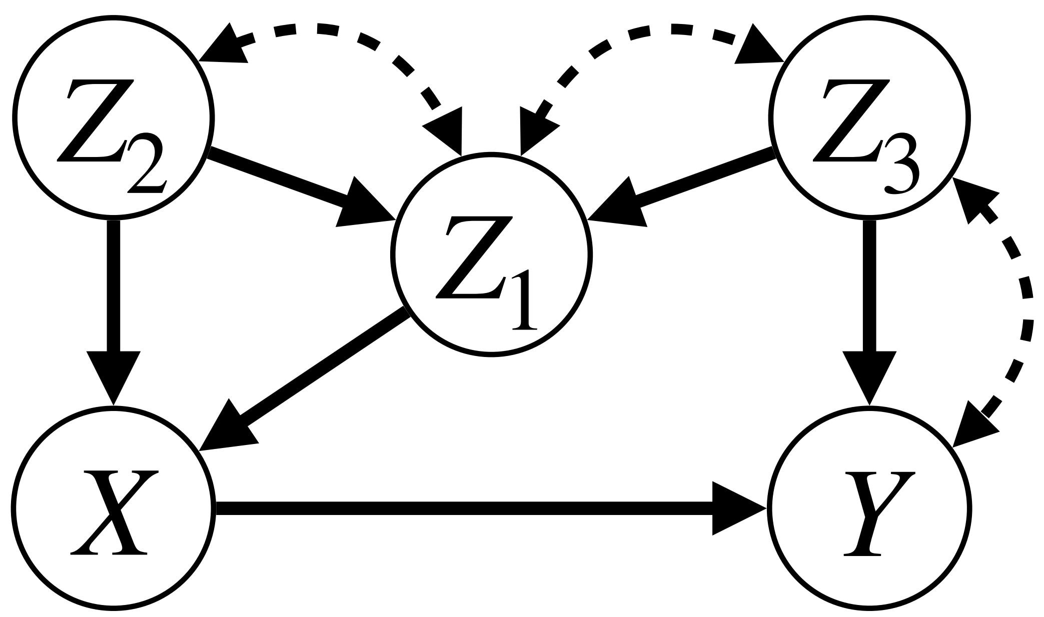
Identification Via Adjustment over Parents

Let G be a causal graph with all parents observed.

Then, the effect of $\mathbf{X}$ on $\mathbf{Y}$ is given by:

$$P(\mathbf{y} \mid do(\mathbf{x})) = \sum_{\mathbf{pa_x}} P\left(\mathbf{y} \mid \mathbf{x}, \mathbf{pa_x}\right) P\left(\mathbf{pa_x}\right)$$

Proof follows from the truncated factorization for Markovian models. Try at home!



$$Pa_x = \{Z_1, Z_2\}$$

$$\mathbf{X} = \{X\}$$
$$\mathbf{Y} = \{Y\}$$
$$\mathbf{Pa_X} = \{Z_1, Z_2\}$$

$$P(y \mid do(x)) = \sum_{z_1, z_2} P\left(y \mid x, z_1, z_2\right) P\left(z_1, z_2\right)$$

Identification via Backdoor Criterion

In $G_{\underline{\mathbf{X}}}$, all non-backdoor paths are severed

Let $\mathbf{X}$ be a set of treatment variables and $\mathbf{Y}$ a set of outcome variables in the causal graph G.

If there exists a set $\mathbf{Z}$ such that:

1. $\mathbf{Z}$ d-separates $\mathbf{X}$ and $\mathbf{Y}$ in the graph $G_{\underline{\mathbf{X}}}$, i.e., the graph resulting from cutting the arrows out of $\mathbf{X}$

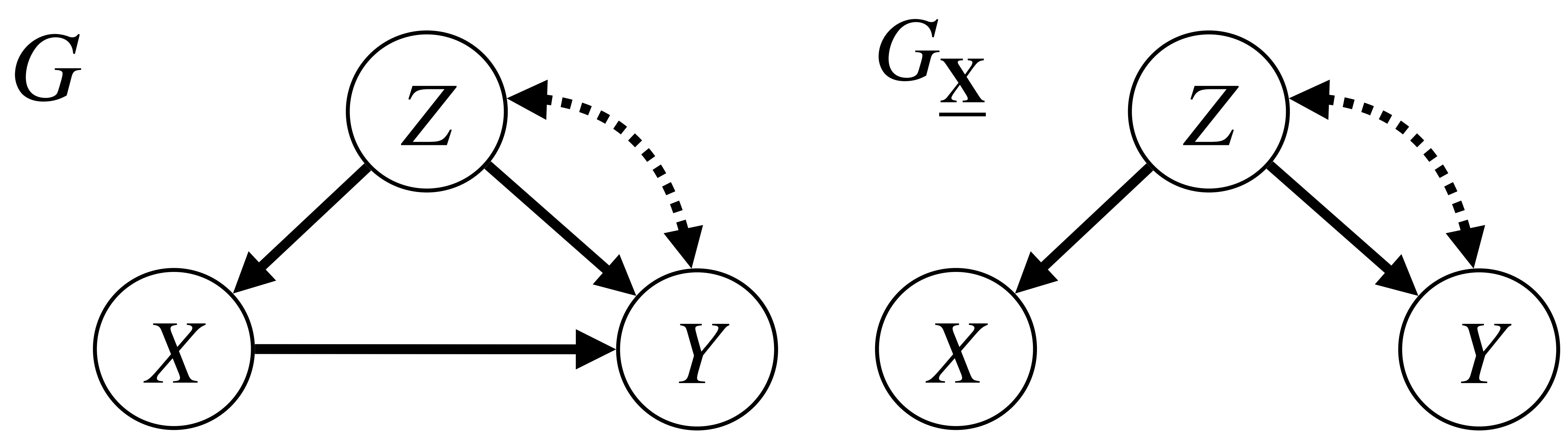
2. no node in $\mathbf{Z}$ is a descendant of a variable $X \in \mathbf{X}$ in G (all variables in $\mathbf{Z}$ are pre-treatment)

Then, $\mathbf{Z}$ satisfies the ***backdoor criterion*** for $(\mathbf{X}, \mathbf{Y})$ and, then the effect of $\mathbf{X}$ on $\mathbf{Y}$ is given by:

$$P(\mathbf{y} \mid do(\mathbf{x})) = \sum_{\mathbf{z}} P\left(\mathbf{y} \mid \mathbf{x}, \mathbf{z}\right) P\left(\mathbf{z}\right)$$

$\mathbf{X} = \{X\}$
$\mathbf{Y} = \{Y\}$
$\mathbf{Z} = \{Z\}$

$\mathbf{Z}$, a set of covariates, admissible for backdoor adjustment



Judea Pearl. Comment: Graphical models, causality and intervention. Stat. Sci., 8:266–269, 1993.

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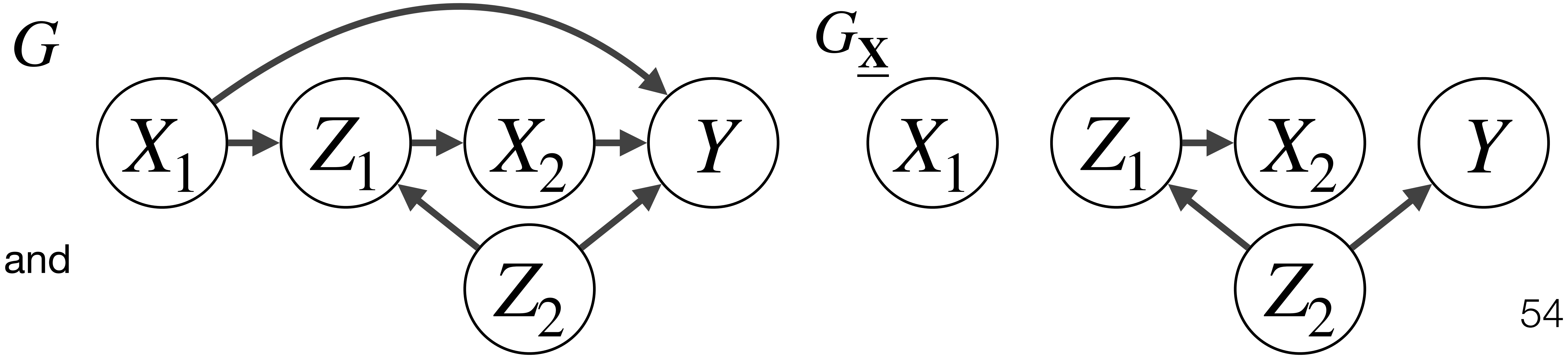
$$\mathbf{X} = \{X_1, X_2\}$$
$$\mathbf{Y} = \{Y\}$$
$$\mathbf{Z} = \{Z_2\}$$

$\mathbf{Z}$, a set of covariates, admissible for backdoor adjustment

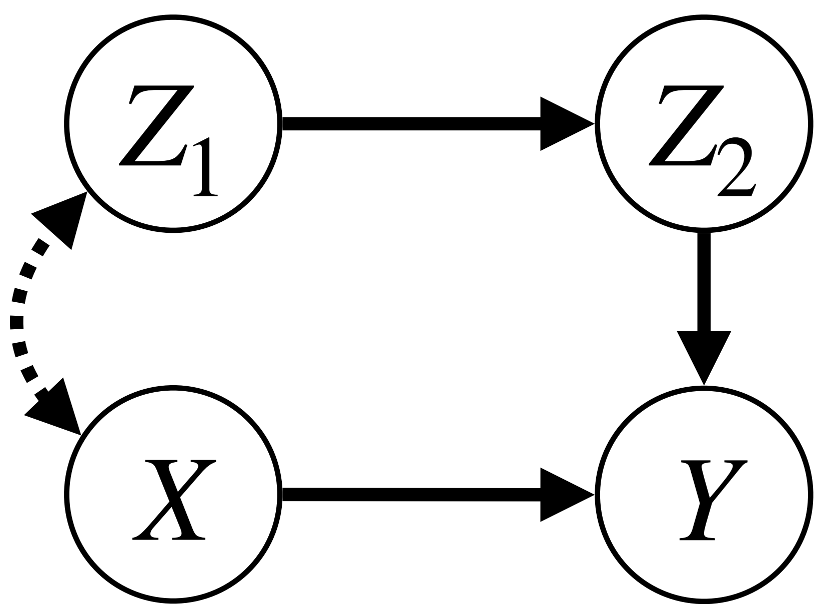


Judea Pearl. Comment: Graphical models, causality and intervention. Stat. Sci., 8:266–269, 1993.

Admissible Sets for BD Adjustment

$\mathbf{Z}$ satisfies the ***backdoor criterion*** for or $(\mathbf{X}, \mathbf{Y})$ in the causal graph G if:

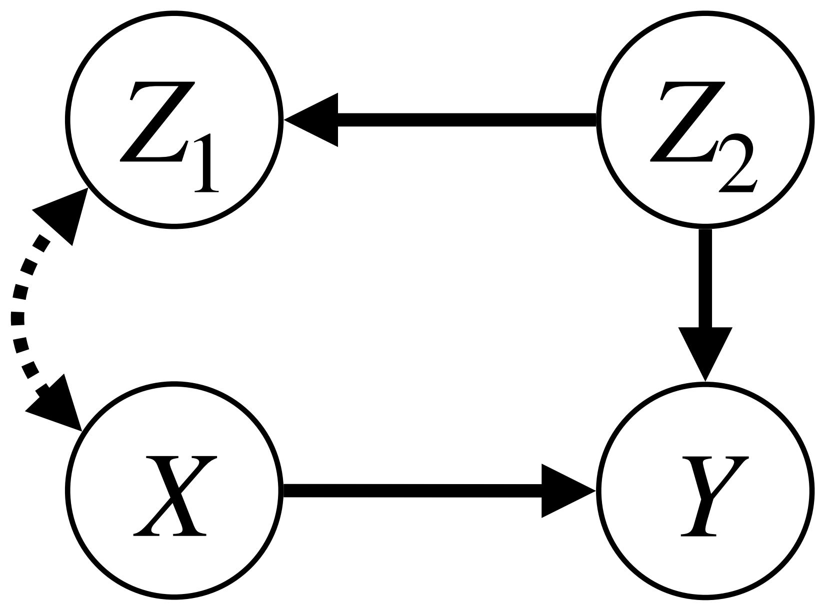
1. $\mathbf{Z}$ d-separates $\mathbf{X}$ and $\mathbf{Y}$ in the graph $G_{\underline{\mathbf{X}}}$, i.e., the graph resulting from cutting the arrows out of $\mathbf{X}$
2. no node in $\mathbf{Z}$ is a descendant of a variable $X \in \mathbf{X}$ in G (all variables in $\mathbf{Z}$ are pre-treatment)



Minimal BD Adjustment Sets: $\{Z_1\}$, $\{Z_2\}$,

$\{Z_1, Z_2\}$

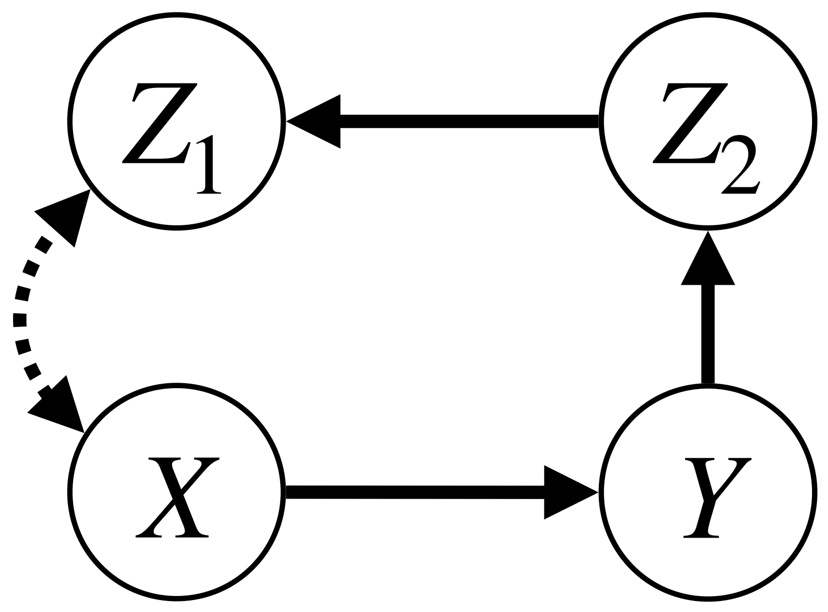
$$P(y \mid do(x)) = \sum_{z_1} P\left(y \mid x, z_1\right) P\left(z_1\right)$$



$\{\}$,

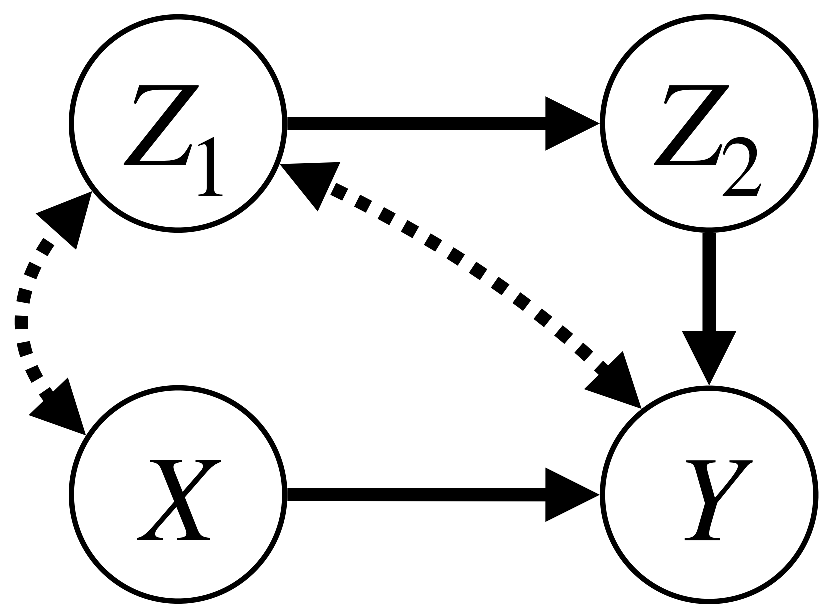
$\{Z_2\}$,
$\{Z_1, Z_2\}$

$$P(y \mid do(x)) = P\left(y \mid x\right)$$



$\{\}$

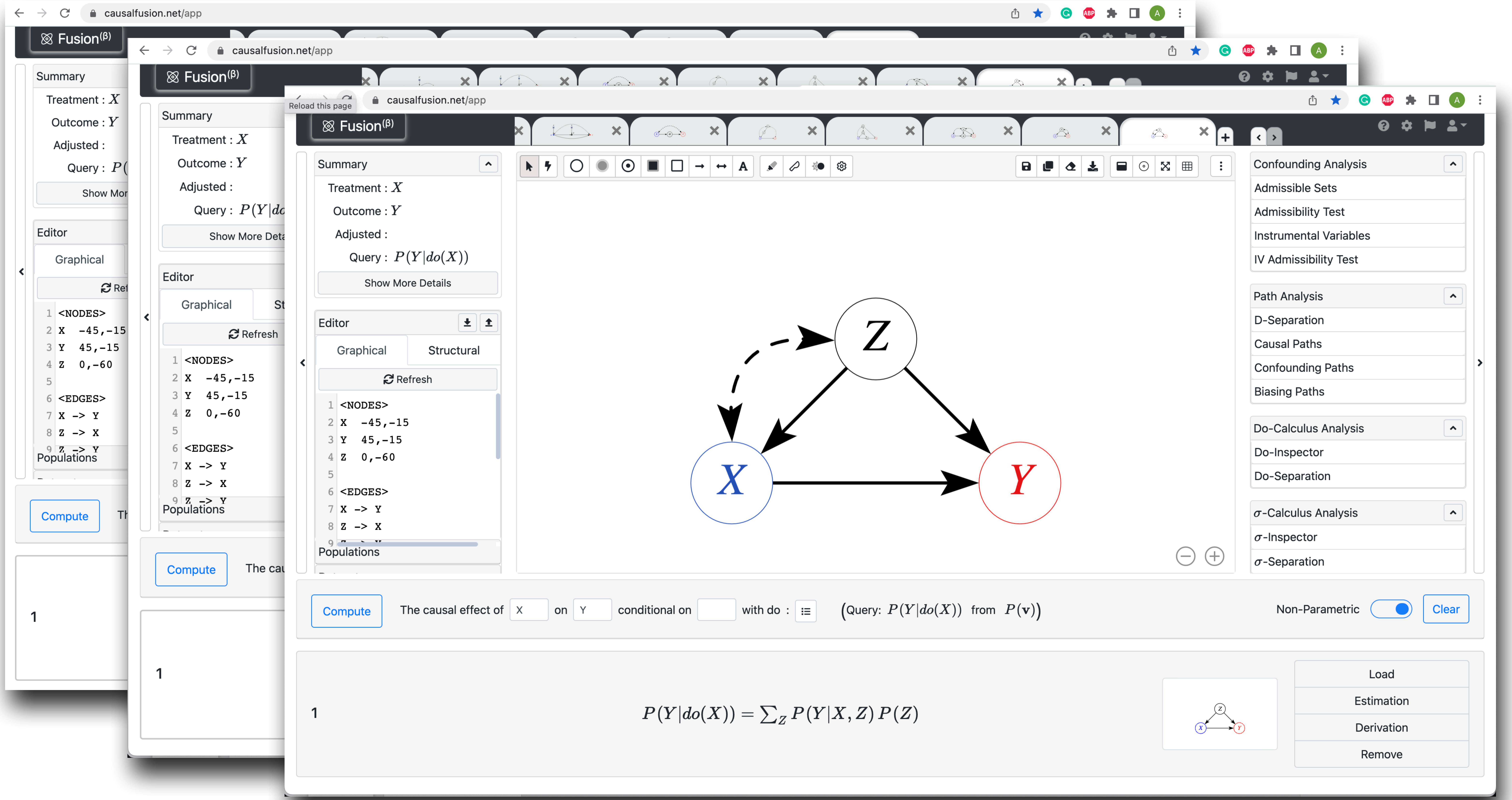
$$P(y \mid do(x)) = P\left(y \mid x\right)$$



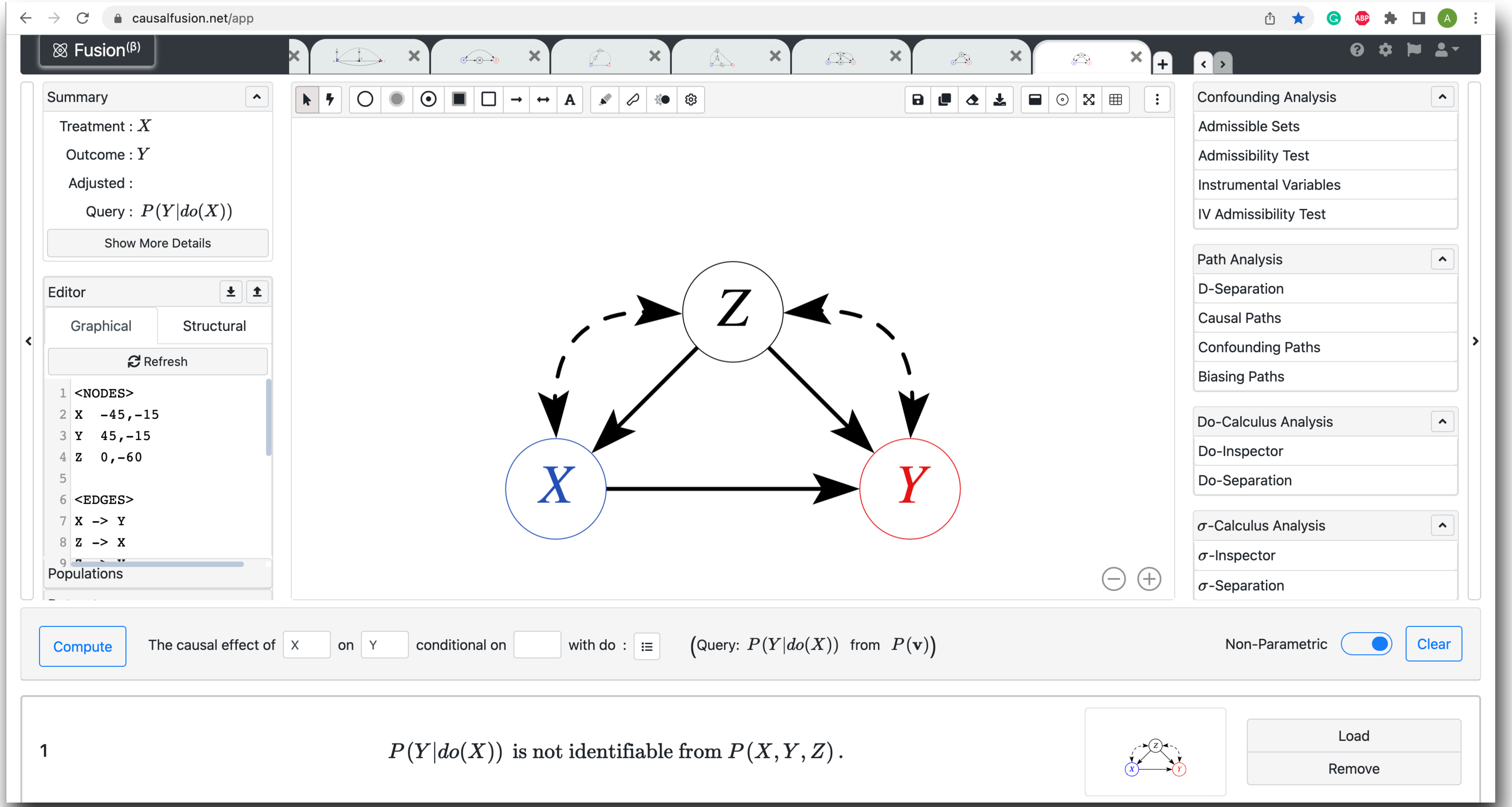
There is no BD Adjustment Set!

$P(y \mid do(x))$ is non-identifiable

http://causalfusion.net



http://causalfusion.net

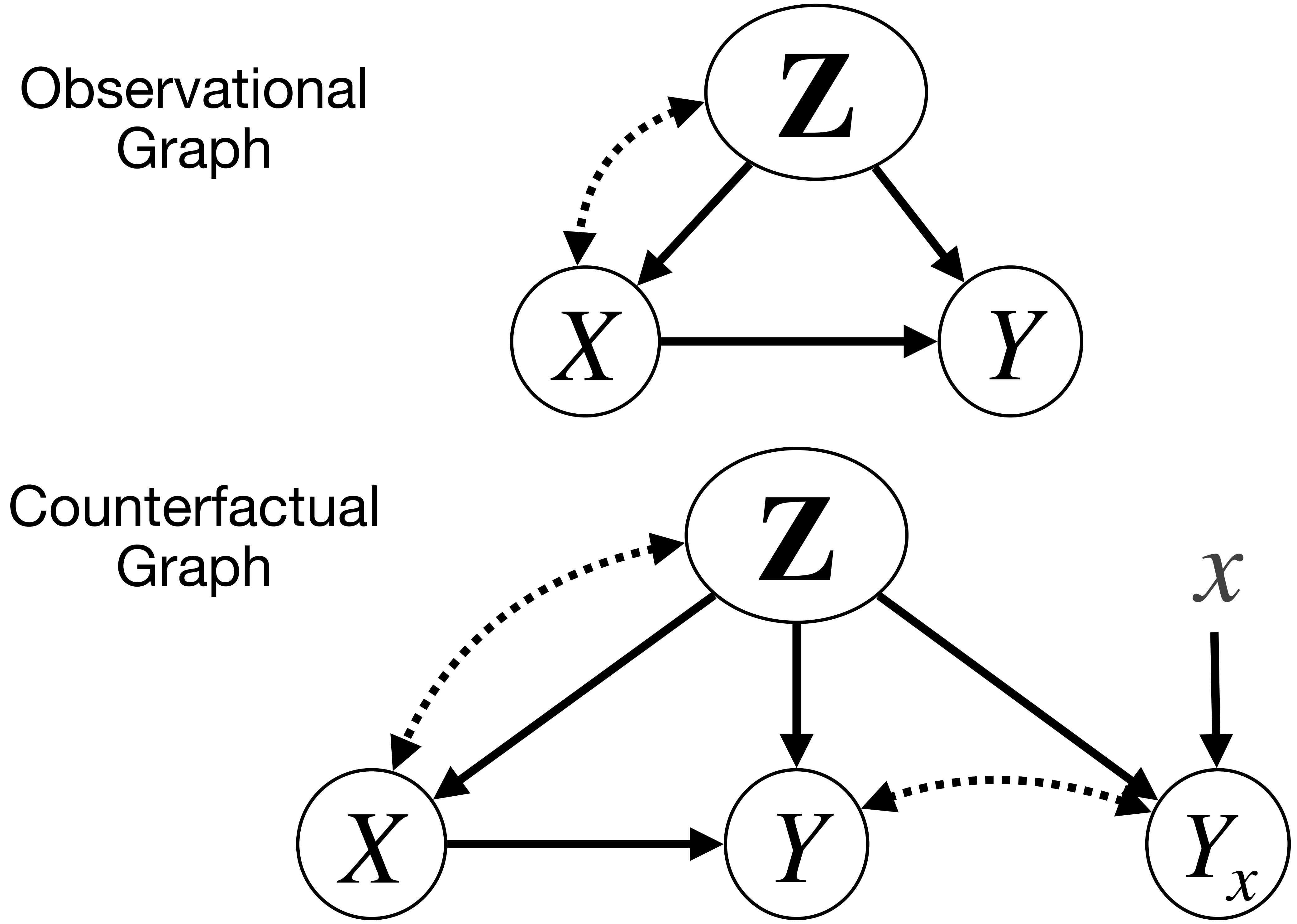


Counterfactual Interpretation of Backdoor

Theorem 4.3.1, Pearl's Primer Book

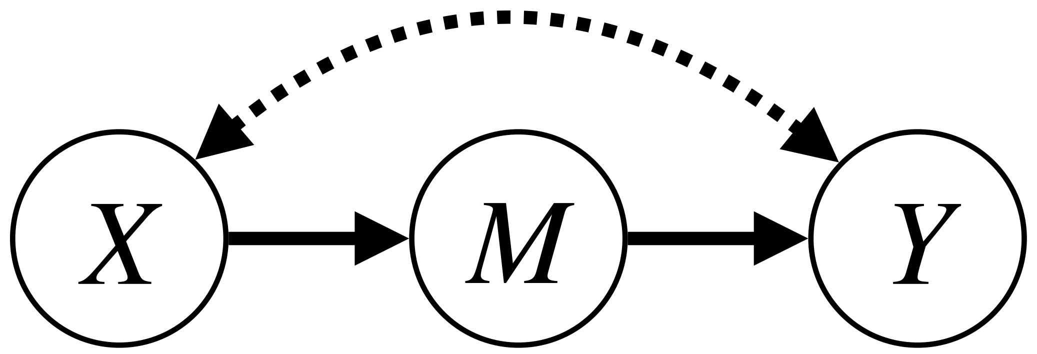
Theorem: If a set $\mathbf{Z}$ satisfies the *backdoor criterion* w.r.t. the ordered pair (X, Y), then, for all x, it holds that $Y_x \perp\!\!\!\perp X \mid \mathbf{Z}$.

Although the satisfiability of $\mathbf{Z}$ to the *backdoor criterion* can be tested given a causal diagram or a PAG, the condition $Y_x \perp\!\!\!\perp X \mid \mathbf{Z}$ is sometimes framed as an assumption, referred to as **(conditional) ignorability**, **exchangeability** or **unconfoundedness**.



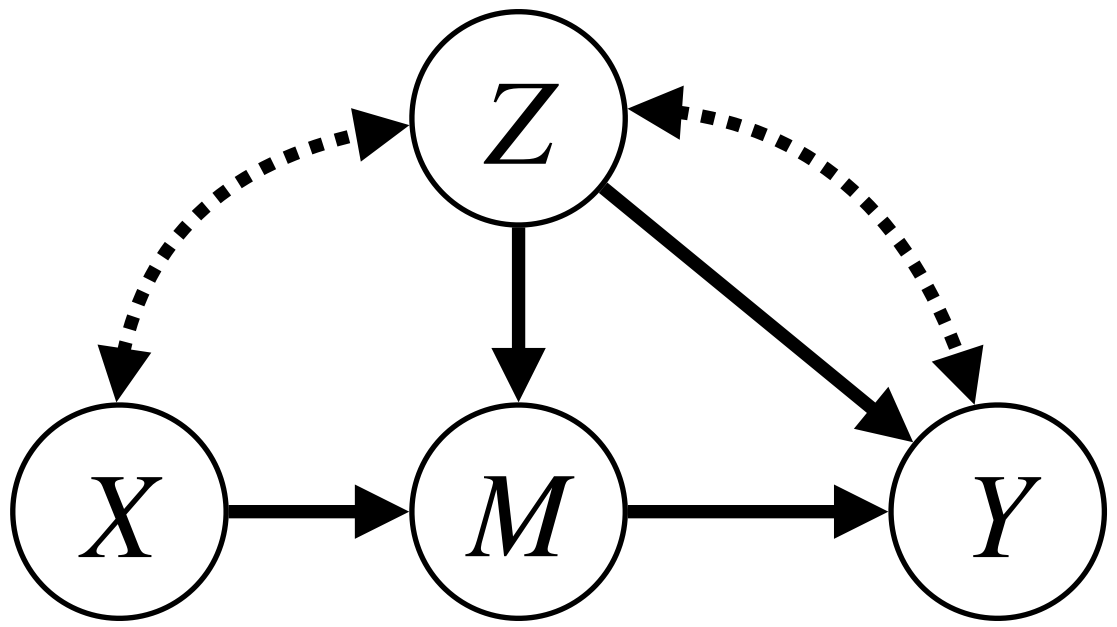
$Y_x \perp\!\!\!\perp X \mid Z$

Many Scenarios Beyond Adjustment!

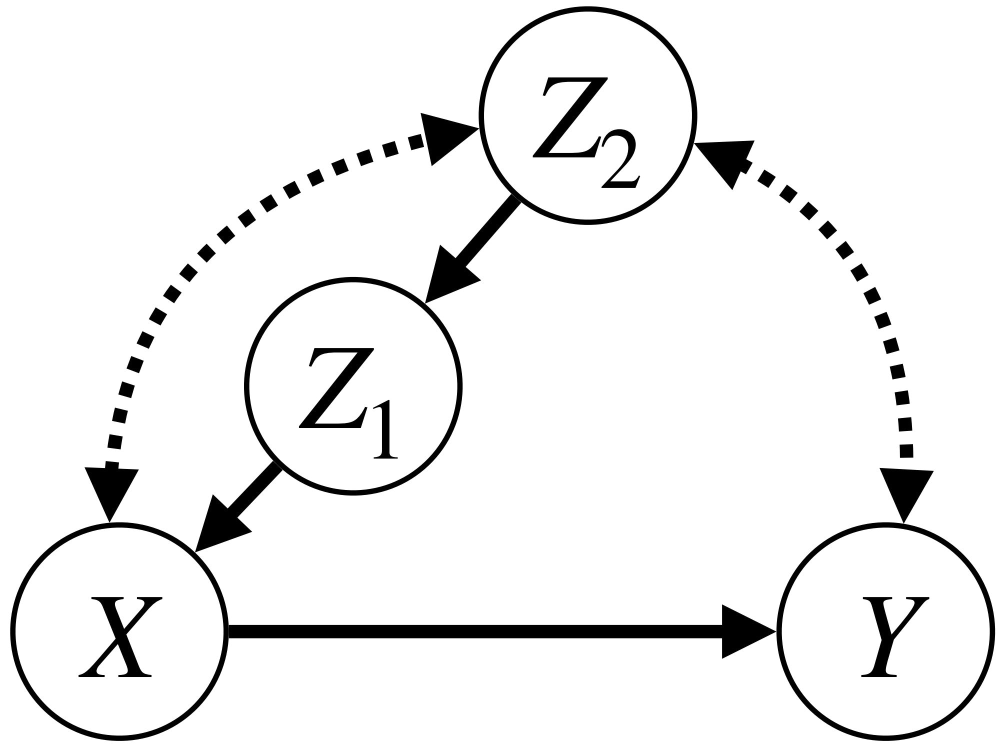


$$P(y \mid do(x)) = \sum_{m} P(m \mid x) \sum_{x'} P(y \mid m, x')P(x')$$

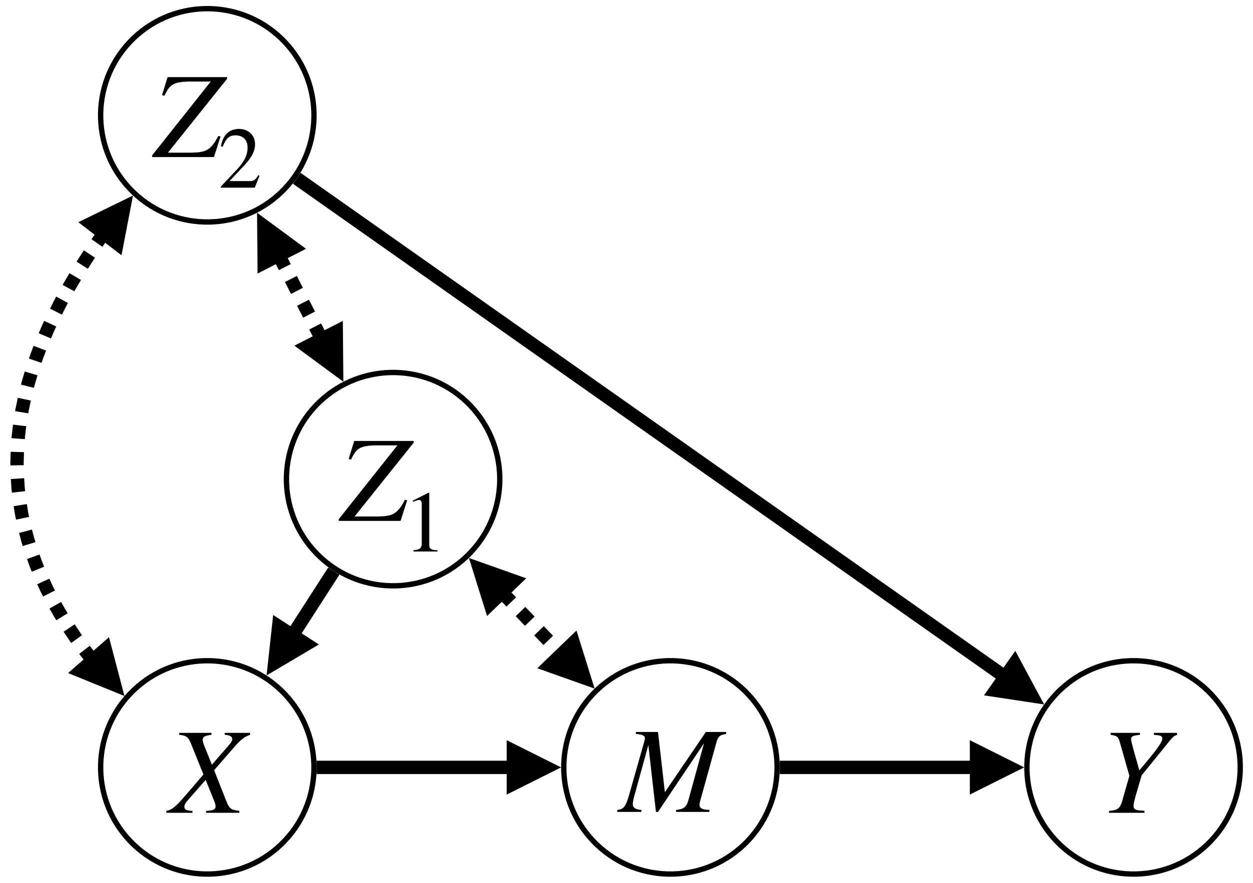
Front-Door



Conditional Front-Door



Napkin



Unnamed

$$E(y \mid do(x)) = \sum_{m,z} P(m \mid x, z)P(z \mid x) \sum_{x'} E(y \mid m, x', z)P(x' \mid z)$$

$$P(y \mid do(x)) = \frac{\sum_{z_2} P(x, y \mid z_1, z_2)P(z_2)}{\sum_{z_2} P(x \mid z_1, z_2)P(z_2)}$$

$$P(y \mid do(x)) = \sum_{z_2, z_3} P(y \mid x, z_1, z_2, z_3)P(z_2) \sum_{z_1} P(z_3 \mid x, z_1)P(z_1)$$

And many others….

Tools for Causal Identification

1. Markovian Models (No Unobserved Confounders)
    i. Truncated Factorization / G-computation or G-formula
2. Adjustment over Parents (No Unobserved Parents)
3. Non-Markovian Models (Under the Presence of Unobserved Confounders)
    i. Graphical criteria (Backdoor Adjustment, Generalized Adjustment, Front-door Adjustment)
    ii. Do-Calculus (a.k.a Causal Calculus)
    iii. Identify Algorithm (a.k.a. ID algorithm)

Pearl, J. (2000). Causality: Models, Reasoning, and Inference. Cambridge University Press, New York. http://dx.doi.org/10.1017/CBO9780511803161

Jin Tian. Studies in causal reasoning and learning. PhD thesis, University of California, Los Angeles, 2002.

Advances on Effect Identification given a Causal Diagram

Identification from observational and experimental data:

Lee, S., Correa, J., and Bareinboim, E. (2019). General identifiability with arbitrary surrogate experiments. In *Proceedings of the 35th Conference on Uncertainty in Artificial Intelligence*, volume 35, Tel Aviv, Israel. AUAI Press.

J. Correa, S. Lee, E. Bareinboim. (2021) Nested Counterfactual Identification from Arbitrary Surrogate Experiments. In Proceedings of the 35th Annual Conference on Neural Information Processing Systems

Identification of stochastic/soft (and possibly imperfect) interventions:

Correa, J. and Bareinboim, E. (2020). A calculus for stochastic interventions: Causal effect identification and surrogate experiments. In *Proceedings of the 34th AAAI Conference on Artificial Intelligence*, New York, NY. AAAI Press.

Identification and Estimation via Deep Neural Networks:

Xia, K., Lee, K.-Z., Bengio, Y., and Bareinboim, E. (2021). The causal-neural connection: Expressiveness, learnability, and inference. *Advances in Neural Information Processing Systems*, 34.

Xia, K., Pan, Y.,and Bareinboim, E. (2023) Neural Causal Models for Counterfactual Identification and Estimation. In Proceedings of the 11th International Conference on Learning Representations.

Coding Exercises

Causality Tutorial:

- Available at https://github.com/adele/Causality-Tutorial/ → Nordic ProbAI 2024
- **Google Colab Notebook:** (Link)

Check Part I:

1. Causal Modeling
2. Causal Effect Identification from Causal Diagrams
    1. Adjustment Criterion -- pcalg R package
    2. ID Algorithm -- causaleffect R package

What if domain knowledge does not allow you construct a causal diagram?

Super-Exponential Growth

The space of DAGs grows super-exponentially with the number *n* of variables, as shown by the following recurrence relation (Robinson, 1973):

$$|DAG(n)| = \sum_{i=1}^{n} \binom{n}{1} 2^{i(n-i)} |DAG(n-1)|$$

n	$\lvert DAG(n) \rvert$
2	3
3	27
4	729
5	59,049
6	1.4349×10^{7}
7	1.0460×10^{10}
8	2.2877×10^{13}

Inference trough enumeration
is not a good idea!

Super-Exponential Growth

The space of ADMGs also grows super-exponentially with the number *n* of variables, and it is much bigger than the space of DAGs:

$$|ADMG(n)| = |DAG(n)| \times 2^{n(n-1)/2}$$

$$|ADMG(n)| \gg |DAG(n)|$$

n	$\|DAG(n)\|$	$\|ADMG(n)\|$
2	3	6
3	27	216
4	729	46,656
5	59,049	6.0457×10^7
6	1.4349×10^7	4.7019×10^{11}
7	1.0460×10^{10}	2.1936×10^{16}
8	2.2877×10^{13}	6.1410×10^{21}

Learning the Markov Equivalence Class

Causal Discovery:

Many models are statistically indistinguishable without additional parametric / distributional assumptions.

In non-parametric settings, causal discovery algorithms can only learn a graphical representation of its *Markov equivalence class* (MEC)!

Fast Causal Inference (FCI): Sound and complete causal discovery algorithm, even in the presence of unobserved confounders and selection bias.

Zhang, J. (2008). On the completeness of orientation rules for causal discovery in the presence of latent confounders and selection bias. *Artificial Intelligence*, 172(16):1873–1896. Link

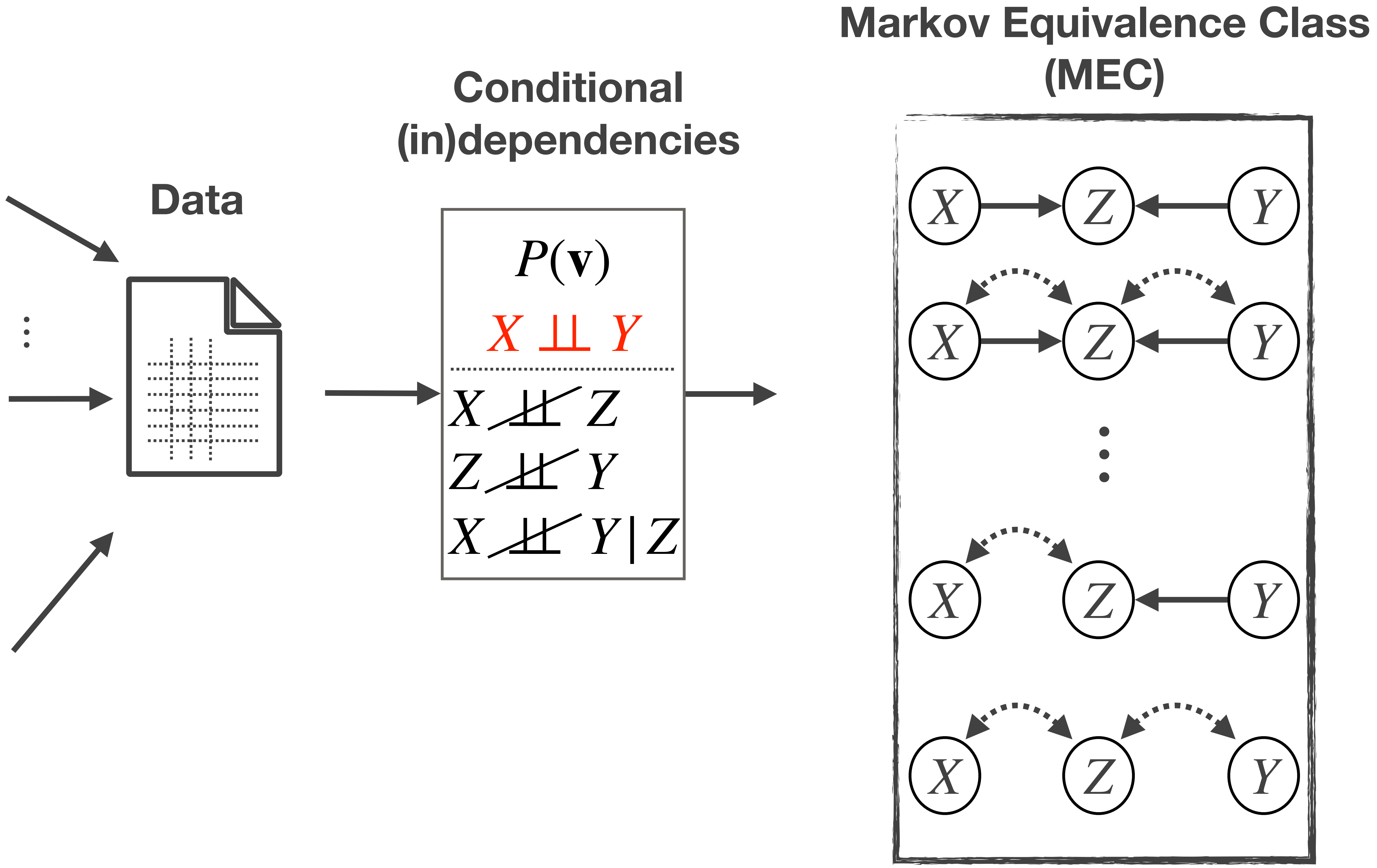
Causal Discovery: Learning Structural Invariances

$$
\mathcal{M}_1 = \begin{cases} \mathbf{V} = \{X, Y, Z\} \\ \mathbf{U} = \{U_x, U_Y, U_Z\} \\ \mathcal{F} = \begin{cases} X \leftarrow f_X(U_X) \\ Z \leftarrow f_Z(X, Y, U_Z) \\ Y \leftarrow f_Y(U_Y) \end{cases} \\ P(\mathbf{U}) \end{cases}
$$

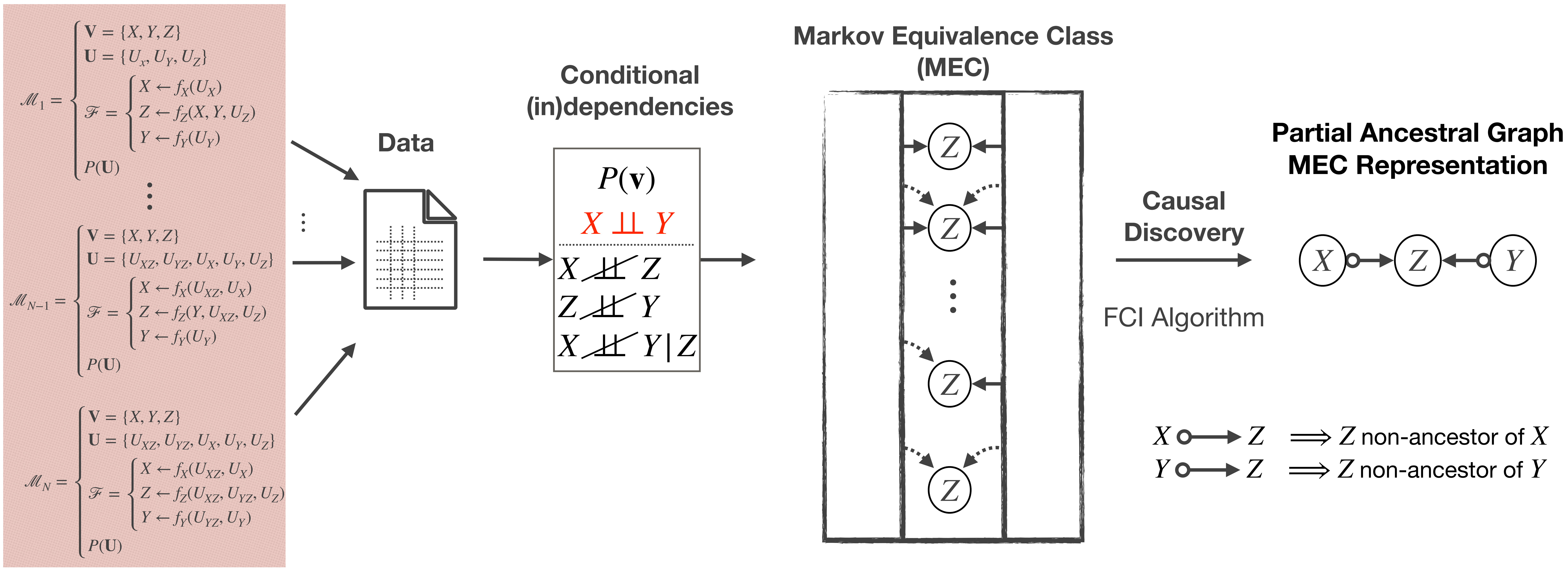
$$\vdots$$

$$
\mathcal{M}_{N-1} = \begin{cases} \mathbf{V} = \{X, Y, Z\} \\ \mathbf{U} = \{U_{XZ}, U_{YZ}, U_X, U_Y, U_Z\} \\ \mathcal{F} = \begin{cases} X \leftarrow f_X(U_{XZ}, U_X) \\ Z \leftarrow f_Z(Y, U_{XZ}, U_Z) \\ Y \leftarrow f_Y(U_Y) \end{cases} \\ P(\mathbf{U}) \end{cases}
$$

$$
\mathcal{M}_N = \begin{cases} \mathbf{V} = \{X, Y, Z\} \\ \mathbf{U} = \{U_{XZ}, U_{YZ}, U_X, U_Y, U_Z\} \\ \mathcal{F} = \begin{cases} X \leftarrow f_X(U_{XZ}, U_X) \\ Z \leftarrow f_Z(U_{XZ}, U_{YZ}, U_Z) \\ Y \leftarrow f_Y(U_{YZ}, U_Y) \end{cases} \\ P(\mathbf{U}) \end{cases}
$$



Causal Discovery: Learning Structural Invariances

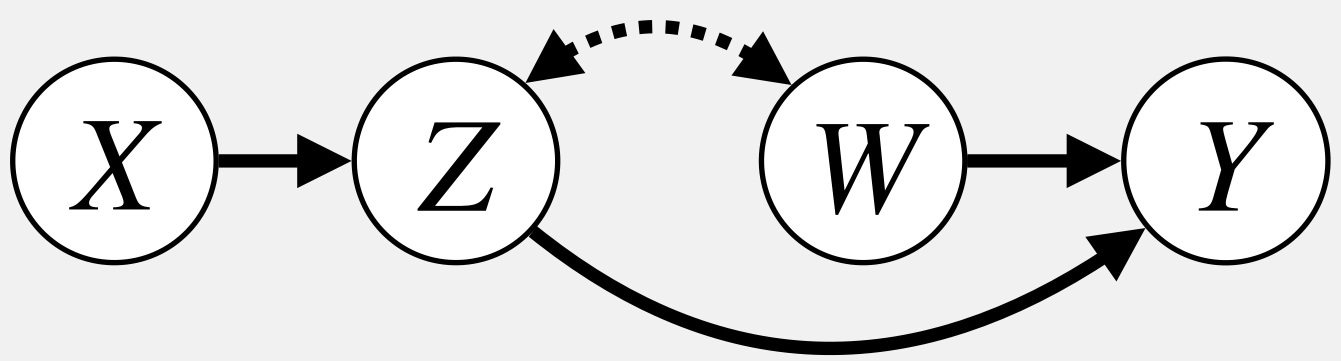


Zhang, J. (2008). On the completeness of orientation rules for causal discovery in the presence of latent confounders and selection bias. *Artificial Intelligence*, 172(16):1873–1896. [Link]

FCI Algorithm - Pipeline

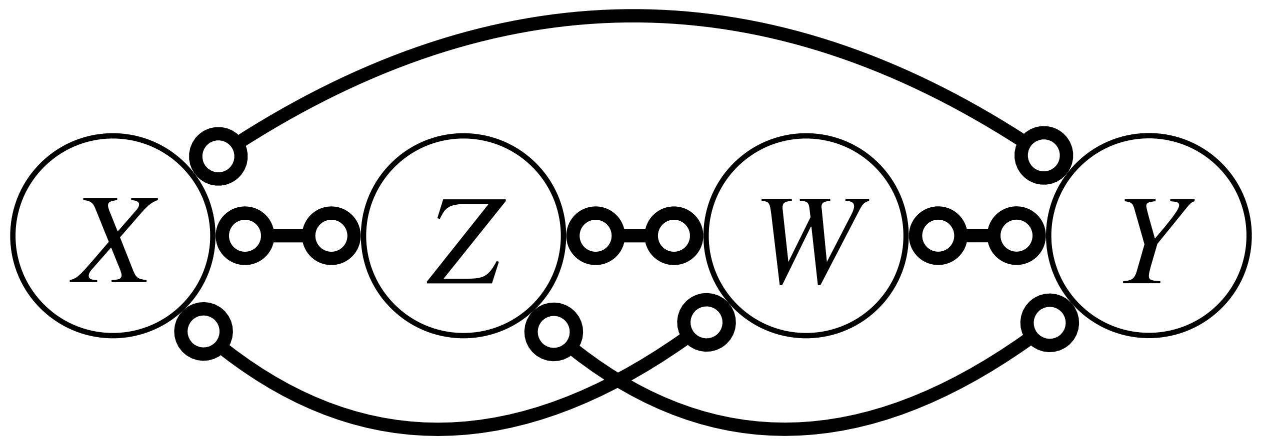
Unknown Reality

True causal diagram



$X \perp\!\!\!\perp W$
$X \perp\!\!\!\perp Y \mid Z, W$

Implied by the ADMG using d-separation

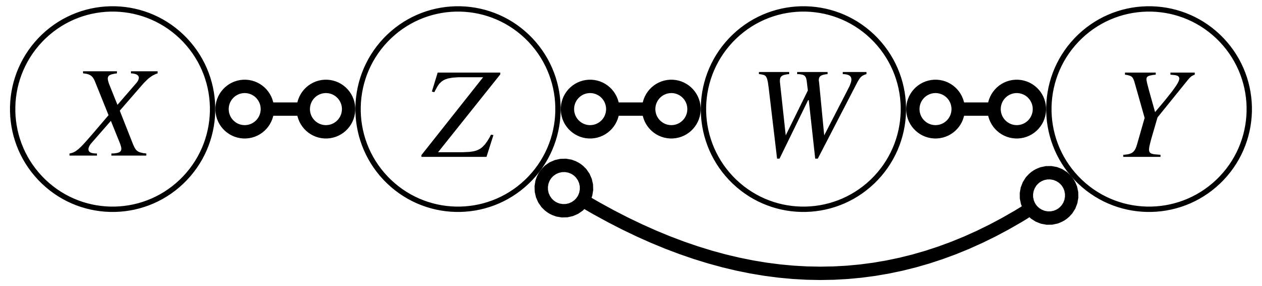


Complete Graph

Conditional Independence Tests

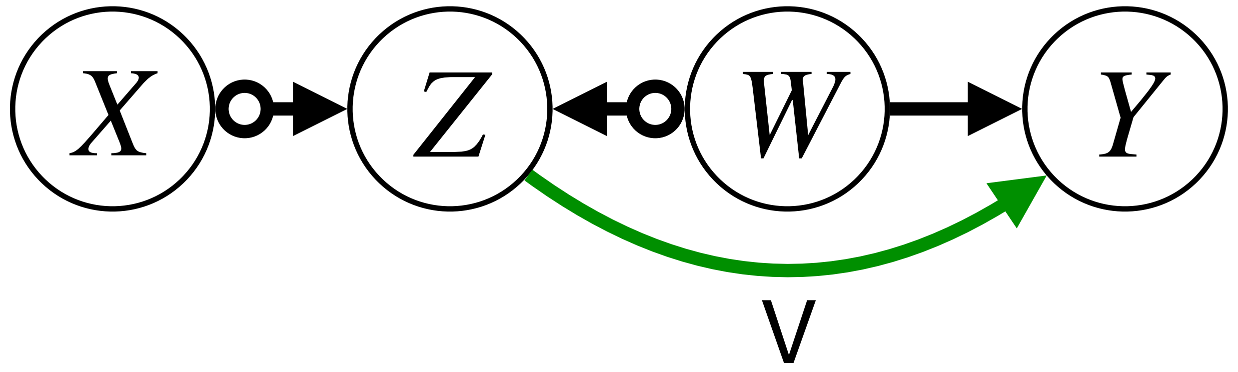
$X \perp\!\!\!\perp W$
$X \perp\!\!\!\perp Y \mid Z, W$

By **faithfulness**, are correctly observed in the data



Skeleton

FCI Rules
$(R1) - (R10)$



Partial Ancestral Graph (PAG)

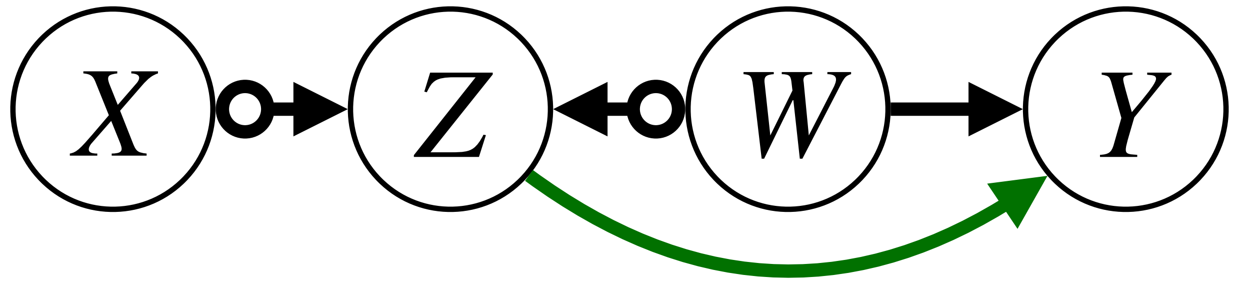
Implied by the PAG using m-separation

$X \perp\!\!\!\perp W$
$X \perp\!\!\!\perp Y \mid Z, W$

$A \circ\!\!\rightarrow B \implies$ B non-ancestor of A
$A \rightarrow B \implies$ A ancestror of B
$A \leftrightarrow B \implies$ spurious association
$A - B \implies$ selection bias

Z is not an ancestor of X or W.
Z and W are ancestors of Y.
Z is not confounded with Y.

PAG: Representation of the Markov Equivalence Class



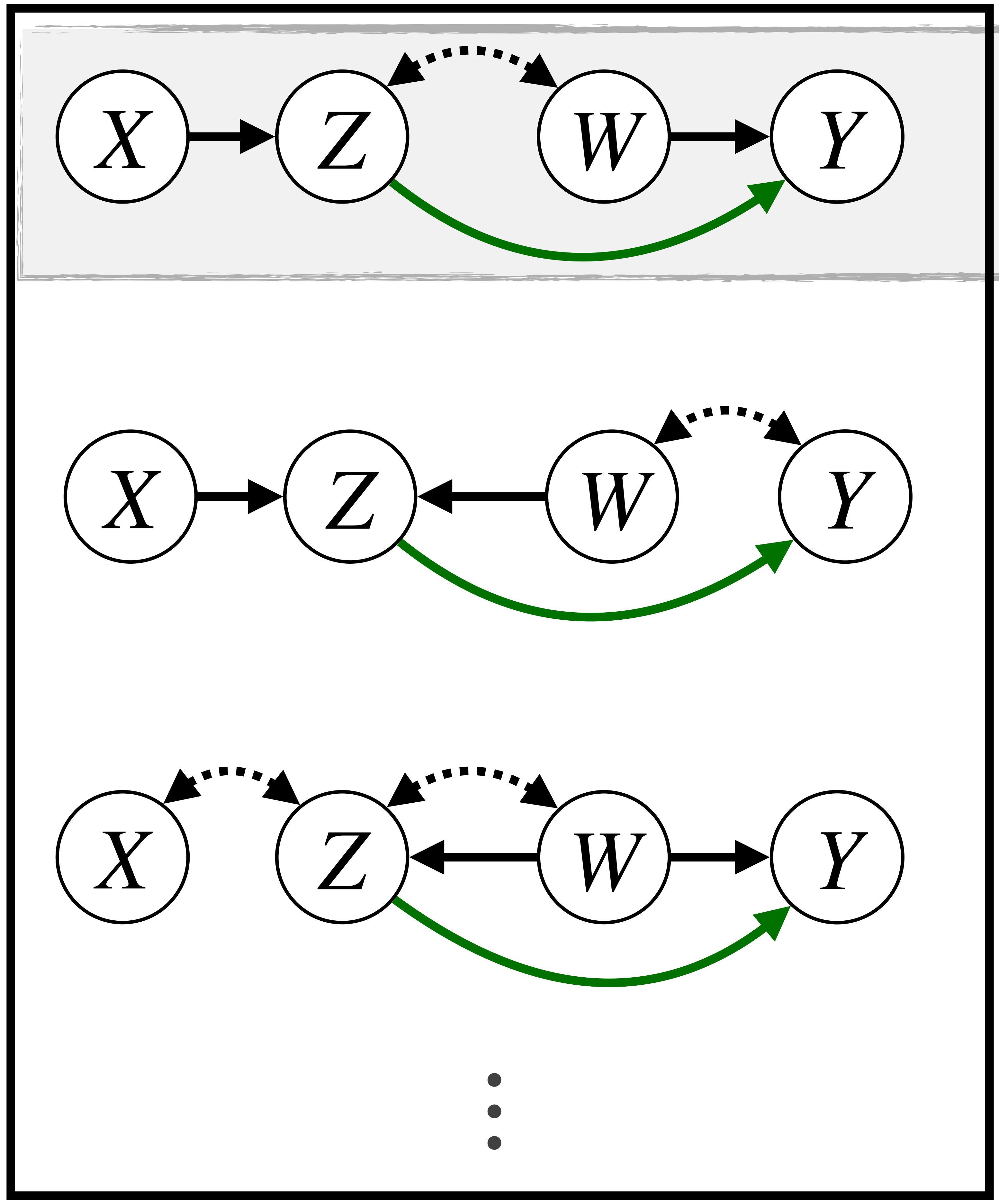
V

Partial Ancestral Graph
(PAG)

Z **is not an ancestor of** X **or** W**.**

Z **and** W **are ancestors of** Y**.**

Z **is not confounded with** Y**.**



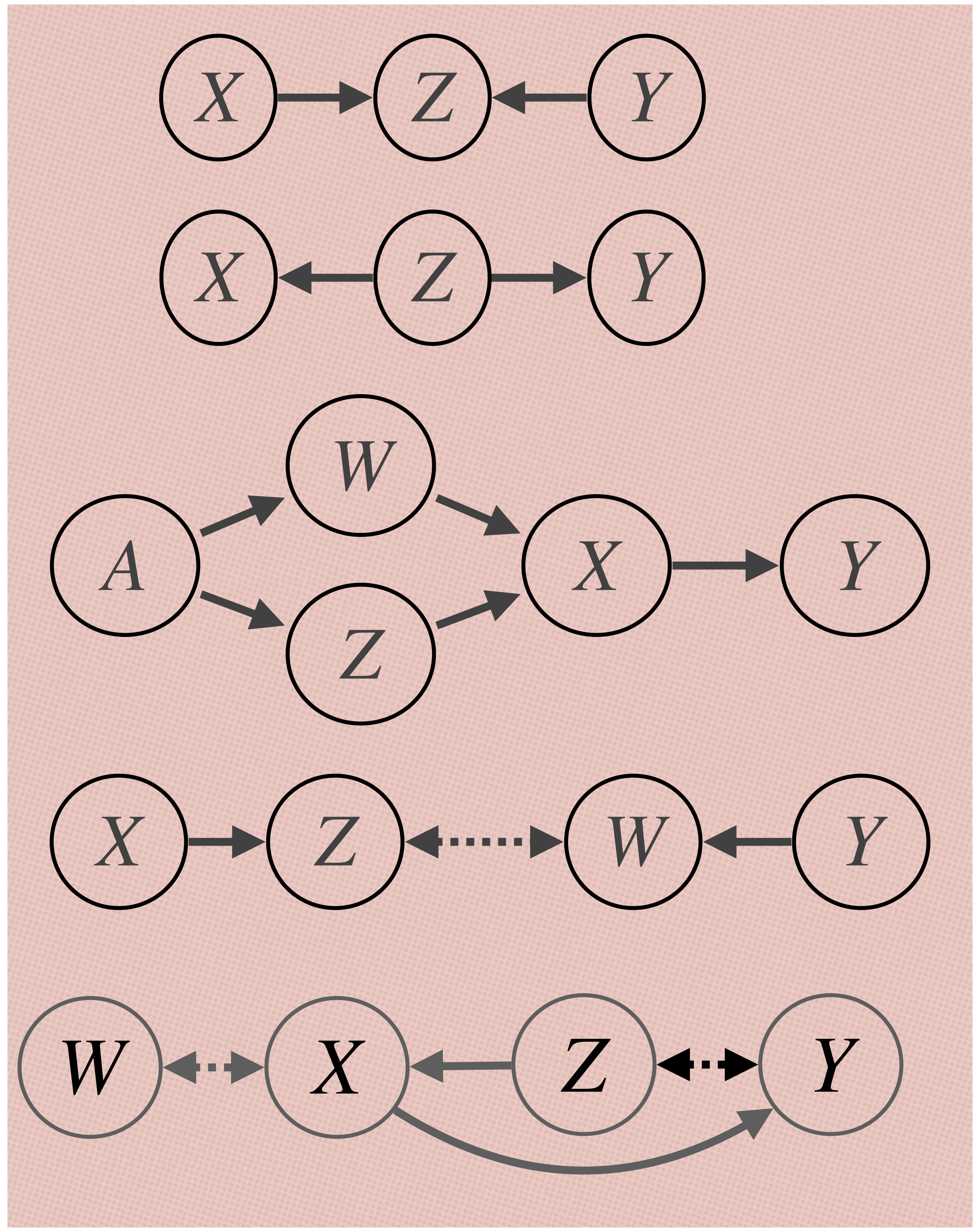
True (unknown) causal diagram

$X \perp\!\!\!\perp W$

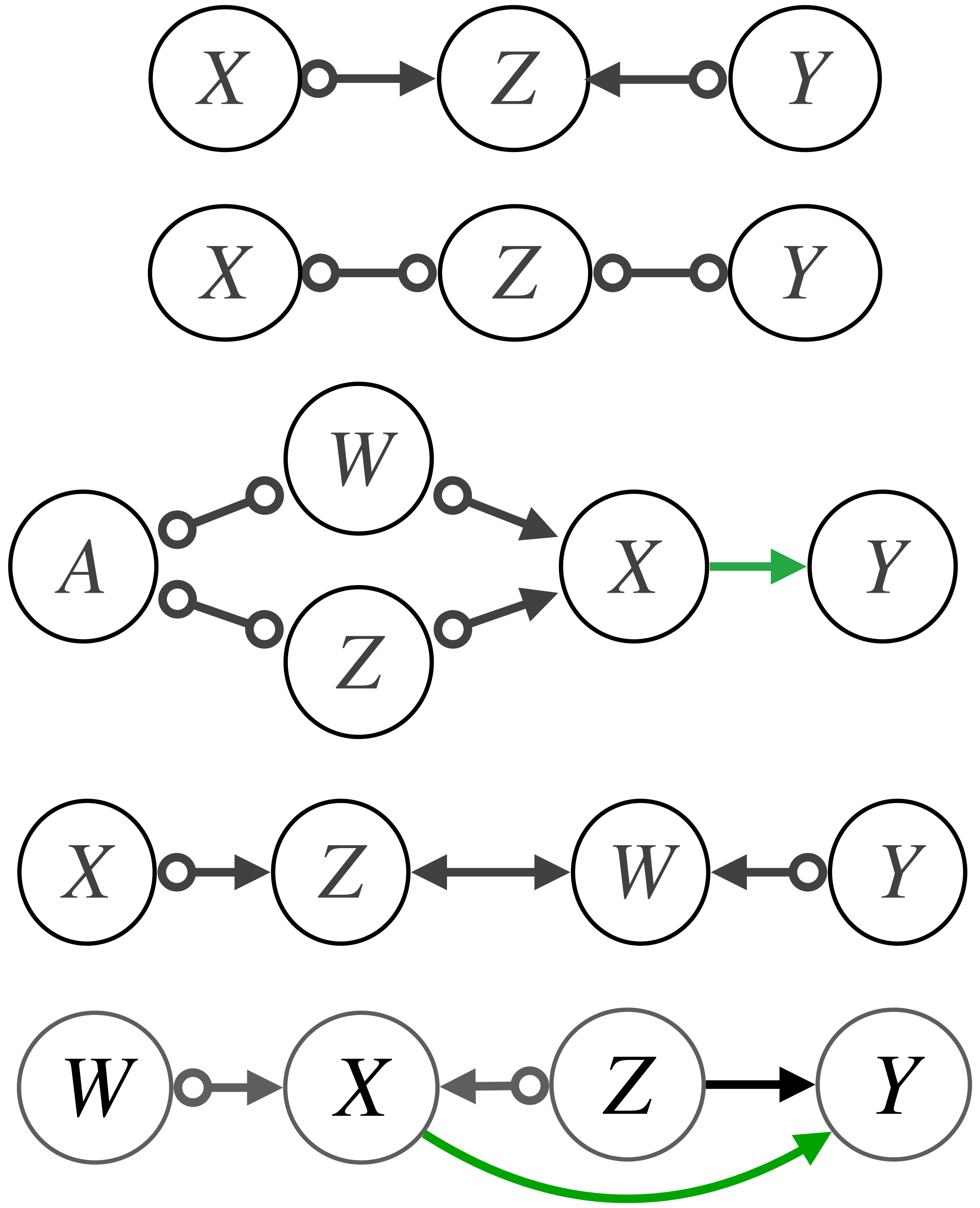
$X \perp\!\!\!\perp Y \mid Z, W$

Fast Causal Inference (FCI) Algorithm

Underlying Causal Diagram



Partial Ancestral Graph



Conditional Independence Tests

Gaussian errors and independent observations: partial correlation test

Fisher, R.A. (1921). *On the Probable Error of a Coefficient of Correlation Deduced from a Small Sample.*
R package: https://cran.r-project.org/web/packages/pcalg/

Kernel-based non-parametric test:

Zhang, K., Peters, J., Janzing, D., & Schölkopf, B. (2012). *Kernel-based conditional independence test and application in causal discovery.* In: Uncertainty in artificial intelligence. AUAI Press; 2011. p.804–13
R package: https://cran.r-project.org/web/packages/CondIndTests

Continuous (conditional Gaussian) or Discrete (Binary, Ordinal, Multinomial) - Linear Regression

- Tsagris, M., Borboudakis, G., Lagani, V. *et al.* (2018) Constraint-based causal discovery with mixed data. *Int J Data Sci Anal* **6**, 19–30. (Link)
- R package: https://cran.r-project.org/web/packages/MXM/

Gaussian errors and correlated observations (family data) :

Ribeiro A.H., Soler J.M.P. (2020). *Learning Genetic and environmental graphical models from family data*, Statistics in Medicine.
R package: https://github.com/adele/FamilyBasedPGMs

Causal Identification from PAGs

Can we identify causal effects from the equivalence class?

Effect Identification:

For Covariate Adjustment, we can use the Generalized Adjustment Criterion.

Recently, we proposed complete calculus and algorithms for the identification of marginal and conditional causal effect in PAGs!

Perkovic, E., Textor, J. C., Kalisch, M., & Maathuis, M. H. (2018). Complete graphical characterization and construction of adjustment sets in Markov equivalence classes of ancestral graphs. Journal of Machine Learning Research 18 (2018) 1-62

Jaber A., **Ribeiro A. H.,** Zhang, J., Bareinboim, E. (2022) Causal Identification under Markov Equivalence - Calculus, Algorithm, and Completeness. In Proceedings of the 36th Annual Conference on Neural Information Processing Systems, NeurIPS. (Link)

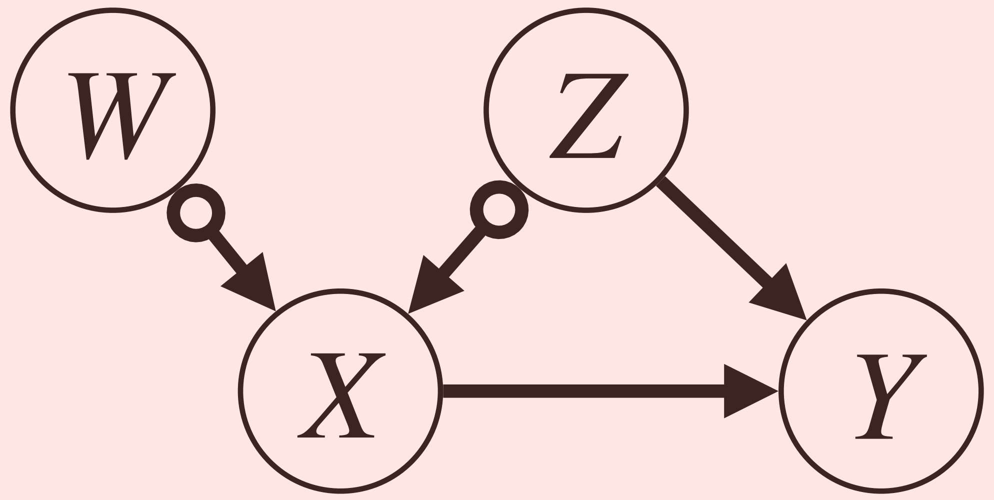
Effect Identification in Markov Equivalence Classes

Can be constructed in a fully data-driven way!

1 **Query**

$P(y \mid do(x))$

2 **PAG**



3 **Data**

$P(x, y, z, w)$ **Observational Distribution**

Inference Engine

Solution

yes / no

$$P(y \mid do(x)) = \sum_{z} P\left(y \mid x, z\right) P\left(z\right)$$

Inferred (Interventional) Distribution ← Available (Observational) Distribution

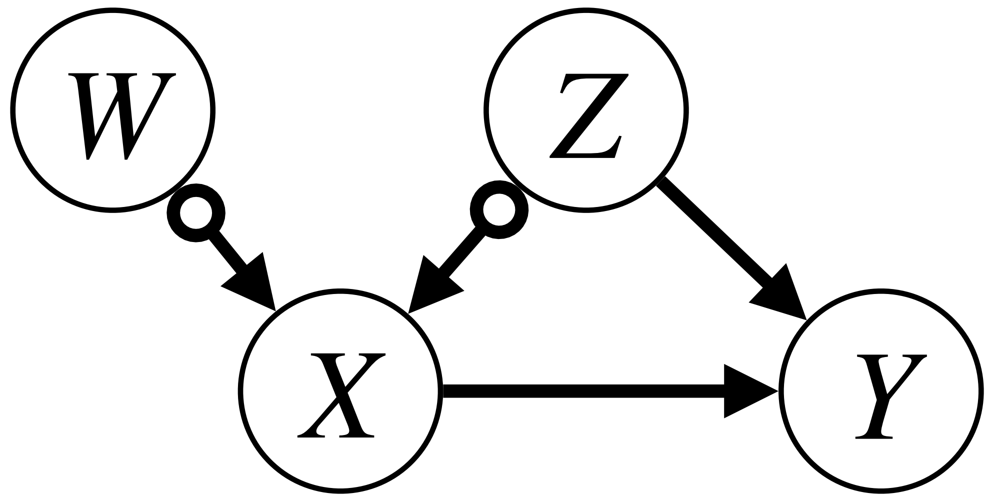
Identification via Adjustment in Markov Equivalence Classes

Identification is possible only when the Generalized Adjustment Criterion applies.

1 **Query**

$P(y\,|\,do(x))$

2 **PAG**



3 **Data**

$P(x, y, z, w)$ **Observational Distribution**

Adjustment Criterion

Solution

yes / no

$$P(y\,|\,do(x)) = \sum_z P\left(y\,|\,x, z\right) P\left(z\right)$$

Inferred (Interventional) Distribution ← Available (Observational) Distribution

Perkovic, E., Textor, J. C., Kalisch, M., & Maathuis, M. H. (2018). Complete graphical characterization and construction of adjustment sets in Markov equivalence classes of ancestral graphs. Journal of Machine Learning Research 18 (2018) 1-62

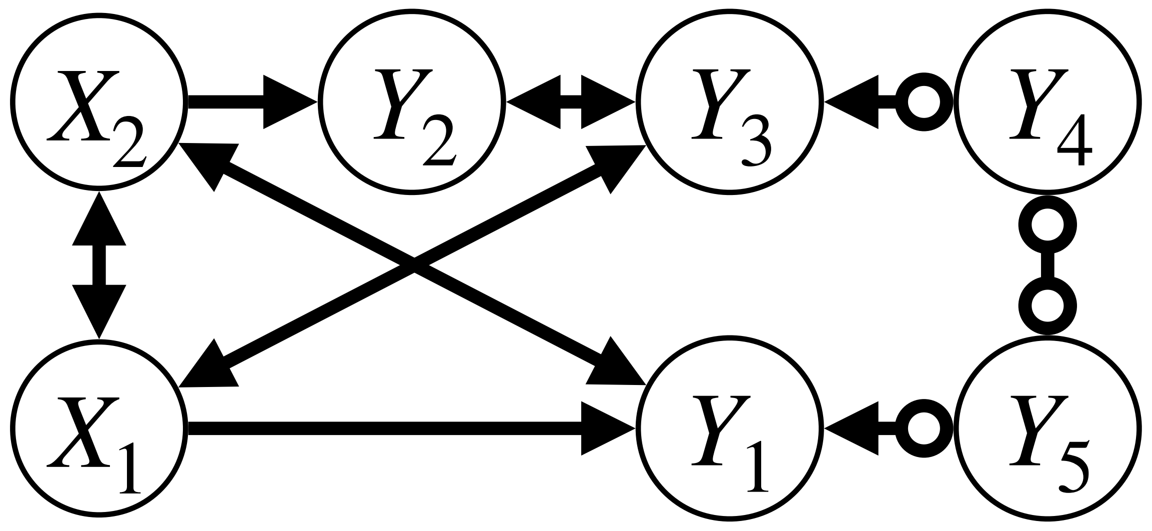
General Identification in Markov Equivalence Classes

Complete algorithms,
available at the *PAGId* R package:
https://github.com/adele/PAGId

1 **Query**

$P(y \mid do(x))$

2 **PAG**



3 **Data**

$P(x, y, z, w)$ **Observational Distribution**

IDP / CIDP

Solution
yes / no

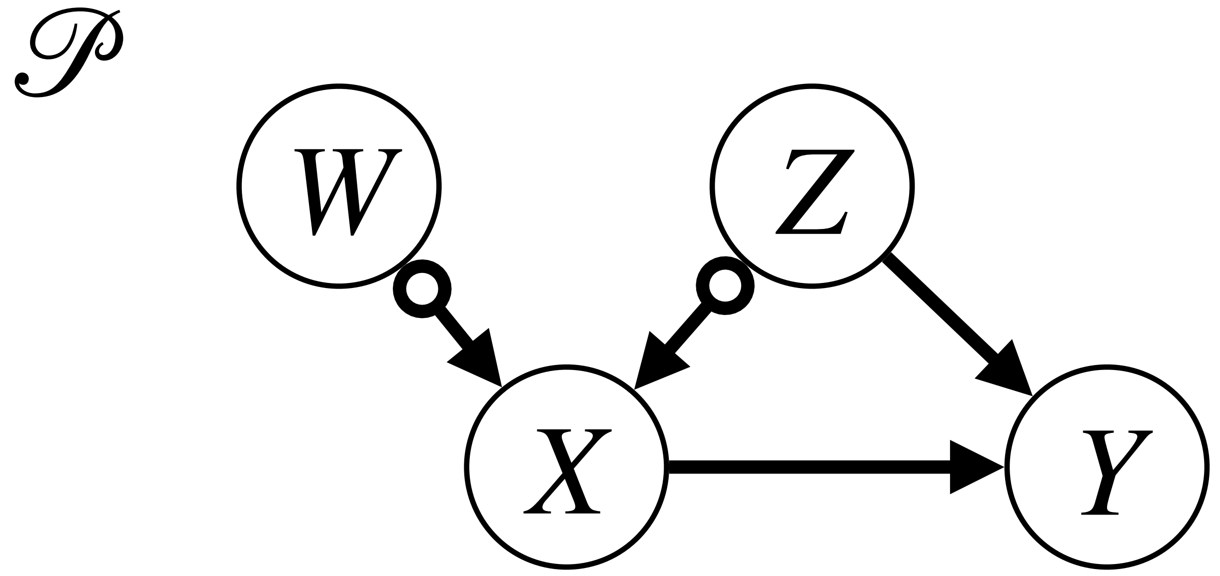
$$P(y \mid do(x)) = \frac{P(y_1, y_4, y_5 \mid x_1) \,.\, P(y_2 y_3, y_4, y_5 \mid x_2)}{P(y_4, y_5)}$$

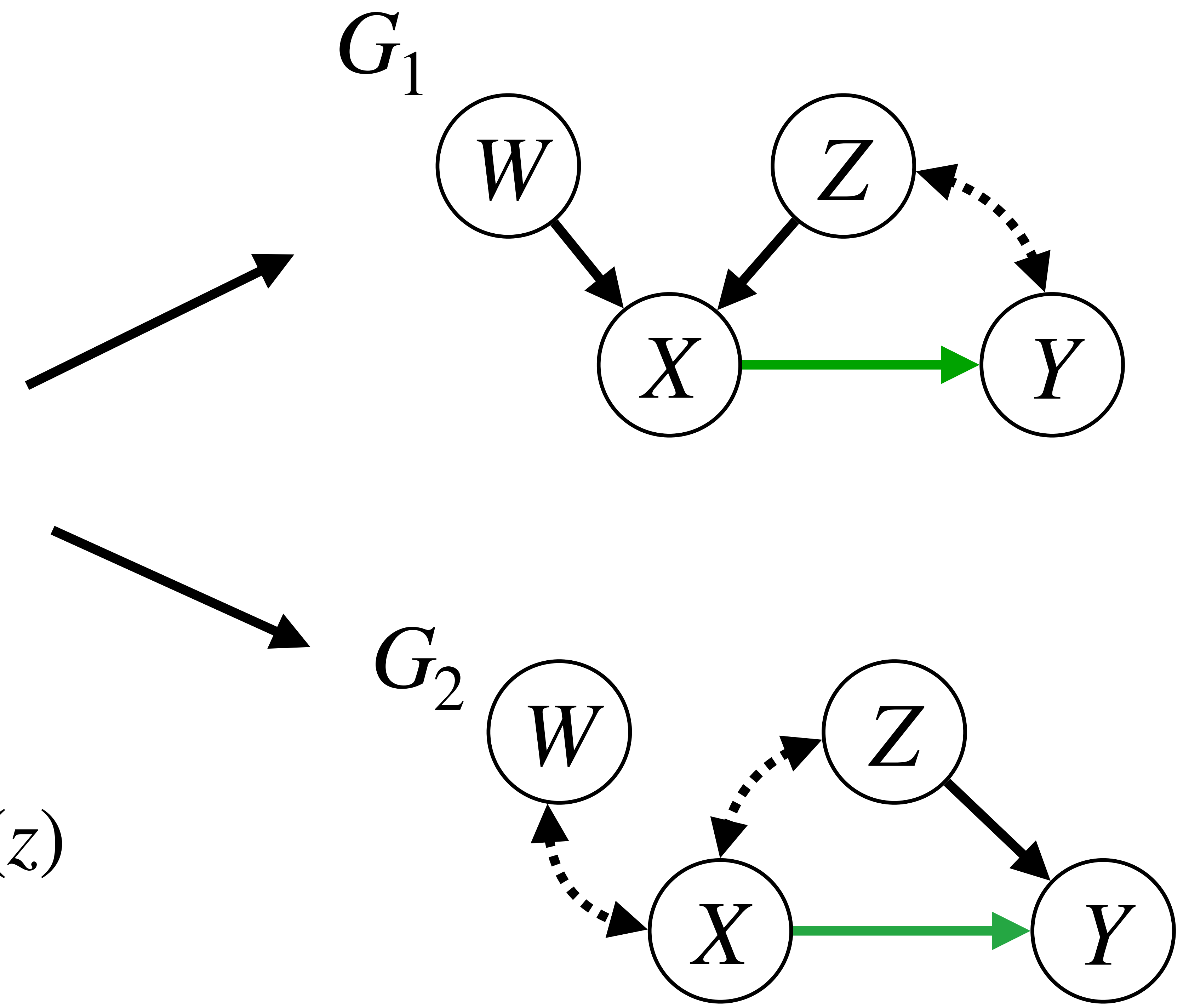
Inferred (Interventional) Distribution ← Available (Observational) Distribution

👍

Jaber A., **Ribeiro A. H.,** Zhang, J., Bareinboim, E. (2022) Causal Identification under Markov Equivalence - Calculus, Algorithm, and Completeness. In Proceedings of the 36th Annual Conference on Neural Information Processing Systems (NeurIPS 2022).

Effect Identifiabiliy given a PAG





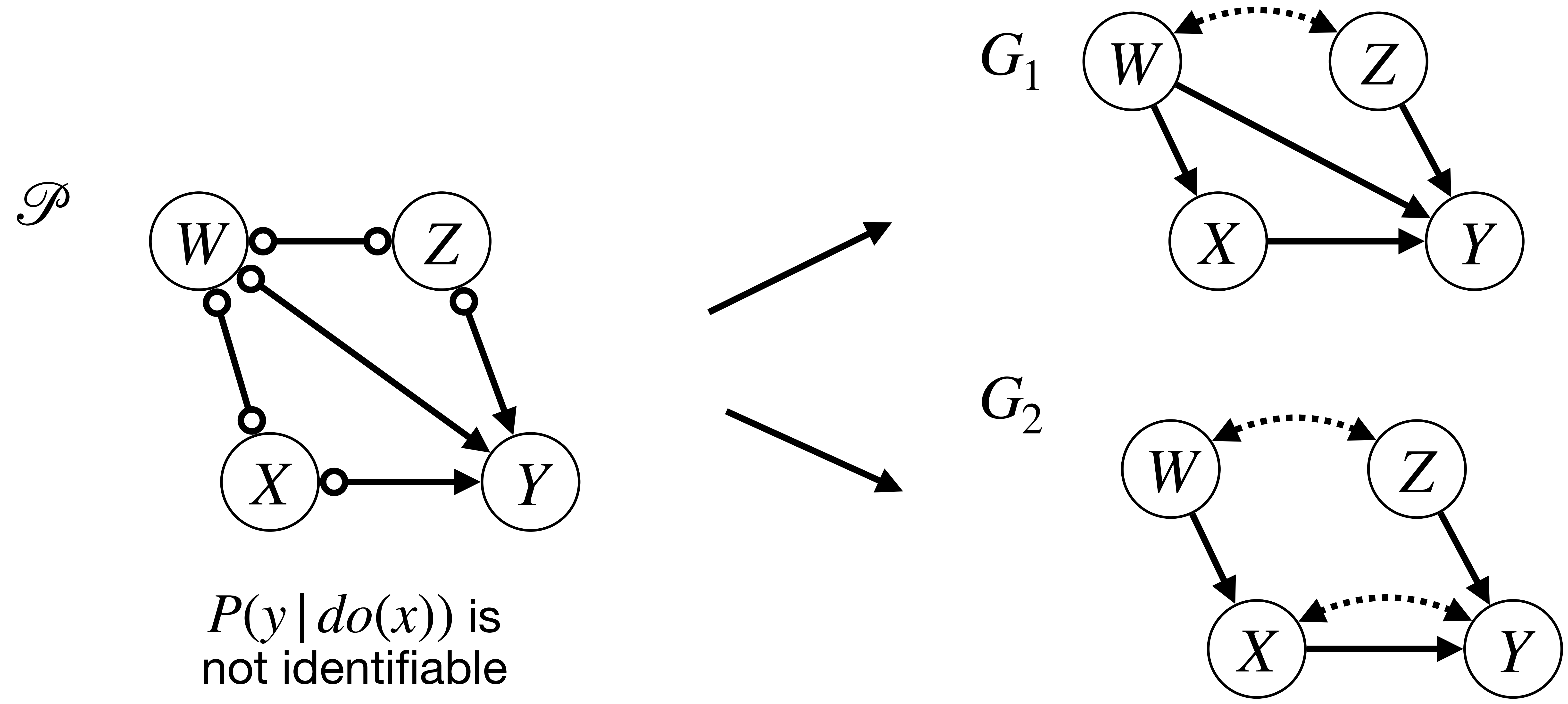
$$P(y \mid do(x)) = \sum_z P(y \mid x, z) P(z)$$

$$P(y \mid do(x)) = \sum_z P(y \mid x, z) P(z)$$

$$P(y \mid do(x)) = \sum_z P\left(y \mid x, z\right) P\left(z\right)$$

An effect identifiable in a PAG $\mathscr{P}$ is identifiable in all causal diagrams G in the Markov Equivalence Class using the same identification formula!

Effect Non-Identifiabiliy given a PAG



$$P(y \mid do(x)) = \sum_z P(y \mid x, z) P(z)$$

$P(y \mid do(x))$ is not identifiable

An effect not identifiable in a PAG $\mathscr{P}$ is not identifiable in at least one causal diagrams G in the Markov Equivalence Class

Many other Topics in Causal Inference

1. Causal Discovery with Cycles
2. Causal Discovery from Time-Series Data
3. Causal Discovery in Linear Models
4. Causal Discovery with Prior Knowledge
5. Causal Discovery for Additive Noise Models
6. Causal Discovery with Interventional Data
7. Data-Driven Covariate Selection for Adjustment
8. Partial Effect Identification
9. Fairness and Mediation Analysis
10. Individual Treatment Effect (ITE) Estimation
11. Many more…

Causal Discovery with Cycles

October 2021

Foundations of structural causal models with cycles and latent variables

Stephan Bongers, Patrick Forré, Jonas Peters, Joris M. Mooij

Author Affiliations +

Ann. Statist. 49(5): 2885-2915 (October 2021). DOI: 10.1214/21-AOS2064

DE GRUYTER

Research Article

S. Bongers*, T. Blom, and J.M. Mooij

Causal Modeling of Dynamical Systems

Structural Dynamical Causal Models
(SDCMs)

Establishing Markov Equivalence in Cyclic Directed Graphs

Tom Claassen[1] Joris M. Mooij[2]

[1]Institute for Computing and Information Sciences, Radboud University, Nijmegen, Netherlands
[2]Korteweg-deVries Institute, University of Amsterdam, Amsterdam, Netherlands

Claassen, T. &; Mooij, J.M.. (2023). *Establishing Markov equivalence in cyclic directed graphs. Proceedings of the Thirty-Ninth Conference on Uncertainty in Artificial Intelligence*, PMLR 216:433-442, 2023.

Causal Discovery from Time-Series Data

High-recall causal discovery for autocorrelated time series with latent confounders

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07745 Jena, Germany
jakob.runge@dlr.de

Gerhardus, A., & Runge, J. (2020). High-recall causal discovery for autocorrelated time series with latent confounders. *Advances in Neural Information Processing Systems (NeurIPS 2020)*, *33*, 12615-12625.

CHARACTERIZATION OF CAUSAL ANCESTRAL GRAPHS FOR TIME SERIES WITH LATENT CONFOUNDERS

BY ANDREAS GERHARDUS[1,a]

[1] *German Aerospace Center, Institute of Data Science,* [a] *andreas.gerhardus@dlr.de*

Causal Discovery and Identification in Linear Models

ParceLiNGAM: A causal ordering method robust against latent confounders

Tatsuya Tashiro[*], Shohei Shimizu[†], Aapo Hyvärinen[‡] and Takashi Washio[§]

Tashiro, T., Shimizu, S., Hyvärinen, A., & Washio, T. (2014). ParceLiNGAM: A causal ordering method robust against latent confounders. *Neural computation*, *26*(1), 57-83.

Causal Discovery with Unobserved Confounding and Non-Gaussian Data

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Causal Effect Identification in LiNGAM Models with Latent Confounders

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Causal Discovery with Prior Knowledge

Sound and Complete Causal Identification with Latent Variables Given Local Background Knowledge

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Sound and complete causal identification with latent variables given local background knowledge

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Causal Discovery for Additive Noise Models

Beyond the Markov Equivalence Class: Extending Causal Discovery under Latent Confounding

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Causal Discovery with Interventional Data

Causal Discovery from Soft Interventions with Unknown Targets: Characterization and Learning

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Causal discovery from observational and interventional data across multiple environments

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Data-Driven Covariate Selection for Adjustment

Finding Valid Adjustments under Non-ignorability with Minimal DAG Knowledge

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Differentiable Causal Backdoor Discovery

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Partial Effect Identification

Stochastic Causal Programming for Bounding Treatment Effects

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Kirtan Padh, Jakob Zeitler, David Watson, Matt Kusner, Ricardo Silva, Niki Kilbertus; *Proceedings of the Second Conference on Causal Learning and Reasoning,* PMLR 213:142-176

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Fairness and Mediation Analysis

A Causal Framework for Decomposing Spurious Variations

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D. Plecko, E. Bareinboim. A Causal Framework for Decomposing Spurious Variations. In *Proceedings of the 37th Annual Conference on Neural Information Processing Systems* — NeurIPS-23.

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Causal Fairness Analysis

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Individual Treatment Effect (ITE) Estimation

Generalization Bounds and Representation Learning for Estimation of Potential Outcomes and Causal Effects

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Estimating individual treatment effect: generalization bounds and algorithms

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Learning Representations for Counterfactual Inference

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Causal Representation Learning

Toward Causal Representation Learning

This article reviews fundamental concepts of causal inference and relates them to crucial open problems of machine learning, including transfer learning and generalization, thereby assaying how causality can contribute to modern machine learning research.

By Bernhard Schölkopf, Francesco Locatello, Stefan Bauer, Nan Rosemary Ke, Nal Kalchbrenner, Anirudh Goyal, and Yoshua Bengio

Schölkopf, B., Locatello, F., Bauer, S., Ke, N. R., Kalchbrenner, N., Goyal, A., & Bengio, Y. (2021). Toward causal representation learning. *Proceedings of the IEEE*, *109*(5), 612-634.

Coarse-grained causal models:

Causal Consistency of Structural Equation Models

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Causal Effect Identification in Cluster DAGs

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Neural Causal Abstractions

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And many more….

1. Effect Identificationf from surrogate experimental distributions such as $P(\mathbf{y} \mid do(\mathbf{w}))$
2. Identification of effects from soft-interventions:
    i. Hard / do-interventions vs soft / σ-interventions.
3. Counterfactual Identification:
    1. Axioms and Identification Algorithms of $\mathscr{L}_3$ quantities, such as $P(\mathbf{y}_{\mathbf{x}} \mid \mathbf{x}', \mathbf{z})$.
4. More on estimation of the adjustment formula:
    1. Variety of frameworks leveraging machine learning
    2. Doubly robustness
    3. Balancing
5. Optimal Causal Experimental Design
6. Causal Reinforcement Learning

Coding Exercises

Causality Tutorial:

- Available at https://github.com/adele/Causality-Tutorial/ → Nordic ProbAI 2024
- **Google Colab Notebook:** (Link)

Check Part II:

1. Causal Discovery using FCI
2. Causal Effect Identification from PAGs

Thank you! :)

Feel free to reach out to me if you have any questions:

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