# Side by Side

## A comparisons of BNs and MNs

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*https://probabll.github.io*

# BNs and MNs

Compact representations of joint distributions over collections of random variables (the nodes of the BN/MN structure).

Statistical properties of these distributions can be shaped by how rvs relate to one another, which we code via edges in the BN/MN structure.

These distributions can be parameterised (expressed in terms of a compact number of numerical parameters) that interact in a factorisation.

# Edges

Asymmetric statistical prediction

Mutual affinity

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Asymmetric statistical prediction

In a BN, the parents of a node predict (statistically) the node's outcome

Mutual affinity

In MNs, connected nodes exhibit patterns of mutual affinity.

# Parameters

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A parameter in an MN is the value that one specific factor, say $\phi_2$, associates with an assignment of a specific clique, say $(B = b^1, C = C^0)$.

But a clique may be in the scope of multiple factors, and the variables in the clique may be part of other cliques.

Because of all these levels of interaction, there is no way to interpret a single parameter, such as $\phi_2(B = b^1, C = C^0)$, as proportional to a marginal or conditional probability.

Interpretable as conditional probabilities

A parameter in a BN is the probability of an assignment of a node, say $G = g^1$, given an assignment of its parents, say $(D = d^1, I = i^1)$.

# Parameterisation

A collection $\mathcal{C}$ of CPD factors, one per node, each CPD in $\mathcal{C}$ is a local model of a node given its parent nodes in $\mathcal{G}$

A collection $\Phi$ of non-negative factors that cover all nodes and all edges of $\mathcal{H}$ at least once but possibly many times, each factor in $\Phi$ must have a clique of $\mathcal{H}$ as its scope.

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A BN defined by $(\mathcal{G}, \mathcal{C})$.

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An MN defined by $(\mathcal{H}, \Phi)$.

# Graph to Factorisation

Non-unique, unless we know the factors' scopes.

Uniquely determined by the graph structure.

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Non-unique, unless we know the factors' scopes.

The MN structure does not identify the factorisation, but when we know at least the factor's scopes we can state it

$$P_{\boldsymbol{\Phi}}(X_1, \ldots, X_n) \propto \prod_{\phi_i \in \boldsymbol{\Phi}} \phi_i(\mathsf{D}_i) \qquad \text{where } \mathsf{D}_i = \text{Scope}[\phi_i]$$

Uniquely determined by the graph structure.

The BN structure is unambiguous in terms of factorisation.

$$P_{\mathcal{C}}(X_1, \ldots, X_n) = \prod_i P(X_i | \text{Pa}_{\mathcal{G}}(X_i))$$

# Factorisation to Graph

Uniquely determined by local structure of CPD factors.

Uniquely determined by knowledge of the scope of all factors.

# Factorisation to Graph

Uniquely determined by local structure of CPD factors.
In BNs, each CPD factor in $\mathcal{C}$ tells that we must introduce a collection of nodes (parents and child) in a specific relation (parents point to child).

Uniquely determined by knowledge of the scope of all factors.
In MNs, the collection of factors $\Phi$ obeys strict rules: every node is covered by one or more factors, every edge (pair of nodes) is covered by one or more factors, every 2 rvs in the scope of a factor must be connected by an edge.

# Direct Influence

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This is true for both BNs and MNs.

# Patterns of Indirect Influence

Marginal independence **but** conditional dependence.

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Marginal independence **but** conditional dependence.
In a BN v-structure $A \to C \leftarrow B$,
$A$ and $B$ cannot influence one another (marginal independence) but conditioning activates influence (conditional dependence).

Marginal dependence **but** conditional independence.
In a BN chain $A \to C \to B$ or in a BN fork $A \leftarrow C \to B$,
$A$ and $B$ influence one another (marginal dependence) but conditioning blocks influence (conditional independence).

MNs are also like that. In the chain $A—B—C$ or cycle $A—B—C—D—A$,
$A$ and $C$ influence one another (marginal dependence) but conditioning on $(B, D)$ blocks influence (conditional independence).

# Midterm Skills Checklist

An overview of necessary skills:

- Recognise graph from factorisation.
- Factorise and recognise factorisations.
- Parameterise using tabular CPDs and non-negative factors.
- Compute probability queries from tabular view of (unnormalised) joint distribution and/or via operations on the factorised distribution.
- Recognise forks, chains, v-structures, cycles and their implications.
- Analyse direct and indirect influence.
- Test (in)dependence numerically or via graphical (d)-separation.
- Use graphical structure and/or factor operations for reasoning.

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This is only a high-level view of skills, don't take it as exhaustive.

# Midterm Rules

- Work individually and without consulting sources.
- You may use a basic calculator but no other device.

# References