A1.5.3. Speed limit

energy and speed in a particle accelerator

According to classical physics, charged particles move in electric fields in a way that increases its kinetic energy in the same amount that decreases its potential energy.

$E = \frac{1}{2} mv^2 = qV \quad \rightarrow \quad v^2 = 2qV / m$

for a proton (in SI units), $q = 1.6 \times 10^{-19}$ C $m = 1.7 \times 10^{-27}$ kg,

so $q / m = 10^8$ C / kg.

If $V = 1$ V, we have $v^2 = 2 \cdot 10^8$, so that $v = 1.5 \times 10^4$ m / s = 15 km / s

Therefore, applying a potential difference of one Volt to a proton, it stores an energy of 1 eV and speeds up to acquire a speed of 15 km / s.

As the classical kinetic energy depends on the square of the speed, we have that the speed increases as the square root of the increase in potential difference. That is, if we increase the potential difference by a factor of 100, the speed increases by a factor of 10.

Applying 100 V, its speed will increase 10 times, up to 150 km / s, and so on.

A circular accelerator at CERN called LHC *(Large Hadron Collider*) can apply up to one TV (10^{12} V) to protons in a revolving ring of 30 km.

According to the previous rule, the particles would reach a speed of 15 million km / s, ie 50 times the speed of light.

We will see that this is not so.

The time it takes for the protons to go around the circular ring (which can be measured) is inversely proportional to the speed that they have.

Thus, applying a potential of 4 V, Protons carry an energy of 4 eV, and they will have a speed of 30 km / s, and they will take 1 second to go around the full circle.

If we increase the energy 100 times, up to 400 eV, its speed is increased 10 times, so it will take 10 times less to turn around: 1/10 s = 0.1 s.

If we keep increasing the potential one million times, up to 400 MeV = 0.4 GeV, its speed would increase 1000 times and it would take one thousandth of the previous time, that is: 0´10 ms = 10^{-4} s = 100 μs .

At this time, its speed would be equal to that of light, since

v = e / t = 30 km / 10^{-4} s = 300.000 km/s

the time measured in reality is 140 microseconds, so they did not get to the speed of light.

From there, we can increase the power up to 4 TeV (ie, by a factor of 10,000), and yet, the time it takes to turn around (which for Classical mechanics would be reduced by a factor 100, ie 1 μs) does not reach down to 100 microseconds.

Despite the huge amount of energy they store, the particles fail to reach the speed limit as predicted by the Relativity theory.