C1.2.3. Lorentz

After various attempts to explain the result of Michelson (see Annex 1: A1.3.4 and A1.4.1) it was Lorentz who proposed a new kind of spacetime transformation in which the speed of light was kept invariant.

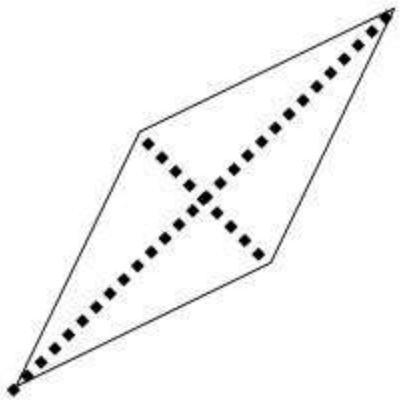


Figura 1.19

In Figure 1.19 we can see how this new transformation consists of a rhombus tipped 45 degrees.

The same as in the transformation of Galileo, the surface of the rhombus is equal to the original square. For this, one of the diagonals is stretched and the other one contracted in exactly the same proportion. As the surface of a rhombus is half the product of its diagonals, the result does not vary.

It is important to highlight three aspects of this figure:

Though one of the diagonals is stretched in relation to the other, the speed of light is the same in both. Just remember that the speed is given by the inclination, and the two diagonals have the same inclination of 45º. Another way to see it is if we realize that at the great diagonal, which is three times longer than the other, the space covered by light is three times larger, while it also takes the triple time to cross it, so the speed is the same in the two diagonals.

-The main difference with Galilean transformation is that, now, the base no longer remains horizontal, but leaning the same way that the side of the square. Precisely in this fact are based the counterintuitive aspects of the Theory of Special Relativity (SRT).

-In The Galilean transformation the invariant was a line that was horizontal. As a result, time was all the same all over the universe. In Lorentz transformation appears a different invariant, which are the diagonals. Thus, it appears a new absolute in the universe, which is the speed of light.

In connection with the construction and interpretation of the Lorentz transformation, see Annex 1: A1.4.3.

Once the geometric shape of the Lorentz transformation is established, we will explore which are its consequences on the measurement of physical quantities that we saw earlier.

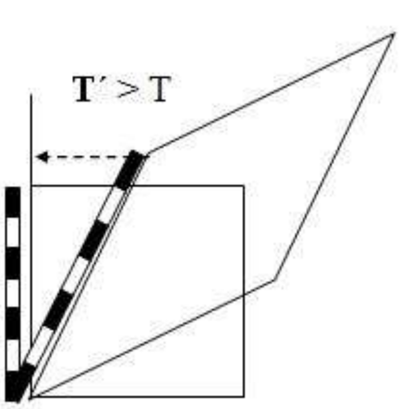


Figura 1.20

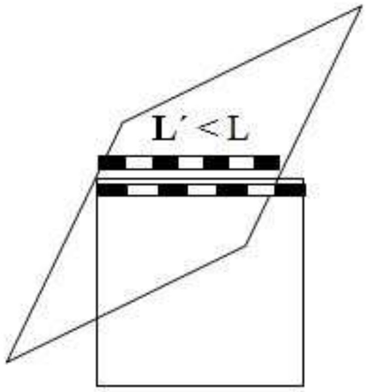


Figura 1.21

One can see in Figure 1.20, that time, now, does no longer measure the same in the original reference system as in the transformed one. The time in the moving RS suffers a slowdown which is known as *time dilation*, and is the first of the four relativistic effects that we plan to view. It is interesting to note that the figures offer not only a qualitative picture of this effect (and the following), but they can be used to set numerical values. In this case, comparing the two scales, we can see that 6 units of time in the new RS measure the same as 7 units in the original RS, ie, time dilates at 7/6. If we apply the formula for the time dilation factor *(gamma* factor, see the end of A1.4.3) to this figure, in which we observ that the relative speed of the RS is half the speed of light, we would get about the same amount (in relation to measures of physical quantities in the Lorentz transformation, see Annex 1: A1.4.4).

Applying similar considerations to Figure 1.21, we can see the phenomenon of *length contraction*, whereby the lengths, in a reference system in motion, are smaller than at rest. It is important to notice that in this case, the measurement of the length of a stick involves measuring the horizontal separation between its ends, which are generally two lines with the same inclination. In this case, they correspond to the side edges of the unit cell, both in the case of the initial square as in the rhombus (Lorentz transformation).

In the following diagram (Figure 1.22) we can observe the relativistic effect of speed limit. Many people know more or less that we can not go over the speed of light, but they are generally unaware of the reasons. It is thought that it is a kind of technical limitation, as it was at the time of the sound barrier for the planes, and that the progress of the technique will overcome it some day. Also usually they think that it is an impossibility to measure speeds in excess of 300 000 km / s, which would be a kind of *infinite* for all measures.

In the picture we can visualize the existence and characteristics of this limit.

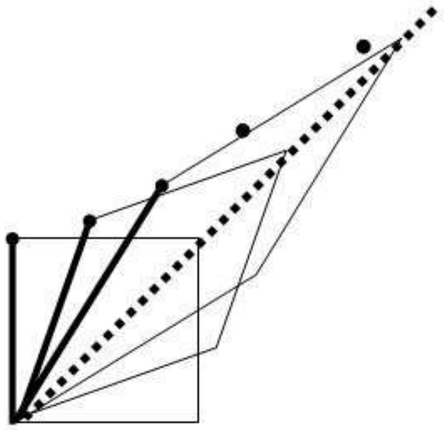


Figura 1.22

As the reference system (which should always be able to be associated with a real object) has a higher speed, its left side keeps leaning. By way of the Lorentz transformation, the base of the figure also tilts upward, in the same proportion. The two lines, therefore, tend to join on the diagonal points (speed of light). But we also know that the rhombus which delimit the two lines must always maintain a constant surface area, so the much they stretch they never reach to close at all, and it is therefore impossible for a reference system to move at the speed of light with respect to another.

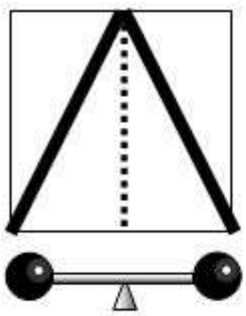


Figura 1.23

This is an essential property of spacetime, just as the impossibility of reaching an infinite speed was a fundamental characteristic of the Galilean transformation.

However, this speed limit only affects the movement of the reference system, and therefore, the movement of objects, energy or information, which are elements that can be associated with a certain reference system.

But it is possible to find a speed that does not have this limit, for instance the wave that would be done in a sports stadium by the public. The speed of this wave is given by the delay between a viewer and the next, and if all the spectators would rise at once the “wave” would have an infinite speed. The same happens with the waves of quantum state, in which were newly discovered speeds faster than light, without this being a refutation of SR as it has been pretended, since no material, energy or information can be transported by them.

To recognize the effect of the Lorentz transformation on the mass, we will go to the same starting figure of an inelastic symmetric collision which has been already used for the Galilean transformation (Figure 1.23). One can see that the speed with which the two equal masses approach, in this particular case, is half the speed of light. The center of mass is represented by the vertical dotted line, indicating that it is at rest (which corresponds with the fact that the situation is totally symmetrical both from the point of view of the masses as from their kinetic energies).

We now observe the same event from a reference system that moves with the mass of the right, that is, at half the speed of light to the left (Figure 1.24). To do this, we apply the corresponding Lorentz transformation to the starting square, and we draw lines between the corresponding points (lower vertices and midpoints of the sides). In a first observation, we find that the line of the right mass is now vertical, indicating that, from the new system of reference, it does not move, as was expected. We can also observe that the CDM, which was previously at rest, now moves at half the speed of light to the right, which was also to be expected.

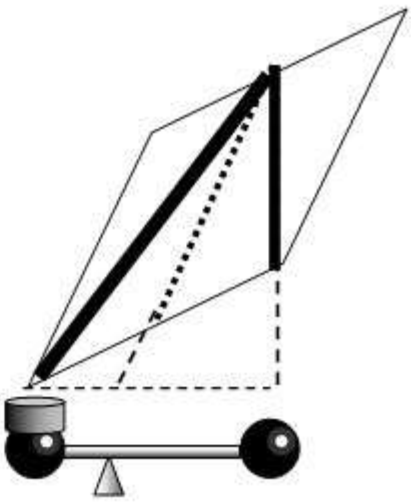
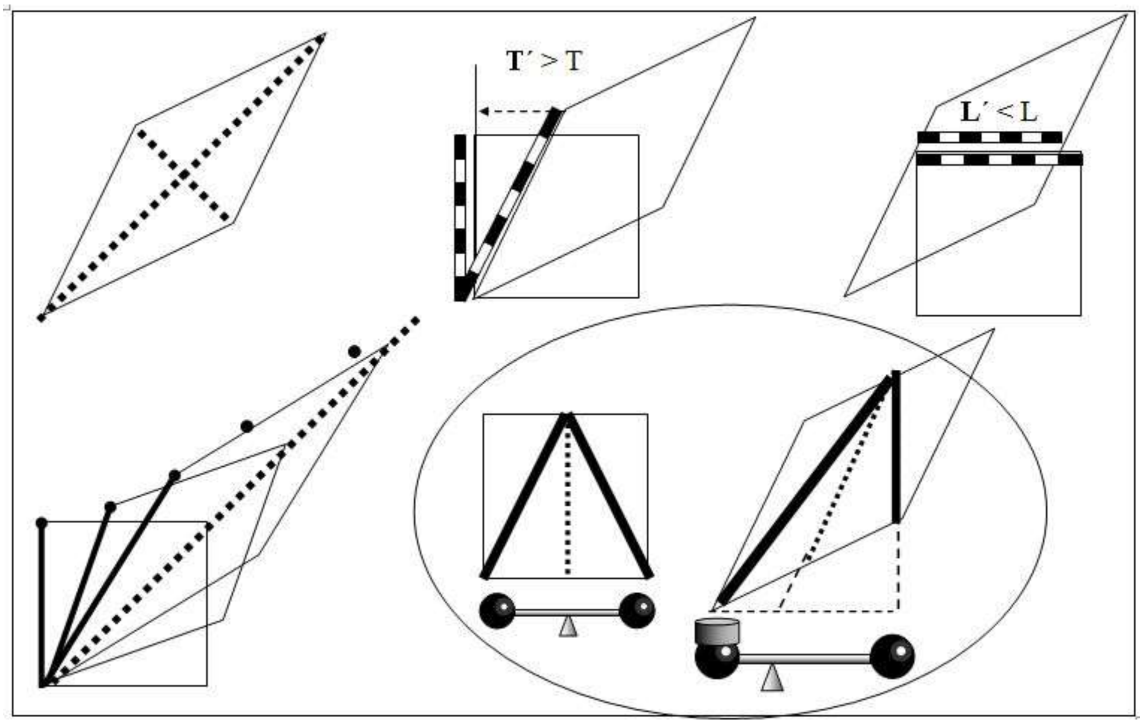


Figura 1.24

The most important aspect results from applying the construction of a mass balance in a horizontal line at the bottom of Figure 1.24. We note that, now, the CDM is no longer located at the midpoint between the two masses. This indicates that "something" moves the CDM to the left.

The situation now is no longer symmetric, since the mass of the right no longer moves, and, therefore, it has no kinetic energy, while the left mass takes an even greater speed than before, that is, it has kinetic energy. It can be also shown graphically (see Annex 1: A1.5.4) that the value of the kinetic energy of the left mass is exactly equal to an additional mass which moved the CDM to the point where it is now located. This is the relativistic phenomenon known as *equivalence of mass and energy*,which can be expressed by saying that energy has the same inertial features than the mass, or that mass and energy are two forms of the same physical quantity. The result can be also expressed through a formula that seems almost trivial: $E = m$

If we consider that to change the units of mass to energy it is necessary to multiply by a speed squared, and that in the diagrams being used the speed of light is the unit ($c = 1$), we can write the formula as follows: $E = m \cdot 1^{2}$. Finally, in other units in which the speed of light is not unity, the formula is: $E = mc^{2}$, which is the well-known formula of Einstein, considered as one of the most mysterious and difficult to understand in physics, and we are now able to interpret it as a geometric result of Lorentz transformation (the information on the Einstein formula can be extended in Section A1.5.4.6 of Annex 1).



E = m (c = 1) to **E = mc²**

Table 1.3: spacetime display for Special Relativity

In Table 1.3 we meet the figures seen so far, in this way to have a synthetic representation of the special theory of relativity by spacetime diagrams.

At the top left of the table 1.3 the geometric shape of the Lorentz transformation is represented (rhombus tipped 45), which also tells us that the speed of light (dotted line) is always on the diagonal.

In the two remaining figures from the top we can observe the relativistic phenomena of time dilation and length contraction. The presence of measurement scales for length and time is intended to emphasize the fact that each and every one of the diagrams used are not only qualitative, but entirely valid quantitative results can be obtained.

At the bottom left we observe resulting figure of applying the same change of RS in succession. As the figure is being stretched diagonally due to the slope of the base of the rhombus, we check the impossibility of exceeding the speed of light, which is, therefore, the relativistic limit for speeds between SRI.

We also note in the bottom right that the figures of levers and collisions are collected in a single "balloon" Compared with the corresponding box in Figure 1.2, we see now that the kinetic energy displaces the CDM in a manner which is equivalent to an additional mass.

The formulas underneath these figures allow us to understand that the famous Einstein's expression $E = mc^2$ is actually a change of units, this expression becomes almost trivial in the units we've been using: $E = m$.

This set of figures is referenced with the name of *Lorentz* to indicate that this Dutch scientist was the first to propose the mathematical expression of the transformation that bears his name, although he not come to establish its profound physical implications, what did later Albert Einstein.

With this explanation the visualization of relativistic phenomena using sàcetime diagrams comes to an end. What follows is a series of checks and realistic applications of these phenomena, to emphasize their physical and real (and not purely geometric) nature.