stAngles

In the **Euclidean 2D** plane we can associate an Angle to every vector in the following way:

Vector: $v = (v_1, v_2)$ $\rightarrow$ **Angle: $\varphi = \text{atan}\,(v_2/v_1)$**

The **unit vector** for this direction can then be written as

$u_v = (\cos\varphi , \sin\varphi)$

In the **Minkowskian (1,1)D** plane we can also associate an stAngle to every vector in an analogous way:

Vector: $v = (v_1, v_0)$ $\rightarrow$ **stAngle: $\varphi = \text{atanh}\,[\,(v_0/v_1)^{stType}\,]$**

where atanh is the inverse hyperbolic tangent function, and

using $v^2 = v_1^2 - v_0^2$

if $v^2 = 0$ $\rightarrow$ stType = 0 , otherwise stType = v^2 / abs(v^2)

The resulting values for stType are:

stTipe = +1 for spacelike vectors ($v_1 > v_0$)

stTipe = - 1 for timelike vectors ($v_1 < v_0$)

stTipe = 0 for lightlike vectors ($v_1 = v_0$)

The **stUnitVector** in the minkowskian case can be written as

$u_v = (\cosh\varphi , \sinh\varphi)$ for spacelike vectors (stType = +1)

$u_v = (\sinh\varphi , \cosh\varphi)$ for timelike vectors (stType = -1)

u_v is nonexistent for lightlike vectors (stType = 0)

Geometric interpretation of Angles as Areas

Euclidean Plane

-The Angle (in rads) is the measure of the Arc Length of a Unit Circle.

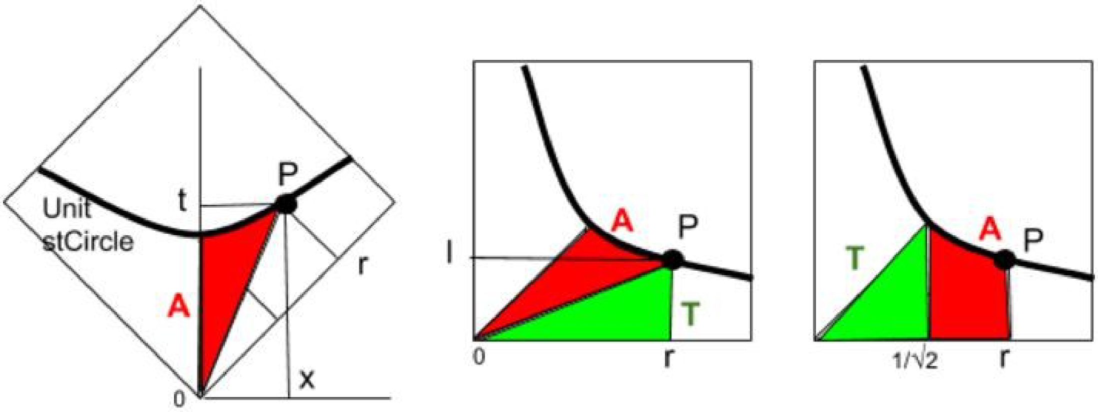
-The Angle is also the **double of the Area** enclosed by a Unit Circle Sector.

For example, the Angle of a whole circle is 2π, which is the length of its unit circumference, $2\pi\cdot r$, and two times the area of the unit circle: $A = \pi\cdot r^2$ ($r = 1$)

Minkowski Spacetime

-The stAngle is the measure of the Arc stLength of a Unit stCircle.

-The stAngle is also the double of the Area enclosed by a Unit stCircle Sector.



In the figure, point P(x,t) on a Unit stCircle (left) marks a stCircular Sector (red) whose Area is A.

Rotating the figure 45º , the equation of the stCircle, in the new coordinates (r,l) , is: $r \cdot l = ½$.

It is easy to check at the right figures that the area of both green triangles has the same value ($T = r \cdot l/2 = ¼$).

The Area A, thus, is equal to the integral of the hyperbolic curve between the values $1/\sqrt{2}$ and r.

Integrating the equation $l = ½\, r^{-1}$ yields $A = \int l dr = ½ \int r^{-1} dr = ½ \ln r$

Between $1/\sqrt{2}$ and r, we get $A = ½ [\ln r - \ln(1/\sqrt{2})] = ½ \ln (r\sqrt{2})$.

Applying $\varphi = 2A$, we have $\varphi = \ln (r\sqrt{2})$

Getting back to the original coordinates, $r = 1/\sqrt{2}\,(x+y)$, so

$\varphi = \ln (x+y)$

To see that this is in accordance with the usual definition, we transform this expression to get $x+t = e^{\varphi}$

Recalling that $x = \sinh \varphi = ½ (e^{\varphi} - e^{-\varphi})$ and

$t = \cosh \varphi = ½ (e^{\varphi} + e^{-\varphi})$

$x + t = \sinh \varphi + \cosh \varphi = e^{\varphi}$