Hands-on Science

Brightening our Future

Edited by
Manuel Filipe P. C. Martins Costa
José Benito Vázquez Dorrío

The Hands-on Science Network

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FOREWORD

CONTENTS

New Light to Relativity with Levers and Sticks

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Abstract. We propose the use of simple physical devices like balances, levers and sticks to understand modern concepts of relativistic physics like mass/energy equivalence or the generation of electricity by induction. The approach is based on the geometric formulation of relativity in spacetime by Minkowski.

To explain mass/energy equivalence we propose the use of ancient scales together with spacetime diagrams to gradually recognise the effect of the lever inclination on the equilibrium point and therefore on the relationship between mass and energy. For the explanation of electromagnetic induction we will show how the movement of a conductor stick, together with the relativistic effect of length contraction are responsible for the creation of the induction current.

Keywords. mass/energy equivalence, relativity, spacetime, electromagnetic induction.

1. Introduction

Hermann Minkowski, after Albert Einstein published his papers about Special Relativity (SR), stated that this theory has an essential geometric nature, with space and time joined together into a new physical entity which he called „spacetime“. Spacetime is endowed with a new kind of geometry, in which the speed of light plays the role of an „universal absolute“.

We propose to use this striking geometrical feature to develop teaching strategies to present relativity and its effects in a mainly visual way, thus allowing students to concentrate in the understanding of the reasons and consequences of this new kind of physical geometry.

The author has presented this idea as an interdisciplinar teaching unit for sophomore students [1]. It has been also the subject of a PhD these, whose content was presented at the ESERA Conference in Istambul on 2009 [2], or, more recently, as a series of videos with animations which were presented at the 11th Conference on Hands-on-Science that took place on 2014 in Aveiro [3].

The aim of this paper is to enlarge the scope of physical effects that can be explained with this visual methodology, as well as the type of materials that can be used for this purpose.

We will focus on two physical effects: Mass/energy equivalence, with levers as the main didatic material for it, and electromagnetic effects, where we will use rods and sticks to explain how they can be derived directly from the relativistic length contraction.

2. Mass/energy equivalence with levers

The equivalence of mass and energy, represented by the famous formula $E = mc^2$, which is considered one of the most famous formulas of all times and an icon of relativity, can be easily interpreted and understood with the help of levers and balances.

The balance was one of the first technological advances of humankind, and its development and diffussion is being currently object of intensive study as an early example of innovation processes [4], [5].

2.1. Balances with fixed fulcrum

The equal-armed balance (Fig.1) was already known by the ancient Egyptians.

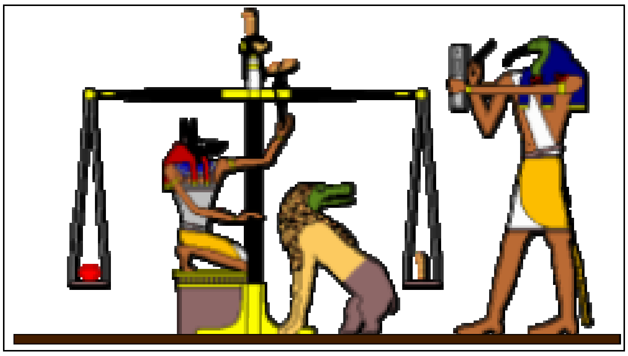

Figure 1. Egyptian equal-armed balance

This type of balance needs a precise set of increasing weights, which difficulted their transportation and their use was restricted mainly to closed places.

The development of an unequal armed balance seems to presuppose knowledge of the lever rule. The familiar steelyard or Roman

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balance (Fig. 2) has a fixed fulcrum and a counterpoise weight moving along its arm. This balance needs only one counterpoise instead of a uniform set of weights. For this reason it could be easily carried.

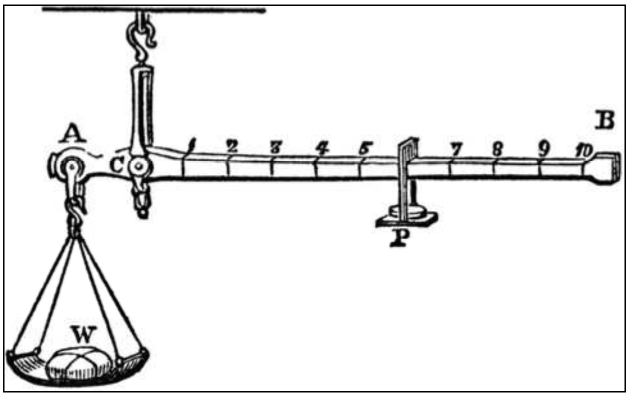

Figure 2. Roman unequal-armed balance

It was presumably the most widespread and frequently used mechanical precision instrument in antiquity and late antiquity [4], and it has also been the most common type of scales in China since the Han dynasty of the 2nd century BC [6].

2.2. Balances with mobile fulcrum

In this type of balance, known as the *bismar* in Medieval Europe (Fig. 3), the counterpoise weight is fixed, and balance was achieved by moving the fulcrum, which was normally a simple loop of chord [7].

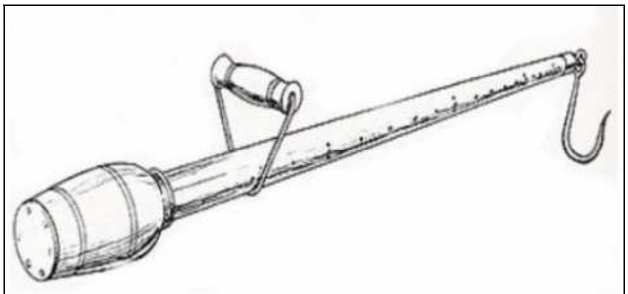

Figure 3. Bismar balance

The first mention in the Western world to this balance is in the Aristotelian *Mechanical Problems* (problem 20), which were probably written by Archytas of Tarentum [8]. The operation of this balance can be understood by the laws of the levers as stated by Archimedes. This is a static type of justification, where the fulcrum is placed at the equilibrium point between both masses [7].

Bismar balances had the advantage of their simplicity of construction and use. In fact, any stick with a fixed weight on one end (such as tools like hoes or hammers, cookware like dippers or pans, or even weapons like spears or clubs) can be converted to a bismar balance by simply placing a hook at the other end [9].

We will use a qualitative rule which applies to these balances. It states that gretaer masses drag the fulcrum to them. In fact, an infinite mass would place the fulcrum directly at the hook.

2.3. Inclination balances

The idea of measuring weights by the inclination of the balance arm has been used to develop several types of inclination scales or pendulum scales, such as letter scales (Fig. 4), which can have a basis (to be placed on a table) or a ring (to be suspended from it).

Figure 4. Letter scales

Their operation is based on the concept of stable equilibrium, which can be achieved when the center of masses of a rigid system lies under its hanging point (this is how a pendulum operates).

We will resort, for didactic purposes, to devices that are not associated with the concept of a balance, such as for example a clothes hanger (Fig. 5) or any similar object.

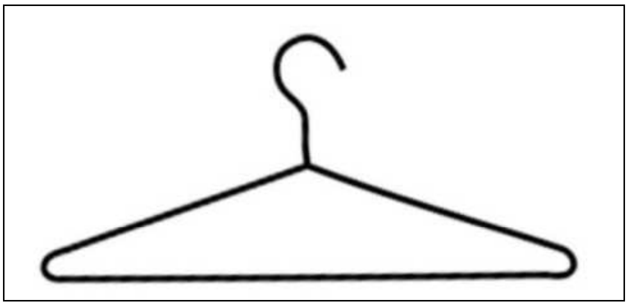

Figure 5. Clothes hanger

The hanger has a fixed point which is placed appreciably over the base line (the „stick“ of the imaginary balance). The fulcrum is not a physical object, but the vertical projection of this hanging point to the stick. If the right mass is greater than the left one (Fig. 6), the hanger will be inclined to the right, thus displacing the fulcrum also to the right (where the greater mass is placed), which is what the qualitative rule stated at 2.3 predicts.

For small inclinations (or, equivalently, low mass unequalities) it is possible to establish a simple direct relation between mass and inclination.

The baseline (Fig. 7) has a length L and the height of the hanging point over the baseline is H. If we place two masses m and M, we can say that M = m(1+d). The displacement of the fulcrum will have a value of x, and the length of the arms will be now equal to L/2+x and L/2-x respectively (Fig. 8).

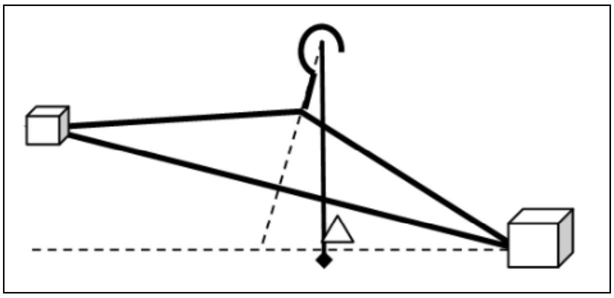

Figure 6. Hanger as a lever (unequal masses)

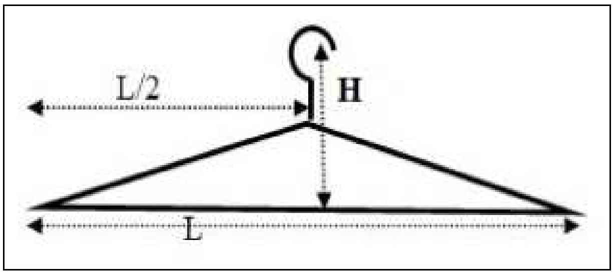

Figure 7. Hanger with special measures

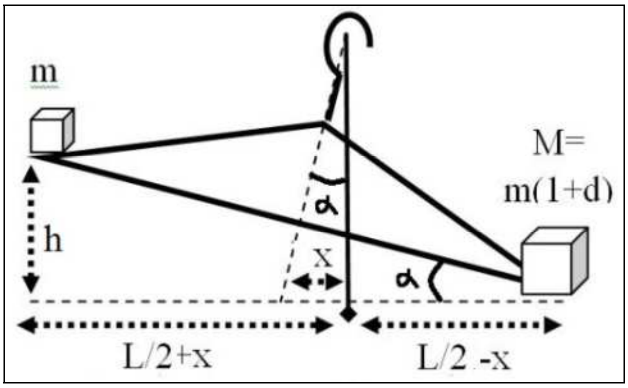

Figure 8. Law of the lever

The law of the lever states that m(L/2+x) = m(1+d)(L/2-x) (1)

L/2+x=L/2-x+dL/2-dx

(2+d)x=dL/2 and x=(0.5*dL)/(2+d) (2)

In the special case where the difference between M and m is low, d will be small compared to 2, and we get the approximate result

x= dL/4 (3)

Comparing triangles we see that the vertical distance h between m and M due to the balance inclination can be expressed as

$h=xLH^{-1}=0.25*dL^2H^{-1}$ or $d=4*HL^{-2}h$ (4)

This would allow us to use the factor $4*HL^{-2}$ to create a vertical scale and place it at the left side of the balance, where we could read directly the relative difference between both masses (d).

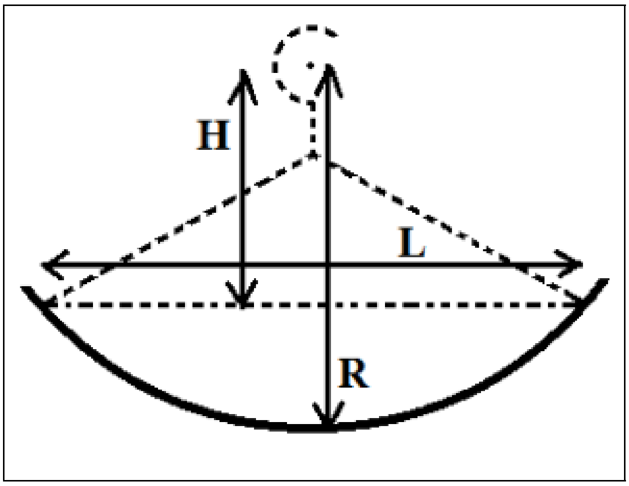

Figure 9. Arc of circumference in hanger

We can even replace the imaginary fulcrum by a real object. This object has to go by itself exactly to the equlibrium point, and this can be achieved by substituting the straight baseline of the hanger by an arc of a circunference, with radius R and chord L (Fig. 9).

A small ball rolling over this curved line (Fig. 10) would show the position of the fulcrum.

The curved base can even be used to hold up the system over a horizontal surface, in which case the hanger would not be necessary (Fig. 11).

If we want this baseline to stand by itself on a table, we should use a surface created by revolution of the curve around a central point (Fig. 12).

With a small ball moving freely on it, two masses can be compared if we place them on opposite borders of the dish. The system will be inclined in the direction of the gretaer mass (Fig. 14), and the displacement of the ball from the central point will be a measure of the ratio between both masses (neglecting the disk´s mass).

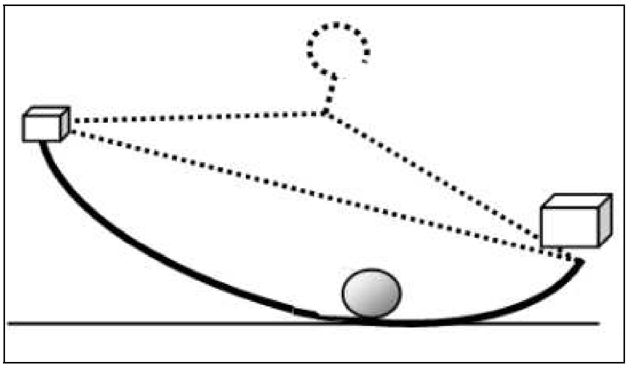

Figure 10. Ball showing the fulcrum

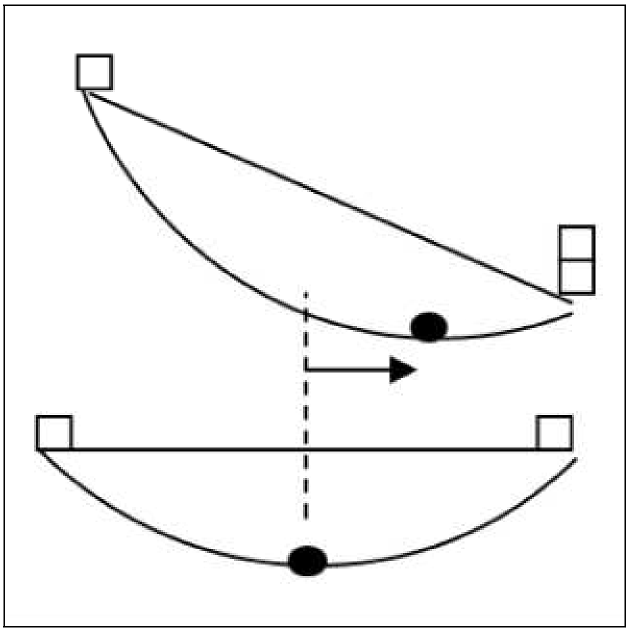

Figure 11. Curved lever without hanger

2.4. Spacetime balance: inelastic collision

It is possible to compare the masses of two colliding particles at the spacetime graph of an inelastic collision [3]. Placing space as the horizontal dimension and time as the vertical one, the collission of two masses with opposite velocities will be seen as a symmetric (isosceles) triangle (Fig. 15).

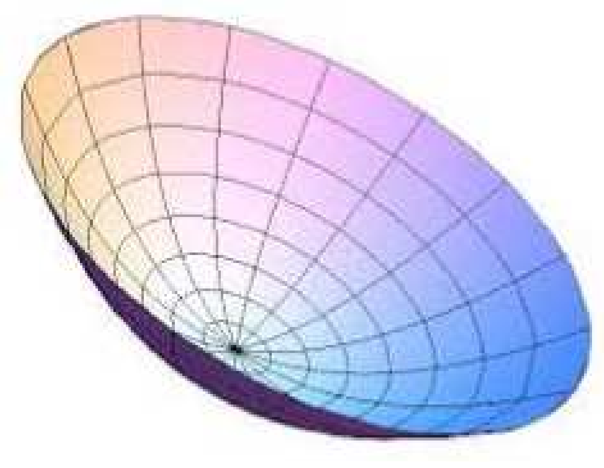

Figure 12. Surface of revolution

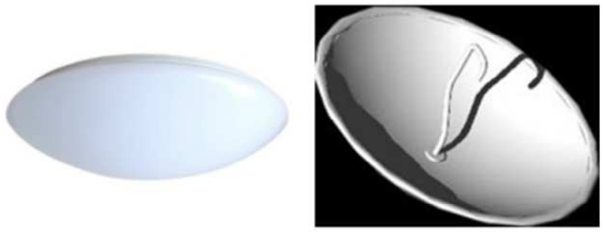

Figure 13. Wall light, satellite dish

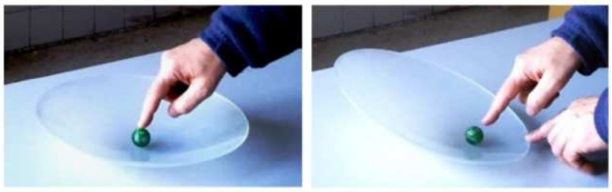

Figure 14. Curved surface with rolling fulcrum

After the collision, both masses will follow the same path together (Fig. 16, wide black line). We can draw this line back in time (dashed line), and it will represent the center of masses at any given time (small triangle).

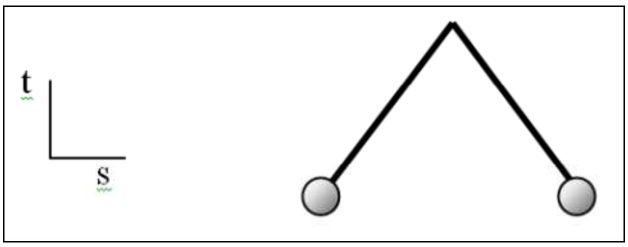

Figure 15. Symmetric collision in spacetime

We can add an imaginary rod, which is represented in the figures by a solid horizontal line joining both masses (Fig. 17). A horizontal line in spacetime can be interpreted as a „slice of reality“, where the measurement takes place.

The center of masses follows the law of the lever, and we can see that its position allows us to compare the values of the masses (they are equal at the left figure, whilst at the right figure the mass at the right is greater than the mass at the left). If one of the masses is known, this comparison allows us to measure the mass of the other particle using the law of the lever, and we have a spacetime balance.

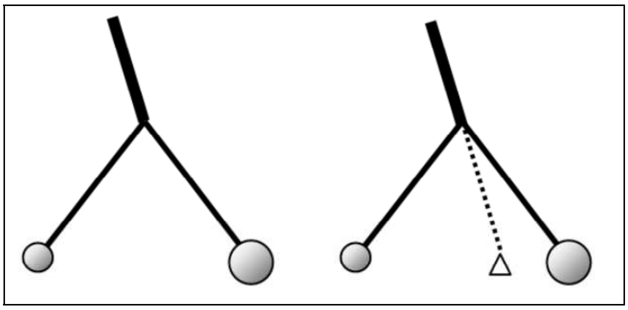

Figure 16. Center of masses and fulcrum

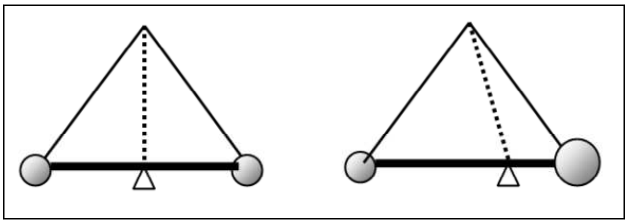

Figure 17

2.5. Energy: pendulum

In order to place mass and energy into the same diagram, we will resort to the pendulum, which had been studied already by Galileo, Huyguens and Newton as a tool to develop their mechanical ideas [10].

The ballistic pendulum is a device where a mass m collides at a speed v with a pendulum at rest, and the vertical deviation of both masses after the inelastic collision is used to establish the velocity v (Fig. 18).

In the previous figure we have made the assumption that the mass of the pendulum is negligible compared to m, and we have chosen units where m, L and g are all equal to 1. This establishes automatically the unit of time as $u_t=(L/g)^{1/2}$. For example, if we have a pendulum with L = 1m, taking $g=9.8\ m/s^2$ as the unit of acceleration would render $u_t = 0.32$ s.

Under these circumstances, the energy conservation Ec = Ep and the laws for both energies (Ep=mgh, $Ec=1/2mv^2$), as well as the geometric property $h=1/2x^2$, (which is valid for small deviations x compared with L) return the interesting result v = x. We have thus a way to establish and measure the velocity v of the incoming bullet directly from the distance x travelled by the pendulum after the collision.

With the chosen units for m and g, we have also that E = h. This special pendulum, therefore, measures both energy and velocity (or equivalently, momentum, since p = mv = 1, because m = 1).

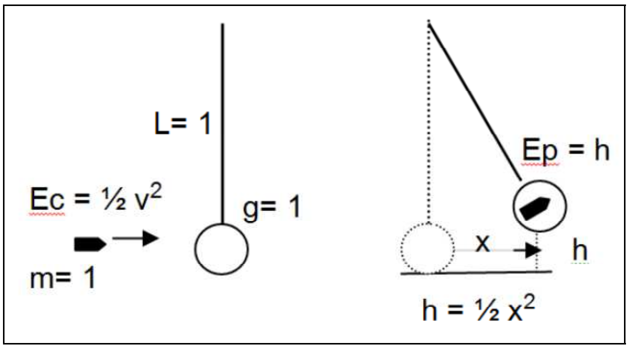

Figure 18

But L is also equal to 1, and thus the inclination of the pendulum from the vertical line will coincide with the velocity of the particle when represented in a spacetime diagram. This is an interesting way to connect spacetime with momentum/energy diagrams, as can be seen in Fig. 19.

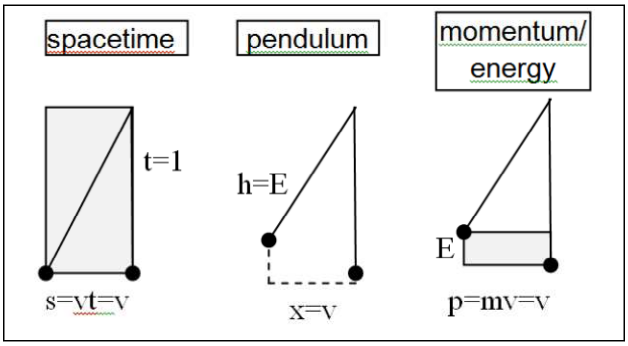

Figure 19. From spacetime to energy

These figures, although apparently very similar, are drawn in different spaces: the left figure is drawn in spacetime (and its lines represent the movement of the bullet and the pendulum at rest), the central figure is in „normal“ (two-dimensional) space (the lines represent the rods of two pendulums, the vertical being at rest and the inclined coming to collid with the same speed as the bullet), and the figure to the right is drawn in the new momentum/energy space (in this case, the lines are there only to compare with the other figures).

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2.6. Mass, energy and relativity

After having established our spacetime balance, together with the geometric equivalence between spacetime and energy/momentum spaces, we can proceed to the following theoretical question, which Einstein already established as the title for one of its famous articles in 1905: *»Ist die Trägheit eines Körpers von seinem Energieinhalt abhängig?«* (Does the inertia of a body depend upon its energy-content?). This article [11] was the origin of the mass/energy equivalence relations, with the worldwide famous formula $E=mc^2$.

We will introduce mass and energy into our spacetime balance and look after their behaviour under a spacetime transformation. Uniform movements are represented as straight lines in spacetime, and any transformation between inertial reference systems must keep them straight. As a consequence, the transformations will have the property of being linear. The determinant of a linear transformation measures the rate of change in the surface area, and the determinant of two successive transformations is the product of their determinants (in particular, the determinant of the inverse transformation is the inverse of its determinant). The principle of space isotropy states that all spatial directions have the same properties. To obtain the inverse transformation of a given one, we must simply go in the opposite direction. As a consequence, the spacetime area must be conserved in inertial transformations (because otherwise it would change differently in one direction and in the opposite one, contradicting the isotropy principle). The conservation of spacetime area is a fundamental property of every relativistic transformation between reference systems, and we will use it as a geometric property of the spacetime transformations.

We will begin with a very symmetrical kind of inelastic collision, where two identical particles collide with opposite velocities. In this special situation, it is straightforward to verify that the center of mass, due to the symmetry of the figure, must stand at its center (Fig. 20, left side). This corresponds to the condition of both particles having the same mass.

If we view the same collision from the reference system of the mass coming from the left (Fig. 20, right side), this means that its spacetime line will be vertical (any particle is at rest in its own reference system), whilst the other mass comes from the right with greater speed. Regarding energy, the left mass will have no kinetic energy, and the same is not true for the right mass. The symmetry of the situation is therefore broken, and we can now try to find an answer to Einstein´s question in the following visual way: The right mass has energy, but the left one has no energy. Any change in the inertia of the right mass due to its energy content should be observed as a displacement of the centre of mass.

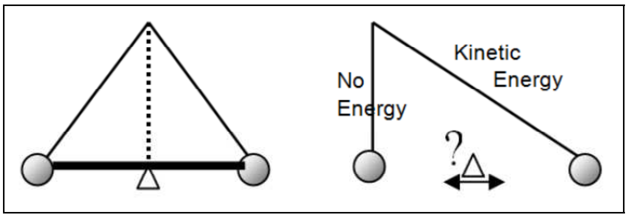

Figure 20. Einstein´s question in spacetime

Classical relativity is based on the galilean transformation, which depends on the concept of a universal time. This means that the (horizontal) baseline of the figure remains unchanged (Fig. 21).

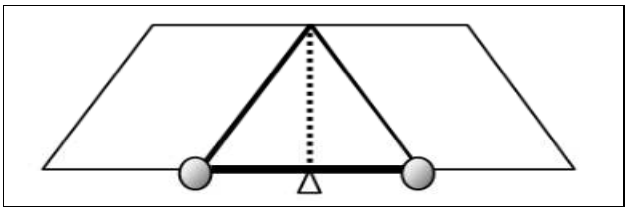

Figure 21. Classical symmetric collision

The centre of mass lies therefore always in the middle of both masses (Fig. 22), and the answer to Einstein´s question from classical physics is crearly a negative one: The energy does not displace the centre of mass, because mass and energy are clearly different magnitudes and they cannot be added in classical physics.

We have drawn spacetime cells as lateral parallelograms for every particle to see more clearly the effect of the spacetime transformation. It is possible, for example, to recognize that the spacetime area of these cells remains constant during the transformations, since the area of a parallelogram is given by the product of its base by its height, and both remain unchanged.

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2.7. Mass and energy in special relatlvity: equivalent magnitudes

Special relativity is based on the Lorentz transformation, where the horizontal baseline, due to the relative movement, gets an inclination which in natural units (where c = 1) is equal to the inclination of the line of the particle with respect to the vertical.

It is possible to compare the effect of inclination on a square and a rhombus which derive from a given square of size 1, if both figures share a common side, which derives from an inclination of the square´s side by a factor v (Fig. 23).

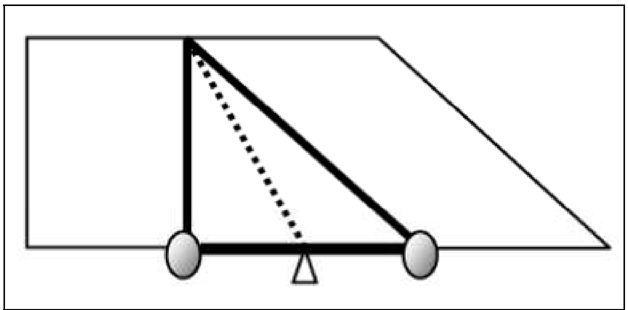

Figure 22. Galilean transformation

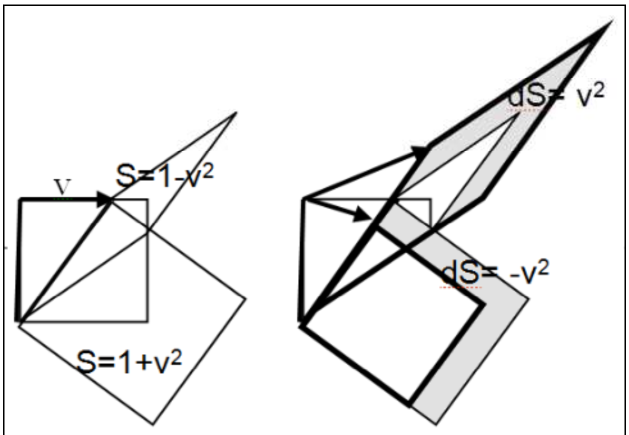

Figure 23. Inclined square and rhombus

The inclined square will have now a size greater than 1 ($S=1+v^2$), and the rhombus´ size will be smaller ($S=1-v^2$).

If the size of these figures must keep its initial value of 1, we must compress the square and expand the rhombus. In both cases, the change in size is the same: v^2. For small values of v, this change in size can be viewed as the substraction (for the inclined square) or addition (for the rhombus) of two strips with approximate length of 1 and width of $v^2/2$ (grey shadows on the right side of Fig. 23).

In the general case, the square will describe a circumference, and the vertical contraction (Fig. 24) will be given by the cosine of the inclination angle. For the rhombus (Lorentz transformation), the line described is a equilateral hyperbola, and the vertical dilation is given by a hyperbolic cosine. The function cosh v, for small values of v, can be approximated by $1+ v^2/2$, as we have seen.

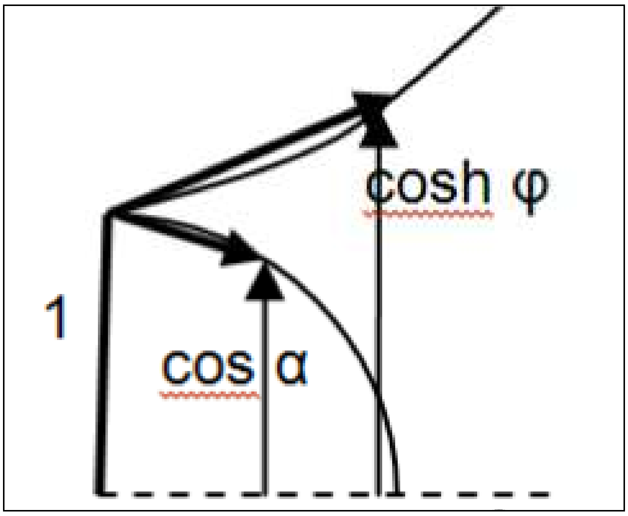

Figure 24. Circular and hyperbolic functions

As a consequence of this, the Lorentz transformation will show an enlargement on the vertical component of the timelike lines, which is called „time dilation". The combined effects of time dilation and base inclination will create a new geometrical meaning for energy, as we will see. We will begin with the spacetime diagram of the symmetric collision with the spacetime cells (Fig. 25).

The change to the reference system of one of the masses is called a „boost", and its geometric representation is a Lorentz transformation (Fig. 26).

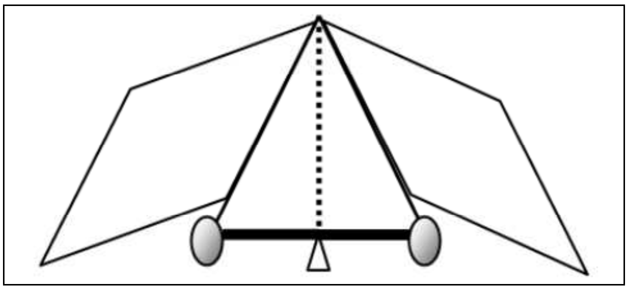

Figure 25. Relativistic symmetric collision

We can notice several differences between the classical case (Fig. 22) and the relativistic case (Fig. 26). The latter shows again the center of masses at the middle of the inclined baseline (dashed line). But the spacetime balance must be constructed, as we have seen, with an (imaginary) horizontal rod, because reality is horizontal in spacetime. The position of the center of masses in this rod is no more in the middle of both masses, but it is displaced to the right. The situation is no more symmetric, and the mass coming from the right has a kinetic energy which the other mass lacks (it is at rest in its own reference frame). This dragging action of the energy on the center of masses is equivalent to an additional mass. In this case, the answer to Einstein´s question is: *„YES, the inertial properties of the mass have changed due to its energy“* [12].

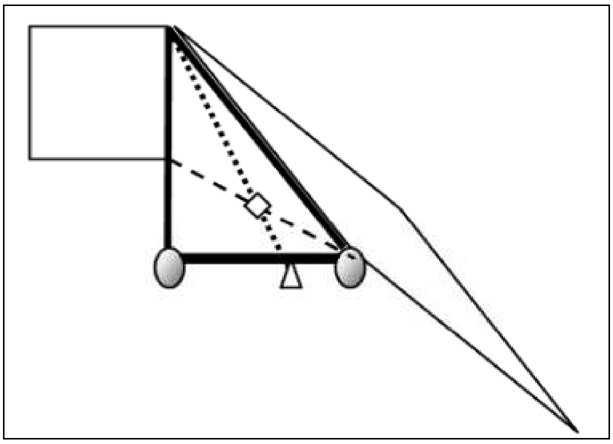

Figure 26. Symmetric collision after the boost

We can even try to quantify the magnitude of this effect (at least in the low-velocity case) using the relations we have seen earlier for the inclined levers. In section 2.4 equation 4 tells us that the vertical displacement h is proportional to the (relative) additional mass d, the proportionality factor depending on the measures of the balance. The spacetime diagram in Fig. 27 shows also that the energy is proportional to the vertical displacement h, which is in visual accordance with Fig 19 for the pendulum.

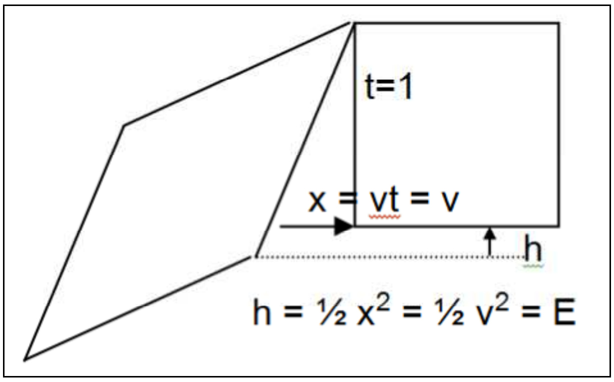

Figure 27. Spacetime collision with energies

All these relations give us a strong visual and intuitive supoport for the claim that energy displaces the center of mass in the same way as an additional mass would do. We are using natural units with c = 1, and Einstein´s formula in these units looks much more unsurprising:

$$E = m$$

We have now the possibility to explain the most famous formula of modern physics in a visual way using levers and diagrams.

3. Electromagnetic effects with rods

The mutual force between parallel currents (known as Ampère´s Law) can be explained as a consequence of the relativistic effect of length contraction when applied to the moving charges, and some textbooks already use this approach [13]. In the same way, we will show how the movement of a conductor stick, together with the relativistic effect of length contraction, are responsible for the creation of an induction current (Faraday-Lenz Law).

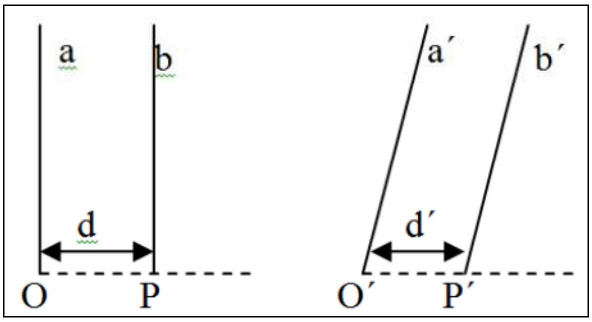

Figure 28. Length contraction

3.1. Length contraction

The Lorentz transformation, when applied to a certain length which is parallel to the relative velocity, creates a relativistic effect which is knwon as the length contraction. We can view it as the (horizontal in spacetime) distance d between two parallel lines (a, b) corresponding to two charged particles in relative rest. This is the distance d in Fig. 28 (left) between points O and P in their own reference system.

If the charges are moving all with the same speed v (Fig. 28, right side) then the new horizontal distance will be d´= d/γ. The factor 1/γ is called the „length contraction factor“. It is always lower than 1, and in the low-speed limit (as shown in section 2.7) it can be approximated by

$$\gamma^{-1}=1-0.5*v^2 \quad (5)$$

3.2. Line current I

We will use a very simple model for a line current. It consists of two sets of particles with identical positive and negative charges, which are uniformly spaced over a very long straight line where they move with constant opposite velocities. A charge density λ_0 is defined dividing the individual charges by their separation. The positive charges will have a density $\lambda_+ = \lambda_0$ whilst the negative charges show a density $\lambda_- = -\lambda_0$.

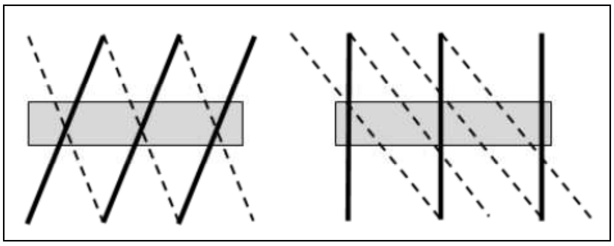

Figure 29. Current and boost

The total (net) charge density is obtained as $\lambda = \lambda_+ + \lambda_- = \lambda_0 - \lambda_0 = 0$. This means that for an outside observer the line current appears to have no net charge at all.

The positive and negative charges move with opposite velocities $v_+ = v$ and $v_- = -v$. These movements produce two currents:

$I_+ = v_+ \lambda_+ = v\lambda_0$ for the positive charges, and $I_- = v_- \lambda_- = (-v)(-\lambda) = v\lambda_0$ for the negative charges. There will be a net current as a result of both opposite movements:

$$I = I_+ + I_- = 2 v\lambda_0 \quad (6)$$

3.3. Ampère Law

If we place two line currents I_1 and I_2 parallel to each other at a distance r, the experimental evidence obtained by Ampère states that they feel a mutual force which is attractive if both currents flow in the same direction, and repulsive if their directions are opposite. The force by unit of length F/L is given by

$$F/L=\mu_0 I_1 I_2/2\pi r \quad (7)$$

It is possible to explain this force as a result of the relativistic effect of length contraction.

In natural units, where the speed of light is equal to 1 (c=1), the relation between the magnetic and electric constants μ_0 and ε_0, $\mu_0 \cdot \varepsilon_0=1/c^2 = 1$ implies that $\mu_0=1/\varepsilon_0$. We define the intensities $I_1 = 2v\lambda_1$ and $I_2 = 2v\lambda_2$. With these units and definitions, the force obtained experimentally by Ampère can be expressed as

$$F/L=4v^2\lambda_1 \lambda_2/2\pi\varepsilon_0 r \quad (8)$$

In Fig. 29 (left) we can see the model of line current in a spacetime diagram. The positive charges (solid lines) go to the right and the negative charges (dashed lines) flow to the left. They are equally spaced, so that the net charge is zero (we can count 3 positive and 3 negative charges in the horizontal slice of spacetime).

At the right, we see the same current as observed from the reference system of the positive charges in the second conductor. This corresponds to a boost that places the positive charges at rest, while the negative charges have greater speed to the left. The length contraction applies in this case to the negative charges, and they are more tightly spaced (we can count 4 negative charges but only 3 positive charges in the spacetime slice). There is now an excess of negative charge in conductor 1, and since the reference system corresponds to the positive charges of the second conductor, they will feel an attraction, as was observed by Ampère.

To quantify this effect, we should notice that the negative charges (in the low speed approximation) will have a velocity 2v, and the legth contraction (Eq. 5) will have a value of

$$\gamma^{-1}=1-2*v^2 \quad (9)$$

The charge density is inverse to the distance between charges, and the inverse of $1-2v^2$, in this approximation, can be taken as being $1+2v^2$. Therefore, if the density of the positive charges in the first conductor is $\lambda_+ = \lambda_1$, the negative charges will have a greater density: $\lambda_- = -\lambda_1 (1+2v^2)$. The total charge density has now a nonzero value:

$$\lambda = \lambda_+ + \lambda_- = \lambda_1 - \lambda_1(1+2v^2)= - 2 \lambda_1 v^2 \quad (10)$$

The first conductor is felt therefore (by the positive charges of the second one) as a charged rod with a negative density $-2\lambda_1 v^2$. This produces an electric field with a value of

$$E=2v^2\lambda_1 /2\pi\varepsilon_0 r \quad (11)$$

A segment of length L from the second conductor, whose charge density is λ_2, will have a total positive charge $q=L\lambda_2$, and it will be attracted by the first conductor with a force

$$F_+=qE=2v^2\lambda_1 \lambda_2/2\pi\varepsilon_0 r \quad (12)$$

The negative charges from the second conductor, in their own reference system, will see the negative charges in conductor 1 at rest, and the positive charges in movement, so that they will be contracted. There will be an excess of positive charges, which will exert an attractive force F_- with the same value as F_+. The segment of length L from the second conductor will thus feel a total attraction with a value of

$$F=2F_+=4Lv^2\lambda_1 \lambda_2/2\pi\varepsilon_0 r \quad (13)$$

Which corresponds exactly with Ampères Law as written in Eq. (9).

3.4. Electromagnets

The possibility of explaining Ampère´s Law as a direct consequence from the relativistic effect of length contraction opens the gate for the introduction of magnetic effects as a sensible evidence in favor of relativity in earlier courses.

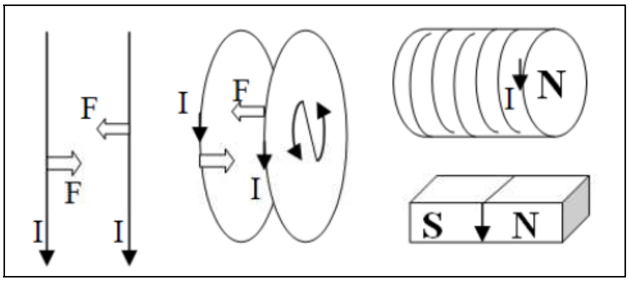

Figure 30. From currents to magnets

To understand the operation of magnets and magnetic motors, we can simply resort to the pair of attracting parallel currents, and turn them around to form circles, which will also attract one another. A collection of such circular currents is a solenoid, and a magnet can be substituted by a solenoid with the appropriate orientation (Fig.30).

Magneric effects can be very impressive and diverse, and they can be presented in any laboratory from the most simple case of a self-made electromagnet to more complex ones, such as electric motors (Fig. 31).

As we have seen, all these phenomena can be explained very intuitively using only the attractive or repulsive forces between parallel currents, which ultimately derive (even quantitatively) from the relativistic effect of length contraction.

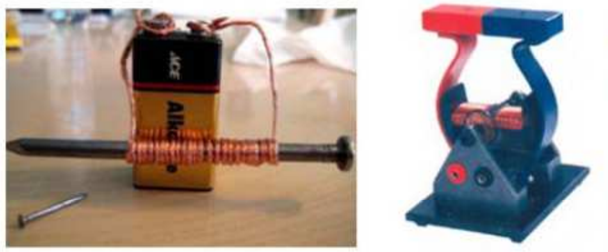

Figure 31. Electromagnet and electric motor

3.5. Plane current K

It is also possible to explain the generation of electric currents by induction resorting only to relativistic effects. We will focus here on the special case of an uniforma and static magnetic field. This can be ideally achieved by an unlimited planar sheet of current, which can be also viewed as an infinite set of parallel line currents lying on a plane. If the intensity of every line current has a value of **I** and there are N lines per unit of width, the plane current is defined as **K = NI**.

The magnetic field created by this plane current has a value

$$B = \mu_0 IN/2 = \mu_0 K/2, \quad (14)$$

which is uniform and does not depend on the distance from the sheet of current.

We can model a plane current in the same way as we did in section 3.2 for linear currents. We consider now that the plane current is made up of two uniform plane sheets of opposite charges moving with opposite velocities. Defining the surface density of charges as σ_0, we will have two cahrge deinsities $\sigma_+ = \sigma_0$ and $\sigma_- = -\sigma_0$ which cancel mutually to produce a total charge density $\sigma=0$.

Positive and negative charges move again with opposite velocities $v_+ = v$ and $v_- = -v$, producing two plane currents $K_+ = v_+ \sigma_+ = v\sigma_0$ and $K_- = v_- \sigma_- = v\sigma_0$. As a result of both opposite movements there will be a net current:

$$K = K_+ + K_- = 2 v\sigma_0 \quad (15)$$

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3.6. Induction (Faraday Law)

If a conductor of length L lies perpendicular to a uniform magnetic field B and is forced to move with a velocity v_2 which is perpendicular both to the field and the conductor, an electromotive force (emf) Є is generated with a value of Є = BvL (Fig. 32).

This is a consequence of Faraday´s Law, and it is also in accordance with Lenz´s Law Є = - dΦ/dt, where Φ is the magnetic flux over a closed surface **S** (Φ=**B·S**) and the minus sign indicates that the induced current is opposed to the change in magnetic flux.

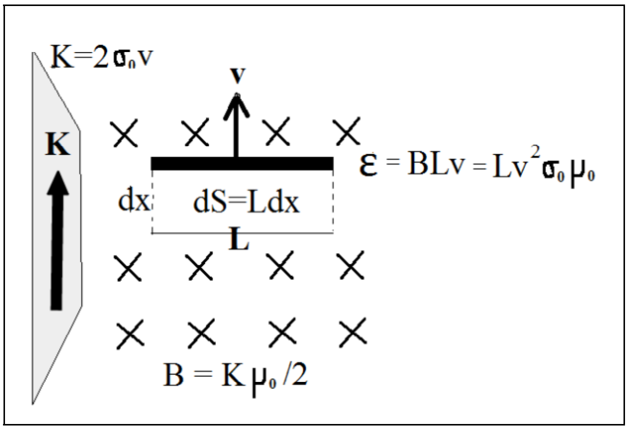

Figure 32. Sheet of current and emf

To show the identity of both expressions, we recall that the magnetic field B is perpendicular to the surface S. As a result, the magnetic flux will be given by Φ = BS, and its time derivative dΦ/dt = SdB/dt + BdS/dt. We consider the case where B is constant, so dΦ/dt = BdS/dt. The conductor sweeps a surface dS = Ldx with a velocity v_2 = dx/dt, and thus dS/dt=Lv, which immediately renders the desired result:

$$d\Phi/dt = BdS/dt = BLv. \quad (16)$$

An electromotive force Є on a conductor of length L can be seen also as the result of a uniform electric field E pointing alongside L. The relation is Є = LE, or E = Є/L. (17)

Equations (14) to (17), together with $\mu_0=1/\varepsilon_0$, give the following expression for the field E:

$$E = Є/L = Bv = \mu_0 Kv /2 = v^2\sigma_0 /\varepsilon_0 \quad (18)$$

We will try to justify equation (18) as a result of the relativistic length contraction when applied to the negative charges (Fig. 33).

We make the assumption that the rod moves with the same speed v as the current charges (we could always assure this by adjusting the surface density accordingly). Then, from the rod´s reference system, the positive charges would be at rest and the negative charges would move with a speed 2v. The same as in (10), if the density of the positive charges in the current sheet is $\sigma_+ = \sigma_0$, the negative charges will have a greater density: $\sigma_- = -\sigma_0\,(1+2v_1^2)$. The total surface charge density has now a nonzero value:

$$\sigma = \sigma_+ + \sigma_- = \sigma_0 - \sigma_0\,(1+2v^2) = -2\sigma_0 v^2 \quad (19)$$

The electric field due to a plane charged with a surface density σ is $E = \sigma /2\varepsilon_0$, which using (19) gives $E = \sigma_0 v^2/\varepsilon_0$ which is identical to the desired expression (18).

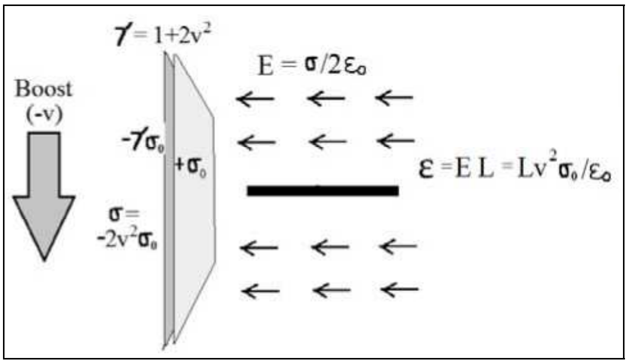

Figure 33. Boost and resulting electric field

3.7. Electricity production

Electromotive forces (or induction currents) are responsible for the production of electricity in almost every generator. In this way, it is possible to state that relativity is the ultimate cause that allows artificial electricity to exist. This can be very easily illustrated at any elemental laboratory using a didactic dynamo or alternator, as well as with a bycicle lamp (Fig. 34).

The relativistic explanation for the induction of electromotive forces, as well as for magnets and motors, has two great advantages from a didactic point of view: It is conceptually simple, and at the same time it makes relativity a plausible and real phenomenon, not of an esotheric nature, but related instead to the very foundations of our highly techified societies. The fact that the same effect of length contraction can be used to explain a broad scope of electromagnetic effects as well as the equivalence between mass and energy, is an additional advantage of this innovative didactic approach. Its visual essence, derived from the fact that relativity is embedded in the geometry of spacetime, adds even more interest to it from a constructivist viewpoint.

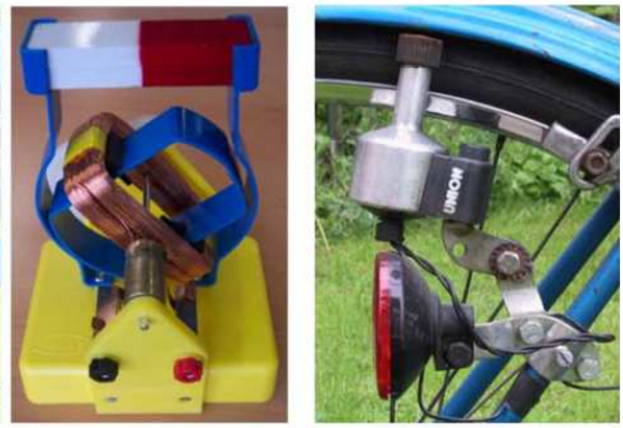

Figure 34. Didactic and bycicle dynamos

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