Probability

LLE – Mathematics and Statistics Skills

Single Event

1. On a fair six-sided die, what is the probability of rolling a four?

   ○ 6 ⊗ $\frac{1}{6}$ ○ $\frac{1}{2}$ ○ $\frac{1}{4}$

   Solution

   There are six total outcomes and only and only 1 option has a four

2. On a standard fair six-sided die, what is the probability of rolling a prime number?

   ○ $\frac{1}{6}$ ○ $\frac{2}{3}$ ⊗ $\frac{1}{2}$ ○ 0

   Solution

   There are six total outcomes and there are three primes (2, 3, and 5).

3. A card set has cards that contain: 2 red squares, 3 blue rectangles, 5 red triangles, 1 blue pentagon, and 1 green pentagon. What is the probability of picking, at random, a red card?

   ○ $\frac{1}{6}$ ○ $\frac{5}{12}$ ○ $\frac{1}{2}$ ⊗ $\frac{7}{12}$

   Solution

   There are 12 total outcomes, of which there are 2 red squares and 5 red triangles.

4. A card set has cards that contain: 2 red squares, 3 blue rectangles, 5 red triangles, 1 blue pentagon, and 1 green pentagon. What is the probability of picking, at random, a card with a shape of less than 5 sides?

⊗ $\frac{5}{6}$ ○ $\frac{1}{6}$ ○ 1 ○ 0

Solution

There are 12 total outcomes, of which there are 10 shapes with less than 5 sides (triangles, squares, and rectangles).

5. A bag contains 20 red counters and 30 blue counters. A counter is selected, recorded, then returned to the bag. If 15 draws are made, what is the expected number of blue counters?

   Answer: _____ **9**

   Solution

   There is a probability of 3/5 for selecting a blue counter each time. Multiply this by 15.

6. A bus is late to my stop with a probability of 0.28. In a 14-day period, how many times would I expect the bus to be on time? Give answer correct to 2 decimal places.

   Answer: _____ **10.08**

   Solution

   There is a probability of 0.72 $(1 - 0.28)$ of being on time. Multiply this by 14.

Contingency or Two-way Tables

1. Use the information given in the table to complete the remaining frequencies of a company's profile of division and level of employment.

Division/Level	Junior	Senior	Manager	TOTAL
Manufacturing	15	8	**7**	30
Marketing	20	**8**	4	32
Sales	**15**	**9**	**4**	28
TOTAL	50	25	15	90

Solution

Use the row totals to calculate the missing manufacturing and marketing frequencies. Then use the column totals to work out the missing sales frequencies.

2. Using the table above; what is the probability that a randomly selected person works in manufacturing?

○ $\frac{1}{5}$ ⊗ $\frac{1}{3}$ ○ $\frac{1}{2}$ ○ 1

Solution

30 out of 90 people work in manufacturing. This is one-third.

3. Using the table above; what is the probability that a randomly selected person is senior and in sales?

⊗ $\frac{1}{10}$ ○ $\frac{14}{45}$ ○ $\frac{5}{18}$ ○ $\frac{4}{45}$

Solution

Senior and sales contains 9 people out of 90. This is one-tenth.

4. Using the table above; what is the probability that a randomly selected person is not a manager?

○ $\frac{3}{18}$ ○ $\frac{5}{18}$ ○ $\frac{1}{2}$ ⊗ $\frac{15}{18}$

Solution

75 people are not managers, 75 out of 90 is 15/18.

5. Using the table above; given that a person is a junior member, what

is the probability that they work in marketing?

○ $\frac{16}{45}$ ○ $\frac{5}{9}$ ○ $\frac{3}{18}$ ⊗ $\frac{2}{5}$

Solution

Only looking at the 50 people who are junior. Out of these 50 people, 20 work in marketing, so 20/50.

6. Using the table above; given a person is a senior member, what is the probability that they work in sales? Give your answer as a decimal number between 0 and 1.

Answer: _____ **0.36**

Solution

Only looking at the 25 people who are senior, 9 of these work in sales, so 9/25.

7. The company 'The Pecs Bar & Gym' has different levels of membership, called Basic, Standard, Premium. There are 522 members on a Basic membership and 828 members on the Standard membership. There is also a swimming pool, and members can either book the pool through reception (known as Normal) or can pay to have a VIP membership (which means they don't need to book). There are 1260 customers who have Normal pool access. The company has a total of 1800 customers.

   (a) Given the information above, and in the table below, complete the two-way table.

Membership/Pool	Basic	Standard	Premium	TOTAL
Normal pool	450	**540**	**270**	**1260**
VIP pool	**72**	**288**	180	**540**
TOTAL	**522**	**828**	**450**	**1800**

Solution

Fill in the information from the blurb first, then use the fact that rows add up to the row totals and columns add up to the column totals.

(b) A member is selected at random, from the information given above. Find the probability, as a decimal between 0 and 1 rounded to 3 decimal places if needed, that this member:

i. has a Premium membership

Answer: _____ **0.25**

Solution

There are 450 premium out of 1800.

ii. has a Standard or Premium membership

Answer: _____ **0.71**

Solution

There are a total of 1278 who have standard or premium (828 and 450) out of 1800

iii. has a Standard membership and Normal Pool booking

Answer: _____ **0.3**

Solution

The number of people who are standard and normal pool is 540.

iv. has a VIP Pool booking given that they have Basic membership

Answer: _____ **0.138**

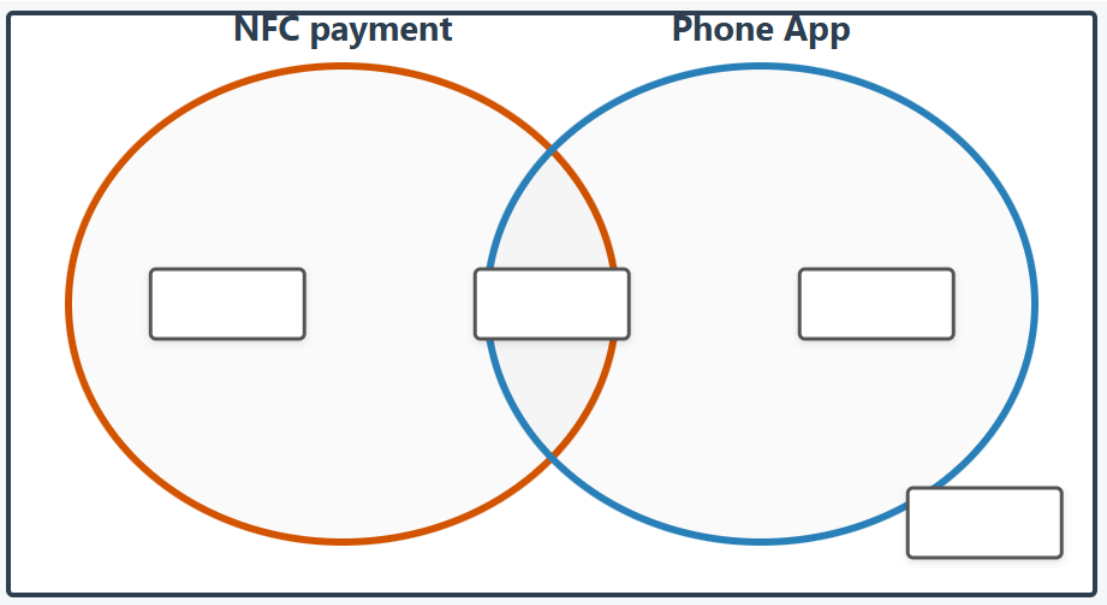
Solution

We are aware that they have basic membership, so only now out of 522. Look at the number of VIP pool and basic memberships there are.

Venn Diagrams

At the bar of the company 'The Pecs Bar & Gym', customers can pay for items via different means. 30% of customers now use NFC on their smartphone to pay at the bar. The company also has an app that gives news, information and offers to customers, 70% of customers have installed this app. 20% of customers use their smartphones to pay and have the company app.

1. Complete the Venn diagram, showing the probabilities as decimal numbers between 0 and 1.



Solution

Fill in the intersection of 0.2 first. The work out the remainder of NFC $(0.3 - 0.2 = 0.1)$ and the remainder of app $(0.7 - 0.2 = 0.5)$. All 4 numbers need to sum to 1.

2. Using the Venn diagram, or otherwise, calculate the following probability (as a decimal number to no more than 3 decimal places) that a customer:

   (a) does not use the NFC function and does not use the app

   Answer: _____ **0.2**

Solution

This is the area outside of the circles.

(b) uses their phone for at least one of the NFC or app means

Answer: _____ **0.8**

Solution

This is the total area inside of the circles.

(c) uses their NFC but does not have the app

Answer: _____ **0.1**

Solution

This is the NFC circle outside of the app circle.

(d) uses their NFC given that they have the app installed

Answer: _____ **0.286**

Solution

We know here that we are in the app circle, which is 0.7, and 0.2 of this 0.7 is inside the NFC circle.

(e) uses the app given that they use NFC to pay

Answer: _____ **0.667**

Solution

It is now out of 0.3 (the NFC circle), and 0.2 of this is the app too.

3. Are the events using NFC and using the phone app independent of one another?

Answer: _____ **no**

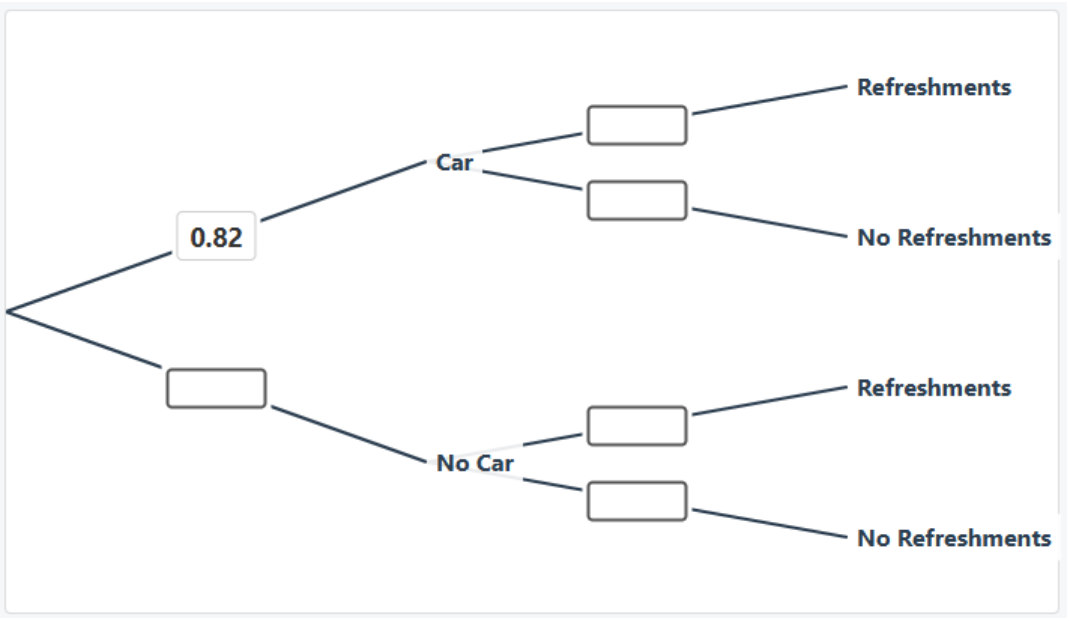
Solution

Probability of NFC is 0.3 and the probability of NFC given they use the app is 0.286. Not equal, so not independent.

Tree Diagrams

A company records and finds that 82% of its members make use of the car park when they visit. Of these 82%, 23% regularly use the café for refreshments during the visit. For those who do not use the car park 64% obtain refreshments.

1. Fill in the missing probabilities as decimals between 0 and 1.



Solution

No Car: $1-0.82$. For the car branch we will have 0.23 and $1-0.23$ and for the no car branch we will start with 0.64.

2. Find the probability that a randomly selected customer drives by car and has refreshments from the café. Give your answer as a decimal correct to 4 decimal places.

Answer: _____ **0.1886**

Solution

We multiply along the branches so need 0.82×0.23

3. Find the probability that a randomly selected customer has refreshments from the café. Give your answer as a decimal correct to 4 decimal

places.

Answer: _____ **0.3038**

Solution

Refreshments can either be car and refreshements or they could be no car and refreshments. We have 0.1886, so need to find the no car and refreshments in a similar way, then add 0.1886 to this answer for no car and refreshments.

4. Given that a customer has had refreshments from the café, what is the probability that they did not use the car?

Answer: _____ **0.3792**

Solution

We have the probability of refreshments from question 3, so this will be our denominator. The numerator is going to be the no car and refreshments, which was also worked out as part of question 3.

Probability Rules

1. Given the following information:

$$P(A) = \frac{2}{5} \qquad P(B) = \frac{1}{4} \qquad P(A \cap B) = \frac{1}{10}$$

(a) What is $P(A \cup B)$?

○ $\frac{13}{20}$ ⊗ $\frac{11}{20}$ ○ $\frac{3}{7}$ ○ $\frac{1}{10}$

Solution

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cup B) = \frac{2}{5} + \frac{1}{4} - \frac{1}{10}$.

Make sure to use a common denominator when adding and subtracting fractions, or use a calculator.

(b) What is $P(A' \cap B')$

○ $\frac{11}{20}$ ○ $\frac{9}{10}$ ⊗ $\frac{9}{20}$ ○ $\frac{1}{2}$

Solution

$P(A' \cap B') = 1 - P(A \cup B)$.
Make sure to use a common denominator when adding and subtracting fractions, or use a calculator.

(c) Are the events A and B independent?

Answer: _____ **yes**

Solution

Work out $P(A) \times P(B)$ and compare to $P(A \cap B)$.
For independence: $P(A \cap B) = P(A)P(B)$

2. There are two events A and B such that:

$$P(M) = \frac{1}{2} \qquad P(N) = \frac{1}{3} \qquad P(M|N) = \frac{1}{4}$$

(a) Calculate $P(M$ AND $N)$

⊗ $\frac{1}{12}$ ○ $\frac{1}{6}$ ○ $\frac{1}{8}$ ○ $\frac{1}{4}$

Solution

Use $P(M \cap N) = P(M|N) \times P(N)$

(b) Calculate $P(M$ OR $N)$

⊗ $\frac{3}{4}$ ○ $\frac{2}{5}$ ○ $\frac{5}{6}$ ○ $\frac{1}{4}$

Solution

Use $P(M \cup N) = P(M) + P(N) - P(M \cap N)$ and previous answer.

(c) You have another event R, such that M and R are independent. It is also given that:

$$P(M \text{ AND } R) = \frac{1}{12} \qquad P(N \text{ or } R) = \frac{1}{2}$$

Are the events N and R mutually exclusive?

Answer: _____ **yes**

Solution

Independent M and R gives

$P(M \cap R) = P(M) \times P(R)$

$\implies \frac{1}{12} = \frac{1}{2} \times P(R)$

$\implies P(R) = \frac{1}{6}$

$P(N \cup R) = P(N) + P(R) - P(N \cap R)$

$\implies \frac{1}{2} = \frac{1}{3} + \frac{1}{6} - P(N \cap R)$

$\implies P(N \cap R) = 0$