Distributions

LLE – Mathematics and Statistics Skills

1 Frequency Table

A bar sells scratch cards, for £3, where the customer can win between £0 and £10, in whole number of £. The winnings record over the past 250 cards sold is given as a frequency table.

1. Complete the relative and cumulative relative frequency columns, to 3 decimal places.

Winnings, £	Frequency	Relative Frequency	Cumulative RF
0	48	0.192	0.192
1	60	0.24	0.432
2	42		
3	31		
4	33		
5	12		
6	8		
7	9		
8	5		
9	1		
10	1		

2. Identify the mode, by finding the payout with the highest frequency.

Answer: _____

3. Use the cumulative relative frequencies to find the median payout (50th percentile).

Answer: _____

4. Find the lower quartile (25th percentile).

Answer: _____

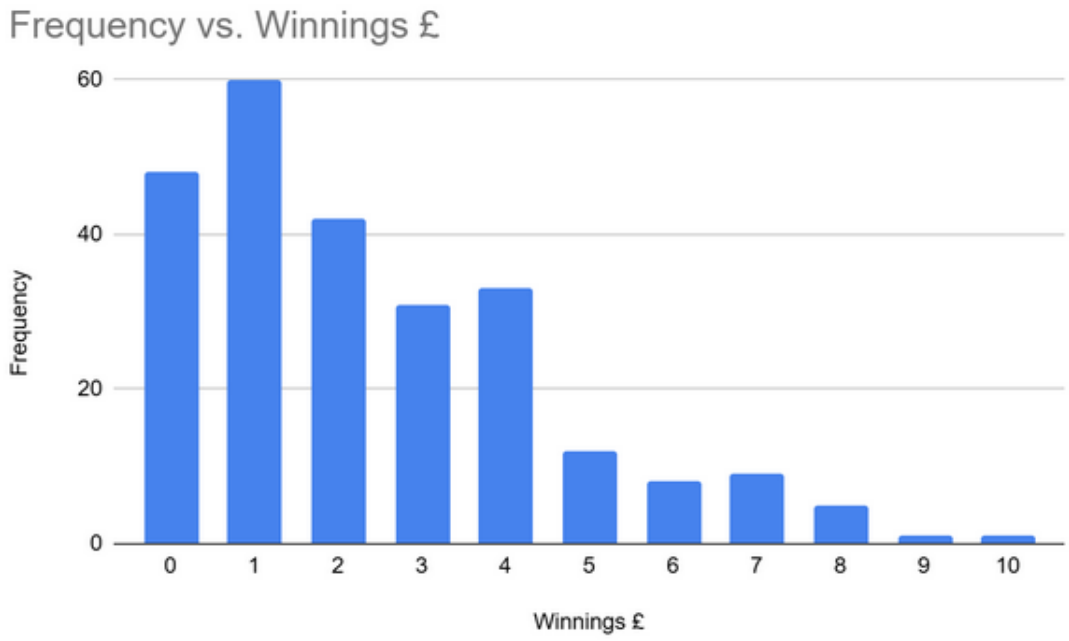
5. Find the upper quartile (75th percentile).

Answer: _____

6. Find the 90th percentile.

Answer: _____

A bar chart of the frequencies of winnings is shown:



7. Describe the skew of the distribution (left-skew, right-skew, or symmetric) based on the shape of the bar chart.

○ Left-skew ○ Right-skew ○ Symmetric

8. Calculate the total revenue on scratchcards (£3 each), total amount paid out in winnings (from table), in order to work out **the profit made**.

Answer: _____

2 Chebyshev's Rule

Chebyshev's Theorem provides an estimate for the minimum proportion of values that lie within a given number of standard deviations from the mean.

For any given distribution, the minimum proportion of observations that lie within k standard deviation of the mean ($k > 1$), is given by:

$$1 - \frac{1}{k^2}$$

1. What is the minimum proportion (giving answer as a percentage) of observations that lie within $\sqrt{2}$ standard deviations of the mean?

Answer: _____

2. What is the maxmimum proportion (giving answer as a percentage) of observations that lie beyond 2 standard deviations of the mean?

Answer: _____

3. A distribution has a mean value of 50 and a standard deviation of 5. What is the minimum proportion of observations (giving answer as a percentage) that are between 40 and 60?

Answer: _____

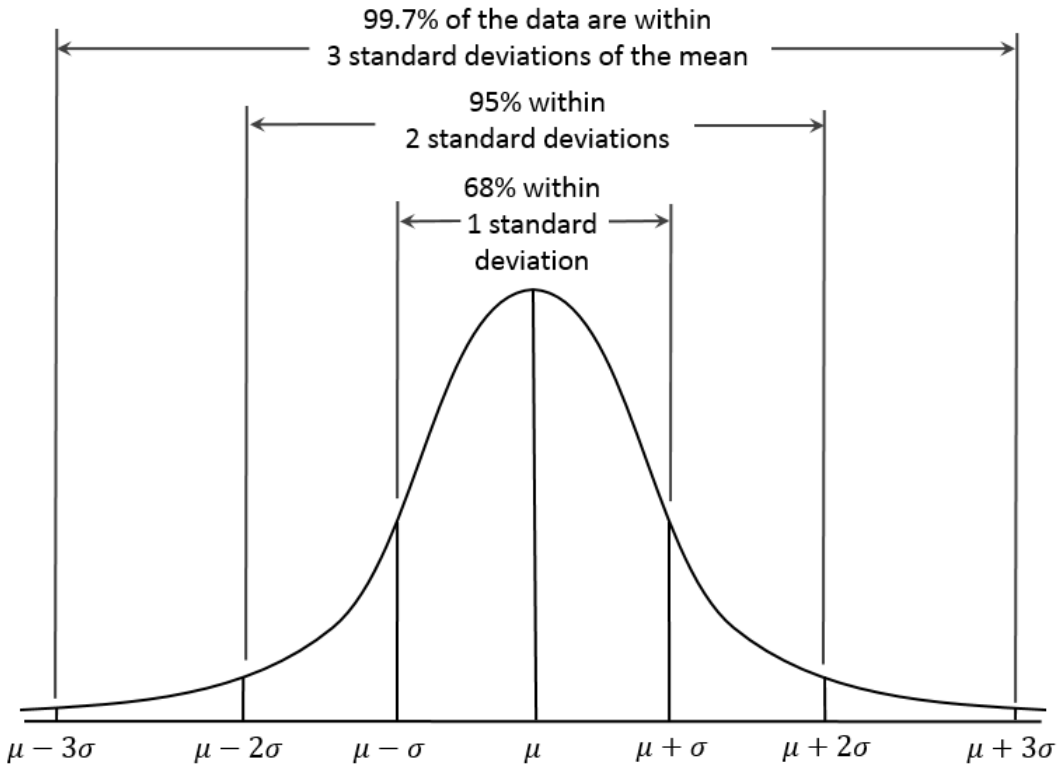
4. A distribution has a mean value of 60 and a standard deviation of 20. What is the minimum proportion of observations that are between 30 and 90?

○ 40 % ○ 44 % ○ 50 % ○ 56 % ○ Not enough information

3 Empirical Rule for Normal Distributions

Distributions that are described as normally distributed have a particular shape and have defined characteristics.

The curve is perfectly symmetrical around its centre, meaning the left side is a mirror image of the right.
About 68% of data falls within 1 standard deviation (±σ) of the mean.
About 95% falls within 2 standard deviations (±2σ).
About 99.7% falls within 3 standard deviations (±3σ).



1. Approximately what proportion of data lie above 2 standard deviations above the mean, given the data are normally distributed?

○ 2.5 % ○ 5 % ○ 47.5 % ○ 95 % ○ 97.5 %

2. Approximately what proportion of data lie above 1 standard deviations below the mean, given the data are normally distributed?

○ 17 % ○ 34 % ○ 68 % ○ 83 % ○ Not possible to tell

3. An office coffee machine distributes liquids, in a normal distribution, that have a mean volume of 250 ml and a standard deviation of 6 ml. What percentage of coffees will be between 238 and 262 ml?

Answer: _____

4. An office coffee machine distributes liquids, in a normal distribution, that have a mean volume of 250 ml and a standard deviation of 6 ml. What percentage of coffees will be over 268 ml?

Answer: _____

5. The bulb on a photocopier has a mean life of 16000 hours and a variance of 250000 square hours, and the life is normally distributed. What percentage of bulbs last longer than 15000 hours?

Answer: _____