Rearranging Equations - Solutions

LLE – Mathematics and Statistics
ECO-5007A

Skills - Rearranging Equations

In each of the following expressions, make x the subject of the equation.

1. $x + y = 12$

   Subtract y from both sides:
   $x = 12 - y$

2. $5x - 4 = y$

   Add 4 to both sides:
   $5x = y + 4$
   Divide by 5:
   $x = \frac{y+4}{5}$

3. $ax - 3by = 9$

   Add $3by$ to both sides:
   $ax = 9 + 3by$
   Divide by a:
   $x = \frac{9+3by}{a}$

4. $y = mx + c$

   Subtract c from both sides:

$y - c = mx$

Divide by m:

$x = \frac{y-c}{m}$

5. $2 + 5xy = 6$

   Subtract 2 from both sides:

   $5xy = 6 - 2$

   $5xy = 4$

   Divide by $5y$:

   $x = \frac{4}{5y}$

6. $9x + 6y = 4x + 1$

   Subtract $4x$ from both sides:

   $9x - 4x + 6y = 1$

   $5x + 6y = 1$

   Subtract $6y$ from both sides:

   $5x = 1 - 6y$

   Divide by 5:

   $x = \frac{1-6y}{5}$

7. $ax + by = 4x + 1$

   Subtract $4x$ from both sides:

   $ax - 4x + by = 1$

   Subtract by from both sides:

   $ax - 4x = 1 - by$

   Factor out x:

   $x(a - 4) = 1 - by$

   Divide by $(a - 4)$:

   $x = \frac{1-by}{a-4}$

8. $ax - by = bx - 8$

   Subtract bx from both sides:

   $ax - bx - by = -8$

   Add by to both sides:

   $ax - bx = by - 8$

   Factor out x:

   $x(a - b) = by - 8$

   Divide by $(a - b)$:

   $x = \frac{by-8}{a-b}$

9. $\frac{2ty + mx}{ty - mx} = 1$

   Multiply both sides by $(ty - mx)$:

   $2ty + mx = 1 \times (ty - mx)$

   $2ty + mx = ty - mx$

   Add mx to both sides:

   $2ty + 2mx = ty$

   Subtract $2ty$ from both sides:

   $2mx = -ty$

   Divide by $2m$:

   $x = \frac{-ty}{2m}$

10. $\frac{ty + mx}{ty - 3mx} = k$

    Multiply both sides by $(ty - 3mx)$:

    $ty + mx = k(ty - 3mx)$

    $ty + mx = kty - 3kmx$

    Subtract mx from both sides:

    $ty = kty - 3kmx - mx$

    Subtract kty from both sides:

    $ty - kty = -3kmx - mx$

    Factor out ty on the left and $-x$ on the right:

$ty(1-k) = -x(3km+m)$

$ty(1-k) = -xm(3k+1)$

Divide by $-m(3k+1)$:

$x = \frac{ty(1-k)}{-m(3k+1)}$

$x = \frac{ty(k-1)}{m(3k+1)}$ (multiplying top and bottom by -1 for cleaner form)

11. $9x^2 = 4y$

Divide by 9:

$x^2 = \frac{4y}{9}$

Take the square root of both sides:

$x = \pm\sqrt{\frac{4y}{9}}$

$x = \pm\frac{\sqrt{4}\sqrt{y}}{\sqrt{9}}$

$x = \pm\frac{2\sqrt{y}}{3}$

12. $x^3 - 5m = t$

Add $5m$ to both sides:

$x^3 = t + 5m$

Take the cube root of both sides:

$x = \sqrt[3]{t+5m}$ or $x = (t+5m)^{\frac{1}{3}}$

13. $x^2 - 2x = 15$

Subtract 15 from both sides to get quadratic form $ax^2 + bx + c = 0$:

$x^2 - 2x - 15 = 0$

Factor the quadratic:

$(x-5)(x+3) = 0$

Set each factor to zero:

$x - 5 = 0 \implies x = 5$

$x + 3 = 0 \implies x = -3$

So, $x = 5$ or $x = -3$.

14. $x^2 = 2 - qx$

    Rearrange to $ax^2 + bx + c = 0$:
    $x^2 + qx - 2 = 0$
    Using the quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ with $a = 1, b = q, c = -2$:
    $x = \frac{-q \pm \sqrt{q^2 - 4(1)(-2)}}{2(1)}$
    $x = \frac{-q \pm \sqrt{q^2 + 8}}{2}$

15. $x^2 - 8 = kx$

    Rearrange to $ax^2 + bx + c = 0$:
    $x^2 - kx - 8 = 0$
    Using the quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ with $a = 1, b = -k, c = -8$:
    $x = \frac{-(-k) \pm \sqrt{(-k)^2 - 4(1)(-8)}}{2(1)}$
    $x = \frac{k \pm \sqrt{k^2 + 32}}{2}$

Rearranging with Indices

Practice Problems:

Rearrange the following to make x the subject:

1. $x^{\frac{3}{4}} = y^{\frac{1}{2}}$

    Raise both sides to the power of $\frac{4}{3}$:
    $(x^{\frac{3}{4}})^{\frac{4}{3}} = (y^{\frac{1}{2}})^{\frac{4}{3}}$
    $x = y^{\frac{1}{2} \times \frac{4}{3}}$
    $x = y^{\frac{4}{6}}$
    $x = y^{\frac{2}{3}}$

2. $x^{\frac{3}{4}} = y^{\frac{1}{2}} - 2$

   Raise both sides to the power of $\frac{4}{3}$:
   $(x^{\frac{3}{4}})^{\frac{4}{3}} = (y^{\frac{1}{2}} - 2)^{\frac{4}{3}}$
   $x = (y^{\frac{1}{2}} - 2)^{\frac{4}{3}}$

3. $5x^8 - y = 0$

   Add y to both sides:
   $5x^8 = y$
   Divide by 5:
   $x^8 = \frac{y}{5}$
   Take the 8^{th} root of both sides:
   $x = \pm \left(\frac{y}{5}\right)^{\frac{1}{8}}$

4. $x^{\frac{3}{4}} y = 8my$

   Assuming $y \neq 0$, divide both sides by y:
   $x^{\frac{3}{4}} = 8m$
   Raise both sides to the power of $\frac{4}{3}$:
   $(x^{\frac{3}{4}})^{\frac{4}{3}} = (8m)^{\frac{4}{3}}$
   $x = 8^{\frac{4}{3}} m^{\frac{4}{3}}$
   $x = (\sqrt[3]{8})^4 m^{\frac{4}{3}}$
   $x = 2^4 m^{\frac{4}{3}}$
   $x = 16m^{\frac{4}{3}}$

5. $2x^{2.5} = 64$

   $2.5 = \frac{5}{2}$
   $2x^{\frac{5}{2}} = 64$
   Divide by 2:
   $x^{\frac{5}{2}} = 32$
   Raise both sides to the power of $\frac{2}{5}$:

$(x^{\frac{5}{2}})^{\frac{2}{5}} = 32^{\frac{2}{5}}$
$x = (\sqrt[5]{32})^2$
$x = 2^2$
$x = 4$

6. $x^2y^3x^5y^2 = a$

Group like terms and use $x^a x^b = x^{a+b}$:
$x^{2+5}y^{3+2} = a$
$x^7y^5 = a$
Divide by y^5:
$x^7 = \frac{a}{y^5}$
Take the 7^{th} root of both sides:
$x = \left(\frac{a}{y^5}\right)^{\frac{1}{7}}$ or $x = \frac{a^{\frac{1}{7}}}{y^{\frac{5}{7}}}$

7. $0.5x^{-0.5}y^{-0.5} \times 0.5x^{-0.5}y^{0.5} = 4$

Multiply the numerical coefficients and group variables:
$(0.5 \times 0.5) \times (x^{-0.5}x^{-0.5}) \times (y^{-0.5}y^{0.5}) = 4$
$0.25 \times x^{(-0.5)+(-0.5)} \times y^{(-0.5)+(0.5)} = 4$
$0.25x^{-1}y^0 = 4$
Since $y^0 = 1$ (for $y \neq 0$):
$0.25x^{-1} = 4$
$0.25 \times \frac{1}{x} = 4$
$\frac{0.25}{x} = 4$
Multiply by x:
$0.25 = 4x$
Divide by 4:
$x = \frac{0.25}{4}$
$x = \frac{1/4}{4}$
$x = \frac{1}{16}$

8. $0.8y^{-0.2}x^{0.2} \times 0.2y^{0.8}x^{-0.8} = 10$

Group numerical coefficients and variables:
$(0.8 \times 0.2) \times (y^{-0.2}y^{0.8}) \times (x^{0.2}x^{-0.8}) = 10$
$0.16 \times y^{(-0.2)+(0.8)} \times x^{(0.2)+(-0.8)} = 10$
$0.16y^{0.6}x^{-0.6} = 10$
$0.16y^{0.6}\frac{1}{x^{0.6}} = 10$
$\frac{0.16y^{0.6}}{x^{0.6}} = 10$
Multiply by $x^{0.6}$:
$0.16y^{0.6} = 10x^{0.6}$
Divide by 10:
$\frac{0.16y^{0.6}}{10} = x^{0.6}$
$0.016y^{0.6} = x^{0.6}$
Raise both sides to the power of $\frac{1}{0.6}$ (or $\frac{5}{3}$):
$(0.016y^{0.6})^{\frac{1}{0.6}} = (x^{0.6})^{\frac{1}{0.6}}$
$x = (0.016)^{\frac{1}{0.6}}(y^{0.6})^{\frac{1}{0.6}}$
$x = (0.016)^{\frac{5}{3}}y$
$x \approx 0.0883y$

9. $\frac{\alpha y^{\alpha-1}x^{\beta}}{\beta y^{\alpha}x^{\beta-1}} = k$

Separate terms and use $\frac{x^a}{x^b} = x^{a-b}$:
$\frac{\alpha}{\beta} \times \frac{y^{\alpha-1}}{y^{\alpha}} \times \frac{x^{\beta}}{x^{\beta-1}} = k$
$\frac{\alpha}{\beta} \times y^{(\alpha-1)-\alpha} \times x^{\beta-(\beta-1)} = k$
$\frac{\alpha}{\beta} \times y^{-1} \times x^{1} = k$
$\frac{\alpha}{\beta} \times \frac{1}{y} \times x = k$
$\frac{\alpha x}{\beta y} = k$
Multiply by βy:
$\alpha x = k\beta y$
Divide by α:
$x = \frac{k\beta y}{\alpha}$