Workshop: Differentiation Chain Rule

LLE – Mathematics and Statistics Skills

Power-type Compositions

These are expressions that look like:

$$y = (3x^2 + 5)^4$$

The function $3x^2 + 5$ is inside a 'to the power 4' function.

Define a 'dummy' variable (say u) to represent the 'inside' function:

$$u = 3x^2 + 5$$

The chain rule can be applied:

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

So:

$$\frac{dy}{dx} = 4(3x^2 + 5)^3 \cdot 6 = 24(3x^2 + 5)^3$$

For the example above:

$$\text{Inner: } u = 3x^2 + 5 \quad \Rightarrow \quad \frac{du}{dx} = 6x$$

$$\text{Outer: } y = u^4 \quad \Rightarrow \quad \frac{dy}{du} = 4u^3$$

$$\Rightarrow \frac{dy}{dx} = 4u^3 \cdot 6x = 24x(3x^2 + 5)^3$$

Find the derivative of:

1. $y = (4x^2 + 3)^5$
2. $y = (7x^3 + 2)^4$
3. $y = (5x^2 + x)^3$
4. $y = (1 - 2x)^6$
5. $y = (2x - 3)^7$
6. $y = (x^2 + 5x - 3)^8$
7. $y = (4x^3 - x^2 + 2)^5$

Exponentials with Composed Arguments

These are expressions that look like $y = e^{2x^2+1}$. Here the $2x^2 + 1$ is inside the exponential function.

Use the dummy variable u, so that:

$$u = 2x^2 + 1$$

The chain rule can be applied:

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

So:

$$\frac{dy}{dx} = e^{2x^2+1} \cdot 4x = 4xe^{2x^2+1}$$

For the example above:

$$\text{Inner: } u = 2x^2 + 1 \quad \Rightarrow \quad \frac{du}{dx} = 4x$$

$$\text{Outer: } y = e^u \quad \Rightarrow \quad \frac{dy}{du} = e^u$$

$$\Rightarrow \frac{dy}{dx} = e^{2x^2+1} \cdot 4x$$

Find the derivative of:

8. $y = e^{3x^2+4}$
9. $y = e^{x^3+1}$
10. $y = e^{5x^4+2x}$
11. $y = e^{7x-3}$
12. $y = e^{x^2-4x+1}$
13. $y = e^{x^5-x^2+3}$
14. $y = e^{2x^3-x+4}$

Mixed Practice

Use the examples above and your understanding of the chain rule to differentiate:

15. $y = (5x^2 + 4)^3$
16. $y = e^{x^2+4x+2}$
17. $y = (2x^3 - x^2 + x)^6$
18. $y = e^{4x^3-x^2+x}$
19. $y = (3x^2 + 1)^5$
20. $y = e^{x^4+5x}$