Differentiation Rules

LLE – Mathematics and Statistics Skills

Power Rule

Given the function $y = ax^n$, the derivative is:

$$\frac{dy}{dx} = nax^{n-1}$$

Use this rule to differentiate:

1. $y = 5x^4$
2. $y = 7x^2$
3. $y = \frac{2}{3}x^6$

These indices facts will be useful for the next questions:

$$x^{-n} = \frac{1}{x^n}$$

$$x^{\frac{1}{n}} = \sqrt[n]{x}$$

$$x^a \times x^b = x^{a+b}$$

$$\frac{x^a}{x^b} = x^{a-b}$$

$$(x^a)^b = x^{ab}$$

Use the power rule to differentiate:

4. $y = \frac{3x^3}{x^5}$
5. $y = \frac{10}{x^2}$

6. $y = 2x^{\frac{2}{3}}$
7. $y = 4\sqrt{x}$
8. $y = \frac{2}{3x^2}$
9. $y = \frac{10}{x}$

The power rule can be used to show:

$$y = ax, \quad \frac{dy}{dx} = a$$

$$y = a, \quad \frac{dy}{dx} = 0$$

Put this altogether and differentiate:

10. $y = 5x^2 + 4x + 3$
11. $y = x^4 - 2x^{-5} + \frac{1}{x}$
12. $y = 1 + 2\sqrt{x} + \frac{2}{5x} + \frac{3}{4x^2} + \frac{5}{2x^3}$

Exponentials

Given the function $y = ae^x$, the derivative is:

$$\frac{dy}{dx} = ae^x$$

Not many questions here, differentiate:

1. $y = 5e^x$

Logarithms

Given the function $y = a \ln x$, the derivative is:

$$\frac{dy}{dx} = \frac{a}{x}$$

Differentiate:

1. $y = 5 \ln x$
2. $y = \frac{\ln x}{2} - 3e^x$

The following log laws will be useful:

$$\ln(AB) = \ln A + \ln B$$

$$\ln\left(\frac{A}{B}\right) = \ln A - \ln B$$

$$\ln(A^n) = n \ln A$$

Differentiate:

3. $y = \ln 4x$
4. $y = \ln 0.443x$
5. $y = \ln \frac{2}{x}$
6. $y = \ln x^{10}$
7. $y = 6 \ln \frac{1}{2}x$

Mixed Questions

For each question find the requested derivative:

1. $y = 5x^3 + x^2 - 4x + 1 + 3e^x + \ln 2x \quad \frac{dy}{dx} =$
2. $y = 2x + 5\sqrt{x} - \frac{4}{x} + \frac{3}{x^2} \quad \frac{dy}{dx} =$
3. $y = 5t^2 - 2t^{\frac{3}{4}} + 6 \ln 0.1t \quad \frac{dy}{dt} =$
4. $P = 6e^n - 4 + \frac{5}{2}n - \frac{6}{\sqrt{n}} \quad \frac{dP}{dn} =$
5. $G = 5s^2(3s - 4) \quad \frac{dG}{ds} =$

By substituting into the function and into the derived function, find the value of the function and the value of the gradient at the given x values:

6. $y = 5x^2 - 12x + 2, \quad x = 2$
7. $y = 4x^3 + x + 10, \quad x = -1$
8. $y = x^2 + 5x - 1, \quad x = -2.5$
9. $y = 4e^x - 10\sqrt{x}, \quad x = 2$
10. $y = 5 - \frac{5}{x} + \frac{5}{x^2}, \quad x = 1$