Solutions - Returns to Scale

LLE Mathematics and Statistics

For each question try doubling input $Q(2K, 2L)$ and see the effect on output.

1. $Q(2K, 2L) = 6K + 4L = 2(3K + 2L) = 2Q(K, L)$
   Doubling input doubles output, so constant returns to scale.

2. $Q(2K, 2L) = 0.2(2K)(2L) = 0.2 \times 2 \times 2KL = 0.8KL = 4Q(K, L)$
   Doubling input more than doubles output, so increasing returns to scale.

3. $Q(2K, 2L) = \frac{(2K)(2L)}{2} = 2KL = 4Q(K, L)$ Doubling input more than doubles output, so increasing returns to scale.

4. $Q(2K, 2L) = (2K)^2(2L)^2 = (4K^2)(4L^2) = 16K^2L^2 = 16Q(K, L)$
   Doubling input more than doubles output, so increasing returns to scale.

5. $Q(2K, 2L) = 2^{0.5}K^{0.5}2^{0.5}L^{0.5} = 2^{0.5}2^{0.5}K^{0.5}L^{0.5} = 2Q(K, L)$
   Doubling input doubles output, so constant returns to scale.

6. $Q(2K, 2L) = 2^{0.9}K^{0.9}2^{0.1}L^{0.1} = 2Q(K, L)$
   Doubling input doubles output, so constant returns to scale.

7. $Q(2K, 2L) = 2^{0.2}K^{0.2}2^{0.6}K^{0.6} = 2^{0.8}Q(K, L) \approx 1.74Q(K, L)$
   Doubling input leads to a less than doubling of output, therefore decreasing returns to scale.

8. $Q(2K, 2L) = 2^{0.8}2^{0.3}K^{0.8}L^{0.3} = 2^{1.1}Q(K, L) \approx 2.14Q(K, L)$
   Doubling input more than doubles output, so increasing returns to scale.

9. $Q(2K, 2L) = 2^{\alpha+\beta} Q(K, L)$

(a) $\alpha + \beta = 1 \implies Q(2K, 2L) = 2Q(K, L)$ constant

(b) $\alpha + \beta < 1 \implies 2^{\alpha+\beta} < 2 \implies Q(2K, 2L) < 2Q(K, L)$ decreasing

(c) $\alpha + \beta > 1 \implies 2^{\alpha+\beta} > 2 \implies Q(2K, 2L) > 2Q(K, L)$ increasing