Workshop: Differentiation Product and Quotient Rules – Solutions

LLE - Mathematics and Statistics

Product Rule

1. $y = 5xe^x$

   $u = 5x$, $v = e^x$
   $\frac{du}{dx} = 5$, $\frac{dv}{dx} = e^x$

   $$\frac{dy}{dx} = 5e^x + 5xe^x = 5e^x(1 + x)$$

2. $y = x^3 \ln x$

   $u = x^3$, $v = \ln x$
   $\frac{du}{dx} = 3x^2$, $\frac{dv}{dx} = \frac{1}{x}$

   $$\frac{dy}{dx} = 3x^2 \ln x + x^2$$

3. $y = 4e^x \ln 4x$

   $u = 4e^x$, $v = \ln 4x$
   $\frac{du}{dx} = 4e^x$, $\frac{dv}{dx} = \frac{1}{x}$

   $$\frac{dy}{dx} = 4e^x \ln 4x + \frac{4e^x}{x}$$

4. $y = (6x^2 + 4x + 1) \ln x^2$

   $u = 6x^2 + 4x + 1$, $v = \ln x^2 = 2 \ln x$
   $\frac{du}{dx} = 12x + 4$, $\frac{dv}{dx} = \frac{2}{x}$

$$\frac{dy}{dx} = (12x+4)(2\ln x) + (6x^2+4x+1)\cdot\frac{2}{x}$$

5. $y = (5x - \frac{3}{x^2})e^x$

   $u = 5x - \frac{3}{x^2}$, $v = e^x$
   $\frac{du}{dx} = 5 + \frac{6}{x^3}$, $\frac{dv}{dx} = e^x$

$$\begin{aligned}\frac{dy}{dx} &= \left(5+\frac{6}{x^3}\right)e^x + \left(5x-\frac{3}{x^2}\right)e^x \\ &= e^x\left(10x+\frac{6}{x^3}-\frac{3}{x^2}\right)\end{aligned}$$

Quotient Rule

6. $y = \frac{5x+2}{x-4}$

   $u = 5x+2$, $v = x-4$
   $\frac{du}{dx} = 5$, $\frac{dv}{dx} = 1$

$$\frac{dy}{dx} = \frac{(x-4)(5)-(5x+2)(1)}{(x-4)^2} = \frac{5x-20-5x-2}{(x-4)^2} = \frac{-22}{(x-4)^2}$$

7. $y = \frac{e^x}{x^2+2x}$

   $u = e^x$, $v = x^2+2x$
   $\frac{du}{dx} = e^x$, $\frac{dv}{dx} = 2x+2$

$$\frac{dy}{dx} = \frac{(x^2+2x)e^x - e^x(2x+2)}{(x^2+2x)^2} = \frac{e^x(x^2+2x-2x-2)}{(x^2+2x)^2} = \frac{e^x(x^2-2)}{(x^2+2x)^2}$$

8. $y = \frac{\ln 4x}{x^2+5}$

   $u = \ln 4x = \ln 4 + \ln x$, $v = x^2+5$
   $\frac{du}{dx} = \frac{1}{x}$, $\frac{dv}{dx} = 2x$

$$\frac{dy}{dx} = \frac{(x^2+5)\cdot\frac{1}{x} - \ln 4x\cdot 2x}{(x^2+5)^2}$$

9. $y = \frac{\sqrt{x}}{e^x}$

$u = x^{1/2}$, $v = e^x$
$\frac{du}{dx} = \frac{1}{2\sqrt{x}}$, $\frac{dv}{dx} = e^x$

$$\frac{dy}{dx} = \frac{e^x \cdot \frac{1}{2\sqrt{x}} - x^{1/2}e^x}{(e^x)^2} = \frac{\frac{1}{2\sqrt{x}} - x^{1/2}}{e^x}$$

Mixed

10. $y = e^{4x}(5x + 2)$

$u = e^{4x}$, $v = 5x + 2$
$\frac{du}{dx} = 4e^{4x}$, $\frac{dv}{dx} = 5$

$$\frac{dy}{dx} = 4e^{4x}(5x + 2) + 5e^{4x} = e^{4x}(20x + 8 + 5) = e^{4x}(20x + 13)$$

11. $y = (5x - 1)^4 \ln x$

$u = (5x - 1)^4$, $v = \ln x$
$\frac{du}{dx} = 20(5x - 1)^3$, $\frac{dv}{dx} = \frac{1}{x}$

$$\frac{dy}{dx} = 20(5x - 1)^3 \ln x + \frac{(5x - 1)^4}{x}$$

12. $y = \frac{2x+5}{e^{9x}}$

$u = 2x + 5$, $v = e^{9x}$
$\frac{du}{dx} = 2$, $\frac{dv}{dx} = 9e^{9x}$

$$\frac{dy}{dx} = \frac{e^{9x}(2) - (2x + 5)(9e^{9x})}{e^{18x}} = \frac{2e^{9x} - 18xe^{9x} - 45e^{9x}}{e^{18x}}$$
$$= \frac{e^{9x}(2 - 18x - 45)}{e^{18x}} = e^{-9x}(2 - 18x - 45)$$

13. $y = (x^2 + 3x + 1)e^x \ln x$

First product: $u = (x^2 + 3x + 1)$, $v = e^x \ln x$

Differentiate second product: $a = e^x$, $b = \ln x$

$\frac{da}{dx} = e^x$, $\frac{db}{dx} = \frac{1}{x}$

$\frac{dv}{dx} = e^x \ln x + \frac{e^x}{x}$

$\frac{dy}{dx} = (2x+3)e^x \ln x + (x^2+3x+1)(e^x \ln x + \frac{e^x}{x})$

14. $y = \frac{5xe^x}{x+2}$

$u = 5xe^x$, $v = x+2$

$\frac{du}{dx} = 5e^x + 5xe^x$, $\frac{dv}{dx} = 1$

$$\frac{dy}{dx} = \frac{(x+2)(5e^x + 5xe^x) - 5xe^x}{(x+2)^2}$$

15. $y = \frac{6x-7}{x^2+1} - 5x \ln(2x-5)$

First term: Quotient rule with $u = 6x-7$, $v = x^2+1$

$\frac{du}{dx} = 6$, $\frac{dv}{dx} = 2x$

$$\frac{dy_1}{dx} = \frac{(x^2+1)(6) - (6x-7)(2x)}{(x^2+1)^2}$$

Second term: Product rule

$u = 5x$, $v = \ln(2x-5)$

$\frac{du}{dx} = 5$, $\frac{dv}{dx} = \frac{2}{2x-5}$

$$\frac{dy_2}{dx} = 5 \ln(2x-5) + \frac{10x}{2x-5}$$

Final derivative:

$$\frac{dy}{dx} = \frac{6x^2 + 6 - 12x^2 + 14x}{(x^2+1)^2} - \left(5 \ln(2x-5) + \frac{10x}{2x-5}\right)$$