Chain Rule – Solutions

UEA LEARNING ENHANCEMENT TEAM

MATHEMATICS & STATISTICS

Power-type Compositions

Find the derivative of:

1. $y = (4x^2 + 3)^5$

   Let $u = 4x^2 + 3$, then $\frac{du}{dx} = 8x$

   $y = u^5 \Rightarrow \frac{dy}{du} = 5u^4$

   $$\frac{dy}{dx} = 5(4x^2 + 3)^4 \cdot 8x = 40x(4x^2 + 3)^4$$

2. $y = (7x^3 + 2)^4$

   Let $u = 7x^3 + 2$, then $\frac{du}{dx} = 21x^2$

   $y = u^4 \Rightarrow \frac{dy}{du} = 4u^3$

   $$\frac{dy}{dx} = 4(7x^3 + 2)^3 \cdot 21x^2 = 84x^2(7x^3 + 2)^3$$

3. $y = (5x^2 + x)^3$

   Let $u = 5x^2 + x$, then $\frac{du}{dx} = 10x + 1$

   $y = u^3 \Rightarrow \frac{dy}{du} = 3u^2$

   $$\frac{dy}{dx} = 3(5x^2 + x)^2(10x + 1)$$

4. $y = (1 - 2x)^6$

   Let $u = 1 - 2x$, then $\frac{du}{dx} = -2$

$$y = u^6 \Rightarrow \frac{dy}{du} = 6u^5$$

$$\frac{dy}{dx} = 6(1 - 2x)^5 \cdot (-2) = -12(1 - 2x)^5$$

5. $y = (2x - 3)^7$

   Let $u = 2x - 3$, then $\frac{du}{dx} = 2$

   $y = u^7 \Rightarrow \frac{dy}{du} = 7u^6$

   $$\frac{dy}{dx} = 7(2x - 3)^6 \cdot 2 = 14(2x - 3)^6$$

6. $y = (x^2 + 5x - 3)^8$

   Let $u = x^2 + 5x - 3$, then $\frac{du}{dx} = 2x + 5$

   $y = u^8 \Rightarrow \frac{dy}{du} = 8u^7$

   $$\frac{dy}{dx} = 8(x^2 + 5x - 3)^7(2x + 5)$$

7. $y = (4x^3 - x^2 + 2)^5$

   Let $u = 4x^3 - x^2 + 2$, then $\frac{du}{dx} = 12x^2 - 2x$

   $y = u^5 \Rightarrow \frac{dy}{du} = 5u^4$

   $$\frac{dy}{dx} = 5(4x^3 - x^2 + 2)^4(12x^2 - 2x)$$

Exponentials with Composed Arguments

Find the derivative of:

8. $y = e^{3x^2 + 4}$

   Let $u = 3x^2 + 4$, then $\frac{du}{dx} = 6x$

   $y = e^u \Rightarrow \frac{dy}{du} = e^u$

$$\frac{dy}{dx} = e^{3x^2+4} \cdot 6x = 6xe^{3x^2+4}$$

9. $y = e^{x^3+1}$

   Let $u = x^3 + 1$, then $\frac{du}{dx} = 3x^2$

   $$\frac{dy}{dx} = 3x^2 e^{x^3+1}$$

10. $y = e^{5x^4+2x}$

    Let $u = 5x^4 + 2x$, then $\frac{du}{dx} = 20x^3 + 2$

    $$\frac{dy}{dx} = (20x^3 + 2)e^{5x^4+2x}$$

11. $y = e^{7x-3}$

    Let $u = 7x - 3$, then $\frac{du}{dx} = 7$

    $$\frac{dy}{dx} = 7e^{7x-3}$$

12. $y = e^{x^2-4x+1}$

    Let $u = x^2 - 4x + 1$, then $\frac{du}{dx} = 2x - 4$

    $$\frac{dy}{dx} = (2x - 4)e^{x^2-4x+1}$$

13. $y = e^{x^5-x^2+3}$

    Let $u = x^5 - x^2 + 3$, then $\frac{du}{dx} = 5x^4 - 2x$

    $$\frac{dy}{dx} = (5x^4 - 2x)e^{x^5-x^2+3}$$

14. $y = e^{2x^3-x+4}$

    Let $u = 2x^3 - x + 4$, then $\frac{du}{dx} = 6x^2 - 1$

$$\frac{dy}{dx} = (6x^2 - 1)e^{2x^3 - x + 4}$$

Mixed Practice

15. $y = (5x^2 + 4)^3$

    Let $u = 5x^2 + 4$, then $\frac{du}{dx} = 10x$

    $$\frac{dy}{dx} = 3(5x^2 + 4)^2 \cdot 10x = 30x(5x^2 + 4)^2$$

16. $y = e^{x^2 + 4x + 2}$

    Let $u = x^2 + 4x + 2$, then $\frac{du}{dx} = 2x + 4$

    $$\frac{dy}{dx} = (2x + 4)e^{x^2 + 4x + 2}$$

17. $y = (2x^3 - x^2 + x)^6$

    Let $u = 2x^3 - x^2 + x$, then $\frac{du}{dx} = 6x^2 - 2x + 1$

    $$\frac{dy}{dx} = 6(2x^3 - x^2 + x)^5(6x^2 - 2x + 1)$$

18. $y = e^{4x^3 - x^2 + x}$

    Let $u = 4x^3 - x^2 + x$, then $\frac{du}{dx} = 12x^2 - 2x + 1$

    $$\frac{dy}{dx} = (12x^2 - 2x + 1)e^{4x^3 - x^2 + x}$$

19. $y = (3x^2 + 1)^5$

    Let $u = 3x^2 + 1$, then $\frac{du}{dx} = 6x$

    $$\frac{dy}{dx} = 5(3x^2 + 1)^4 \cdot 6x = 30x(3x^2 + 1)^4$$

20. $y = e^{x^4+5x}$

Let $u = x^4 + 5x$, then $\frac{du}{dx} = 4x^3 + 5$

$$\frac{dy}{dx} = (4x^3 + 5)e^{x^4+5x}$$