Solutions - Optimal Production

LLE Mathematics and Statistics

1. If q_A are produced on machine A, the cost of production is

$$TC_A = 10 + 20q_A$$

If q_B are produced on machine B, the cost of production is

$$TC_B = 2 + q_B^2$$

The company are about to produce 100 of the item.

(a) All A: $TC = 10 + 20 \times 100 = 2010$

(b) All B: $TC = 2 + 100^2 = 10002$

(c) Minimise costs $TC_A + TC_B$ subject to $q_A + q_B = 100$:
Lagrangian:

$$L = 10 + 20q_A + 2 + q_B^2 + \lambda(100 - q_A - q_B)$$

FOC:

$$L_{q_A} = 20 - \lambda = 0 \quad (1)$$
$$L_{q_B} = 2q_B - \lambda = 0 \quad (2)$$
$$L_\lambda = 100 - q_A - q_B = 0 \quad (3)$$

From equation (1) we have

$$\lambda = 20$$

Putting this into equation (2) gives

$$2q_B = 20 \implies q_B = 10$$

Putting this into equation (3) gives

$$100 - q_A - 10 = 0 \implies q_A = 90$$

Total cost for minimisation:

$$TC = TC_A + TC_B = 10 + 20 \times 90 + 2 + 10^2 = 1912$$

2.

$$TC_G = 80 + 10g^{1.5}$$

$$TC_H = 100 + 12h^{1.25}$$

Firm produces 420 items, cost of manufacturing:

(a) All are at G: $TC = 80 + 10 \times 420^{1.5} \approx 86154$

(b) All are at H: $TC = 100 + 12 \times 420^{1.25} \approx 22916$

(c) Minimise the total cost and use both G and H
$TC = 80 + 10g^{1.5} + 100 + 12h^{1.25}$ subject to $g + h = 420$

Lagrangian:

$$L = 80 + 10g^{1.5} + 100 + 12h^{1.25} + \lambda(420 - g - h)$$

FOC:

$$L_g = 15g^{0.5} - \lambda = 0 \quad (4)$$

$$L_h = 15g^{0.25} - \lambda = 0 \quad (5)$$

$$L_\lambda = 420 - g - h = 0 \quad (6)$$

Using equations (4) and (5) we have:

$$15g^{0.5} = \lambda \tag{7}$$
$$15h^{0.25} = \lambda \tag{8}$$

Dividing equation (7) by equation (8) and rearranging:

$$\frac{15g^{0.5}}{15h^{0.25}} = 1 \tag{9}$$
$$g^{0.5} = h^{0.25} \tag{10}$$

raising to power 4: $$g^2 = h \tag{11}$$

Substitute for h from (11) into (6):

$$\begin{aligned} 420 - g - g^2 &= 0 \\ \implies g^2 + g - 420 &= 0 \\ \implies (g - 20)(g + 21) &= 0 \\ \implies g &= 20 \text{ since } g = -21 \text{ is not sensible} \\ \implies h &= 400 \end{aligned}$$

So, total cost will be:

$$TC = 80 + 10 \times 20^{1.5} + 100 + 12 \times 400^{1.25} \approx 22451$$