

Workshop: Differentiation Product and Quotient Rules

LLE Mathematics and Statistics

Product Rule

Functions that are made of other functions multiplied together can be differentiated using the product rule.

$$y = f(x) \cdot g(x)$$

For example, you may have the function:

$$y = x^2 e^x$$

Use 'dummy' variables to represent the two functions:

$$u = f(x), \quad v = g(x)$$

Then:

$$\frac{dy}{dx} = v \cdot \frac{du}{dx} + u \cdot \frac{dv}{dx}$$

So, in the example:

$$u = x^2, \quad v = e^x, \quad \frac{du}{dx} = 2x, \quad \frac{dv}{dx} = e^x$$

$$\frac{dy}{dx} = 2xe^x + x^2e^x = xe^x(2 + x)$$

Differentiate the following:

1. $y = 5xe^x$

2. $y = x^3 \ln x$

3. $y = 4e^x \ln 4x$

4. $y = (6x^2 + 4x + 1) \ln x^2$

5. $y = (5x - \frac{3}{x^2})e^x$

Quotient Rule

When a function is produced by a function divided by another function, it can be differentiated with the quotient rule.

$$y = \frac{u}{v}, \quad \frac{dy}{dx} = \frac{v \cdot \frac{du}{dx} - u \cdot \frac{dv}{dx}}{v^2}$$

For example:

$$y = \frac{x^2 + 5}{2x - 1}$$

$$u = x^2 + 5, \quad v = 2x - 1$$

$$\frac{du}{dx} = 2x, \quad \frac{dv}{dx} = 2$$

$$\frac{dy}{dx} = \frac{(2x - 1)(2x) - (x^2 + 5)(2)}{(2x - 1)^2} = \frac{4x^2 - 2x - 2x^2 - 10}{(2x - 1)^2}$$

$$\frac{dy}{dx} = \frac{2x^2 - 2x - 10}{(2x - 1)^2}$$

Differentiate the following:

$$6. y = \frac{5x+2}{x-4}$$

$$7. y = \frac{e^x}{x^2+2x}$$

$$8. y = \frac{\ln 4x}{x^2+5}$$

$$9. y = \frac{\sqrt{x}}{e^x}$$

Mixed

The following questions use product and quotient rule. They also make use of the chain rule. Find the derivative:

$$10. y = e^{4x}(5x + 2)$$

$$11. y = (5x - 1)^4 \ln x$$

$$12. y = \frac{2x+5}{e^{9x}}$$

$$13. y = (x^2 + 3x + 1)e^x \ln x$$

$$14. y = \frac{5xe^x}{x+2}$$

$$15. y = \frac{6x-7}{x^2+1} - 5x \ln(2x - 5)$$