Differentiation Rules - Solutions

LLE – Mathematics and Statistics Skills

Power Rule

Given the function $y = ax^n$, the derivative is:

$$\frac{dy}{dx} = nax^{n-1}$$

Use this rule to differentiate:

1. $y = 5x^4$

$$\frac{dy}{dx} = 20x^3$$

2. $y = 7x^2$

$$\frac{dy}{dx} = 14x$$

3. $y = \frac{2}{3}x^6$

$$\frac{dy}{dx} = 4x^5$$

Use the power rule to differentiate:

4. $y = \frac{3x^3}{x^5}$

$$y = 3x^{-2}, \quad \frac{dy}{dx} = -6x^{-3}$$

5. $y = \frac{10}{x^2}$

$$y = 10x^{-2}, \quad \frac{dy}{dx} = -20x^{-3}$$

6. $y = 2x^{\frac{2}{3}}$

$$\frac{dy}{dx} = \frac{4}{3}x^{-\frac{1}{3}}$$

7. $y = 4\sqrt{x}$

$$y = 4x^{1/2}, \quad \frac{dy}{dx} = 2x^{-1/2}$$

8. $y = \frac{2}{3x^2}$

$$y = \frac{2}{3}x^{-2}, \quad \frac{dy}{dx} = -\frac{4}{3}x^{-3}$$

9. $y = \frac{10}{x}$

$$y = 10x^{-1}, \quad \frac{dy}{dx} = -10x^{-2}$$

10. $y = 5x^2 + 4x + 3$

$$\frac{dy}{dx} = 10x + 4$$

11. $y = x^4 - 2x^{-5} + \frac{1}{x}$

$$\frac{dy}{dx} = 4x^3 + 10x^{-6} - x^{-2}$$

12. $y = 1 + 2\sqrt{x} + \frac{2}{5x} + \frac{3}{4x^2} + \frac{5}{2x^3}$

$$\frac{dy}{dx} = x^{-1/2} - \frac{2}{5}x^{-2} - \frac{3}{2}x^{-3} - \frac{15}{2}x^{-4}$$

Exponentials

1. $y = 5e^x$

$$\frac{dy}{dx} = 5e^x$$

Logarithms

Differentiate:

1. $y = 5 \ln x$

$$\frac{dy}{dx} = \frac{5}{x}$$

2. $y = \frac{\ln x}{2} - 3e^x$

$$\frac{dy}{dx} = \frac{1}{2x} - 3e^x$$

3. $y = \ln 4x$

$$y = \ln 4 + \ln x, \quad \frac{dy}{dx} = \frac{1}{x}$$

4. $y = \ln 0.443x$

$$y = \ln 0.443 + \ln x, \quad \frac{dy}{dx} = \frac{1}{x}$$

5. $y = \ln \frac{2}{x}$

$$y = \ln 2 - \ln x, \quad \frac{dy}{dx} = -\frac{1}{x}$$

6. $y = \ln x^{10}$

$$y = 10 \ln x, \quad \frac{dy}{dx} = \frac{10}{x}$$

7. $y = 6 \ln \frac{1}{2}x$

$$y = 6(\ln \tfrac{1}{2} + \ln x), \quad \frac{dy}{dx} = \frac{6}{x}$$

Mixed Questions

For each question find the requested derivative:

1. $y = 5x^3 + x^2 - 4x + 1 + 3e^x + \ln 2x \quad \frac{dy}{dx} =$

$$\frac{dy}{dx} = 15x^2 + 2x - 4 + 3e^x + \frac{1}{x}$$

2. $y = 2x + 5\sqrt{x} - \frac{4}{x} + \frac{3}{x^2} \quad \frac{dy}{dx} =$

$$\frac{dy}{dx} = 2 + \frac{5}{2\sqrt{x}} + \frac{4}{x^2} - \frac{6}{x^3}$$

3. $y = 5t^2 - 2t^{\frac{3}{4}} + 6 \ln 0.1t \quad \frac{dy}{dt} =$

$$\frac{dy}{dt} = 10t - \frac{3}{2}t^{-1/4} + \frac{6}{t}$$

4. $P = 6e^n - 4 + \frac{5}{2}n - \frac{6}{\sqrt{n}} \quad \frac{dP}{dn} =$

$$\frac{dP}{dn} = 6e^n + \frac{5}{2} + 3n^{-3/2}$$

5. $G = 5s^2(3s - 4) \quad \frac{dG}{ds} =$

$$G = 15s^3 - 20s^2, \quad \frac{dG}{ds} = 45s^2 - 40s$$

By substituting into the function and into the derived function, find the value of the function and the value of the gradient at the given x values:

6. $y = 5x^2 - 12x + 2, \quad x = 2$

$$\frac{dy}{dx} = 10x - 12$$
$$y = 0, \quad \frac{dy}{dx} = 8$$

7. $y = 4x^3 + x + 10, \quad x = -1$

$$\frac{dy}{dx} = 12x^2 + 1$$
$$y = 5, \quad \frac{dy}{dx} = 13$$

8. $y = x^2 + 5x - 1, \quad x = -2.5$

$$\frac{dy}{dx} = 2x + 5$$
$$y = -7.25, \quad \frac{dy}{dx} = 0$$

9. $y = 4e^x - 10\sqrt{x}, \quad x = 2$

$$\frac{dy}{dx} = 4e^x - \frac{5}{\sqrt{x}}$$

$$y \approx 15.42, \quad \frac{dy}{dx} \approx 26.02$$

10. $y = 5 - \frac{5}{x} + \frac{5}{x^2}, \quad x = 1$

$$\frac{dy}{dx} = \frac{5}{x^2} - \frac{10}{x^3}$$

$$y = 5, \quad \frac{dy}{dx} = -5$$