Partial Differentiation

LLE – Mathematics and Statistics Skills

Partial Derivatives

1. For each of the following, find:

$$\frac{\partial z}{\partial x}, \quad \frac{\partial z}{\partial y}, \quad \frac{\partial^2 z}{\partial x^2}, \quad \frac{\partial^2 z}{\partial y^2}, \quad \frac{\partial^2 z}{\partial x \partial y}, \quad \frac{\partial^2 z}{\partial y \partial x}$$

(a) $z = 5x + 2y$

(b) $z = 5x^3 + 3y^2 + 4$

(c) $z = 8xy$

(d) $z = 2xy^{0.5}$

(e) $z = e^{2x} + 4e^y$

(f) $z = x^2 + 2xy + y^2$

(g) $z = 10x^2y^3 - 2x - 4y$

(h) $z = 5x^2 + 2x + 5y^3 - 2y^2$

(i) $z = 10x^4y^3$

(j) $z = 2e^{5x} - 4x^2y^2 + \ln(3xy)$

(k) $z = x^3 \ln y$

(l) $z = y(5 - 2x)$

(m) $z = y(5 - 2x)^2$

2. Given the demand function:

$$Q = 50 - 2P + 0.5I$$

Q is quantity demanded, P is price of good, and I is the income level of consumers.

(a) Find $\frac{\partial Q}{\partial P}$ and $\frac{\partial Q}{\partial I}$

(b) Interpret the meanings of these partial derivatives

(c) Suppose income changes by 100, what is the change in demand?

3. Given $Q(L, K) = AL^\alpha K^{1-\alpha}$, where Q is output, L is labour, and K is capital.

(a) Find the partial derivatives $\frac{\partial Q}{\partial L}$ and $\frac{\partial Q}{\partial K}$.

(b) Let $A = 1$ and $\alpha = 0.3$. Evaluate your expressions at $L = 10$, $K = 5$.

(c) Repeat for $L = 20$, $K = 5$. How does the value of $\frac{\partial Q}{\partial L}$ compare to before? Explain your answer using the formula from part (a).

First Order Conditions

4. For each of the functions $f(x, y)$ below, find:

(i) f_x and f_y

(ii) the values of x, y, and $f(x, y)$ where both $f_x = 0$ and $f_y = 0$

(a) $f(x, y) = x^2 + y^2$

(b) $f(x, y) = x^2 - 2x - y^2 - 4y - 3$

(c) $f(x, y) = x^2 y - 2xy^2 + 3xy + 4$