Hicksian Demand and Expenditure Functions

LLE – Mathematics and Statistics Skills

Questions

1. Consider a consumer with the utility function $U(x, y) = xy$, where x and y are two goods. Let p_x and p_y denote their respective prices, and let $\bar{U}$ be the target level of utility.

   (a) Set up and solve the expenditure minimization problem using a Lagrangian to derive the Hicksian demand functions for x and y, $x_H(p_x, p_y, \bar{U})$ and $y_H(p_x, p_y, \bar{U})$.

   (b) Use the Hicksian demand functions to derive the general expenditure function, $E(p_x, p_y, \bar{U})$.

   (c) Using your solutions from parts (a) and (b) above, find the initial Hicksian demands, initial minimum expenditure, new Hicksian demands, new minimum expenditure, and compensating variation (CV) for the following situations:

      (i) Target utility $\bar{U} = 100$. Initial prices $p_x = 1, p_y = 1$. The price of x then increases to $p_x = 4$, with p_y remaining 1.

      (ii) Target utility $\bar{U} = 1000$. Initial prices $p_x = 1, p_y = 1$. The price of x then increases to $p_x = 5$, with p_y remaining 1.

      (iii) Target utility $\bar{U} = 36$. Initial prices $p_x = 1, p_y = 2$. The price of x then increases to $p_x = 3$, with p_y remaining 2.

      (iv) Target utility $\bar{U} = 256$. Initial prices $p_x = 1, p_y = 1$. The price of y then increases to $p_y = 4$, with p_x remaining 1.

2. Consider a consumer with the utility function $U(x, y) = x^{1/2}y^{1/2}$. Let p_x and p_y denote the prices of x and y, and let $\bar{U}$ be the target level of utility.

   (a) Set up and solve the expenditure minimization problem using a Lagrangian to derive the Hicksian demand functions for x and y, $x_H(p_x, p_y, \bar{U})$ and $y_H(p_x, p_y, \bar{U})$.

   (b) Use the Hicksian demand functions to derive the general expenditure function, $E(p_x, p_y, \bar{U})$.

   (c) Using your solutions from parts (a) and (b) above, find the initial Hicksian demands, initial minimum expenditure, new Hicksian demands, new minimum expenditure, and compensating variation (CV) for the following situations:

      (i) Target utility $\bar{U} = 16$. Initial prices $p_x = 1, p_y = 1$. The price of x then increases to $p_x = 4$, with p_y remaining 1.

      (ii) Target utility $\bar{U} = 25$. Initial prices $p_x = 2, p_y = 2$. The price of y then increases to $p_y = 8$, with p_x remaining 2.

      (iii) Target utility $\bar{U} = 81$. Initial prices $p_x = 3, p_y = 1$. The price of y then increases to $p_y = 3$, with p_x remaining 3.

3. Consider a consumer with the utility function $U(x, y) = x^{0.4}y^{0.6}$. Let p_x and p_y denote the prices of x and y, and let $\bar{U}$ be the target level of utility.

   (a) Set up and solve the expenditure minimization problem using a Lagrangian to derive the Hicksian demand functions for x and y, $x_H(p_x, p_y, \bar{U})$ and $y_H(p_x, p_y, \bar{U})$.

   (b) Use the Hicksian demand functions to derive the general expenditure function, $E(p_x, p_y, \bar{U})$.

   (c) Using your solutions from parts (a) and (b) above, find the initial Hicksian demands, initial minimum expenditure, new

Hicksian demands, new minimum expenditure, and compensating variation (CV) for the following situations:

(i) Target utility $\bar{U} = 20$. Initial prices $p_x = 1, p_y = 2$. The price of x then increases to $p_x = 2$, with p_y remaining 2.

(ii) Target utility $\bar{U} = 50$. Initial prices $p_x = 4, p_y = 2$. The price of x then decreases to $p_x = 1$, with p_y remaining 2.

(iii) Target utility $\bar{U} = 100$. Initial prices $p_x = 2, p_y = 1$. The price of y then increases to $p_y = 3$, with p_x remaining 2.