T Tests

LET Mathematics & Statistics

T-tests are used to compare a sample mean with a population mean, or to compare two sample means, to infer whether there are differences in the population.

One-sample t-test

A one-sample t-test is used to test whether a sample mean is different from a hypothesised mean.

1. The mean IQ is known to be 100. A researcher wants to test whether people in Norwich have a different mean than 100. They collect a sample of 10 students and give them an IQ test. The results were:

   110, 98, 103, 99, 95, 112, 120, 100, 112, 101

   (a) Find the mean IQ score for the sample of Norwich people.

   Solution

   sample mean $= \frac{110+98+103+99+95+112+120+100+112+101}{10} = 105$

   (b) How many IQ points difference is there between this sample mean and the population mean (100)?

   Solution

   $105 - 100 = 5$

Standard Error of the Mean

When estimating a population mean from a sample mean there will be error in our estimate (the standard error). The standard error of the mean is related to how spread out the data are and how big the sample is.

$$\text{standard error of the mean} = \frac{s}{\sqrt{n}}$$

Where s is the sample standard deviation and n is the sample size.

Higher standard deviations (greater spread in the sample) lead to higher standard error.

Higher sample sizes lead to lower standard error.

2. Given that the standard deviation of the sample above is 7.6, calculate the standard error of the mean. Give your answer correct to 2 decimal places.

Solution

SEM $= \frac{7.6}{\sqrt{10}} = 2.40$

The t Statistic

The t statistic can be thought of as a type of standardised measure of difference between two values. The bigger the t statistic, the bigger the difference. For a one-sample t-test it can be calculated with the formula:

$$t = \frac{\bar{x} - \mu}{\text{SEM}}$$

Where $\bar{x}$ is the sample mean and μ is the population mean.

Another importance calculation is the degrees of freedom (df). For a one-sample t-test the degrees of freedom are sample size minus 1.

3. For the data above:

   (a) Calculate the t-statistic for the difference between the sample mean and the hypothesised mean of 100. Given you answer correct to 2 decimal places.

   Solution

   $t = \frac{105-100}{2.4} = 2.08$

(b) Calculate the degrees of freedom of this t statistic

Solution

df $= 10 - 1 = 9$

p-values

Using the t-statistic and the degrees of freedom, a p-value can be calculated. The p-value can be used to determine whether there is evidence of a difference in the population. Conventionally, p-values of less than 0.05 are considered evidence of a difference.

A p-value of less than 0.05 is considered evidence of a difference at the 5% level

4. For this IQ experiment: $t = 2.08$, df $= 9$, $p = 0.0673$

   Would these statistics give evidence that the people of Norwich have a significantly different IQ to 100, at the 5% level.

   Solution

   No, since for this case $p = .0673 > 0.05$, there is not enough evidence of a difference from 100.

5. A similar study is conducted using a sample of students from UEA. The question is whether UEA students have a different IQ than 100. A sample of size 30 is taken, and statistics are calculated

   Sample mean = 105; Sample SD = 7.6

   (a) Calculate the standard error of the mean. Give your answer correct to 2 decimal places.

   Solution

   SEM $= \frac{7.6}{\sqrt{30}} = 1.39$

   (b) Calculate the t-statistic for the difference between the sample mean and the hypothesised mean of 100 from the UEA data. Given you answer correct to 2 decimal places.

Solution

$t = \frac{105-100}{1.39} = 3.60$

(c) The results come out as t = 3.60, df = 29, p = 0.00116. Is there evidence of a difference, at the 5% level, between UEA IQ and the value of 100?

○ Not enough evidence of a difference in IQ from 100

⊗ **There is evidence of a difference in IQ from 100**

(d) Given that there was a difference of 5 in both studies and the standard deviation of the two samples were both 7.6, why do you think that the p-value was smaller in the second study?

Solution

The sample size, n, in the second case is bigger

Independent-samples T-test

T-tests can also be used to determine whether there is evidence of a difference between two groups.

For example, you may be looking to see if there is a difference in a mobility score between a group that received treatment and a group that did not receive treatment. Again, you are looking for p-values to be lower than a value (typically 0.05) to say there is evidence of a difference.

For an independent-samples t-test:

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\text{SE of difference}}$$

Where $\bar{x}_1$ and $\bar{x}_2$ are the sample means of groups 1 and 2.

For an independent-samples t-test:

$$\text{df} = n_1 + n_2 - 2$$

Where n_1 and n_2 are the sample sizes of groups 1 and 2.

6. A treatment group and a control group are compared on systolic blood pressure levels. There are 20 patients in both groups.

Mean (treatment) = 109

Mean (control) = 122

(a) Calculate the number of degrees of freedom

Solution

$20 + 20 - 2 = 38$

(b) Calculate the difference between the two sample means

Solution

$122 - 109 = 13$

(c) It is found that

$$t = 2.03, p = .0493$$

Given the information above, is there evidence of a difference in blood pressures between the control and the treatment groups?

Solution

Yes, since for this case $p = .0493 < 0.05$, there is evidence of a difference between the two groups, at the 5% level.