Straight Lines

LET Mathematics & Statistics

This workshop focuses on gradients and the equation of a straight line

1 Gradient

Gradient is the measure of how steep something is. For a straight-line, the gradient is the same throughout, straight-lines do not get steeper or shallower.

The **gradient of a straight-line** is a numerical value that tells you how far up (if a positive value) or down (if a negative value) when you move one unit horizontally to the right. It can be calculated as:

$$\text{Gradient} = \frac{\Delta y}{\Delta x}$$

Δ represents the amount of change.

If the points with coordinates (x_1, y_1) and (x_2, y_2) lie on a straight-line, the gradient will be calculated as:

$$\text{Gradient} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

1. Find the gradient of the straight-line that passes through the points $(1, 1)$ and $(5, 13)$

   Solution

   $\frac{13-1}{5-1} = 3$

2. Find the gradient of the straight-line that passes through the points $(2, -1)$ and $(10, 3)$

   Solution

   $\frac{3-(-1)}{10-2} = 0.5$

3. Find the gradient of the straight-line that passes through the points $(5, 18)$ and $(7, 8)$

> **Solution**
>
> $\frac{8-18}{7-5} = -5$

2 Equation of a Straight Line

The general equation of a straight line is:

$$y = mx + c$$

Here, m is the **gradient of the straight-line** and c is the y-**intercept**, which is the value of y when $x = 0$ and the point on the y-axis where the line crosses it.

Any function that is written in this form, *or can be rearranged to be in this form*, are straight-lines.

4. What is the gradient and y-intercept of the straight-line $y = 5x + 1$

> **Solution**
>
> Already in the correct format, gradient = 5, y-intercept = $(0, 1)$

5. What is the gradient and y-intercept of the straight-line $y = 15 - 2x$

> **Solution**
>
> Already in the correct format, gradient = -2, y-intercept = $(0, 15)$

6. What is the gradient and y-intercept of the straight-line $4y = 4x + 12$

> **Solution**
>
> Rearrange to $y = x + 3$; gradient = 1, y-intercept = $(0, 3)$

7. What is the gradient and y-intercept of the straight-line
   $4x - 2y - 9 = 0$

> **Solution**
>
> Rearranging $4x - 9 = 2y \implies 2x - \frac{9}{2} = y$
>
> gradient = 2, y-intercept = $(0, \frac{9}{2})$

8. What is the gradient and y-intercept of the straight-line $2x + 3y = -7$

> **Solution**
>
> Rearranging $3y = -2x - 7 \implies y = \frac{-2}{3}x - \frac{7}{3}$
>
> gradient = $-\frac{2}{3}$, y-intercept = $(0, -\frac{7}{3})$

3 Finding the Equation of a Straight-Line

If you have two points on a straight-line, you can find its equation by:

Step 1: Find the gradient between the two points

Step 2: Writing the partial answer $y = mx + c$, using your value of the gradient for m

Step 3: Substitute either of the coordinates in for x and y into $y = mx + c$ and solve for c

9. Find the equation of the straight-line that passes through the points (1,6) and (3,10)

> **Solution**
>
> $m = \frac{10-6}{3-1} = 2$
> $y = 2x + c$
> $6 = 2(1) + c \implies c = 4$
>
> $$y = 2x + 4$$

10. Find the equation of the straight-line that passes through the points (5,12) and (12,-9)

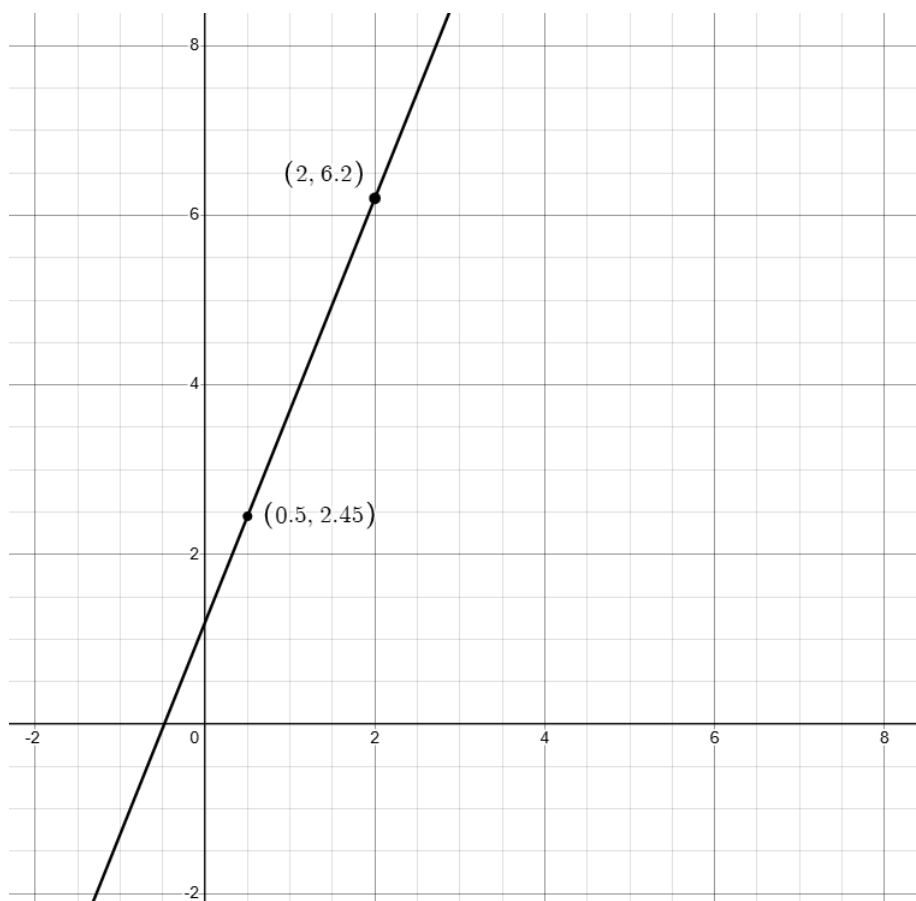11. Find the equation of the straight-line shown in the image below

12. Use your answer from question 11 to write the equation of this line in the form

$$ax + by + c = 0$$

Where a, b, and c are all integers (whole numbers).

Solution

$y = 2.5x + 1.2$

Multiply all terms by 10: $10y = 25x + 12$

Subtract $10y$ from both sides: $0 = 25x + 12 - 10y$

$$25x - 10y + 12 = 0$$

Note also that $-25x + 10y - 12 = 0$ is also a correct solution.