

# Probability

## LET Mathematics & Statistics

1. A small company of 20 people runs a weekly raffle where there are always 2 prizes. Each employee gets 1 raffle entry for every year they have worked there, so during your first year you don't have a ticket in the draw. Before the draw today the combined total years people have worked there is 80 years.

A ticket is drawn, the winner picks their prize, the winning ticket is returned to the draw and a second winner is drawn.

- (a) In a weekly draw, what is the probability of winning the first prize:
  - i. for an employee that's been there for 2 years?

**Solution**

$$P(2\text{yr 1st prize}) = \frac{2}{80} = \frac{1}{40} = 0.025$$

- ii. for an employee that's been there for 9 months?

**Solution**

$$P(0\text{yr 1st prize}) = \frac{0}{80} = 0$$

- (b) A particular employee has been working there for 8 years. What is the probability that in a weekly draw that
  - i. they win the first prize

**Solution**

$$P(8\text{yr 1st prize}) = \frac{8}{80} = \frac{1}{10} = 0.1$$

- ii. given they win the first prize they then win the second prize?

**Solution**

$P(8\text{yr 2nd prize given 1st prize}) = \frac{8}{80} = \frac{1}{10} = 0.1$   
since return of ticket means 2nd prize is independent of first prize

- iii. they win both the first and second prizes?

**Solution**

$$\begin{aligned}P(1\text{st and 2nd prize}) &= \\P(1\text{st prize}) \times P(2\text{nd prize given 1st prize}) &= \\ \frac{1}{10} \times \frac{1}{10} &= \frac{1}{100} = 1\%\end{aligned}$$

- (c) The company decides to change the nature of the draw, and the first prize winning ticket is no longer returned to the draw. Work out the probability, for the individuals given, of winning.
- Employee has worked there for 4 years, probability of them winning first prize.

**Solution**

$$P(1\text{st prize}) = \frac{4}{80} = \frac{1}{20} = 0.05 = 5\%$$

- Given an employee of 4 years has just won first prize, the probability of them also winning second prize.

**Solution**

$$\begin{aligned}P(2\text{nd prize given 1st prize}) \\= \frac{3}{79} = 0.038 = 3.8\%\end{aligned}$$

Due to 1 ticket being removed and 1 less ticket chance of winning

- An employee of 8 years, probability of them winning both first and second prizes.

**Solution**

$$\begin{aligned}P(1\text{st prize and 2nd prize}) \\= \frac{8}{80} \times \frac{7}{79} = \frac{7}{790} = 0.0089 = 0.89\%\end{aligned}$$

- iv. An employee of 1 year, probability of them winning the second prize.

**Solution**

Since only 1 ticket, needs to not win the first prize.

$$P(\text{Not win 1st prize and win 2nd prize}) =$$

$$\frac{79}{80} \times \frac{1}{79} = \frac{1}{80} = 0.0125 = 1.25\%$$

- v. An employee of 5 years, probability of them winning one of the prizes but not both.

**Solution**

$$P(\text{1st AND not 2nd}) = \frac{5}{80} \times \frac{75}{79} = \frac{75}{1264}$$

$$P(\text{not 1st AND 2nd}) = \frac{75}{80} \times \frac{5}{79} = \frac{75}{1264}$$

$$P(\text{One not both}) = \frac{75}{1264} + \frac{75}{1264} = \frac{75}{632} = 0.119 = 11.9\%$$

- (d) The number of employees based on the number of complete years they've worked there is shown in the table.

Years	0	1	2	3	4	5	6	7	8
Employees	4	1	1	1	4	3	2	1	3

What is the probability that in a weekly draw there are no prizes to employees who have been there more than 5 years?

**Solution**

At the first draw there are  $2 \times 6 + 1 \times 7 + 3 \times 8 = 43$  tickets for those who've been there more than 5 years, so  $80 - 43 = 37$  tickets are non-winners.

$$P(\text{no prize any over 5 yr}) = \frac{37}{80} \times \frac{36}{79} = \frac{333}{1580} = 0.211 = 21.1\%$$

2. Below is an incomplete two-way table showing the preferences of 500 people to two operating systems (Pear and Robot) and their self-declared interest in technology (low, medium, high).

Operating System	Tech Interest			Total
	Low	Medium	High	
Pear	95	210	45	350
Robot	20	90	40	150
<b>Total</b>	<b>115</b>	300	<b>85</b>	<b>500</b>

- (a) Complete the two-way table
- (b) What is the probability that a randomly selected person:
- uses the Pear operating system?

**Solution**

$$P(\text{Pear}) = \frac{350}{500} = \frac{7}{10} = 0.7$$

- has a medium interest in technology?

**Solution**

$$P(\text{Medium}) = \frac{300}{500} = \frac{3}{5} = 0.6$$

- has a medium interest and uses the Robot operating system?

**Solution**

$$P(\text{Medium AND Robot}) = \frac{90}{500} = \frac{9}{50} = 0.18$$

- has a low or medium interest and uses the Pear operating system?

**Solution**

$$P(\text{Low/medium AND Pear}) = \frac{95+210}{500} = \frac{61}{100} = 0.61$$

- has a high technology interest, given that they use the Robot operating system?

**Solution**

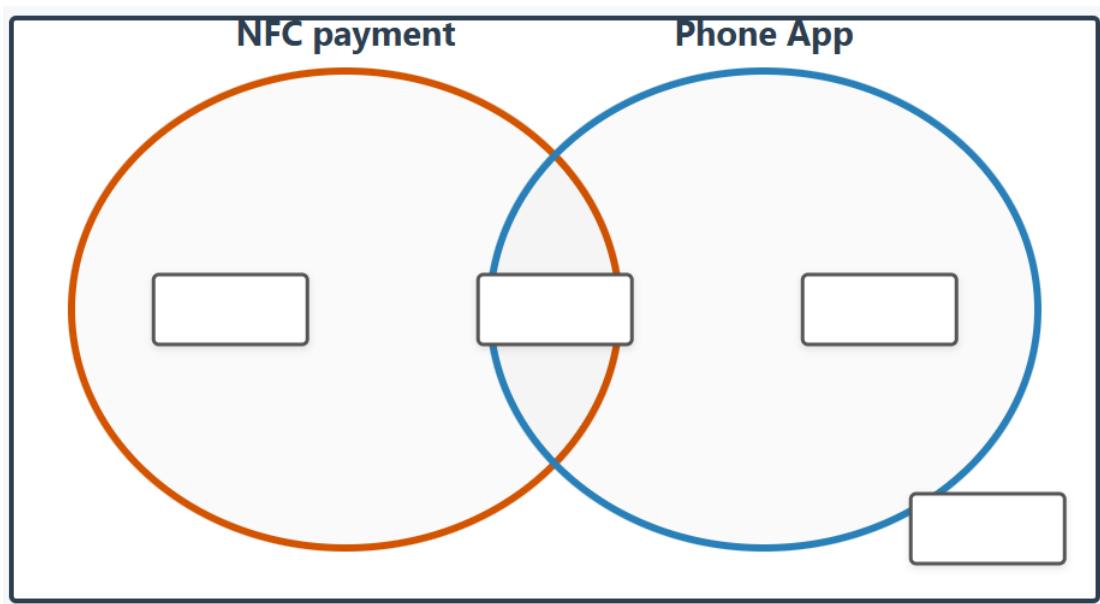
$$P(\text{High GIVEN Robot}) = \frac{40}{150} = \frac{4}{15} = 0.27$$

- uses the Robot operating system, given they have a low technology interest?

**Solution**

$$P(\text{Robot GIVEN Low}) = \frac{20}{115} = \frac{4}{23} = 0.17$$

3. At the bar of the company ‘The Pecs Bar & Gym’, customers can pay for items via different means. 30% of customers now use NFC on their smartphone to pay at the bar. The company also has an app that gives news, information and offers to customers, 70% of customers have installed this app. 20% of customers use their smartphones to pay and have the company app.
- (a) Complete the Venn diagram, showing the probabilities as decimal numbers between 0 and 1.

**Solution**

Fill in the intersection of 0.2 first. Then work out the remainder of NFC ( $0.3 - 0.2 = 0.1$ ) and the remainder of app ( $0.7 - 0.2 = 0.5$ ). All 4 numbers need to sum to 1.

- (b) Using the Venn diagram, or otherwise, calculate the following probability (as a decimal number to no more than 3 decimal places) that a customer:
- does not use the NFC function and does not use the app

**Solution**

This is the area outside of the circles, 0.2.

- ii. uses their phone for at least one of the NFC or app means

**Solution**

This is the total area inside of the circles, 0.8.

- iii. uses their NFC but does not have the app

**Solution**

This is the NFC circle outside of the app circle, 0.1.

- iv. uses their NFC given that they have the app installed

**Solution**

We know here that we are in the app circle, which is 0.7, and 0.2 of this 0.7 is inside the NFC circle.  $\frac{0.2}{0.7} = 0.286$

- v. uses the app given that they use NFC to pay

**Solution**

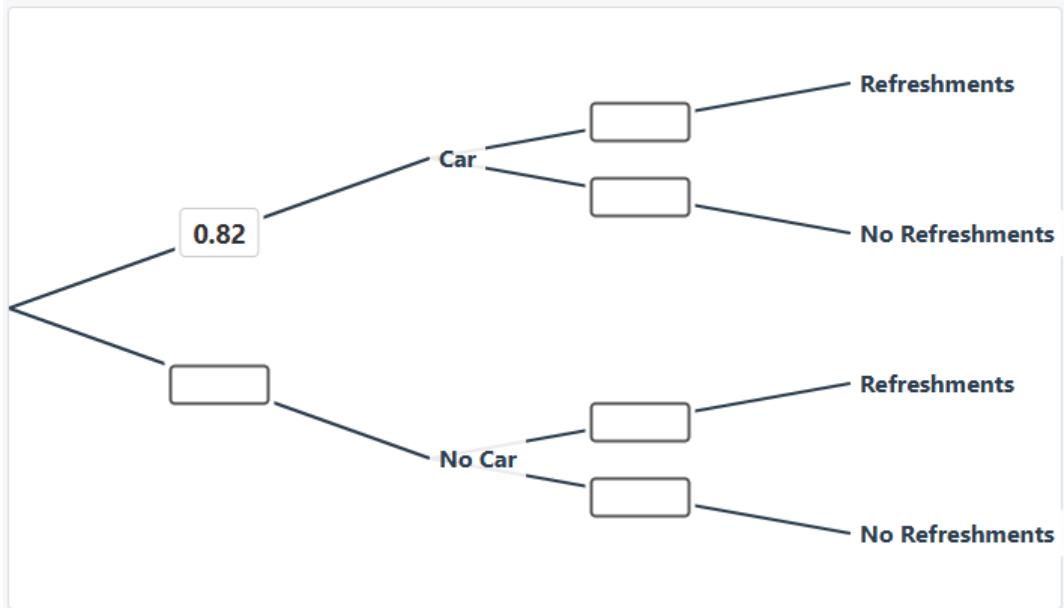
It is now out of 0.3 (the NFC circle), and 0.2 of this is the app too,  $\frac{0.2}{0.3} = 0.667$ .

- (c) Are the events using NFC and using the phone app independent of one another?

**Solution**

Probability of NFC is 0.3 and the probability of NFC given they use the app is 0.286. Not equal, so not independent.

- 4. A company records and finds that 82% of its members make use of the car park when they visit. Of these 82%, 23% regularly use the café for refreshments during the visit. For those who do not use the car park 64% obtain refreshments.
  - (a) Fill in the missing probabilities as decimals between 0 and 1.



### Solution

No Car:  $1 - 0.82$ . For the car branch we will have 0.23 and  $1 - 0.23$  and for the no car branch we will start with 0.64.

- (b) Find the probability that a randomly selected customer drives by car and has refreshments from the café. Give your answer as a decimal correct to 4 decimal places.

### Solution

We multiply along the branches so need  $0.82 \times 0.23 = 0.1886$

- (c) Find the probability that a randomly selected customer has refreshments from the café. Give your answer as a decimal correct to 4 decimal places.

### Solution

Refreshments can either be car and refreshments or they could be no car and refreshments. We have 0.1886, so need to find the no car and refreshments in a similar way, then add 0.1886 to this answer for no car and refreshments. 0.3038.

- (d) Given that a customer has had refreshments from the café, what is the probability that they did not use the car?

### Solution

We have the probability of refreshments from part c, so this will be our denominator. The numerator is going to be the no car and refreshments, which was also worked out as part of c. 0.3792