Straight Line Graphs – Solutions

LLE – Mathematics and Statistics

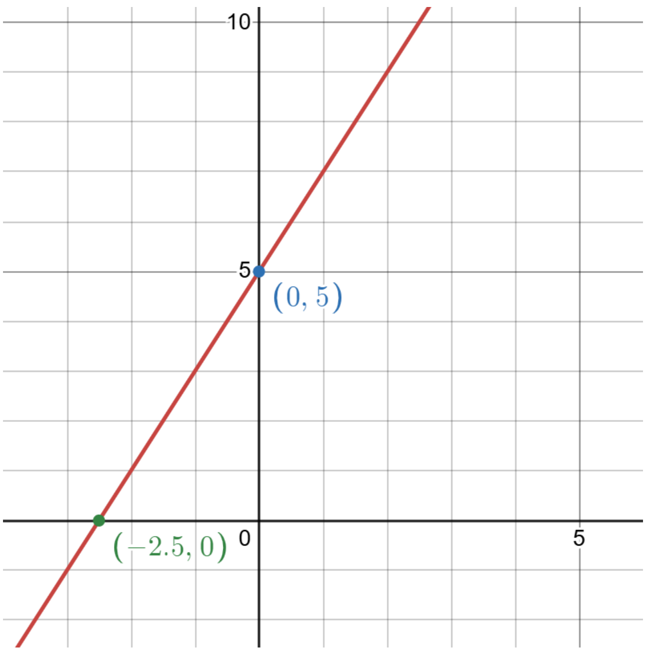
1. Given the straight line function $y = 2x + 5$:

   (a) *Solution:* The equation is in the form $y = mx + c$, where m is the gradient.
   Answer: The gradient is 2.

   (b) *Solution:* Set $x = 0$: $y = 2(0) + 5 = 5$.
   Answer: The y-intercept is at (0, 5).

   (c) *Solution:* Set $y = 0$: $0 = 2x + 5 \implies -5 = 2x \implies x = -2.5$.
   Answer: The x-intercept is at (-2.5, 0).

   (d) *Solution:* Draw a straight line that passes through the y-axis at (0, 5) and the x-axis at (-2.5, 0).

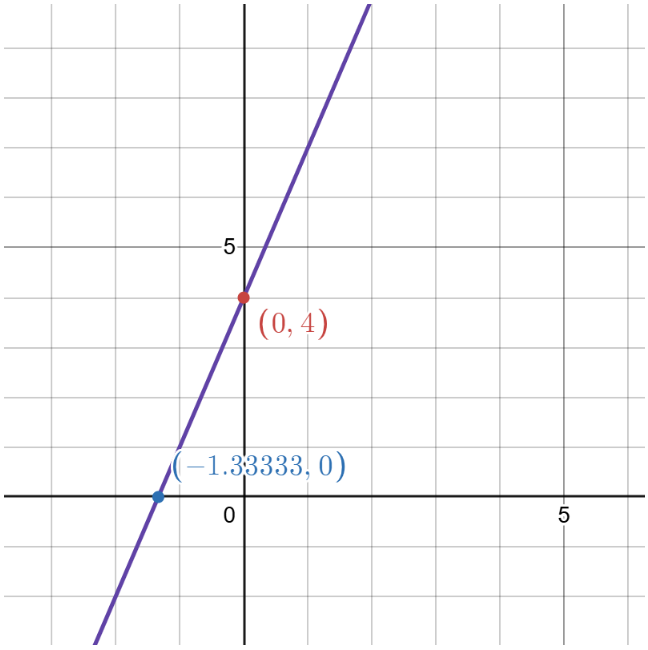


2. For each of the functions given:

   (a) $2y = 6x + 8$
   Solution: a) Divide all terms by 2 to get $y = 3x + 4$.

b) Yes, this is a straight line.

c) The gradient is 3, the y-intercept is (0, 4), and the x-intercept is (-4/3, 0).

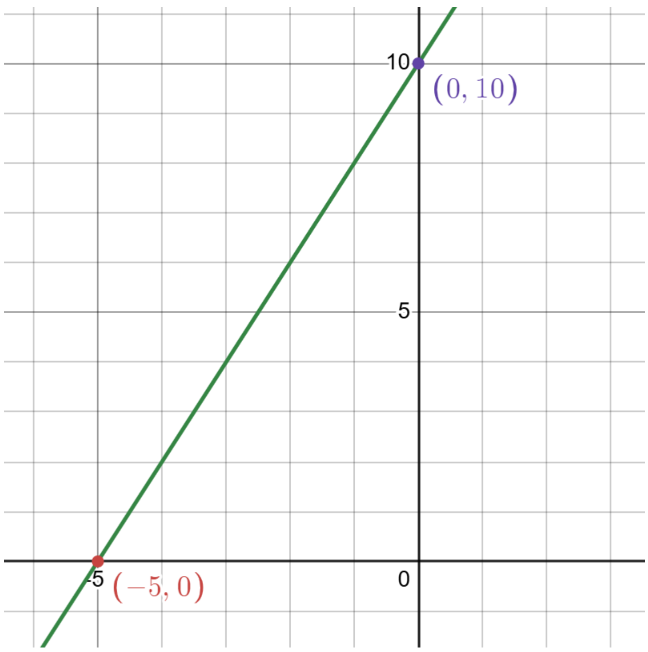


(b) $y - 2x = 10$

Solution: a) Add 2x to both sides to get $y = 2x + 10$.

b) Yes, this is a straight line.

c) The gradient is 2, the y-intercept is (0, 10), and the x-intercept is (-5, 0).



(c) $xy - 5 = 10$

Solution: a) Rearrange to get $y = \frac{15}{x}$.

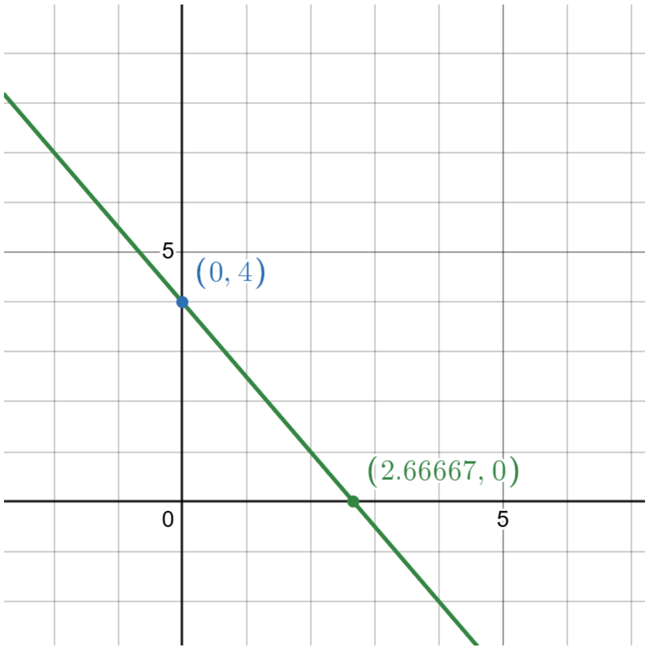
b) No, this is not a straight line because x is in the denominator.

(d) $3x + 2y - 8 = 0$

Solution: a) Rearrange to get $y = -\frac{3}{2}x + 4$.

b) Yes, this is a straight line.

c) The gradient is -3/2, the y-intercept is (0, 4), and the x-intercept is (8/3, 0).



(e) $x^2 - 3y = 6$

Solution: a) Rearrange to get $y = \frac{x^2}{3} - 2$.

b) No, the x^2 term means it is a parabola, not a straight line.

(f) $x^2 - 3y^2 = 6$

Solution: a) Rearrange to get $y = \pm\sqrt{\frac{x^2-6}{3}}$.

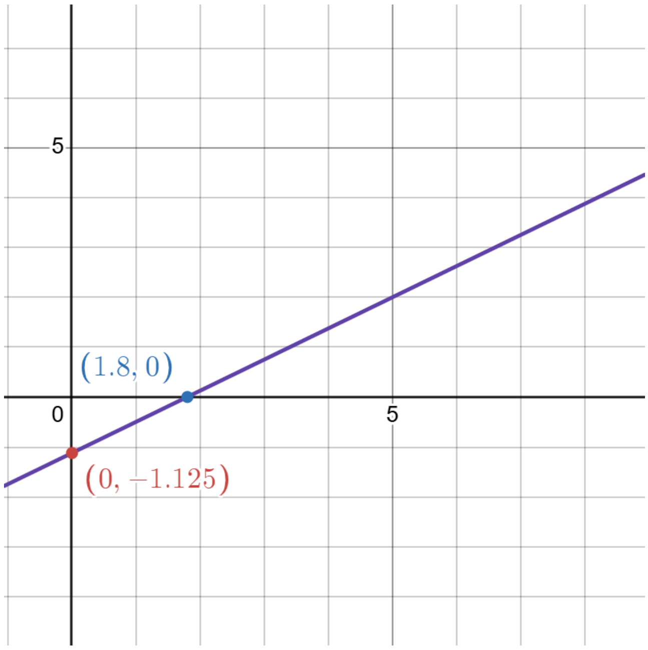
b) No, the squared terms mean this is not a straight line.

(g) $5x - 8y = 9$

Solution: a) Rearrange to get $y = \frac{5}{8}x - \frac{9}{8}$.

b) Yes, this is a straight line.

c) The gradient is 5/8, the y-intercept is (0, -9/8), and the x-intercept is (9/5, 0).

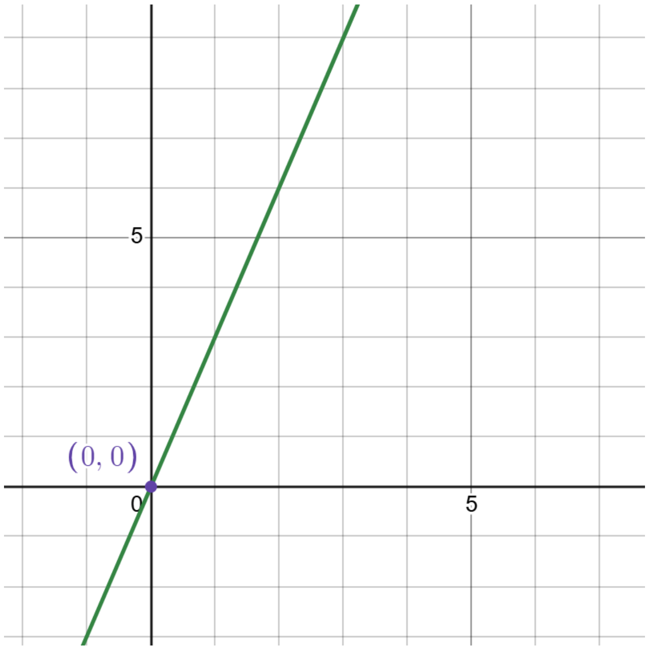


(h) $\frac{2y-x}{5} = x$

Solution: a) Rearrange to get $y = 3x$.

b) Yes, this is a straight line.

c) The gradient is 3, and it passes through the origin (0, 0).

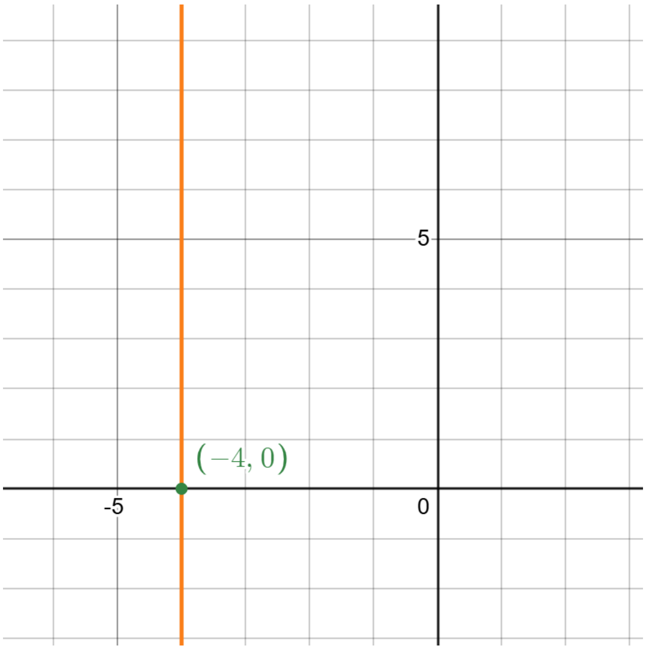


(i) $x(3y - 1) = 3xy + 4$

Solution: a) Expand and simplify to get $-x = 4$, or $x = -4$.

b) Yes, this is a vertical straight line.

c) The gradient is undefined, there is no y-intercept, and the x-intercept is (-4, 0).

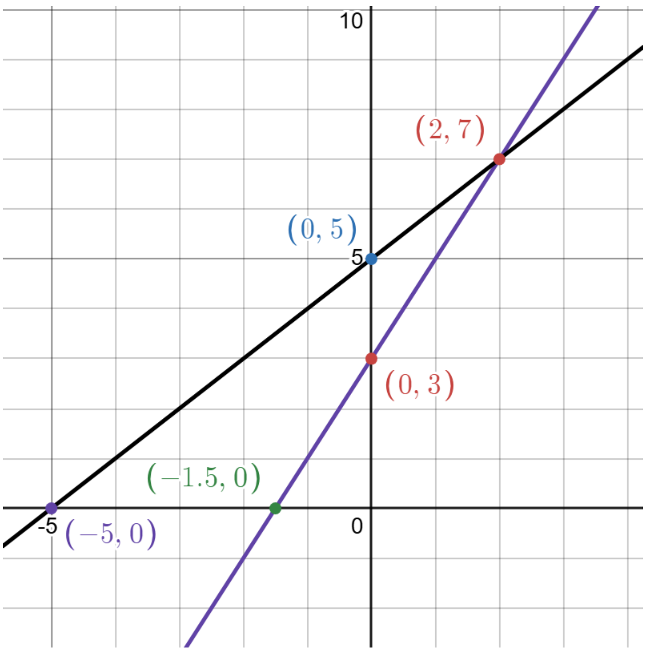


3. For each pair of straight lines:

   (a) $y = 2x + 3$ and $y = x + 5$

   Solution: b) Set equations equal: $2x + 3 = x + 5 \implies x = 2$. Substitute back to find $y = 7$.

   c) The intersection point is (2, 7).

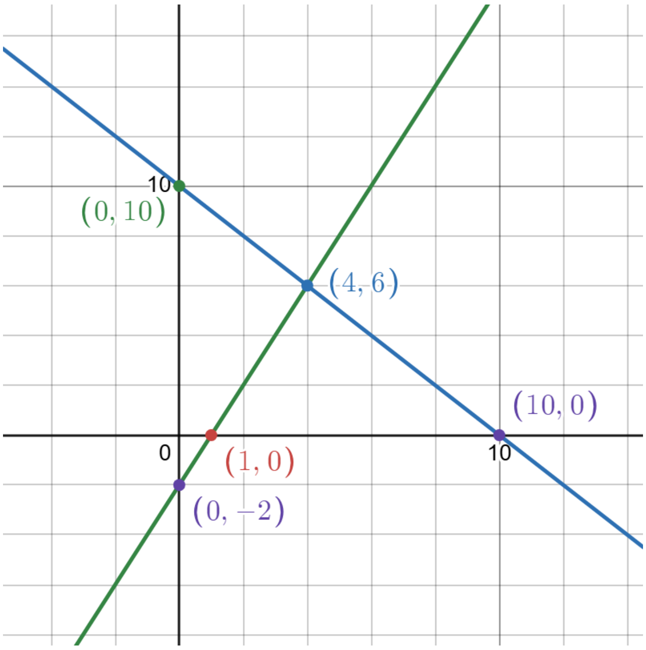


   (b) $x + y = 10$ and $2x - y = 2$

   Solution: b) Add equations to eliminate y: $3x = 12 \implies x = 4$.

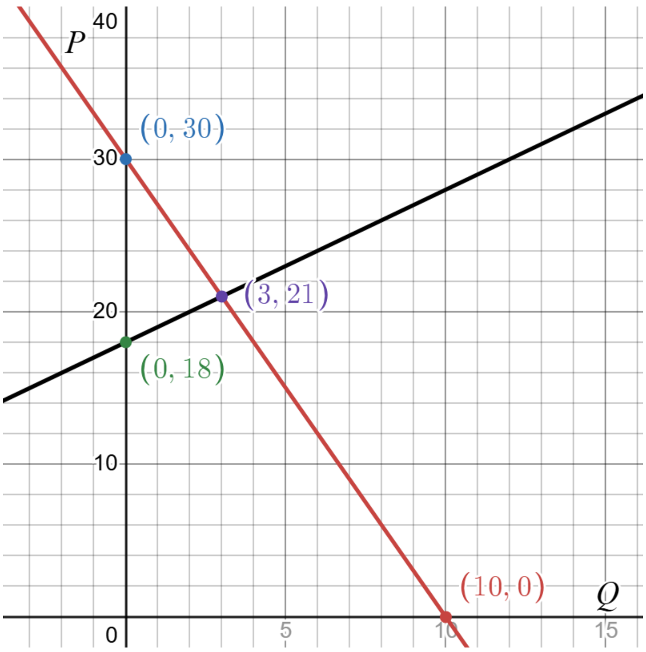
Substitute back to find $y = 6$.

c) The intersection point is (4, 6).



4. *Solution:* Rearrange demand to $P = 30 - 3Q$. Set supply equal to demand: $Q + 18 = 30 - 3Q \implies 4Q = 12 \implies Q = 3$. Substitute into the supply equation to find $P = 3 + 18 = 21$.

   Answer: Equilibrium is at Quantity=3, Price=21.



5. Solutions for the gym pricing problem:

(a) *Solution:* This has no starting cost and a positive gradient of £20.
Answer: Graph A

(b) *Solution:* This has a starting cost of £20 (the y-intercept) and a positive gradient of £12.
Answer: Graph B

(c) *Solution:* Graph C has a zero gradient, so there is no cost per session, and a positive intercept, so a one-off cost.
Answer: Graph C: For example, pay a one-off fee of £60 and attend as many sessions as you like (perhaps in a given time-frame).

(d) **Answer:** $C = 20E$

(e) **Answer:** $C = 12E + 20$

(f) *Solution:* Structure 1: $C = 20 \times 4 = 80$.
Structure 2: $C = (12 \times 4) + 20 = 48 + 20 = 68$.
Answer: Structure 1: £80. Structure 2: £68.

(g) *Solution:* Structure 1: $100 \div 20 = 5$.
Structure 2: First pay the £20 fee, leaving £80. $80 \div 12 \approx 6.67$, so 6 complete entries.
Answer: Structure 1: 5 times. Structure 2: 6 times.

(h) *Solution:* Set equations equal: $20E = 12E + 20 \implies 8E = 20 \implies E = 2.5$.
Answer: The cost is the same at 2.5 entries. For 3 or more entries, the membership structure is cheaper.