Probability - Solutions

LLE – Mathematics and Statistics

1. The table shows a summary of data from the Pecs Bar and Gym, relating to session attended and whether the customer came to the session by car or not.

Customer Frequencies by Session and Journey

Session	**Car**	**No Car**	**Total**
Free Weights	3	1	**4**
Cardio Machines	4	1	**5**
Fitness Class	2	2	**4**
Bar	1	2	**3**
Total	**10**	**6**	**16**

(a) Completed two-way table

(b) A person is selected at random. Probability that:

i. they went to the fitness class
Total in Fitness Class is 4. Total people is 16.

$$P(\text{fitness}) = \frac{4}{16} = \frac{1}{4} = 0.25$$

ii. they came by car
10 came by car

$$P(\text{car}) = \frac{10}{16} = \frac{5}{8} = 0.625$$

iii. they used the cardio machines and came by car
Number in Cardio and Car is 4

$$P(\text{cardio and car}) = \frac{1}{4} = 0.25$$

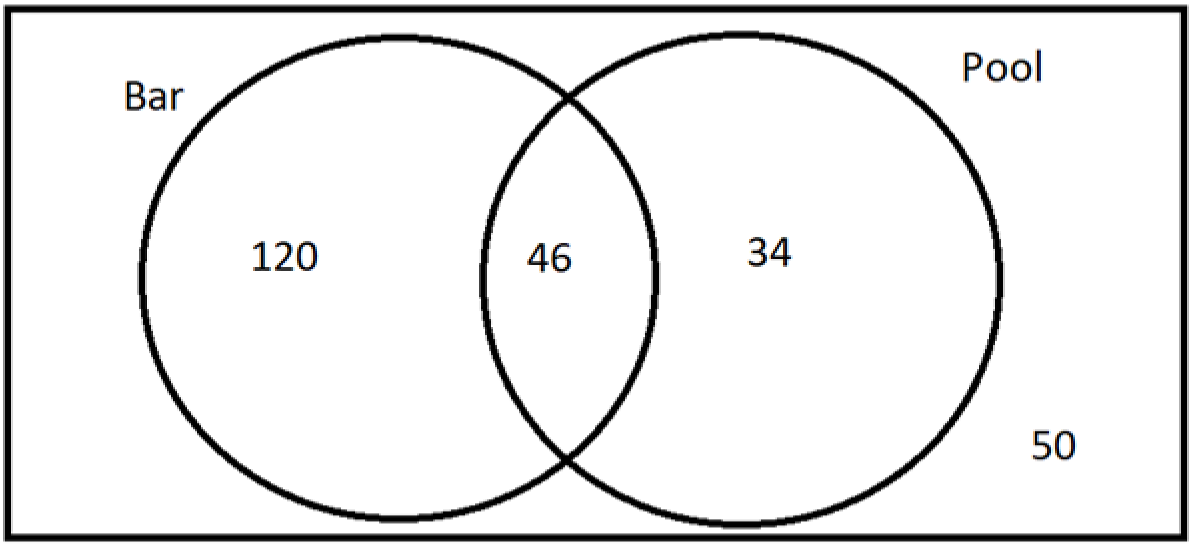
iv. they did not use free weights and did not come by car
The number of people in this group is (Cardio, No Car) + (Fitness, No Car) + (Bar, No Car) = 1 + 2 + 2 = 5

$$P(\text{no weight and no car}) = \frac{5}{16} = 0.3125$$

(c) Given that a randomly selected person came by car, what is the probability they went to the fitness class?
This is a conditional probability. The total is now only the people who came by car (10). Of those 10, 2 went to the fitness class.

$$P(\text{fitness given car}) = P(\text{fitness}|\text{car}) = \frac{2}{10} = \frac{1}{5} = 0.2$$

2. The Pecs Bar and Gym carry out a larger-scale study of its facility usage. The diagram below summarises some of the findings.



(a) This is a Venn diagram

(b) How many people were included in the study?
Add all the numbers in the diagram: 120 (Bar only) + 46 (Both) + 34 (Pool only) + 50 (Neither)

$$120 + 46 + 34 + 50 = 250$$

(c) What is the probability that a randomly selected person used the pool?
First, find the total number of people who used the Pool: 34 (Pool only) + 46 (Both) = 80

$$P(\text{pool}) = \frac{80}{250} = \frac{8}{25} = 0.32$$

(d) What is the probability that a randomly selected person used both the pool and the bar?
The number in the overlapping section is 46.

$$P(\text{pool AND bar}) = P(\text{pool} \cap \text{bar}) = \frac{46}{250} = \frac{23}{125} = 0.184$$

(e) What is the probability that a randomly selected person did not use the bar?
Total who used the Bar = 120 + 46 = 166.
Total who did not use the Bar = 250 - 166 = 84

$$P(\text{NOT bar}) = P(\overline{\text{bar}}) = \frac{84}{250} = \frac{42}{125} = 0.336$$

(f) What is the probability that a randomly selected person used the pool or the bar?
Can use formula

$$P(\text{pool OR bar}) = P(\text{pool}) + P(\text{bar}) - P(\text{pool AND bar})$$

$$P(\text{pool} \cup \text{bar}) = \frac{80}{250} + \frac{166}{250} - \frac{46}{250} = \frac{200}{250} = \frac{4}{5} = 0.8$$

(g) Given that a person used the bar, what is the probability that they used the pool?
The total is now only the people who used the bar (120 + 46 = 166).

Of these 166 people, 46 also used the pool.

$$P(\text{pool}|\text{bar}) = \frac{46}{166} = \frac{23}{83} \approx 0.277$$

(h) Are the events of attending the pool and using the bar independent?
For events to be independent, P(Pool | Bar) must equal P(Pool).
From part (g), P(Pool | Bar) ≈ 0.277.
From part (c), P(Pool) = 0.32.
Since these are not equal, the events are not independent.

(i) Two people are randomly selected from the group. Probability that

i. they are two people who both used the pool?

$$P(\text{1st used Pool}) = \frac{80}{250}$$

$$P(\text{2nd used Pool, given 1st did}) = \frac{79}{249}$$

$$P(\text{both used pool}) = \frac{80}{250} \times \frac{79}{249} \approx 0.101$$

ii. at least one of the two people used the pool?
This is equal to $1 - P(\text{neither used pool})$

$$P(\text{neither used pool}) = \frac{170}{250} \times \frac{169}{249}$$

$$P(\text{at least one used pool}) = 1 - \frac{170}{250} \times \frac{169}{249} \approx 0.539$$

3. Customers can use lockers while enjoying the facilities of the Pecs Bar and Gym. The management of the locker keys is quite messy; all the keys are just randomly left in a drawer.

   There are 90 green tagged keys, numbered 1 to 90. There are 60 red tagged keys, numbered 1 to 60.

The first customer of the day appear and then they ask for a locker. The receptionist reaches into the drawer and pulls out a key.

(a) Probability that they get a green tagged key?
First, find the total number of keys:
Total Keys = 90 (Green) + 60 (Red) = 150.
The number of green keys is 90.

$$P(\text{Green}) = \frac{90}{150} = \frac{3}{5} = 0.6$$

(b) Probability that they get a key numbered over 50?
First, find how many keys of each colour are numbered over 50:
Green keys > 50 = 90 - 50 = 40.
Red keys > 50 = 60 - 50 = 10.
Total keys > 50 = 40 + 10 = 50.
The probability is this number divided by the total:

$$P(> 50) = \frac{50}{150} = \frac{1}{3} \approx 0.333$$

It's a slow day. The customer does their thing and returns the key to reception. No other customers have entered during this time. Just as the reception thinks about closing early for lunch, a group of three people make their way into the building. They all want lockers.

(c) Probability that all three get green tags?
This is sampling without replacement. We multiply the probabilities for each successive event:

$$P(\text{all green}) = \frac{90}{150} \times \frac{89}{149} \times \frac{88}{148} \approx 0.213$$

(d) Probability that all three get tags of the same colour?
We need to find the probability of all three getting green OR

all three getting red.

$$P(\text{all green}) \approx 0.213$$

$$P(\text{all red}) = \frac{60}{150} \times \frac{59}{149} \times \frac{58}{148} \approx 0.062$$
$$P(\text{same colour}) \approx 0.213 + 0.062 = 0.275$$

4. The company gets its equipment from two supplies, supplier X and supplier Y, with 70% of the equipment coming from supplier X. Evidence suggests that supplier's X equipment has a 5% chance of failing in the first year. The figure for supplier Y is 8%.

   (a) Probability that a randomly selected piece of equipment:

      i. is from supplier X and fails in the first year
         Multiply the probability of being from Supplier X by the probability of it failing, given it's from X.

$$P(X) \times P(\text{Fail}|X) = 0.7 \times 0.05 = 0.035$$

      ii. fails in the first year
         Find the probability of being from Y and failing

$$P(Y) \times P(\text{Fail}|Y) = 0.3 \times 0.08 = 0.024$$

         Add the two failure probabilities together

$$P(\text{fail}) = 0.035 + 0.024 = 0.059$$

   (b) Given that a piece of equipment fails during the first year, the probability:

      i. it was from supplier X
         This is a conditional probability. We use the formula

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$P(X|\text{fail}) = \frac{P(X \cap \text{fail})}{P(\text{fail})} = \frac{0.035}{0.059} \approx 0.593$$

ii. it was from supplier Y

$$P(Y|\text{fail}) = \frac{P(Y \cap \text{fail})}{P(\text{fail})} = \frac{0.024}{0.059} \approx 0.407$$

5. The bar staff at the Pecs Bar and Gym have a game they play with customers. The bar staff member picks a card at random and will offer a free drink if the customer can guess what it is.

   (a) What is the probability that the customer guesses the correct card?
   There is one correct card out of 52 possibilities.

$$P(\text{correct}) = \frac{1}{52}$$

   The customer is unlikely to win. To help the customer, they are allowed to ask a single 'yes/no' question, from the list below, which the bar member will answer honestly, before they guess. The choices of questions are:

   - Is the card a red card?
   - Is the card a club?
   - Is the card the 5 of diamond

   (b) Work out the new probabilities of guessing correctly for each of these initial questions.

$$P(Win) = P(Win|Yes)P(Yes) + P(Win|No)P(No)$$

   "Is it red?":

$$P(Win) = \frac{1}{26} \times \frac{26}{52} + \frac{1}{26} \times \frac{26}{52} = \frac{1}{26}$$

”Is it a club?”:

$$P(Win) = \frac{1}{13} \times \frac{13}{52} + \frac{1}{39} \times \frac{39}{52} = \frac{1}{26}$$

”Is it the 5 of diamonds?”:

$$P(Win) = 1 \times \frac{1}{52} + \frac{1}{51} \times \frac{51}{52} = \frac{1}{26}$$