

Solutions - Algebra Skills

LLE – Mathematics and Statistics

Expanding Brackets

1. For each of the following, expand and simplify

(a) $5(4x + 3) = 20x + 15$

(b) $2x(3 + 4y) = 6x + 8xy$

(c) $5(2x + 1) + x(3x - 2) = 10x + 5 + 3x^2 - 2x = 3x^2 + 8x + 5$

(d) $x(4x + y) + y(3x - 1) = 4x^2 + xy + 3xy - y = 4x^2 + 4xy - y$

(e) $(x + 5)(x + 2) = x^2 + 2x + 5x + 10 = x^2 + 7x + 10$

(f) $(3x - 4)(2x + 1) = 6x^2 + 3x - 8x - 4 = 6x^2 - 5x - 4$

Solving Linear Equations

2. For each equations below, solve for x

(a) $4x - 1 = 21 \implies 4x = 22 \implies x = \frac{22}{4} = \frac{11}{2}$

(b) $18 - 7x = 4 \implies 18 = 4 + 7x \implies 14 = 7x \implies x = 2$

(c) $5(2x - 3) = 11 \implies 10x - 15 = 11 \implies 10x = 26 \implies x = \frac{13}{5}$

(d)

$$5x - 3 = 3x + 7$$

$$5x - 3x = 7 + 3$$

$$2x = 10$$

$$x = 5$$

(e)

$$\begin{aligned}9x + 7 &= 5x - 9 \\9x - 5x &= -9 - 7 \\4x &= -16 \\x &= -4\end{aligned}$$

(f)

$$\begin{aligned}\frac{10}{x+1} &= 4 \\10 &= 4(x+1) \\10 &= 4x + 4 \\6 &= 4x \\x &= \frac{3}{2}\end{aligned}$$

Rearranging Expressions

3. Make the variable in the square brackets the subject of the formula

(a) $[x]$

$$\begin{aligned}y &= 3x + 5 \\y - 5 &= 3x \\x &= \frac{y - 5}{3}\end{aligned}$$

(b) $[t]$

$$\begin{aligned}4t - 8s &= 20 \\4t &= 20 + 8s \\t &= \frac{20 + 8s}{4} = 5 + 2s\end{aligned}$$

(c) $[Q]$

$$P = 4Q + 3$$

$$P - 3 = 4Q$$

$$Q = \frac{P - 3}{4}$$

(d) $[Q]$

$$P = 20 - 2Q$$

$$2Q = 20 - P$$

$$Q = \frac{20 - P}{2}$$

(e) $[P]$

$$\frac{P + Q}{5} = 2$$

$$P + Q = 10$$

$$P = 10 - Q$$

(f) $[Q]$

$$5(Q - 3) = 10P$$

$$Q - 3 = 2P$$

$$Q = 2P + 3$$

(g) $[T]$

$$MV = PT$$

$$T = \frac{MV}{P}$$

(h) $[L]$

$$Q = 0.25L^2K^3$$
$$\frac{4Q}{K^3} = L^2$$
$$L = \sqrt{\frac{4Q}{K^3}}$$

(i) $[M]$

$$S = \frac{1}{1-M}$$
$$1-M = \frac{1}{S}$$
$$M = 1 - \frac{1}{S} \text{ or } M = \frac{S-1}{S}$$

(j) $[x]$

$$z = \frac{x - \mu}{\sigma}$$
$$z\sigma = x - \mu$$
$$x = \mu + z\sigma$$

(k) $[\mu]$

$$z = \frac{x - \mu}{\sigma}$$
$$z\sigma = x - \mu$$
$$\mu = x - z\sigma$$

(l) $[E(X)]$

$$Var(X) = E(X^2) - (E(X))^2$$
$$(E(X))^2 = E(X^2) - Var(X)$$
$$E(X) = \sqrt{E(X^2) - Var(X)}$$

(m) $[P(A \cap B)]$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = P(A) + P(B) - P(A \cup B)$$

(n) $[P(B)]$

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

$$P(B) = \frac{P(A \cap B)}{P(A | B)}$$

Laws of Indices

4. Use the laws of indices to simplify the following expressions

(a) $x^5 x^{10} = x^{5+10} = x^{15}$

(b) $\frac{t \cdot t^9}{t^6} = t^{1+9-6} = t^4$

(c) $(xy^2)^5 = x^5 y^{2 \times 5} = x^5 y^{10}$

(d) $y^4 t y t^{-5} = y^4 y t t^{-5} = y^5 t^{-4}$ or $\frac{y^5}{t^4}$

(e) $xy^2 \sqrt{xy} = xy^2 x^{\frac{1}{2}} y^{\frac{1}{2}} = x^{\frac{3}{2}} y^{\frac{5}{2}}$

(f) $\frac{xy^2}{\sqrt{xy}} = \frac{xy^2}{x^{\frac{1}{2}} y^{\frac{1}{2}}} = x^{\frac{1}{2}} y^{\frac{3}{2}}$

(g) $\frac{a^4 b^{-4} a^8}{(a^{-3} b)^{-4}} = \frac{a^4 b^{-4} a^8}{a^{12} b^{-4}} = a^0 b^0 = 1$

Factorising

5. Fully factorise the following expressions

(a) $10t + 25 = 5(2t + 5)$

(b) $8x - 32y = 8(x - 4)$

(c) $7pt + 12pk = p(7t + 12k)$

$$(d) 5y^4 - 8xy^2 = y^2(5y^2 - 8x)$$

$$(e) 12xy^3 + 30x^3y^3 = 6xy^3(2 + 5x^2)$$

$$(f) 9a^2b + 27a^3b^2 = 9a^2b(1 + 3ab)$$

$$(g) 16ab^2c - 4a^2b^2c^3 + 12a^5b^2c^4 = 4ab^2c(4 - ac^2 + 3a^4c^3)$$

6. Use factorising to simplify these algebraic fractions

$$(a) \frac{5x+20}{10} = \frac{5(x+4)}{10} = \frac{x+4}{2}$$

$$(b) \frac{12y-30}{2y-5} = \frac{6(2y-5)}{2y-5} = 6$$

$$(c) \frac{8t+40}{6t+30} = \frac{8(t+5)}{6(t+5)} = \frac{8}{6} = \frac{4}{3}$$

$$(d) \frac{8x^3-8x^2}{16x-16} = \frac{8x^2(x-1)}{8(x-1)} = x^2$$

$$(e) \frac{10a^2b}{a^2b-ab} = \frac{10a^2b}{ab(a-1)} = \frac{10a}{a-1}$$

Algebraic Fractions

7. Simplify the following algebraic fractions

$$(a) \frac{x}{5} + \frac{2x}{5} = \frac{x+2x}{5} = \frac{3x}{5}$$

$$(b) \frac{5}{x} + \frac{2}{x} = \frac{5+2}{x} = \frac{7}{x}$$

$$(c) \frac{10}{x} - \frac{5}{2x} = \frac{20}{2x} - \frac{5}{2x} = \frac{15}{2x}$$

$$(d) \frac{2}{x} + \frac{1}{y} = \frac{2y}{xy} + \frac{x}{xy} = \frac{2y+x}{xy}$$

$$(e) \frac{2a}{b} \times \frac{3c}{5d} = \frac{2a \times 3c}{b \times 5d} = \frac{6ac}{5bd}$$

$$(f) \frac{5m}{2p} \times \frac{4m}{3k} = \frac{20m^2}{6pk} = \frac{10m^2}{3pk}$$

$$(g) \frac{8t}{5k} \times \frac{10k}{5t^2} = \frac{80tk}{25t^3} = \frac{16k}{5t^2}$$

$$(h) \frac{8x}{3} \div \frac{2y}{5x} = \frac{8x}{3} \times \frac{5x}{2y} = \frac{40x^2}{6y} = \frac{20x^2}{3y}$$

$$(i) \frac{x+1}{5} \div \frac{2x+2}{3} = \frac{x+1}{5} \times \frac{3}{2x+2} = \frac{3(x+1)}{5(2x+2)} = \frac{3(x+1)}{5 \times 2(x+1)} = \frac{3}{10}$$

$$(j) \frac{5}{x} \left(\frac{x^2}{4} + \frac{x}{5} \right) = \frac{5}{x} \left(\frac{5x^2}{20} + \frac{4x}{20} \right) = \frac{5}{x} \times \frac{5x^2+4x}{20} = \frac{5}{x} \times \frac{x(5x+4)}{20} = \frac{5x+4}{4}$$