A graphical-model semantics for probabilistic programs

Hugo Paquet (University of Oxford)

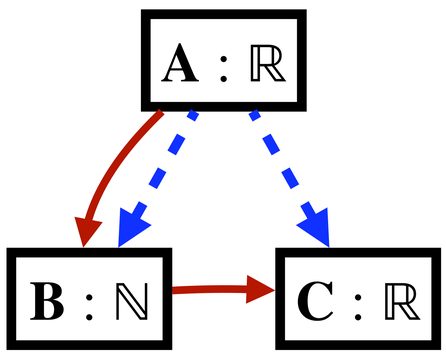
Probabilistic models are often encoded as **Bayesian networks.**

This work is about a semantics for probabilistic programs where programs are interpreted as a generalized kind of Bayesian network.

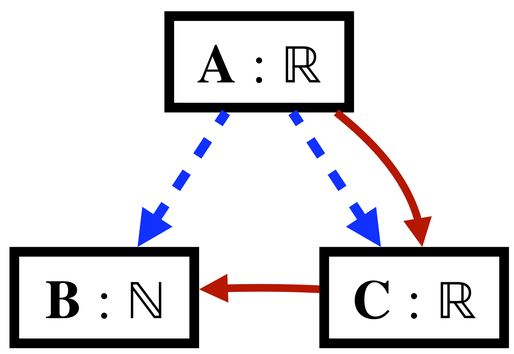
Separating data flow & control flow

Reordering lines gives different representations:

```
A <- sample (normal 0 1)
B <- sample (poisson A)
C <- sample (normal A 1)
```




```
A <- sample (normal 0 1)
C <- sample (normal A 1)
B <- sample (poisson A)
```



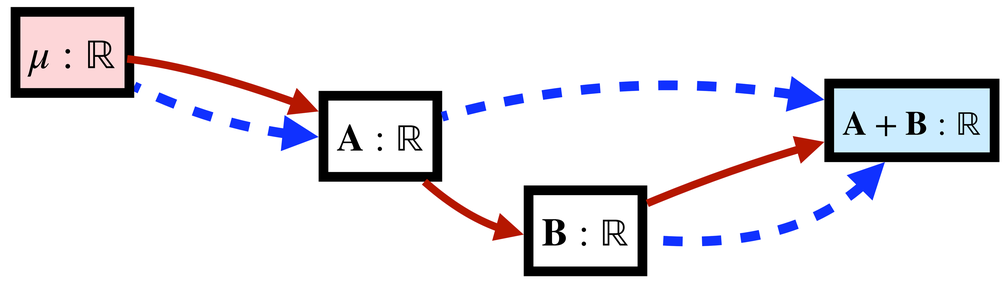
Compositionality

Use ideas from **game semantics** and **event structures** to represent programs with free variables.

$\mu : \mathbb{R} \vdash$

```
A <- sample (normal μ 1)
B <- sample (normal 0 1)
return (A + B)
```

$: \mathbb{R}$

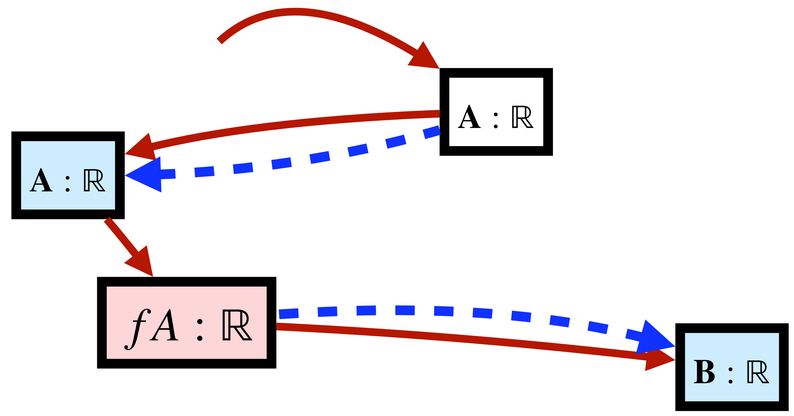


$f : \mathbb{R} \to \mathbb{R} \vdash$

```
A <- sample (normal 1 2)
B <- f A
return B
```

$: \mathbb{R}$

The program interacts with the argument f



- Representations can be composed to recover a regular Bayesian network + control flow.
- Possible representation for conditional branching using **conflict** in event structures
- Support for **linear types**, e.g $!(\mathbb{R} \multimap \mathbb{R})$ to represent non-parametric processes