$$\mathbb{E}_{p(x|y)}[f(x)] = \int f(x)p(x|y)\mathrm{d}x$$

$$= \int f(x)p(x|y)\frac{q(x)}{q(x)}\mathrm{d}x$$

$$= \mathbb{E}_{q(x)}\left[f(x)\frac{p(x|y)}{q(x)}\right]$$