Trajectory planning for Quadrotor Using Linear Model Predictive Control

This script defines a continuous-time nonlinear quadrotor model and generates a state function and its Jacobian function as a linearization for the state space reprezentation.

https://sal.aalto.fi/publications/pdf-files/eluu11_public.pdf

Pre-start configuration

```
close all
clear all
set(0,'defaulttextinterpreter','latex')
set(0,'DefaultAxesFontSize',1)
```

Add path to the solver

```
cd(fileparts(matlab.desktop.editor.getActiveFilename))
addpath('/home/prochazo/Documents/MATLAB/osqp-matlab');
addpath 'C:\Users\mropr\Documents\git\qpOASES\interfaces\matlab'
```

```
Warning: Name is nonexistent or not a directory: /home/prochazo/git/landing_uav_on_usv_matlab/
MPC_MATLAB/MPC/C
Warning: Name is nonexistent or not a directory: /home/prochazo/git/landing_uav_on_usv_matlab/
MPC_MATLAB/MPC/\Users\mropr\Documents\git\qpOASES\interfaces\matlab
```

Choose program input/output variables

```
use_wamv_trajectory = 0; % to
z_offset = 0;
if z_offset > 0
    fprintf("The reference for z was applied: %i\n",z_offset)
end

Ts = 0.1;     %Sampling time 0.05
fprintf("Sampling time: %f\n", Ts);
```

```
Sampling time: 0.100000
```

```
N = 15; % Prediction horizon 36
fprintf("Size of the prediction horizon is: %i\n", N);
```

```
Size of the prediction horizon is: 15
```

```

% Q = diag([30 30 40 1 1 30 1 1 1 1 1 50]); % tracking weight matrix
Q = diag([30 30 40 1 1 50 1 1 1 1 1 1]); % tracking weight matrix
% P = diag([30 30 40 1 1 30 1 1 1 1 1 50]); % tracking weight matrix
P = diag([0 0 0 0 0 0 0 0 0 0 0 0]); % tracking weight matrix
R = 0.1*eye(4); % input increment weight matrix
```

setup usage of the constraints

```
use_soft_constraints = true;
```

```
use_slew_rate = false;
```

Create symbolix variables for states, manipulated variables and parameters

```
syms xt(t) yt(t) zt(t) phit(t) thetat(t) psit(t) xdott(t) ydott(t)...
    zdott(t) phidott(t) thetadott(t) psidott(t)
syms u1 u2 u3 u4 Ixx Iyy Izz k l m b g
syms x y z phi theta psi xdot ydot zdot phidot thetadot psidot
```

The twelve states for the quadrotor are:

$$[x, y, z, \phi, \theta, \psi, \dot{x}, \dot{y}, \dot{z}, \dot{\phi}, \dot{\theta}, \dot{\psi}],$$

where

- $[x, y, z]$ denote the positions in the inertial frame
- Angle positions $[\phi, \theta, \psi]$ are in the order of roll, pitch, and yaw
- The remaining states are the velocities of the positions and angles

and the others are

- ui: squared angular velocity of rotor *i*
- g: gravitation acceleration
- b: drag constant - propeller constant used for torque
- k: lift constant
- l: distance between rotor and center of gravity
- Iii: diagonal elements of inertia matrix

Group symbolic variables

```
statet = {
    xt(t) yt(t) zt(t) phit(t) thetat(t) psit(t) xdott(t) ...
    ydott(t) zdott(t) phidott(t) thetadott(t) psidott(t)};
state = {
    x y z phi theta psi xdot ydot zdot phidot thetadot psidot};
    state_diff = {diff(xt(t),t), diff(yt(t),t), diff(zt(t),t), ...
    diff(phit(t),t), diff(thetat(t),t), diff(psit(t),t)};
state_dot = {
    xdot ydot zdot phidot thetadot psidot};
control = [
    u1, u2, u3, u4];
```

Set values for dynamics parameters

The difinition of the values was taken from *mrs_simulation/models* .

```
gVal = 9.81; % gravitation acceleration
% mass = 4.903; % real-drone weight
```

```
mass = 3.5; % drone's weight in simulation
d = 0.325; % arm's length
lVal = d*cos(deg2rad(45));
kVal = 24.5;
bVal = 0.7;

% test values
% kVal = 15.46;
% bVal = 0.1;
% kVal = 23;
% bVal = 0.016;
% bVal = 0.8;

inertia_body_radius = 0.25;
inertia_body_height = 0.15; %0.05;
rotor_mass = 0; % 0.005 % kg zatim tuto hmotnost zanedbam
motor_mass = 0.148; % kg
propeller_mass = 0.0202;

IxxVal = 2*((rotor_mass+motor_mass+propeller_mass)*lVal^2) + (mass - (rotor_mass+motor_
IyyVal = 2*((rotor_mass+motor_mass+propeller_mass)*lVal^2) + (mass - (rotor_mass+motor_
IzzVal = 4*((rotor_mass+motor_mass+propeller_mass)*lVal^2) + (mass - (rotor_mass+motor_

% TODO: add weight of the motors like is aproximated in Dynamic Modelling
% of a Quadrotor Aerial Vehicle with Nonlinear Inputs -good paper = DONE

paramValues = [IxxVal IyyVal IzzVal kVal lVal mass bVal gVal];

% working point for linearization
u_work = 0.48*[1 1 1 1]; % minimum rotation^2 of the rotors to be in hover
state_work = zeros(1,numel(state));
```

Transformation matrix for angular velocities from inertial frame to body frame

```
W = [1, 0, -sin(thetat);
    0, cos(phit), cos(thetat)*sin(phit);
    0, -sin(phit), cos(thetat)*cos(phit)];
```

R-ZYX Euler

```
Rz = [cos(psit), -sin(psit), 0;
    sin(psit), cos(psit), 0;
    0, 0, 1];
Ry = [cos(thetat), 0, sin(thetat);
    0, 1, 0;
    -sin(thetat), 0, cos(thetat)];
Rx = [1, 0, 0;
    0, cos(phit), -sin(phit);
    0, sin(phit), cos(phit)];
```

Rotation matrix from body frame to inertial frame

```
Rzyx = Rz*Ry*Rx;
```

Jacobian (relates body frame to inertial frame velocities)

```
I = [Ixx, 0, 0; 0, Iyy, 0; 0, 0, Izz];
J = W.'*I*W;
```

Coriolis forces

```
dJ_dt = diff(J);
dJ_dt = subs(dJ_dt,[state_diff statet],[state_dot state]);
h_dot_J = [phidott(t), thetadott(t), psidott(t)]*J;
h_dot_J = subs(h_dot_J,[state_diff statet],[state_dot state]);
grad_temp_h = jacobian(h_dot_J,[phi theta psi]);
C = dJ_dt - 1/2*grad_temp_h;
```

Total thrust

```
T = k*(u1+u2+u3+u4);
```

Torques in the direction of phi, theta, psi

```
tau_beta = [l*k*(- u1 - u2 + u3 + u4); l*k*(- u1 - u4 + u2 + u3); b*(- u1 + u2 - u3 + u
```

UAV's dynamics

```
f(1) = xdott;
f(2) = ydott;
f(3) = zdott;
f(4) = phidott;
f(5) = thetadott;
f(6) = psidott;
```

Equations for COM configuration

```
f(7:9) = -g*[0;0;1] + Rzyx*[0;0;T]/m;
```

Euler Lagrange equations for angular dynamics

```
f(10:12) = inv(J)*(tau_beta - C*[phidott(t); thetadott(t); psidott(t)]);
```

Replace parameters and drop time dependence

```
f = subs(f, [Ixx Iyy Izz k l m b g], paramValues);
f = subs(f,statet,state);
f = simplify(f);
```

Calculate linearization from the UAV's nonlinear model

```
Ac = jacobian(f,[state{:}]);
Ac = subs(Ac, state,state_work);
Ac = double(subs(Ac, control,u_work));

Bc = jacobian(f,control);
Bc = subs(Bc, state,state_work);
Bc = double(subs(Bc, control,u_work));

Cc = diag(ones(1,12));
% Cc(7,7) = sqrt(3)/2;
% Cc(7,8) = 1/2;
% Cc(8,7) = 1/2;
% Cc(8,8) = sqrt(3)/2;

Dc = zeros(12,4);

sys=ss(Ac,Bc,Cc,Dc);
```

Discretize the model of the UAV

```
model = c2d(sys,Ts);

% Extract the matrices of the discrete state-space description
A = model.A;
B = model.B;
C = model.C;

n = size(A,1); % number of states
m = size(B,2); % number of inputs
p = size(C,1); % number of outputs
```

LMPC

The control inputs for the quadrotor are the squared angular velocities of the four rotors:

$[\omega_1^2, \omega_2^2, \omega_3^2, \omega_4^2]$.

These control inputs create force, torque, and thrust in the direction of the body z-axis. In this example, every state is measurable, and the control inputs are constrained to be within [-0.5,0.5] $\left(\frac{\text{rad}}{s}\right)^2$. Because it was linearized around ω^2 = 0.55, which is the rotation speed when the copter is hovering.

```
umax = (1-0.35)*[1 1 1 1]';
umin = -0.35*[1 1 1 1]';
```

Augment model

```
Atilde = [A B;
```

```
          zeros(m,n) eye(m)];
Btilde = [B;
          eye(m)];
Ctilde = [C zeros(p,m)];
```

Calculate sequential form

```
%% second part of the equations
[r, c] = size(Btilde);
ps = zeros(r*N,c);
for i=1:N
    ps(1+(i-1)*r:i*r,:) = Atilde^(i-1)*Btilde;
end
[~, c] = size(ps);

% C double bar
Cdbar = zeros(N*r,N*c);
for i = 0:(N-1)
    Cdbar(:,1+c*i:m+c*i) = [zeros(i*r,m);ps(1:end-i*r,:)];
end
[r, c] = size(Atilde);

% A double hat
Adhat = zeros(r*N,c);
for i = 1:N
    Adhat((i-1)*r+1:(i)*r,:)=Atilde^i;
end

RCell = repmat({R}, 1, N);
TCell = repmat({Q*Ctilde}, 1, N-1);
QCell = repmat({Ctilde'*Q*Ctilde}, 1, N-1);

Rdbar = blkdiag(RCell{:});
Qdbar = blkdiag(QCell{:},Ctilde'*P*Ctilde);
Tdbar = blkdiag(TCell{:},P*Ctilde);
```

Compute H and F matrices

```
H = Cdbar.' * Qdbar * Cdbar + Rdbar;
F = [(Adhat.' * Qdbar * Cdbar); -Tdbar * Cdbar];
```

The experimental code which only uses states that are choosen follows ...

```
% idx = 1;
% for y=0:N-1
%     for z=[1,2,3,13,14,15,16]
%         Cdbar_tmp(idx,:) = Cdbar(y*(n+m)+z,:);
%         Adhat_tmp(idx,:) = Adhat(y*(n+m)+z,:);
%         idx = idx+1;
%     end
% end
% RCell = repmat({R}, 1, N);
```

```
% TCell = repmat({eye(3)*eye(3)}, 1, N-1);
% QCell = repmat({eye(3)'*eye(3)*eye(3)}, 1, N-1);
%
% Rdbar = blkdiag(RCell{:});
% Qdbar = blkdiag(QCell{:},eye(3)'*eye(3)*eye(3));
% Tdbar = blkdiag(TCell{:},eye(3)*eye(3));
%
% H = Cdbar_tmp.' * Qdbar * Cdbar_tmp + Rdbar;
% F = [(Adhat_tmp.' * Qdbar * Cdbar_tmp); -Tdbar * Cdbar_tmp];
```

Calculate constraints

This computation is only to constrain the inputs but is not used since we need to add constraint also on states like are the angles.

To prepare the matrixes we derived this form from following equation, for the maximum $G\Delta u <= x_{max} - S[x(t), u(t)]^T$ and for the minimum $-G\Delta u <= -x_{min} + S[x(t), u(t)]^T$. To limit the inputs we limit the input states, which are part of the system since we aougmented the states space representation matrices.

The matrix G is in bottom triangual shape ....

Hard constraints

```
G_out_max = [Cdbar];
S_out_max = [-Adhat];

% W_out_max = repmat([Inf, Inf, Inf, deg2rad(45) deg2rad(45) Inf 2.5882 2.5882 5 5 5 5
W_out_max = repmat([Inf, Inf, Inf, deg2rad(45) deg2rad(45) Inf 1 1 1.5 5 5 5 umax']',N,
% W_out_min = repmat(-[-Inf, -Inf, -Inf, -deg2rad(45) -deg2rad(45) -Inf -2.5882 -2.5882
W_out_min = repmat(-[-Inf, -Inf, -Inf, -deg2rad(45) -deg2rad(45) -Inf -1 -1 -1.5 -5 -5

G_out = [G_out_max;-G_out_max];
W_out = [W_out_max; W_out_min];
S_out = [S_out_max;-S_out_max];
```

Soft constraints

The clasic hard constrain is $G\Delta\, u <= x_{\max} - S\left[x(t), u(t)\right]^T$ but for the soft constraints we need to add another slack variable directly into the objective function and also change the constraints in a following way.

$$[G, -I] \cdot [\Delta\, u, m]^T <= W - S[x(t), u(t)]^T$$

It is also necessary to limit the size of the slack variable, therefore

$$\begin{bmatrix} G & -I \\ -G & -I \\ 0 & I \\ 0 & -I \end{bmatrix} \cdot [\Delta\, u, m]^T <= \begin{bmatrix} W_{\max} \\ W_{\min} \\ W_{s_{\max}} \\ W_{s_{\min}} \end{bmatrix} - \begin{bmatrix} S \\ -S \\ 0 \\ 0 \end{bmatrix} [x(t) \quad u(t)]^T$$

```
G_out_soft = [
    Cdbar  -eye(size(Cdbar,1));
   -Cdbar   -eye(size(Cdbar,1));
    zeros(size(Cdbar,1),N*size(Btilde,2))      eye(size(Cdbar,1));
```

```
    zeros(size(Cdbar,1),N*size(Btilde,2))      -eye(size(Cdbar,1))
    ];

W_out_soft = [
    W_out_max;
    W_out_min;
    10*ones(1,size(Btilde,1)*N)';
    zeros(1,size(Btilde,1)*N)'
    ];

S_out_soft = [
    -Adhat;
    Adhat;
    zeros(size(Btilde,1),size(Btilde,1)*N)';
    zeros(size(Btilde,1),size(Btilde,1)*N)'
    ];

if use_soft_constraints
    M = Qdbar;                 % weight on slack variables
    L = ones(1,16*N);          % missdesign of the slack variables

    H = blkdiag(H,M);
end
```

Slew rate

It is also possible to limit the input signal. Moreover, in this case by limiting the input values we limit the change of the input, because the system was ugmented and as input is used the change of the input.

```
ub = 0.1*ones(m*N,1);    % Upper bounds
lb = -0.1*ones(m*N,1);   % Lower bounds

G_in = [eye(N*4);-eye(N*4)];
W_in = [ub;-lb];
S_in = [zeros(N*m*2,n),[-repmat(zeros(m),N,1);repmat(zeros(m),N,1)]];
```

Comlete the matrices and prepare them for the optimisation.

```
if use_soft_constraints
    G = [G_out_soft];
    W = [W_out_soft];
    S = [S_out_soft];
else
    G = [G_out];
    W = [W_out];
    S = [S_out];
end

if use_slew_rate
    G = [G; G_in]
```

```
    W = [W; W_in]
    S = [S; S_in]
end
```

short_model

Experimental code which decrease the number of states of the UAV's model

```
N_states = 12;
% A_short = A(1:N_states,1:N_states);
% B_short = B(1:N_states,:);
% C_short = C(1:N_states,1:N_states);
%
% n_short = size(A_short,1); % number of states
% m = size(B_short,2); % number of inputs
% p_short = size(C_short,1); % number of outputs
%
% Q = Q(1:N_states,1:N_states);
% Q = diag([30 30 40 1 1 50 1 1 1 1 1 1]);
% P = P(1:N_states,1:N_states);
%
% % Augmented model
% Atilde = [A_short B_short;
%           zeros(m,n_short) eye(m)];
% Btilde = [B_short;
%           eye(m)];
% Ctilde = [C_short zeros(p_short,m)];
%
% %% second part of the equations
% [r, c] = size(Btilde);
% ps = zeros(r*N,c);
% for i=1:N
%     ps(1+(i-1)*r:i*r,:) = Atilde^(i-1)*Btilde;
% end
% [~, c] = size(ps);
%
% % C double bar
% Cdbar = zeros(N*r,N*c);
% for i = 0:(N-1)
%     Cdbar(:,1+c*i:m+c*i) = [zeros(i*r,m);ps(1:end-i*r,:)];
% end
% [r, c] = size(Atilde);
%
% % A double hat
% Adhat = zeros(r*N,c);
% for i = 1:N
%     Adhat((i-1)*r+1:(i)*r,:)=Atilde^i;
% end
%
% RCell = repmat({R}, 1, N);
% TCell = repmat({Q*Ctilde}, 1, N-1);
% QCell = repmat({Ctilde'*Q*Ctilde}, 1, N-1);
%
```

```
% Rdbar = blkdiag(RCell{:});
% Qdbar = blkdiag(QCell{:},Ctilde'*P*Ctilde);
% Tdbar = blkdiag(TCell{:},P*Ctilde);
%
% H = Cdbar.' * Qdbar * Cdbar + Rdbar;
% F = [(Adhat.' * Qdbar * Cdbar); -Tdbar * Cdbar];
```

Simulation

Generate the reference signal

```
    if use_wamv_trajectory == true
        [Duration, ref] = WamvReferenceTrajectory(N,z_offset);
        t = 0:Ts:Duration; % time vector
    else
        Duration = 20; % Final time
        t = 0:Ts:Duration; % time vector
        ref = QuadrotorReferenceTrajectory([t, Duration+Ts:Ts:Duration+(N*Ts)])';
    end
```

Intial conditions

```
x0 = zeros(n,1);
x0(1) = ref(1,1);
x0(2) = ref(1,2);
x0(3) = ref(1,3);

x0(8) = -1.5;

u0 = zeros(m,1);
xtilde0 = [x0; u0];
```

Initialize the matrices where the simulation results will be stored

```
x = zeros(n, numel(t));
y = zeros(p, numel(t));
u = zeros(m, numel(t));

x(:,1) = x0;
y(:,1) = C*x0;
uprev = u0;
```

Initialize OSQP (uncomment if OSQP is used)

```
mosqp = osqp;
settings = mosqp.default_settings();
settings.verbose = 0;
settings.eps_abs = 1e-05;
settings.eps_rel = 1e-05;
settings.polish = 1;
settings.check_termination = 10;
settings.max_iter = 4000;
% settings.warm_start = 1;
% settings.time_limit = 0.018;
```

```
if use_soft_constraints
    mosqp.setup(H/2+H'/2, [[x(1:N_states,i)' uprev' vec(ref(1:N, 1:N_states)')']*F, L],
else
    mosqp.setup(H/2+H'/2, [x(1:N_states,i)' uprev' vec(ref(1:N, 1:N_states)')']*F, G, -
end

%% in case of qpOASES
% myOptions = qpOASES_options;
% myOptions.maxIter = 1000;
% myOptions.maxCpuTime = 0.1;
```

Simulation for-cycle

```
% hbar = waitbar(0,'Simulation Progress');
for i = 1:(numel(t)-1)
    tic;
    % Reference signal over the current prediction window
    rk = ref(i:i+N-1, 1:N_states)';

    % Solve the QP
    % by quadprog from Optimization Toolbox ...
%     qp_res = quadprog(H/2+H'/2, [x(:,i)' uprev' rk(:)']*F, G, W+S*[x(:,i); uprev], [
%     qp_res = quadprog(H/2+H'/2, [x(:,i)' uprev' rk(:)']*F, G, W+S*[x(:,i); uprev], [

    % or by qpOASES ...
%     [qp_res,fval,exitflag,iter,lambda,auxOutput] = qpOASES(H/2+H'/2, F'*[x(:,i)' upre

    % or by OSQP
    if use_soft_constraints
        mosqp.update('q', [[x(1:N_states,i)' uprev' rk(:)']*F, L], 'u', W+S*[x(:,i); up
    else
        mosqp.update('q', [x(1:N_states,i)' uprev' rk(:)']*F, 'u', W+S*[x(:,i); uprev])
    end
    results = mosqp.solve();
    qp_res = results.x;

    % Extract the incement in u
    duk = qp_res(1:m,1);

    u(:,i) = uprev + duk;

    % Simulate the system
    x(:,i+1) = A*x(:,i) + B*u(:,i);
    y(:,i+1) = C*x(:,i+1);

    uprev = u(:,i);
%     waitbar(i*Ts/Duration,hbar);

    time_c(i) = toc;
end
% close(hbar)
```

Summation of the working point

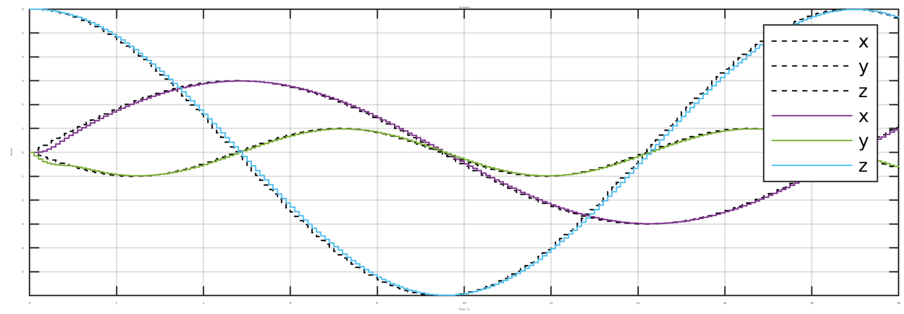
```
u = u_work'+u;
ref(:,3) = ref(:,3)-z_offset;
```

Plot the results (just for your convenience)

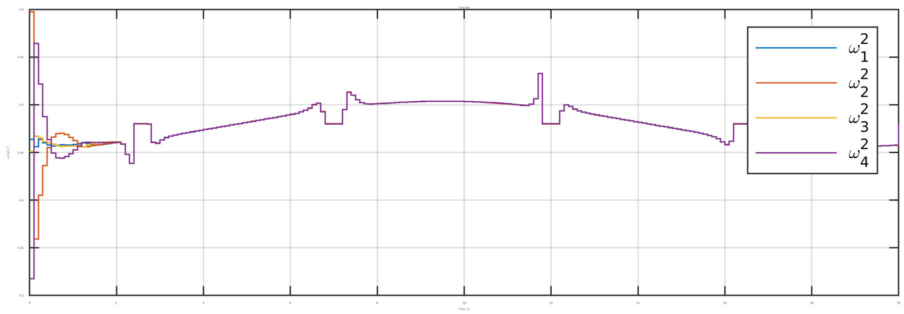
```
figure(1)
x1 = subplot(211);
stairs(t, ref(1:numel(t),1:3), 'k--');
hold on
% stairs(t, ref(1:numel(t),4:6), 'k:');
stairs(t, y(1:3,:)');
hold off
title('Outputs')
xlabel('Time (s)')
ylabel('States')
grid on
axis tight
legend('x','y','z', 'x','y','z')

x2 = subplot(212);
stairs(t, u');
title('Controls')
xlabel('Time (s)')
ylabel('$\omega (rad/s)^2$')
grid on
legend(' \omega_1^2 ',' \omega_2^2 ',' \omega_3^2 ',' \omega_4^2 ')

linkaxes([x1 x2],'x');
```

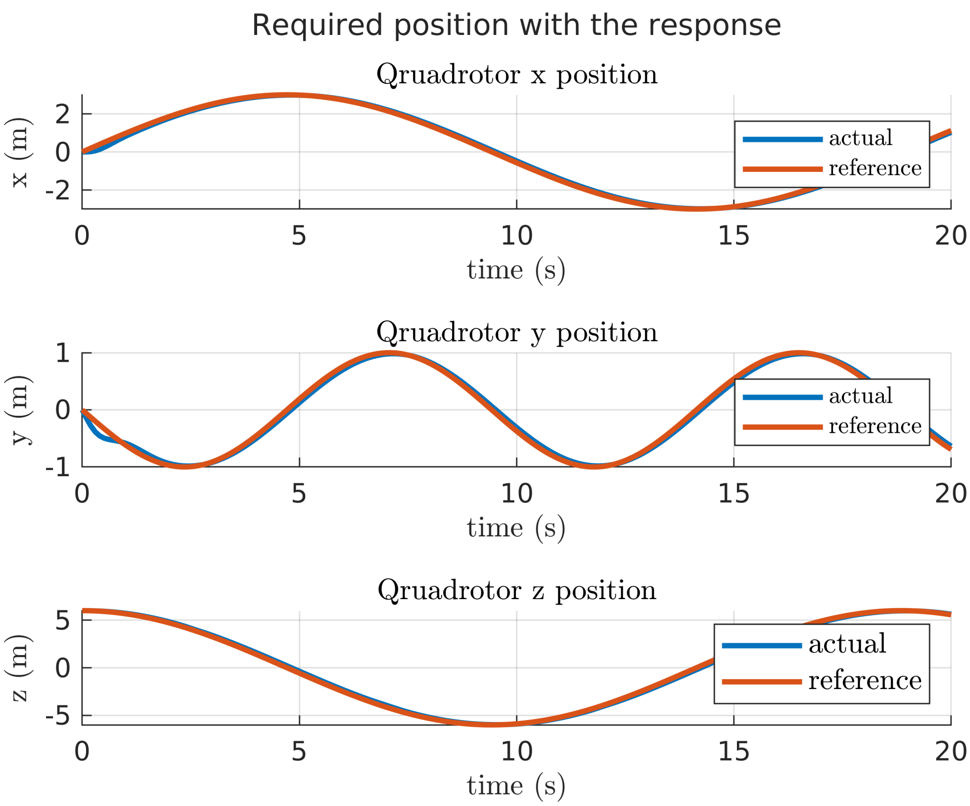


x
y
z
x
y
z

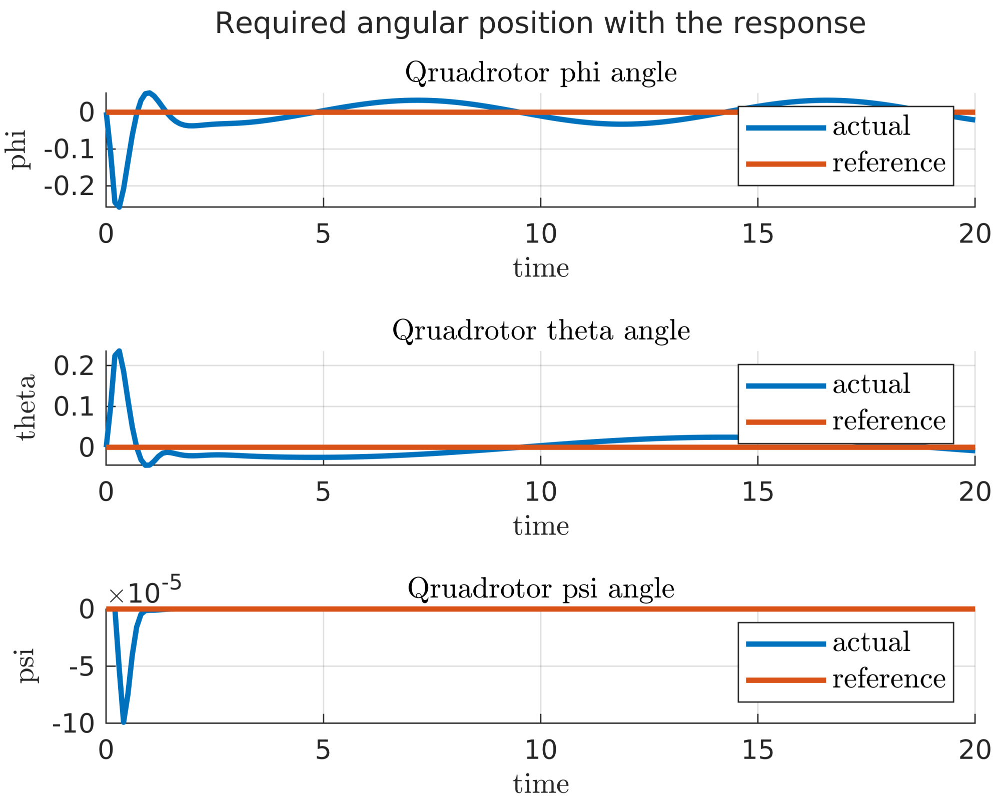


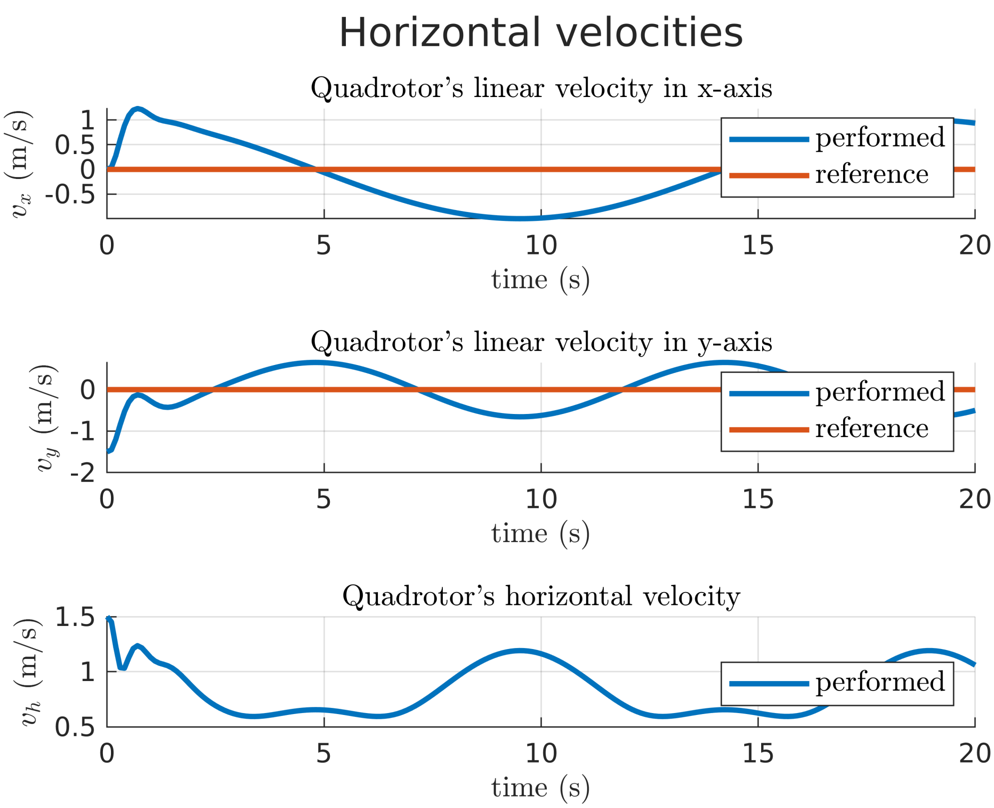
ω_1^2
ω_2^2
ω_3^2
ω_4^2

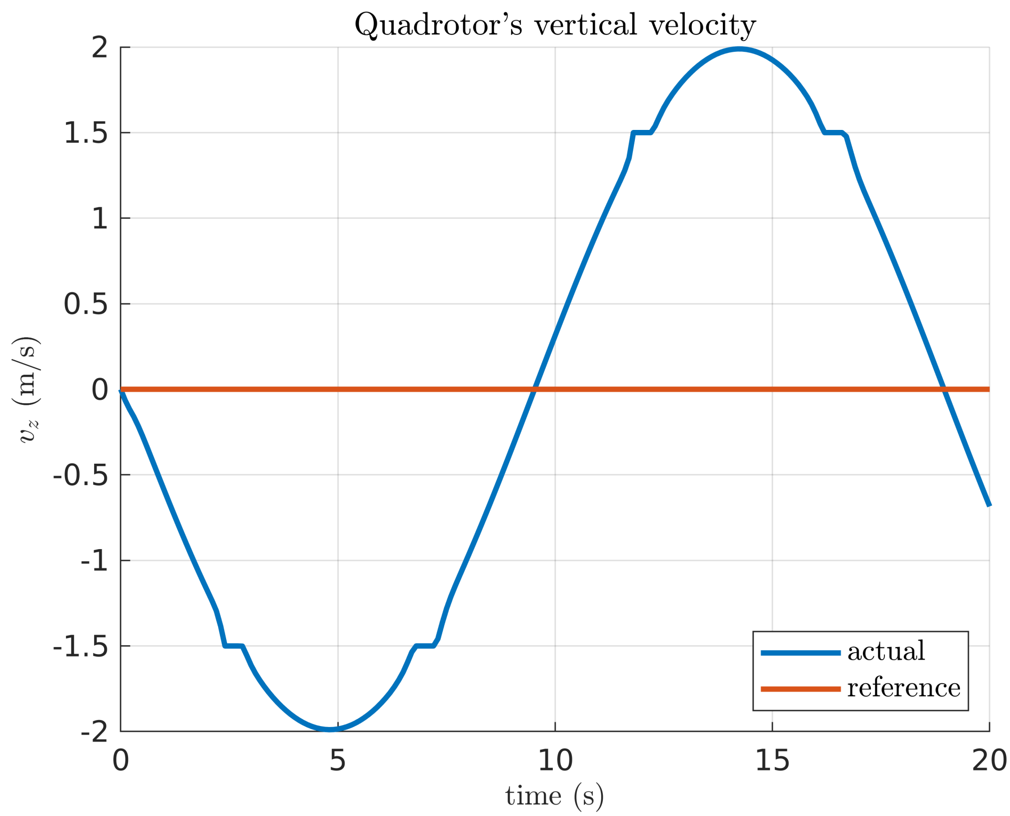

```
plotResult;
```

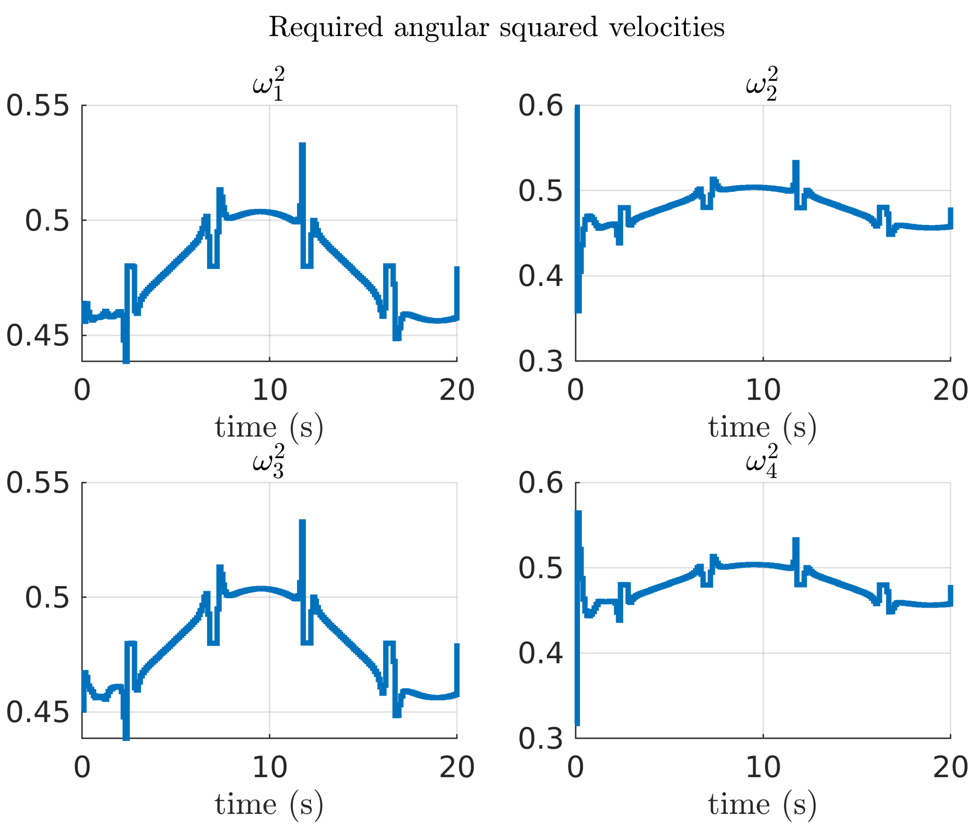


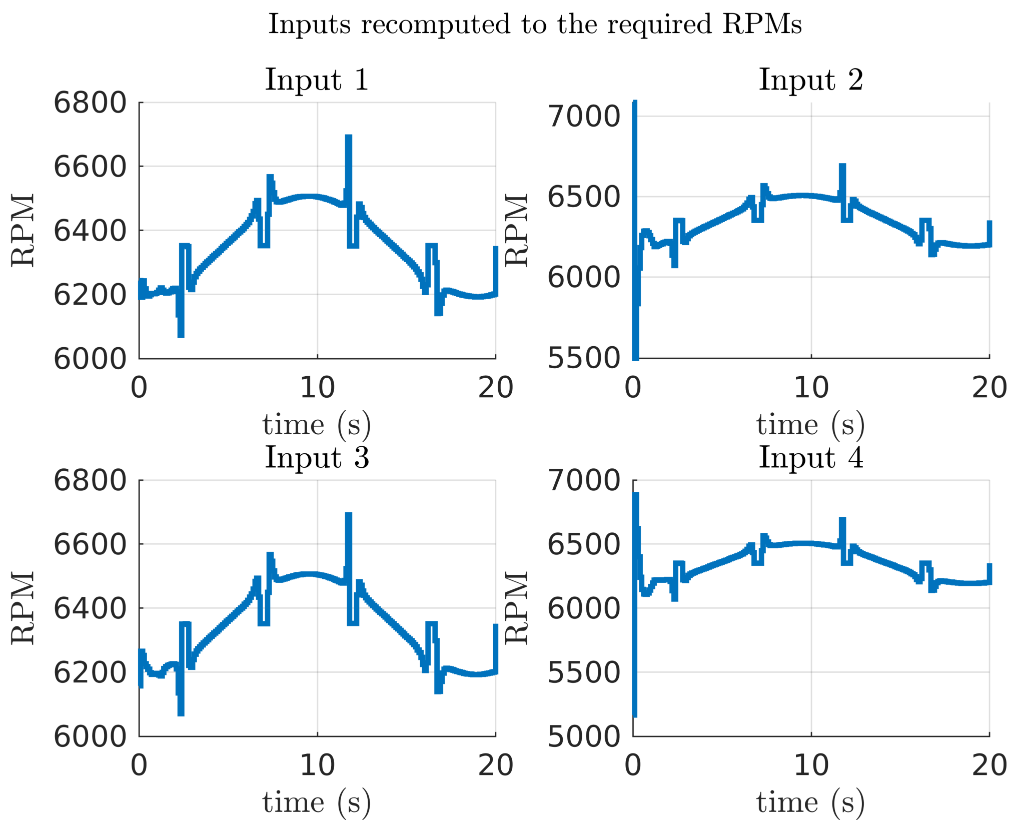
Required position with the response
Qruadrotor x position
x (m)
2
0
-2
0
5
10
15
20
time (s)
actual
reference
Qruadrotor y position
y (m)
1
0
-1
0
5
10
15
20
time (s)
actual
reference
Qruadrotor z position
z (m)
5
0
-5
0
5
10
15
20
time (s)
actual
reference


Required angular position with the response
Qruadrotor phi angle
phi
time
actual
reference
Qruadrotor theta angle
theta
time
actual
reference
Qruadrotor psi angle
psi
×10^-5
time
actual
reference


Horizontal velocities
Quadrotor's linear velocity in x-axis
v_x (m/s)
time (s)
performed
reference
Quadrotor's linear velocity in y-axis
v_y (m/s)
time (s)
performed
reference
Quadrotor's horizontal velocity
v_h (m/s)
time (s)
performed

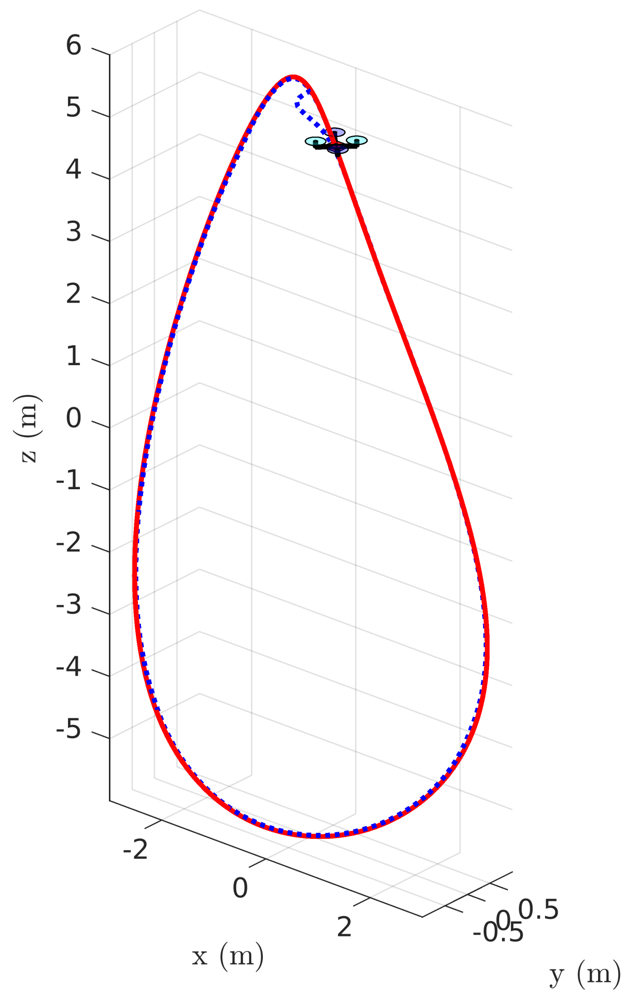

Quadrotor's vertical velocity
v_z (m/s)
time (s)
actual
reference

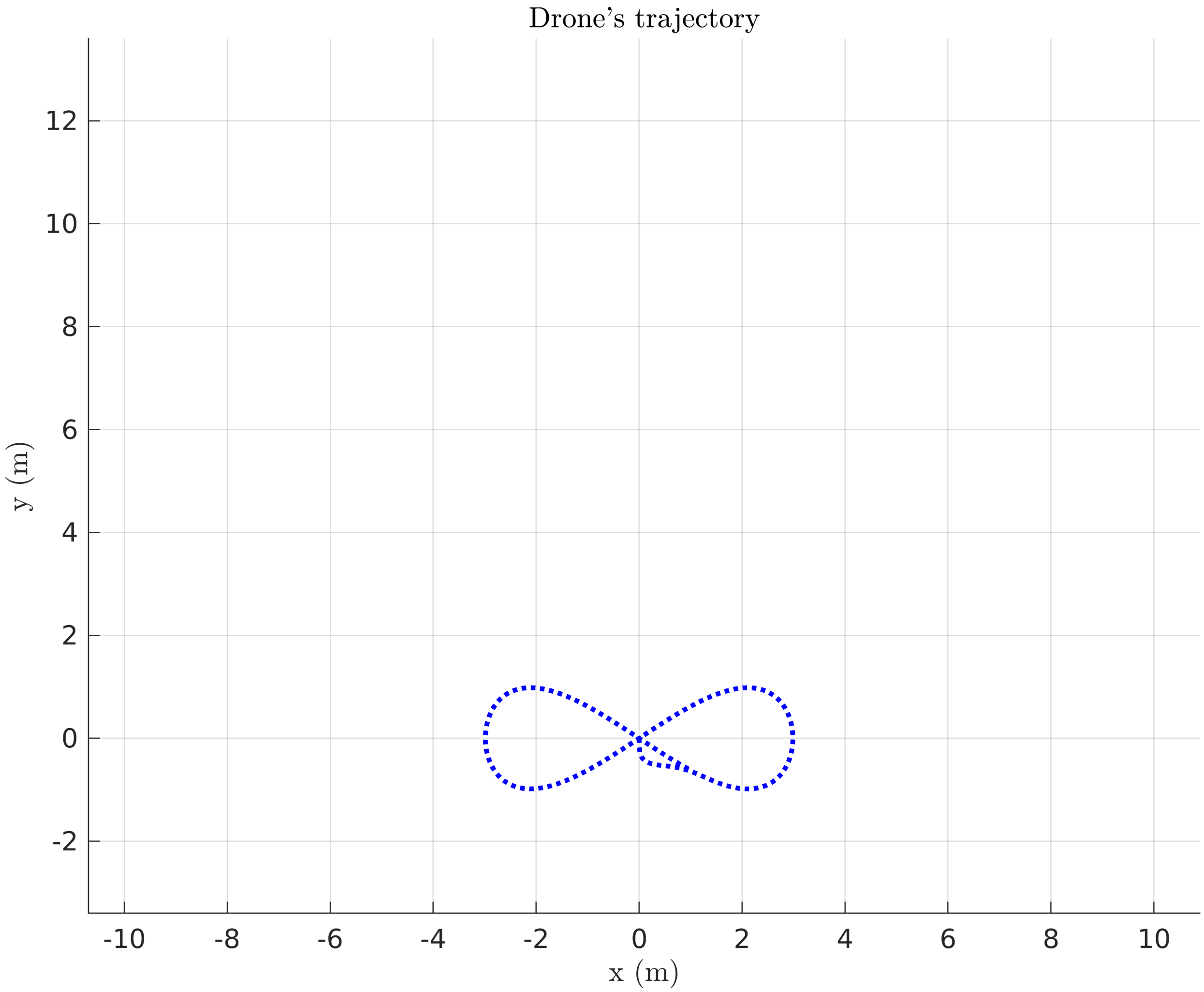

Required angular squared velocities
ω_1^2
ω_2^2
ω_3^2
ω_4^2
time (s)
time (s)
time (s)
time (s)




```
% animate;
```

```
drone_Animation(y(1,:)',y(2,:)',y(3,:)',y(4,:)',y(5,:)',y(6,:)', t, ref)
```


6
5
4
3
2
1
0
-1
-2
-3
-4
-5
z (m)
-2
0
2
x (m)
-0.5
0
0.5
y (m)


Drone's trajectory
y (m)
x (m)
12
10
8
6
4
2
0
-2
-10
-8
-6
-4
-2
0
2
4
6
8
10