Q. Interprete $\frac{dy}{dx}$ geometrically.

Sol^n:

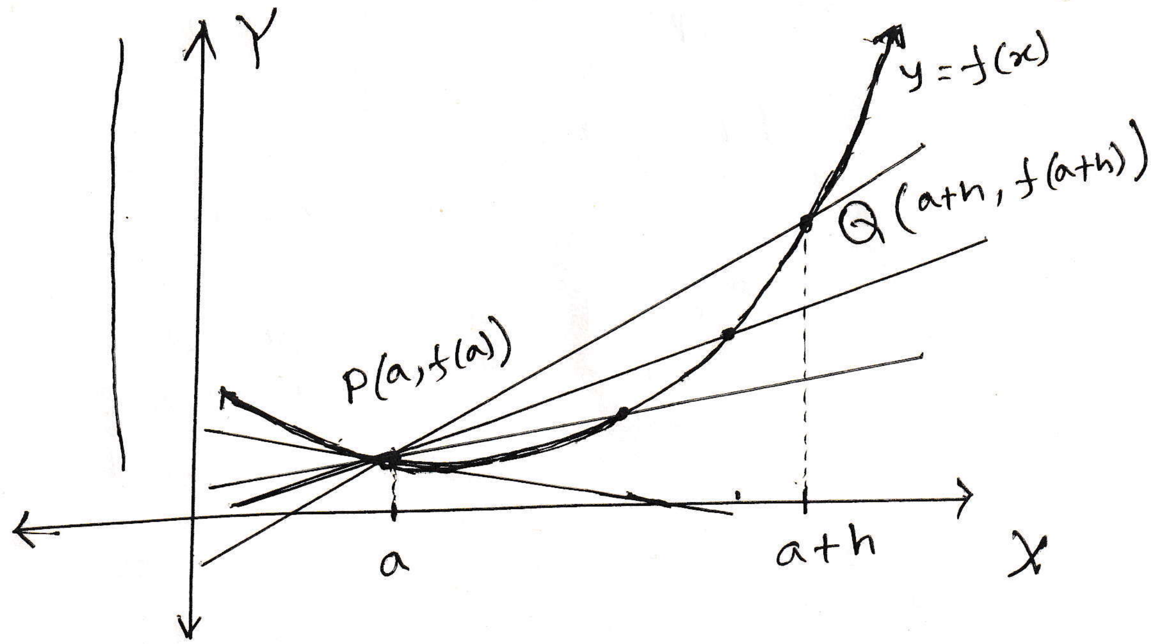

Fig - 1

From Fig -1 we see that $\frac{f(a+h)-f(a)}{(a+h)-a} = \frac{f(a+h)-f(a)}{h}$ is the slop of the line joining P(a, f(a)) and Q(a+h, f(a+h)). As $h \to 0$, P remains fixed and Q moves along the curve toward P. Thus the line becomes a tangent line to the curve at P(a, f(a)).

Hence $f'(a) = \left.\frac{dy}{dx}\right|_{x=a} = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$ is the slope of the tangent line at P to the curve $y = f(x)$.

—x—

Q. Find $\frac{dy}{dx}$ from $(\cos x)^y = (\sin y)^x$.

<u>Solⁿ:</u> Given,

$$(\cos x)^y = (\sin y)^x$$

$$\therefore \quad y \log(\cos x) = x \log(\sin y)$$

$$\therefore \quad y \cdot \frac{1}{\cos x}(-\sin x) + \log(\cos x)\frac{dy}{dx}$$

$$= x \cdot \frac{1}{\sin y} \cdot \cos y \frac{dy}{dx} + \log(\sin y)$$

$$\Rightarrow \{\log(\cos x) - x \cot y\}\frac{dy}{dx} = \log(\sin y) + y \tan x$$

$$\therefore \quad \frac{dy}{dx} = \frac{\log(\sin y) + y\tan x}{\log(\cos x) - x\cot y}$$ <u>Answer</u>

Q. Find $\frac{dy}{dx}$ from $x^3y + xy^3 = 2$.

<u>Solⁿ:</u> ~~Sol~~ Given

$$x^3y + xy^3 = 2$$

$$\therefore \quad 3x^2y + x^3\frac{dy}{dx} + y^3 + 3xy^2\frac{dy}{dx} = 0$$

$$\Rightarrow (x^3 + 3xy^2)\frac{dy}{dx} = -(y^3 + 3x^2y)$$

$$\therefore \quad \frac{dy}{dx} = \frac{-y(y^2+3x^2)}{x(x^2+3y^2)}$$ <u>Answer</u>

Q. Differentiate $\tan^{-1}\frac{\sqrt{1+x^2}-1}{x}$ with respect to $\tan^{-1}x$.

Sol^n: Let $u = \tan^{-1}\frac{\sqrt{1+x^2}-1}{x}$ and $v = \tan^{-1}x$

We have to find $\frac{du}{dv}$.

Put $x = \tan\theta$ then $\theta = \tan^{-1}x$

$$\therefore \quad u = \tan^{-1}\frac{\sqrt{1+\tan^2\theta}-1}{\tan\theta}$$

$$= \tan^{-1}\left(\frac{\sec\theta - 1}{\tan\theta}\right)$$

$$= \tan^{-1}\left(\frac{\frac{1}{\cos\theta} - 1}{\frac{\sin\theta}{\cos\theta}}\right)$$

$$= \tan^{-1}\left(\frac{1-\cos\theta}{\cancel{\cos\theta}} \times \frac{\cancel{\cos\theta}}{\sin\theta}\right)$$

$$= \tan^{-1}\left(\frac{2\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\right)$$

$$= \tan^{-1}\tan\frac{\theta}{2}$$

$$= \frac{\theta}{2}$$

$$= \frac{1}{2}\tan^{-1}x = \frac{1}{2}v$$

$$\therefore \frac{du}{dv} = \frac{1}{2}$$ Answer

Q. Differentiate $\tan^{-1}x$ with respect to x^2.

Sol^n: Let $u = \tan^{-1}x$ and $v = x^2$.

$$\therefore \frac{du}{dx} = \frac{1}{1+x^2} \quad \text{and} \quad \frac{dv}{dx} = 2x$$

$$\therefore \frac{du}{dv} = \frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{\frac{1}{1+x^2}}{2x} = \frac{1}{2x(1+x^2)}.$$

Answer

. Differentiate $y = \cos\{2\sin^{-1}(\cos x)\}$ with respect to x.

Sol^n: Given $y = \cos\{2\sin^{-1}(\cos x)\}$

$2\sin^{-1}x$

$= \sin^{-1}(2x\sqrt{1-x^2})$

Put $\cancel{z = \cos x}$ $\sin^{-1}(\cos x) = z$

$$\therefore \cos x = \sin z$$

Now
$$\sin 2z = 2\sin z\cos z$$
$$= 2\sin z\sqrt{1-\sin^2 z}$$
$$= 2\cos x\sqrt{1-\cos^2 x}$$
$$= 2\cos x\sin x$$
$$= \sin 2x$$

$$\therefore 2z = \sin^{-1}\sin 2x = 2x$$

$$\therefore 2\sin^{-1}(\cos x) = 2x$$

$$\therefore y = \cos 2x \qquad \therefore \frac{dy}{dx} = -2\sin 2x$$

Ans

. Differentiate $y = x^{\sin^{-1}x}$ with respect to $\sin^{-1}x$.

Sol^n: Given $y = x^{\sin^{-1}x}$

$$\therefore \log y = \sin^{-1}x \log x$$

$$\therefore \frac{1}{y}\frac{dy}{dx} = \frac{\sin^{-1}x}{x} + \frac{\log x}{\sqrt{1-x^2}}$$

$$\therefore \frac{dy}{dx} = x^{\sin^{-1}x}\left(\frac{\sin^{-1}x}{x} + \frac{\log x}{\sqrt{1-x^2}}\right)$$

Let $z = \sin^{-1}x$

$$\therefore \frac{dz}{dx} = \frac{1}{\sqrt{1-x^2}}$$

$$\therefore \frac{dy}{dz} = \frac{\frac{dy}{dx}}{\frac{dz}{dx}}$$

$$= \frac{x^{\sin^{-1}x}\left(\frac{\sqrt{1-x^2}\sin^{-1}x + x\log x}{x\sqrt{1-x^2}}\right)}{\frac{1}{\sqrt{1-x^2}}}$$

$$= x^{\sin^{-1}x - 1}\left(\sqrt{1-x^2}\sin^{-1}x + x\log x\right)$$

Answer

Q. A circular plate of metal expands by heat so that its radius increases at the rate of 0.25 cm/sec. Find the rate at which the surface area is increasing when the radius is 7 cm.

Solⁿ: Let r be the radius of the circular plate.

Then the area of the circular plate is,

$$A = \pi r^2$$

Given that $\frac{dr}{dt} = 0.25$ cm/sec

$$\therefore \frac{dA}{dt} = 2\pi r \frac{dr}{dt} \quad cm^2/sec$$

$$= 2\pi(7).(0.25) \quad [\because r = 7 cm]$$

$$= 3.5\pi \ cm^2/sec$$

Answer

Q. If in the rectilinear motion of a particle $s = ut + \frac{1}{2} f t^2$, where u and f are constants, then prove that the velocity at time t is $u + ft$ and ~~the~~ acceleration is f.

Sol^n: Given, $s = ut + \frac{1}{2} f t^2$

$\therefore$ The velocity, $v = \frac{ds}{dt} = u + \frac{1}{2} f \cdot 2t$

$= u + ft$

Proved

Acceleration $= \frac{dv}{dt} = 0 + f$

$= f$

Proved