Limit of a function:

A function $f(x)$ is said to possesses a limit L at $x=a$ if for any $\varepsilon > 0$ (however small) there exists a $\delta > 0$ such that $|f(x)-L| < \varepsilon$ whenever $0 < |x-a| < \delta$.

Mathematically, $\lim_{x \to a} f(x) = L$.

Q. Use the precise definition of limit to show that $\lim_{x \to 2} (4x-5) = 3$.

Soln: Here $a=2$, $f(x) = 4x-5$ and $L=3$

Now, $|f(x)-L| = |4x-5-3| = |4x-8| = 4|x-2|$

Thus $|f(x)-L| < \varepsilon \Rightarrow 4|x-2| < \varepsilon \Rightarrow |x-2| < \frac{\varepsilon}{4}$

Choose $\delta = \frac{\varepsilon}{4}$, then

$|f(x)-L| < \varepsilon \Rightarrow |x-a| < \delta$

Thus $\lim_{x \to a} f(x) = L$

i.e. $\lim_{x \to 2} (4x-5) = 3$.