Function: A function $f: X \longrightarrow Y$ is a correspondance by which each element of X corresponds to a unique element of Y.

Domain: If $f: X \to Y$ is a function, then the set X is called the domain of the function f. It is denoted by D_f.

Range: If $f: X \longrightarrow Y$ is a function, then the set $\{f(x) : x \in X\}$ is called the range of the function f. It is denoted by R_f.

Note: If $A, B \subseteq \mathbb{R}$ and $f: A \to B$ then the domain of the ~~dh~~ definition A is the set of all real numbers for which f is defined. The range is the set $\{f(a) : a \in A\}$.

2. Find the domain of the definition and range of the following functions. Also draw their graphs

(a) $f(x) = x$, (b) $f(x) = |x|$, (c) $f(x) = 3$, (d) $f(x) = 2x - 1$

(e) $f(x) = [x]$, (f) $f(x) = |x+1| + |x|$, (g) $f(x) = \frac{|x|}{x}$,

(h) $f(x) = \begin{cases} x+2 & \text{if } -1 < x < 0 \\ x & \text{if } 0 \leq x < 1 \end{cases}$,

(i) $f(x) = \begin{cases} 2-x & \text{if } 0 < x < 2 \\ x-1 & \text{if } 2 \leq x < 4 \end{cases}$.

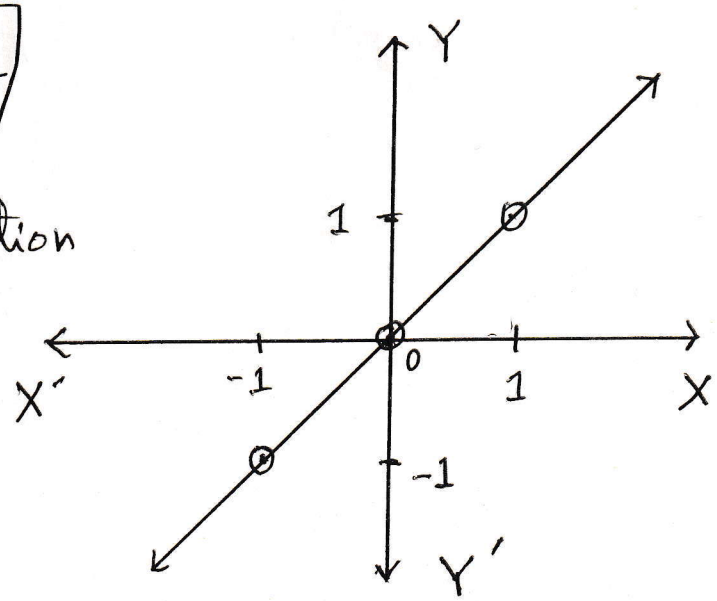
Soln: (a) Given, $f(x) = x$

$\therefore D_f = \mathbb{R}$ and $R_f = \mathbb{R}$.

Now, form a table for different values of x and corresponding values of $y = f(x)$.

x	-1	0	1
$y = f(x)$	-1	0	1

The graph of the function is as follows:



(b) Given, $f(x) = |x| = \begin{cases} x & \text{if } x > 0 \\ 0 & \text{if } x = 0 \\ -x & \text{if } x < 0 \end{cases}$

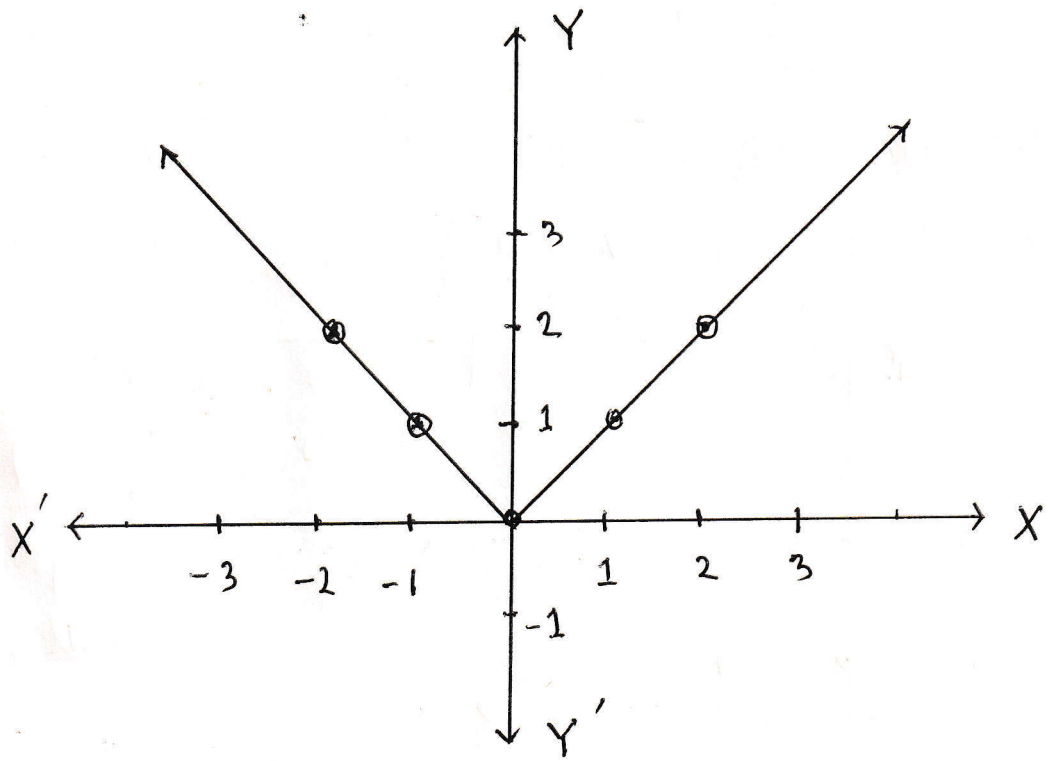
Domain of the function $D_f = \mathbb{R}$

Range of the function $R_f = [0, \infty)$

Form a table for different values of x and corresponding values of $y = f(x)$.

x	-2	-1	0	1	2
$y = f(x)$	2	1	0	1	2

The graph of the function is as follows:



(c) Given, $f(x) = 3$.

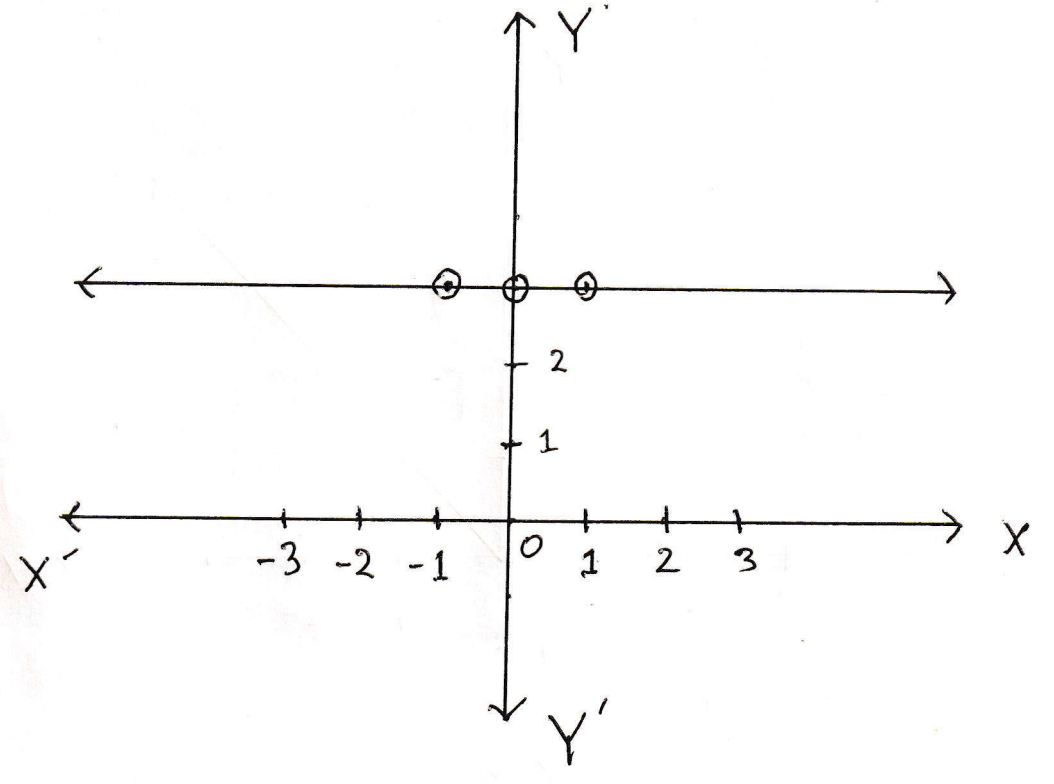
Domain of the function $D_f = \mathbb{R}$

Range of the function $R_f = \{3\}$

Form a table for different values of x and corresponding values of $y = f(x)$.

x	-1	0	1
$y = f(x)$	3	3	3

The graph is as follows:



(d) Given $f(x) = 2x - 1$

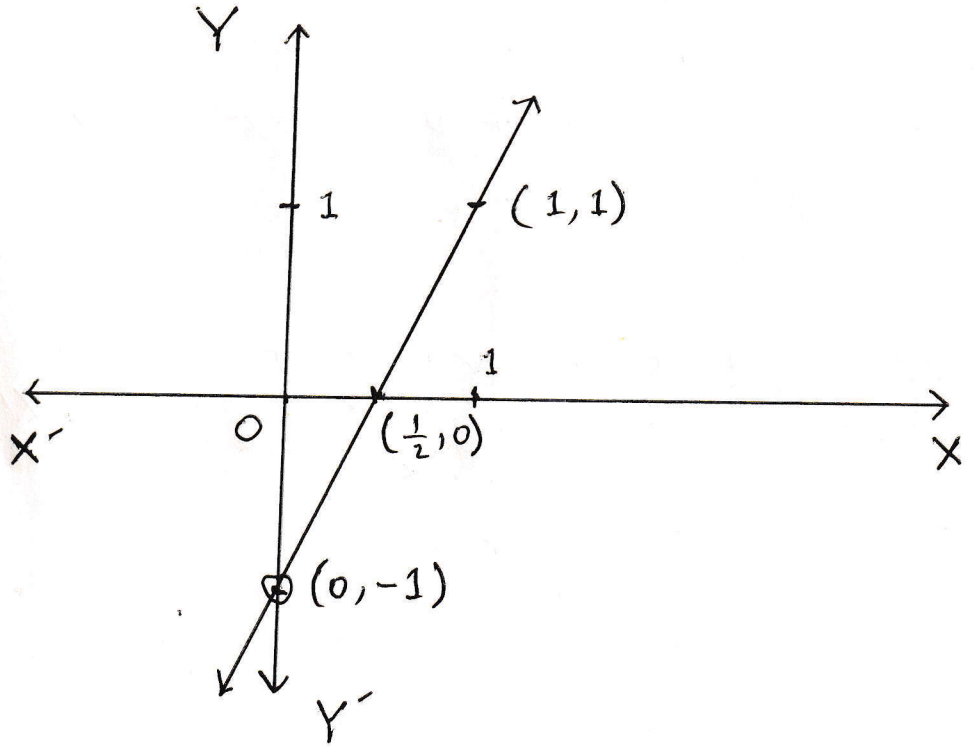
Domain of the function $D_f = \mathbb{R}$.

Range of the function $R_f = \mathbb{R}$.

Form a table for different values of x and corresponding values of $y = f(x)$.

x	$\frac{1}{2}$	0	1
$y = f(x)$	0	-1	1

The graph is as follows:



(e) $f(x) = [x]$ = The greatest integer less than or equal to x.

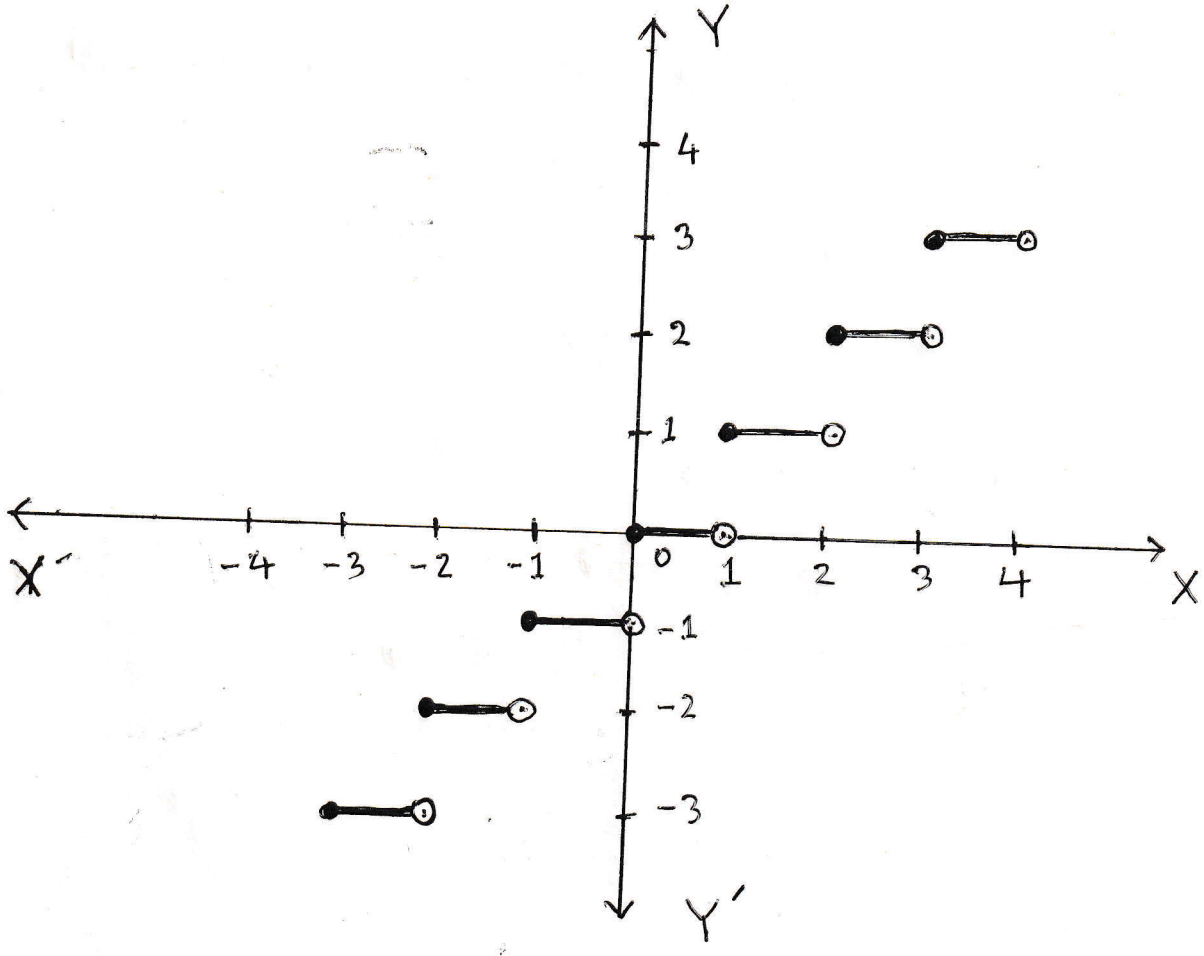
Domain of the function $D_f = \mathbb{R}$

Range of the function $R_f = \mathbb{Z}$

Form a table for different values of x and the corresponding values $y = f(x)$.

x	-3	-2.5	-2.1	-2	-1.5	-1.1	-1	-.5	0	.5	.9	1	1.5	2	2.5	3	3.9
$y = f(x)$	-3	-3	-3	-2	-2	-2	-1	-1	0	0	0	1	1	2	2	3	3

The graph is as follows:



(f) $f(x) = |x+1| + |x|$

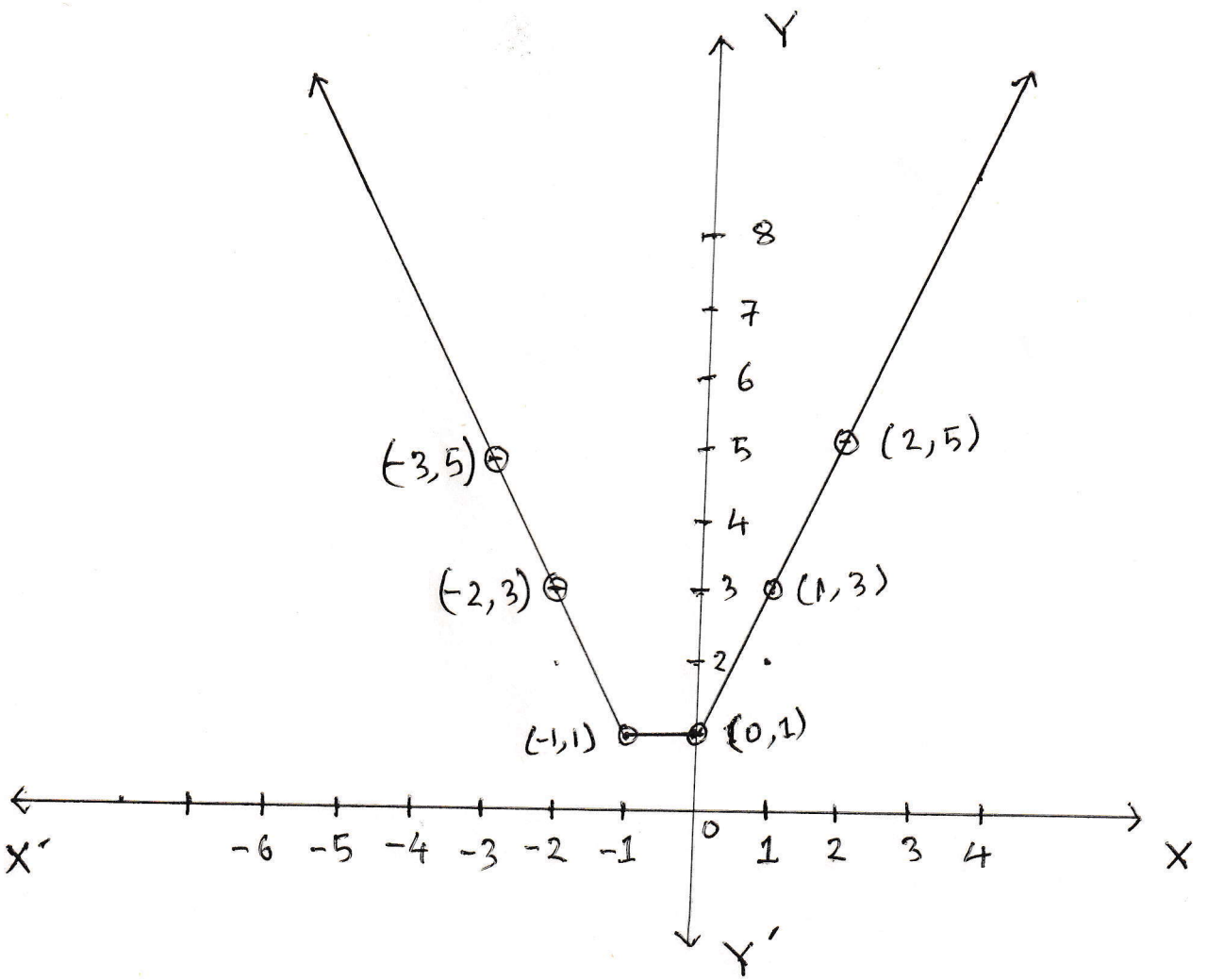
Domain of the function $D_f = \mathbb{R}$

Range of the function $R_f = [1, \infty)$.

Form a table for different values of x and the corresponding values of $y = f(x)$.

	-3	-2	-1	-0.5	0	1	2	
	5	3	1	1	1	3	5	

The graph is as follows:



(g) $f(x) = \frac{|x|}{x} = \begin{cases} 1, & \text{if } x > 0 \\ -1, & \text{if } x < 0 \end{cases}$

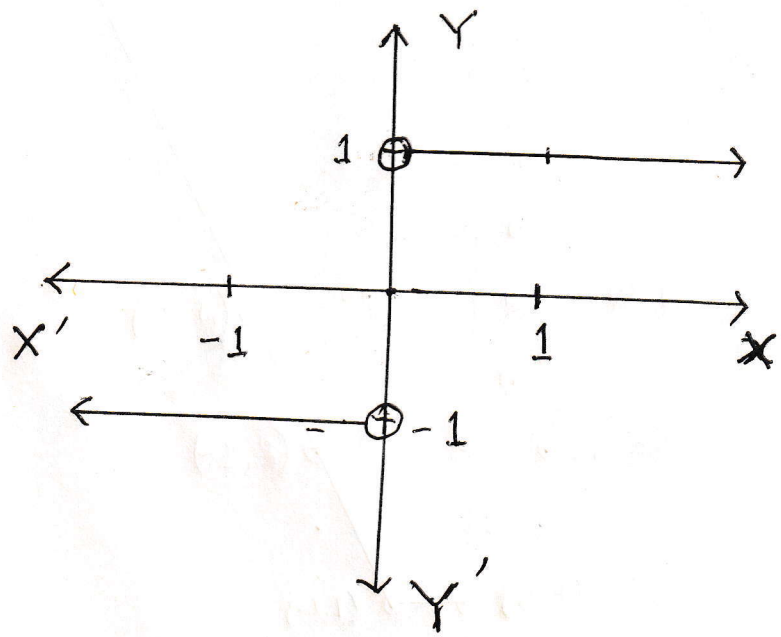
Domain of the function $D_f = \mathbb{R} \setminus \{0\}$.

Range of the function $R_f = \{-1, 1\}$.

Form a table for different values of x and the corresponding values of $y = f(x)$

x	-1	$-.01$	0.1	1
$y = f(x)$	-1	-1	1	1

The graph is as follows:



(h) Soln: Given $f(x) = \begin{cases} x+2 & \text{if } -1 \le x < 0 \\ x & \text{if } 0 \le x < 1 \end{cases}$

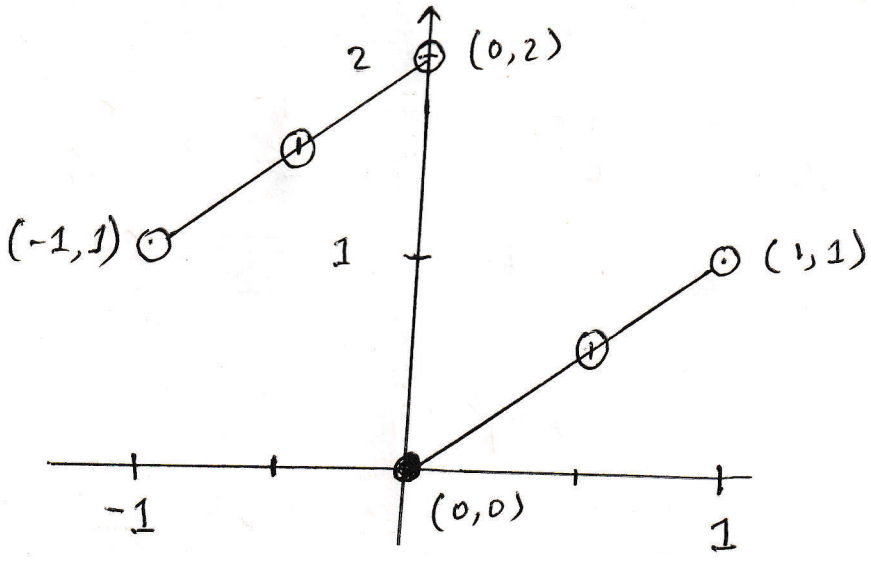
Domain of the function $D_f = (-1, 0) \cup [0, 1) = (-1, 1)$

Range of the function $R_f = (1, 2) \cup [0, 1)$

$= [0, 2) \setminus \{1\}$

Form a table for different values of x and the corresponding values of $y = f(x)$

x	-0.9	-.5	-.1	0	0.5	0.9
$y = f(x)$	1.1	1.5	1.9	0	0.5	0.9



(i) Soln: Given, $f(x) = \begin{cases} 2-x & \text{if } 0 < x < 2 \\ x-1 & \text{if } 2 \leq x < 4 \end{cases}$

Domain of the function $D_f = (0,2) \cup [2,4) = (0,4)$

Range of the function $R_f = (0,2) \cup [1,3) = (0,3)$

Form the following table for different values of x and the corresponding values of $y = f(x)$.

x	0.1	1	1.9	2	3	3.9
$y = f(x)$	1.9	1	0.1	1	2	2.9