Section B

Test of hypothesis:

Question

1. → Contingency table ——— (1)
2. → General test of hypothesis
3. → Application

Contingency table:

$r \geq 2$
$c \geq 2$

	(rA)	$\dots (iA) \dots$	$(2A)$	$(1A)$

(2)

- row, column heading মান না থাকলে, বলতে হবে, row and column have (mutually exclusive).

Contingency Table (CT):

Q. Manifold classification?

$(A_1 B_1)$ ← bracket numerical value

	A_1	A_2	...	A_j	...	A_r	
B_1	(A_1B_1)						(B_1) ← marginal total
B_2							(B_2)
⋮							⋮
B_i							(B_i)
⋮							⋮
B							(B)
	(A_1)	(A_2)	...	(A_j)	...	(A_r)	N

observed frequency

$$\sum_{i=1}^{r} (A_i) = \sum_{j=1}^{s} (B_j) = N \text{ (grand total)}$$

* row, column এর মধ্যে সম্পর্ক বের করতে হবে।

H_0 = Row and column are independent

✳ Test statistic

✳ Contingency table এ χ^2 test করা।
or,
attribute test χ^2 " "

✳ Expected frequency বের করতে হবে।

↑ r × s সংখ্যক।

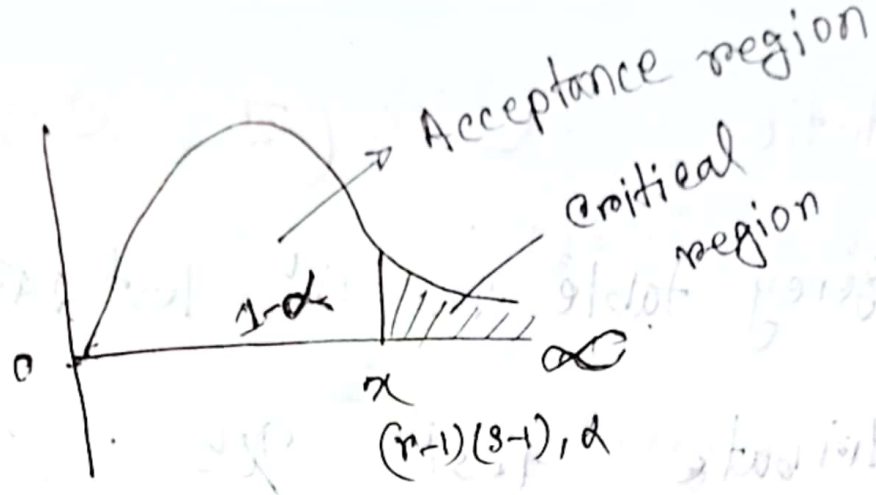
$E_{11} = \frac{(A_1)(B_1)}{N}$

$= \frac{120 \times 150}{1000}$

$= 18$

$\chi^2 = \sum_{i=1}^{r} \sum_{j=1}^{s} \frac{\{(A_i B_i) - (A_i B_i)_0\}^2}{(A_i B_i)_0}$ $\sim \chi^2_{dist}$

$\sim \chi^2_{(r-1, s-1)}$



$$\int_0^{\infty} f(\chi^2)\, d\chi^2 = 1$$

$$\int_{\chi}^{\infty} f(\chi^2)\, d\chi^2 = \alpha \quad ; \quad < 1$$

level of significance :

α 100 % $= 0.05 \times 100 = 5\%$ ← practical

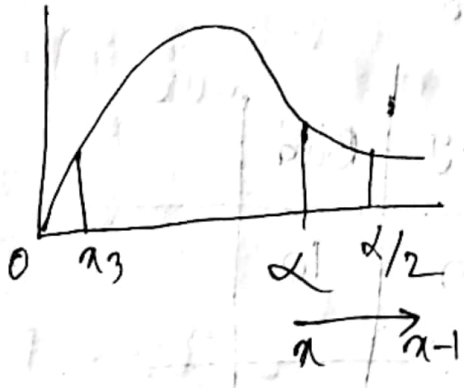
α ← theoritical.

Tab. χ^2

D.F → $(r-1)(s-1)$

level of significance → α

- Cal $\chi^2 \geq$ Tabulated $\chi^2_{(r-1)(s-1),\alpha}$



(i) $\int_{\chi_{-1}}^{\alpha} f(\chi^2)\, d\chi^2 = \alpha/2$

(ii) $\int_{0}^{\chi_3} f(\chi^2)\, d\chi^2 = \alpha/2$

2×2 contingency table:

Yates' correction :

2×2 table-এ a, b, c, d এর যেকোনো একটি 5 এর চেয়ে ছোট হয়, তবে 2×2 টুল ফলো করে না।

Ex.

Gender	Good	Avg	Bad
Male	13	9	10
Female	12	15	17

H.S

Q. Do you think health status is N=76 student independent of gender ?

$\alpha = 0.05$

Soln:

	Good	Avg	Bad	
Male	13	9	10	32
Female	12	15	17	44
	25	24	27	76

Hypothesis:

H_0 : There is no relationship between health status and Gender

H_1 : There is a relationship between health status and Gender

Test statistic:

Soln: Question — 2:

At first we have set up the following hypothesis

H_0 : There is no relationship between recommendation and types of burger.

vs. H_1 : H_0 is not true.

The test statistic:

$$\chi^2 = \sum_{i=1}^{3} \sum_{j=1}^{2} \frac{\sum (O_{ij} - E_{ij})^2}{E_{ij}}$$

Here,

E_{11} =

Accept null

$$P\{x \in \bar{W} \mid H_1\} = \beta$$

$$P\{x \in W \mid H_1\} = 1-\beta$$

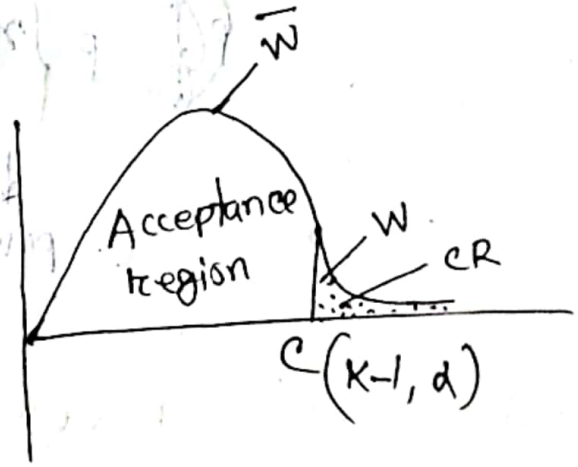
Decision:

True statement	Reject H_0	Accept H_0
H_0 true	Type I error	Right Decision
H_0 false	Right Decision	Type II error

Type I error and Type II error:

Rejecting H_0 when H_0 is true, is called type I error. $\rightarrow x \in W / H_0$

And Accept H_0 when H_1 is true, is called type II error.



The probability of typ-I error is called the size of the test. It is also called the level of significance and is denoted by α. Mathematically we can write,

$$P\{x \in W / H_0\} = \alpha$$

$$P\{x \in \bar{W} / H_0\} = 1-\alpha$$

$$P\{x \in \bar{W} / H_1\} = \beta$$

$$P\{x \in W / H_1\} = 1-\beta$$

↓

power of the test.

* what is power.

* What is p-value?

$$P\{x \in W / H_0\} = \int_c^{\alpha} f(x)\, dx = \alpha$$

$$\int_{\text{cal. value}}^{\infty} f(x)\, dx = p\text{-value}$$

cal. value $\geq$ tabulat

Now, compare between $p \leftrightarrow \alpha$

if, $p \leq \alpha$ then we reject H_0

Ex: Battery service, 4000h

H_0 : μ = 4000 (μ → mean)

H_1 : $\mu \neq 4000$

: 4, 6, 2, 2, 2, 3, 3, 5, 4, 3, 2, 1

$$\bar{x} = \frac{37}{n=12} = 3.1$$

Test statistic:

(i) If sample size ≥ 30 →

OR, pop^n variance known →

z-test.

(ii) Otherwise, t-test.

(*) sample size < 30, but, pop^n ~~sample~~ variance know, → z-test.

(*) Sample size < 30 and pop^n variance unknown

t-test

$$Z = \frac{\bar{X} - \mu_0}{\sqrt{\sigma^2/n}} \sim Z\text{-statistic}$$

Here,

$\bar{X} = 3.1$

$\mu = 4.0$

$\sigma = 1.6$

$n = 12$

$$= \frac{3.1 - 4}{\sqrt{\frac{1.6}{12}}}$$

$$= -2.51$$

cal. $Z = -2.51$

$|\text{cal. } Z| = 2.51$

5% level of sig.

1 tail = 1.64

2 tail = 1.96

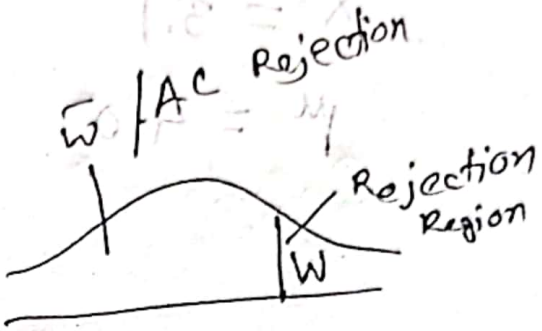
Practice class:

- Reject H_0 when H_0 is true

$$X = x \in W / H_0$$

- $P\{X = x \in W / H_0\}$

$$= \alpha$$



- $P\{X = x \in \bar{W} / H_0\} = 1-\alpha$

- Accept H_0 when H_1 is false

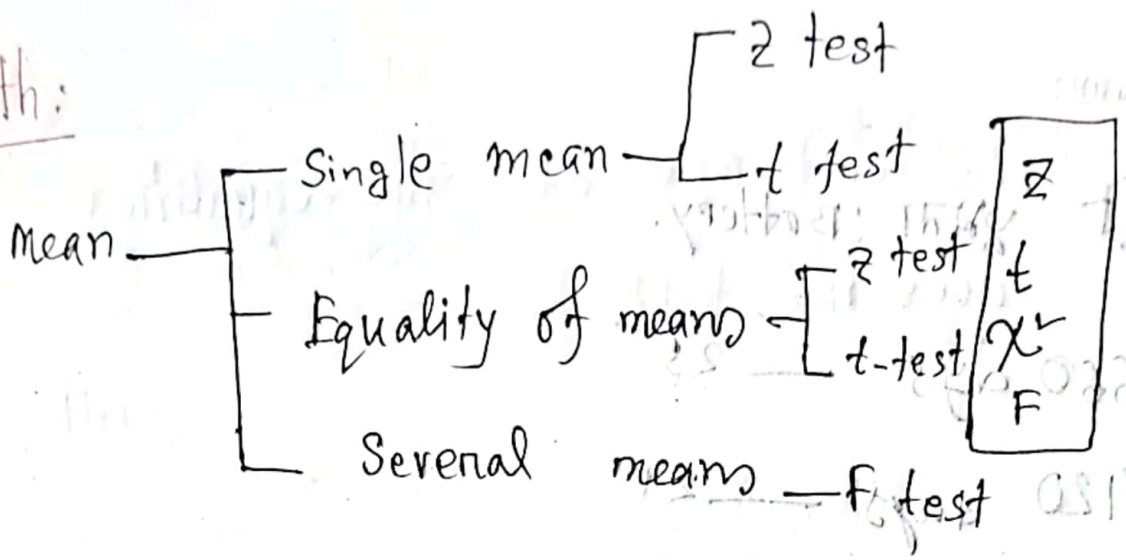
$$X : x \in \bar{W} / H_1$$

$$P\{X : x \in \bar{W} / H_1\} = \beta$$

$$P\{X : x \in W / H_1\} = 1-\beta \rightarrow \text{Power}$$

* Particular point এর জন্য → Power
* All Renge এর জন্য → Power function

Math:



☐ At least 630 days.

H_0 : $\mu \geq 630$

H_1 : $\mu < 630$

↑ one tail

$\neq$ → two tail

z :

$n \geq 30$ (or) variance known (population variance)

t

$n < 30$ (and) variance unknown.

Equality of mean:

দুইটি কোম্পানির তুলনা : Battery.

$\bar{x}_1 = 660$ days — 25

$\bar{x}_2 = 7120$ days — 21

☐ ২য় Product এর lifetime কত?

$H_0 : \mu_1 = \mu_2$

$H_1 : \mu_1 < \mu_2$ → left tail

☐ দুইটার সমান কিনা?

$H_0 : \mu_1 = \mu_2$

$H_1 : \mu_1 \neq \mu_2$ — 2tail

$\mu_1 > \mu_2$ → right tail

Mean ... variance ... single variance ... equality ... several ...

Test for correlation coefficient:

1. Zero - correlation ⟶ t test
2. Correlation / Single corr. → t test
3. Equality of corr. → z test
4. Several cor. coefficient → χ^2

Q. Describe the test procedure to test the following hypothesis.

Single i. $H_0 : \mu = 0$ vs $H_1 : \mu \neq 0$

Equality of ii) $H_0 : \sigma_1^2 = \sigma_2^2$ vs $H_1 : \sigma_1^2 > \sigma_2^2$

Several cor. coeff iii) $H_0 : \rho_1 = \rho_2 = \dots = \rho_k$ vs H_1: H_0 is not true

ii) or,

H_1 : At least one of the equalities does not hold.