University of Rajshahi

Department of Computer Science and Engineering

B. Sc. (Engg) Part-II Odd Semester Examination 2021

Course: MATH-2131 (Differential Equations and Optimization)

Full Marks: 52.5 Duration: 3(Three) Hours

Answer 06(Six) questions taking any 03(Three) questions from each part

Section-A

1. a) Define the order and degree of a differential equation with examples. Find the differential equation of all circles passing through the origin and having their centers on the x-axis. 2.75
   b) Form the differential equation of the family of parabolas with focus at the origin and axes along the x-axis. 3
   c) Solve $y = x^2p^2 - px$ where $p \equiv \frac{dy}{dx}$. 3

2. a) Solve $\cos(x + y)\, dy = dx$. 3
   b) Solve $\frac{dy}{dx} = x^3y^3 - xy$. 2.75
   c) Define Bernoulli differential equation. Solve $\frac{dy}{dx} + y = xy^3$. 3

3. a) Solve $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 3y = 2e^x - 10cosx$ by using method of undetermined coefficients. 3
   b) Solve the differential equation $(D^2 - 4D + 4)y = 3x^2e^{2x}sin2x$ by operator method. 2.75
   c) Solve $(D^2 + 4)y = cos^2x$. 3

4. a) Define regular singular point. Find the general solution $\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2 + 2)y = 0$ in power of x about $x_0 = 0$. 4
   b) Write Helmholtz's equation and solve it. 4.75

Section-B

5. a) What is an optimization problem? Describe unconstrained and constrained optimization problems. 3
   b) Define: linear programming problem, nonlinear programming problem, and convex optimization problem. 2.75
   c) What do you mean by minimizer of a function? Define local and global minimizer of a function f. Let $f: \mathbb{R} \to \mathbb{R}$ be a function defined by $f(x) = (x + 1)^2 + 3$ find the arg min $f(x)$. 3
   d)

6. a) Define feasible direction of a vector and directional derivative of a function f. Let $f: \mathbb{R}^2 \to \mathbb{R}$ be defined by $f(x) = x^T \begin{pmatrix} 1 & 2 \\ 4 & 7 \end{pmatrix} x + x^T \begin{pmatrix} 3 \\ 5 \end{pmatrix} + 6$. Find the directional derivative of f at $(1,1)^T$ with respect to a unit vector in the direction of maximal rate of increase. 3.75
   b) Define *gradient and Hessian matrix* of $f: \mathbb{R}^n \to \mathbb{R}$. State First-Order Necessary Condition. Suppose there are two basestation antennas, one for the primary basestation and another for the neighboring basestation. Both antennas are transmitting signals to the mobile user at equal power. However, the power of the received signal as measured by the mobile is the reciprocal of the squared distance from the associated antenna. Find the position of the mobile that maximizes the signal-to-interference ratio, which is the ratio of the received signal power from the primary basestation to the received signal power from the neighboring basestation. 3
   c) Let $f: \mathbb{R}^n \to \mathbb{R}$ be the continuously differentiable function and $\{x_k\}$ be a sequence generated by Steepest descent algorithm. Prove that, for each k, the vectors $x_{k+1} - x_k$ is orthogonal to the vector $x_{k+2} - x_{k+1}$. 2

7. a) Define the epigraph of a function and a convex function. Let $f:\Omega \subseteq \mathbb{R}^n \to \mathbb{R}$ be a convex function on Ω. Prove that Ω is a convex set. 3.75

b) Let $f:\Omega \subseteq \mathbb{R}^n \to \mathbb{R}$ be a quadratic form which is given by $f(x) = x^T Q x, Q \in \mathbb{R}^{n\times n}, Q = Q^T$. Prove that f is convex on Ω if and only if $(x-y)^T Q(x-y) \geq 0$, for all $x, y \in \Omega$. 3

c) Let $f:\Omega \subseteq \mathbb{R}^n \to \mathbb{R}$ be a function defined by $f(x) = x_1 x_2$. Is f convex on $\Omega = \{x: x_1 \geq 0, x_2 \geq 0\}$? 2

8. a) Define a unimodal function. Use Golden section search method to find the value of x that minimizes $f(x) = x^4 - 14x^3 + 60x^2 - 70x$ in the range $[0,2]$. Locate this value of x to within a range of 0.3. 3

b) Define Lagrangian function. Prove that the Lagrange condition for a local minimizer x^* can be represented using the Lagrangian function as $Dl(x^*, \lambda^*) = 0^T$ for some λ^*, where the derivative operation D is with respect to the entire argument $(x^T, \lambda^T)^T$. 3.75

c) What is a sequential quadratic programming (SQP)? Construct SQP for the nonlinear optimization problem. 2

Section B

5. a) Define ordinary differential equations. Solve the homogeneous differential equation $\frac{dy}{dx} = \frac{3y^2 - x^2}{2xy}$. **2.75**

   b) Define Bernoulli differential equations. Identify the differential equation $\frac{dy}{dx} + \frac{3y}{x} = 6x^2$ and solve it. **3.00**

   c) The population x of a certain city satisfies the logistic law $\frac{dx}{dt} - \frac{1}{100}x = -\frac{1}{10^8}x^2$, where time t is measured in years. Find the population of the city at any time. **3.00**

6. a) Define initial value problems. A circuit has in series a constant electromotive force of 40 V, a resistor of 10Ω, and an inductor of 0.2 H. Find the current at time $t > 0$ if the initial current is zero. **2.75**

   b) Define Clairaut's equation. Find a one-parameter family of solutions of the equation $y = px + p^2$, where $p \equiv \frac{dy}{dx}$. Find singular solution, if exists, of the given equation which is not a member of the one-parameter family of solutions. **3.00**

   c) Use the method of undetermined coefficients to solve the differential equation $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 3y = 2e^x - 10\sin x$. **3.00**

7. a) Define Reccati's equation. State under what conditions, Reccati's equation reduces to a linear and Bernoulli's equation? Solve: $\frac{dy}{dx} + y = xy^3$ **3.00**

   b) Solve $(D^2 + 4)y = x^2e^{2x}$ by operator method. **2.75**

   c) Define regular singular point and examine the regular singular point of $x^2y'' - xy' + 8(x^2 - 1)y = 0$ **3.00**

8. a) Convert the differential equation $\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$ with the initial conditions $y(0) = 1$, $y'(0) = 0$ into an integral equation. Finally, identify your obtained integral equation. **2.75**

   b) An LTIC system is in zero state. Its response $y(t)$ is described by the differential equation $\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 6y = \frac{df}{dt} + f(t)$. Find the response of the system if the input is given by $f(t) = 3e^{-5t}u(t)$ **2.00**

   c) Solve the following one-dimensional heat equation by the method of separation of variables **4.00**

   $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, $0 < x < 1, t \geq 0$

   subject to the initial profile

   i) $u(x, 0) = 1,\ 0 < x < 1$ and the Dirichlet boundary conditions

   ii) $u(0, t) = 0, t \geq 0$ and

   iii) $u(1, t) = 0, t \geq 0$.

University of Rajshahi
Department of Computer Science and Engineering
B. Sc. (Engg.) Part-2 Odd Semester, Examination-2019
Course: MATH-2111 (Matrices and Differential Equations)

Full Marks: 52.5 Time: 3Hours

[Answer six questions taking any three from each section]

Section A

1. (a) Define diagonal matrix, identity matrix, scalar matrix and skew symmetric matrix with examples. 2.75
   (b) Define periodic matrix. Write the period of an idempotent matrix. If AB=A and BA=B, show that A and B are idempotent. 3
   (c) Show that every square matrix with complex elements can be uniquely expressed as the sum of a Hermitian and a skew Hermitian matrices. 3

2. (a) Define inverse of a matrix. If A and B are non-singular matrices, show that $(AB)^{-1} = B^{-1}A^{-1}$. 2.75
   (b) Show that $A^{-1} = \frac{adj\,A}{|A|}$ and hence find the inverse of $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 4 \\ 1 & 4 & 3 \end{bmatrix}$. 3
   (c) If A and B are matrices of order nxn, show that adj (AB)=(adj B)(adj A). 3

3. (a) Define rank of matrices. Reduce $\begin{bmatrix} 1 & 2 & -1 & 4 \\ 2 & 4 & 3 & 5 \\ -1 & -2 & 6 & -7 \end{bmatrix}$ to the canonical form and find its rank. 2.75
   (b) Define normal forms of a matrix. Reduce the matrix A=$\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$ to normal form and find its rank. 3
   (c) Determine the value of a so that the following system in unknowns x, y and z has (i) no solution, (ii) more than one solution, (iii) a unique solution: 3
   $x+y-z=1,\ 2x+3y+az=3,\ x+ay+3z=2$.

4. (a) Find the solutions of the system of equations: 2.75
   $x_1+x_2+x_3+x_4=0$
   $x_1+3x_2+2x_3+4x_4=0$
   $2x_1+x_3-x_4=0$
   (b) Define characteristic roots. If $\mu_1, \ldots, \mu_n$ are the characteristic roots of the n-square matrix A, prove that $\mu_1-\lambda, \ldots, \mu_n-\lambda$ are the roots of the characteristic equation of $A-\lambda I_n$. 3
   (c) Find eigen values and eigen vectors of $A = \begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}$, and an invertible matrix P such that $P^{-1}AP$ is diagonal. 3

Section B

5. (a) Define ~~differential~~ differential equations, its order and its degree with example. 2.75
   (b) Form the differential equation of the circle represented by $y^2-2ay+x^2=a^2$, a being arbitrary constant. Also, identify the differential equation. 3
   (c) What is first order exact differential equation? Solve $(2x\cos y+3x^2y)dx+(x^3-x^2\sin y-y)dy=0,\ y(0)=2$. 3

6. (a) Define homogeneous ODE. Solve the DE $\left(x^3 + y^2\sqrt{x^2+y^2}\right)dx - xy\sqrt{x^2+y^2}\,dy = 0$. 3
   (b) Define Bernoulli differential equation. Solve the DE $\frac{dy}{dx} + y = xy^3$. 3
   (c) $y+px=x^4p^2$, where $p \equiv \frac{dy}{dx}$. 2.75

7. (a) Solve $y'' - 6y' + 25 = 0,\ y(0) = -3,\ y'(0) = -1$. 2.75
   (b) Solve $(D^4+2D^3-3D^2)y=3e^{2x}+4\sin x$ by operator method. 3
   (c) Find a series solution of $2xy'' - xy' + (x-5)y = 0$ in some interval 0<x<R using the method of Frobenius. 3

8. (a) Solve $p\cos(x+y)+q\sin(x-y)=z$ by Lagrange method. 3.75
   (b) Eliminate a, b from $z=(x^2+a)(y^2+b)$. 2
   (c) Apply Charpit's method to solve $p=(qy+z)^2$. 3

Section-B

5.(a) Find the differential equations of all circles which have their centres on x axis and have a given radius. 3

(b) Solve the differential equation: $\frac{dy}{dx} = x^3y^3 - xy$. 3

(c) Prove that the differential equation $(\sin x \cos y + e^{2x})dx + (\cos x \sin y + \tan y)\, dy = 0$ is exact and solve it. 2.75

6. Solve the following differential equations:

(a) $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + y = \sin 2x$. 3

(b) $(D^3 - 7D - 6)y = e^{2x}\, x^2$. 3

(c) $x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} - 4y = x^4$. 2.75

7.(a) Solve the differential equation by the method of variation of parameters: $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = x^2e^x$. 4

(b) Solve the differential equation by the method of operator factorization: $[xD^2 + (1-x)D - 2(1+x)]y = e^{-x}\,(1-6x)$. 4.75

8.(a) Solve the partial differential equations by Charpit's method $z^2 = pqxy$. 4

(b) Solve the partial differential equation $x(y^2+z)p - y(x^2+z)q = z(x^2-y^2)$ by Lagrange's method and hence find its integral surface containing the straight line $x + y = 0,\ z = 1$. 4.75

Section-B

5.(a) Define order and degree of differential equation. Find the order and degree of the differential equation 2.75

$$2\frac{d^3y}{dx^3} + 3\left(\frac{d^2y}{dx^2}\right)^4 + \frac{dy}{dx} + y = \sin 4x$$

(b) Define homogeneous ODE. Solve the ODE $(2xy + 3y^2)dx - (2xy + x^2)dy = 0$ 3

(c) Define exact differential equation. Solve $3x(xy-2)\,dx + (x^3 + 2y)dy = 0$ 3

6.(a) Define Bernoulli's equation. Solve $\frac{dy}{dx} + \frac{y}{x}logy = \frac{y}{x^2}(logy)^2$ 2.75

(b) Solve *(i)* $y + px = p^2x^4$ *(ii)* $y = 2px + y^2p^3$ where $p = \frac{dy}{dx}$ 3+3

7.(a) Solve *(i)* $(D^3 - D^2 - 6D)y = 1 + x^2$ *(ii)* $(D^2 + 4)y = cosx$ 3+3

(b) Using variation of parameters to find the general solution $4y'' - 4y' - 8y = 8e^{-t}$ 2.75

8.(a) Define regular singular point. Find the general solution of $\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2 + 2)y = 0$ in power of x about $x_0 = 0$ 5.75

(b) Define Laplace transformation. Fine the solution of $f''(t) + 3f'(t) + 2f(t) = 4t$ where $f(0) = f'(0) = 0$ using Laplace transformation. 3

University of Rajshahi

Department of Computer Science and Engineering

B. Sc. (Engg.) Part-2 Odd Semester Examination-2016

Course: MATH2111 (Matrix and Differential Equation)

Full Marks: 52.5 Duration: 3(Three) Hours

Answer 06(Six) questions taking any 03(Three) questions from each part.

Part-A

1. (a) Let **A** and **B** be matrices of order $m \times n$ and $n \times p$ respectively. Prove that $(\mathbf{AB})' = \mathbf{B}'\mathbf{A}'$. 3

   (b) Define symmetric matrix and skew symmetric matrix. Give an example of each kind. Prove that the diagonal elements of a skew symmetric matrix are all zero. 3

   (c) Let **A** and **B** be n-square non-singular matrices. Prove that **AB** is non-singular and $(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$. 2.75

2. (a) For what value of λ, the system of equations fail to have a solution: 3

$$3x - y + \lambda z = 1$$
$$2x + y + z = 2$$
$$x + 2y - \lambda z = -1$$

   (b) State Cayley-Hamilton theorem and verify it for the matrix $\begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix}$. 3

   (c) Define linear dependence and linear independence of a set of vectors. Determine whether or not the vectors [1,2,3], [2,3,4], [3,5,7] are linearly dependent. 2.75

3. (a) Apply Cramer's rule to solve the equations: 4

$$x + y + z = 1,\ x + 2y + z = 2,\ x + y + 2z = 0$$

   (b) Determine the values of a and b so that the system of equations 4.75

   $x + 2y + z = 1,\ 3x + y + 2z = b,\ ax - y + 4z = b^2$ has

   (i) a unique solution, (ii) no solution and (iii) many solutions.

4. (a) For any square matrix **A**, define $adj(\overset{A}{\cancel{B}})$. Prove that $\mathbf{A}adj(\mathbf{A}) = |\mathbf{A}|\mathbf{I}$. 3

   (b) Find the adjoint and inverse of the matrix $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$. 3

   (c) Reduce the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 5 & 7 \end{bmatrix}$ to echelon form and find its rank. 2.75

Part-B

5. (a) Define *degree* and *order*. Find the differential equation of the family of circles 3

$$x^2 + y^2 + 2gx + 2fy + c = 0.$$

   (b) Identify and solve $(x^2 + y^2)dx - 2xydy = 0$. 3

   (c) Solve by the variation of parameters: $y'' + y = cos^2(x)$ 2.75

6. Solve the following differential equations:

(a) $$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = xe^{-x}.$$ 2.75

(b) $$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 4y = e^x \cos(x).$$ 3

(c) $$\frac{d^2y}{dx^2} + a^2y = x\cos(ax).$$ 3

7. Define regular singular points. Find the general solution of $\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$ by series method. Test the convergency of the series. 8.75

8. (a) Solve $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, $u(x,0) = 3\sin(2\pi x)$, $u(0,t) = 0$, $u(1,t) = 0$ where $0 < x < 1$, $t > 0$ by Laplace transformation. 4.75

(b) Write down Helmholtz's equation and solve it. 4

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University of Rajshahi
Department of Computer Science & Engineering
B.Sc. (Engg.) Part-II Odd Semester Examination 2015
Course: MATH-2111 (Matrix and Differential Equation)
Full Marks: 52.5 Duration: 3(Three) Hours
Answer 6 (Six) questions taking any 3(Three) from each part

Part-A

1. a) Define matrix multiplication. Prove that matrix multiplication is associative. 3
   b) Prove that every square matrix can be expressed uniquely as a sum of a symmetric and a skew symmetric matrix. 3
   c) If **A** and **B** are n-square matrices then prove that **A** and **B** commute if and only if **A-kI** and **B-kI** commute for scalar k. 2.75

2. a) For any square matrix A and B, prove that $adj(AB)=adj(B)adj(A)$. 3
   b) Find the inverse of $A=\begin{pmatrix}1 & 3 & 3\\ 1 & 4 & 3\\ 1 & 3 & 4\end{pmatrix}$, using elementary row operations. 3
   c) Define linear dependence and independence of a set of vectors. Determine whether or not the following vectors are linearly dependent $x_1=[1, 2, -3, 4]$, $x_2=[3, -1, 2, 1]$, $x_3=[1, -5, 8, -7]$ 2.75

3. a) Determine the value of k such that the system in unknowns x, y, z has (i) a unique solution (ii) no solution, (iii)more than one solution 3
$$x+y+kz=2$$
$$3x+4y+2z=k$$
$$2x+3y-z=1$$
   b) Reduce the matrix $A=\begin{pmatrix}0 & 2 & 3 & 4\\ 2 & 3 & 5 & 4\\ 4 & 8 & 13 & 12\end{pmatrix}$ into echelon form and determine its rank. 2.75
   c) Define an eigenvalue and associated vector of a square matrix. Find eigenvalues and associated eigenvectors of the matrix $A=\begin{pmatrix}1 & 2\\ 3 & 2\end{pmatrix}$. 3

4. a) If x_1 and x_2 are eigenvectors of a matrix A belonging to m eigenvalue λ then prove that any linear combination $c_1x_1+c_2x_2$ of x_1 and x_2 is also an eigenvector of A belonging to the eigenvalue λ provided $c_1x_1+c_2x_2\neq 0$. 3
   b) If A is an n-square matrix. Prove that **A** and **A′** have the same eigenvalues. 2.75
   c) Let $A=\begin{pmatrix}4 & -3\\ 2 & -1\end{pmatrix}$. Find a nonsingular matrix **P** such that $\mathbf{P}^{-1}\mathbf{AP}$ is diagonal. 3

Part-B

5. a) Define variable separable equation. Find an explicit solution of the initial value problem $x^2\frac{dy}{dx}=y-xy, y(-1)=-1$ by separating variabless. 3
   b) Find the general solution of the differential equation $y'+3x^2y=x^2$. Give the largest interval over which the solution is defined. Is there any transient term in the general solution? 3
   c) Solve the equation by using an appropriate substitution $x\frac{dy}{dx}+y=\frac{1}{y^2}$. 2.75

6. Solve the following differential equations:
   a) $y''+4y'+4y=2x+6$ 3
   b) $y''-y=x^2e^x+5$ 3
   c) $y''-y'-12y=e^{4x}$ 2.75

7. a) Use annihilator operator to solve $y'' - y' - 12y = e^{4x}$ 3
   b) Byvariation of parameters solve the equation $2y'' + 2y' - 8y = 2e^{-2x} - e^{-x}$ 3
   c) Solve the Cauchy-Euler equation $3x^2y'' + 6xy' + y = 0$ 2.75

8. a) Solve the IVP using Laplace transform$y'' - 3y' + 2y = e^{-4t}, y(0) = 1, y'(0) = 5$ 4
   b) Find the series solution of $y'' + \cos x\, y = 0$ 4.75

University of Rajshahi

Department of Computer Science and Engineering

B. Sc. Engg. Part-II Odd Semester, Examination-2014

Course: Math2111 (Matrices and Differential Equations)

Full Marks: 52.5 Time: 3 Hours

Answer **six (06)** questions taking **three (03)** from each part

PART-A

1.(a) Define ODE and solve $x \sin y\, dx + (x^2+1)\cos y\, dy = 0$ 2.75

(b) Define homogeneous differential equation and solve $(x^3+y^3)dx - 3xy^2 dy = 0$ 3

(c) Define exact differential equation and solve $(6xy+2y^2-5)dx + (3x^2+4xy-6)dy = 0$ 3

2.(a) Define linear ODE and solve $\frac{dy}{dx} + 3y = 3x^2 e^{-3x}$ 3

(b) Identify and solve $dy + (4y - 8y^{-3})xdx = 0$ 2.75

(c) Find the family of oblique trajectories that intersect the family of straight line $y = cx$ at an angle 45^0. 3

3.(a) Use the method of undetermined coefficient to find the general solution of the differential equation $(D^2+4D+4)y = 6\sin 3x$ 4.75

(b) Solve $(D^2+4)y = x^2 e^{2x}$ by the operator method. 4

4.(a) Find the series solution of $y''+xy'+(x^2+2)y = 0$ 5.75

(b) Find the solution of $x''(t) - 4x'(t) + 4x(t) = 0$ with the boundary condition x(0)=0, x'(0)=1 using Laplace transformation. 3

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University of Rajshahi
Department of Computer Science and Engineering
B.Sc. (Engg.) Part-II Odd Semester Exam - 2013
Course: MATH-2111 (Matrices and Differential Equations)
Full Marks: 52.5 Time: 4 Hours
(Answer any 6 questions taking 3 from each part)

Part A

1.(a) Define degree and order of a differential equation with examples. Find the differential equation of all straight lines at unit distance from the origin. 3

(b) Solve $\cos(x+y)\,dy = dx$ 2.75

(c) Define orthogonal trajectories. Find it for the family $(x^2+y^2)=c^2$ of circles. 3

2.(a) Solve $(x^2+y^2)\,dx + xy\,dy = 0$ 3

(b) Solve $dy/dx + y = xy^3$ 2.75

(c) Solve $(2x+3y+1)\,dx + (4x+6y+1)\,dy = 0$ 3

3.(a) Solve $(D^2+1)\,y = \cos x$, where $D=d/dx$ 2.75

(b) Solve $(D^2+4)\,y = x^2\sin 2x$, where $D=d/dx$ 3

(c) Solve $(D^3+2D^2-D-2)\,y = e^x+x^2$, where $D=d/dx$ 3

4. Define ordinary point, singular point and regular point of differential equation. Solve $xy_2+y_1+xy = 0$ in series. 8.75

Part B

5.(a) Define Symmetric matrix. If $AB = BA$, then prove that $(AB)^2 = A^2B^2$. 2.75

(b) Prove that every Symmetric matrix can be uniquely expressed as the sum of a symmetric and a skew-symmetric matrix. 3

(c) If A is an idempotent matrix, then prove that $(I-A)(I+A) = 0$ implies that $A=I$. 3

6.(a) Define a nilpotent matrix. Show that $A = \begin{bmatrix} 1 & 1 & 1 \\ 5 & 2 & 6 \\ -2 & -1 & -6 \end{bmatrix}$ is a nilpotent matrix of order 3. 3

(b) Define the rank of a matrix. Find the rank of the matrix 3

$$A = \begin{bmatrix} 2 & 3 & 4 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{bmatrix}$$

(c) Reduce the matrix into its normal form: 2.75

$$\begin{bmatrix} 1 & 2 & 0 & -1 \\ 3 & 4 & 1 & 2 \\ -2 & 3 & 2 & 5 \end{bmatrix}$$

University of Rajshahi
Department of Computer Science and Engineering
B.Sc. Engineering 2nd year, 1st Semester Examination- 2012
Course Code: MATH-2111, Course Title: Matrices and Differential Equations

Full Marks: $52\frac{1}{2}$ Time: 4 Hours

[N. B. Answer ***three*** questions from each part]

Part-A

1. (a) Define adjoint of a matrix. If A be an n-square matrix, then prove that $A(AdjA) = (adjA).A = |A|.I$, where I is the identity matrix of order n. 3
   (b) If A is matrix of order n and rank n-1, then prove that adjA is of rank 1. 3
   (c) If A is n-square, then prove that $|adj(adjA)| = |A|^{(n-1)^2}$ $2\frac{3}{4}$

2. (a) Define elementary transformations and the normal form of a matrix. Reduce the matrix $\begin{bmatrix} -2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$ to normal form and hence define its rank. 3
   (b) Define inverse of a matrix. Show that the inverse of a matrix, if it exists, must be unique. 3
   (c) Define transpose of a matrix. Prove that $(AB)' = B'A'$. $2\frac{3}{4}$

3. (a) Find the inverse of $A = \begin{bmatrix} 2 & 4 & 3 & 2 \\ 3 & 6 & 5 & 2 \\ 2 & 5 & 2 & -3 \\ 4 & 5 & 14 & 14 \end{bmatrix}$ using only row transformation to reduce A to I. 3
   (b) What do you mean by consistency and inconsistency of a system of non homogeneous linear equation? $2\frac{3}{4}$
   (c) Solve the system of equations $x+y+z=6, 2x+3y-2z=2, 5x+y+2z=13$, by the matrix method. 3

4. (a) Find the eigenvalues and eigenvectors of the matrix $\begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -4 & -3 \end{bmatrix}$. $4\frac{3}{4}$
   (b) State and prove Cayley Hamilton theorem. 4

Part-B

5. (a) Define degree and order of ODE. Solve the equation $(y\sec^2 x + \sec x \tan x)dx + (\tan x + 2y)dy = 0$ 3
   (b) Define first order homogeneous ODE. Find general solution of $(x^3 + y^2\sqrt{x^2+y^2})dx - xy\sqrt{x^2+y^2}dy = 0$ 3
   (c) Define Bernoulli equation . Find the general solution of $dy + (4y - 8y^{-3})xdx = 0$ $2\frac{3}{4}$

6. (a) Define orthogonal and oblique trajectories of a given family of curves. Find the orthogonal trajectories of the family of curves $y = x - 1 + ce^{-x}$ 4

(b) Given that $y = x$ is a solution of $(x^2 - x + 1)\frac{d^2y}{d^2x} - (x^2 + x)\frac{dy}{dx} + (x+1)y = 0$ $4\frac{3}{4}$

Find a linearly independent solution by reducing the order. Write the general solution.

7. (a) Find general solution of the differential equation $4\frac{3}{4}$

$\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} + 3\frac{dy}{dx} - 5y = 5\sin 2x + 10x^2 + 3x + 7$ by the method of undetermined coefficients.

(b) Find general solution of the differential equation $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = \frac{e^{-x}}{x}$ by the method of variation of parameters. 4

8. (a) Define ordinary point, singular point and regular singular point of differential equation. Use the method of Frobenius to find solutions of the differential equation $8\frac{3}{4}$

$2x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + (x-5)y = 0$ in some interval $0 < x < R$.