

Predicate Logic Conti....

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Properties of Statements:

- **Satisfiable:** A statement is satisfiable if there is some interpretation for **which it is true**.
- **Contradiction:** A sentence is contradictory (unsatisfiable) if there is **no interpretation** for **which, it is true**.
- **Valid:** A sentence is valid **if it is true for every interpretation**.
- **Equivalence:** Two sentences are equivalent if they have the **same truth value** under every interpretations.

Conti...

- **Logical Consequences:** A sentence is a logical consequence of another if it is satisfied by **all interpretations** which satisfy the first.
- **A valid statement is satisfiable, and a contradictory statement is invalid.**

Example: On The above definitions:

- P is satisfiable but **not valid** since an interpretation **that assigns false to P** assigns **false to the sentence P** .
- $P \vee \sim P$ is **valid** since every interpretation results in a value of **true for $(P \vee \sim P)$** .
- $P \& \sim P$ is a **contradiction** since every interpretation results in a value of **false for $(P \& \sim P)$** .

Conti...

- **P** and $\sim(\sim P)$ are **equivalent** since each has the **same truth values** under every interpretation.
- **P** is a **logical consequence** of $(P \ \& \ Q)$ since any interpretation for which **$(P \ \& \ Q)$ is true, **P** is also true.**

Theorem

- **Theorem 4.1:** The sentence s is a logical consequence of s_1, s_2, \dots, s_n if and only if $s_1 \& s_2 \& s_3 \dots \& s_n \rightarrow s$ is valid.
- **Proof:** Theorem 4.1 can be seen by first noting that if s is a logical consequence of s_1, s_2, \dots, s_n , then for any interpretation I in which $s_1 \& s_2 \& s_3, \dots, \& s_n \rightarrow s$ is true.
- on the other hand, if $s_1 \& s_2 \& s_3, \dots, \& s_n \rightarrow s$ is valid, then for any interpretation I if $s_1 \& s_2 \& s_3, \dots, \& s_n$ is true, **s is also true.**

Theorem

- **Theorem 4.2:** The sentence s is a logical consequence of s_1, s_2, \dots, s_n if and only if $s_1 \& s_2 \& s_3, \dots \& s_n \& \sim s$ is **inconsistent**.
- **Proof:** The proof of theorem 4.2 follows directly from **theorem 4.1** since s is a logical consequence of s_1, s_2, \dots, s_n if and only if $s_1 \& s_2 \& s_3, \dots \& s_n \rightarrow s$ is valid, that is, if and only if $\sim(s_1 \& s_2 \& s_3, \dots \& s_n \rightarrow s)$ is inconsistent.

Conti

- But

$$\begin{aligned}\sim(s_1 \& s_2 \& s_3 , \dots \& s_n \rightarrow s) &= \sim(\sim(s_1 \& s_2 \& s_3 , \dots \& s_n) \vee s) \\ &\quad \text{[By Conditional Elimination]} \\ &= \sim\sim(s_1 \& s_2 \& s_3 , \dots \& s_n) \& \sim s \\ &\quad \text{[By De Morgan's Law]} \\ &= s_1 \& s_2 \& s_3 , \dots \& s_n \& \sim s\end{aligned}$$

- When s is a logical consequence of s_1, s_2, \dots, s_n , the formula $s_1 \& s_2 \& s_3, \dots \& s_n \rightarrow s$ is called a theorem, with s is the conclusion.
- When **s** is a logical consequence of the set **$S = \{s_1, s_2, \dots, s_n\}$** we will also set **$S$** logically implies or logically entails **S** , written **$S \vdash s$** .

Table 4.2 lists some of the important laws of PL (Some Equivalence Laws)

Name of Laws	Statements
Idempotency	$P \vee P = P$ $P \& P = P$
Associativity	$(P \vee Q) \vee R = P \vee (Q \vee R)$ $(P \& Q) \& R = P \& (Q \& R)$
Commutativity	$P \vee Q = Q \vee P$ $P \& Q = Q \& P$ $P \leftrightarrow Q = Q \leftrightarrow P$
Distributivity	$P \& (Q \vee R) = (P \& Q) \vee (P \& R)$ $P \vee (Q \& R) = (P \vee Q) \& (P \vee R)$
De Morgan's Laws	$\sim(P \vee Q) = \sim P \& \sim Q$ $\sim(P \& Q) = \sim P \vee \sim Q$
Conditional Elimination	$P \rightarrow Q = \sim P \vee Q$
Bi-conditional Elimination	$P \leftrightarrow Q = (P \rightarrow Q) \& (Q \rightarrow P)$

Example

- Show that $P \rightarrow Q$ is equivalent to $\sim P \vee Q$ and that $P \leftrightarrow Q$ is equivalent to the expression $(P \rightarrow Q) \& (Q \rightarrow P)$.
- The truth table 4.3 is given bellow.

TABLE 4.3 : Truth table for equitant sentences

P	Q	$\sim P$	$(\sim P \vee Q)$	$(P \rightarrow Q)$	$(Q \rightarrow P)$	$(P \rightarrow Q) \& (Q \rightarrow P)$
T	T	F	T	T	T	T
T	F	F	F	F	T	F
F	T	T	T	T	F	F
F	F	T	T	T	T	T

Inference Rules

- The inference rules of PL provide the means to perform logical proofs or deductions.
- Few Such Rules are as follows:
 - Modus ponens
 - Chain Rule

Modus Ponens:

- From P and $P \rightarrow Q$ infer Q . This sometimes written as

- $$\begin{array}{c} P \\ P \rightarrow Q \\ \hline Q \end{array}$$

Example For Modus Ponens:

- Given: (Joe is a father)
- And: (Joe is a father) \rightarrow (Joe has a child)
- Conclude: (Joe has a child)

Chain Rule

- Form $P \rightarrow Q$ and $Q \rightarrow R$, infer $P \rightarrow R$.

- Or

- $P \rightarrow Q$
- $Q \rightarrow R$
- ---
- $P \rightarrow R$

Example for Chain Rule

- **Given:** (programmer likes LISP) \rightarrow (programmer hates COBOL)
- **and :** (programmer hates COBOL) \rightarrow (programmer likes recursion)
- **Conclude:** (programmer likes LISP) \rightarrow (programmer likes recursion)

- LISP \rightarrow **List Processing**
- COBOL \rightarrow **Common Business Oriented Language**
- Prolog \rightarrow **Programming in Logic**

Assignment-3

1. Construct a **truth Table** for the expression **$(A \& (A \vee B))$** .
2. Determine whether each of the following sentences is
 - (a) Satisfiable
 - (b) Contradictory
 - (c) Valid

Assignment-3 Conti...

- (i) $S_1: (P \& Q) \vee \sim(P \& Q)$
- (ii) $S_2: (P \vee Q) \rightarrow (P \& Q)$
- (iii) $S_3: (P \vee Q) \rightarrow R \vee \sim Q$
- (iv) $S_4: (P \vee Q) \& (P \vee \sim Q) \vee P$
- (v) $S_5: P \rightarrow Q \rightarrow \sim P$
- (vi) $S_6: P \vee Q \& \sim P \vee \sim Q \& P$

• **Thanks**