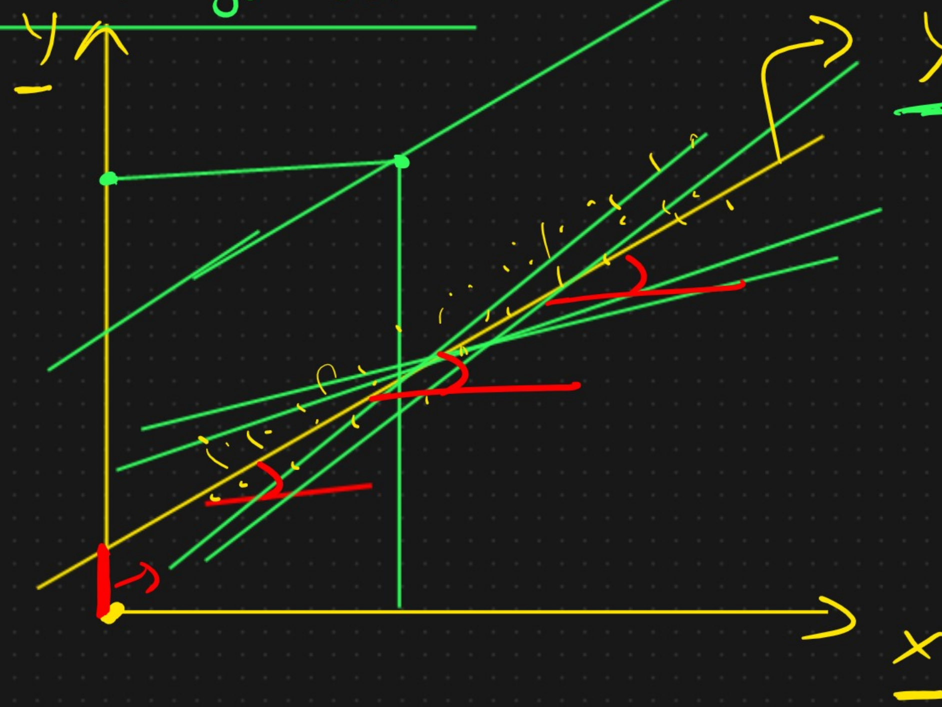
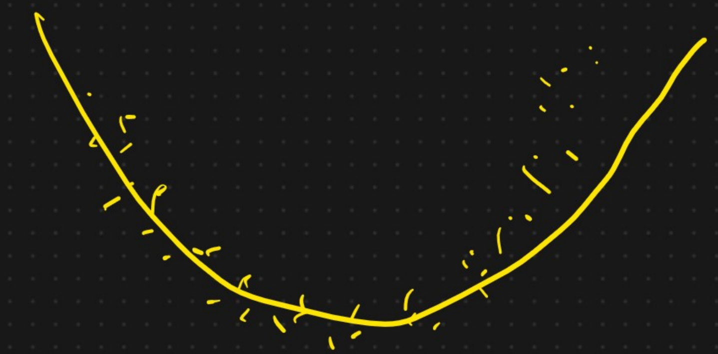
ML

linear Regression



$$y = mx + c$$



$m = 5 \ , \ c = 2$

$$y = 5x + 2$$

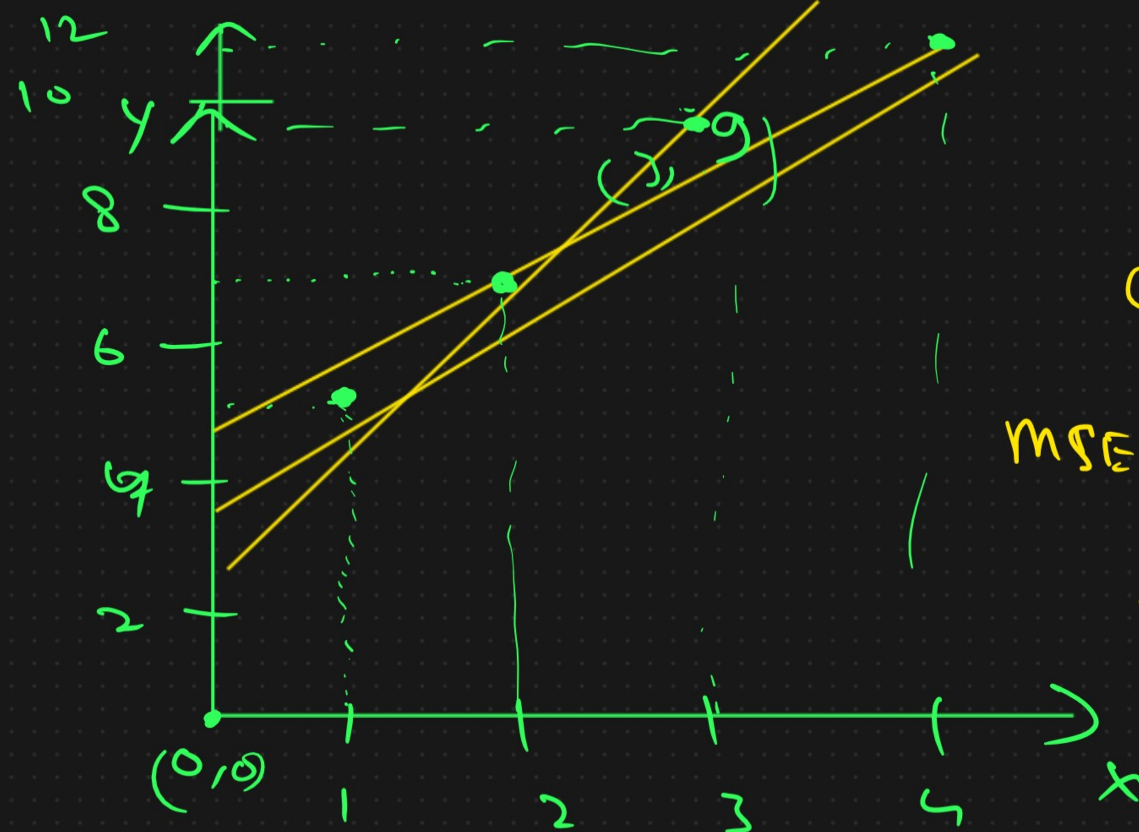
$$y = mx + c$$

$x = 0 \quad y = c$

$c = 0 \quad m = \left(\frac{y}{x}\right)$

x	y	$\hat{y}$ ($m=2$, $c=1$)	$\hat{y}$ ($m=0$, $c=0$)
1	5	3	0
2	7	5	0
3	9	7	0
4	11	9	0

4   69



$$y = mx + c$$

$$y = m_1x_1 + m_2x_2 + \dots + m_nx_n + c$$

$$MSE = \frac{1}{4}\left((5-3)^2 + (7-5)^2 + (9-7)^2 + (11-9)^2\right)$$

$$= \frac{1}{4}(4+4+4+4)$$

$$= \frac{16}{4} = 4$$

Loss Function

$$MSE = \frac{1}{n}\sum_{i=1}^{n}(y - \hat{y})^2$$

$(y = 2x + 1)$

$$m = 2, \quad c = 1$$

$$y = 2x + 1$$

$$y_{x=1} = 3$$

$$y_{x=2} = 2\times 2 + 1 = 5$$

$$y_{x=3} = 2\times 3 + 1 = 7$$

$$y_{x=4} = 2\times 4 + 1 = 9$$

$$m = 0, \quad c = 0$$

$$y = 0x + 0$$

$$y = 0$$

$$MSE = \frac{1}{4}\left((5-0)^2 + (7-0)^2 + (9-0)^2 + (11-0)^2\right)$$

$$= \frac{1}{4}\left[5^2 + 7^2 + 9^2 + 11^2\right)$$

$$= 69$$

(0, 0)

learning Rate

$$m_n = m_0 - \eta\left(\frac{\partial MSE}{\partial m}\right) \quad (1)$$

$$c_n = c_0 - \eta\left(\frac{\partial MSE}{\partial c}\right) \quad (2)$$

$$\frac{\partial y}{\partial x} = \frac{\partial y}{\partial z}\cdot\frac{\partial z}{\partial x}$$

$$x^n = n x^{n-1}$$

$$MSE = \frac{1}{n} \sum_{i=1}^{n} (y - \hat{y})^2 \qquad E = \underline{(y - (mx + c))}$$

$$MSE = \frac{1}{n} \sum \underbrace{\left((y - (mx_i + c))\right)^2}_{E}$$

$$MSE = \frac{1}{n} \sum (E)^2$$

$$\frac{\partial MSE}{\partial m} = \frac{1}{n} \sum 2E \left(\frac{\partial E}{\partial m}\right) \xrightarrow{} -x \quad = \frac{1}{n} \sum 2E(-x)$$

$$\frac{\partial E}{\partial m} = \frac{\partial}{\partial m} (y - (mx + c))$$

$$\frac{\partial E}{\partial m} = \frac{\partial}{\partial m} (y - mx - c) = -x$$

$$\frac{\partial MSE}{\partial m} = \frac{1}{n} \sum 2E(-x)$$

$$= \frac{1}{n} \sum 2(y - (mx + c))(-x)$$

$$= \frac{1}{n} \sum (2y - 2mx - 2c)(-x)$$

$$= \frac{1}{n} \sum (-2xy + 2x^2 m + 2xc)$$

$$= \frac{-2}{n} \sum x(y - mx - c)$$

$$= \frac{-2}{n} \sum x(y - \underline{(mx+c)})$$

$$\boxed{\left(\frac{\partial MSE}{\partial m}\right)} = \frac{-2}{n} \sum x_i (y_i - \hat{y}_i)$$

$$m_n = m_0 - \eta \left(\frac{\partial MSE}{\partial m}\right)$$

0.1, 0.001, 0.01

$$\underline{m_n} = m_0 - \eta \left(\frac{-2}{n} \sum x(y - \hat{y})\right)$$

$$= 0 - 0.1 \left(\frac{-2}{4} \left[1(5-0) + 2(7-0) + 3(9-0) + 4(11-0)\right]\right)$$

$$\underline{m_n} = -0.1 \left(\frac{-2}{4} [5 + 14 + 27 + 44]\right)$$

$= \frac{4.5}{0.1} =$

$c_n = c_0 - \eta\left(\frac{\partial MSE}{\partial c}\right)$

$\frac{\partial MSE}{\partial c} = \frac{\partial}{\partial c} \frac{1}{n} \Sigma (y - \hat{y})^2$

$= \frac{\partial}{\partial c}\left[\frac{1}{n} \Sigma \left(\underbrace{y - (mx+c)}_{E}\right)^2\right]$

$= \frac{\partial}{\partial c}\left[\frac{1}{n} \Sigma (E)^2\right]$

$\left(\frac{\partial E}{\partial c}\right) = \frac{\partial}{\partial c}\left(y - (mx+c)\right)$

$= \frac{\partial}{\partial c}(y - mx - c)$

$= -1$

$\left(\frac{\partial MSE}{\partial c}\right) = \frac{1}{n} \Sigma 2E \cdot \left(\frac{\partial E}{\partial c}\right)$

$= \frac{1}{n} \Sigma (2E)(-1)$

$= -\frac{1}{n} \Sigma (2E)$

$$= \frac{-2}{n} \Sigma \left(y - (mx + c) \right)$$

$$= \frac{-2}{n} \Sigma (y - \hat{y})$$

$$\left(\frac{\partial MSE}{\partial c} \right) = \left(\frac{-2}{n} \Sigma (y - \hat{y}) \right)$$

$$c_n = c_0 - \eta \left(\frac{\partial MSE}{\partial c} \right)$$

$$= c_0 - \eta \left(\frac{-2}{n} \Sigma (y - \hat{y}) \right)$$

$$= c_0 + \eta \left(\frac{2}{n} \Sigma (y - \hat{y}) \right)$$

$$\left[c_n = 0 + 0.1 \left(\frac{2}{4} \left((5-0) + (7-0) + (9-0) + (11-0) \right) \right) \right]$$

$c_n = 1.6$

$m_n = 4.5$

$y = 4.5x + 1.6 \rightarrow 3 \rightarrow$ model

$$MSE = \frac{1}{n} \sum (y - \hat{y})^2$$

x	y	$\hat{y}$	
1	5	6.1	
2	7	10.6	
3	9	15.1	
4	11	19.6	

$$MSE = \frac{1}{4}\left[(5-6.1)^2 + (7-10.6)^2 + (9-15.1)^2 + (11-19.6)^2\right]$$

$$= 31.33$$

$$\left(\frac{\partial MSE}{\partial c}\right)$$

$$c_n = 0 + 0.1\,[1.6] \rightarrow \text{(change)}$$

$100 \rightarrow c_n = 1600$

$.01 \rightarrow c_n = 0.16$

$MSE_{m=0,\, c=0} = 69$

$MSE_{m=4.5,\, c=1.6} = 31.33$

$$m_n = 4.5 - \eta \frac{\partial MSE}{\partial m}$$

$(0.00000000001) \rightarrow \eta$

$$c_n = 1.6 - \eta \left(\frac{\partial MSE}{\partial c}\right)$$

$(0.1,\ 0.001,\ 0.01)$

$$y = mx + c$$

$$\underline{MSE} = \frac{1}{n} \sum_{i=1}^{n} (y - \hat{y})^2$$

$$c_n = c_0 - \eta \frac{\partial MSE}{\partial c}$$

$$m_n = m_0 - \eta \frac{\partial MSE}{\partial c}$$