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Is Time-Discounting Hyperbolic or Subadditive?

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Abstract

Subadditive time discounting means that discounting over a delay is greater when the delay is divided into subintervals than when it is left undivided. This may produce the most important result usually attributed to *hyperbolic* discounting: declining impatience, or the inverse relationship between the discount rate and the magnitude of the delay. Three choice experiments were conducted to test for subadditive discounting, and to determine whether it is sufficient to explain declining impatience. All three experiments showed strong evidence of subadditive discounting, but there was no evidence of declining impatience. I conclude by questioning whether hyperbolic discounting is a plausible account of time preference.

Key words: decision making, intertemporal choice, hyperbolic discounting, support theory

JEL Classification: A12, D81, D90, J22

Introduction

The expected utility of an option is a function of the *value* it will have when and if it is received, the *probability* that it will be received, and the *delay* between the moment of choice and the moment of receipt. Recent studies have shown that judgments of both value and probability are *subadditive*, meaning that the price put on a good, or the probability assigned to an event, is greater if the good or event is first divided into parts which are evaluated individually, with the individual evaluations being summed, than if it is evaluated in its entirety. In this paper, I investigate whether the influence of delay on choice is similarly subadditive.

Kahneman and Knetsch (1992) were amongst the first to demonstrate subadditive pricing. They found that if a public good is decomposed into parts, then the willingness-to-pay for each part is frequently identical to the willingness-to-pay for the whole.[1] One implication of this ‘embedding effect’ is that the total willingness-to-pay for a good depends greatly on whether its parts are priced separately or as a bundle—the more parts there are, the greater the total price. This effect has been replicated dozens of times for non-market goods (like pollution abatement) and has recently been demonstrated even for market goods like meals and peanut butter (Bateman et al., 1997; Frederick and Fischhoff, 1997, 1998). In a variant on this finding, Weber, Eisenführ, and von Winterfeldt (1988) showed that the impact of an attribute on subjective value is increased when it is divided into parts, such as dividing 'job security' into the separate attributes 'low risk of bankruptcy' and 'cannot be fired.'

Subadditive probability judgments have similarly been demonstrated many times in a variety of contexts (e.g., Ayton, 1997; Cohen, Dearnaley and Hansel, 1956; Peterson and Pitz, 1988; Rottenstreich and Tversky, 1997; Tversky and Koehler, 1994; Wright and Whalley, 1983). In one memorable study by Fischhoff, Slovic and Lichtenstein (1978), expert mechanics assigned probabilities to the branches of fault trees that displayed all the reasons why a car might not start. In their 'pruned' fault trees several branches were collapsed into one. The judged probability of a reason was higher if it had a separate branch than if it was part of a branch that subsumed several reasons. The effect of subadditive probability on valuation was demonstrated by Starmer and Sugden (1993), who showed that a prize contingent on either of two independent events each with probability $1/6$ was preferred to one contingent on a single event with probability $1/3$ (c.f., Humphrey, 1995, 1996).

Time discounting, the effect of delay on expected utility, may also be subadditive. Consider someone judging the present value of an outcome to be received in one year. He or she can separately discount for each of the 12 months in the year, or discount once for the unbroken one year. Additive discounting means that the present value is independent of how the year is divided, while subadditive discounting means that the total discounting is greater when the year is divided into months.

Subadditive discounting could have important implications for theories of intertemporal choice. A frequently replicated finding in previous studies has been *declining impatience*, meaning that the average discount rate decreases as the delay increases. However, this finding is based on studies that confound *delay* (the period between the present and when an outcome occurs) with the *interval* between two outcomes. This is significant because declining impatience is the major qualitative prediction made by the hyperbolic time discount function, which has been widely adopted as a description of both human and animal discounting (Ainslie and Haslam, 1992; Kirby, 1997). Hyperbolic discounting attributes declining impatience to delay, but subadditive discounting explains it as a function of the inter-outcome interval.

The main objective of this paper is to determine whether time discounting is subadditive and, if so, to establish whether this can account for declining impatience. I report the results of three experiments that test for subadditive discounting using an experimental design that varies the number of intervals into which a delay is divided. I conclude that time discounting is subadditive, and that this can account for previous findings of declining impatience.

1. Theory

1.1. Additive intertemporal choice

This section shows that conventional discount functions, meaning those which treat discounting is a function of delay only, are subadditive. I begin with some notation. I will treat time as divisible into discrete *periods*. The future begins at 0, the start of period 1, and continues onward. A *discount function* is one which associates with each period i a *discount factor* Δ_i. That is, if an amount V which was to be received at the start of period j is now delayed to the end of that period, its value changes to:

$$V_{(j-1)\to j} = V\Delta_j. \tag{1}$$

The arrow subscript depicts the endpoints of the jth period, which span the interval from the end of period $j-1$ to the end of period j. In general, arrow subscripts will be used to designate the beginning and end of a discounting interval.

An interval that begins at 0 is a *delay*. The *present value* of a delayed amount V is the value that it has in the present, given that it has been delayed from the start of the period 1 (0) to the end of period T:

$$V_{0\to T} = V\prod_{i=1}^{T}\Delta_i. \tag{2}$$

In other words, a decision maker will be indifferent between receiving $V_{0\to T}$ now or V at the end of period T.

Most analyses of time preference assume that people prefer to receive good things as soon as possible. This implies a discount factor between 0 and 1 for all periods, although in principle it can take any non-negative value. Moreover, most economic treatments of time discounting assume that it is exponential. This means that $\Delta_i = \Delta_j$ (designated δ) for all periods i and j. Regardless of these details—whether the discount factor is the same for all periods, and whether it is more or less than 1—discounting will be *additive*. This means that the total discounting over an interval is independent of how the interval is divided. To see this, consider what happens if we subdivide the delay from $0 \to T$ into two subintervals from $0 \to T'$ and from $T' \to T$. The present value after the first delay is given by:

$$V_{0\to T'} = V\prod_{i=1}^{T'}\Delta_i. \tag{3}$$

This value is further discounted from $T' \to T$, so that:

$$V_{0\to T'\to T} = V_{0\to T'} \prod_{i=T'+1}^{T} \Delta_i = \left[V \prod_{i=1}^{T'} \Delta_i \right] \prod_{i=T'+1}^{T} \Delta_i$$
$$= V \prod_{i=1}^{T} \Delta_i = V_{0\to T} \qquad (4)$$

This is the same as the present value (Eq. 2) when the interval is undivided.

The exponential discount function represents the preferences of a 'rational' decision maker, meaning one whose preferences are stationary and dynamically consistent. Although few economists have maintained that people consistently employ exponential discounting,[2] this has not prevented the theoretical possibility from being subjected to empirical attack (e.g., Ainslie, 1975; Benzion et al., 1982; Kirby, 1997; Loewenstein and Thaler, 1989; Thaler, 1981). Most of these attacks have revolved around a single important anomaly, *declining impatience*, which means that the discount factor increases with delay (i.e., $\Delta_{i-1} < \Delta_i$, for all i). Figure 1 shows a discount function that illustrates declining impatience: the ratio between the subjective value of an option available immediately or an otherwise identical option available in one week is greater than the ratio between its value when delayed by one week and when delayed by two weeks.

Several alternative discount functions have been proposed to account for declining impatience. The best known of these is the *hyperbolic* discount function according to which present value is inversely related to a linear function of delay. In its simplest form (Mazur, 1984; Mazur and Herrnstein, 1988), the one-period

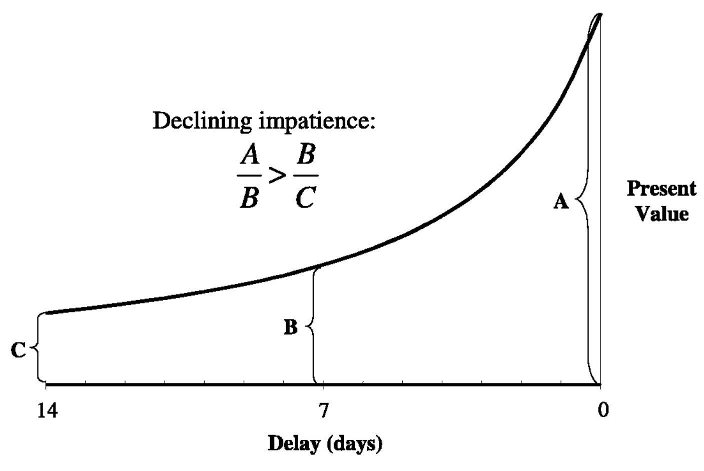


Figure 1. Hyperbolic discount function illustrating declining impatience.

discount factor is:

$$\Delta_j = \left(\frac{1 + k(j-1)}{1 + kj}\right), \tag{5}$$

where k is a *hyperbolic discount parameter*, assumed to be positive. Many generalizations and variations of this function have been proposed (Ainslie, 1975; Green et al., 1994; Harvey, 1994; Laibson, 1997; Loewenstein and Prelec, 1992; O'Donoghue and Rabin, 2000), but Eq. 5 has been shown to do a good job of accounting for discounting over unbroken delays (e.g., Kirby, 1997; Rachlin and Raineri, 1992).

Hyperbolic discounting is additive over any subdivision of a delay. To illustrate, discounting over a delay that is divided into two subintervals, from 0 through T' and then from T' through T, is:

$$\begin{aligned} V_{0\to T'\to T} &= V\left(\frac{1}{1+kT'}\right)\left(\frac{1+kT'}{1+kT}\right) \\ &= V\left(\frac{1}{1+kT}\right) = V_{0\to T} \end{aligned} \tag{6}$$

To facilitate the subsequent discussion, I will use the term *discount fraction* to refer to the proportional change in value that an outcome undergoes over a specified interval. For example,

$$f_{T'\to T} = \frac{V_{T'}}{V_T} \tag{7}$$

When $V_{T'}$ and V_T are two amounts, available at T' and T respectively, between which the decision maker is indifferent. Note that the discount fraction is analogous to a discount factor for an arbitrary interval. If discounting is hyperbolic, for instance, then $f_{T'\to T} = (1 + kT')/(1 + kT)$. For an interval which has been partitioned into a set of subintervals, the discount fraction for the divided interval is the product of the subinterval discount fractions. For instance, when the delay $0 \to T$ is divided into two subintervals:

$$f_{0\to T'\to T} > f_{0\to T'} \times f_{T'\to T}, \tag{8}$$

subadditivity means that $f_{0\to T} > f_{0\to T'\to T}$. That is, discounting over an interval is greater when it is calculated in 'installments' than when it is done in one operation. In the next section I suggest why time discounting is likely to be subadditive.

1.2. Reasons for subadditive intertemporal choice

Several mechanisms have been proposed to explain subadditivity in pricing and probability judgments, at least two of which are likely to influence intertemporal choice. The first is based on cognitive processes such as attention and memory, while the second is 'non-psychological' and suggests that subadditivity will occur whenever judgments and evaluations contain error.

The first class of explanation is exemplified by Tversky and Koehler's (1994) *support theory*, which holds that features are given decision weight in proportion to the attention they receive. When an object or event is subdivided, each part is paid more attention than if it is part of a larger whole. For probability judgments, Tversky and Koehler (p. 549) propose that explicit attention to a sub-event can 'remind people of possibilities that would not have been considered otherwise,' and (of particular importance for intertemporal choices) 'the explicit mention of an outcome tends to enhance its salience and hence its support.' There is no obvious way that intertemporal choices would be based on memory, but it is likely that the salience of a delay is increased by drawing attention to its parts. The imagined pain of two days waiting, for instance, might be increased if the days are contemplated separately rather than together.

The second class of explanation is based on a factor that arises whenever people make quantitative judgments or ratings. Subjective estimates of all kinds are typically biased in the direction of the midpoint of whatever range is being estimated (Woodworth and Schlosberg, 1954; Poulton, 1989), leading to overestimates of small quantities and underestimates of large ones. Stevens and Greenbaum (1966) labeled this the 'regression effect,' evoking the statistical phenomenon of regression-to-the-mean. Regression-to-the-mean, which arises whenever error-prone measurements are made (Nesselroade, Stigler and Baltes, 1980), has been advanced as an explanation for subadditive probability judgments (Mulford and Dawes, 1999; Varey, Mellers and Birnbaum, 1990). The basic idea is that subjective probability judgments typically contain some error, and consequently judgments of high probability events are typically too low, and judgments of low probability events are too high.

A similar regression effect may occur in intertemporal choice. When a measurement scale is left-truncated, the direction of errors for estimates of small quantities will tend to be positive (leading to overestimation); and when they are right truncated, the error for large quantities will tend to be negative (leading to underestimation). Imagine a decision maker deciding what amount $\hat{V}_T$ (available at T) has the same subjective value as $V_{T'}$ (at T') where $T' < T$. If the time interval $(T' < T)$ is small, the 'true' premium required by time will also be small. Underestimates of that premium are bounded on the right by the lowest possible value $V_{T'}$ (the decision maker won't accept less than $V_{T'}$) but relatively unbounded on the left ($\hat{V}_T$ can be very large). The net effect of any error will be biased upward, and the smaller the true premium, the greater the expected positive value of this error. This will lead to subadditivity. A corresponding argument can be made for estimates of $\hat{V}_{T'}$ that will be equivalent to a known value V_T. Now there are two bounds, 0 on the left and V_T on the right. The V_T bound will have its biggest impact on small delays (leading to overestimates of the true discount rate) and the 0 bound will have its biggest impact on long delays (leading to underestimates). Again, the net effect will be subadditivity.

In short, at least two compelling explanations for subadditivity in other domains are applicable to time discounting. It is not the purpose of the present paper to distinguish between explanations for subadditive discounting, but to establish whether it occurs, and to investigate its implications for the theory of hyperbolic discounting. As discussed in the next section, subadditive discounting can look like declining impatience.

1.3. Declining impatience and subadditive discounting

The underlying theory of hyperbolic discounting is that the impact of a given change in delay increases the closer it occurs to the present. To cite Kirby (1997), the value of a reward increases 'by an increasingly larger proportion per unit time as the reward approaches (p. 54).' In Figure 1, this is represented by the fact that the slope of the present value line increases at an accelerating rate as it approaches the point of zero delay. The primary evidence for hyperbolic discounting of monetary rewards comes from studies in which participants either give the present value of alternatives delayed by varying amounts (Benzion, Rapaport and Yagil, 1989; Chapman and Elstein, 1995; Kirby, 1997; Kirby and Marakovic, 1995; Raineri and Rachlin, 1993; Thaler, 1981) or choose between pairs of alternatives comprised of one amount available immediately and another amount available following a delay (Green et al., 1994; Green, Myerson and McFadden, 1997; Holcomb and Nelson, 1992; Richards et al., 1999). In these studies discount rates are derived from the simultaneous variation of two factors: the *delay*, or the time intervening between now and when the later outcome is to occur; and the *interval*, or the time intervening between the two outcomes. For instance, a study might compare the effects of the $0 \rightarrow 6$ month, and $0 \rightarrow 12$ month delays. Not only is 6 months earlier than 12 months (this is the delay), but the interval between 0 and 6 months is shorter than that between 0 and 12 months. The standard finding is that the longer the delay/interval, the lower the discount rate. This is interpreted as support for hyperbolic discounting, which is a theory about *delay*.

Subadditive discounting, however, means that the discount rate will be greater the shorter the *interval*. Imagine that the delay $0 \rightarrow T$ is divided into two *equal* intervals, 0 to $\frac{T}{2}$, and $\frac{T}{2}$ to T. Subadditivity means that

$$f_{0 \rightarrow T} > f_{0 \rightarrow \frac{T}{2} \rightarrow T} = f_{0 \rightarrow \frac{T}{2}} \times f_{\frac{T}{2} \rightarrow T}. \tag{9}$$

This implies either or both of the following:

$$\sqrt{f_{0 \rightarrow T}} > f_{0 \rightarrow \frac{T}{2}} \tag{10}$$

$$\sqrt{f_{0 \rightarrow T}} > f_{\frac{T}{2} \rightarrow T} \tag{11}$$

Eq. 10 is usually interpreted as declining impatience—the average rate of discounting per period is lower for longer delays. Although in principal this inequality is itself sufficient to predict subadditive discounting, it can only do so if the $0 \rightarrow \frac{T}{2}$ delay results in more discounting than predicted by hyperbolic discounting. Eq. 11 is inconsistent with declining impatience, which holds that there will be more discounting over the first period than over the second ($\sqrt{f_{0 \rightarrow T}} < f_{(T/2) \rightarrow T}$).

Subadditive discounting predicts that the shorter the delay, the greater the discount rate over that delay. It may, therefore, be the sole cause of many observations consistent with hyperbolic discounting, in which case the discount fraction for an interval of given length would be independent of when the interval begins ($f_{0 \rightarrow (T/2)} = f_{(T/2) \rightarrow T}$). On the other hand, subadditive discounting may occur *in addition* to hyperbolic discounting. These contrasting predictions can be tested by disentangling the effects of delay and interval, as is done in the experiments reported below.

2. Hypotheses and overview of experiments

Before describing the experiments and the hypotheses, a more convenient notation for discount fractions is introduced. Consider a delay $0 \rightarrow T$ divided into n subintervals. The discount fraction for the divided delay will be denoted:

$$f_{T \cdot n} = \prod_{i=1}^{n} f_{T \cdot n \cdot i}, \tag{12}$$

where i represents the specific subinterval. The symbol $f_{T \cdot 3}$ denotes the discount fraction when the delay is divided into three equal subintervals, and $f_{T \cdot 3 \cdot 1}$, $f_{T \cdot 3 \cdot 2}$, and $f_{T \cdot 3 \cdot 3}$ denote the discount fractions for each of its subintervals.

In the experiments, an interval was decomposed into one or three subintervals. Participants chose between pairs of delayed alternatives. The two delays were held constant but one of the amounts was adjusted in a titration procedure which zeroed in on an 'indifference point' where the two delayed amounts were subjectively equivalent. For instance, it would be possible to infer that the subject was indifferent between \$500 in six months, and \$700 in a year. Discount fractions for divided intervals were obtained in the manner specified in Eq. 12.

Hypothesis 1, *subadditive discounting*, is that the more subintervals into which a delay is divided, the smaller the overall discount fraction. In this study, delays were either divided into three or left undivided, so that the hypothesis was:

H1: $f_{T \cdot 3} > f_{T \cdot 1}$.

According to Hypothesis 2 there is *true declining impatience* which may or may not operate in addition to subadditive discounting. This means that the discount fractions for subintervals will increase along with delay, or that later subintervals will show less discounting than earlier ones:

$$\text{H2: } f_{T\cdot 3\cdot 1} < f_{T\cdot 3\cdot 2} < f_{T\cdot 3\cdot 3}.$$

H2 predicts that (i) there will be a main effect of interval onset, and (ii) this will be associated with an increasing linear trend. If declining impatience is entirely due to subadditive discounting, there will be no systematic relationship between discount fraction and delay. Hypothesis 2 describes the main prediction of hyperbolic discounting.

Hypothesis 3 tests a further assumption of the one-parameter version of hyperbolic discounting. If Eq. 5 is a good approximation of time discounting then the parameter k will be the same for all delays, independent of their length. One possible basis of subadditivity, however, is that discounting for shorter delays is greater than expected: $k_{T\cdot 3\cdot 1} > k_{T\cdot 1}$. Alternatively, it may be that these parameters are equal, as predicted by hyperbolic discounting, but that the discount parameters in later periods are greater than expected. Hypothesis 3 states what would be true if a one-parameter hyperbolic discounting model (Eq. 5) can account for discounting over delays (i.e., intervals that start immediately):

$$\text{H3: } k_{T\cdot 1.1} = k_{T\cdot 3\cdot 1}.$$

According to Hypothesis 4, the *absolute interval assumption*, longer delays will lead to more discounting. That is:

$$\text{H4: } f_{T\cdot 1} < f_{T\cdot 3\cdot 1}.$$

Any account of time preference that assumes positive time preference would make this prediction, so the test of H4 was more a manipulation check than a test of theory: if H4 was rejected, this would cast doubt on the experiment and not the theory.

3. Experiment 1

3.1. Method

3.1.2. Subjects. Subjects were 35 students and staff from the University of Leeds: 59% were female, with a mean age of 23 (range from 18 to 35). They received £2 in vouchers and a chocolate bar for participating. (£1 is worth about $1.50 US.)

3.1.3. Procedure. The experiment was conducted on a computer. Subjects chose between a larger-later (LL) and a smaller-sooner (SS) amount. Once the choice had been made, one of these amounts (the *variable amount*) was adjusted and they chose again. The successive choices were designed to bring the two amounts toward an indifference point, where the SS and LL amounts would have the same present value and a discount fraction could be obtained by taking the ratio SS/LL. I use the term 'choice sequence' to designate a set of choices leading to an indifference point.

Each subject responded to 16 test choice sequences which were preceded by a practice sequence, and presented in random order. The test sequences were constructed according to a 4 (*interval*) $\times$ 2 (variable amount *timing*) by 2 (variable amount on-screen *location*) design. The *intervals* were either an unbroken 24-month interval or three 8-month subintervals designated using exact months and years: the unbroken interval was from 'February 2000' to 'February 2002,' and the subintervals were 'February 2000' to 'October 2000,' 'October 2000' to 'June 2001' and 'June 2001' to 'February 2002.'

The timing and location manipulations were included to ensure that the results were reliable. *Timing* designates whether the SS or LL amount was adjusted following each choice. Each set of delays was repeated once with the SS amount being adjusted (SS-*variable*) and once with the LL amount being adjusted (LL-*variable*). The *fixed* amount was always £500.

Location was varied by repeating each choice sequence with the variable amount on the right and on the left of the screen. The location variable served two purposes. First, it yielded two separate measures of each indifference point estimate which were then combined to yield a more stable measure of the true indifference point. Second, it varied the onscreen presentation of information to ensure that subjects 'paid attention' to the task.

The variable amount was adjusted in response to choices using a 'splitting the difference' procedure, by which each successive amount was found at the midpoint between the highest value they had judged as too low (called *highup*), and the lowest value they had judged as too high (*lowdown*), rounded down to the nearest multiple of 10 (e.g., 245 would become 240). The method can be illustrated with some example choices. At the beginning of an LL-variable choice sequence with LL on the right of the screen, a subject would choose between:

Amount:	£500	£1000
When received:	Feb 2000	Oct 2000.

At the start of the choice sequence, highup was given a starting value equal to SS (£500). If the subject preferred SS, LL would be adjusted upwards by half of the difference between LL and the current value of highup (i.e., LL = LL + (LL − highup)/2), so that the next choice would be between:

Amount:	£500	£1250
When received:	Feb 2000	Oct 2000.

Since the subject had indicated that £1000 in October was too low, this would become the new value for highup. If the subject then chose LL, the variable amount would be adjusted downward by half the difference between £1,000 (highup) and £1,250 (lowdown), rounded down to the nearest £10:

Amount:	£500	£1120
When received:	Feb 2000	Oct 2000.

This process would continue until the difference between highup and lowdown was less than £10. An *indifference value*—the value of the variable amount at the indifference point—would then be estimated as:

$$\frac{\text{lowdown} + \text{highup}}{2}.$$

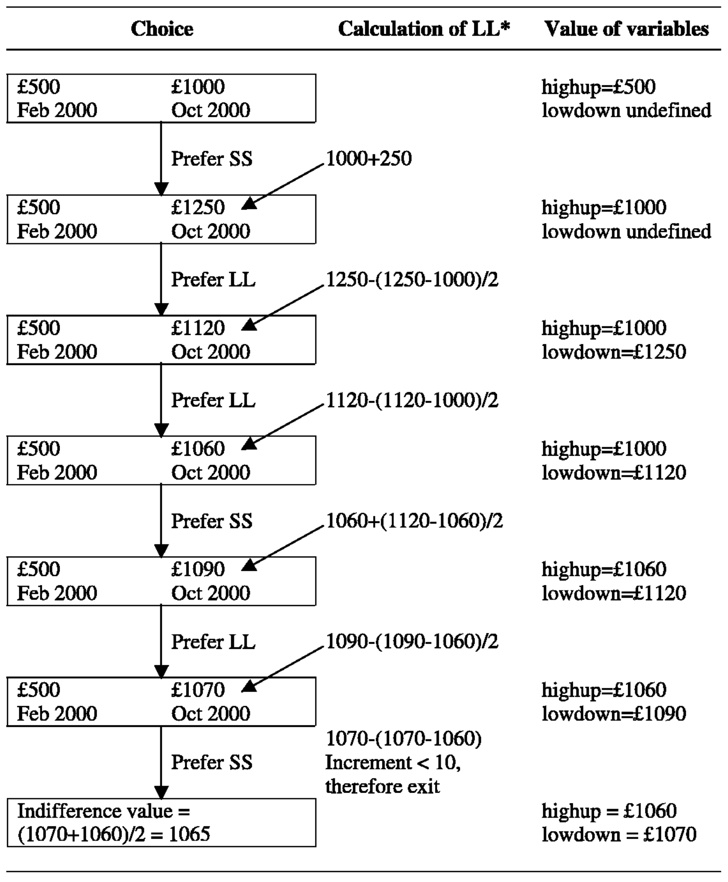
In the LL-variable condition, possible indifference values ranged from £505 to £2495 (in £10 increments); in the SS-variable condition, they varied from £5 to £495. An illustrative choice sequence, completing the one started here, is shown in Figure 2.

In Experiment 1 and 2 a choice sequence was ended if (a) an indifference point was reached, or (b) a series of choices were made that would bring the indifference value above a ceiling of £2,500. Because the unconstrained indifference value of any subject whose choices reached this ceiling were unknown, these subjects were excluded from the analysis. Data collection continued until there were 32 subjects whose indifference values were all below this ceiling. In Experiment 1, three subjects were excluded in this way. To demonstrate that none of the results reported below are due to excluding these subjects, Table 1 lists the median values, based on all subjects, for the discount fractions that appear in the analyses of all experiments. For every important comparison, the pattern of the medians and means is identical.

3.2. Results

3.2.1. Subadditive discounting (H1). The mean discount fractions for both subdivisions of the two year interval, and for both timings are depicted in Figure 3. There was clear evidence of subadditive discounting, with discount fractions being much lower when the interval was subdivided into three. This was confirmed with a 2 (number of intervals) $\times$ 2 (timing) repeated measures ANOVA, which revealed a strong main effect of number of intervals ($F[1, 31] = 36.7$, $p < .001$). A further index of subadditivity is the proportion of subjects for whom $f_{T \cdot 1} > f_{T \cdot 3}$: 91% in the LL-variable and 78% in the SS-variable condition.

In addition to subadditive discounting, there was a main effect of timing ($F[1, 31] = 5.9$, $p < .05$), with discount fractions being substantially lower in the SS-variable condition. This is a manifestation of the well known finding that discount rates are inversely related to the amount being discounted (e.g., Green et



* All values are rounded down to the nearest multiple of 10.

Figure 2. Sample choice sequence illustrating methods used to find indifference points in all experiments. Highup is the highest value which the respondent has judged to be too low, and Lowdown is the lowest value judged to be too high.

al., 1997; Kirby, 1997; Thaler, 1981). Both the SS and the LL values were smaller in the SS-variable condition, when discounting was greatest, than in the LL-variable condition. The timing effect was small relative to the subadditivity effect: the 95% confidence interval for the impact of subadditivity on the discount fraction (i.e., the difference $f_{T\cdot 1} - f_{T\cdot 3}$) was 0.08 to 0.17; the corresponding interval for the timing effect was 0.006 to 0.07.

3.2.2. True declining impatience (H2). Figure 4 shows no evidence of declining impatience, defined as $f_{T\cdot 3\cdot 1} < f_{T\cdot 3\cdot 2} < f_{T\cdot 3\cdot 3}$. Indeed, the discount fraction was greatest for the first interval, suggesting *increasing* impatience. A 3 (interval

Table 1. Median discount fractions for all conditions of experiments 1–3[a]

Timing[b]	Discount fraction				
	$f_{T\cdot 1}$	$f_{T\cdot 3}$	$f_{T\cdot 3\cdot 1}$	$f_{T\cdot 3\cdot 2}$	$f_{T\cdot 3\cdot 3}$
Exp 1					
SS	0.47	0.39	0.74	0.68	0.67
LL	0.60	0.39	0.81	0.71	0.72
Exp 2					
SS	0.43	0.32	0.68	0.71	0.64
LL	0.56	0.28	0.72	0.64	0.67
Exp 3					
SS	0.62	0.36	0.73	0.68	0.64
LL	0.61	0.36	0.72	0.68	0.68

[a] H1: $f_{T\cdot 1} > f_{T\cdot 3}$; H2: $f_{T\cdot 3\cdot 1} > f_{T\cdot 3\cdot 2} > f_{T\cdot 3\cdot 3}$; H4: $f_{T\cdot 1} > f_{T\cdot 3\cdot 1}$.
[b] SS (Smaller Sooner), earlier and smaller amount adjusted following each choice; LL (Larger Later), later and larger amount adjusted.

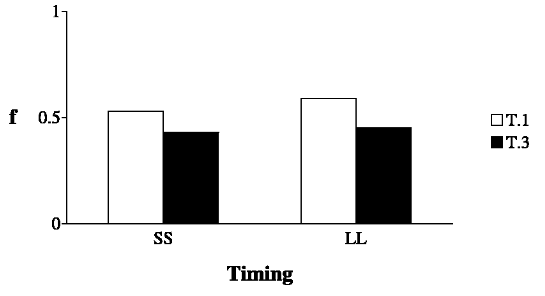


Figure 3. Test of H1 (Subadditivity) in Experiment 1. $T \cdot 1$ is the discount fraction when the interval is undivided (1 interval), and $T \cdot 3$ is when it is divided into 3 subintervals.

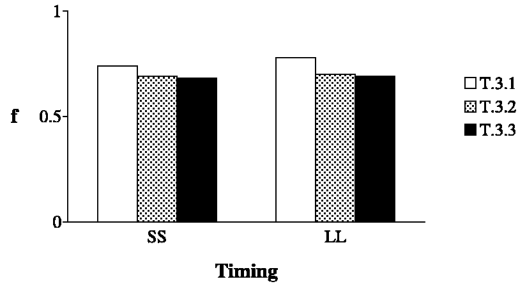


Figure 4. Test of H2 (True declining impatience) in Experiment 1. $T \cdot 3 \cdot 1$ is the discount fraction for the first subinterval, $T \cdot 3 \cdot 2$ for the second subinterval, and so on.

onset) × 2 (timing) repeated measures ANOVA showed a significant main effect for onset only ($F[1,31] = 14.5$, $p < .001$). Within-subject contrasts revealed a significant linear ($F = 20.2$) and quadratic ($F = 5.0$) trend confirming what can be seen in the figure: discount rates increased the later the onset (linear trend), but at a declining rate so that the increase from the first interval to the second exceeded that from the second to the third (quadratic trend).

One explanation for increasing impatience is that subjects may have viewed the first interval as shorter than the others. The first interval went from 'February 2000' to 'October 2000,' while the second went from 'October 2000' to June 2000.' The experiment was conducted during February, so there was less of February remaining than any of the other months. Unlike the later intervals, therefore, the first one may have been treated as containing less than eight months (seven months plus whatever remained of February). In Experiments 2 and 3 the methods were changed to ensure that all intervals would clearly be the same length.

3.2.3. Constant value of k (H3). The mean and median values of the discount parameter k for all conditions of Experiments 1 to 3 are given in Table 2. These discount parameters for all intervals $T' \to T$ were obtained by taking[3]:

$$k = \frac{1}{T' - T}\left(\frac{v_T}{v_{T'}} - 1\right) \tag{13}$$

Table 2. Mean and median values of k for all conditions of experiments 1–3[a]

Timing[b]		Discount parameter			
		$k_{T\cdot 1}$	$k_{T\cdot 3\cdot 1}$	$k_{T\cdot 3\cdot 2}$	$k_{T\cdot 3\cdot 3}$
Exp 1					
SS	Mean	0.91	0.81	1.10	1.15
	Median	0.56	0.44	0.54	0.68
LL	Mean	0.52	0.56	0.97	0.96
	Median	0.37	0.35	0.49	0.58
Exp 1					
SS	Mean	0.91	1.56	1.09	1.23
	Median	0.61	0.69	0.60	0.84
LL	Mean	0.56	0.89	1.00	0.82
	Median	0.36	0.56	0.83	0.71
Exp 3					
SS	Mean	0.47	0.70	1.00	0.70
	Median	0.22	0.66	0.95	0.51
LL	Mean	0.53	0.76	1.00	0.89
	Median	0.31	0.60	0.78	0.82

[a] H3: $k_{T\cdot 1} = k_{T\cdot 3\cdot 1}$.
[b] SS (Smaller Sooner), earlier and smaller amount adjusted following each choice; LL (Larger Later), later and larger amount adjusted.

In line with H3, the discount parameter for the first subinterval ($k_{T\cdot 3\cdot 1}$) was approximately equal to that for the undivided interval ($k_{T\cdot 1}$). A 2 × 2 ANOVA showed the familiar effect of timing ($F(1, 31) = 7.4$, $p < .01$), but no effect of interval length ($F < 1$).

If, as just discussed, the first subinterval was treated as encompassing fewer than eight months then this would have the effect of underestimating $k_{T\cdot 3\cdot 1}$ to a greater degree than $k_{t.1}$. Consistent with this suggestion, $k_{T\cdot 3\cdot 2}$ and $k_{T\cdot 3\cdot 3}$ were larger than $k_{T\cdot 1}$. The change of design in Experiments 2 and 3 permitted this possibility to be tested.

3.2.4. Absolute interval assumption (H4). The absolute interval assumption, according to which the shorter the delay the greater the discount fraction, was fully supported. A 2 (interval length) × 2 (timing) ANOVA found a strong effect of interval length ($F[1, 31] = 36.7$, $p < .001$), as well as timing ($F[1, 31] = 5.9$, $p < .02$). Virtually all subjects fit the predicted pattern: 97% in the SS-variable condition, and 88% in the LL-variable condition.

4. Experiment 2

Experiment 2 was a replication of Experiment 1, with two changes made to increase the validity of the results. First, the description of the times when the money would be received was changed to ensure that all intervals were the same length. Second, subjects were given feedback about their 'indifference points' and allowed to change them if they were dissatisfied.

4.1. Method

Subjects were 34 students and staff from University of Leeds who were paid £4 for participation. Their average age was 26 (range from 19 to 49) and 75% were female. Two subjects were dropped from the primary analysis because at least one of their indifference values exceeded the ceiling of £2,500, although they are included in the medians reported in Table 1.

Experiment 2 incorporated two modifications to Experiment 1. First, instead of using specific months the delays were described directly in terms of the number of months delay: the delays were given as '0 months,' '8 months,' '16 months,' and '24 months,' with choices presented on screen as follows:

Amount:	£500	£1120
Delay:	0 months	8 months.

Second, subjects were given the opportunity to indicate whether they agreed or disagreed with the indifference points generated by the computer. The indifference point was presented on the computer screen in the following way:

> Based on your choices, the computer has calculated that you value the following two payoffs equally:
>
> | | | |
> |---|---|---|
> | Payment: | £500 | £960 to £970 |
> | When received: | 0 months | 8 months |
>
> Please press the key marked
> AGREE if you agree
> DISAGREE if you disagree (and want another go)

The F4 and F8 keys were used to designate agreement and disagreement. When subjects disagreed they repeated the choice sequence until they were satisfied.

4.2. Results

The following analyses are restricted to the indifference points which subjects agreed were correct. If all subjects and all trials (including practice trials) are included, 90% of the first indifference points were agreed. If we restrict our attention to the non-practice trials of the 32 subjects whose indifference values never exceeded £2,500, then 94% of the indifference points were agreed.

4.2.1. Subadditive discounting (H1). Figure 5 shows strong evidence of subadditive discounting, which was shown by the great majority of subjects (84% in the SS-variable and 94% in the LL-variable condition). A 2 (number of intervals) × 2 (timing) repeated measures ANOVA showed a main effect of number of intervals ($F[1, 31] = 82.4$, $p < .0001$). This effect appeared to be slightly greater (and more

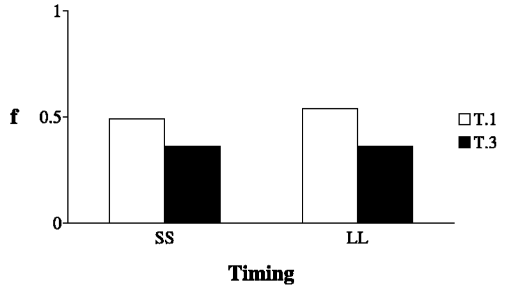


Figure 5. Test of H1 (Subadditivity) in Experiment 2. $T \cdot 1$ is the discount fraction when the interval is undivided (1 interval), and $T \cdot 3$ is when it is divided into 3 subintervals.

reliable) than that observed in Experiment 1—the 95% confidence interval for the difference ($f_{T\cdot 1} - f_{0 \to T\cdot 3}$) was 0.12 to 0.19.

The timing effect approached significance ($F[1, 31] = 3.0$, $p < .1$), and there was an interval-by-timing interaction ($F[1, 31] = 4.5$, $p < .05$). This interaction reflects the fact that discounting was higher in the SS-variable condition only when the interval was undivided.

4.2.2. True declining impatience (H2). The mean discount fraction for all interval onsets and both timings are shown in Figure 6. The effect of increasing impatience observed in Experiment 1 was not replicated, but neither was there any evidence of declining impatience. This was confirmed with a 3 (onset) × 2 (timing) repeated measures ANOVA that showed no main effects. This is what would be expected if subadditivity alone could account for declining impatience.

4.2.3. Constant value of k (H3). As shown in Table 2, The discount parameter was greater for an 8 month delay ($k_{T\cdot 3\cdot 1}$) than for a 24 month delay ($k_{T\cdot 1}$). A 2 (interval length) × 2 (timing) ANOVA revealed a main effect of both length ($F[1, 31] = 4.2$, $p < 0.05$), and timing ($F[1, 31] = 5.6$, $p < 0.05$). The proportion of subjects for whom $k_{T\cdot 3\cdot 1} > k_{T\cdot 1}$ was 63% in the SS-variable condition and 75% in the LL-variable condition.

4.2.4. The absolute interval assumption (H4). This manipulation check was highly significant. A 2 (interval length) × 2 (timing) ANOVA revealed a strong effect of interval length ($F[1, 31] = 64.2$, $p < .0001$), and an effect of timing ($F[1, 31] = 7.7$, $p < .01$), showing greater discounting in the SS-variable condition. The proportion of subjects for whom $f_{T\cdot 3\cdot 1} > f_{T\cdot 1}$ was 84% in the SS-variable condition, and 94% in the LL-variable condition.

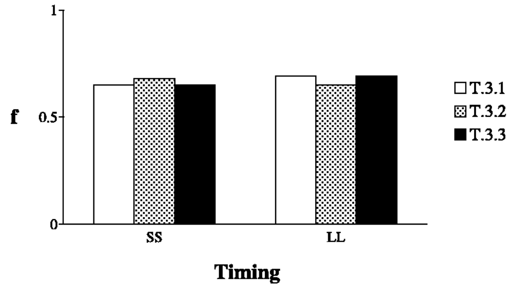


Figure 6. Test of H2 (True declining impatience) in Experiment 2. $T \cdot 1$ is the discount fraction when the interval is undivided (1 interval), and $T \cdot 3$ is when it is divided into 3 subintervals.

5. Experiment 3

Experiment 3 replicated the earlier studies with an additional modification. Kirby (1997) has argued that discounting measures are more reliable when measured using real choices rather than hypothetical ones. This experiment incorporated real choices by following the experiment with a draw in which one of the participants received one of their choices for real.

5.1. Methods

Twenty subjects participated, 45% were female, with a mean age of 31.2 (range from 20 to 48). Subjects were mature members of the University of Leeds community, all of whom worked for a living (either as course tutors, lecturers, researchers or administrative staff), and were thus expected to have had experience trading-off current and future consumption. Of these participants, 16 were included in the final data set, with 4 excluded because at least one of their indifference values exceeded a ceiling.

All subjects were tested in a single one hour period. Following the session, the name of one subject was drawn at random, and this subject received one randomly selected choice 'for real' (I wrote a post-dated check for the delayed amount that they preferred). Because of this 'real choice', the amounts on offer were only one-tenth of those in Experiments 1 and 2—the fixed amount was £50 ($75), and the variable amount could reach a ceiling of £250 ($375). In addition, the indifference points were rounded to the nearest £2, rather than £10 as in the earlier studies. With small amounts rounding to the nearest £10 would have had a significant effect on the precision of the indifference amount estimates.

A second modification was incorporated to enhance task verisimilitude. The payday for each delay was given as an exact date, the last Friday of the month: the specific dates were March 31, 2000 (0 month delay); Sept 29, 2000 (6 months); March 30, 2001 (12 months); and Sept 28, 2001 (18 months). In this study, the dates ranged over 18 months (rather than 24 as in the earlier studies) because: (a) with the real choice element I did not want the time interval to be as long as in earlier studies because people might not know whether they (or I, as the paymaster) would be around for so long; and (b) I wanted to avoid dates that were close to Christmas, which might have distorted preferences for that date (8 months from March would have entailed the last week in November—the traditional pre-Christmas paycheck).

5.2. Results

As in Experiment 2, only those indifference points with which participants concurred were included in the analysis, which in this study included 90% of non-practice trials. The rejection rate was somewhat higher in this study than in Experiment

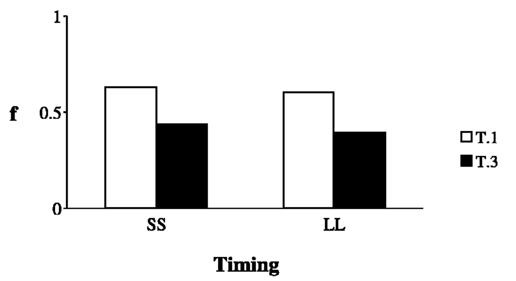


Figure 7. Test of H1 (Subadditivity) in Experiment 3. $T \cdot 1$ is the discount fraction when the interval is undivided (1 interval), and $T \cdot 3$ is when it is divided into 3 subintervals.

2 (10% versus 6%), suggesting that the real money incentive made subjects more vigilant about the accuracy of the calculated indifference points.

5.2.1. Subadditive discounting (H1). Figure 7 shows clear evidence of subadditive intertemporal choice. The prevalence of subadditive discounting was even greater in Experiment 3 than in the earlier ones, being observed in 89% of choices in the SS-variable condition and 100% in the LL-variable condition. A 2 (number of intervals) $\times$ 2 (timing) repeated measures ANOVA confirmed the strong main effect of number of intervals ($F[1, 15] = 60.4$, $p < .0001$). The confidence interval for the difference $f_{T \cdot 1} - f_{T \cdot 3}$ was 0.153 to 0.268. There was no effect of timing.

5.2.2. True declining impatience (H2). No declining impatience was observed in the present study. Indeed, there was slightly less discounting in the earliest interval than in either the second or third. A 3 (interval onset) $\times$ 2 (timing) ANOVA showed a significant effect of onset ($F[2, 14] = 4.05$, $p < .05$). Further analysis confirmed (as can be seen from Figure 8) that this was due to a quadratic trend ($F = 5.2$)—the middle interval (from 6 to 12 months) showed more discounting than the other two.

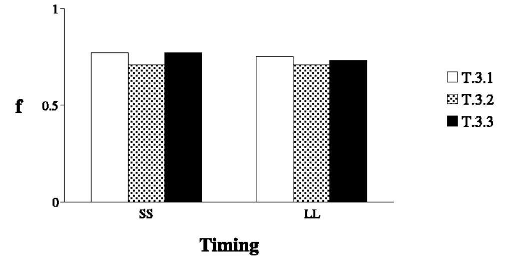


Figure 8. Test of H2 (True declining impatience) in Experiment 3. $T \cdot 1$ is the discount fraction when the interval is undivided (1 interval), and $T \cdot 3$ is when it is divided into 3 subintervals.

5.2.3. Constant value of k (H3). As in Experiment 2, the discount parameter k was higher for the short delay than for the long one. This was confirmed with a 2 (interval length) $\times$ 2 (timing) ANOVA: $f(1, 15) = 12.01$, $p < 0.01$. The proportion of subjects for whom $k_{T\cdot 3\cdot 1} > k_{T\cdot 1}$ was 88% in both timing conditions.

5.2.4. Absolute interval assumption (H4). This manipulation check was highly significant. A 2 (interval length) $\times$ 2 (timing) ANOVA revealed an effect of interval length ($F[1, 15] = 8.4$, $p < .01$). The proportion of subjects for whom $f_{0 \to T\cdot 3\cdot 1} > f_{T\cdot 1}$ was 94% in both timing conditions.

6. Discussion

The results were clear. First, subadditive intertemporal choice was observed in every experiment: when a delay was divided into three, the total discounting over that delay was increased by an average of 40%. Second, there was no evidence of declining impatience: the amount of discounting was equal or lower for earlier intervals than for later intervals. As already discussed, these findings have major implications for a host of previous findings conventionally attributed to hyperbolic discounting. Most experimental support for hyperbolic discounting, particularly for outcomes described as gains and losses of money, comes from studies that confound delay with interval, and consequently may only provide support for subadditive discounting. Without further research much of the evidence for declining impatience, the main prediction of hyperbolic discounting, must be judged as doubtful.

In this discussion I take up two further issues. I first discuss how a discount function might be modified to incorporate the pattern of results observed in these experiments. This is followed by an examination of the relationship between this paper and other findings often attributed to hyperbolic discounting. I will discuss whether my findings are compatible with this research.

6.1. Toward a subadditive discount function

Most qualitative predictions of subadditive discounting are identical to those of hyperbolic discounting. Both predict an inverse relationship between the length of a delay and the average discount rate for that delay. A delay of one year will yield more discounting per-unit-time than will a delay of two years. Both also predict that the discount rate over an interval, no matter when it begins, will be higher the longer the interval. Adding one week to a month is more painful (per-unit-time) than adding four weeks to a month.

In contrast to hyperbolic discounting, however, subadditive discounting predicts that adding four separate one-week intervals to a month will be more painful than adding one four-week interval. Moreover, *exclusive* subadditive discounting (no declining impatience) differs from hyperbolic discounting by predicting that short delays will be just as painful if they start in one week or in one year.

One way that hyperbolic discounting falls short is that it treats intervals as ratios between delays, rather than as intervals proper. As mentioned above (Section 1), the hyperbolic discount fraction for an interval is given as:

$$f_{T' \to T} = \frac{1 + kT'}{1 + kT} \tag{14}$$

This expression is derived by treating discounting from $T' \to T$ as the ratio between discounting from $0 \to T'$ and $0 \to T$. If discounting over intervals is due to their length (or the *difference* between delays) rather than their distance from the present, then interval length $(T - T')$ should be introduced directly into our discounting formula.

A second finding inconsistent with hyperbolic discounting is that estimates of the k parameter were inversely related to interval length. There are numerous ways to accommodate this in an alternative discount function. One way is to interpret it as reflecting time perception. There is ample evidence that people overestimate short intervals and underestimate long ones in both memory and perception (Björkman, 1984; Fraisse, 1963, 1984), and it is plausible that subjective intervals also increase with real intervals at a declining rate.

The following modified function, which summarises the qualitative results observed in this study, is a plausible alternative account of discounting.

$$f_{T' \to T} = \frac{1}{1 + k(T - T')^{s}} \tag{15}$$

Where s $(0 < s \leq 1)$ is a parameter that reflects non-linear time perception. This parameter was suggested by a similar one adopted by Green et al. (1994) and Myerson and Green (1995) to incorporate the fact that present value declines less rapidly than predicted by a hyperbolic discount function. When Myerson and Green fitted the s parameter to their data, the usual result was that $0 < s < 1$.

A variant of the conventional hyperbolic discount function is, however, unnecessary to accommodate the results of these studies if we permit ourselves the luxury of an s parameter reflecting non-linear time perception. The *exponential* discount function modified with an s parameter makes the same qualitative predictions:

$$f_{T' \to T} = \delta^{(T' - T)^{s}}. \tag{16}$$

This function shows subadditivity, but no declining impatience.

The modifications to conventional discounting models above may be a good way to accommodate the results of the present experiments, and may indeed describe *money* discounting. As discussed in the next section, however, both the results and the models are inconsistent with a major finding, often attributed to hyperbolic discounting, involving non-monetary goods.

6.2. Preference reversals and declining impatience

Those familiar with research into preference reversals, or even those who have occasionally struggled with their own weaker impulses, might raise an objection to the suggestion that there is no declining impatience. This objection is based on overwhelming evidence, both in the laboratory and in the field, of dynamic inconsistency in the form of the failure to withstand temptation. Consider an experience of a type common to most dieters: immediately after a full breakfast, I vow to have a light salad for lunch; yet when lunchtime rolls around and I see what's available in the dining room, I order fish-and-chips (while promising to have a salad for supper). Such self-control failures have been widely discussed and explained using the concept of hyperbolic discounting (e.g., Ainslie and Haslam, 1992; Frank, 1988; Nozick, 1993). The idea is that temptation involves a choice between a small benefit in the near future (the SS alternative), and a larger benefit in the more distant future (the LL alternative). This choice, along with hyperbolic discount functions for the alternatives, is depicted in Figure 9. Because of hyperbolic discounting, the present values of the two alternatives cross over, so that the present value of SS exceeds that of LL during some interval preceding consumption of SS. When choices are made during this interval, the SS alternative is often chosen.

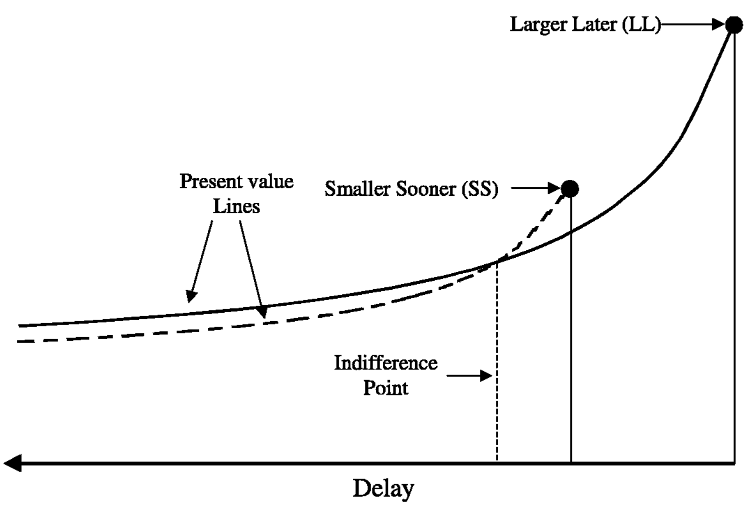


Figure 9. How hyperbolic discounting predicts impulsive choices of smaller-sooner over larger-later options.

Hyperbolic discounting has never, however, been entirely satisfying as an explanation for these failures of self-control (see Hoch and Loewenstein, 1991). It has two major failings. The first and most important is that it does not distinguish between the *kinds* of goods which lead to preference reversals. Preference reversals occur when choosing between experiences (generally *not* money) that will bring immediate or delayed pleasure and pain. Indeed, they appear to be largely restricted to situations involving the satisfaction of what Loewenstein (1996) has labeled 'visceral' states, which include hunger, fatigue, sexual arousal, frustration, pain and even (according to Loewenstein) curiosity. In true Freudian fashion, the desire to eliminate a state of visceral arousal can reduce the influence of more rational processes. Consistent with this, some goods are never associated with self-control failures, while other goods trouble almost everyone at least sometimes. I can illustrate the difference using my own preferences. If you offer me a choice between one pack of computer paper in two hours and two packs in four hours, I will take the two packs. I won't change my mind even if you offer me the one pack immediately, and I won't be tempted even if you are standing in front of me holding the pack in your hand. On the other hand, if you offer me a choice between a hamburger in two hours and a fine dinner in four hours I will take the fine dinner. If you offer me the hamburger immediately, I might well change my mind. This is even more likely if I can see and smell the hamburger right now. There is a crucial difference between my response to the hamburger and the computer paper which is not reflected in the concept of a discount function. Consistent with such anecdotes, virtually all laboratory or field studies of time-based preference reversal are based on comparisons between options that give almost immediate pleasure versus options that are better in the long run (Christensen-Szalanski, 1984; Read and Van Leeuwen, 1998; Read, Loewenstein and Kalyanaraman, 1999).[5]

In the experiments described above I did not offer people choices between different quantities of visceral goods, but between different amounts of *money*. Although money is certainly pleasurable to receive, and thus may have some visceral properties, this is not its primary characteristic. Most of the benefits that come from money are distributed over time. The benefits need not begin when the money is received (it is possible to borrow on the promise of future earnings) and certainly do not end then. The way we discount money need have little or no relationship to the way we discount pleasure or pain.[6] Subadditive intertemporal choice, therefore, which does not predict preference reversals for choices between money, is fully consistent with preference reversals for other goods.

The second failing of hyperbolic discounting is that it is relatively unspecific about the length of the delay preceding the SS alternative and when the preference reversal is likely to occur. It depends on the curvature of the hyperbolic discount function, the relative size of the LL and SS alternatives, and the delay intervening between them. It might occur a month before the SS is to be consumed, a week before, or a second before. Yet anyone who has lapsed knows that we can be much more specific than that. Almost every temptation works by offering immediate pleasure. Dieters don't lapse by planning a big feast tomorrow, but by grabbing a slice of cake *right now*. Alcoholics fall off the wagon when confronted by temptation, not before. Dynamic inconsistency of this sort does not require hyperbolic discounting, but only an *immediacy effect* (Hoch and Loewenstein, 1991). This means that we put a lot more decision weight on immediate relative to delayed pleasure or pain. An immediacy effect is *not* declining impatience, but rather a one-time-only charge for delaying consumption.

An immediacy effect may occur for money. It is certainly consistent with the experiments in this paper. In none of the studies was it implied that the money would be received immediately, even for the earliest possible payoff time. In Experiment 3, when exact payoff dates were given, the earliest payoff date was almost two weeks from the moment of choice. It is possible that an immediacy effect occurred in the days before the moment at which the first choice was anticipated to take place. That is, all the options were devalued because they were not immediate.

6.3. Conclusion

The hyperbolic discounting model has become a dominant paradigm around which both psychological and behavioral-economic studies oftime preference are organized. The studies reported above suggest that the main 'interesting' prediction of this model—that the discount rate declines with delay beyond a one-shot immediacy effect—is incorrect. Moreover, if we go beyond the context of the present studies, hyperbolic discounting fails to predict any further interesting phenomena in intertemporal choice. It cannot accommodate sign effects (Thaler, 1981: discounting is more rapid for gains than for losses), magnitude effects (Kirby, 1997: discounting is more rapid for small amounts than for large amounts), reference point effects (Loewenstein, 1988: discounting is greater when a positive experience is delayed than when it is brought forward), intransitive intertemporal choice (Roelofsma and Read, 2000), a variety of sequence effects (Chapman, 1996; Loewenstein and Prelec, 1993), or any other major result. The weight of evidence against hyperbolic discounting appears strong, and I suggest that researchers should reconsider their allegiance to it as a model of human time preference.

Notes

1. To cite one of Kahneman and Knetsch's (1992) compelling examples, the willingness-to-pay for 'providing parks, pollution control, preservation of wilderness and wildlife, disposal of industrial wastes, and improved *preparedness for disasters*' was no greater than that for one small aspect of preparedness for disasters: 'improving the availability of equipment and trained personnel for rescue operations' (p. 60).
2. Samuelson (1937), credited with the first statement of discounted utility, recognized that 'individuals do not behave in terms of our functions,' when citing phenomena (precommitment) that have since

become a staple piece of evidence for hyperbolic discounting. Other views varying in the degree of skepticism are those of Becker and Mulligan (1997); Böhm-Bawerk (1888/1930); Hammond (1976); Laibson (1997), and Strotz (1955). Loewenstein (1992) provides a historical review of the perspectives of many economists.

3. This equation is correct [based on Eq. (5)] when $T' = 0$. For later intervals, it is based on the assumption that discounting begins and ends with each subinterval. The discount rate for continuous discounting is given by:

$$k_{T' \to T} = \left(\frac{V_T - V_{T'}}{TV_T - T'V_{T'}} \right).$$

For later intervals, which do not begin at 0, this will always give much higher values of k than those found in Table 2.

4. Consider two amounts $V_{T'}$ and V_T which will be available at T' and T respectively and have equal present value. That is:

$$V_{T'} \left(\frac{1}{1 + kT'} \right) = V_T \left(\frac{1}{1 + kT} \right).$$

The discount fraction is given by the ratio $V_{T'}/V_T$, which is equal to Eq. (18), the discount fractions for the two delays.

5. There are two exceptions to this assertion. Kirby and Herrnstein (1995, Experiments 1 and 2) found time-based preference reversals for money. Their experimental design and their results suggests that the preference reversals they observe are likely attributable to an immediacy effect, as discussed below.
6. This is reflected in economic models of time preference which describe preferences over streams of *consumption* or *utility* rather than money (e.g., Fisher, 1930; Koopmans, Diamond and Williamson, 1964; O'Donoghue and Rabin, 2000; Samuelson, 1937; Strotz, 1955).

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