D1EAD – Análise Estatística para Ciência de Dados 2021.1

Data Distributions (Part 3)

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Problem

Suppose the heights of the inhabitants of a city are **normally distributed** with **population standard deviation** of 20 cm.

We measure the heights of 40 randomly chosen people, and get a **mean height** of 1.75 m.

Construct a **confidence interval** for the **population mean** with a **significance level of 5%**.

Let’s recap some concepts

fixed number from population

change from sample to sample

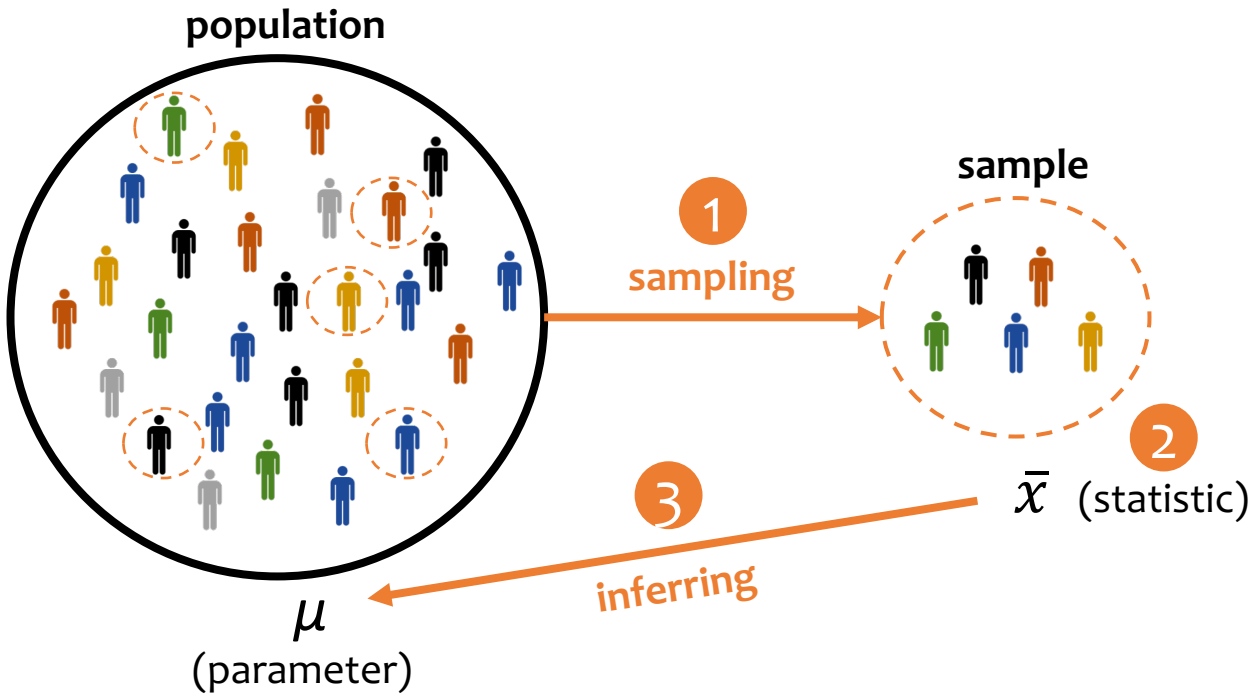
Population Parameter	Sample Statistic	Description
N	n	Number of elements.
μ	$\bar{x}$	Mean
σ	s	Standard deviation
ρ	r	Correlation coefficient.

https://www.youtube.com/watch?v=mtbJbDwqWLE

Estimation

Estimation

The process of using a **sample** to make **inferences** about a **population** is called **statistical inference**.



Types of Estimation

point estimate

single estimate of a population parameter

$\bar{x}$

No statements are made about the quality or precision.

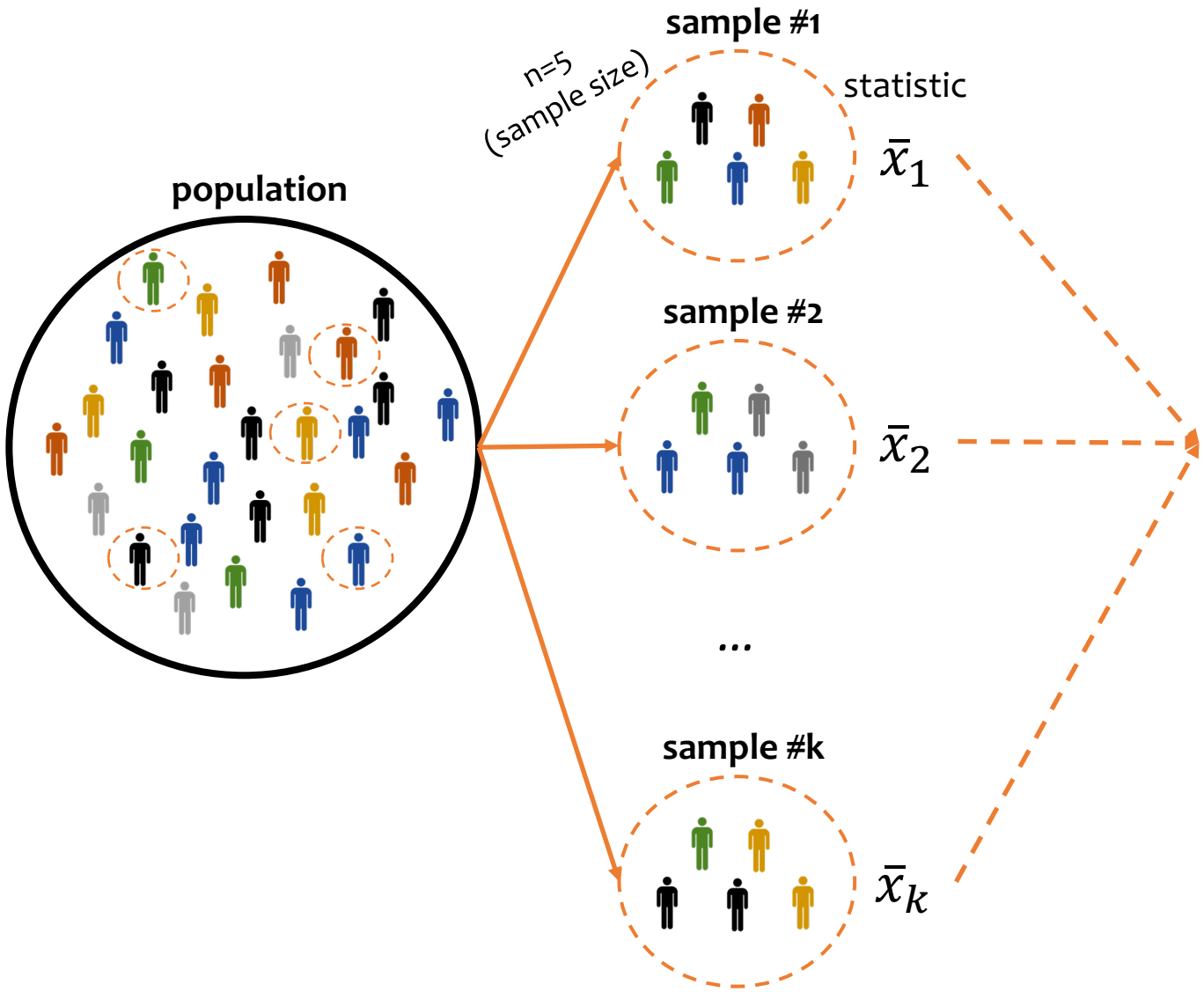
interval estimate

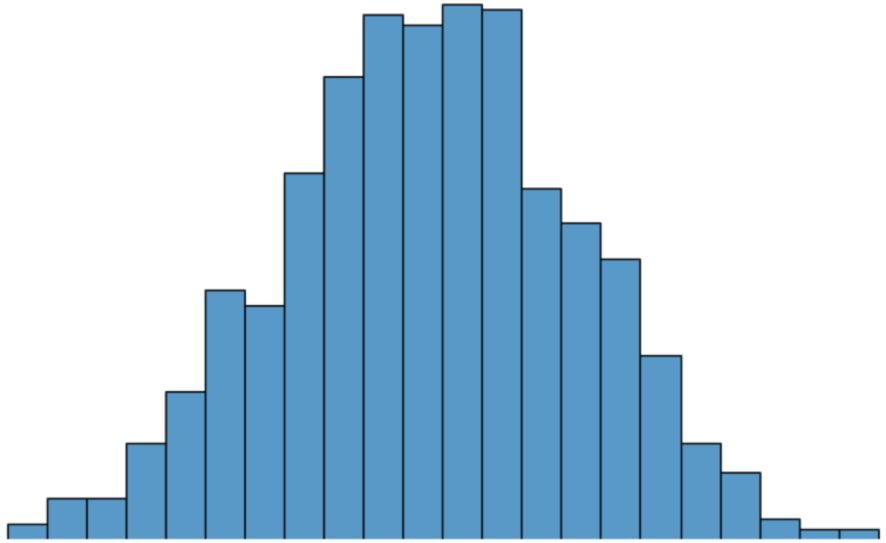
degree of confidence that the **interval** contains the population parameter being estimated

$[\bar{x} - e, \bar{x} + e]$

e.g., with 95% confidence level

Sampling Distribution of a Statistic

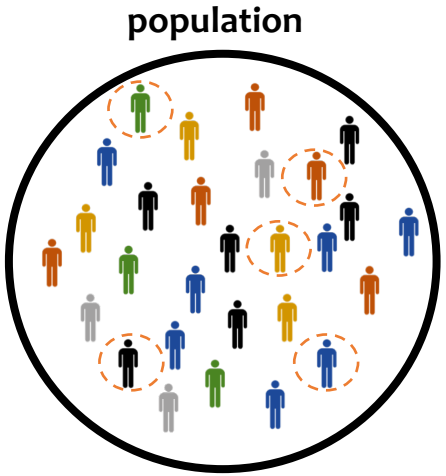


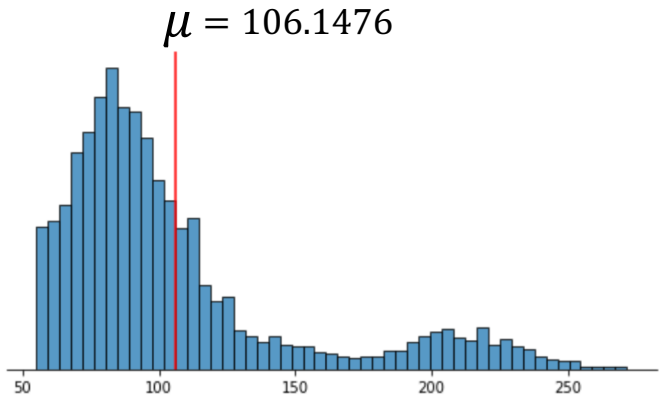


sampling distribution of the sample mean

Central Limit Theorem

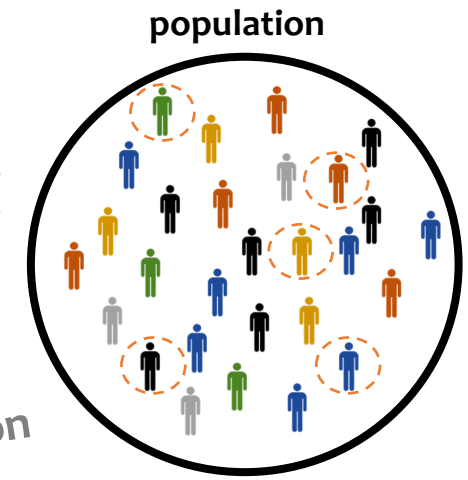
Central Limit Theorem

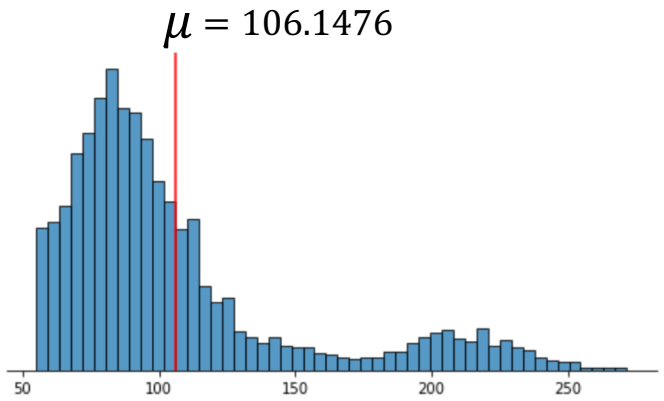




Central Limit Theorem

As the **sample size increases**, the **sampling distribution** of the **mean** approaches a **normal distribution** (no matter the population distribution)





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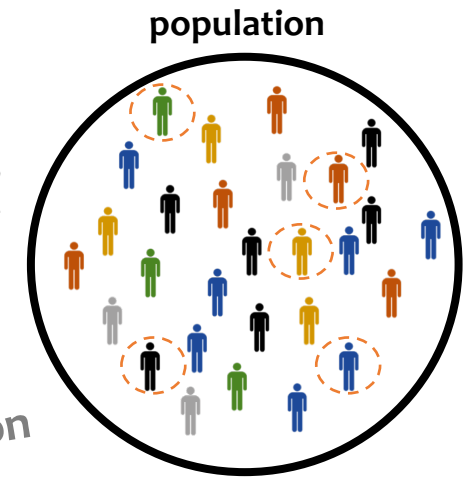
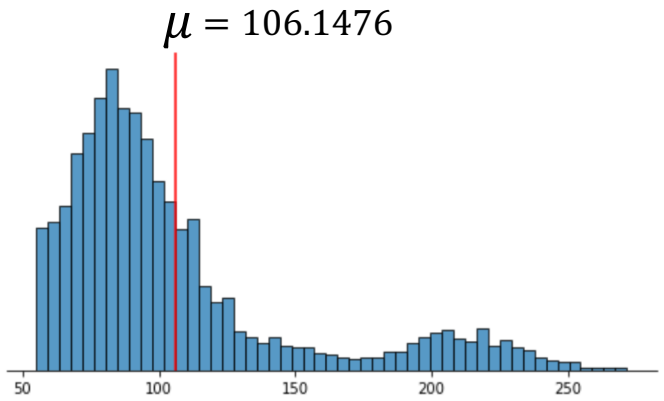
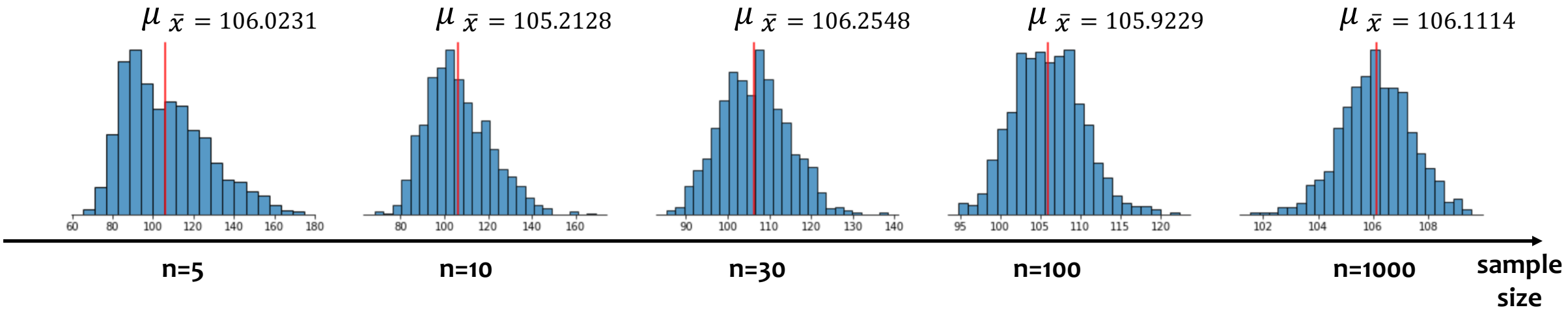




sampling distribution of the mean $\bar{x}$



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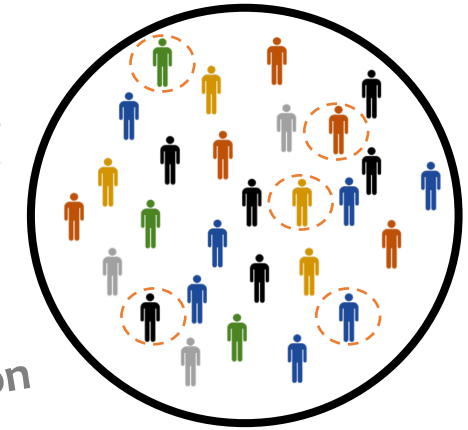
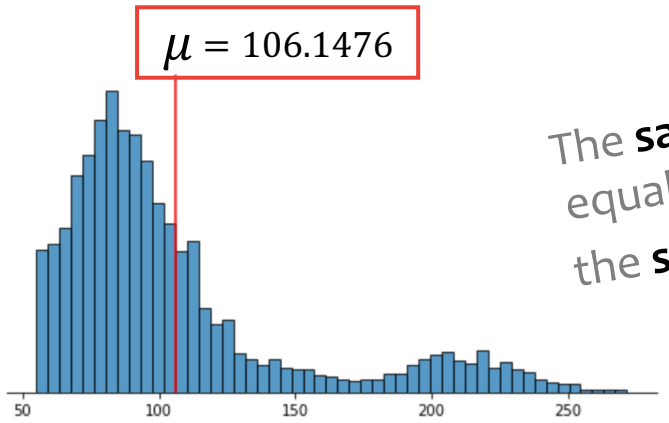
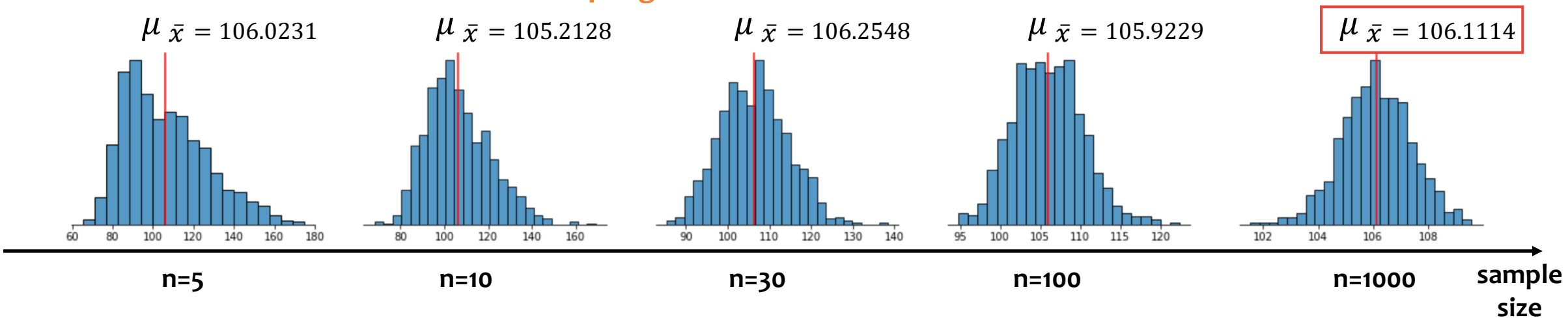




The **sampling distribution's mean** equals the **population mean**, and the **standard deviation** equals $\frac{\sigma}{\sqrt{n}}$.

sampling distribution of the mean $\bar{x}$



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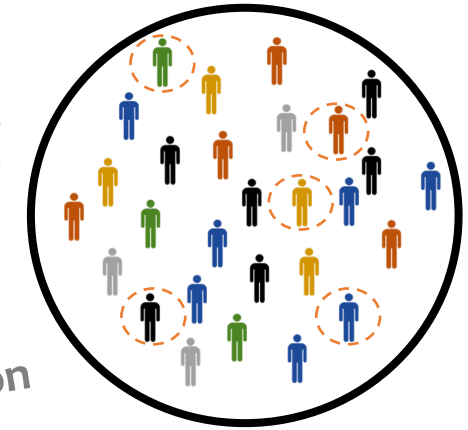
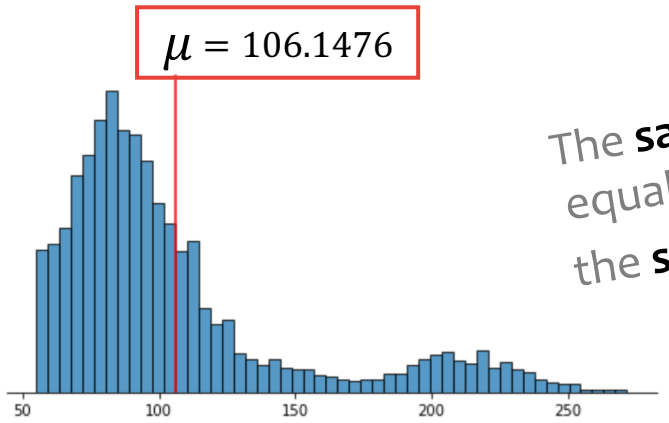
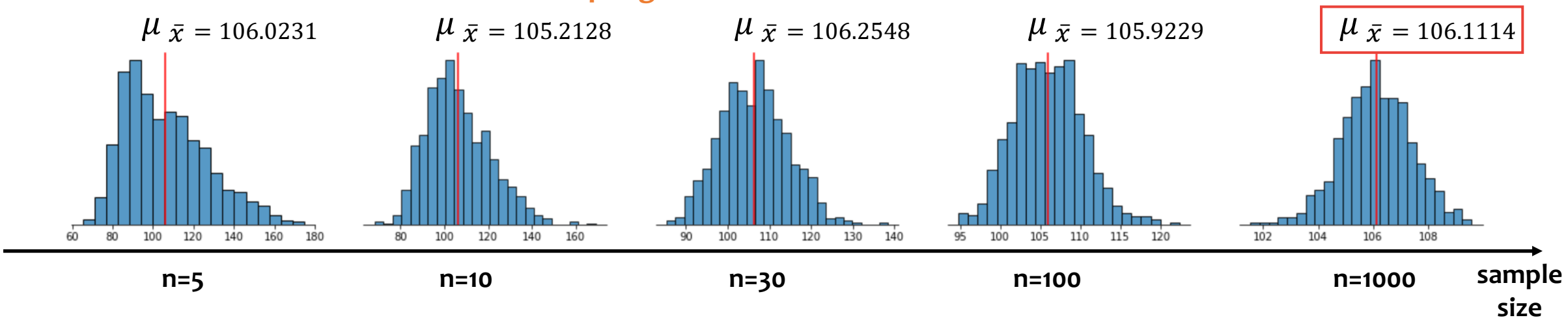




The **sampling distribution's mean** equals the **population mean**, and the **standard deviation** equals $\frac{\sigma}{\sqrt{n}}$.

really useful to **estimate** **population parameters**

sampling distribution of the mean $\bar{x}$



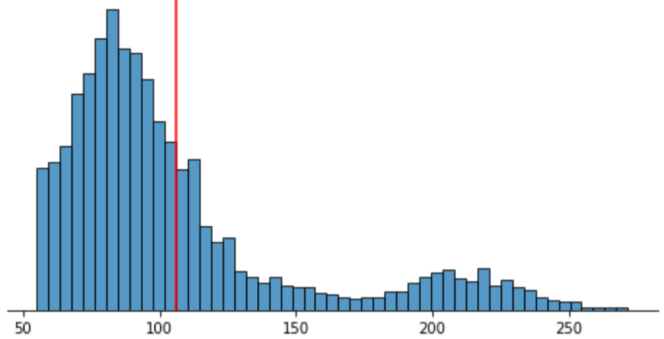
Properties

For a large **enough sample size n**:

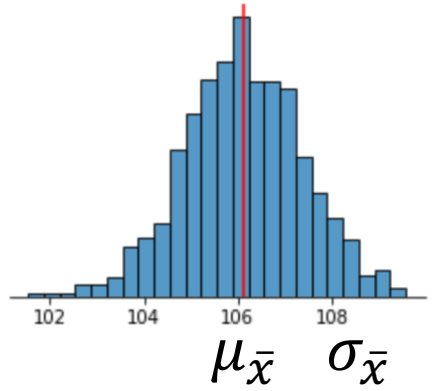
- The sampling distribution for the mean tends to a **normal distribution** (n ≥ 30);
- Sampling distribution's mean = Population mean
  - $\mu_{\bar{x}} = \mu$
- Sampling distribution's standard deviation (**standard error**):
  - $\sigma_{\bar{x}} = SE_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$

how far the **sample mean $\bar{x}$** is likely to be from the **population mean μ**

population distribution

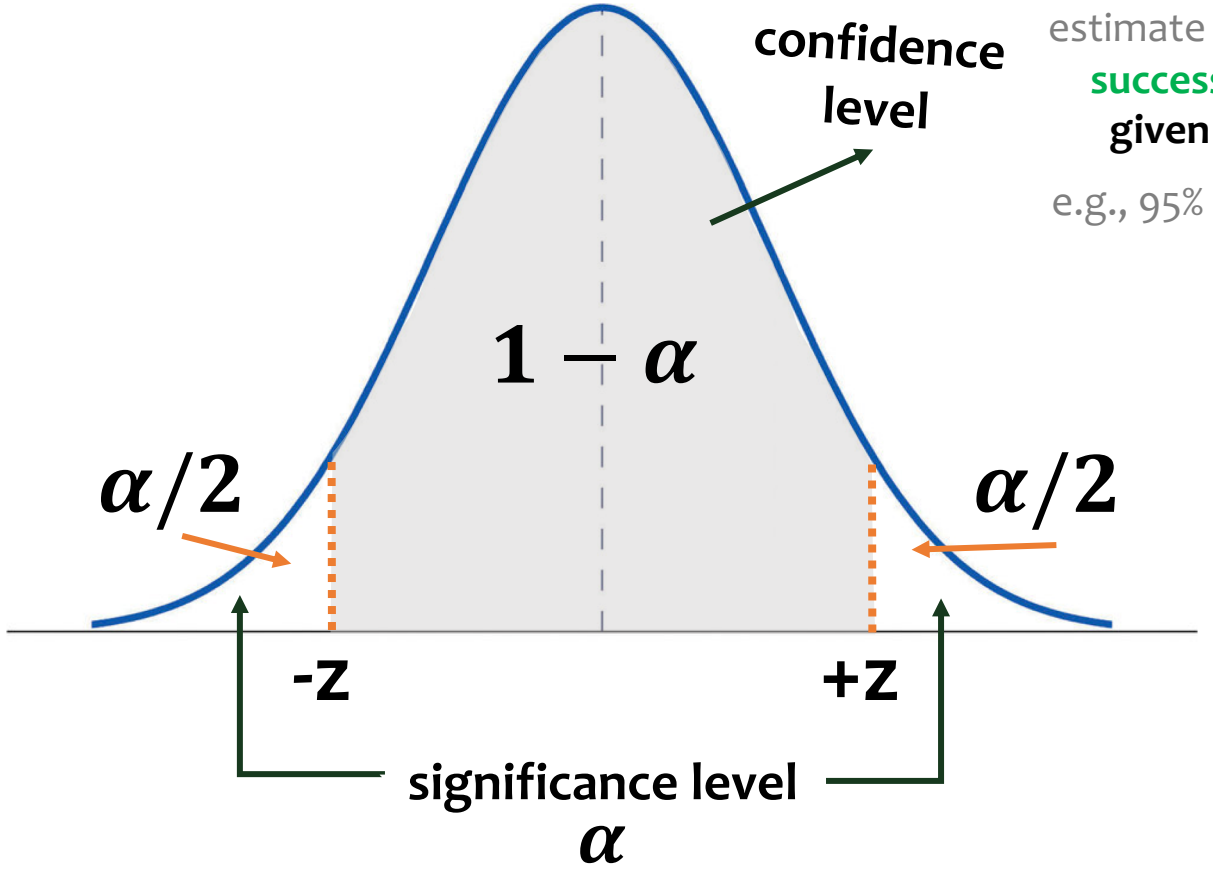


sampling distribution of the mean $\bar{x}$



Confidence and Significance Levels

computed from a **(standard) normal distribution**

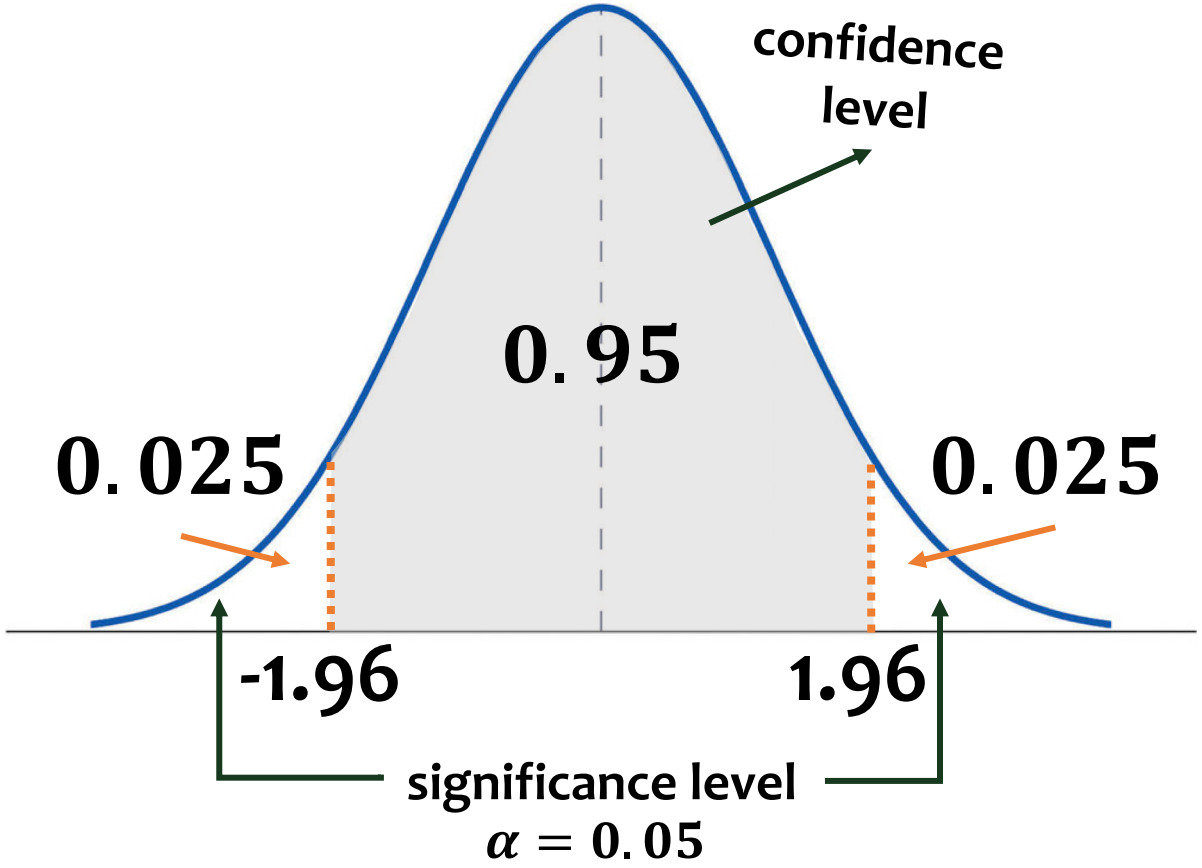


estimate percentage **success** within a **given interval**

e.g., 95% and 99%

estimate percentage **failure** within a **given interval**

When we set a **95% confidence level** in a survey, for example, we assume that there is a **95% probability** that the survey results **represent reality**.



Margin of Error

population
standard deviation

$$e = z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

standard error

critical value

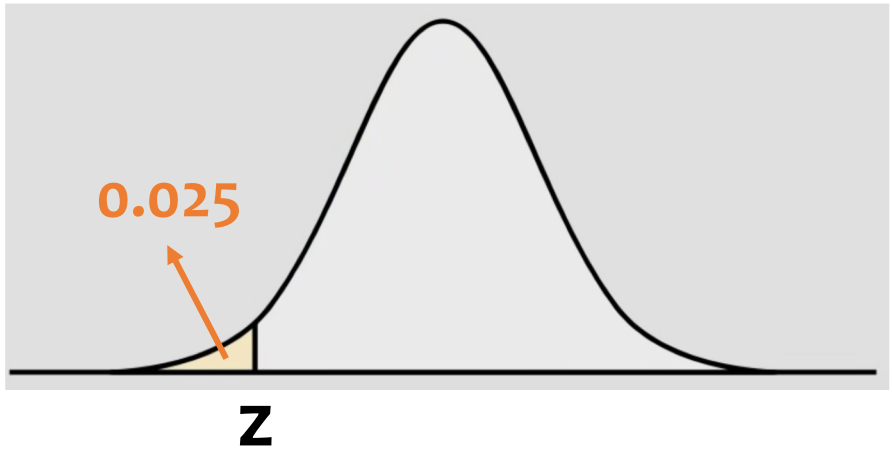
sample size

the z-score for which an area $\alpha/2$ lies.

Margin of Error

95% confidence level: 1 - α = 0.95
significance level: α = 0.05

$z_{\alpha/2} = z_{0.05/2} = z_{0.025}$=???



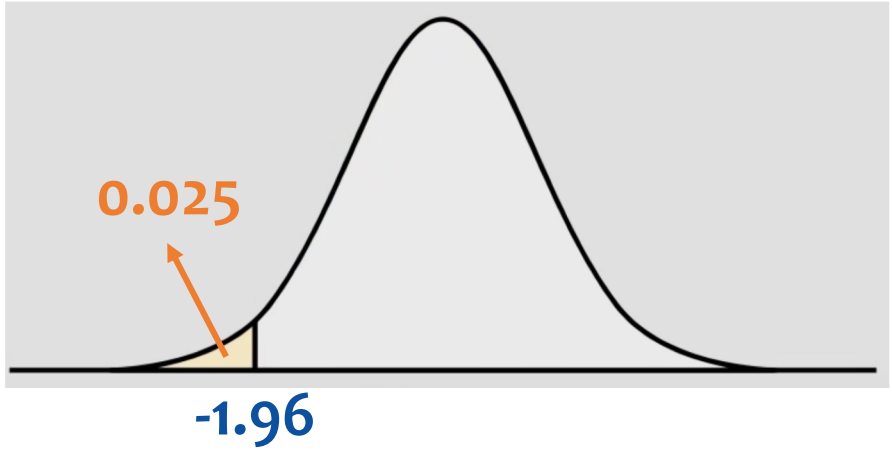
Z-Score Table

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0224
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582

Margin of Error

95% confidence level: 1 - α = 0.95
significance level: α = 0.05

$z_{\alpha/2} = z_{0.05/2} = z_{0.025} =$ **-1.96**



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-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008
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-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051
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-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150
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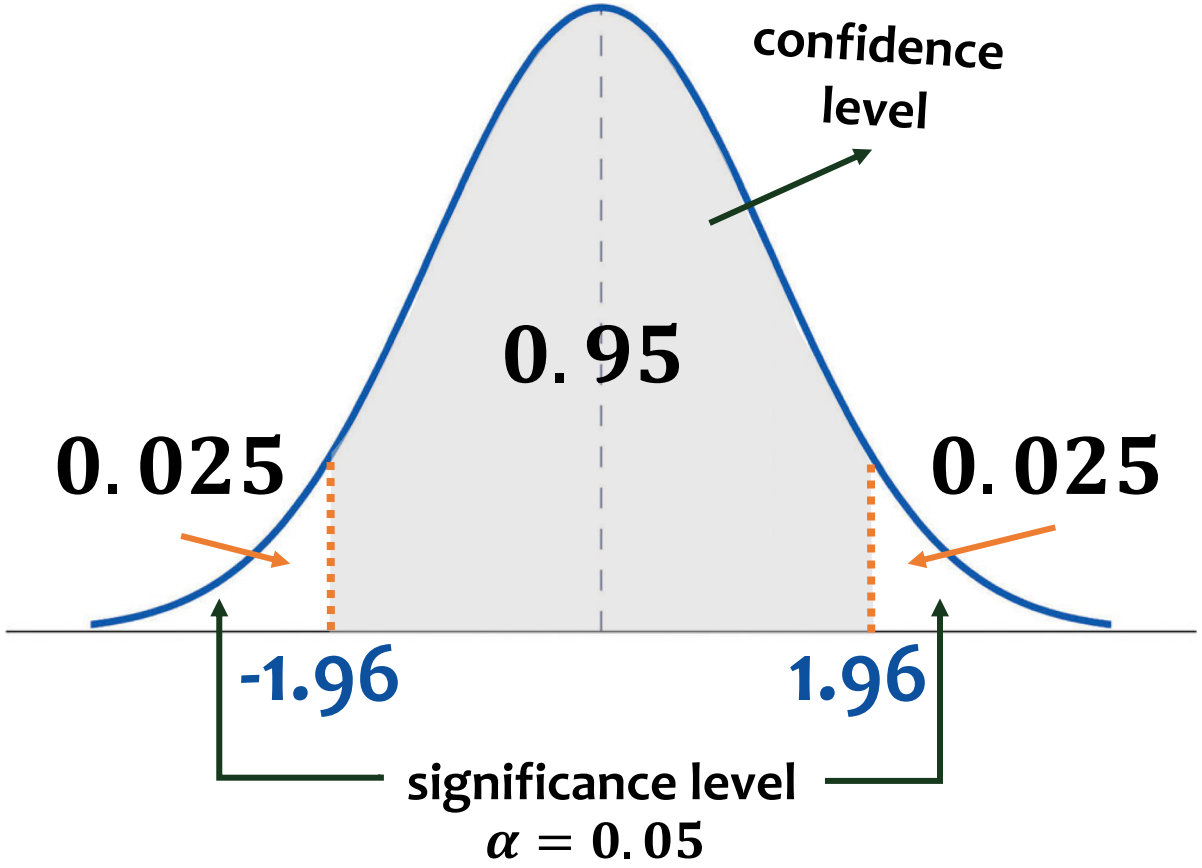
Margin of Error

95% confidence level: 1 - α = 0.95
significance level: α = 0.05

$z_{\alpha/2} = z_{0.05/2} = z_{0.025} =$ **-1.96**

$$e = 1.96\frac{\sigma}{\sqrt{n}}$$



Margin of Error

- A **90% level of confidence** has α = 0.10 and **critical value** of $z_{\alpha/2}$ = 1.64.
- A **95% level of confidence** has α = 0.05 and **critical value** of $z_{\alpha/2}$ = 1.96.
- A **99% level of confidence** has α = 0.01 and **critical value** of $z_{\alpha/2}$ = 2.58.

Confidence Interval for the Mean

Provides an **interval estimate** that contains the **population mean** with a certain **confidence level**.

Known populational standard deviation

$$\mu = \bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

Unknown populational standard deviation

$$\mu \approx \bar{x} \pm z_{\alpha/2} \frac{s}{\sqrt{n}}$$

s → sample standard deviation

Exercise 1

Suppose the heights of the inhabitants of a city are **normally distributed** with **population standard deviation** of 20 cm.

We measure the heights of 40 randomly chosen people, and get a **mean height** of 1.75 m.

Construct a **confidence interval** for the **population mean** with a **significance level of 5%**.

Exercise 2

Given a dataset from stroke patients, we want to study their **mean glucose level**.

For two samples of 100 and 1000 observation, provide a **95% confidence intervals** for the following scenarios:

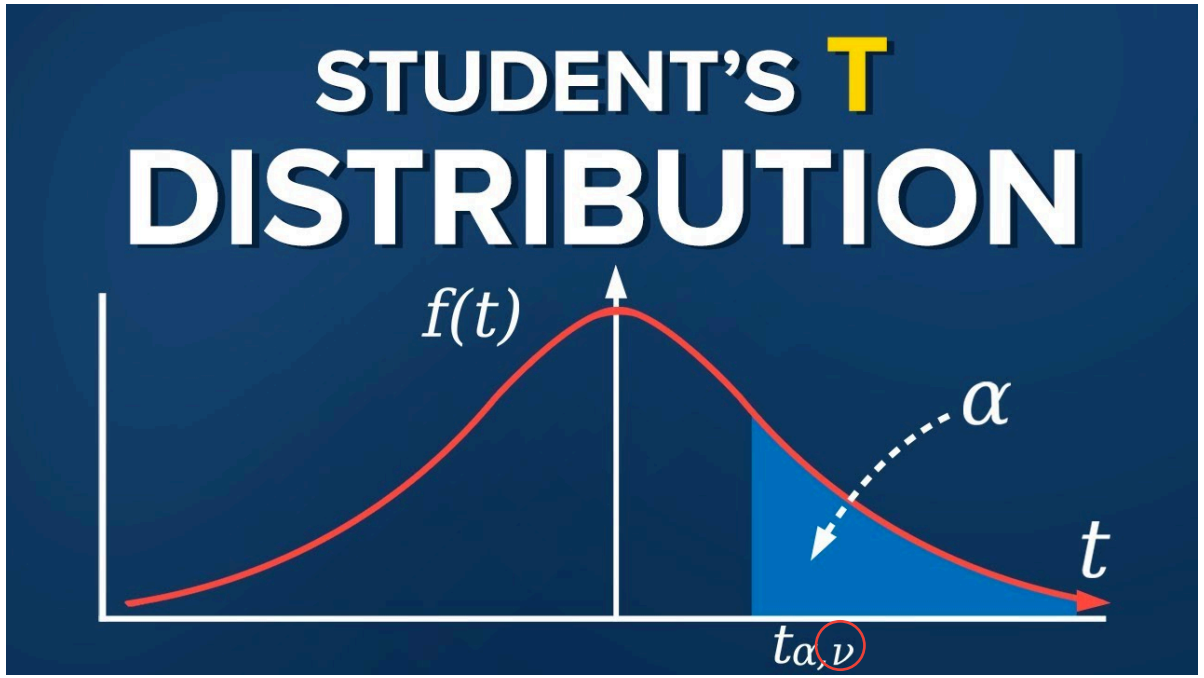
(a) Known population standard deviation, and sample sizes of 100 and 1000

(b) Unknown population standard deviation, and sample sizes of 100 and 1000

Confidence Interval for the Mean (Small Samples)

With **small sample sizes** (n < 30 observations), the distributions of sample means **is not always exactly normal**.

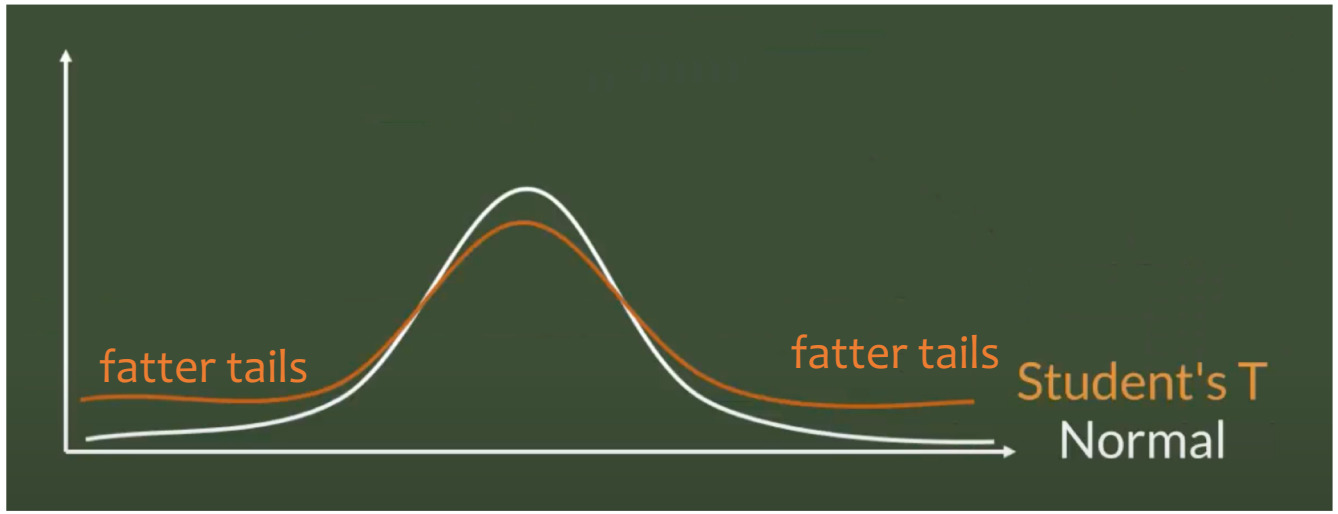
Then, we use a **t-distribution instead** of a **z-distribution (standard normal distribution).**



$$\mu = 0$$
$$\sigma = 1$$

Degrees of freedom
(nº of independent observations dataset)

https://www.youtube.com/watch?v=32CuxWdOlow



$$\mu = 0, \sigma = 1$$

TABLE A-3	*t* Distribution: Critical *t* Values				
	Area in One Tail				
	0.005	0.01	0.025	0.05	0.10
Degrees of Freedom	Area in Two Tails				
	0.01	0.02	0.05	0.10	0.20
1	**63.657**	31.821	**12.706**	6.314	3.078
2	**9.925**	6.965	**4.303**	2.920	1.886
3	**5.841**	4.541	**3.182**	2.353	1.638
4	**4.604**	3.747	**2.776**	2.132	1.533
5	**4.032**	3.365	**2.571**	2.015	1.476
6	**3.707**	3.143	**2.447**	1.943	1.440
7	**3.499**	2.998	**2.365**	1.895	1.415
8	**3.355**	2.896	**2.306**	1.860	1.397
9	**3.250**	2.821	**2.262**	1.833	1.383
10	**3.169**	2.764	**2.228**	1.812	1.372
11	**3.106**	2.718	**2.201**	1.796	1.363
12	**3.055**	2.681	**2.179**	1.782	1.356
13	**3.012**	2.650	**2.160**	1.771	1.350
14	**2.977**	2.624	**2.145**	1.761	1.345
15	**2.947**	2.602	**2.131**	1.753	1.341
16	**2.921**	2.583	**2.120**	1.746	1.337
17	**2.898**	2.567	**2.110**	1.740	1.333
18	**2.878**	2.552	**2.101**	1.734	1.330
19	**2.861**	2.539	**2.093**	1.729	1.328
20	**2.845**	2.528	**2.086**	1.725	1.325
21	**2.831**	2.518	**2.080**	1.721	1.323
22	**2.819**	2.508	**2.074**	1.717	1.321
23	**2.807**	2.500	**2.069**	1.714	1.319

https://www.youtube.com/watch?v=32CuxWdOlow

Confidence Interval for the Mean Mean (Small Samples)

Provides an **interval estimate** that contains the **population mean** with a certain **confidence level**.

Known populational standard deviation

$$\mu = \bar{x} \pm t_{(\frac{\alpha}{2}, n-1)} \frac{\sigma}{\sqrt{n}}$$

Degrees of freedom

Unknown populational standard deviation

$$\mu = \bar{x} \pm t_{(\frac{\alpha}{2}, n-1)} \frac{s}{\sqrt{n}}$$

s → sample standard deviation

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Data Distributions (Part 3)

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