Aprendizado de Máquina e Reconhecimento de Padrões

Naive Bayes

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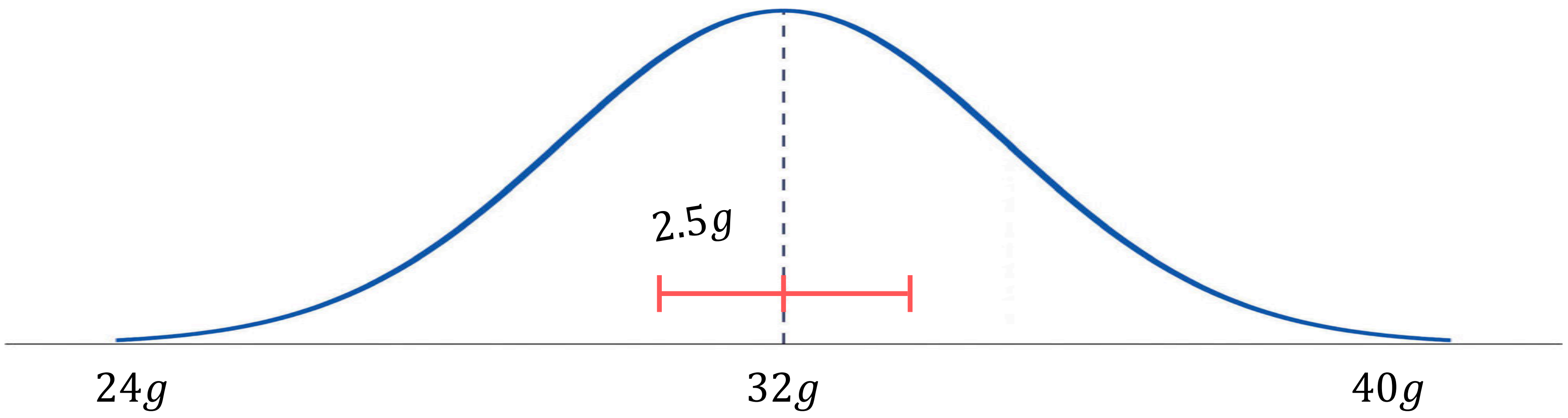
Preliminaries: Probability vs Likelihood

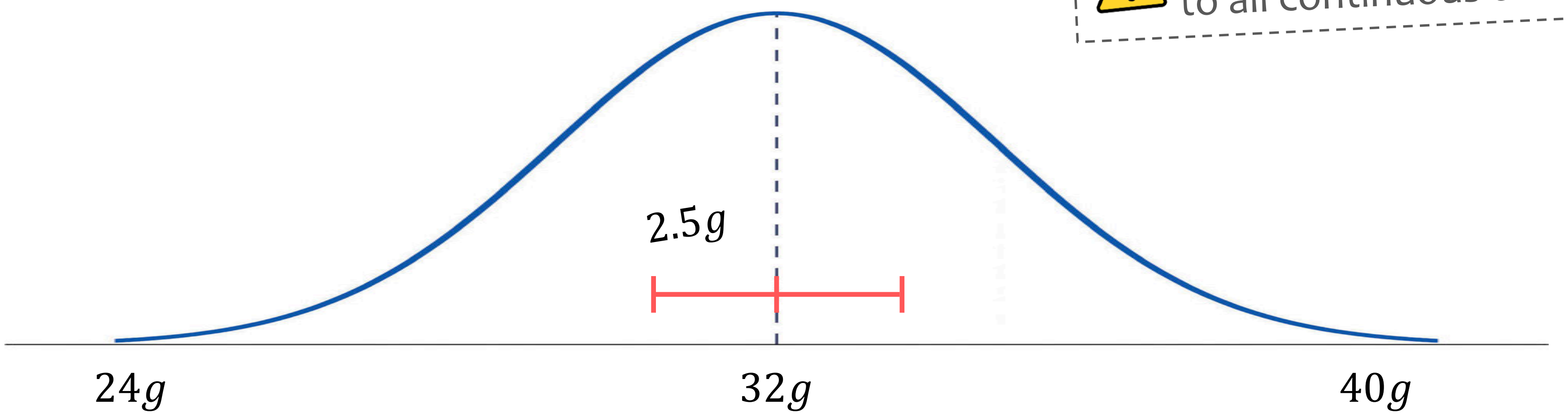
Let's assume that the weights of a mouse population follow a **Normal Distribution** $\mathbf{N(\mu, \sigma) = N(32g, 2.5g)}$



$$\mathbf{N(32g, 2.5g)}$$

Let's assume that the weights of a mouse population follow a **Normal Distribution** $\mathbf{N(\mu, \sigma) = N(32g, 2.5g)}$

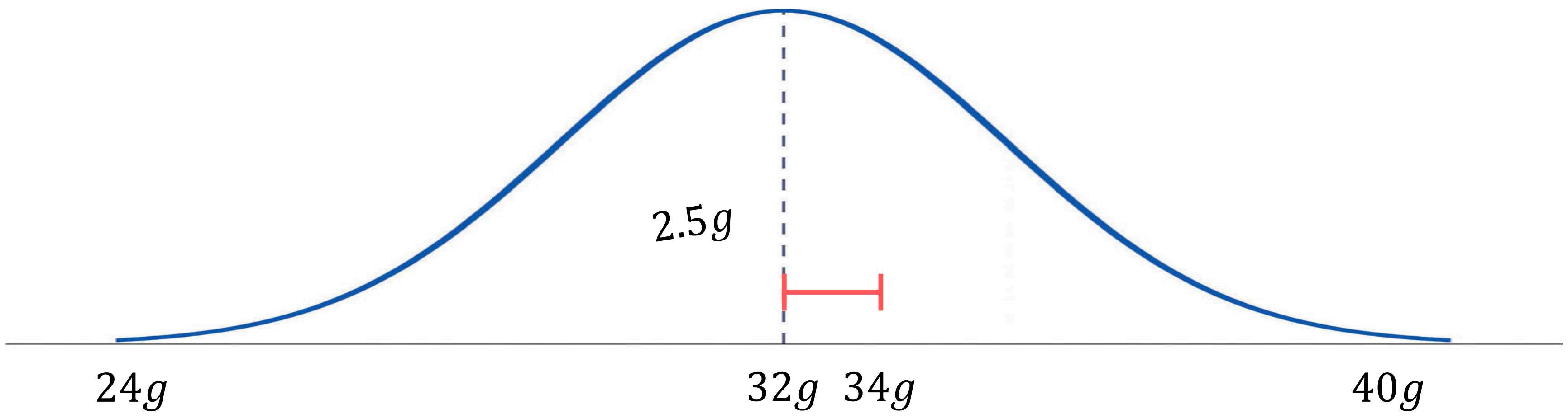
The concepts explained here apply to all continuous distributions.



$$\mathbf{N(32g, 2.5g)}$$

The **probability** that a randomly selected mouse

weighs between **32g and 34g**



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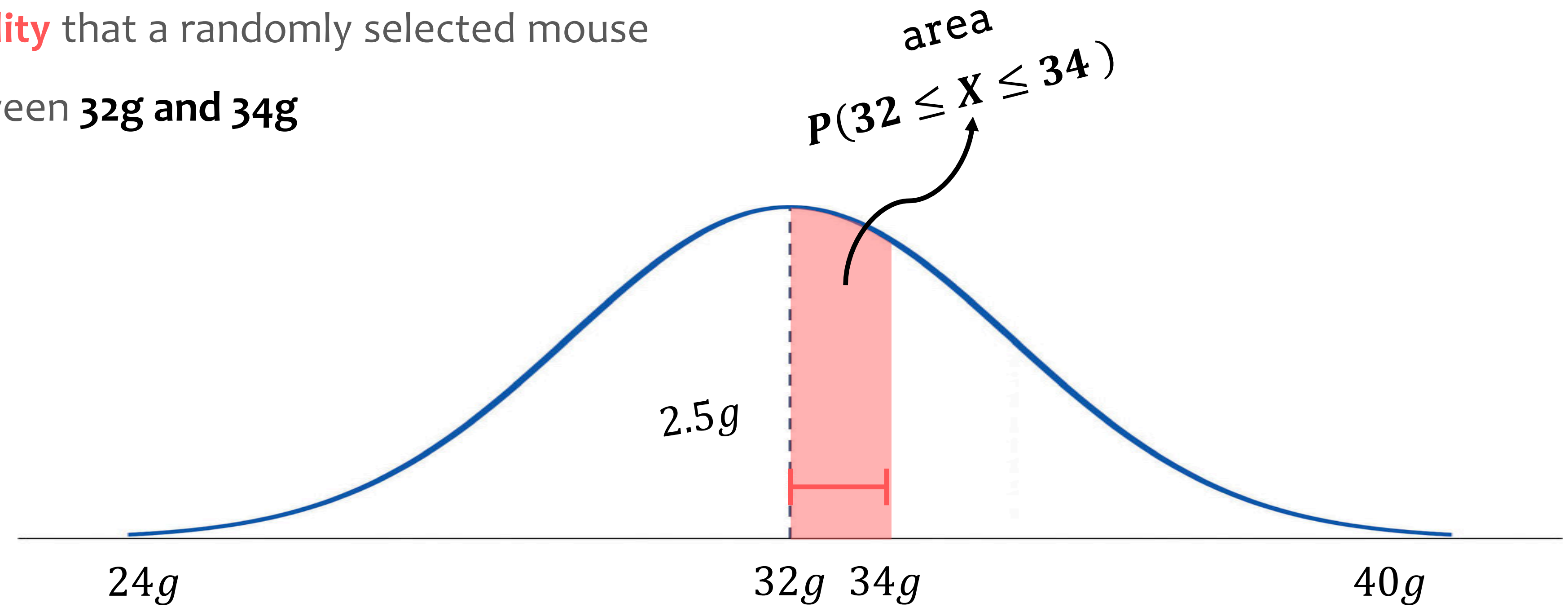
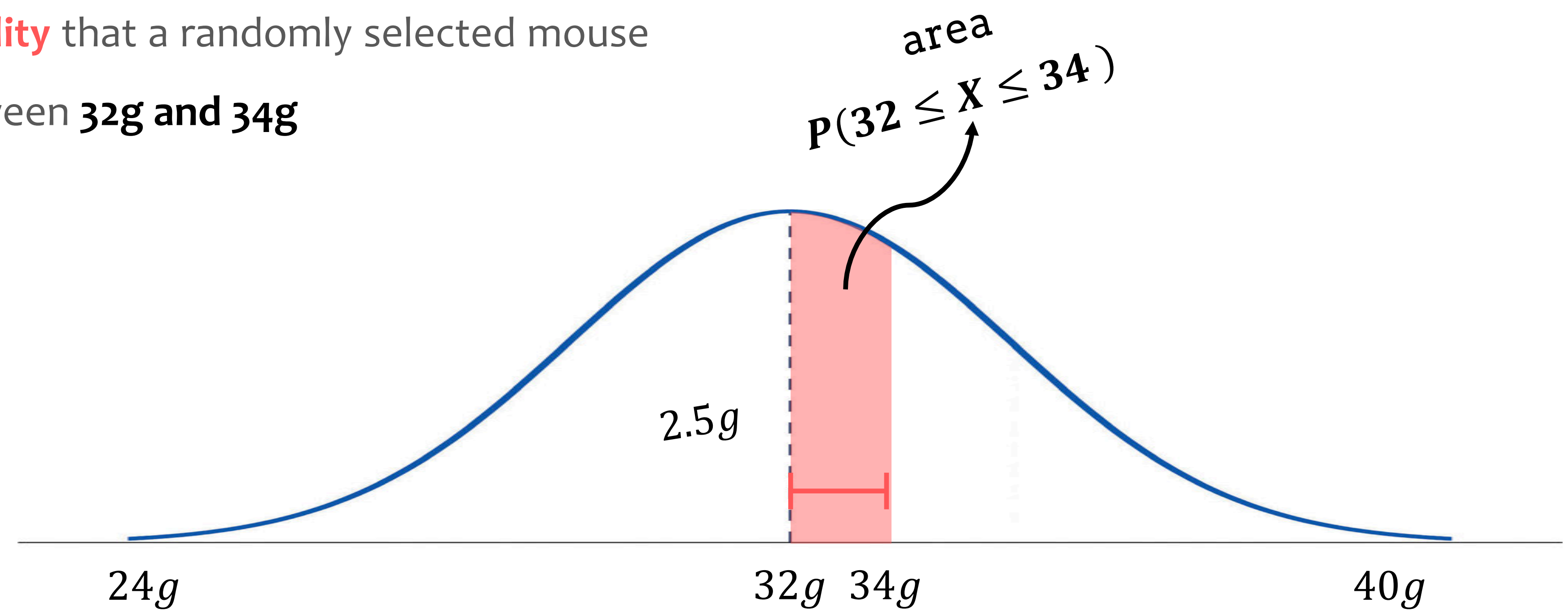


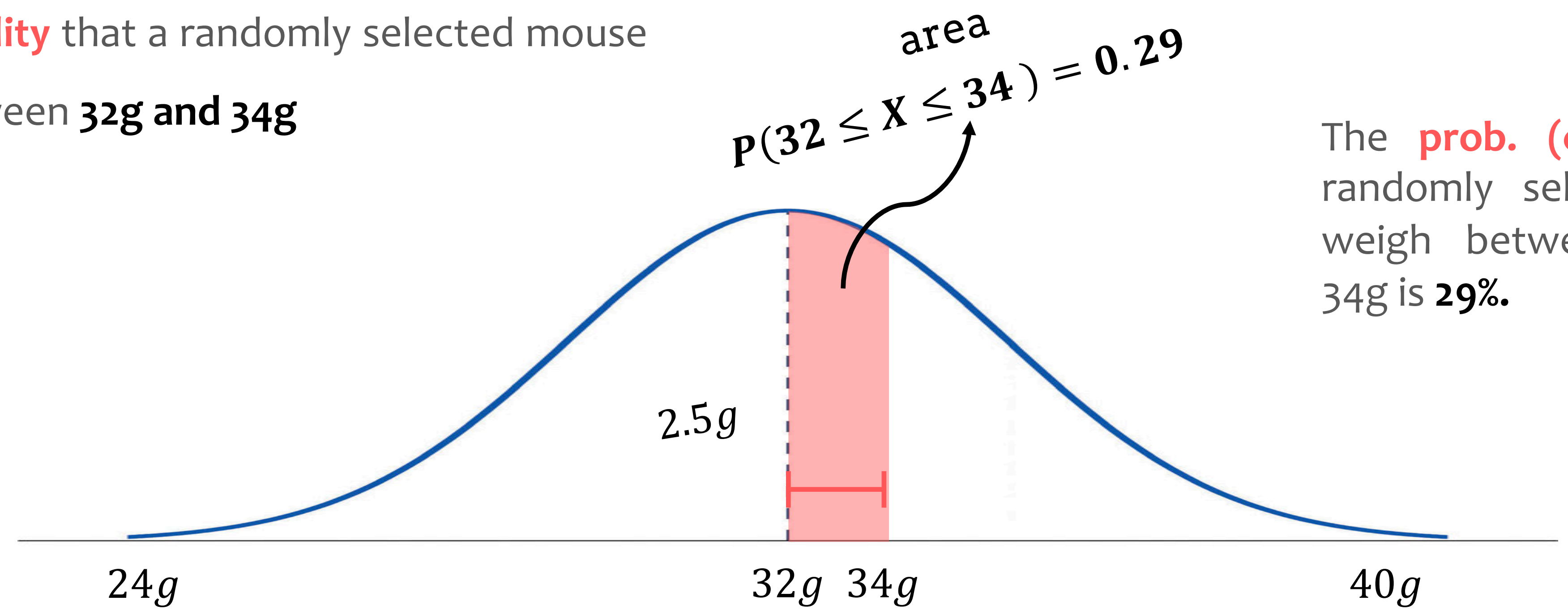
The **probability** that a randomly selected mouse weighs between **32g and 34g**



$$P(a \leq X \leq b\ ) = \int_a^b f(x)dx = \int_a^b \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{1}{2}(\frac{x-\mu}{\sigma})^2}\, dx$$

The **probability** that a randomly selected mouse weighs between **32g and 34g**

In this case, the **area under the curve** is **0.29**

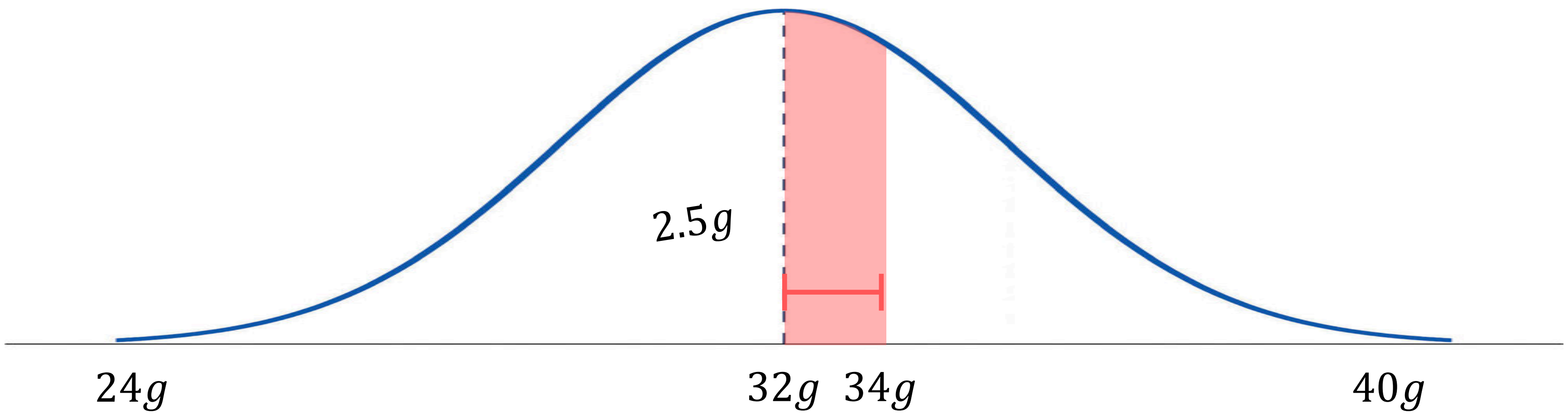


The **prob. (chance)** of a randomly selected mouse weigh between 32g and 34g is **29%.**

$$P(a \leq X \leq b) = \int_a^b f(x)dx = \int_a^b \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{1}{2}(\frac{x-\mu}{\sigma})^2} dx$$

Mathematically, we use the following notation:

$$P(weight\ between\ 32g\ and\ 34g \mid mean = 32g\ and\ std = 2.5g) = 0.29$$



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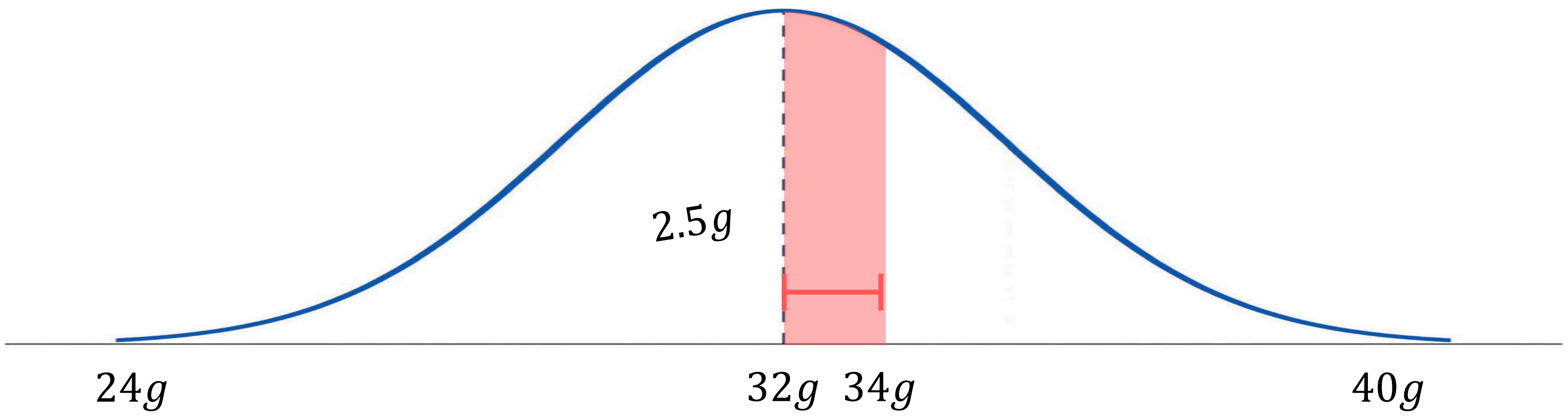

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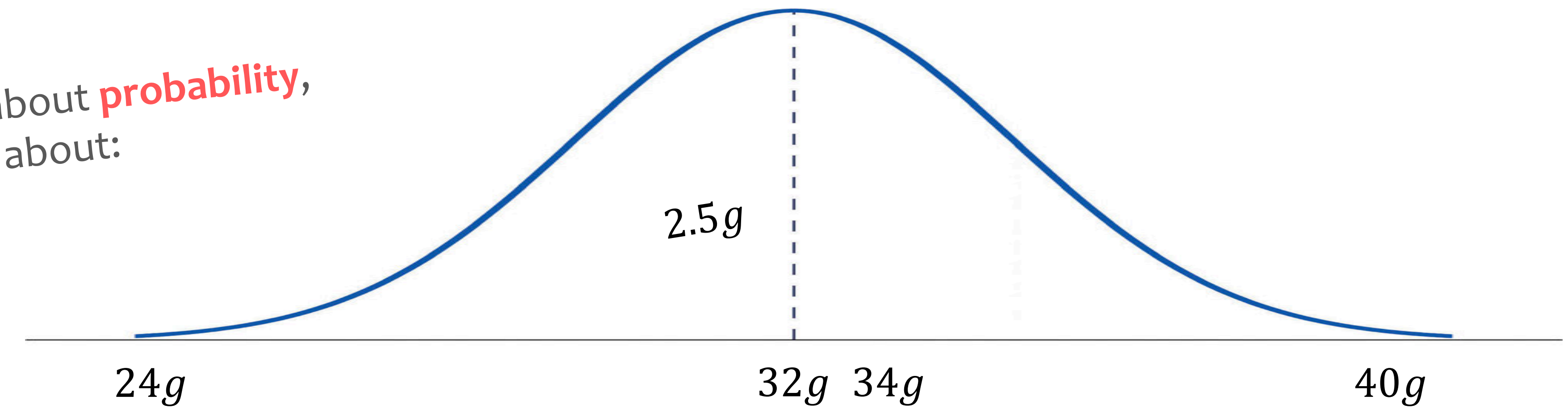
When talking about **probability**,
we are talking about:



A **distribution** described by this side of the equation.

$$P(weight\ between\ 32g\ and\ 34g\ |\ \boxed{mean = 32g\ and\ std = 2.5g}) = 0.29$$

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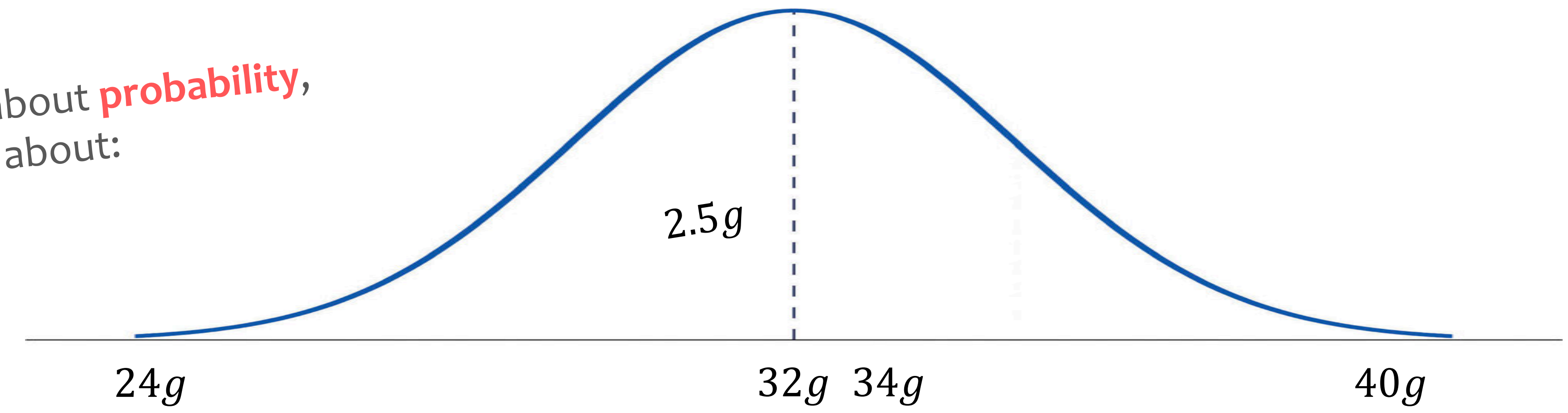

A **distribution** described by this side of the equation.

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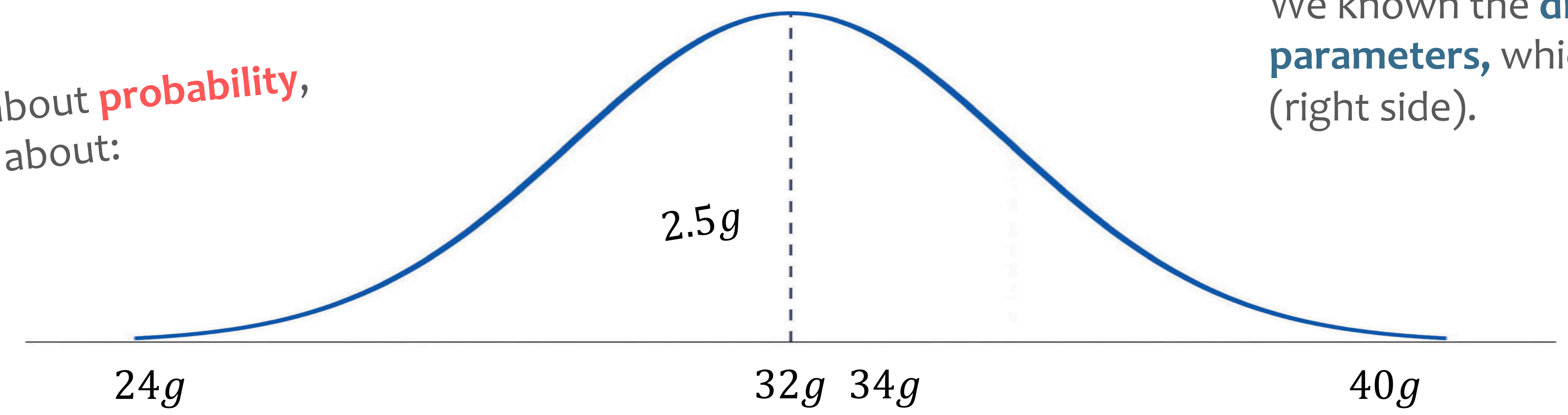

We known the **distribution parameters,** which are **fixed** (right side).

and the **area under the curve**,
described by this side of the equation.

$$P(\boxed{weight\ between\ 32g\ and\ 34g} \mid mean = 32g\ and\ std = 2.5g) = 0.29$$

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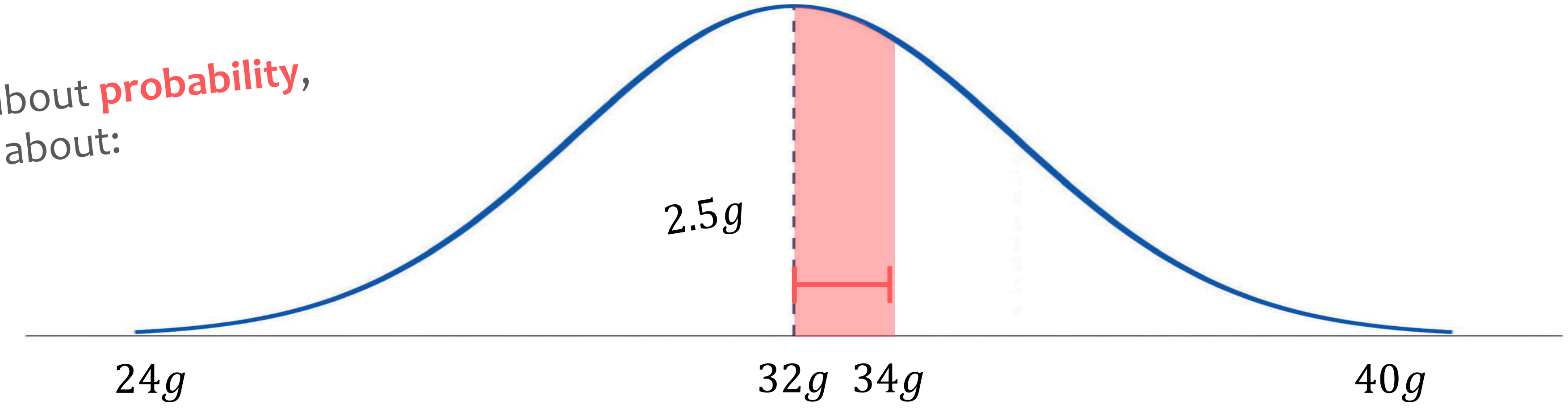

Using the same **distribution...**

$$P(weight\ between\ 32g\ and\ 34g\ |\ \boxed{mean = 32g\ and\ std = 2.5g}) = 0.29$$

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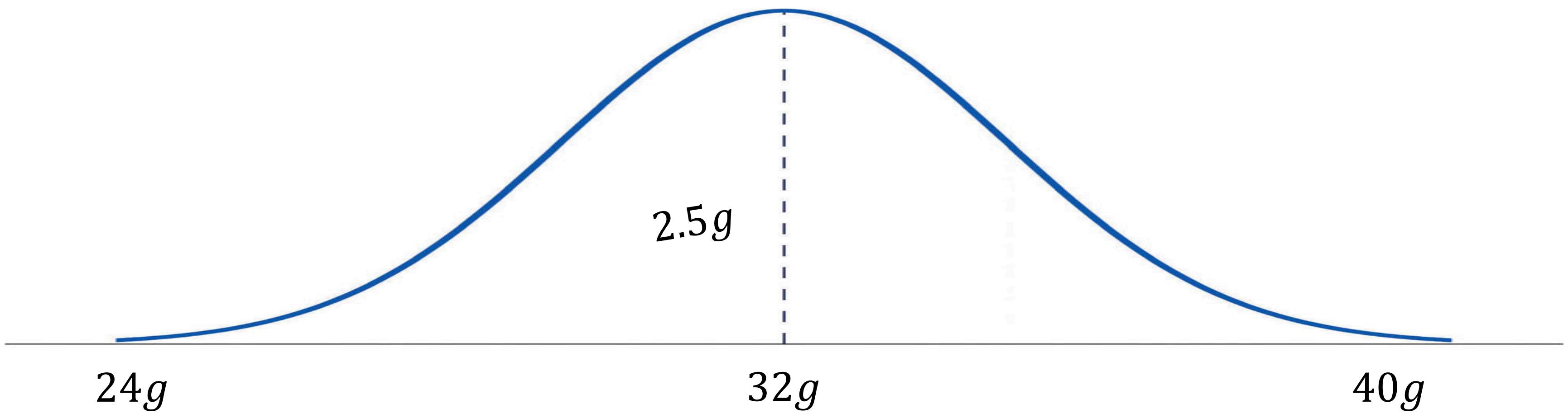
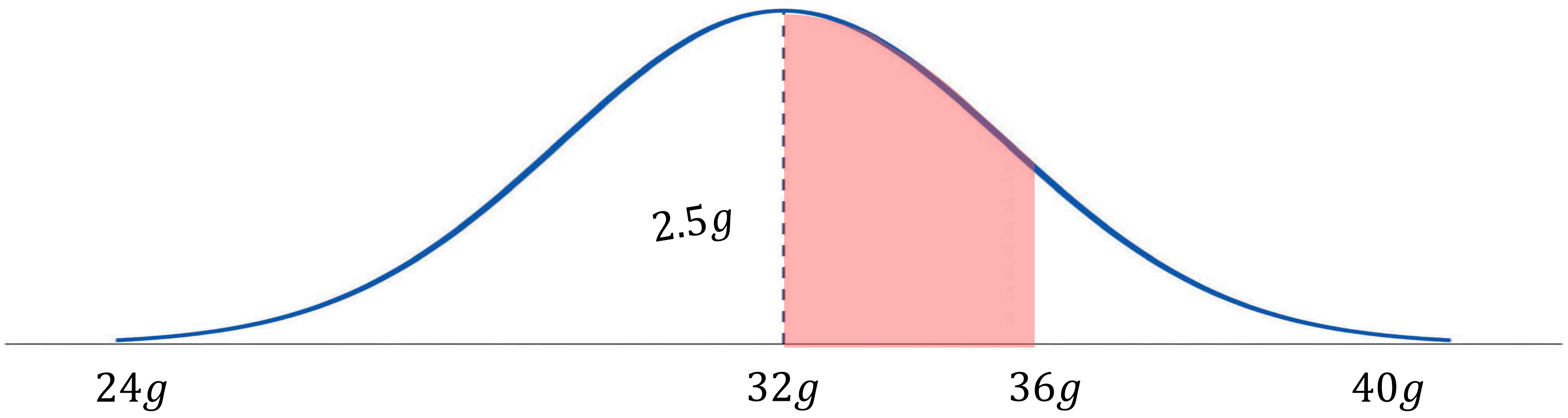


… we can change the left side of the equation to get a **new probability**!

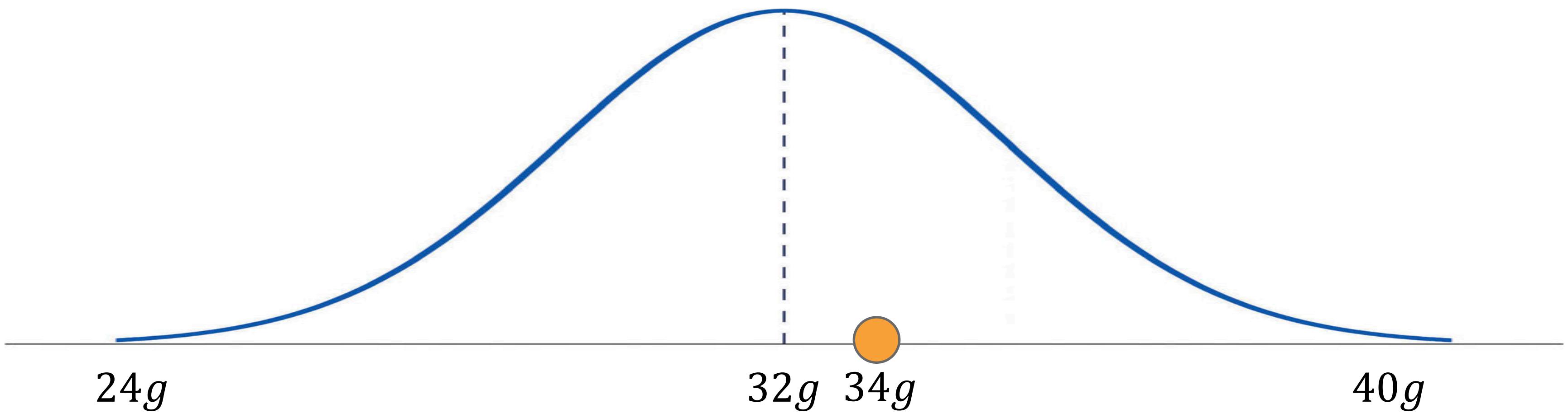
Using the same **distribution…**

$$P(\boxed{weight\ between\ 32g\ and\ 36g} \mid \boxed{mean = 32g\ and\ std = 2.5g}) = 0.44$$

$$P(\boxed{32 \leq X \leq 36} \mid \boxed{\mu = 32, \sigma = 2.5}) = 0.44$$



In case of **likelihood**, we assume that we already have selected the mouse (e.g., weighing 34g).



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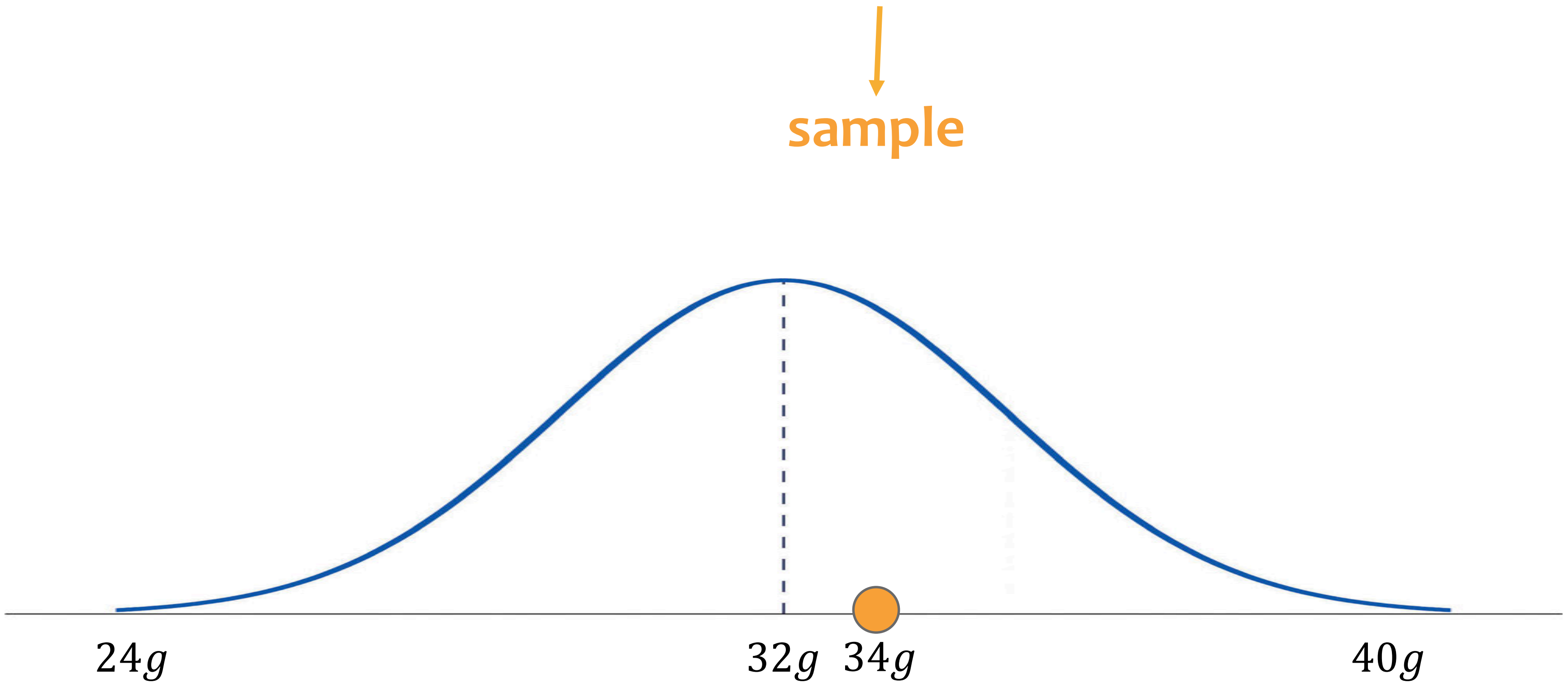

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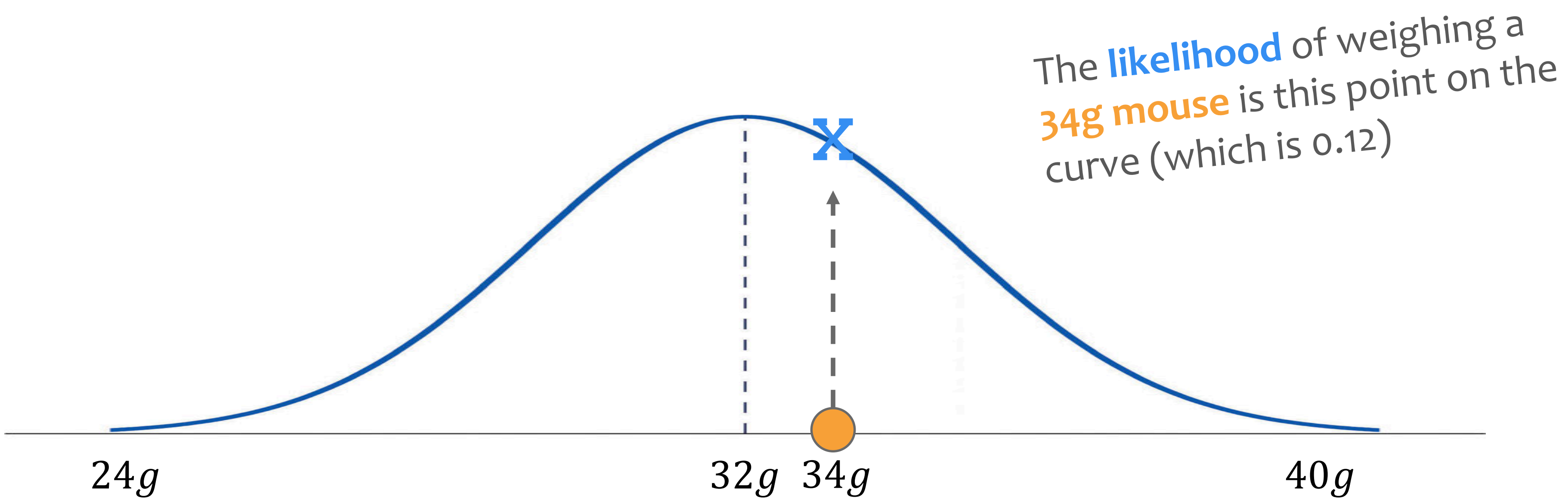

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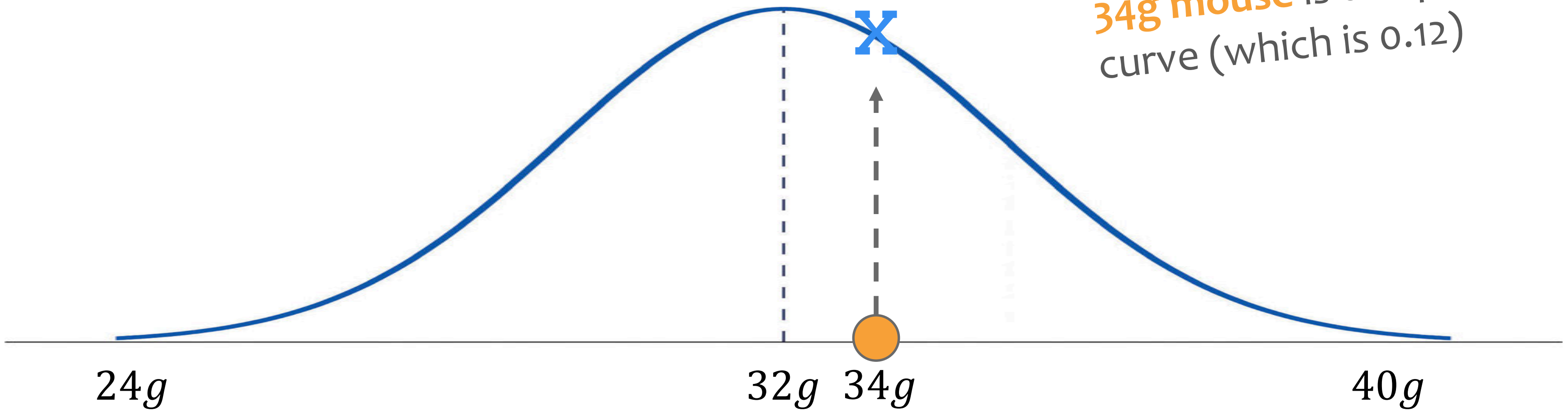
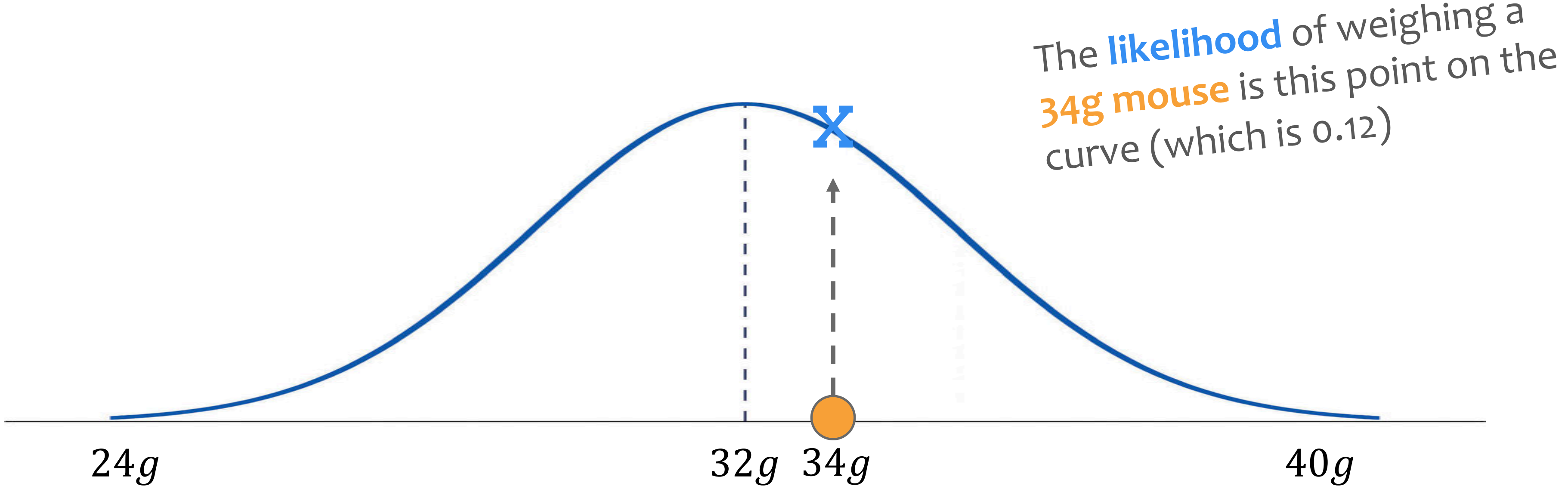


The **likelihood** of weighing a **34g mouse** is this point on the curve (which is 0.12)

We can think **likelihood** as:
"How **likely** (what is the probability) a ***particular*** **population**, described by a given **distribution (parameters),** is to produce an **observed sample**?"

In case of **likelihood**, we assume that we already have selected the mouse (e.g., weighing 34g).



Note we **do not** know the **population/distribution**, only a **sample**.

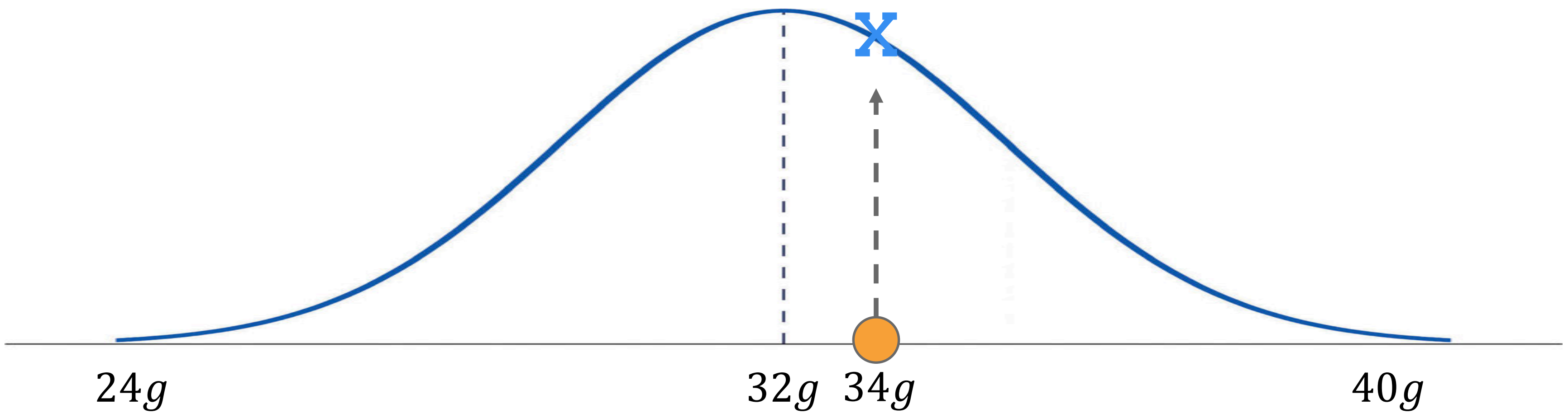
We can think **likelihood** as:
"How **likely** (what is the probability) a ***particular*** **population**, described by a given **distribution (parameters),** is to produce an **observed sample**?"

Mathematically, we use the following notation:

$$L(mean = 32g \; and \; std = 2.5g \mid mouse \; weighs \; 34g\,) = 0.12$$

$$L(\mu = 32, \sigma = 2.5 \mid \mathrm{x} = 34) = 0.12$$

$$\theta = \mu, \sigma$$



The **likelihood** is equal to the **probability density** of the observed outcome given a distribution.

likelihood function

$$\mathrm{L}(\theta|\mathrm{x}_i) = f_\theta(\mathrm{x}_i) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}(\frac{x_i-\mu}{\sigma})^2}$$

$$\mathrm{L}(\mu, \sigma|\mathrm{x}_i) = f_{\mu,\sigma}(\mathrm{x}_i) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}(\frac{x_i-\mu}{\sigma})^2}$$

Mathematically, we use the following notation:

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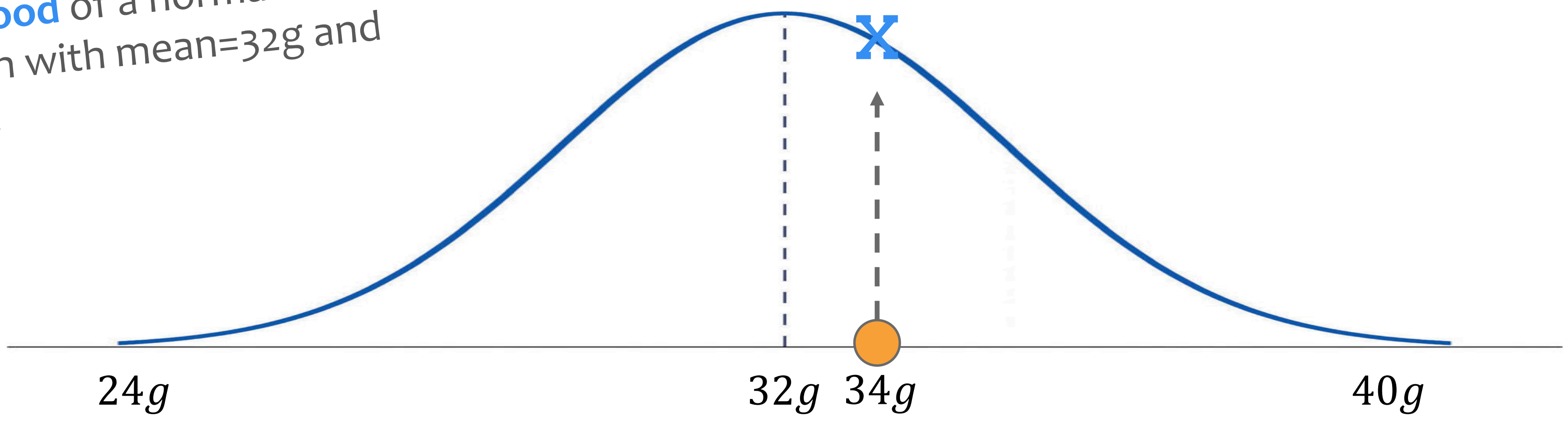
$$L(\boxed{\mu = 32, \sigma = 2.5} \mid \mathrm{x} = 34) = 0.12$$

$$\theta = \mu, \sigma$$

The **likelihood** of a normal distribution with mean=32g and std=2.5g…



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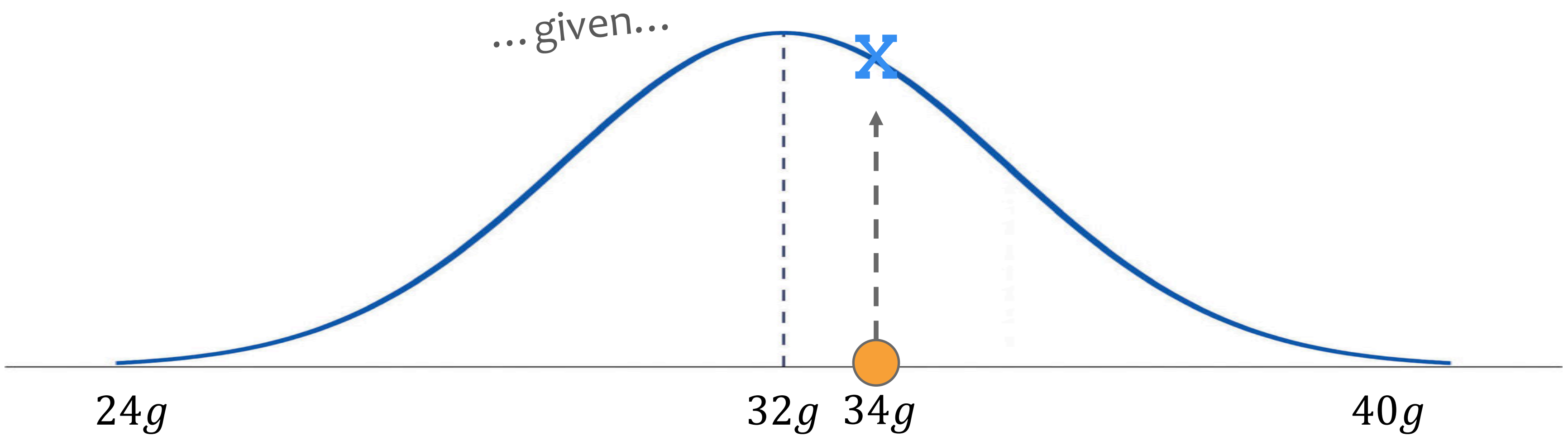

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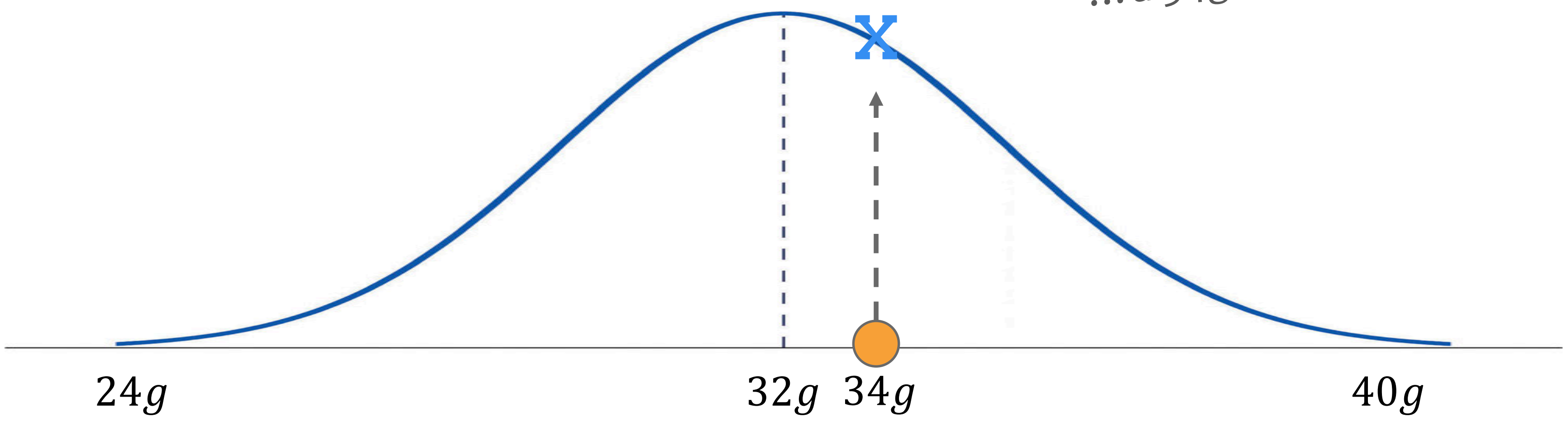
…a 34g mouse



Mathematically, we use the following notation:

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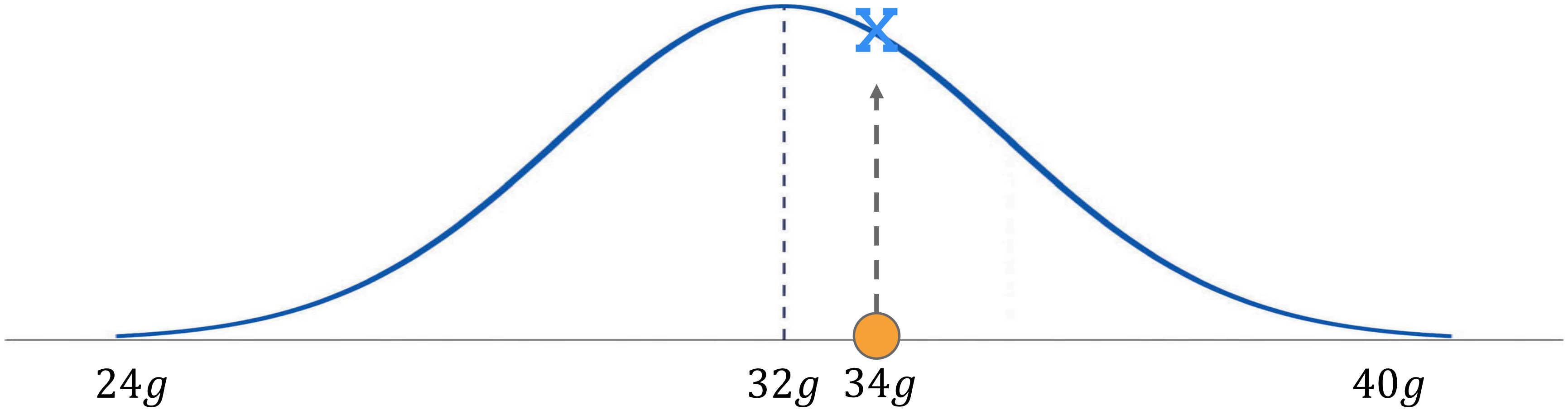
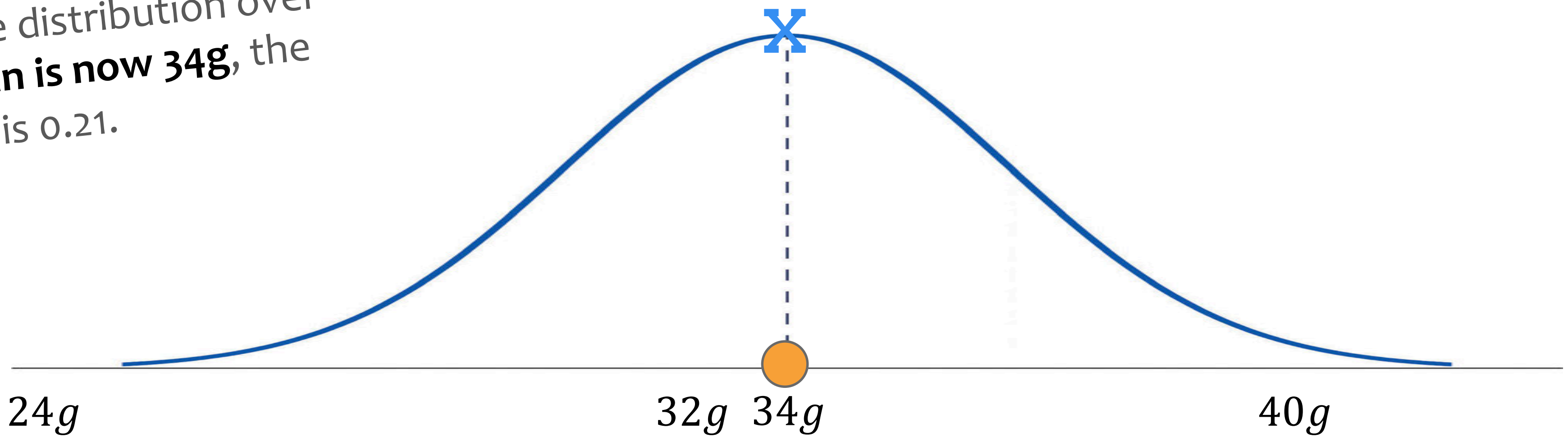


$$L(\boxed{mean = 34g\ and\ std = 2.5g}\ |\ mouse\ weighs\ 34g\ ) = 0.21$$

$$L(\boxed{\mu = 34, \sigma = 2.5}\ |\ \mathrm{x} = 34) = 0.21$$

$$\theta = \mu, \sigma$$

If we **shifted** the distribution over so that the **mean is now 34g**, the **new likelihood** is 0.21.



$$L(mean = 34g \text{ and } std = 2.5g \mid \boxed{mouse\ weighs\ 34g}) = 0.21$$

$$L(\mu = 34, \sigma = 2.5 \boxed{\mid x = 34}) = 0.21$$

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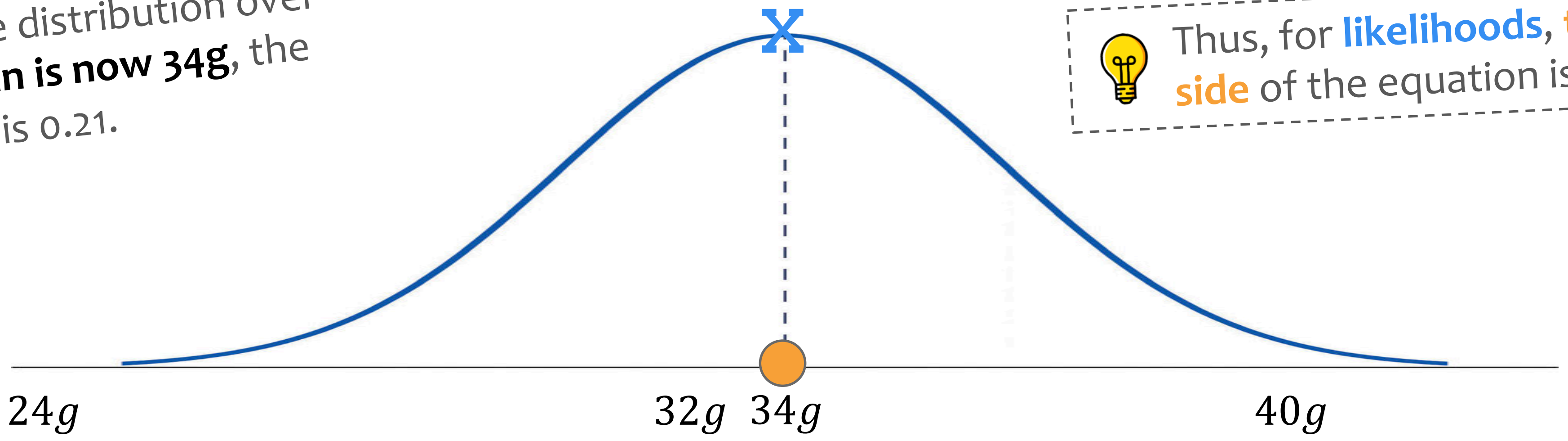

Thus, for **likelihoods**, **the right side** of the equation is **fixed**!

$$L(mean = 34g \ and \ std = 2.5g \mid \boxed{mouse \ weighs \ 34g}) = 0.21$$

$$L(\mu = 34, \sigma = 2.5 \mid \boxed{x = 34}) = 0.21$$

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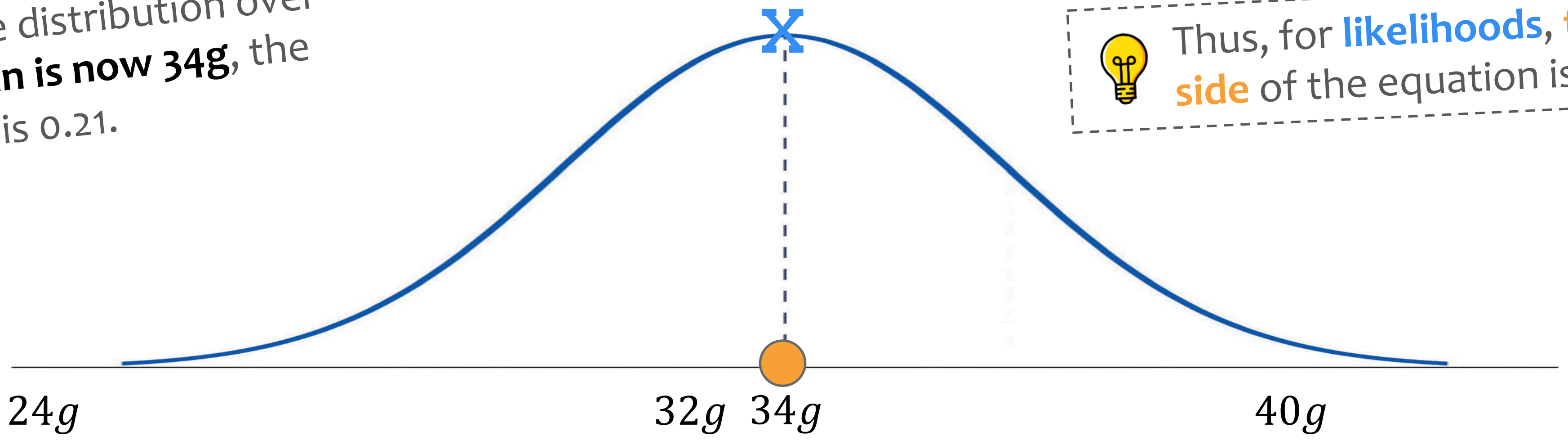


Thus, for **likelihoods**, **the right side** of the equation is **fixed**!

Recalling…

We can think **likelihood** as:
"How **likely** a particular **population** is to produce an **observe sample**."

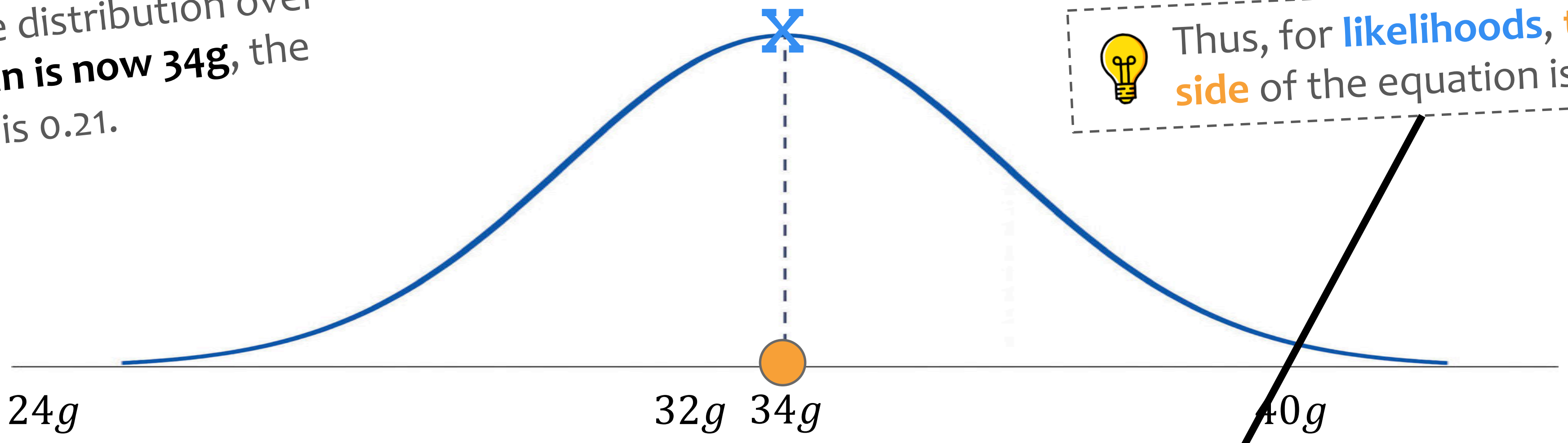
$$L(mean = 34g \; and \; std = 2.5g \mid \boxed{mouse \; weighs \; 34g}) = 0.21$$

$$L(\mu = 34, \sigma = 2.5 \mid \boxed{x = 34}) = 0.21$$

$$\theta = \mu, \sigma$$

If we **shifted** the distribution over so that the **mean is now 34g**, the **new likelihood** is 0.21.

Thus, for **likelihoods**, **the right side** of the equation is **fixed**!



Recalling…

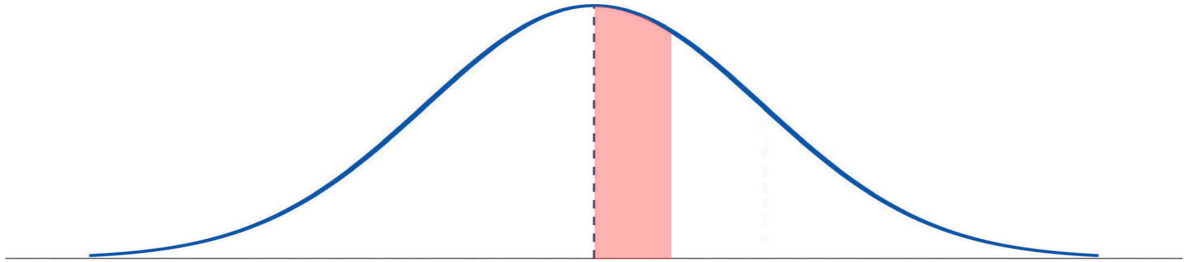
We can think **likelihood** as:
"How **likely** a particular **population** is to produce an **observe sample**."

We can then check **likelihood** for **different populations** (represented by different data distributions) for a given **observe sample**.

Summary

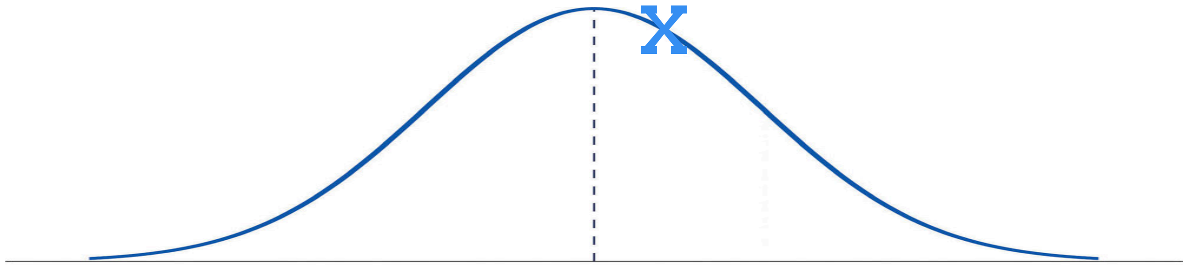
Probabilities are the **areas under** a *fixed distribution.*

$$P(data \mid distribution)$$

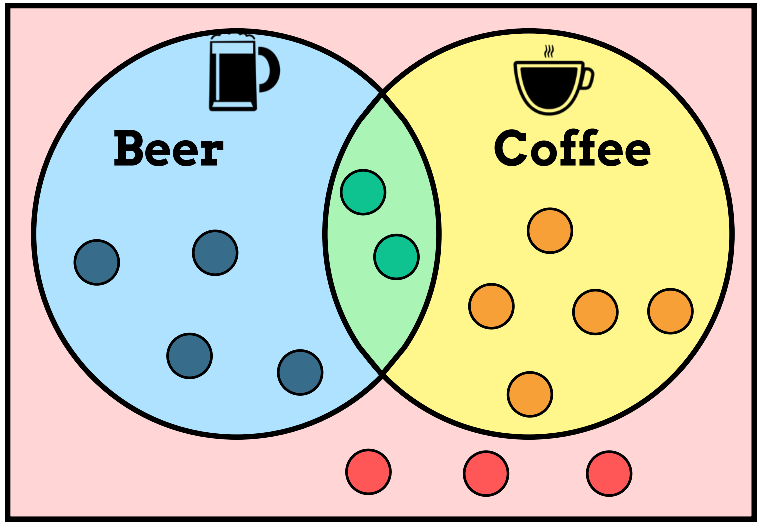


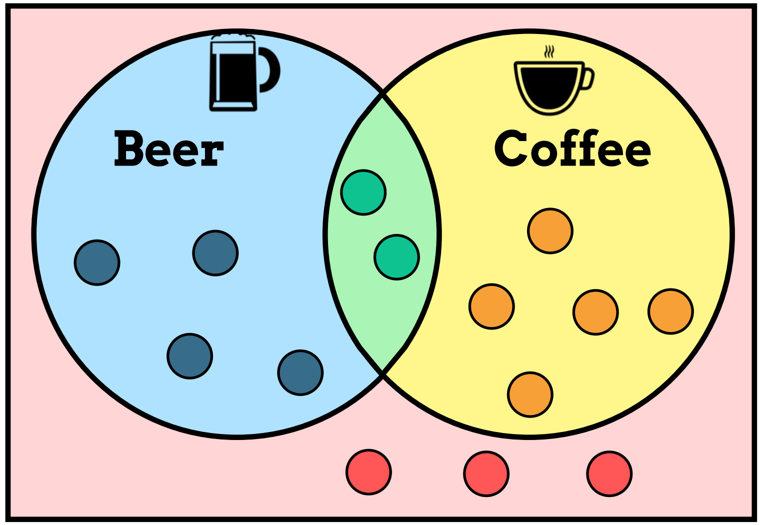
Likelihoods are the **y-axis values** for *fixed data points* with distributions that can be moved.

$$L(distribution \mid data)$$



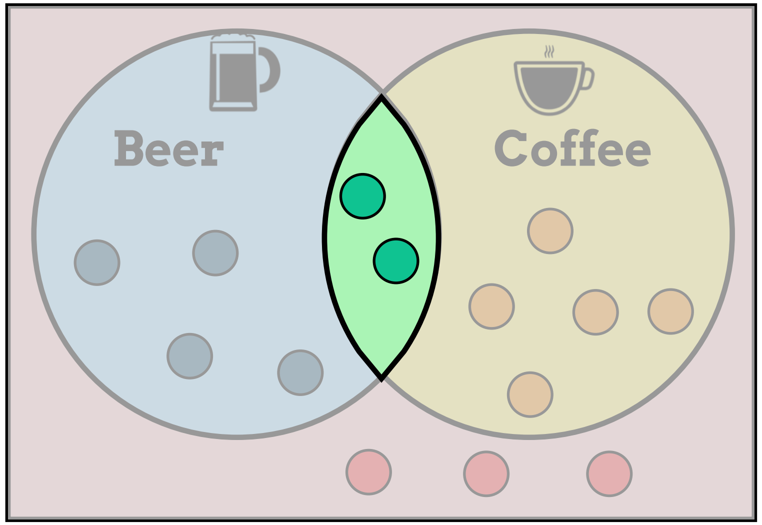
Bayes' Theorem


Beer
Coffee



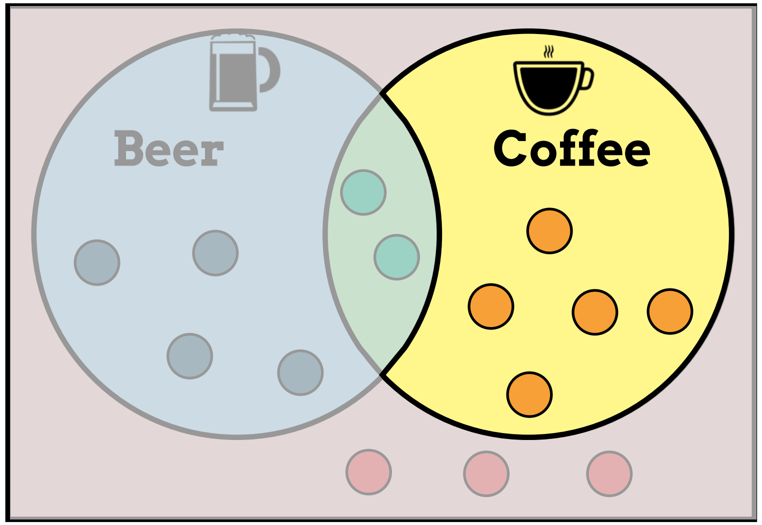
Contingency Table

	Likes Beer	Doesn't Like Beer
Likes Coffee		
Doesn't Like Coffee		



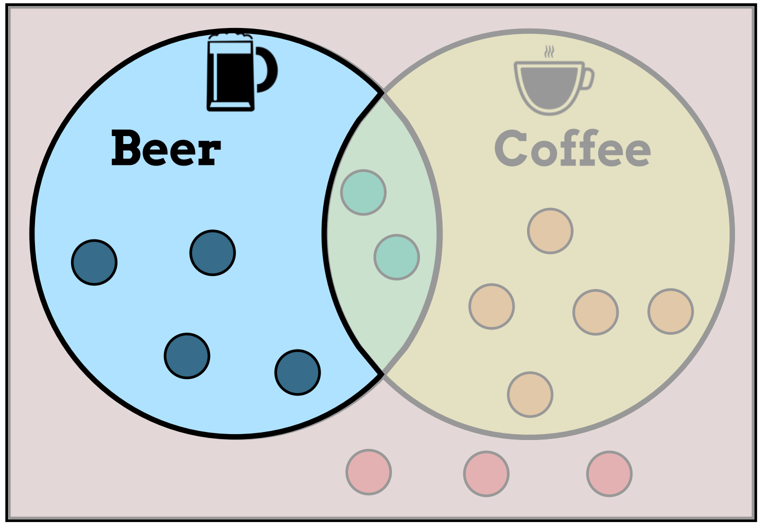
Contingency Table

	Likes Beer	Doesn't Like Beer
Likes Coffee	2 P=2/14	
Doesn't Like Coffee		



Contingency Table

	Likes Beer	Doesn't Like Beer
Likes Coffee	2 P=2/14	5 P=5/14
Doesn't Like Coffee		



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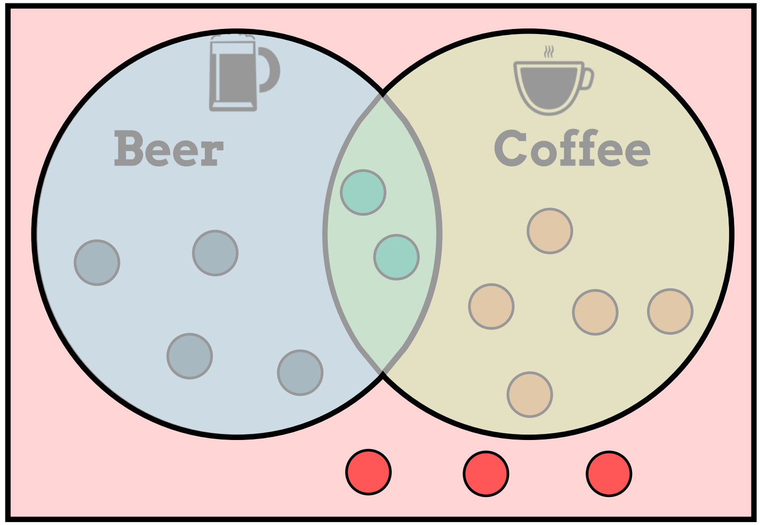
	Likes Beer	Doesn't Like Beer
Likes Coffee	2 P=2/14	5 P=5/14
Doesn't Like Coffee	4 P=4/14	



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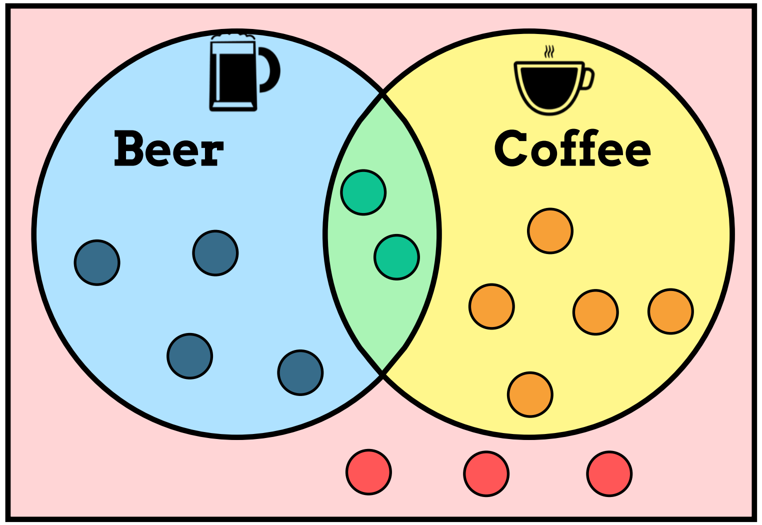
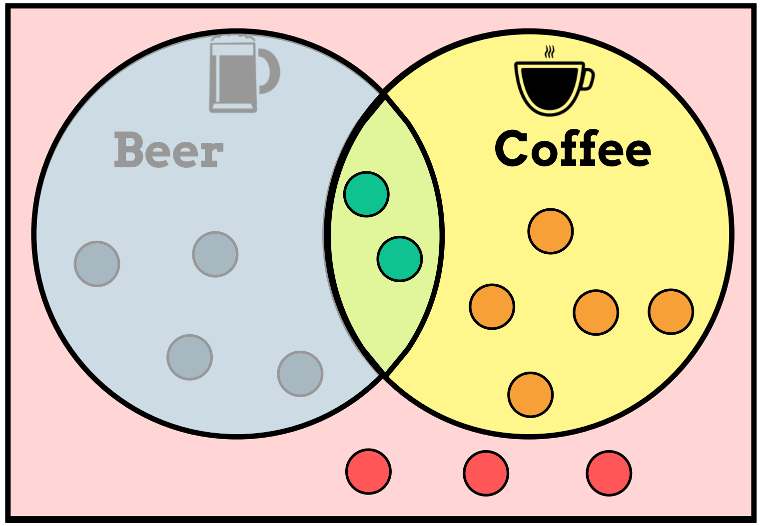


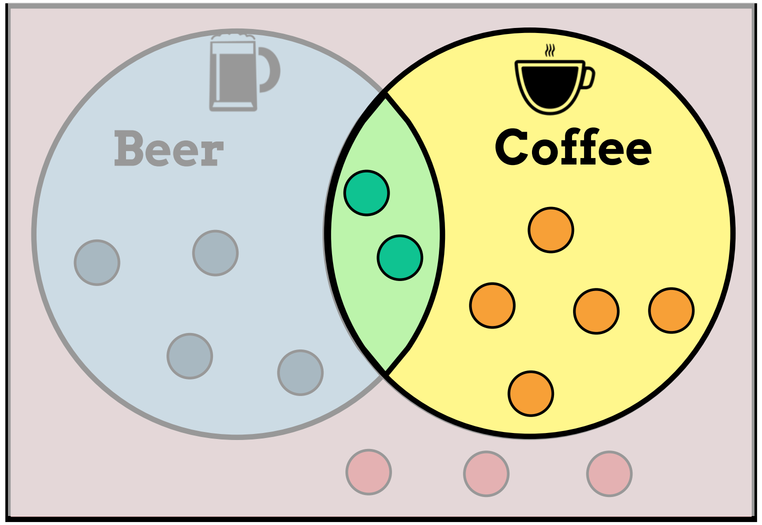
Contingency Table

	Likes Beer	Doesn't Like Beer	Row Total
Likes Coffee	2 P=2/14	5 P=5/14	
Doesn't Like Coffee	4 P=4/14	3 P=3/14	



Contingency Table

	Likes Beer	Doesn't Like Beer	Row Total
Likes Coffee	2 P=2/14	5 P=5/14	2+5=7 P=7/14
Doesn't Like Coffee	4 P=4/14	3 P=3/14	



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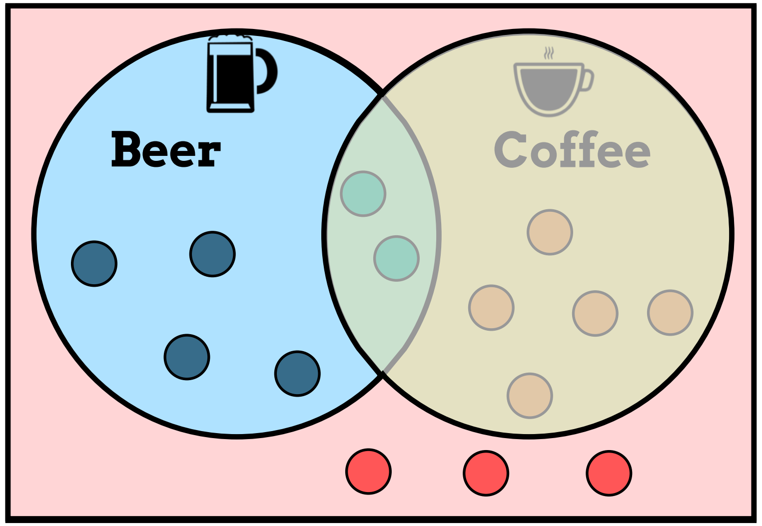

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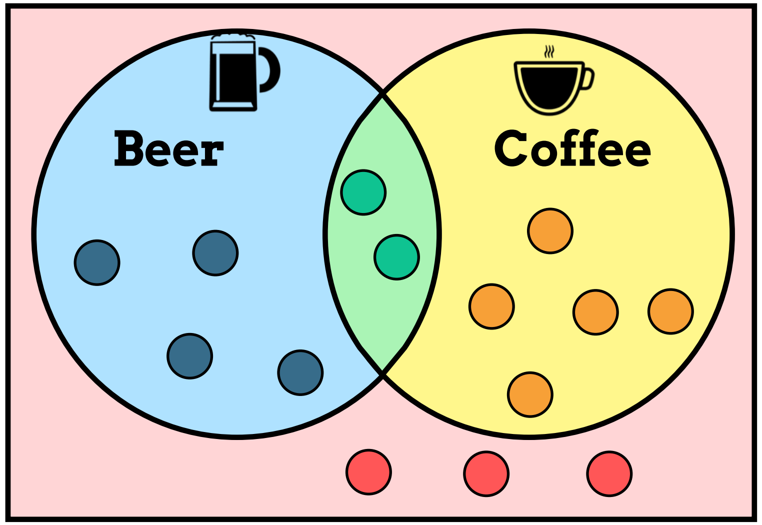

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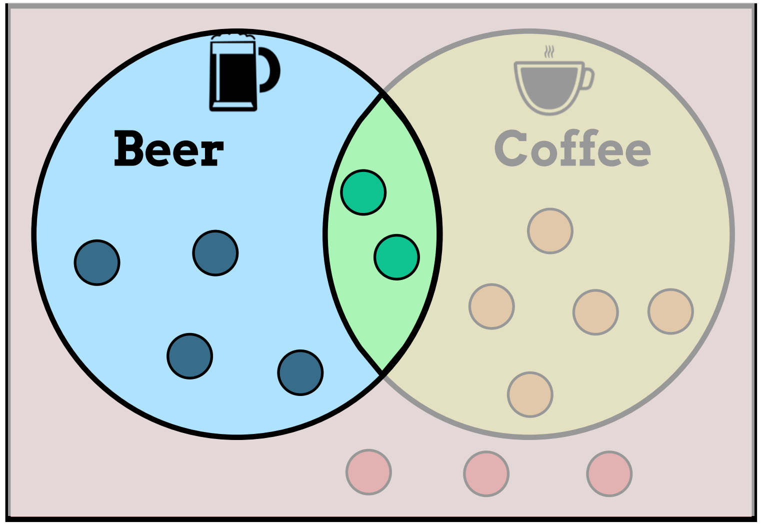
Column Total			



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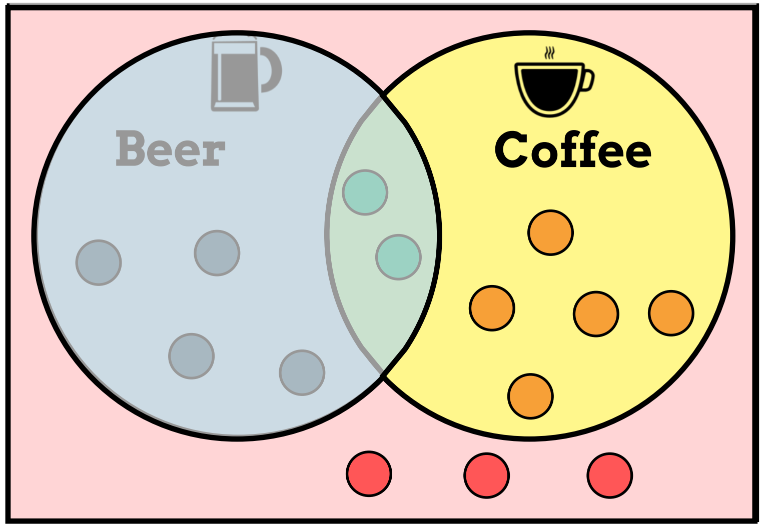
Column Total	2+4=6 P=6/14		



Contingency Table

	Likes Beer	Doesn't Like Beer	Row Total
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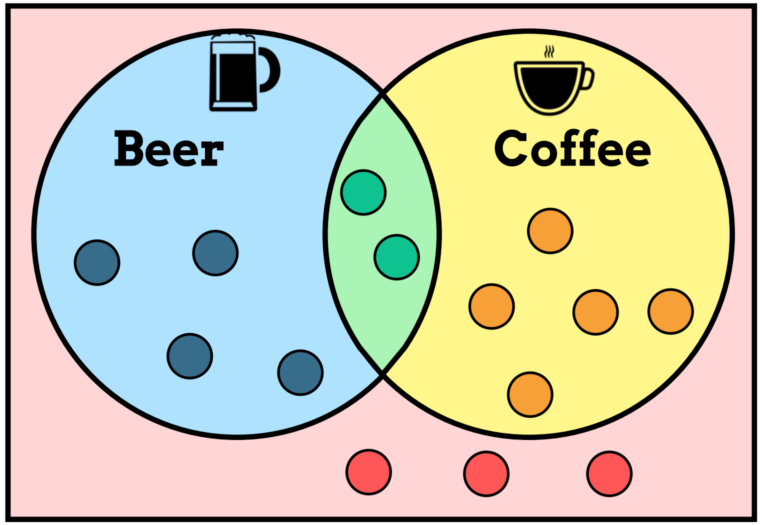
Doesn't Like Coffee	4 P=4/14	3 P=3/14	4+3=7 P=7/14
Column Total	2+4=6 P=6/14	5+3=8 P=8/14	



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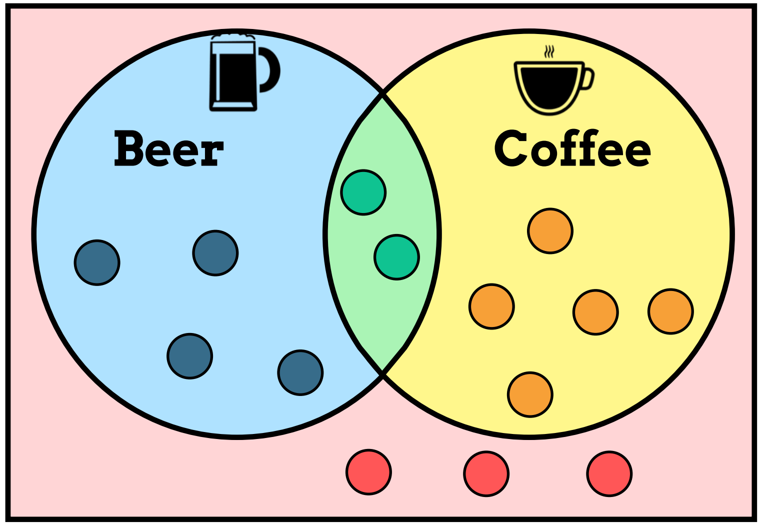

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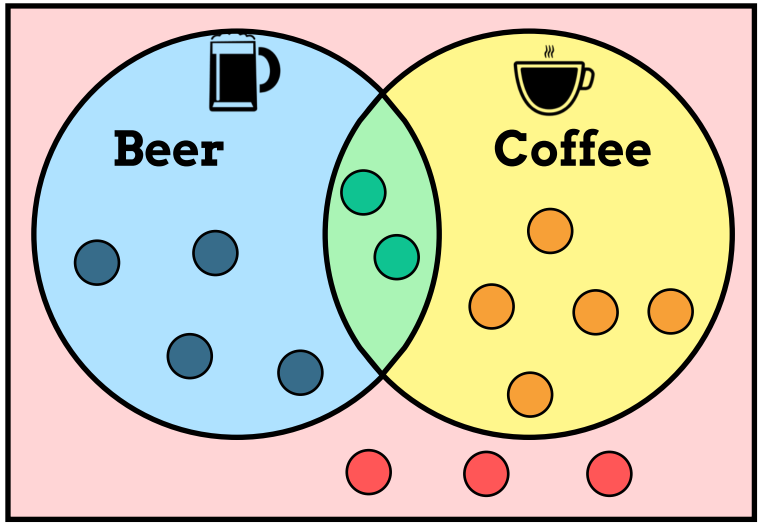
What is the **(conditional) probability** that someone does **not like beer** given that (s)he **likes coffee?**



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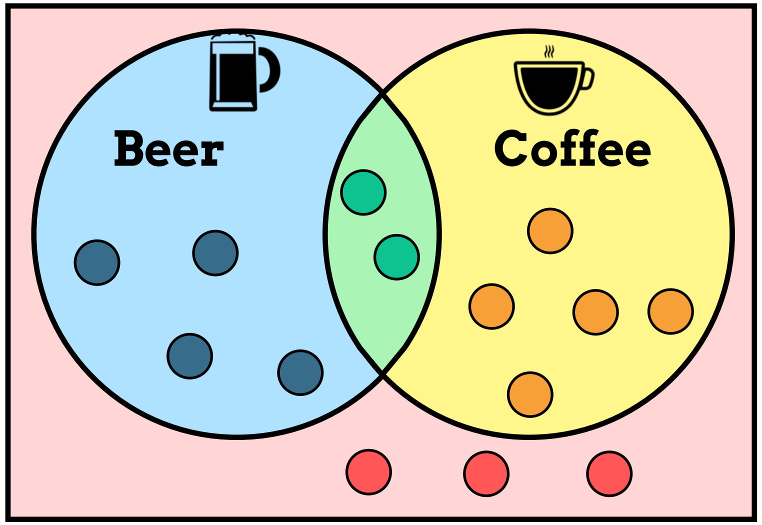
$$\mathrm{P}(\text{no like } \boldsymbol{b} \mid \text{like } \boldsymbol{c}) = ?$$



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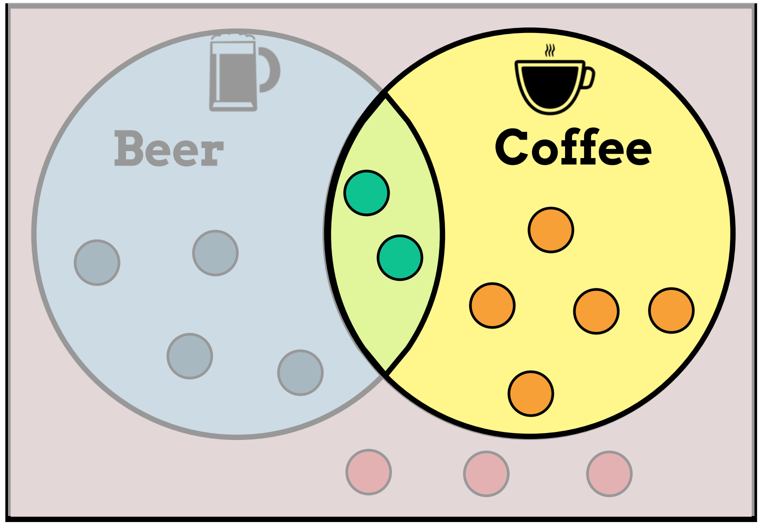
$$P(\text{no like } \boldsymbol{b} \mid \text{like } \boldsymbol{c}) = ?$$

We can rewrite that, being **redundant**, just to simplify our interpretation.

$$P(\text{no like } \boldsymbol{b} \text{ AND like } \boldsymbol{c} \mid \text{like } \boldsymbol{c}) = ?$$



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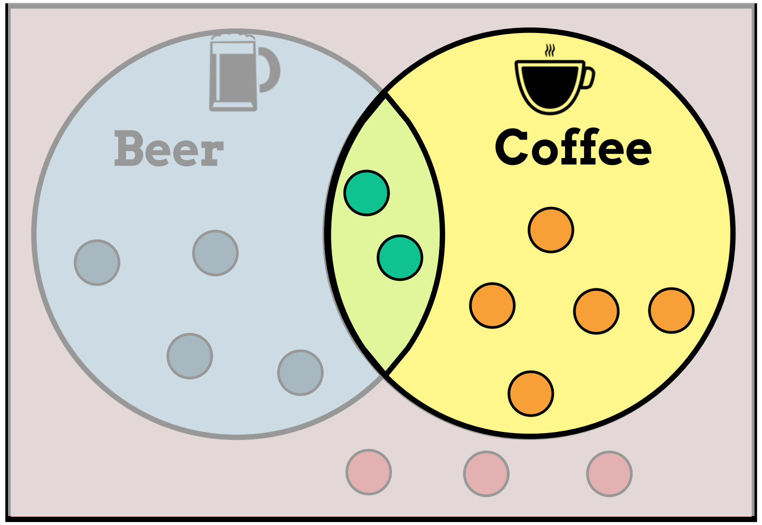
$$\boxed{\mathrm{P}(\text{no like } \boldsymbol{b} \text{ AND like } \boldsymbol{c}} \ \text{like } \boldsymbol{c}) = \boxed{\mathrm{P}(\text{no like } \boldsymbol{b} \text{ AND like } \boldsymbol{c})}$$



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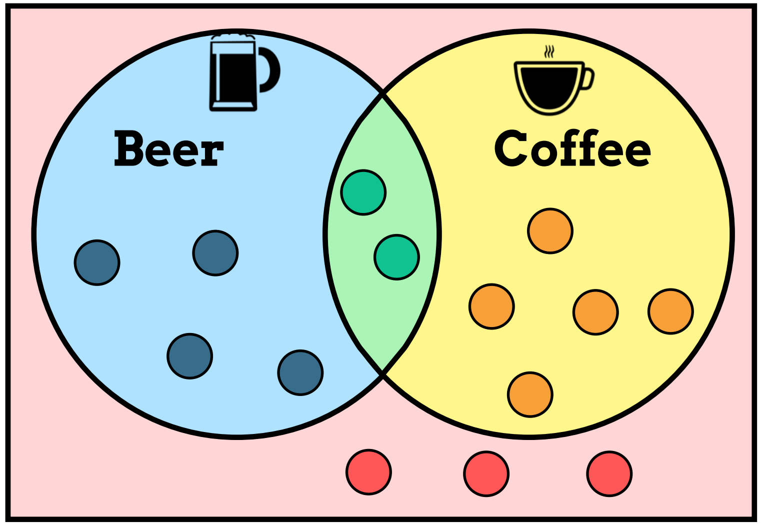
$$\mathrm{P}(\text{no like } \boldsymbol{b} \text{ AND like } \boldsymbol{c} \mid \boxed{\text{like } \boldsymbol{c}}) = \frac{\mathrm{P}(\text{no like } \boldsymbol{b} \text{ AND like } \boldsymbol{c})}{\boxed{\mathrm{P}(\text{like } \boldsymbol{c})}}$$



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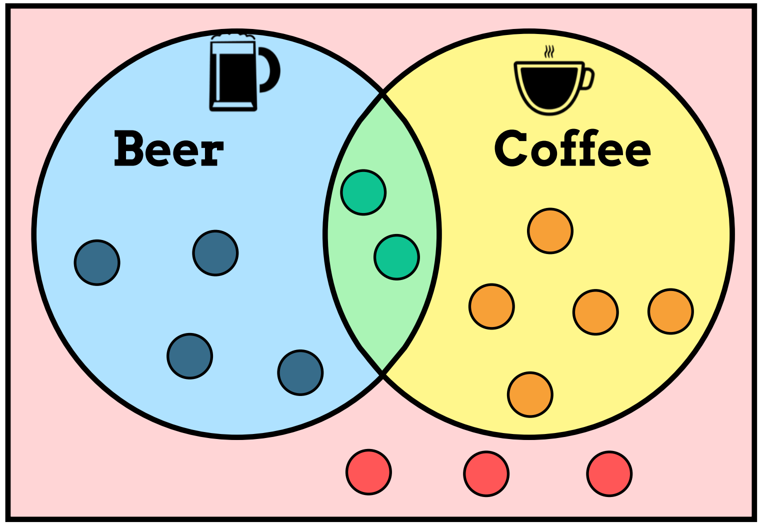
What is the **(conditional) probability** that someone does **not like beer** given that (s)he **likes coffee?**

$$P(\text{no like } \boldsymbol{b} \text{ AND like } \boldsymbol{c} \mid \text{like } \boldsymbol{c}) = \frac{\boxed{5}}{P(\text{like } \boldsymbol{c})}$$



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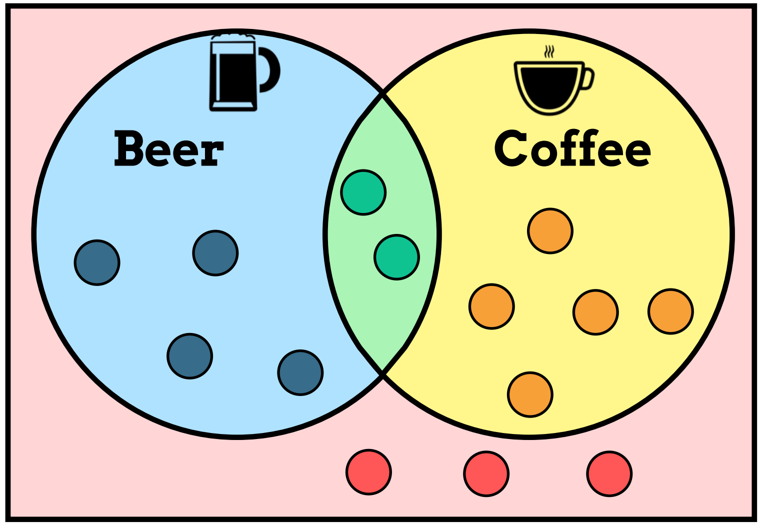

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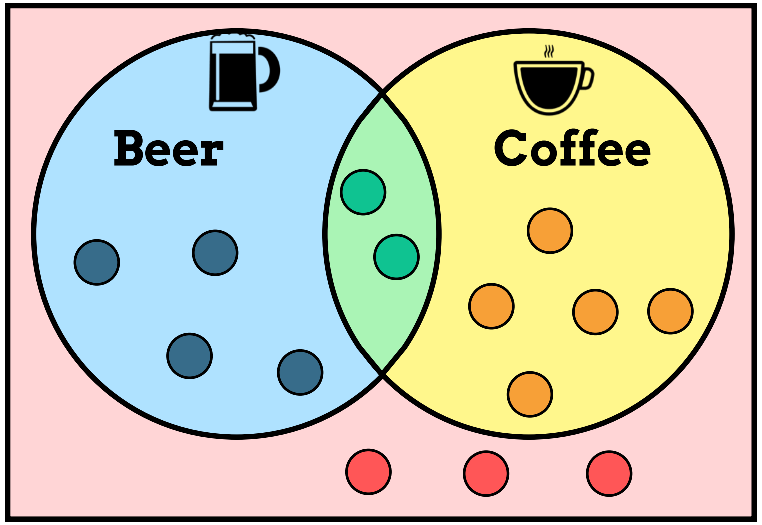
$$\text{P(no like } \boldsymbol{b} \text{ AND like } \boldsymbol{c} \mid \text{like } \boldsymbol{c}) = \frac{5}{7} = \boxed{\mathbf{0.71}}$$



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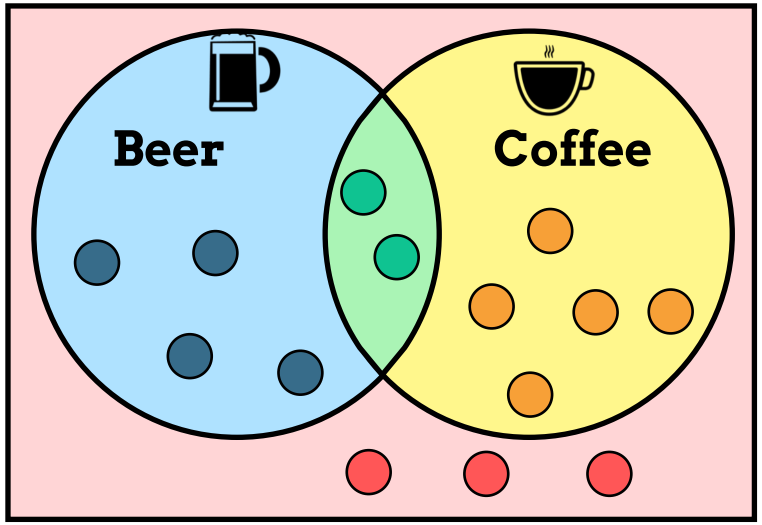

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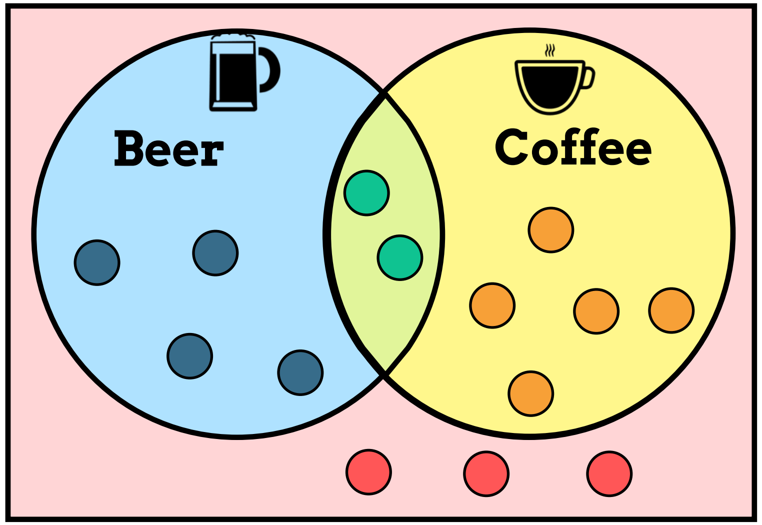

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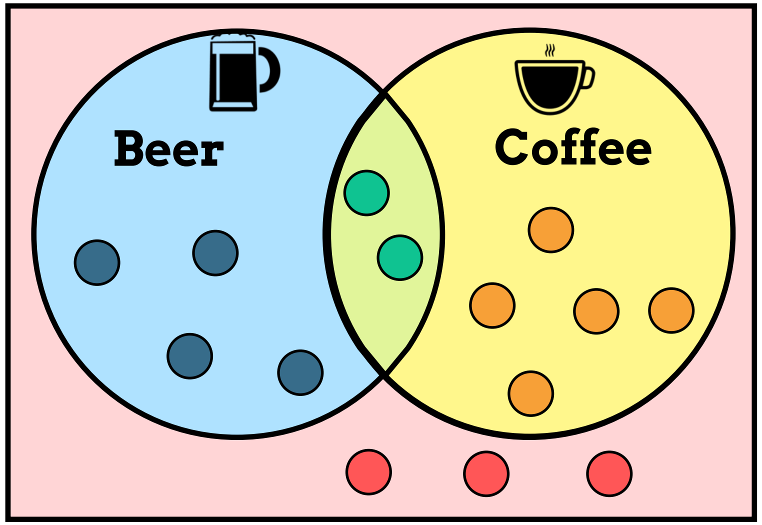
We have just derived the formula of **conditional probability.**



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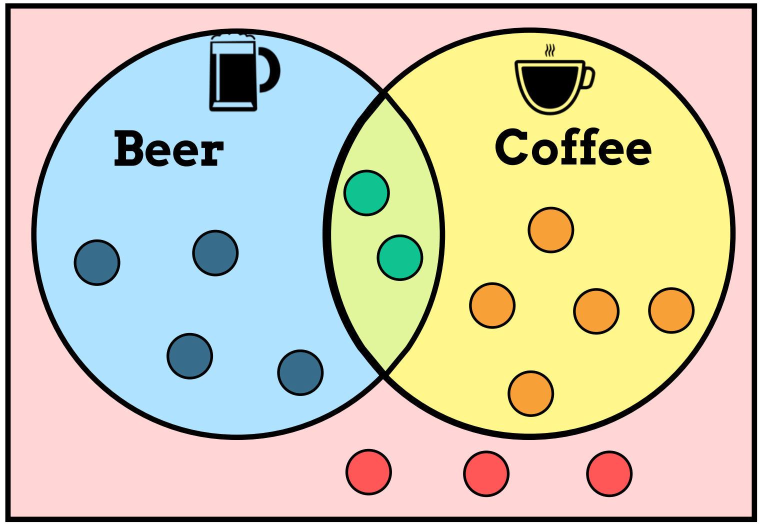

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scaled by the knowledge we already know about the event
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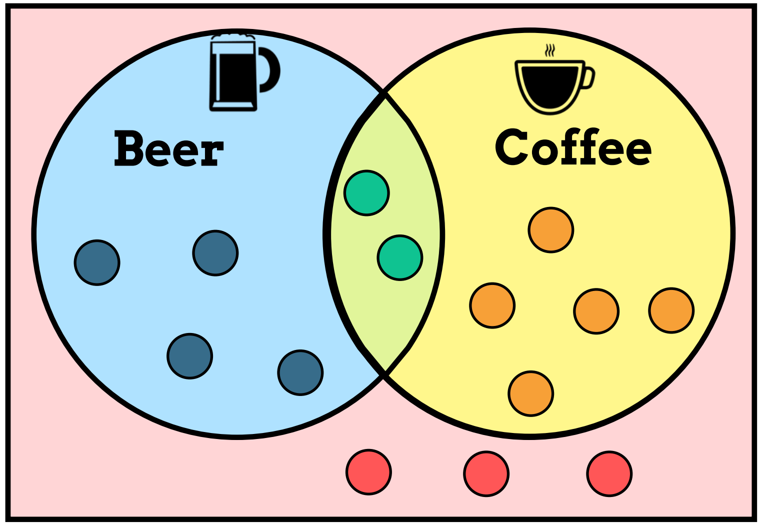

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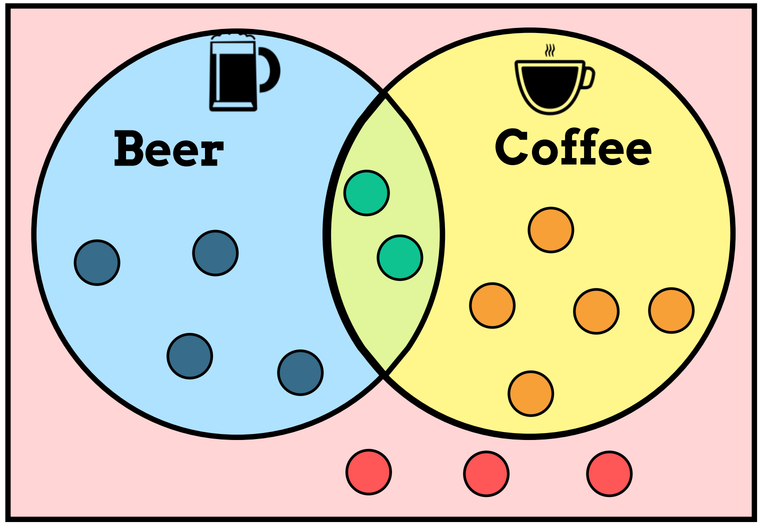
Remember this part is ***redundant***:
"Someone who *does not like beer* and *likes coffee*, given that (s)he *likes coffee*".



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Remember this part is ***redundant***:
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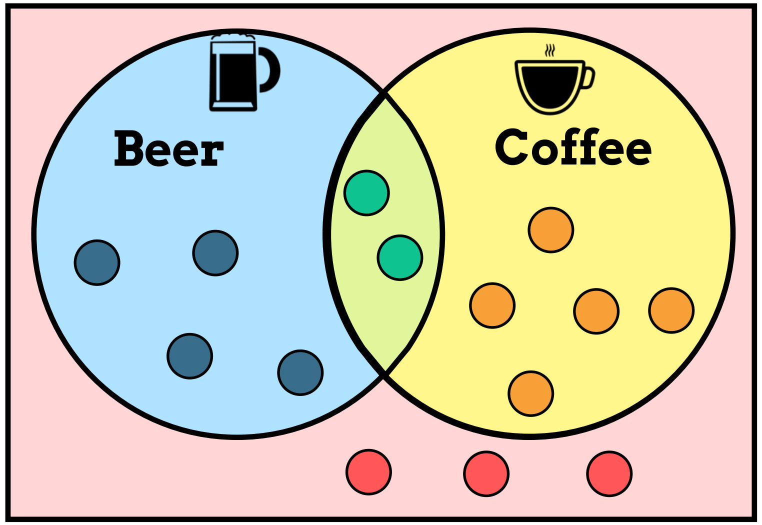
The common and shorten statement does not have that part.



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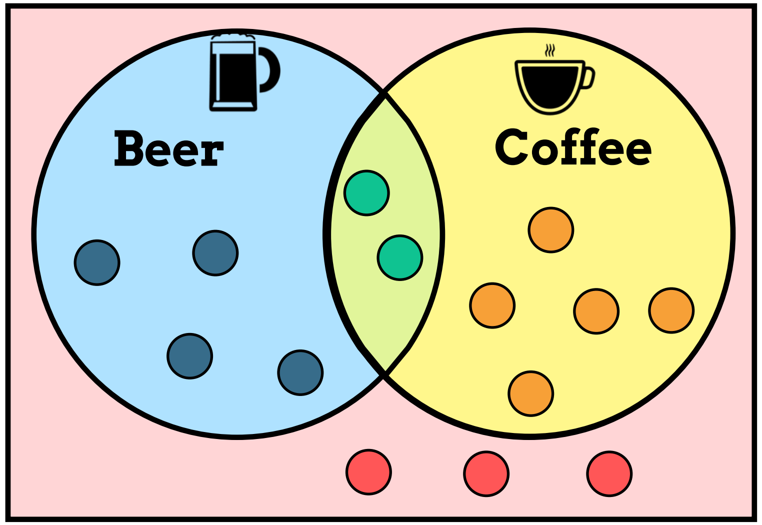
However, the **redundant notation** for **conditional probability** makes it obvious that we want the prob. that an event happens…



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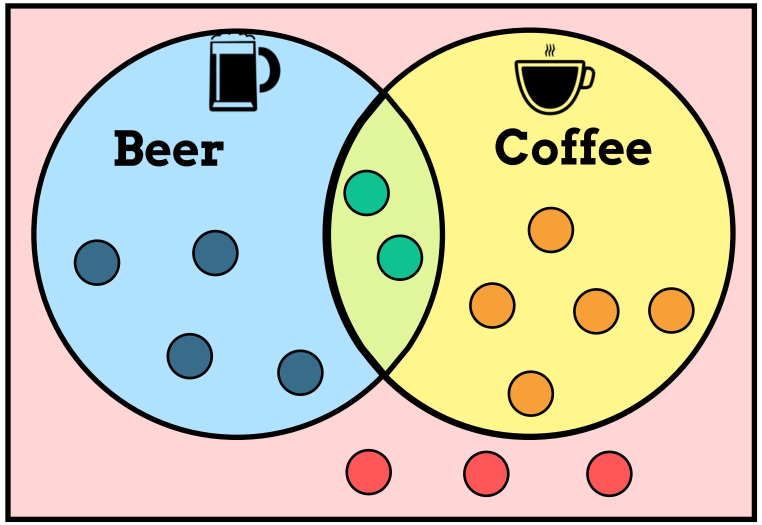

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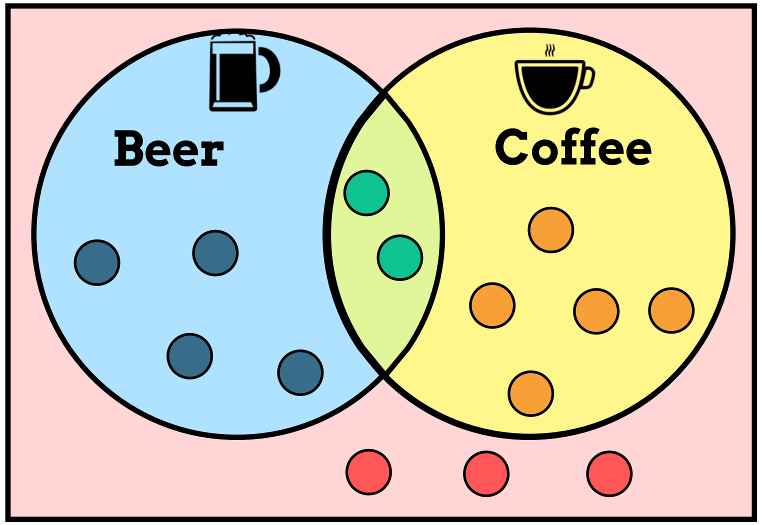
Let's see what happens when we **change** our **a priori** knowledge about the event.

$$\mathrm{P}(\text{no like } \boldsymbol{b} \text{ AND like } \boldsymbol{c} \mid \boxed{\text{like } \boldsymbol{c}}) = \frac{\mathrm{P}(\text{no like } \boldsymbol{b} \text{ AND like } \boldsymbol{c})}{\boxed{\mathrm{P}(\text{like } \boldsymbol{c})}}$$



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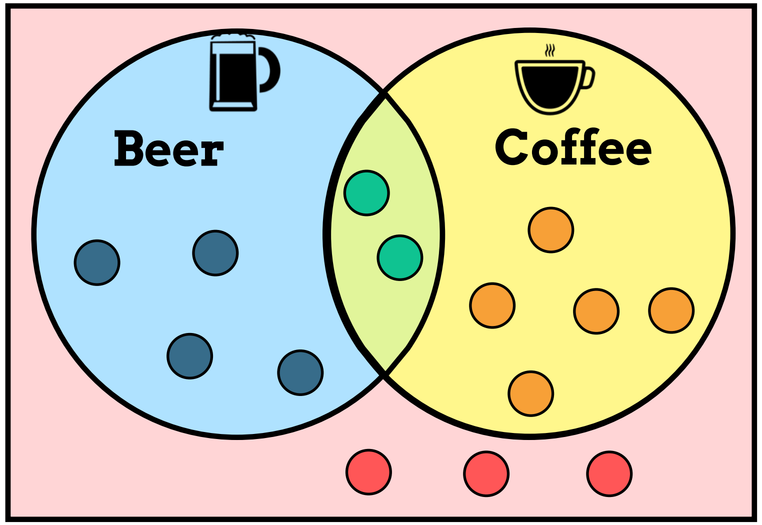

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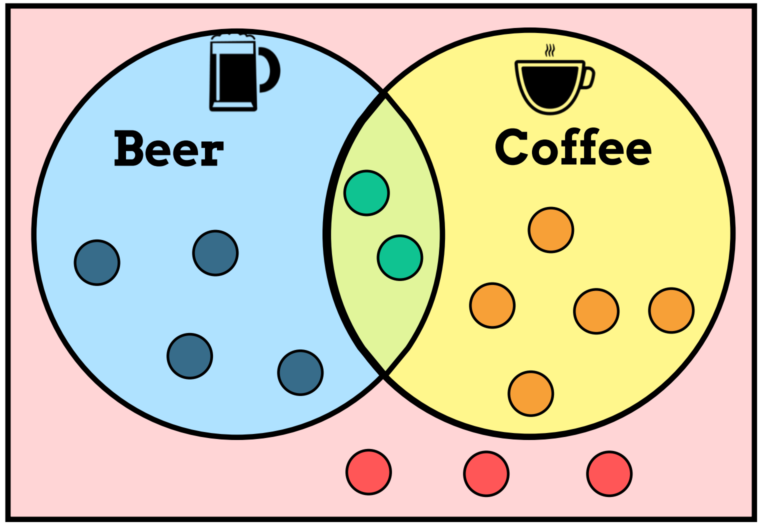
probability that an event will happen
(that someone does not like beer, but likes coffee)…

$$\boxed{\mathrm{P}(\text{no like } \boldsymbol{b} \text{ AND like } \boldsymbol{c}} \text{ no like } \boldsymbol{b}) = \frac{\boxed{\mathrm{P}(\text{no like } \boldsymbol{b} \text{ AND like } \boldsymbol{c})}}{\mathrm{P}(\text{no like } \boldsymbol{b})}$$



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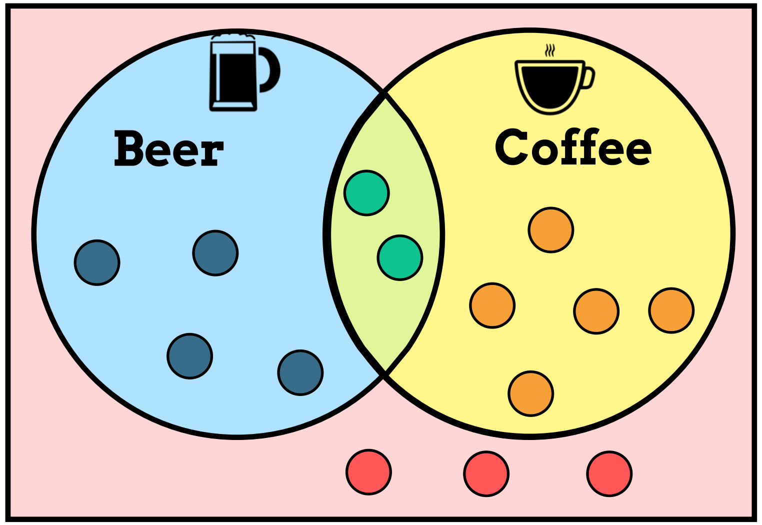
(prob. that someone do not like beer)…

$$P(\text{no like } \boldsymbol{b} \text{ AND like } \boldsymbol{c} \mid \boxed{\text{no like } \boldsymbol{b}}) = \frac{P(\text{no like } \boldsymbol{b} \text{ AND like } \boldsymbol{c})}{\boxed{P(\text{no like } \boldsymbol{b})}}$$



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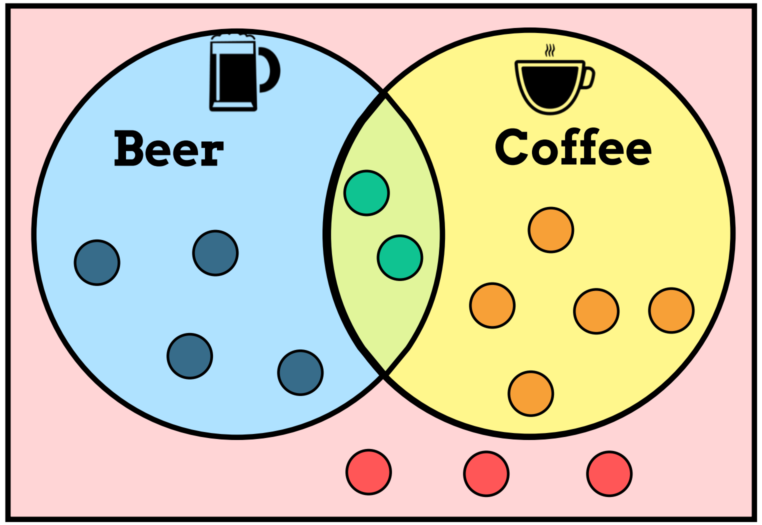
After plugging in the numbers…

$$P(\text{no like } \boldsymbol{b} \text{ AND like } \boldsymbol{c} \mid \text{no like } \boldsymbol{c}) = \boxed{\frac{\frac{5}{14}}{\frac{8}{14}}} = 0.64$$



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Now, let's **compare** both **conditional probabilities**…

$$P(\text{no like } \boldsymbol{b} \text{ AND like } \boldsymbol{c} \mid \text{no like } \boldsymbol{c}) = \frac{\frac{5}{14}}{\frac{8}{14}} = 0.64$$

$$\mathrm{P}(\text{no like } \boldsymbol{b} \text{ AND like } \boldsymbol{c} \mid \text{like } \boldsymbol{c}) = \frac{\mathrm{P}(\text{no like } \boldsymbol{b} \text{ AND like } \boldsymbol{c})}{\mathrm{P}(\text{like } \boldsymbol{c})}$$

$$\mathrm{P}(\text{no like } \boldsymbol{b} \text{ AND like } \boldsymbol{c} \mid \text{no like } \boldsymbol{b}) = \frac{\mathrm{P}(\text{no like } \boldsymbol{b} \text{ AND like } \boldsymbol{c})}{\mathrm{P}(\text{no like } \boldsymbol{b})}$$

In both cases, we want know the **probability of the same event**…

$$\boxed{\mathrm{P}(\text{no like } \boldsymbol{b} \text{ AND like } \boldsymbol{c}} \mid \text{like } \boldsymbol{c}) = \frac{\mathrm{P}(\text{no like } \boldsymbol{b} \text{ AND like } \boldsymbol{c})}{\mathrm{P}(\text{like } \boldsymbol{c})}$$

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In both cases, we want know the **probability of the same event**… which means **the numerators are the same**

$$\boxed{\mathrm{P}(\text{no like } \boldsymbol{b} \text{ AND like } \boldsymbol{c}} \mid \text{like } \boldsymbol{c}) = \frac{\boxed{\mathrm{P}(\text{no like } \boldsymbol{b} \text{ AND like } \boldsymbol{c})}}{\mathrm{P}(\text{like } \boldsymbol{c})}$$

$$\boxed{\mathrm{P}(\text{no like } \boldsymbol{b} \text{ AND like } \boldsymbol{c}} \mid \text{no like } \boldsymbol{b}) = \frac{\boxed{\mathrm{P}(\text{no like } \boldsymbol{b} \text{ AND like } \boldsymbol{c})}}{\mathrm{P}(\text{no like } \boldsymbol{b})}$$

$$P(\text{no like } \boldsymbol{b} \text{ AND like } \boldsymbol{c} \mid \boxed{\text{like } \boldsymbol{c}}) = \frac{P(\text{no like } \boldsymbol{b} \text{ AND like } \boldsymbol{c})}{P(\text{like } \boldsymbol{c})}$$

But, since we have **different knowledge** in each case…

$$P(\text{no like } \boldsymbol{b} \text{ AND like } \boldsymbol{c} \mid \boxed{\text{no like } \boldsymbol{b}}) = \frac{P(\text{no like } \boldsymbol{b} \text{ AND like } \boldsymbol{c})}{P(\text{no like } \boldsymbol{b})}$$

$$P(\text{no like } \boldsymbol{b} \text{ AND like } \boldsymbol{c} \mid \boxed{\text{like } \boldsymbol{c}}) = \frac{P(\text{no like } \boldsymbol{b} \text{ AND like } \boldsymbol{c})}{\boxed{P(\text{like } \boldsymbol{c})}}$$

$$P(\text{no like } \boldsymbol{b} \text{ AND like } \boldsymbol{c} \mid \boxed{\text{no like } \boldsymbol{b}}) = \frac{P(\text{no like } \boldsymbol{b} \text{ AND like } \boldsymbol{c})}{\boxed{P(\text{no like } \boldsymbol{b})}}$$

But, since we have **different knowledge** in each case…
we scale the probabilities of the events differently…

$$\mathrm{P}(\text{no like } \boldsymbol{b} \text{ AND like } \boldsymbol{c} \mid \text{like } \boldsymbol{c}) = \frac{\mathrm{P}(\text{no like } \boldsymbol{b} \text{ AND like } \boldsymbol{c})}{\mathrm{P}(\text{like } \boldsymbol{c})} = \boxed{0.71}$$

$$\mathrm{P}(\text{no like } \boldsymbol{b} \text{ AND like } \boldsymbol{c} \mid \text{no like } \boldsymbol{b}) = \frac{\mathrm{P}(\text{no like } \boldsymbol{b} \text{ AND like } \boldsymbol{c})}{\mathrm{P}(\text{no like } \boldsymbol{b})} = \boxed{0.63}$$

But, since we have **different knowledge** in each case…
we scale the probabilities of the events differently…
getting different **conditional probabilities**

Note that, **the numerator is equal** in both equations…

$$P(\text{no like } \boldsymbol{b} \text{ AND like } \boldsymbol{c} \mid \text{like } \boldsymbol{c}) = \frac{\boxed{P(\text{no like } \boldsymbol{b} \text{ AND like } \boldsymbol{c})}}{P(\text{like } \boldsymbol{c})}$$

$$P(\text{no like } \boldsymbol{b} \text{ AND like } \boldsymbol{c} \mid \text{no like } \boldsymbol{b}) = \frac{\boxed{P(\text{no like } \boldsymbol{b} \text{ AND like } \boldsymbol{c})}}{P(\text{no like } \boldsymbol{b})}$$

$$\text{P(no like } \boldsymbol{b} \text{ AND like } \boldsymbol{c} \mid \text{like } \boldsymbol{c}) = \frac{\boxed{\text{P(no like } \boldsymbol{b} \text{ AND like } \boldsymbol{c})}}{\text{P(like } \boldsymbol{c})}$$

$$\text{P(no like } \boldsymbol{b} \text{ AND like } \boldsymbol{c} \mid \text{no like } \boldsymbol{b}) = \frac{\boxed{\text{P(no like } \boldsymbol{b} \text{ AND like } \boldsymbol{c})}}{\text{P(no like } \boldsymbol{b})}$$

Note that, **the numerator is equal** in both equations…
so that even if we did not know about such a prob., we still could solve the **conditional probability**…

$$\mathrm{P}(\text{no like } \boldsymbol{b} \text{ AND like } \boldsymbol{c} \mid \text{like } \boldsymbol{c}) * \mathrm{P}(\text{like } \boldsymbol{c}) = \boxed{\mathrm{P}(\text{no like } \boldsymbol{b} \text{ AND like } \boldsymbol{c})}$$

$$\mathrm{P}(\text{no like } \boldsymbol{b} \text{ AND like } \boldsymbol{c} \mid \text{no like } \boldsymbol{b}) * \mathrm{P}(\text{no like } \boldsymbol{b}) = \boxed{\mathrm{P}(\text{no like } \boldsymbol{b} \text{ AND like } \boldsymbol{c})}$$

$$\mathrm{P}(\text{no like } \boldsymbol{b} \text{ AND like } \boldsymbol{c} \mid \text{like } \boldsymbol{c}) * \mathrm{P}(\text{like } \boldsymbol{c}) = \boxed{\mathrm{P}(\text{no like } \boldsymbol{b} \text{ AND like } \boldsymbol{c})}$$

$$\mathrm{P}(\text{no like } \boldsymbol{b} \text{ AND like } \boldsymbol{c} \mid \text{no like } \boldsymbol{b}) * \mathrm{P}(\text{no like } \boldsymbol{b}) = \boxed{\mathrm{P}(\text{no like } \boldsymbol{b} \text{ AND like } \boldsymbol{c})}$$

$$\mathrm{P}(\text{no like } \boldsymbol{b} \text{ AND like } \boldsymbol{c} \mid \text{like } \boldsymbol{c}) * \mathrm{P}(\text{like } \boldsymbol{c}) = \mathrm{P}(\text{no like } \boldsymbol{b} \text{ AND like } \boldsymbol{c} \mid \text{no like } \boldsymbol{b}) * \mathrm{P}(\text{no like } \boldsymbol{b})$$

$$\text{P(no like } \boldsymbol{b} \text{ AND like } \boldsymbol{c} \mid \text{like } \boldsymbol{c}) * \text{P(like } \boldsymbol{c}) = \boxed{\text{P(no like } \boldsymbol{b} \text{ AND like } \boldsymbol{c})}$$

$$\text{P(no like } \boldsymbol{b} \text{ AND like } \boldsymbol{c} \mid \text{no like } \boldsymbol{b}) * \text{P(no like } \boldsymbol{b}) = \boxed{\text{P(no like } \boldsymbol{b} \text{ AND like } \boldsymbol{c})}$$

$$\text{P(no like } \boldsymbol{b} \text{ AND like } \boldsymbol{c} \mid \text{like } \boldsymbol{c}) * \text{P(like } \boldsymbol{c}) = \text{P(no like } \boldsymbol{b} \text{ AND like } \boldsymbol{c} \mid \text{no like } \boldsymbol{b}) * \text{P(no like } \boldsymbol{b})$$

$$\boxed{\text{P(no like } \boldsymbol{b} \text{ AND like } \boldsymbol{c} \mid \text{like } \boldsymbol{c}) = \frac{\text{P(no like } \boldsymbol{b} \text{ AND like } \boldsymbol{c} \mid \text{no like } \boldsymbol{b}) * \text{P(no like } \boldsymbol{b})}{\text{P(like } \boldsymbol{c})}}$$

$$P(\text{no like } \boldsymbol{b} \text{ AND like } \boldsymbol{c} \mid \text{like } \boldsymbol{c}) * P(\text{like } \boldsymbol{c}) = \boxed{P(\text{no like } \boldsymbol{b} \text{ AND like } \boldsymbol{c})}$$

$$P(\text{no like } \boldsymbol{b} \text{ AND like } \boldsymbol{c} \mid \text{no like } \boldsymbol{b}) * P(\text{no like } \boldsymbol{b}) = \boxed{P(\text{no like } \boldsymbol{b} \text{ AND like } \boldsymbol{c})}$$

$$P(\text{no like } \boldsymbol{b} \text{ AND like } \boldsymbol{c} \mid \text{like } \boldsymbol{c}) * P(\text{like } \boldsymbol{c}) = P(\text{no like } \boldsymbol{b} \text{ AND like } \boldsymbol{c} \mid \text{no like } \boldsymbol{b}) * P(\text{no like } \boldsymbol{b})$$

$$P(\text{no like } \boldsymbol{b} \text{ AND like } \boldsymbol{c} \mid \text{like } \boldsymbol{c}) = \frac{P(\text{no like } \boldsymbol{b} \text{ AND like } \boldsymbol{c} \mid \text{no like } \boldsymbol{b}) * P(\text{no like } \boldsymbol{b})}{P(\text{like } \boldsymbol{c})}$$

Or

$$P(\text{no like } \boldsymbol{b} \text{ AND like } \boldsymbol{c} \mid \text{no like } \boldsymbol{b}) = \frac{P(\text{no like } \boldsymbol{b} \text{ AND like } \boldsymbol{c} \mid \text{like } \boldsymbol{c}) * P(\text{like } \boldsymbol{c})}{P(\text{no like } \boldsymbol{b})}$$

$$P(\text{no like } \boldsymbol{b} \text{ AND like } \boldsymbol{c} \mid \text{like } \boldsymbol{c}) * P(\text{like } \boldsymbol{c}) = \boxed{P(\text{no like } \boldsymbol{b} \text{ AND like } \boldsymbol{c})}$$

$$P(\text{no like } \boldsymbol{b} \text{ AND like } \boldsymbol{c} \mid \text{no like } \boldsymbol{b}) * P(\text{no like } \boldsymbol{b}) = \boxed{P(\text{no like } \boldsymbol{b} \text{ AND like } \boldsymbol{c})}$$

$$P(\text{no like } \boldsymbol{b} \text{ AND like } \boldsymbol{c} \mid \text{like } \boldsymbol{c}) * P(\text{like } \boldsymbol{c}) = P(\text{no like } \boldsymbol{b} \text{ AND like } \boldsymbol{c} \mid \text{no like } \boldsymbol{b}) * P(\text{no like } \boldsymbol{b})$$

We have just derived the formula **Bayes' Theorem.**

$$P(\text{no like } \boldsymbol{b} \text{ AND like } \boldsymbol{c} \mid \text{like } \boldsymbol{c}) = \frac{P(\text{no like } \boldsymbol{b} \text{ AND like } \boldsymbol{c} \mid \text{no like } \boldsymbol{b}) * P(\text{no like } \boldsymbol{b})}{P(\text{like } \boldsymbol{c})}$$

Or

$$P(\text{no like } \boldsymbol{b} \text{ AND like } \boldsymbol{c} \mid \text{no like } \boldsymbol{b}) = \frac{P(\text{no like } \boldsymbol{b} \text{ AND like } \boldsymbol{c} \mid \text{like } \boldsymbol{c}) * P(\text{like } \boldsymbol{c})}{P(\text{no like } \boldsymbol{b})}$$

B = likes coffee

$$\boxed{\mathrm{P}(\text{no like } \boldsymbol{b} \text{ AND like } \boldsymbol{c} \mid \text{like } \boldsymbol{c})} = \frac{\mathrm{P}(\text{no like } \boldsymbol{b} \text{ AND like } \boldsymbol{c} \mid \text{no like } \boldsymbol{b}) * \mathrm{P}(\text{no like } \boldsymbol{b})}{\mathrm{P}(\text{like } \boldsymbol{c})}$$

Bayes' Theorem tells us that this **conditional probability**, which is based on the knowledge about the person **likes coffee (event B)…**

A = does not like beer
B = likes coffee

$$\text{P}(\text{no like } \boldsymbol{b} \text{ AND like } \boldsymbol{c} \mid \text{like } \boldsymbol{c}) = \frac{\boxed{\text{P}(\text{no like } \boldsymbol{b} \text{ AND like } \boldsymbol{c} \mid \text{no like } \boldsymbol{b})} * \text{P}(\text{no like } \boldsymbol{b})}{\text{P}(\text{like } \boldsymbol{c})}$$

Bayes' Theorem tells us that this **conditional probability**, which is based on the knowledge about the person **likes coffee (event B)…**

…can be derived from this **conditional probability**, which is based on which is based on the knowledge about the person **does not like beer (event A).**

A = does not like beer
B = likes coffee

$$\mathrm{P}(\text{no like } \boldsymbol{b} \text{ AND like } \boldsymbol{c} \mid \text{like } \boldsymbol{c}) = \frac{\mathrm{P}(\text{no like } \boldsymbol{b} \text{ AND like } \boldsymbol{c} \mid \text{no like } \boldsymbol{b}) * \mathrm{P}(\text{no like } \boldsymbol{b})}{\mathrm{P}(\text{like } \boldsymbol{c})}$$

A = does not like beer
B = likes coffee

$$P(\text{no like } \boldsymbol{b} \text{ AND like } \boldsymbol{c} \mid \text{like } \boldsymbol{c}) = \frac{P(\text{no like } \boldsymbol{b} \text{ AND like } \boldsymbol{c} \mid \text{no like } \boldsymbol{b}) * P(\text{no like } \boldsymbol{b})}{P(\text{like } \boldsymbol{c})}$$

$$\downarrow$$

$$P(\mathbf{A} \text{ AND } \mathbf{B} \mid \mathbf{B}) = \frac{P(\mathbf{A} \text{ AND } \mathbf{B} \mid \mathbf{A}) * P(\mathbf{A})}{P(\mathbf{B})}$$

A = does not like beer
B = likes coffee

$$P(\text{no like } \boldsymbol{b} \text{ AND like } \boldsymbol{c} \mid \text{like } \boldsymbol{c}) = \frac{P(\text{no like } \boldsymbol{b} \text{ AND like } \boldsymbol{c} \mid \text{no like } \boldsymbol{b}) * P(\text{no like } \boldsymbol{b})}{P(\text{like } \boldsymbol{c})}$$

$$P(A \text{ AND } B \mid B) = \frac{P(A \text{ AND } B \mid A) * P(A)}{P(B)}$$

Removing redundancy

$$P(A|B) = \frac{P(B \mid A) * P(A)}{P(B)}$$

Bayes' Theorem

Bayes' Theorem

Likelihood
The prob. of B being True, given A.

Prior Probability
The prob. of A being True.

$$P(A|B) = \frac{P(B \mid A) * P(A)}{P(B)}$$

Posterior (*a posteriori prob.*)
The prob. of A being True, given B.

Marginal Likelihood
The prob. of B being True.

Bayes' Theorem

a different way of seeing that

H: Hypothesis

E: Evidence

Likelihood

- The prob. of seeing the **evidence** if the **hypothesis** is true.
- It updates our initial guess through the **evidence** that comes from **data**.

Prior Probability

- The prob. of the **hypothesis** is true.
- Our **initial guess** about the **hypothesis** before any **evidence** is present.
- How probable was our **hypothesis** ***before*** observing the **evidence**?

$$P(H|E) = \frac{P(E|H) * P(H)}{P(E)}$$

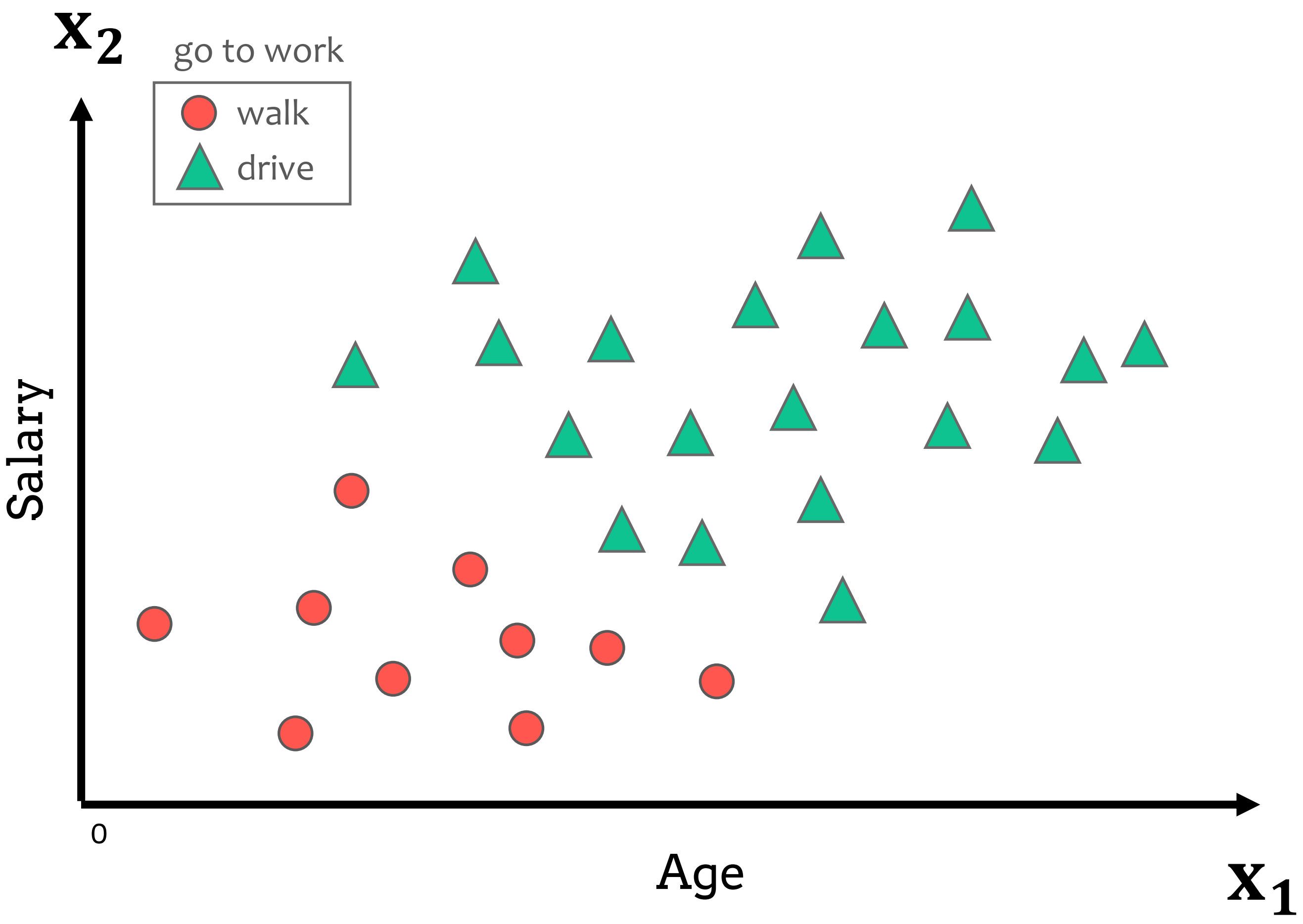
Posterior (*a posteriori prob.*)

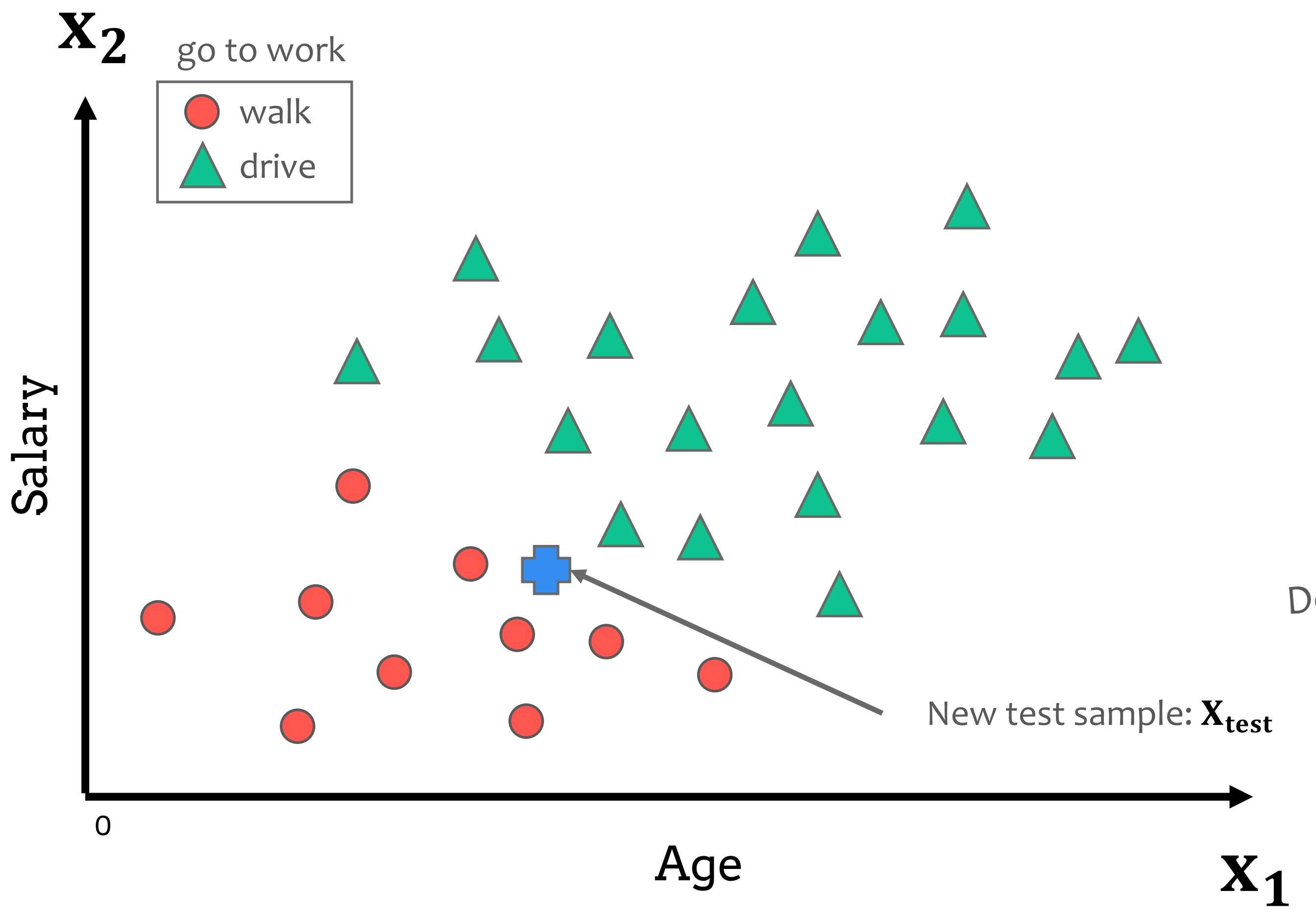
The prob. of the **hypothesis** is true given the observed **evidence**.

Marginal Likelihood

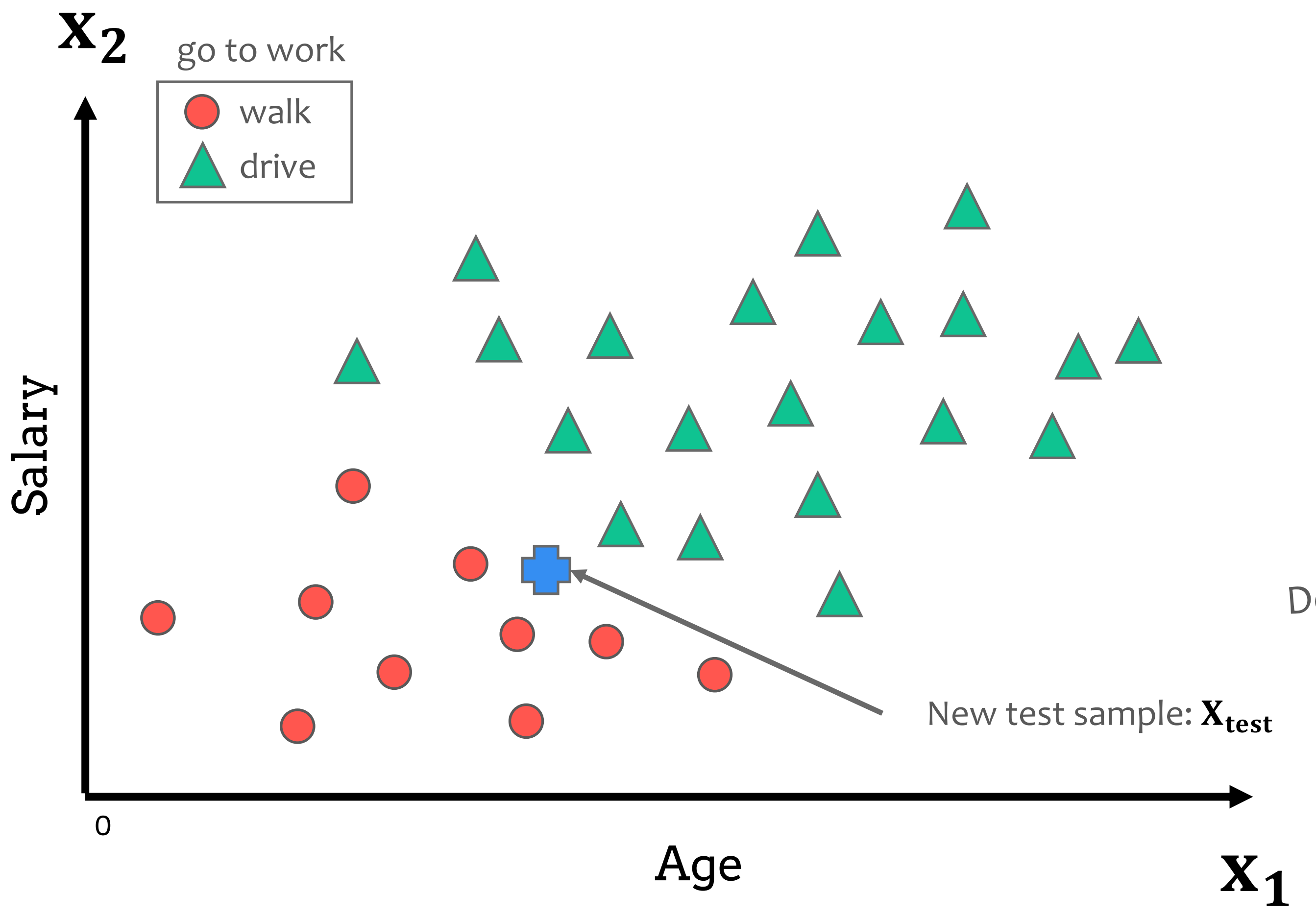
- The prob. of **B** being true.
- The prob. of observing the **evidence**.
- How probable is the new **evidence** under ***all*** possible **hypothesis**?

Naive Bayes Classifier


$\mathbf{x_2}$
go to work
walk
drive
Salary
0
Age
$\mathbf{x_1}$



Does this person go to work by walking or driving?



Let's use **Naive Bayes' Theorem**!

Does this person go to work by **walking** or **driving**?

Naive Bayes Classifier

Bayes' Theorem

Likelihood

Prior probability

$$P(H|E) = \frac{P(E|H) * P(H)}{P(E)}$$

Posterior probability

Marginal Likelihood

Naive Bayes Classifier

Bayes' Theorem

Likelihood

Prior probability

$$P(\text{class}|\text{data}) = \frac{P(\text{data}|\text{class}) * P(\text{class})}{P(\text{data})}$$

Posterior probability

Marginal Likelihood

Naive Bayes Classifier

Step 1

#3 Likelihood

Prior probability #1

class

feats of the new sample

$$P(Walks|x_{test}) = \frac{P(x_{test}|Walks) * P(Walks)}{P(x_{test})}$$

#4 Posterior probability

Marginal Likelihood #2

Naive Bayes Classifier

Step 2

#3 Likelihood

Prior probability #1

class

feats of the new sample

$$P(Drives|x_{test}) = \frac{P(x_{test}|Drives) * P(Drives)}{P(x_{test})}$$

#4 Posterior probability

Marginal Likelihood #2

Naive Bayes Classifier

Step 3

$$P(Walks|\mathrm{x}_{test}) \ \textbf{vs} \ P(Drives|\mathrm{x}_{test})$$

We assign the **new test instance** to the class with **the highest posteriori probability.**

Naive Bayes Classifier

Step 3

Let's do this

$$P(Walks|x_{test}) \textbf{ vs } P(Drives|x_{test})$$

We assign the **new test instance** to the class with **the highest posteriori probability.**

Naive Bayes

#3
Likelihood

Prior probability #1

class　　feats of the new sample

$$P(Walks|x_{test}) = \frac{P(x_{test}|Walks) * P(Walks)}{P(x_{test})}$$

#4 **Posterior probability**

Marginal Likelihood
#2

Naive Bayes

#3
Likelihood

Prior probability #1

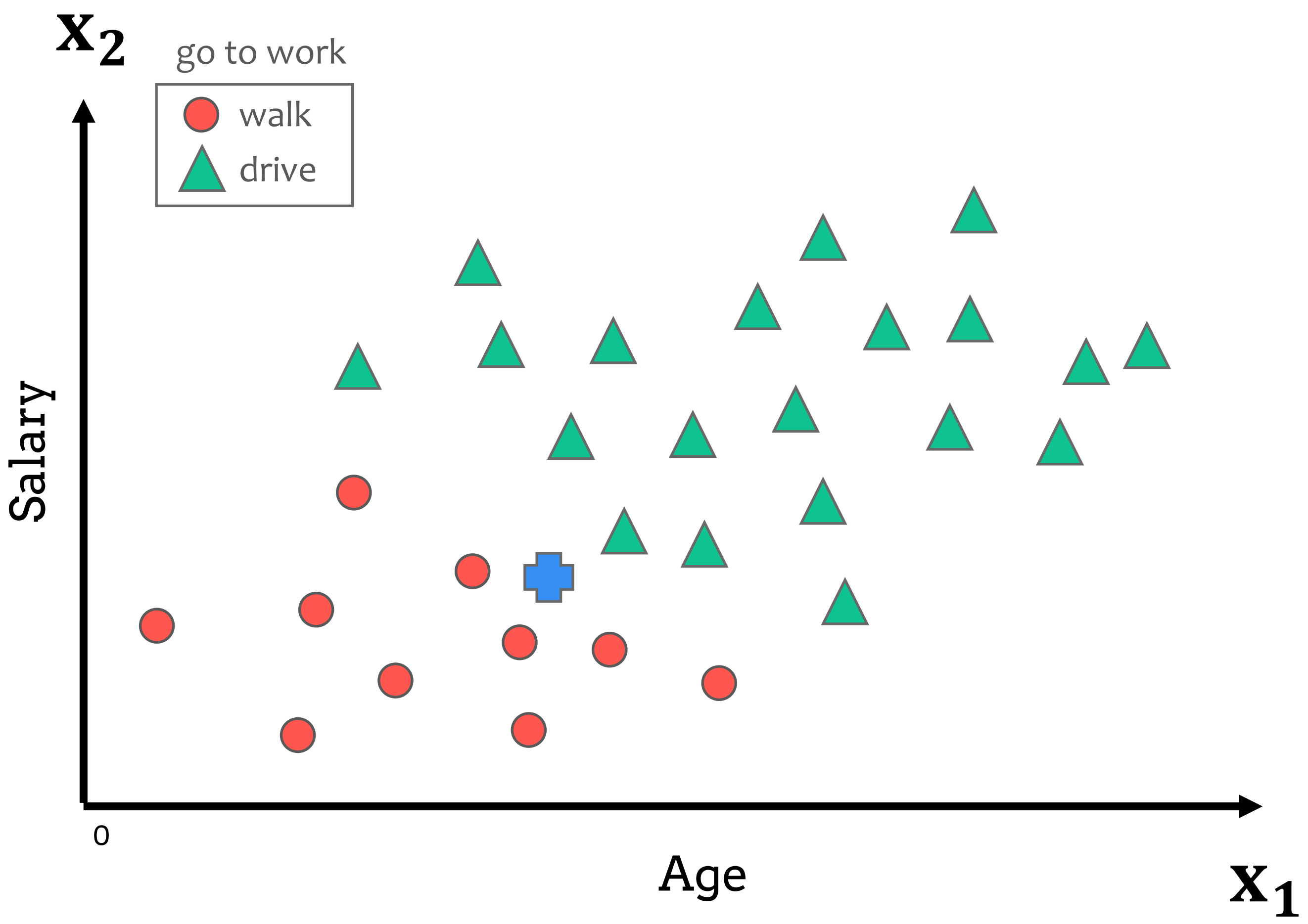
class
feats of the new sample

$$P(Walks|x_{test}) = \frac{P(x_{test}|Walks) * P(Walks)}{P(x_{test})}$$

#4 **Posterior probability**

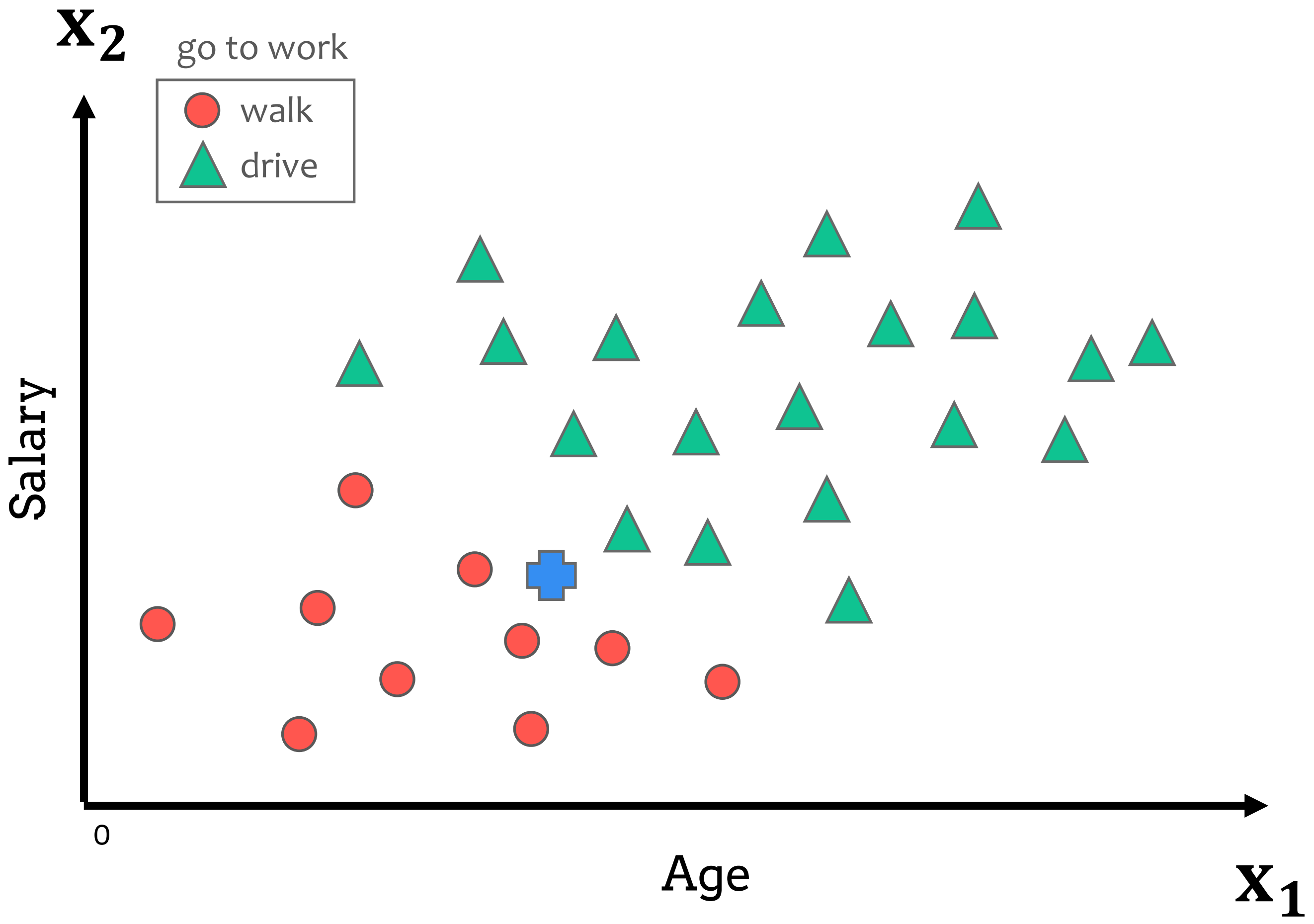
Marginal Likelihood
#2

Naive Bayes: Step 1



#1 - $P(Walks)$

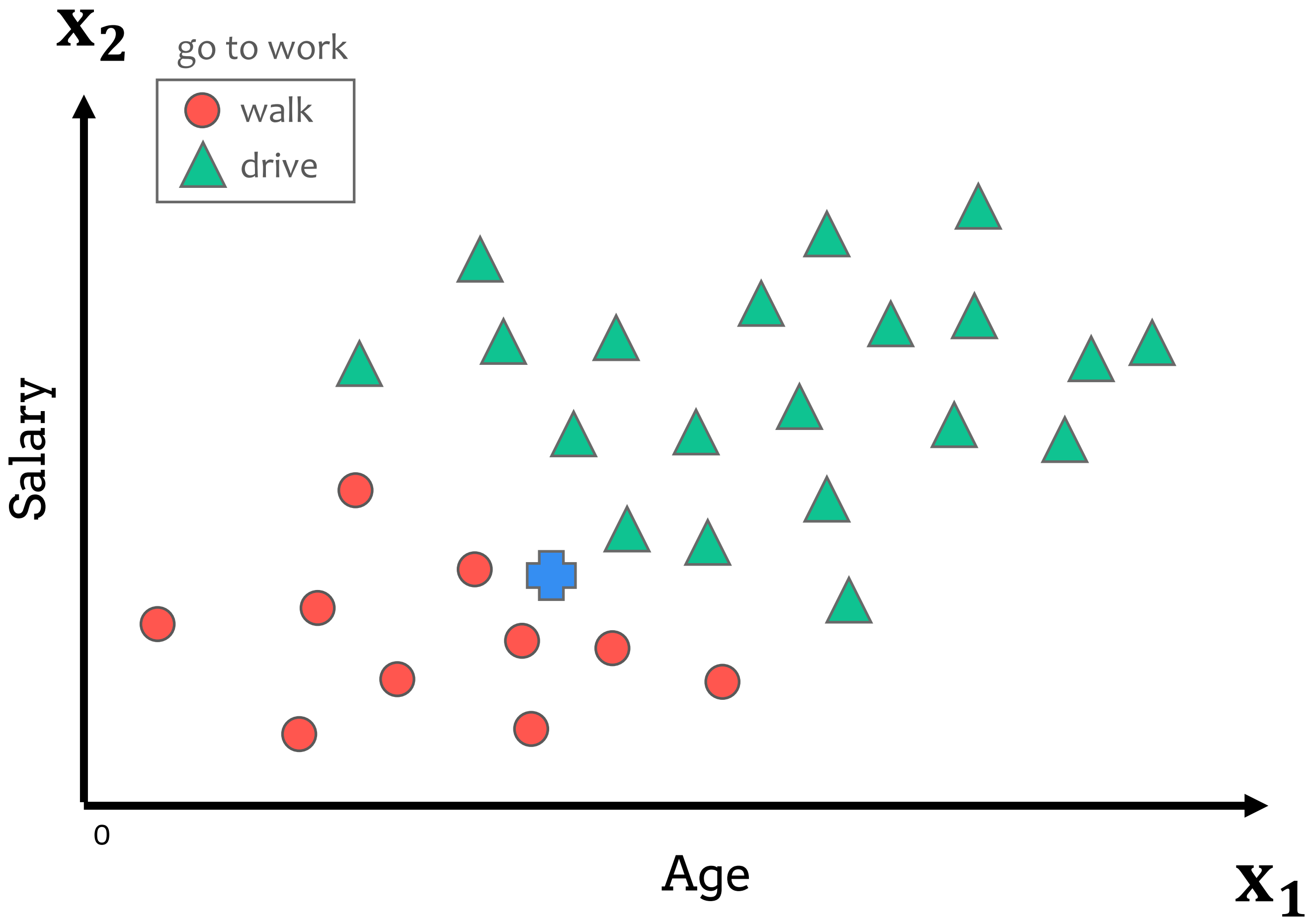
Naive Bayes: Step 1



#1 - $P(Walks)$

This is our **initial guess** that people go to work by **walking**.

Naive Bayes: Step 1

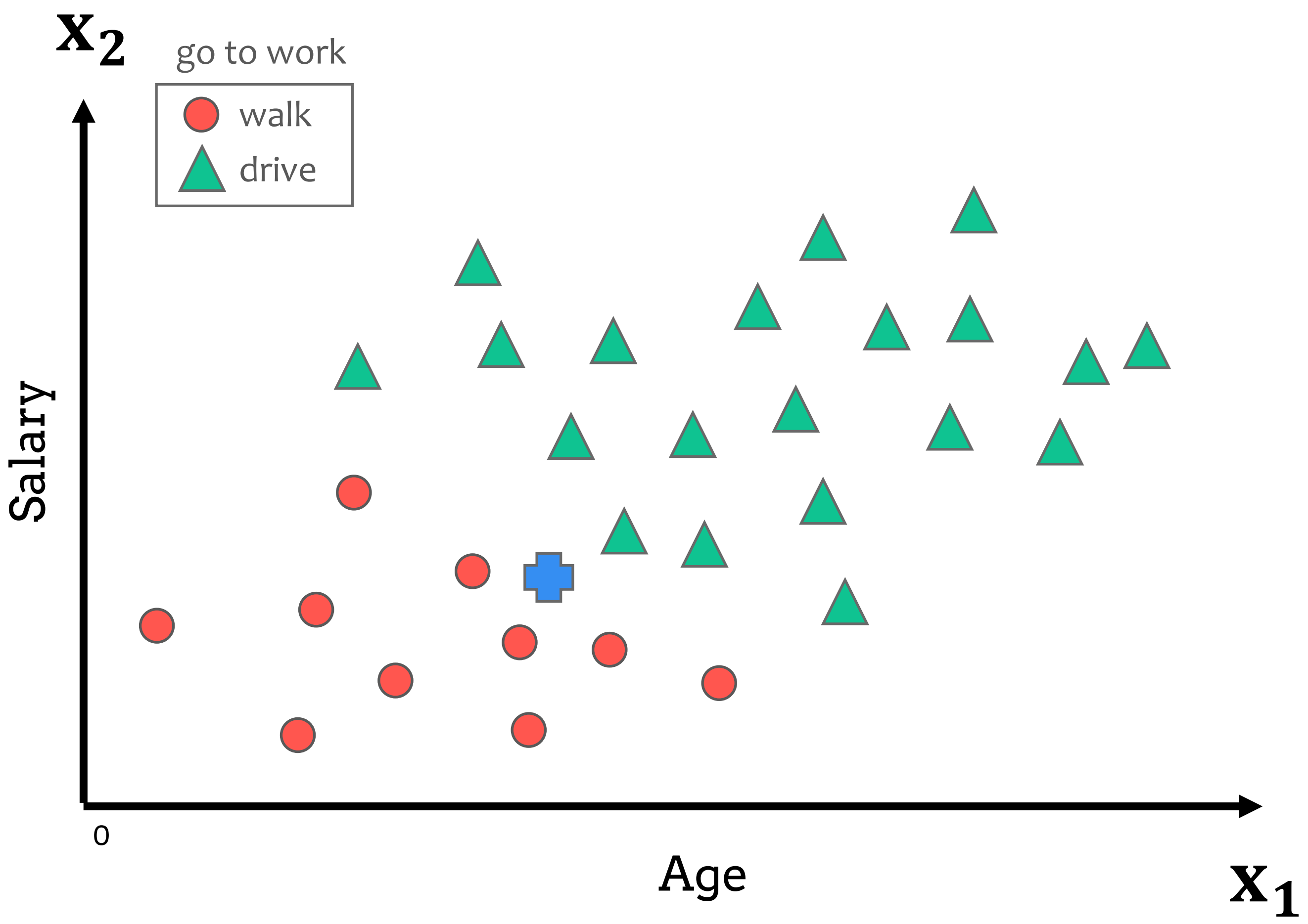


#1 - $P(Walks)$

This is our **initial guess** that people go to work by **walking**.

This can be any **probability**.

Naive Bayes: Step 1



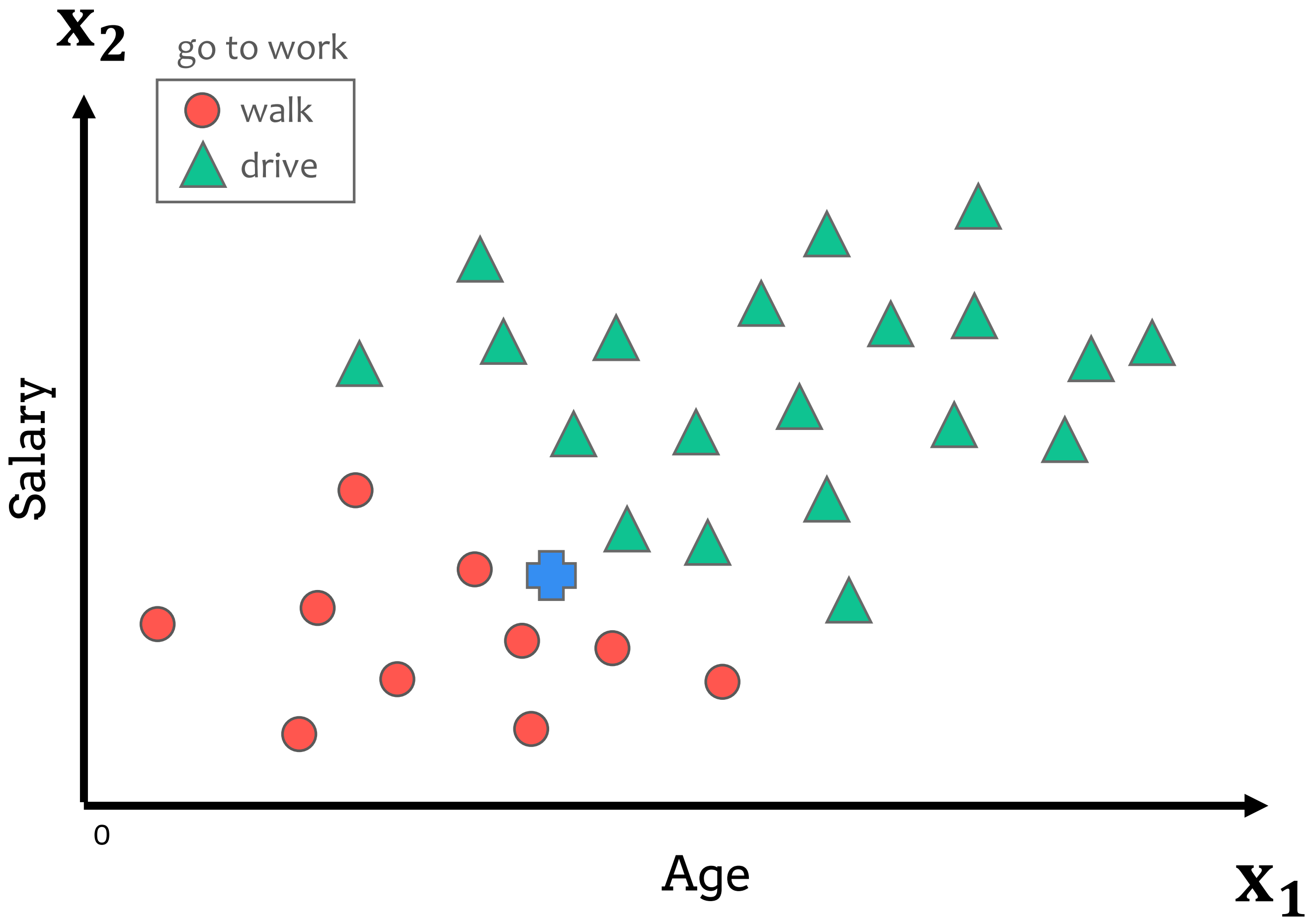
#1 - $P(Walks)$

This is our **initial guess** that people go to work by **walking**.

This can be any **probability**.

However, a **common guess** is estimated from the **training data.**

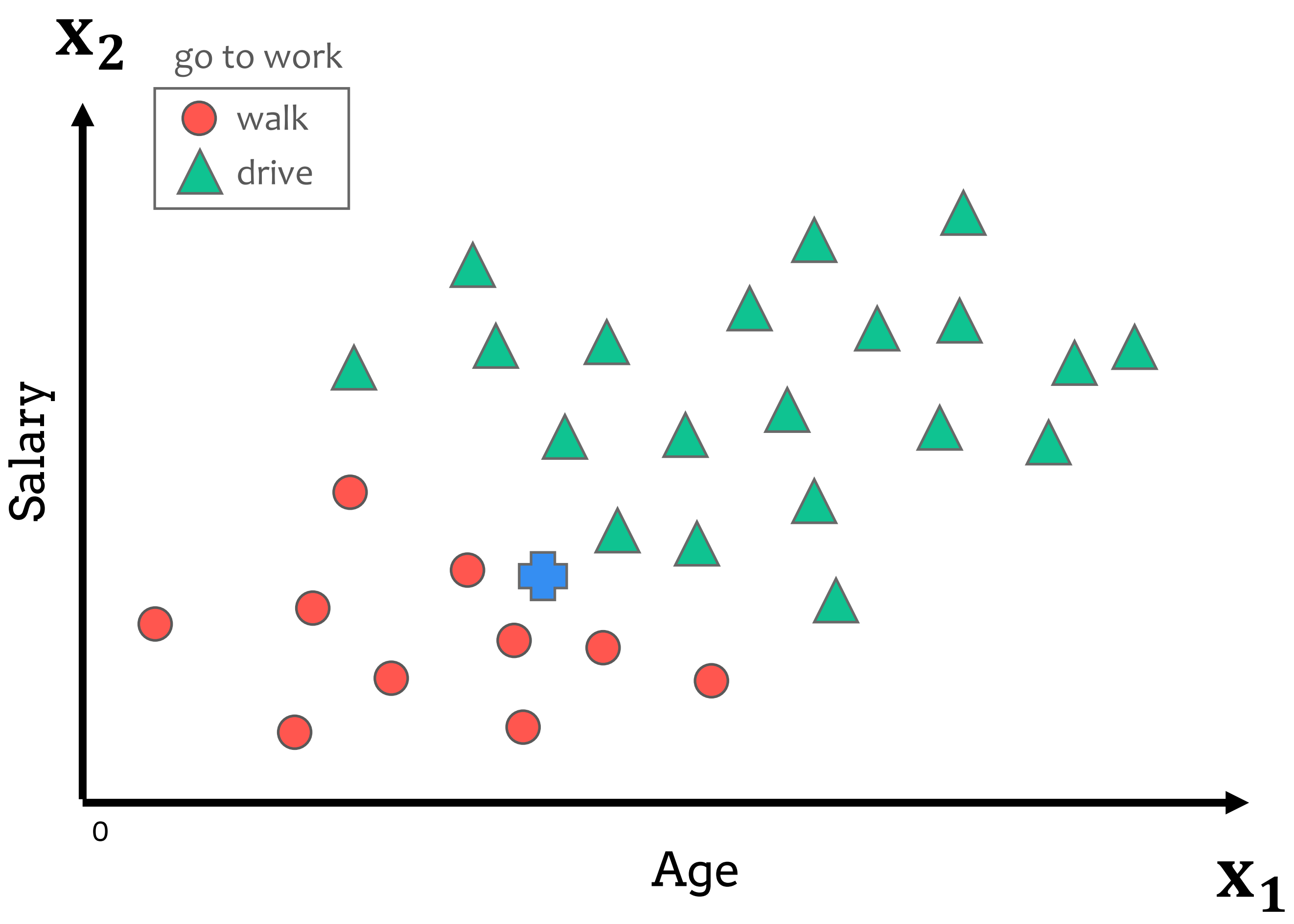
Naive Bayes: Step 1



#1 - $P(Walks)$

$$P(Walks) = \frac{Number\ of\ Walkers}{Total\ Train.Samples}$$

Naive Bayes: Step 1



#1 - $P(Walks)$

$$P(Walks) = \frac{Number\ of\ Walkers}{Total\ Train.Samples}$$

$$P(Walks) = \frac{10}{30}$$

Naive Bayes

#3

Likelihood

Prior probability #1

class

feats of the new sample

$$P(Walks|x_{test}) = \frac{P(x_{test}|Walks) * P(Walks)}{P(x_{test})}$$

#4 Posterior probability

Marginal Likelihood

#2

Naive Bayes

#3 Likelihood

Prior probability #1 (OK!)

class

feats of the new sample

$$P(Walks|x_{test}) = \frac{P(x_{test}|Walks) * P(Walks)}{P(x_{test})}$$

#4 Posterior probability

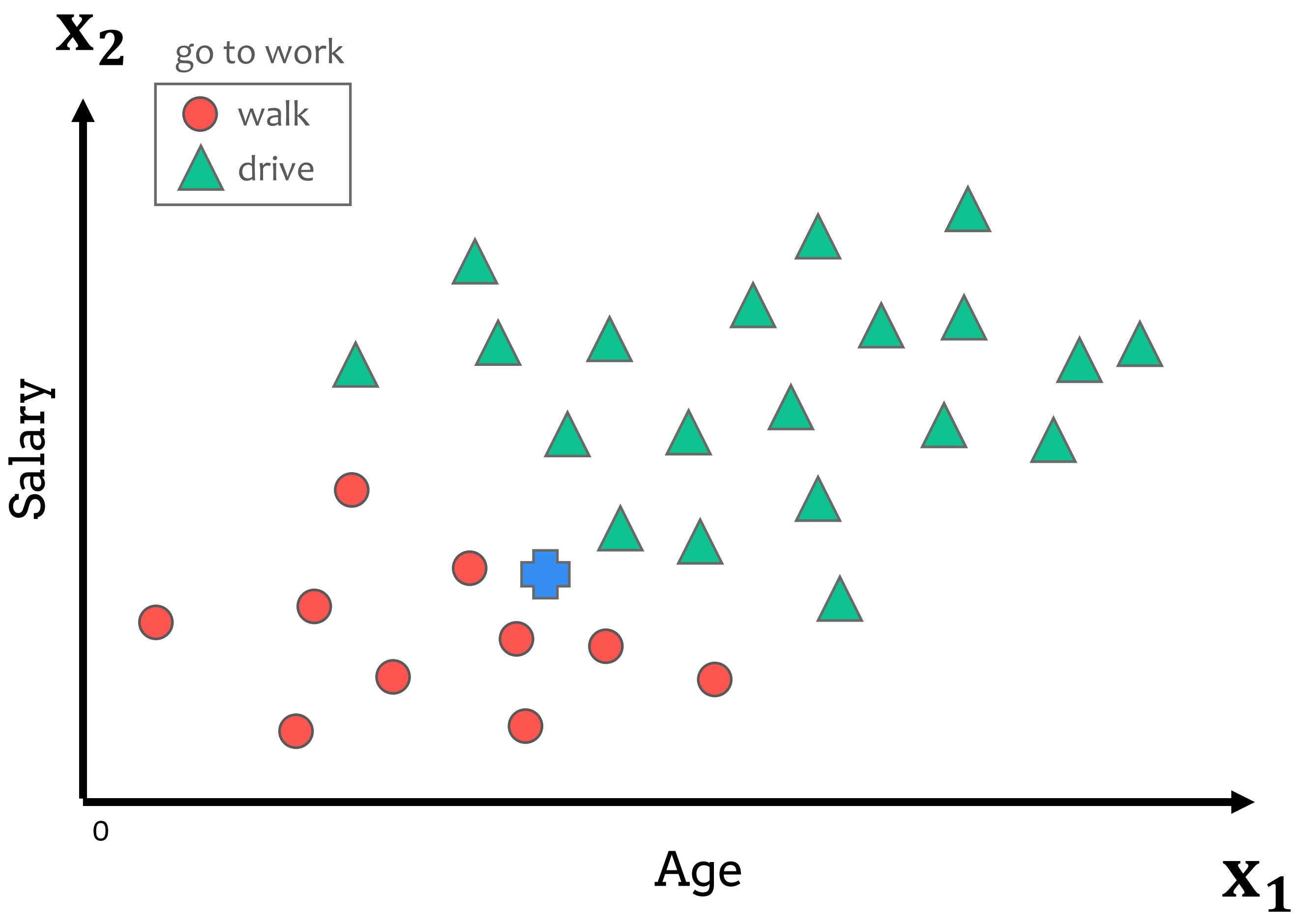
Marginal Likelihood

#2

Naive Bayes: Step 2

#2 - $P(x_{test})$

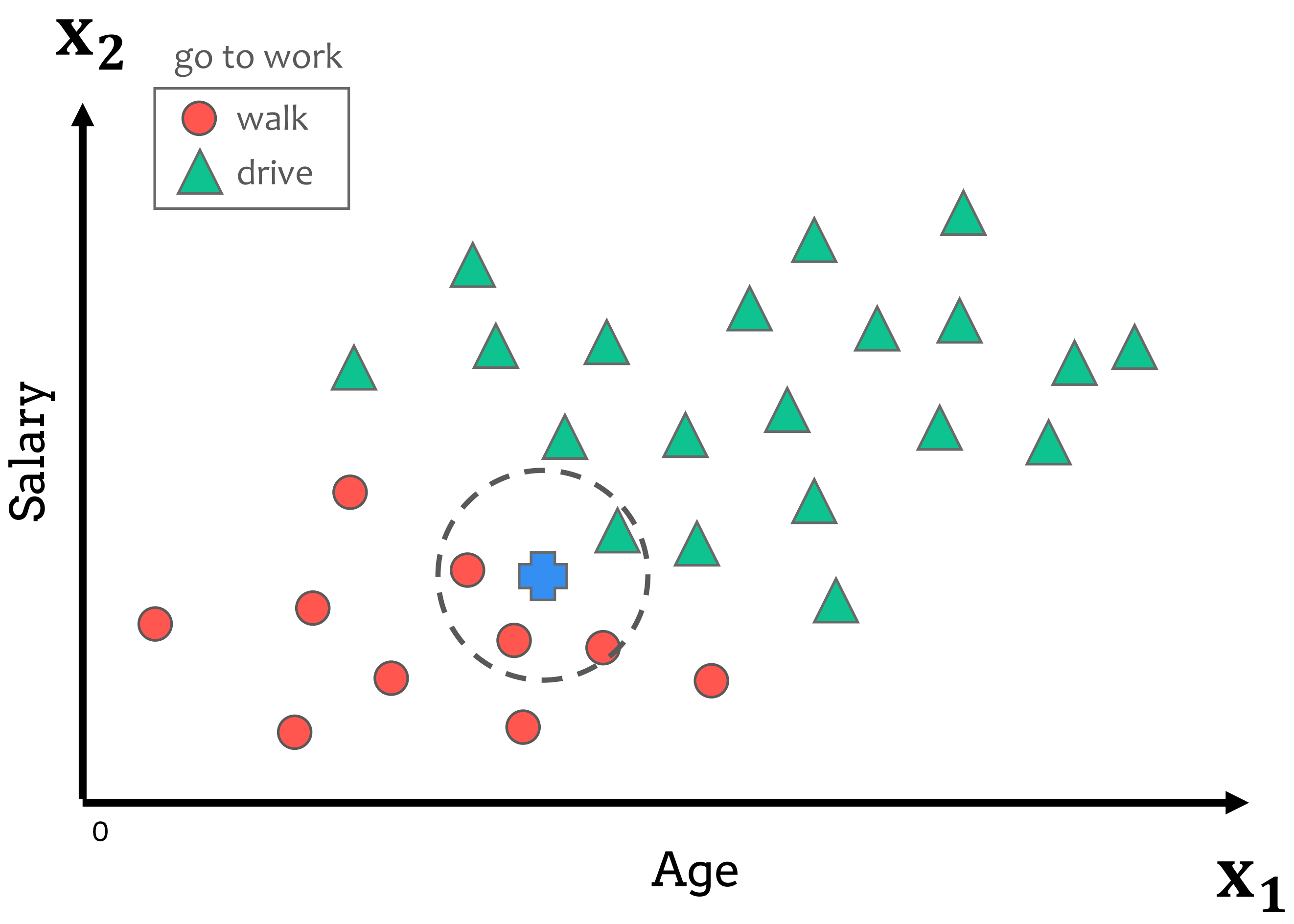
Naive Bayes: Step 2



#2 - $P(x_{test})$

What is the **(prior) probability** of a randomly selected **training sample** from our data will be similar to the **test sample**?

Naive Bayes: Step 2

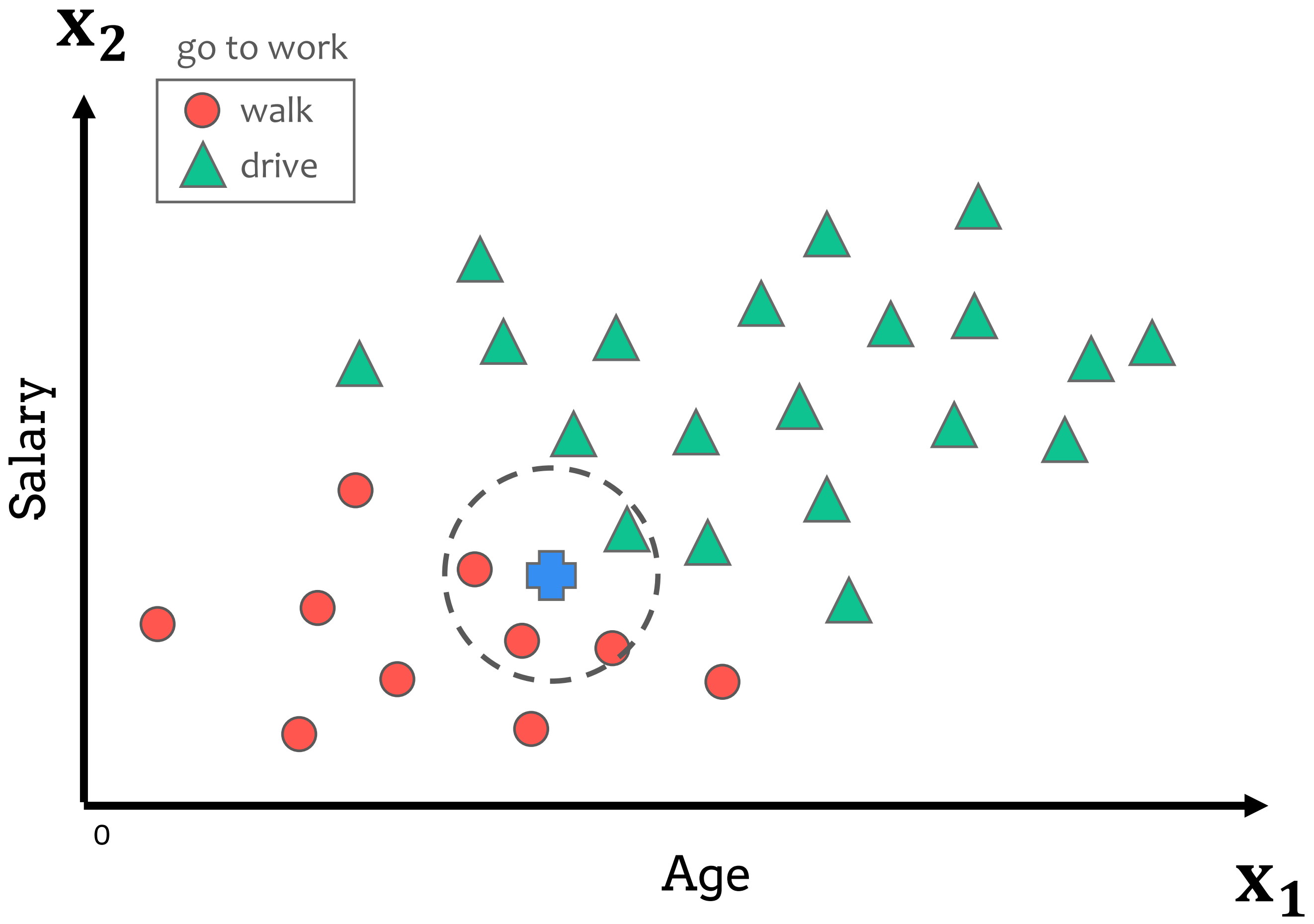


#2 - $P(x_{test})$

What is the **(prior) probability** of a randomly selected **training sample** from our data will be similar to the **test sample**?

Draw a circle (hypersphere) with a given radius around the **test sample**.
The **training samples** inside the circle are **the most similar** to the **test sample**.

Naive Bayes: Step 2



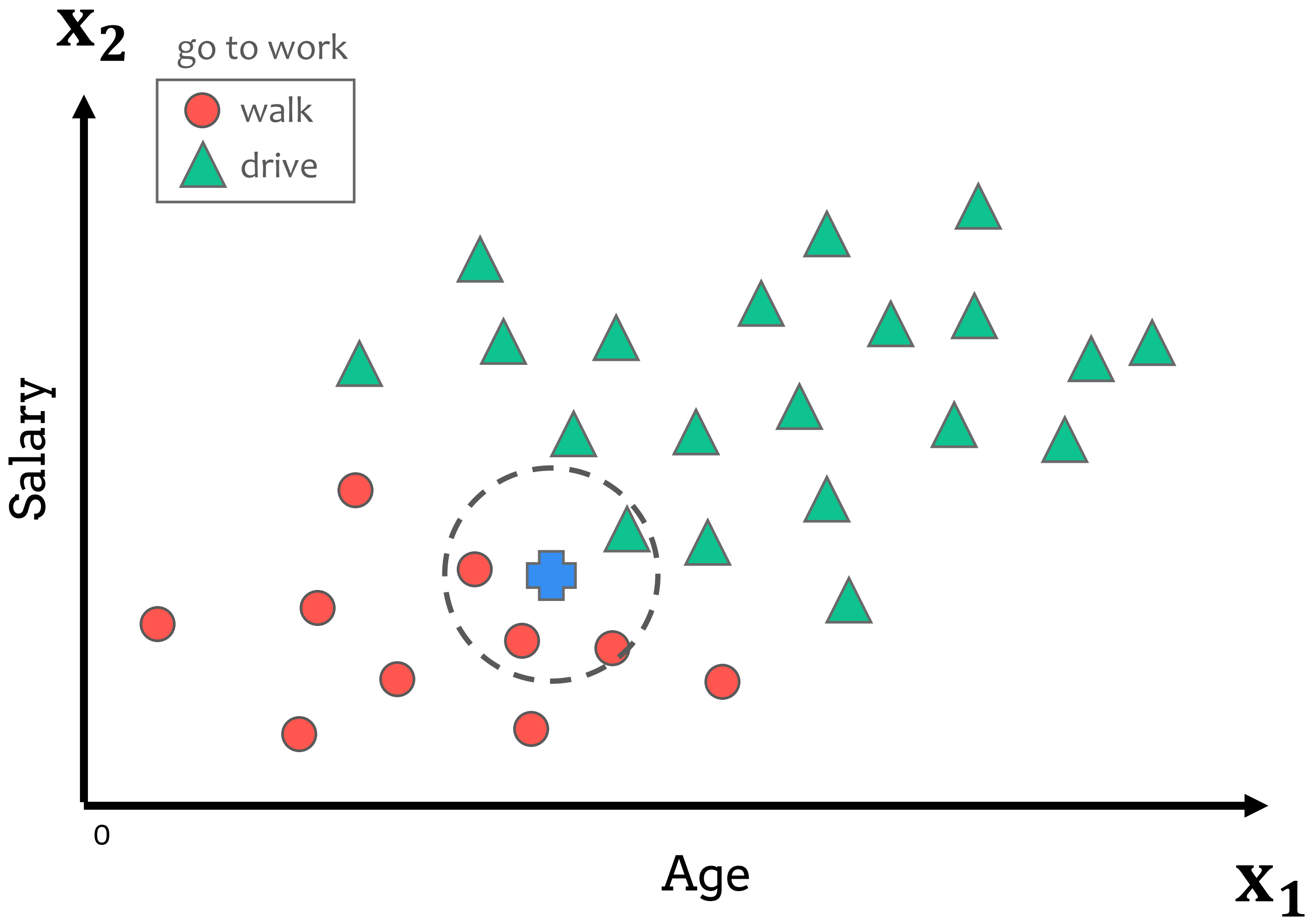
#2 - $P(x_{test})$

What is the **(prior) probability** of a randomly selected **training sample** from our data will be similar to the **test sample**?

Draw a circle (hypersphere) with a given radius around the **test sample**.
The **training samples** inside the circle are **the most similar** to the **test sample**.

Now, count the number of training samples **inside this circle** and divide it by **the total number of training samples**.

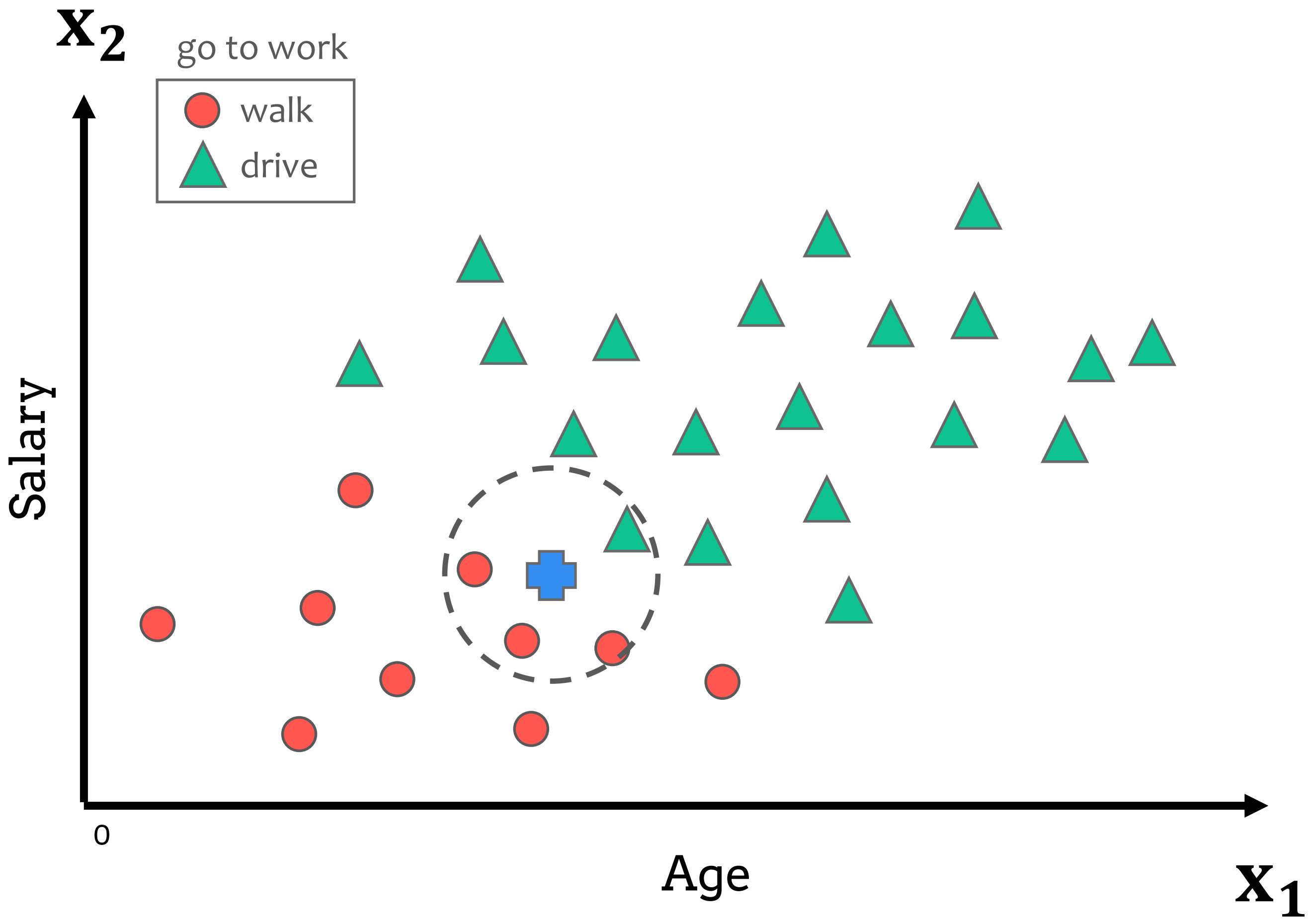
Naive Bayes: Step 2



#2 - $P(\mathbf{x}_{test})$

$$P(\mathbf{x}_{test}) = \frac{Number\ of\ Similar\ Train.Samples}{Total\ Train.Samples}$$

Naive Bayes: Step 2

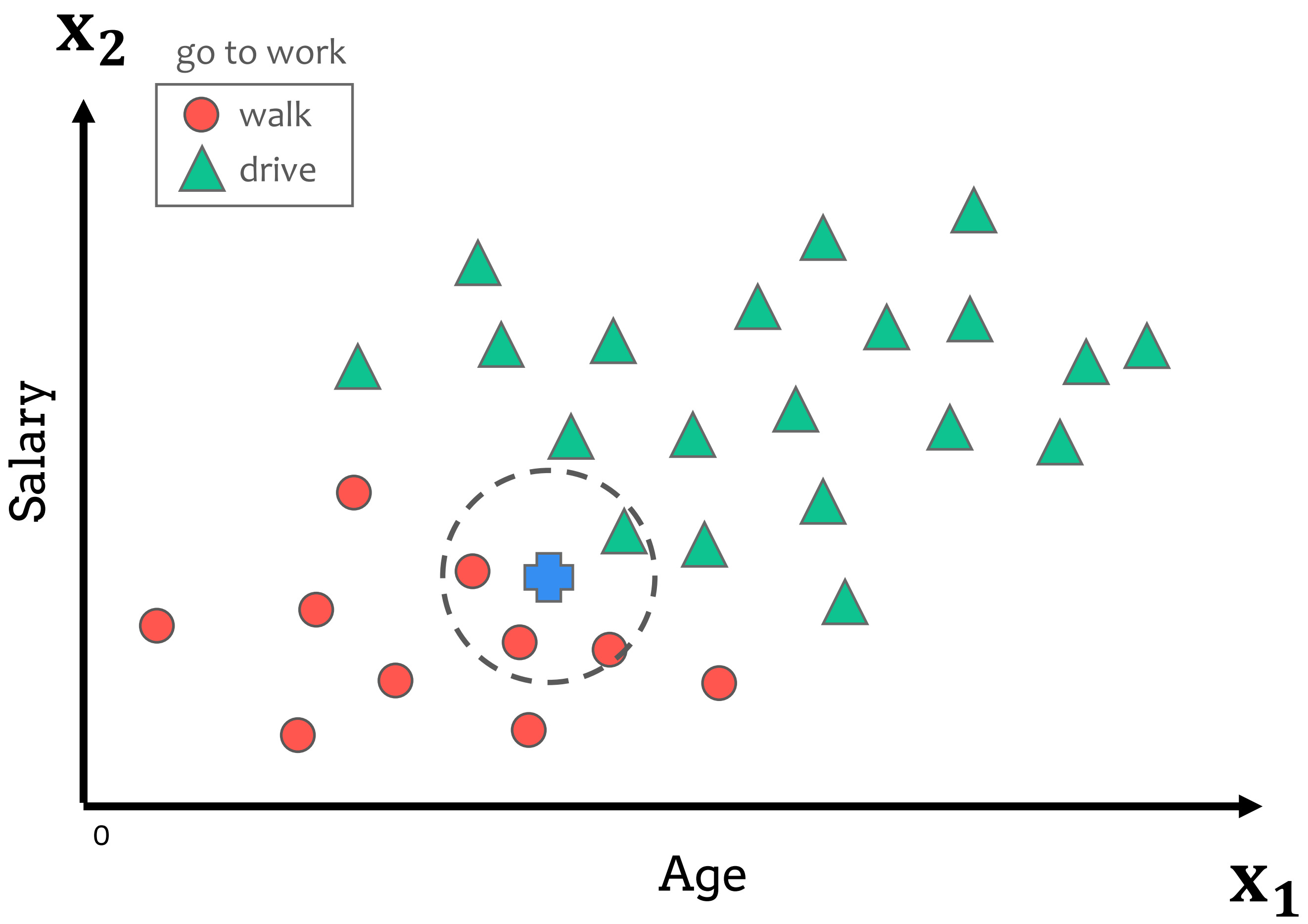


#2 - $P(\mathbf{x}_{\mathbf{test}})$

$$P(\mathbf{x}_{\mathbf{test}}) = \frac{Number\ of\ Similar\ Train.Samples}{Total\ Train.Samples}$$

$$P(\mathbf{x}_{\mathbf{test}}) = \frac{4}{30}$$

Naive Bayes: Step 2



#2 - $\mathrm{P}(\mathbf{x}_{\mathbf{test}})$

$$\mathrm{P}(\mathbf{x}_{\mathbf{test}}) = \frac{Number\ of\ Similar\ Train.Samples}{Total\ Train.Samples}$$

$$\mathrm{P}(\mathbf{x}_{\mathbf{test}}) = \frac{4}{30}$$

We'll see soon we do not need to compute this probability for classification.

Naive Bayes

#3
Likelihood

Prior probability #1 (OK!)

class

feats of the new sample

$$P(Walks|x_{test}) = \frac{P(x_{test}|Walks) * P(Walks)}{P(x_{test})}$$

#4 Posterior probability

Marginal Likelihood
#2

Naive Bayes

#3

Likelihood

Prior probability #1 (OK!)

class

feats of the new sample

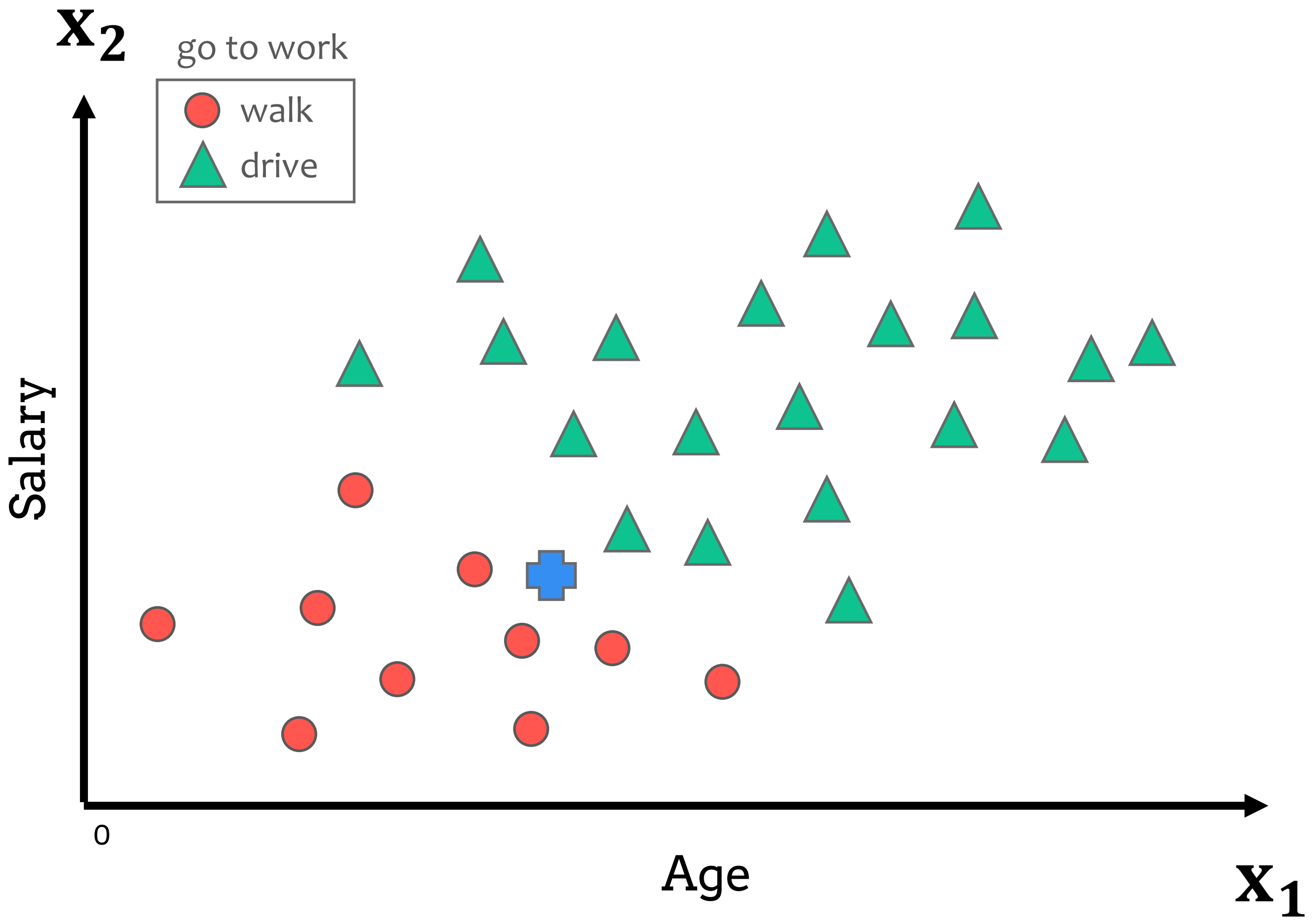
$$P(Walks|x_{test}) = \frac{P(x_{test}|Walks) * P(Walks)}{P(x_{test})}$$

#4 **Posterior probability**

Marginal Likelihood

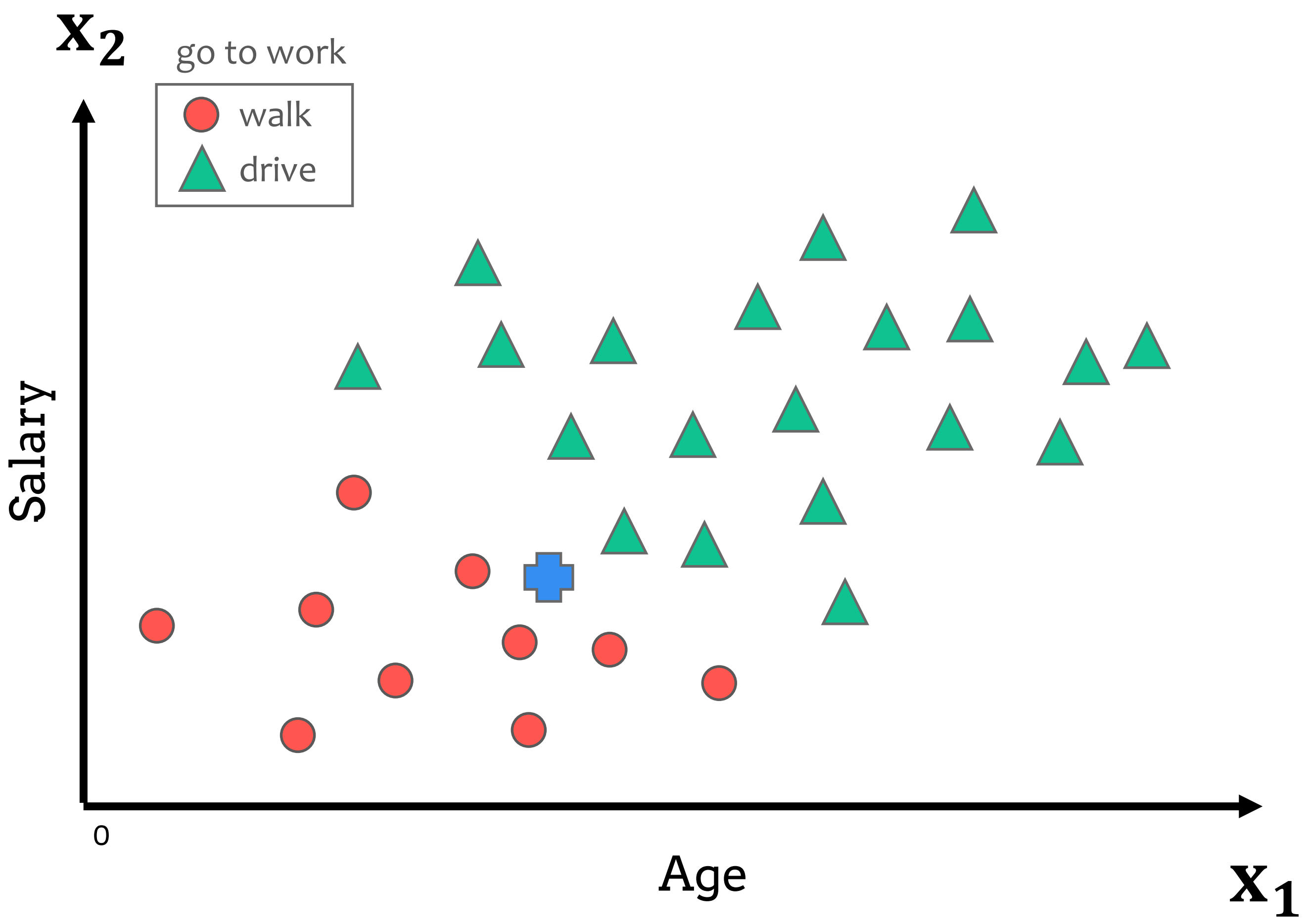
#2 (OK!)

Naive Bayes: Step 3



#3 - $\mathrm{P}(\mathbf{x}_{\mathbf{test}}|Walks)$

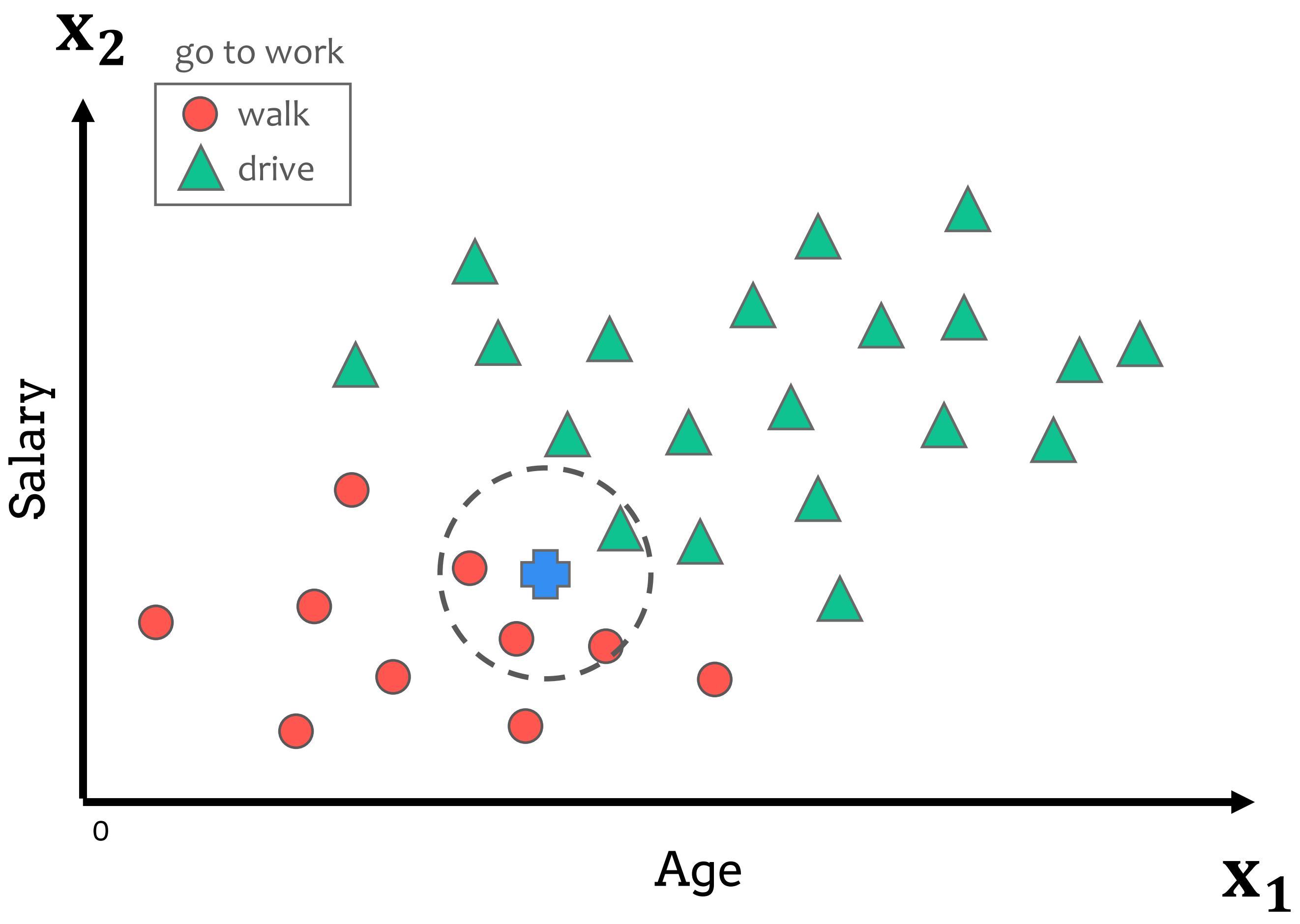
Naive Bayes: Step 3



#3 - $P(x_{test}|Walks)$

What is the **(prior) probability** of a randomly selected **training sample** from our data will be similar to the **test sample**, given it has the class *Walk*?

Naive Bayes: Step 3

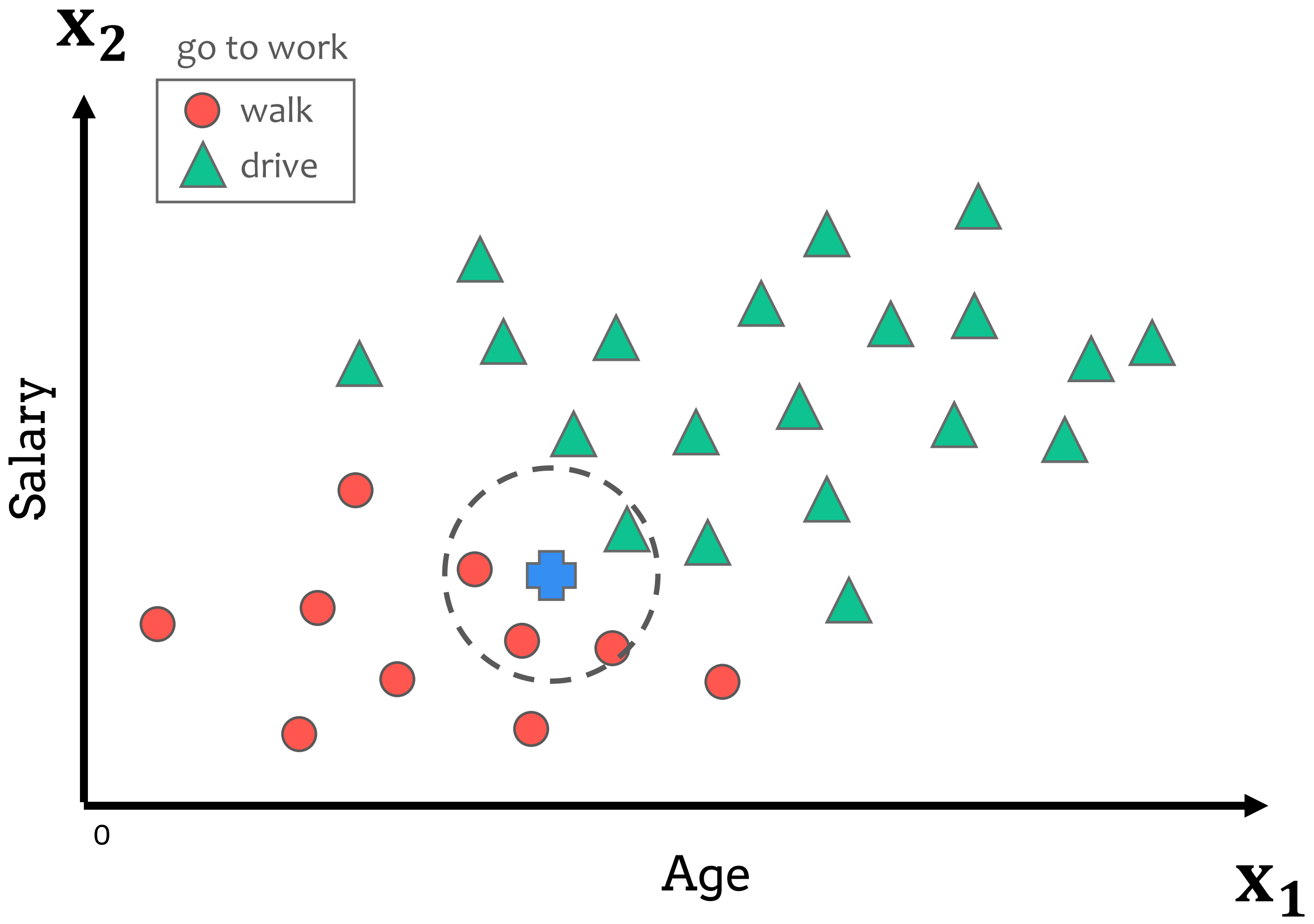


#3 - $P(\mathbf{x}_{\text{test}} | Walks)$

What is the **(prior) probability** of a randomly selected **training sample** from our data will be similar to the **test sample**, given it has the class *Walk*?

Once again, we draw a circle (hypersphere) with a given radius around the **test sample**.

Naive Bayes: Step 3



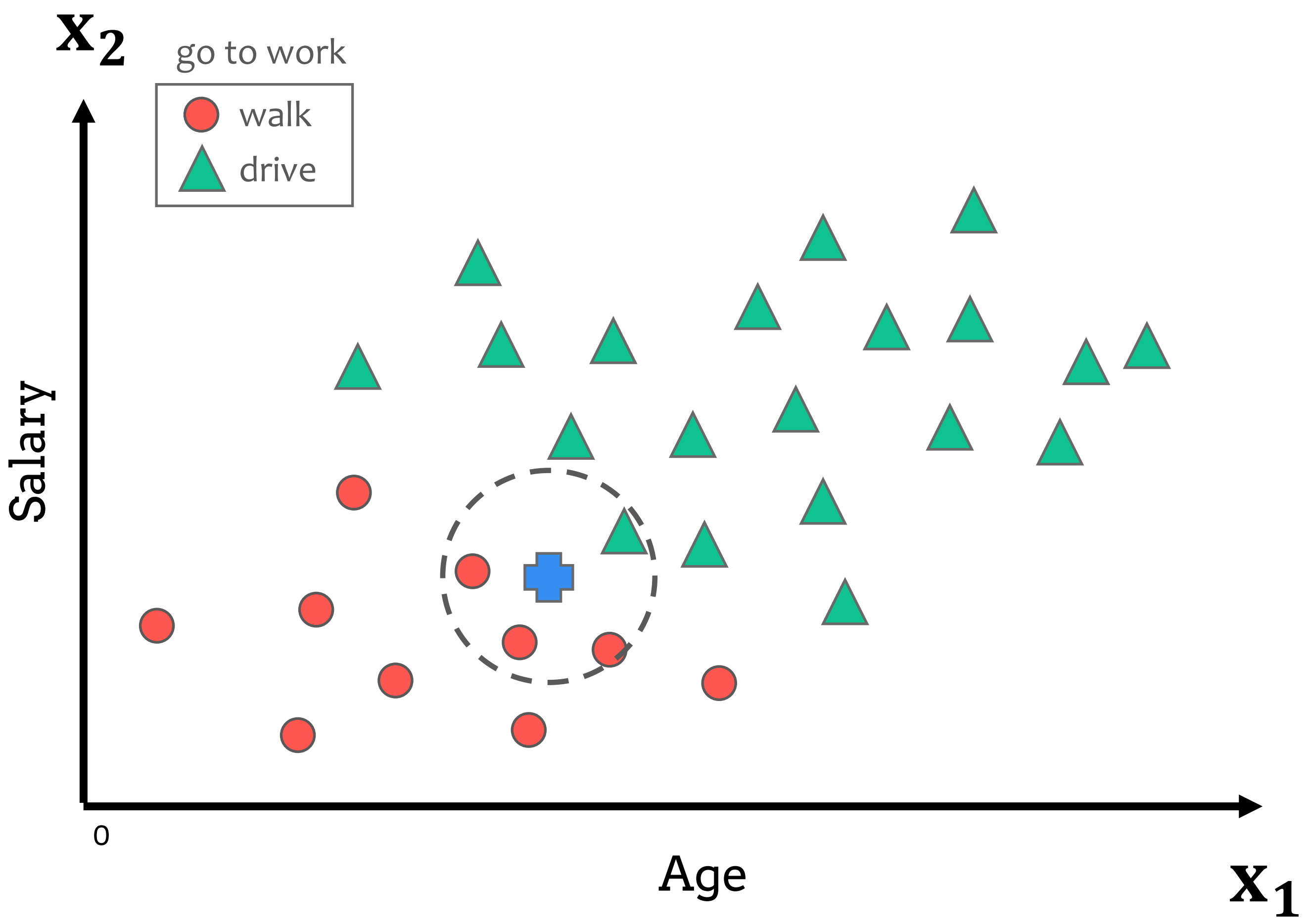
#3 - $P(x_{test}|Walks)$

What is the **(prior) probability** of a randomly selected **training sample** from our data will be similar to the **test sample**, given it has the class *Walk*?

Once again, we draw a circle (hypersphere) with a given radius around the **test sample**.

We then count the number of training samples **inside this circle** with class Walk and divide it by **the total number of training samples** with that class.

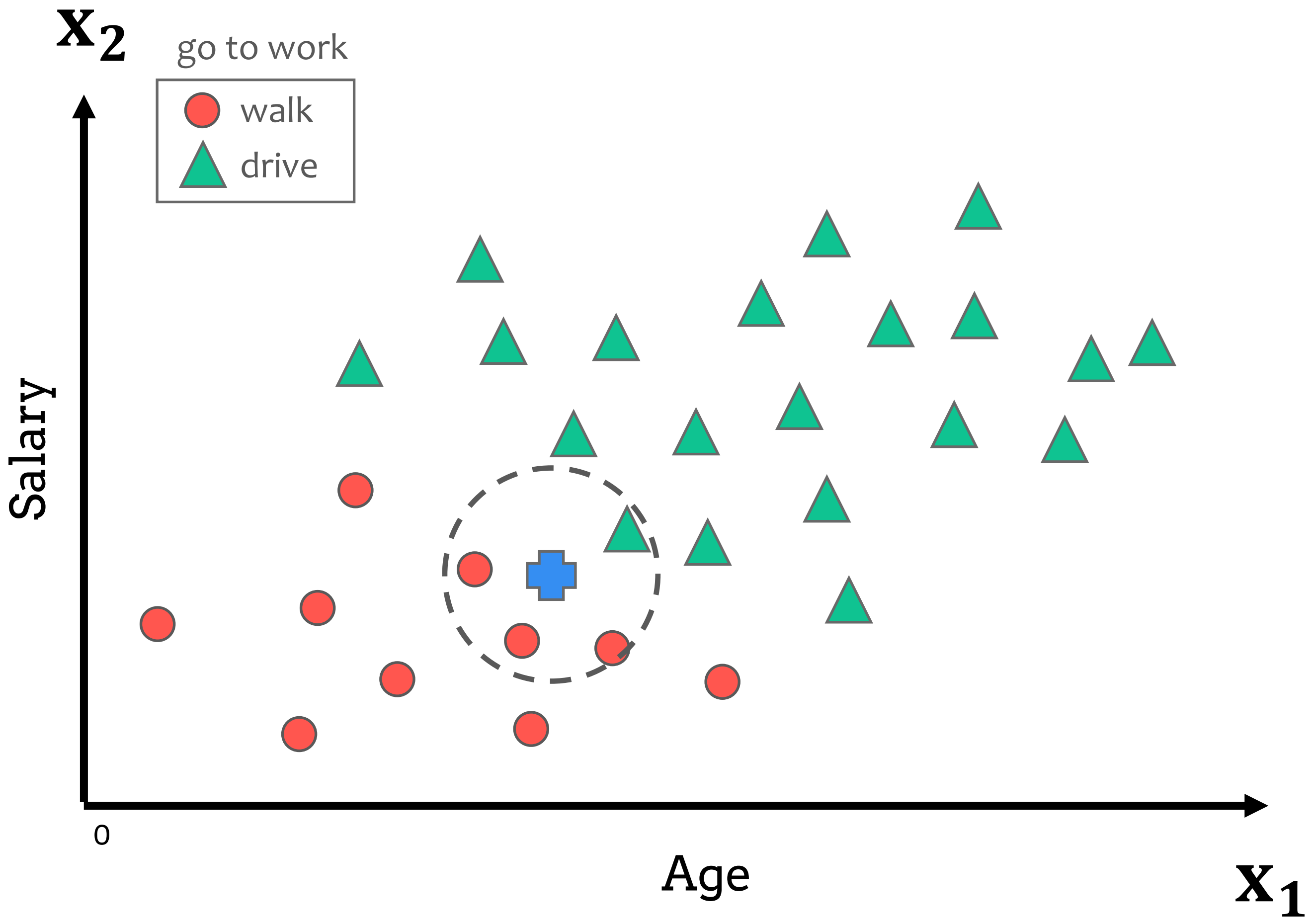
Naive Bayes: Step 3



#3 - $\mathrm{P}(\mathbf{x}_{\mathbf{test}}|Walks)$

$$\mathrm{P}(\mathbf{x}_{\mathbf{test}}|Walks) = \frac{Number\ of\ similar\ train.samples\ with\ class\ Walk}{Total\ train.\ samples\ with\ class\ Walk}$$

Naive Bayes: Step 3



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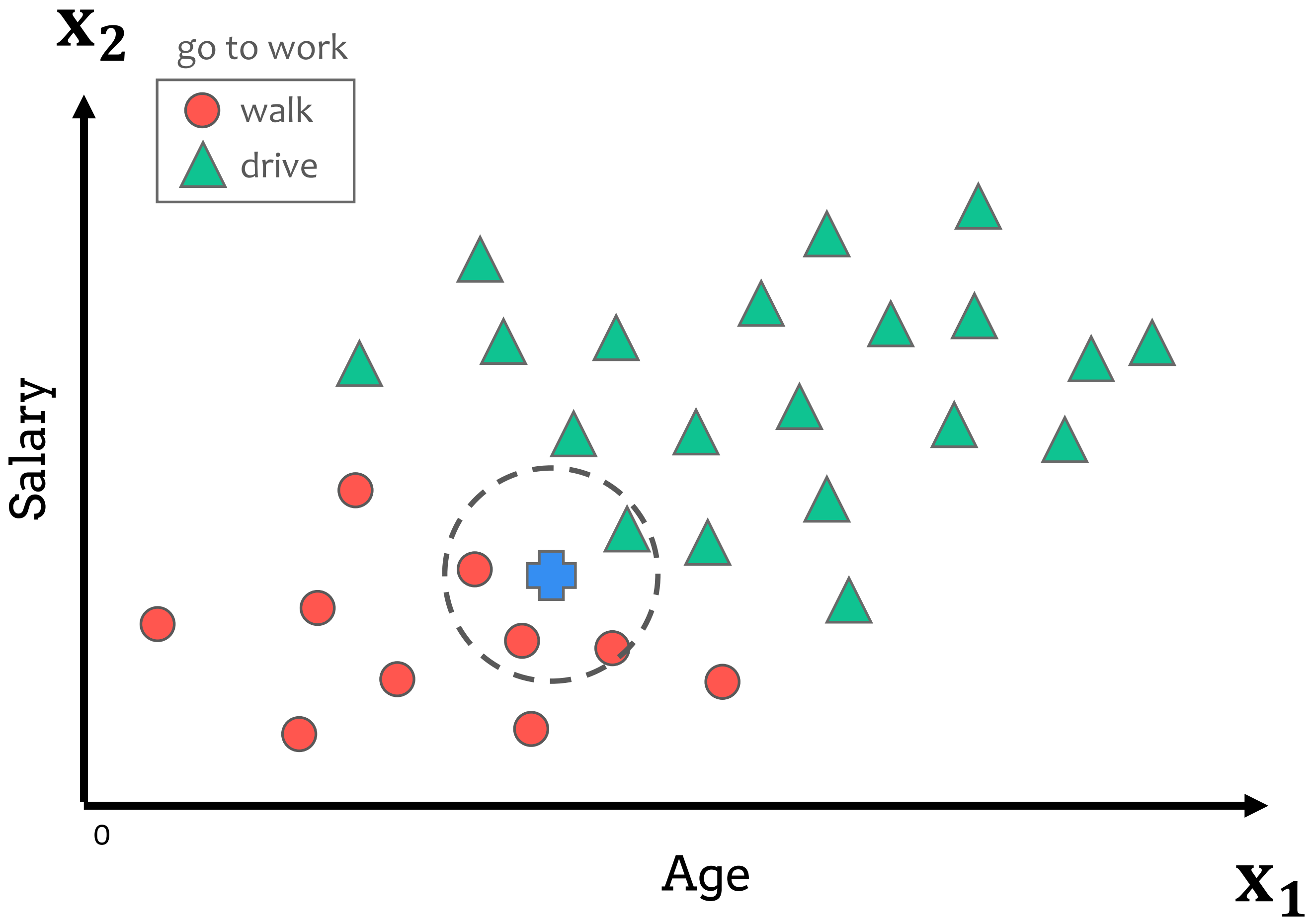

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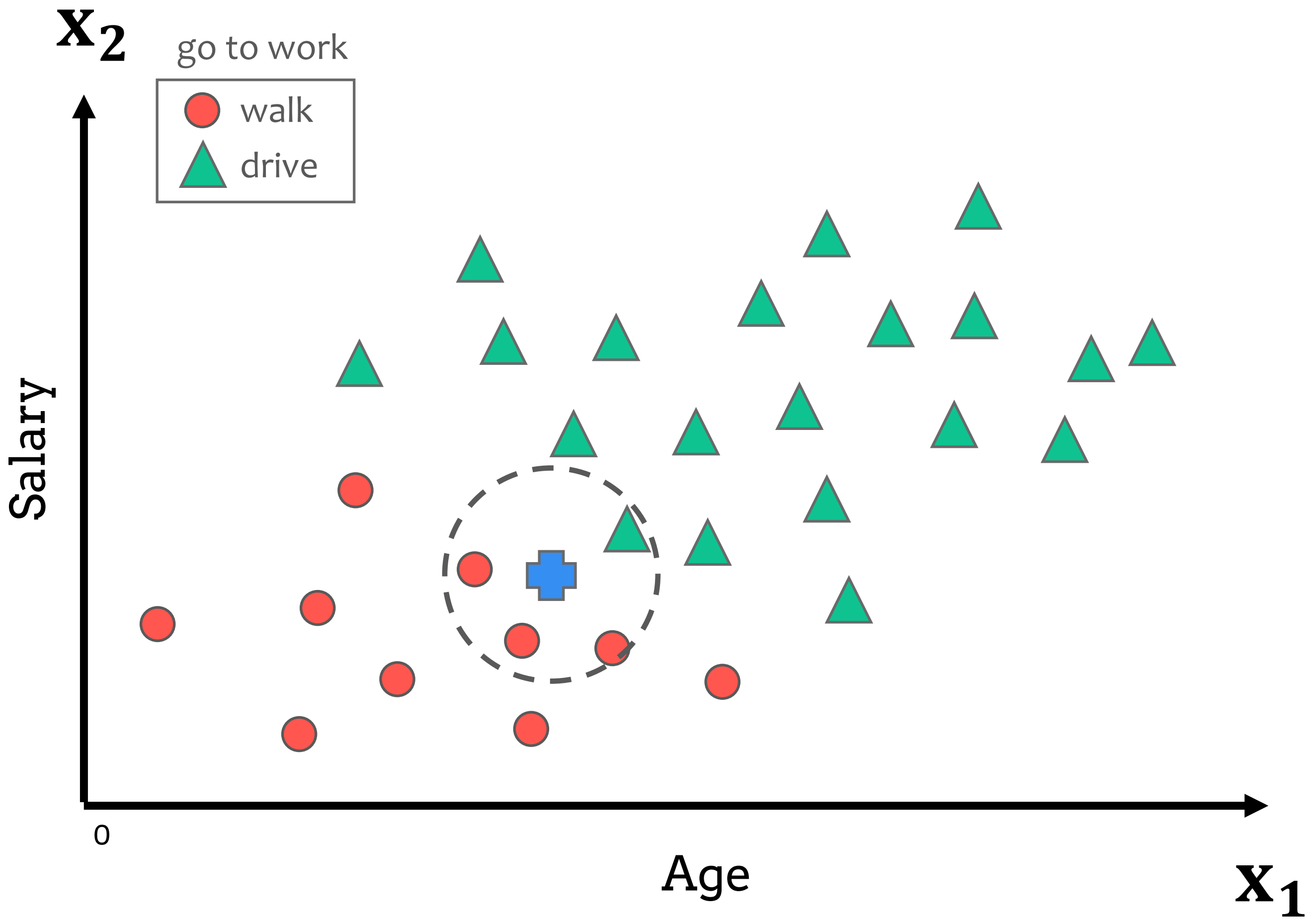
This is an **oversimplified** way to compute the **likelihood** for such a problem, being very sensitive to ***outliers***.
The common way is to assume that **our data follows a given data distribution** (*e.g.*, normal) and then use it to compute the **likelihood**.

Naive Bayes: Step 3



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We'll see soon another way of computing this **likelihood** through Gaussian Distribution.

Naive Bayes

#3

Likelihood

Prior probability **#1 (OK!)**

class feats of the new sample

$$P(\textcolor{red}{Walks}|\mathrm{x}_{test}) = \frac{P(\mathrm{x}_{test}|\textcolor{red}{Walks}) * P(\textcolor{red}{Walks})}{P(\mathrm{x}_{test})}$$

#4 **Posterior probability**

Marginal Likelihood

#2 (OK!)

Naive Bayes

#3 (OK!)

Likelihood

Prior probability #1 (OK!)

class

feats of the new sample

$$P(Walks|x_{test}) = \frac{P(x_{test}|Walks) * P(Walks)}{P(x_{test})}$$

#4 Posterior probability

Marginal Likelihood

#2 (OK!)

Naive Bayes: Step 4

$$P(Walks|x_{test}) = \frac{\frac{3}{10} * \frac{10}{30}}{\frac{4}{30}} = 0.75$$

Naive Bayes

Step 2

Now, repeat the same calculation for the other class: ***Drives***

class

feats of the new sample

#3 Likelihood

Prior probability #1

$$P(\mathit{Drives}|x_{test}) = \frac{P(x_{test}|\mathit{Drives}) * P(\mathit{Drives})}{P(x_{test})}$$

#4 Posterior probability

Marginal Likelihood

#2

Naive Bayes

$$P(Drives|\mathrm{x}_{test}) = \frac{\frac{1}{20} * \frac{20}{30}}{\frac{4}{30}} = 0.25$$

Naive Bayes

Step 3

$$P(Walks|x_{test}) \ \textbf{vs} \ P(Drives|x_{test})$$

Naive Bayes

Step 3

$$P(Walks|\mathrm{x}_{test}) \ \textbf{vs} \ P(Drives|\mathrm{x}_{test})$$

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Step 3

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Naive Bayes

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The **test sample** is classified as **Walker**.

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It considers that **all variables/features** are **mutually independent**, which is not always true.

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For example, in our problem, both features seems **to be correlated**.

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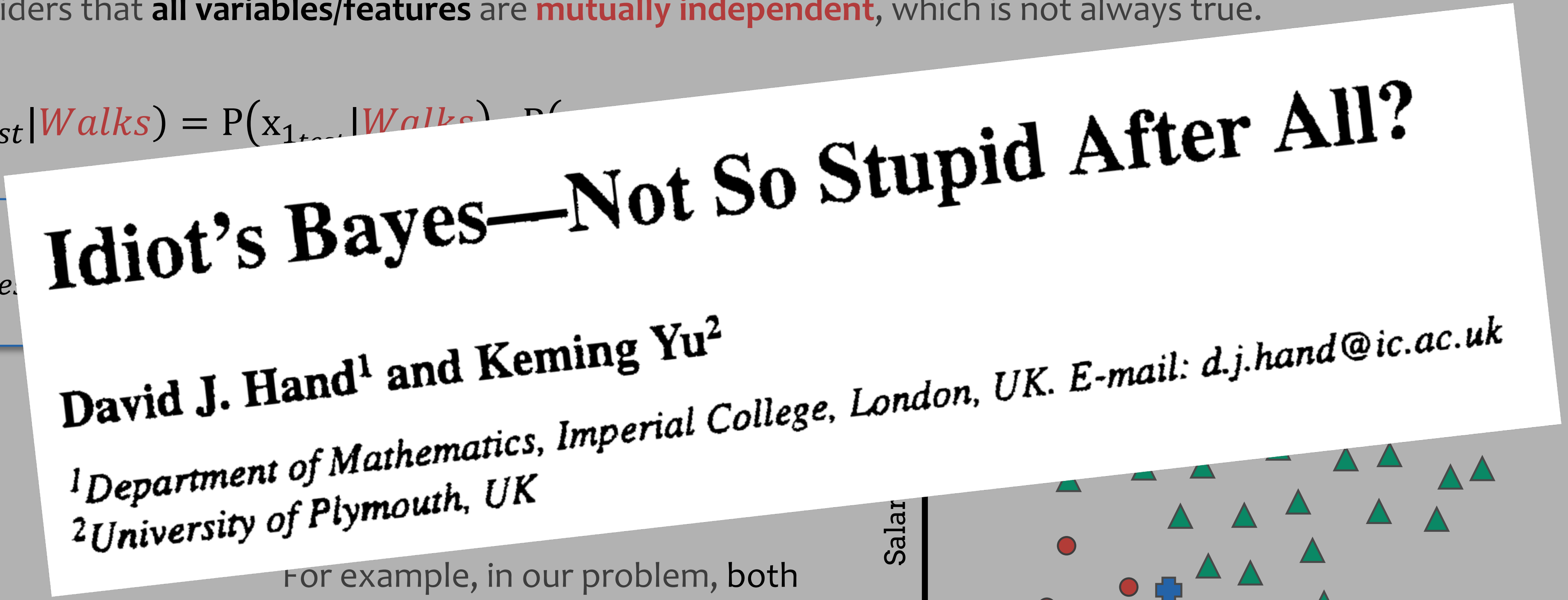
Idiot's Bayes—Not So Stupid After All?

David J. Hand[1] and Keming Yu[2]

[1]Department of Mathematics, Imperial College, London, UK. E-mail: d.j.hand@ic.ac.uk

[2]University of Plymouth, UK

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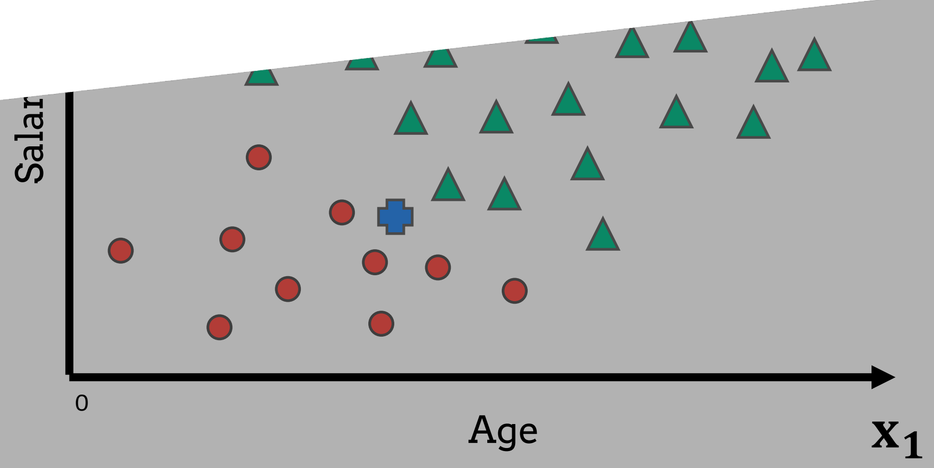

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$\mathrm{P}(Walks|\mathrm{x}_{test})$ **vs** $\mathrm{P}(Drives|\mathrm{x}_{test})$

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Aprendizado de Máquina e Reconhecimento de Padrões

Naive Bayes

Prof. Dr. Samuel Martins (Samuka)
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