Dynamic Pricing in the Presence of Social Learning and Strategic Consumers

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When a product of uncertain quality is first introduced, consumers may be enticed to strategically delay their purchasing decisions in anticipation of the product reviews of their peers. This paper investigates how the presence of social learning interacts with the adoption decisions of strategic consumers and the dynamic-pricing decisions of a monopolist firm, within a simple two-period model. When the firm commits to a price path ex ante (pre-announced pricing), we show that the presence of social learning increases the firm's ex ante expected profit, despite the fact that it exacerbates consumers' tendency to strategically delay their purchase. As opposed to following a price-skimming policy which is always optimal in the absence of social learning, we find that, for most model parameters, the firm will announce an increasing price plan. When the firm does not commit to a price path ex ante (responsive pricing), interestingly, the presence of social learning has no effect on strategic purchasing delays. Under this pricing regime, social learning remains beneficial for the firm and prices may either rise or decline over time, with the latter being ex ante more likely. Furthermore, we illustrate that contrary to results reported in existing literature, in settings characterized by social learning, price-commitment is generally not beneficial for a firm facing strategic consumers.

Key words: Bayesian social learning, strategic consumer behavior, applied game theory, dynamic pricing

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1. Introduction

The term "strategic consumer" is commonly used in the literature to describe a rational and forward-looking consumer, who makes intertemporal purchasing decisions with the goal of maximizing her utility. In its simplest form, strategic behavior may manifest as bargain-seeking behavior, whereby even if the current price of a product is lower than the customer's willingness to pay, she may delay her purchase in anticipation of a future markdown.[1] The importance of forward-looking consumer behavior in shaping firms' pricing decisions has been widely recognized by practitioners and academics alike: to defend against the negative effects of this behavior, firms are investing heavily in price-optimization algorithms (e.g., Schlosser 2004), while the literature has produced a number of managerial insights regarding how firms should adjust their approach to dynamic

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[1] See Li et al. (2013) for empirical evidence of strategic consumer behavior in the air-travel industry.

pricing (e.g., Aviv and Pazgal 2008, Cachon and Feldman 2013, Mersereau and Zhang 2012, Su 2007). Apart from pricing, the effects of strategic consumer behavior also extend to a range of other operational decisions; examples include decisions pertaining to stocking quantities (Liu and van Ryzin 2008), inventory display formats (Yin et al. 2009), the implementation of quick-response and fast-fashion practices (Cachon and Swinney 2009, 2011), the timing of new product launches (Besbes and Lobel 2013), and the employment of advance selling (YU et al. 2013a), to name but a few. Although existing research examines strategic consumer behavior from a variety of perspectives, it generally does not account for cases in which the quality of a new product is ex ante uncertain and, more importantly, for the prominent role of social learning (SL) in resolving this uncertainty.

In reality, many new product introductions are accompanied by quality uncertainty, in particular owing to the ever-increasing complexity of product features. Examples of such products include high-tech consumer electronics (e.g., smart-phones, tablets, computers), media items (e.g., movies, books), and digital products (e.g., computer software, smart-phone apps). In the post-Internet era, online platforms hosting buyer-generated product reviews offer a cheap and straightforward way of reducing quality uncertainty; learning from buyer reviews has now become an integral component of the process by which consumers make their purchasing decisions (e.g., Chevalier and Mayzlin 2006). For the consumers, learning from reviews allows for better-informed purchasing decisions, which in turn reduces the likelihood of ex post negative experiences. For the firm, the SL process can also be beneficial, for instance, by allowing for increased accuracy in forecasting future demand (e.g., Dellarocas et al. 2007). However, the ease with which the modern-day consumer can gain access to buyer reviews also gives rise to a new dimension of strategic consumer behavior: rather than experimenting with a new product themselves, consumers may be enticed to delay their purchasing decisions in anticipation of peer reviews. As a result, both the learning process (in terms of information generation) as well as the firm's performance (in terms of product adoption and profit) may be significantly hampered.

Despite the well-documented importance of managing strategic consumer behavior, our understanding of the effectiveness of alternative operational decisions in settings characterized by SL is extremely limited. In this paper, we take a first step towards developing such an understanding by considering the fundamental problem of uncapacitated dynamic pricing. In particular, we set out to address three research questions. First, how is the behavior of rational forward-looking consumers altered by the opportunity for SL from buyer reviews? More specifically, we aim to gain insight into how the SL process enters consumers' adoption decisions, and identify the relative differences in consumer behavior that result from alternative dynamic-pricing strategies. Second, given the nature of strategic consumer behavior, how should profit-maximizing firms modify their pricing decisions to accommodate the SL process? Here, we are interested in identifying the implications

of SL for the optimal implementation of dynamic pricing, and explaining the underlying drivers of these implications. Third, should firms facing strategic consumers commit to a price path ex ante or adjust prices dynamically in time? The issue of price-commitment is one that arises frequently in the strategic consumer literature. In general, the consensus is that price-commitment may prove beneficial for the firm when consumers are forward-looking (e.g., Aviv and Pazgal 2008); it is of interest to see whether and under which circumstances this finding applies to SL settings.

The model setting considered in this paper is much in the spirit of the seminal paper by Besanko and Winston (1990). There is a monopolist firm selling a new product to a fixed population of strategic consumers, over two periods. Two alternative classes of dynamic-pricing policies may be employed: the firm may either (a) announce the full price path from the beginning of the selling horizon (*pre-announced* pricing) or (b) announce only the first-period price, and delay the second-period price announcement until the beginning of the second period (*responsive* pricing).[2] Consumers are heterogeneous in their preferences for the product and make adoption decisions to maximize their expected utility. Our addition to this simple model, and the focal point of our analysis, is the introduction of ex ante quality uncertainty (faced by both the firm and consumers), which may be partially resolved in the second period by observing the product reviews of first-period buyers (SL).

Because in the presence of SL the product's quality is partially revealed in the second period, the interaction between the firm and consumers is transformed from a game whose outcome can be perfectly anticipated from the onset (in the absence of SL), to one whose outcome is of a probabilistic nature (i.e., a stochastic game). For the firm, the SL process generates demand uncertainty because first-period reviews generate an ex ante probabilistic shift in the second-period demand curve. For the consumers, the SL process generates uncertainty regarding their future assessment of the product, should they choose to delay their purchasing decision. Crucially, both the firm's pricing decisions and the consumers' adoption decisions are complicated by the fact that the generation of product information is *endogenous* to consumers' adoption decisions; for instance, if no sales occur in the first period, then no reviews are generated, and therefore nothing is learned by consumers who delay their purchasing decision.

Under either pricing regime, we show that, conditional on the firm's first-period announcement, the equilibrium in the pricing-purchasing game is unique. To distill the effects of SL on the game

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[2] Pre-announced dynamic pricing is commonly employed in practice indirectly; for instance, firms may set a regular price and offer introductory price-cuts (e.g., via promotional offers or coupons) or charge introductory price-premiums (e.g., for cinema/theater premieres). Note also that maintaining a constant price is a special case of a pre-announced price plan. On the other hand, responsive pricing (sometimes referred to as "contingent pricing") is commonly observed in online commerce; for example, Amazon.com is known to employ complex dynamic-pricing algorithms (*Marketplace* 2012).

between the firm and consumers, we compare the results of our equilibrium analysis against those of a benchmark model in which the firm and consumers remain forward-looking, but in which we "switch off" the SL process (i.e., by cutting off the firm and consumers' access to product reviews). This comparison yields three main sets of insights, which we summarize below.

In terms of the interaction of the SL process with consumers' adoption decisions, we establish the following results. Under pre-announced pricing, the opportunity to learn from product reviews in the second period increases consumers' tendency to delay their purchasing decision. As a result, fewer consumers adopt the product in the first period (relative to the absence of SL), verifying the "free-riding" effect of SL: rather than "gamble" on the performance of the new product, consumers with relatively low ex ante valuations choose to wait until their peers produce reviews. Under responsive pricing, we observe the following interesting phenomenon which stems from the firm's ability to respond to social learning outcomes by adjusting price: consumers who would have purchased early in the absence of SL now have an even stronger tendency to do so, while at the same time consumers who would have delayed their purchase now find it even more appealing to wait. The end result is quite surprising – the adoption strategy employed by consumers in equilibrium is identical to that employed in the absence of SL.

In terms of the effects of SL on the firm's pricing decisions, we identify two significant changes in the optimal implementation of dynamic pricing. When the firm employs pre-announced pricing, in the absence of SL it is always optimal to announce a decreasing price path – a practice known as price-skimming. By contrast, in the presence of SL, the firm finds it optimal to announce an increasing price plan (this is true unless consumers are highly impatient). The intuition underlying this phenomenon is associated with the firm's desire to manage strategic purchasing delays more effectively (by making consumers "pay" for future information), while at the same time extracting high rents in favorable SL scenarios (through the high second-period price). When the firm employs responsive pricing, the first-period price is lower in the presence of SL, while the second-period price is an ex ante random variable. The lower introductory price aims to entice more consumers to purchase early, thereby increasing the potency of the SL process, and allowing the firm to capitalize on its second-period pricing flexibility more effectively. In terms of the structure of the equilibrium price path, we show that the price may either rise or decline in time, with the latter being the ex ante more likely outcome of the game.

Our analysis also indicates that, despite the presence of strategic purchasing delays and the fact that strategic behavior may be amplified by the SL process (i.e., under pre-announced pricing), the overall impact of SL on expected firm profit is always positive. The beneficial effects of SL have been previously established for cases in which consumers are assumed to be non-strategic (e.g., Ifrach et al. 2013); our work generalizes this finding to the case of strategic consumers.

Our third insight pertains to which class of policies is preferred by the firm when facing strategic consumers. In the existing literature, a general finding is that responsive pricing, despite its inherent flexibility, can be sub-optimal for the firm owing to the interplay between the product's price path and the purchasing decisions of the forward-looking consumers (e.g., Aviv and Pazgal 2008, Tang 2006). Our benchmark model concurs with the optimality of pre-announced pricing policies. However, once we introduce SL into the model, our analytical results and numerical experiments indicate that this observation is reversed: in the presence of SL, the firm will prefer a responsive price plan (this is true unless consumers are highly patient and product reviews are uninformative). Furthermore, we find that the presence of SL has the beneficial effect of aligning the firm and customers' preferences for the class of policies that is employed in equilibrium by the firm: in the absence of SL the firm prefers a pre-announced price plan while consumers prefer a responsive price plan; in the presence of SL, a responsive price scheme is preferred by both. As a result, SL ensures that the class of policy selected in equilibrium by the firm is that which achieves higher total welfare.

2. Literature Review

The literature that considers strategic consumer behavior typically assumes that firms employ one of two classes of dynamic pricing policies, either (i) pre-announced or (ii) responsive pricing. For early work focusing on the implications of each of the two classes of policies, the reader is referred to Stokey (1979) and Landsberger and Meilijson (1985) for pre-announced pricing, and to Besanko and Winston (1990) for responsive pricing. Since then, both classes of policies have been used extensively to study various operational decisions; recent examples include Yin et al. (2009), who assume a pre-announced price plan to study the effects of alternative inventory display formats on firm profit, and Cachon and Swinney (2009), who assume a responsive price plan to study the firm's quantity and salvage-pricing decisions. This paper is a first attempt towards understanding the relative effectiveness of pre-announced and responsive pricing when the firm and consumers face quality uncertainty and operate in the presence of SL. As such, our model and analysis are much in the spirit of Landsberger and Meilijson (1985) and Besanko and Winston (1990), in that our focus is on highlighting the effects of SL within a simple model of the interactions between the firm and the consumer population. For each class of policies, we find that the implications of SL are non-obvious, in terms of both consumer behavior as well as firm pricing decisions.

A question of particular interest in our work is which class of policies (i.e., pre-announced or responsive) is preferred by the firm. Responsive price plans generate value because they allow the firm to react optimally to updated information (e.g., demand forecasts, leftover inventory; see Elmaghraby and Keskinocak (2003)). However, when consumers are forward-looking, responsive

pricing may also have adverse effects owing to the interplay between the product's price path and consumers' adoption decisions, as epitomized by the well-known Coase conjecture (Coase 1972). In fact, the general consensus in the literature is that a firm facing strategic consumers will prefer a pre-announced policy (see Cachon and Swinney (2009) for a notable exception). In a multi-period fixed-quantity setting, Dasu and Tong (2010) provide an upper bound for expected revenues under pre-announced and responsive pricing schemes, and observe that a pre-announced price plan with a small number of price changes performs nearly optimally. In a newsvendor model with strategic consumers, Su and Zhang (2008) argue that an endogenous salvage price (i.e., responsive pricing) amplifies consumers' incentive to delay their purchase until the salvage period. Aviv and Pazgal (2008) compare pre-announced and responsive discounts under a more detailed consumer arrival process and find that the firm will generally prefer a pre-announced strategy. In the aforementioned papers, it is assumed that consumers are socially isolated, or equivalently, that no reason exists for consumer interactions to be relevant (e.g., if product quality is known with certainty, then SL as described in this paper is clearly redundant). The model we develop agrees with the consensus (i.e., that pre-announced pricing is optimal for the firm) for the benchmark case in which the firm and consumers operate in the absence of SL. However, interestingly, we find that the equation changes dramatically when SL is accounted for: in the majority of cases, the firm's preference will be reversed from a pre-announced price plan (in the absence of SL) to a responsive price plan (in the presence of SL).

Apart from its contribution to the strategic consumer literature, this paper also adds to a growing stream of literature on "social operations management," which investigates the implications of social interactions between consumers for profit-maximizing firms. Hu et al. (2013) consider a firm selling two substitutable products to a stream of consumers who arrive sequentially and whose purchasing decisions can be influenced by earlier purchases. Candogan et al. (2012) and Hu and Wang (2013) study optimal pricing strategies in social networks with positive externalities. Tereyağoğlu and Veeraraghavan (2012) consider a setting in which consumers may use their purchases to display their social status. The type of social interaction considered in this paper is different: here, consumers interact with each other through buyer reviews with the goal of learning the unobservable quality of a new product; in this respect, our work connects to the SL literature, which we discuss next.

In the SL literature, customers are generally assumed to arrive at the firm sequentially and make once-and-for-all purchasing decisions; in other words, this work typically does not account for strategic consumer behavior. The seminal papers by Banerjee (1992) and Bikhchandani et al. (1992) illustrate that when the actions (e.g., adoption decisions) of the first few agents (e.g., consumers) reveal their private information regarding some unobservable state of the world (e.g., product

quality), subsequent consumers may disregard their own private information and simply mimic the decision of their predecessor. Bose et al. (2006, 2008) illustrate how a monopolist employing dynamic pricing can use its pricing decision to control the amount of information that can be inferred by future consumers from the purchasing decision of the current consumer. Perhaps more relevant to the post-Internet era are models in which SL occurs based on reviews which reveal ex post consumer experiences (as is the case in our model), rather than actions which reveal ex ante private information. Ifrach et al. (2013) study monopoly pricing when consumers report whether their ex post derived utility was positive or negative. Papanastasiou et al. (2013) focus on the implications of SL from buyer reviews on the quantity released by a monopolist during a new product's launch phase. Bergemann and Välimäki (1997) analyze the diffusion of a new product in a duopolistic market when consumers and the firm learn the product's uncertain value from the experiences of previous product adopters. Importantly, the above work does not account for the fact that consumers may initially decide not to purchase the product for strategic reasons (i.e., in order to gain information from product reviews), knowing that they can revisit their decision at a later point in time. By contrast, YU et al. (2013b) allow for such consumer behavior and investigate the firm's optimal responsive pricing strategy. Their equilibrium analysis suggests that the firm may in fact be worse off in the presence of SL, due to the effects of strategic consumer behavior. The focus of our paper is more on highlighting the relative differences between pre-announced and responsive pricing. Furthermore, our approach to modeling the SL process differs substantially and, as a result, so do our insights, even for the particular case of responsive pricing (see §3 and §5.3 for discussions); for instance, we find that despite strategic consumer behavior, the firm is always (ex ante) better off in the presence of SL.

Finally, we note that consumers in our model face uncertainty regarding the intrinsic quality of a new and innovative product but are fully informed about their idiosyncratic preferences. However, consumers in other settings may initially be uninformed about their idiosyncratic preferences for a specific product, and learn these preferences over time (e.g., a traveller may initially be uncertain about his preferences for a ticket on a specific date of travel, but become informed as the date approaches). Since each consumer's preferences may depend on various exogenous factors, extant work has often assumed that this type of uncertainty is resolved exogenously in time. DeGraba (1995) demonstrates that a monopolist may use supply shortages to induce a buying frenzy among uninformed consumers (see also Courty and Nasiry (2013) for a dynamic model of frenzies). Swinney (2011) finds that when consumers learn their preferences over time, the value of quick-response production practices is generally diminished as a result of forward-looking consumer behavior. YU et al. (2013a) and Prasad et al. (2011) investigate whether and how retailers should employ advance selling to uninformed consumers. An exception to the exogenous revelation of preferences

assumed in the above papers is Jing (2011), who considers a responsive pricing problem in which consumers are more likely to become informed about their preferences as the number of early adopters increases. The latter paper shares a common feature with our work, in that the generation of information is endogenous to the adoption decisions of the consumer population; however, the setting we consider differs significantly (quality uncertainty faced by both consumers and firm, as opposed to preference uncertainty faced only by consumers), and therefore so do our model insights (details in §5.3).

3. Model Description

We consider a single firm selling a new product of ex ante uncertain quality, over two periods. The market consists of a continuum of consumers with total mass M normalized to one, and each customer demands at most one unit of the product during the course of the selling season. Customer i's gross utility from purchasing the product comprises two components: a *preference* component, x_i, and a *quality* component, q_i (e.g., Villas-Boas 2004, Li and Hitt 2008). The value of the preference component x_i reflects the customer's idiosyncratic preferences over the product's ex ante observable characteristics (e.g., brand, color). We assume that preference components, x_i, are distributed in the population according to the uniform distribution $U[0,1]$. (The uniform assumption has no significant bearing on our results, but simplifies analysis and exposition.) The quality component q_i represents the product's quality for customer i, which is ex ante uncertain; customers learn the value of q_i only after they purchase and experience the product. We assume that the distribution of ex post quality perceptions in the population is normal, $q_i \sim N(\hat{q}, \sigma_q^2)$, where $\hat{q}$ is the product's *unobservable* mean quality (henceforth referred to simply as product quality) and σ_q captures the degree of heterogeneity in post-purchase quality perceptions (relatively more "niche" products are typically associated with larger σ_q; see Sun (2012)).[3] The wealth-equivalent net benefit of purchasing the product for customer i in period t, $t \in \{1, 2\}$, is defined by $u_{it} = \delta_c^{t-1}(x_i + q_i - p_t)$, where p_t is the price of the product in period t and δ_c $(0 \le \delta_c \le 1)$ is a discount factor that applies to second-period purchases and represents the opportunity cost of first-period product usage. Parameter δ_c may also be interpreted as a measure of customers' patience and therefore as a measure of how "strategic" consumers are (Cachon and Swinney 2009). Throughout our analysis, we say that customers are "myopic" when $\delta_c = 0$, and "infinitely patient" when $\delta_c = 1$.

The product's unobservable quality, $\hat{q}$, is the object of social learning (SL). We assume a symmetric informational structure between the firm and consumers: both parties share a common

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[3] We assume that an individual customer's x_i and q_i components are conditionally independent for simplicity in exposition. Such dependance can be incorporated in our model without changing our model insights: ex post quality perceptions (and therefore product reviews) will be biased by the idiosyncratic preferences of the reviewers, however, rational Bayesian social learners can readily account for this bias provided knowledge of the preference distribution (see Papanastasiou et al. 2013).

and public prior belief over $\hat{q}$.[4] This belief is expressed in our model through the Normal random variable $\tilde{q}_p$, $\tilde{q}_p \sim N(q_p, \sigma_p^2)$, where we fix $q_p = 0$ without loss of generality. All customers who purchase the product in the first period report their ex post derived product quality, q_i, to the rest of the market through product reviews (e.g., via an online reviewing platform).[5] In the beginning of the second period, both the firm and consumers observe the reviews of first-period buyers and update their common belief over the product's mean quality from $\tilde{q}_p$ to $\tilde{q}_u$ according to Bayes' rule.[6] Specifically, if a mass of n_1 customers purchase and review the product in the first period, and the average rating of these reviews is R, then the updated belief, $\tilde{q}_u$, is normally distributed, $\tilde{q}_u \sim N(q_u, \sigma_u^2)$, with mean

$$q_u = \frac{n_1 \gamma}{n_1 \gamma + 1} R, \text{ where } \gamma = \frac{\sigma_p^2}{\sigma_q^2}. \tag{1}$$

(The variance of the posterior belief is given by $\sigma_u^2 = \frac{\sigma_p^2}{n_1 \gamma + 1}$.) The posterior mean q_u is a weighted average between the prior mean $q_p = 0$ and the average rating from first-period reviews R. The weight placed by consumers on R increases with the mass of reviews n_1 (henceforth referred to more naturally as the "number of reviews") and with the ratio γ.[7] Intuitively, a larger number of reviews renders the average rating more credible. The ratio γ is a measure of the degree of ex ante quality uncertainty relative to the uncertainty (noise) in individual product reviews. Notice that when $\gamma = 0$, the SL process is essentially inactive: the updated belief, $\tilde{q}_u$, is identical to the prior belief, $\tilde{q}_p$. This case reflects situations in which SL is either (i) irrelevant, because there is no ex ante quality uncertainty (i.e., $\sigma_p \to 0$) and therefore nothing to be learned from product reviews,

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[4] Since the firm and consumers hold the same prior belief, firm actions in our model cannot convey any additional information on product quality to the consumers (i.e., there is no scope for signalling); this informational structure is commonly assumed in the SL literature to focus attention on the peer-to-peer learning process (e.g., Bergemann and Välimäki 1997, Bose et al. 2006, Bose et al. 2008, YU et al. 2013b). Furthermore, although we do not model expert/critic reviews explicitly, these may take part in forming the public prior belief; Dellarocas et al. (2007) find that there is generally little overlap between the informational content of critic reviews and that of consumer reviews.

[5] It makes no difference in our model whether consumers report directly on quality, net or gross utility. To see why, note that the product's price history and the distribution of preferences in the population are common knowledge. Therefore, rational consumers can still employ (1) to learn product quality (as explained subsequently in the main text), albeit with a simple adjustment performed on the observed average rating R. Furthermore, we may also assume that only a fraction of first-period buyers produce reviews; this has no qualitative bearing on our model insights.

[6] An alternative approach to modeling the SL process has been proposed by Bergemann and Välimäki (1997) (and more recently adopted in YU et al. (2013b)). In that approach, the authors "assume that a statistic on the aggregate performance of the new product" is made available after early purchases, which takes the form of an aggregate signal whose density function is specified by the modeler. Consumers use the realization of the aggregate signal to update their prior, via Bayes' rule. While that approach has qualitatively similar properties to the one used in our analysis, the processes by which reviews are generated and then aggregated into a signal is unclear. By contrast, in our model consumers simply report their own derived quality, and consumers remaining in the market learn directly from these reports, much as SL occurs in real-world settings.

[7] Note that the normalization of the total mass M of consumers to one is inconsequential: for the subsequent analysis to hold for a general mass M, simply redefine γ as $\gamma = M \frac{\sigma_p^2}{\sigma_q^2}$ and consider n_1 to represent the proportion of the market that purchases in the first period.

or (ii) useless, because buyer reviews carry no useful information on product quality for future consumers (i.e., $\sigma_q \to +\infty$). At the other extreme, when $\gamma \to +\infty$, the SL process dominates the updated belief: any positive number of buyer reviews causes the firm and consumers to completely abandon their prior. Throughout our analysis, we refer to γ as the *SL influence parameter*, since larger γ effectively means that the SL process is more influential in shaping the quality perceptions of future consumers.

All of the aforementioned are common knowledge. In addition, each customer has private knowledge of her idiosyncratic preference component, x_i. In the beginning of the selling season, the firm announces either (a) both the first- and second-period prices p_1 and p_2 (*pre-announced* pricing), or (b) only the first-period price p_1, with the second-period price p_2 to be set in the beginning of the second period (*responsive* pricing).[8] Consumers exhibit forward-looking behavior: they observe the firm's announcement and purchase the product in the first period only if the following two conditions hold simultaneously: (i) their expected utility from purchase in the first period is nonnegative, and (ii) their expected utility from purchase in the first period is greater than the expected utility of delaying their purchasing decision. Any customers remaining in the market in the second period purchase a unit provided their expected utility from doing so is positive. The firm seeks to maximize its overall expected profit. For simplicity, in our analysis we assume a firm discount factor of $\delta_f = 1$; our model insights hold qualitatively for any $\delta_f \geq \delta_c$. Furthermore, we normalize production costs to zero and assume that the firm operates in the absence of any binding capacity constraints.[9]

4. Benchmark Case: Forward-Looking Consumers without Social Learning

We begin our analysis by considering a benchmark model in which the firm and consumers operate in the absence of SL. Given our model setup, the absence of SL is operationally equivalent to the case $\gamma = 0$, since in this case consumers' belief over quality remains unaltered throughout the selling season (see (1)). Results presented in this section can be found in existing literature and are stated here in our model's notation for completeness and ease of reference in the subsequent analysis.

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[8] It is beyond the scope of our analysis to model how the firm credibly commits to prices; rather, our goal is to investigate the relative merits of committing to a price path, assuming that such a commitment is feasible (e.g., through repeated interactions with consumers; see Gilbert and Klemperer (2000), Liu and van Ryzin (2008)).

[9] While we make this assumption for simplicity, we also note that it is especially relevant for the case of digital products (e.g., digital music, movies and books, computer software, smart-phone apps); such products have virtually zero marginal production costs and are unlimited in availability by their very nature.

4.1. Benchmark: Pre-Announced Pricing

Under pre-announced pricing, the firm announces a price path $\{p_1, p_2\}$ from the beginning of the selling season. Consumers take this announcement as given and respond by deciding whether and when to buy the product.

Consider first an individual's best response to an arbitrary pre-announced price plan $\{p_1, p_2\}$. Customer i's expected utility from purchase in the first (second) period is $E[u_{i1}] = x_i + q_p - p_1$ ($E[u_{i2}] = x_i + q_u - p_2$). Moreover, in the absence of SL, the belief over product quality remains unchanged throughout the selling season so that $q_p = q_u = 0$. Therefore, customer i chooses to purchase the product in the first period under the two conditions (i) $E[u_{i1}] = x_i - p_1 \geq 0$ and (ii) $E[u_{i1}] \geq \delta_c(x_i - p_2) = E[u_{i2}]$ (see §3). Otherwise, the customer chooses to delay her purchasing decision until the second period, at which time she purchases a unit if $E[u_{i2}] \geq 0$.

It is straightforward to show that for any given pre-announced price plan $\{p_1, p_2\}$, there exists a unique optimal purchasing policy for strategic consumers, such that customer i purchases the product in the first period only if $x_i \geq \tau(p_1, p_2)$, where

$$\tau(p_1, p_2) = \begin{cases} p_1 & \text{if } p_1 \leq p_2, \\ \frac{p_1 - \delta_c p_2}{1 - \delta_c} & \text{if } p_1 > p_2 \text{ and } p_1 - \delta_c p_2 \leq 1 - \delta_c, \\ 1 & \text{if } p_1 > p_2 \text{ and } p_1 - \delta_c p_2 > 1 - \delta_c. \end{cases} \tag{2}$$

When the product has an increasing or constant price plan ($p_1 \leq p_2$), any customer with non-negative first-period expected utility purchases the product, since a delayed purchase incurs a higher (or equal) cost and discounted utility. When the price of the product is decreasing ($p_1 > p_2$), two possible cases arise. If the first-period price is sufficiently close to the second-period price ($p_1 - \delta_c p_2 \leq 1 - \delta_c$), customers with relatively higher expected utility will purchase in the first period in order to avoid discounted second-period utility, while customers with relatively lower expected utility will delay their purchase until the second period. Otherwise, if the second-period price is significantly lower than the first-period price ($p_1 - \delta_c p_2 > 1 - \delta_c$), no customer will purchase the product in the first period. Furthermore, if customer i does not purchase in the first period, then she purchases in the second period provided $x_i \geq p_2$.

Given knowledge of consumers' response to any arbitrary price plan, the firm chooses $\{p_1^*, p_2^*\}$ to maximize its overall profit, given by

$$\pi_{bp}(p_1, p_2) = p_1[1 - \tau(p_1, p_2)] + p_2[\tau(p_1, p_2) - p_2]^+,$$

where we have used the notation $[r]^+ = \max[r, 0]$. The firm's optimal pricing policy, which (along with the consumers' adoption policy, as described above) specifies the unique equilibrium of the pricing-adoption game, is described in Proposition 1 (see also Landsberger and Meilijson (1985)).

PROPOSITION 1. *Under pre-announced pricing, the firm's unique optimal pricing policy is a decreasing price plan with $p_1^* = \frac{2(1-\delta_c)}{4-(1+\delta_c)^2}$ and $p_2^* = \frac{1+\delta_c}{2}p_1^*$. Moreover, p_1^* (p_2^*) is strictly decreasing (increasing) in δ_c, while firm profit $\pi_{bp}(p_1^*, p_2^*)$ is strictly decreasing in δ_c.*

All proofs are relegated to the Appendix. The firm chooses a decreasing price plan and exercises price skimming, but only to the extent permitted by forward-looking consumer behavior. Furthermore, as customers become "more strategic" (i.e., more patient), prices approach each other and firm profit decreases.

4.2. Benchmark: Responsive Pricing

Next, consider the case of responsive pricing. In the first period, the firm announces the first-period price and strategic consumers respond by making first-period purchasing decisions. In the second period, the firm observes the valuations of consumers remaining in the market and announces the second-period price; consumers remaining in the market respond by making second-period purchasing decisions.

Consider first the equilibrium of the second-period subgame. As shown in Besanko and Winston (1990), for any first-period price p_1, consumers in the first period adopt a threshold purchasing policy; therefore, consumers remaining in the market in the second period have total mass $\hat{x}$ and idiosyncratic preference components x_i distributed uniformly $U[0, \hat{x}]$, for some $\hat{x} \in [0, 1]$. Given a second-period price p_2, consumers remaining in the market purchase the product if $E[u_{i2}] = \delta_c(x_i - p_2) \geq 0$. The firm chooses the second-period price p_2^* to maximize $\pi_{br2}(p_2) = p_2(\hat{x} - p_2)$. Therefore, given any $\hat{x}$, there exists a unique equilibrium in the second-period subgame played between the firm and consumers. Specifically, the firm sets the second-period price at $p_2^*(\hat{x}) = \frac{\hat{x}}{2}$, and customers remaining in the market purchase the product provided $x_i > p_2^*$.

In the first period, the firm and consumers anticipate the effects of their actions on the equilibrium of the second-period subgame. Given a first-period price p_1, consumer i forms beliefs (which are correct in equilibrium) over $\hat{x}$ and $p_2^*(\hat{x})$ and purchases only if (i) $E[u_{i1}] = x_i - p_1 \geq 0$ and (ii) $E[u_{i1}] \geq \delta_c(x_i - p_2^*(\hat{x})) = E[u_{i2}]$. More specifically, given any first-period price p_1, there exists a unique optimal purchasing strategy for strategic consumers, such that customer i purchases the product in the first period only if $x_i \geq \chi(p_1)$, where

$$\chi(p_1) = \begin{cases} 1 & \text{if } p_1 > 1 - \frac{\delta_c}{2}, \\ \frac{p_1}{1 - \frac{\delta_c}{2}} & \text{if } p_1 \leq 1 - \frac{\delta_c}{2}. \end{cases} \tag{3}$$

When the product's introductory price is sufficiently high ($p_1 > 1 - \frac{\delta_c}{2}$), all customers prefer to delay their purchase until the second period, knowing that the firm will lower the price of the product significantly in order to maximize its second-period profit. When this is not the case

$(p_1 \leq 1 - \frac{\delta_c}{2})$, high-expected-utility customers purchase in the first period, while relatively lower-expected-utility customers prefer to purchase at the lower second-period price.

The firm anticipates customers' first-period response to any arbitrary price p_1, as well as the outcome of the second-period subgame, and chooses the introductory price p_1^* to maximize its overall profit, given by

$$\pi_{br}(p_1) = p_1[1 - \chi(p_1)] + \frac{\chi(p_1)^2}{4}.$$

The firm's optimal first-period pricing policy, which specifies the unique subgame-perfect equilibrium of the pricing-purchasing game, is described in Proposition 2 (see also Besanko and Winston (1990), but note that in their analysis the firm is assumed to discount at the same rate as the consumers).

PROPOSITION 2. *Under contingent pricing, the firm's unique optimal first-period pricing policy is* $p_1^* = \frac{2}{4a - a^2}$, *where* $a = \frac{1}{1 - \frac{\delta_c}{2}}$. *Moreover, the equilibrium price path satisfies* $p_1^* > p_2^*$ *and* $\pi_{br}(p_1^*)$ *is strictly decreasing in* δ_c.

In equilibrium, the product's price always declines over time, and as customers become more strategic the firm's profit decreases.

5. Forward-Looking Consumers with Social Learning

How does the introduction of SL in the above benchmark model change the consumers' adoption strategy and the firm's pricing decisions? In this section, we consider the game between the firm and consumers when consumers are forward-looking and interact socially through product reviews. We analyze the game under pre-announced and responsive pricing policies in turn, before performing a comparison between the two classes of policies. To facilitate exposition of our results, we begin by illustrating how the SL process manifests in firm and consumer decision-making.

5.1. A Rational Belief over Social Learning

The firm and consumers observe the reviews of first-period buyers and use them to refine their belief over the product's quality, $\hat{q}$, from $\tilde{q}_p$ to $\tilde{q}_u$. In the second period of our model, customers base their purchasing decisions on $\tilde{q}_u$. For instance, if customer i remains in the market for the second period and the product's price is p_2, then she will purchase the product if and only if $E[u_{i2}] = x_i + q_u - p_2 \geq 0$ (recall that q_u is the mean of the updated belief $\tilde{q}_u$; see (1)). Now consider customer i's first-period decision: in order for her to make a decision on whether to purchase the product or delay her purchasing decision, she must form a rational belief over her second-period expected utility. In turn, to achieve the latter it is necessary for her to form a rational belief over the posterior parameter q_u; that is, the posterior mean q_u is viewed in the first period as a random variable, which is realized after the reviews of first-period buyers have been observed by customer i. This rational belief, termed the "pre-posterior" distribution of q_u, is described in Lemma 1.

LEMMA 1. *Suppose that n_1 product reviews are available to customers remaining in the market in the second period. Then the pre-posterior distribution of q_u has a Normal density function with mean zero (i.e., equal to q_p) and standard deviation $\sigma_p \sqrt{\frac{n_1 \gamma}{n_1 \gamma + 1}}$.*

Reassuringly, ex ante, product reviews have no effect, on average, on the mean of customers' belief – had this not been the case, then the prior belief held by consumers would be inconsistent. The standard deviation of the pre-posterior distribution (which measures the extent to which the posterior mean is likely to depart from the prior mean) depends on the amount of information made available to the customer through product reviews, and includes uncertainty regarding both the product's quality $\hat{q}$, as well as the noise in individual buyers' product reviews. Perhaps counter-intuitively, as the number of reviews increases and the information conveyed through these reviews becomes more precise, the variance of the pre-posterior distribution increases. To see why this is the case, note that if $n_1 = 0$, then no additional information is available in the second period, and customers' posterior mean, q_u, is exactly equal to the prior mean, i.e., $q_u = q_p = 0$. On the other hand, as the number of reviews increases, the posterior mean is more likely to depart further from the prior mean, consistent with the pre-posterior distribution having greater variability.

Importantly, in the analysis that follows, the number of reviews generated in the first period (n_1) will be an equilibrium outcome, because it depends directly on customers' first-period adoption decisions. To conclude this section, we introduce the following notation, which will facilitate exposition of our results.

DEFINITION 1. *The probability density function $f(\cdot; z)$ corresponds to a zero-mean Normal random variable of standard deviation $\sigma(z)$, where $\sigma(z) := \sigma_p \sqrt{\frac{(1-z)\gamma}{(1-z)\gamma + 1}}$. Define also $F(\cdot; z)$ as the corresponding cumulative distribution function, and let $\bar{F}(\cdot; z) := 1 - F(\cdot; z)$.*

Note that $f(\cdot; z)$ as described above is not well-defined for the case of $\gamma = 0$. Therefore, in the subsequent analysis, the absence of SL is represented by the limiting case of $\gamma \to 0$.

5.2. Pre-Announced Pricing

In this section, we discuss the equilibrium of the game between the firm and consumers when the firm adopts a pre-announced pricing policy. The sequence of events is similar to that in §4.1, augmented to account for the SL process. In the first period, the firm announces the full price path $\{p_1, p_2\}$. Customers take this announcement as given, and make first-period purchasing decisions. In the second period, customers observe the reviews of first-period buyers, update their beliefs over product quality, and then make second-period purchasing decisions. We first characterize consumers' first- and second-period decisions under any arbitrary price path; we then analyze the firm's pricing problem.

5.2.1. Consumers' Purchasing Strategy To gain an understanding of the process by which consumers make purchasing decisions in the presence of SL, it is instructive to first consider how the actions of individual consumers affect the utility of their peers. In settings characterized by SL, information on product quality is both generated and consumed by the customer population: each additional early purchase generates an additional product review, which in turn enables customers remaining in the market to make an incrementally better-informed purchasing decision. In our model, any individual consumer's expected utility from delaying her purchasing decision (i.e., until the second period) increases with the number of customers who choose to purchase the product early (i.e., in the first period).

LEMMA 2. *For any given pre-announced price plan $\{p_1, p_2\}$, each customer's second-period expected utility is strictly increasing in the number of first-period buyers.*

That is, given a price path $\{p_1, p_2\}$, a better-informed future decision is associated in our model with a higher future expected utility. The equilibrium purchasing strategy adopted by consumers is one characterized by a form of free-riding, since customers are enticed to wait for the information generated by others rather than experiment with the new product themselves. However, this tendency to delay is mediated by the endogenous generation of information: the larger the number of customers who strategically delay their purchase, the less well-informed future decisions will be.

THEOREM 1. *For any given pre-announced price plan $\{p_1, p_2\}$, there exists a unique equilibrium in the purchasing game played between consumers. Specifically:*

- In the first period, customer i purchases the product if and only if $x_i \geq \theta(p_1, p_2)$, where

$$\theta(p_1, p_2) = \begin{cases} 1 & \text{if } p_1 - \delta_c p_2 > 1 - \delta_c, \\ y & \text{if } p_1 - \delta_c p_2 \leq 1 - \delta_c, \end{cases}$$

and $y \in (p_1, 1)$ is the unique solution to the implicit equation

$$y - p_1 = \delta_c \int_{p_2 - y}^{+\infty} (y + q_u - p_2) f(q_u; y) dq_u. \tag{4}$$

- In the second period, customer i purchases the product if and only if $p_2 - q_u \leq x_i < \theta(p_1, p_2)$, where q_u is the realized posterior mean belief over quality.

When the first-period price is significantly higher than the second-period price ($p_1 - \delta_c p_2 > 1 - \delta_c$), we observe what is referred to as "adoption inertia": the significant cost benefit associated with second-period purchases makes all customers choose to defer their purchasing decision, even though second-period decisions will be made without any additional information from product reviews (since no sales occur in the first period). On the contrary, when p_1 is not much higher than p_2, a positive number of customers purchase the product in the first period. The left-hand side of

(4) represents the marginal customer's first-period expected utility from purchase, while the right-hand side represents her expected utility from delaying the purchasing decision. Notice that the lower limit of the integral accounts for the fact that, after observing the reviews of her peers, the customer will only purchase the product if her updated expected utility is positive – delaying the purchasing decision grants customers the right, but not the obligation, to purchase in the second period.

By examining the properties of the solution to (4), we may deduce the following properties of the purchasing equilibrium.

COROLLARY 1. *The number of first-period buyers, $1 - \theta$, is*

- *decreasing (increasing) in the first-period price p_1 (second-period price p_2),*
- *decreasing in customers' discount factor, δ_c,*
- *decreasing in the degree of SL influence, γ.*

Since in the presence of SL customers have an added informational incentive to delay their purchasing decisions, we observe a larger number of strategic delays than in the absence of SL (i.e., $\gamma \to 0$). An increase in the strength of the SL influence (γ) results in an increase in strategic purchasing delays – as SL becomes more influential, consumers become "more strategic."

5.2.2.   Firm's Pricing Policy For the firm, optimizing the pre-announced price plan is a convoluted task owing to the intricate interaction between its pricing decisions, the adoption decisions of strategic consumers, and the ex ante uncertain effects of the SL process on the valuations of second-period consumers. The results of §5.2.1 generate several intriguing questions. How should the firm adjust its pricing decisions to accommodate the SL process in the presence of strategic consumers? Should it even allow for SL to occur, or should it induce adoption inertia (Theorem 1)? Moreover, given that SL amplifies strategic consumer behavior (Corollary 1), is the firm better or worse off in its presence?

Given knowledge of customers' response to any arbitrary pre-announced price plan, the firm chooses $\{p_1^*, p_2^*\}$ to maximize its expected profit, defined by

$$\pi_p(p_1, p_2) = p_1[1 - \theta] + p_2 \left(\int_{p_2 - \theta}^{p_2} [\theta + q_u - p_2] f(q_u; \theta) dq_u + \int_{p_2}^{+\infty} \theta f(q_u; \theta) dq_u \right), \qquad (5)$$

where the dependence of the threshold θ on p_1 and p_2 has been suppressed to simplify notation. The first and second terms correspond to first- and second-period profit, respectively. While first-period profit is deterministic, the firm's second-period profit is ex ante uncertain owing to the demand uncertainty generated by the SL process – depending on the realization of the posterior parameter q_u, either none (low q_u) or a fraction (moderate q_u) or all (high q_u) of the remaining customers purchase the product in the second period.

To gain insight into the firm's problem, we begin with the following result, which allows us to subsequently restrict our attention to policies that generate a strictly positive number of first-period sales and a strictly positive number of expected second-period sales.

PROPOSITION 3. *Under pre-announced pricing, it can never be optimal for the firm to induce adoption inertia in the first period, or to pre-announce a second-period market exit.*

Note that both adoption inertia (i.e., announcing $\{p_1, p_2\}$ such that $p_1 - \delta_c p_2 > 1 - \delta_c$) as well as a pre-announced second-period exit from the market (i.e., announcing $\{p_1, p_2\}$ with $p_2 \to +\infty$) confine all sales to take place in one of the two selling periods; such outcomes are strictly sub-optimal for the firm. Therefore, the significance of Proposition 3 is to establish that the SL process will always be "active" in equilibrium; comparing with Theorem 1, the result implies that a solution to the firm's problem exists and satisfies $p_1^* - \delta_c p_2^* \leq 1 - \delta_c$.

Now let us consider the firm's optimal pricing policy in more detail. To this end, the optimality conditions of problem (5) are, unfortunately, not very informative. To illustrate the effects of SL on the firm's pre-announced price plan, we decompose the overall impact of SL into its two main effects, and analyze the implications of each effect in turn.

1. The *behavioral* effect changes consumers' purchasing behavior in the first period: as γ increases, consumers' informational incentive to delay their purchase increases, resulting in a larger number of strategic purchasing delays.

2. The *valuation* effect shifts the demand curve faced by the firm in the second period: depending on whether (and the extent to which) reviews are favorable or not, the firm faces a population of relatively higher or lower valuations for the product in the second period.

Consider first the implications of the behavioral effect, ignoring for now the valuation effect (that is, suppose for now that the valuations of consumers remaining in the market do not change in the second period). The behavioral effect of SL simply makes consumers more patient; therefore, we may leverage the result of our benchmark model in Proposition 1 and consider what happens when the firm faces consumers who are relatively more patient. This allows us to assert that the behavioral effect of SL causes a decrease in p_1^*, an increase in p_2^*, and a decrease in expected firm profit.

Next, we turn to the implications of the valuation effect, which are less straightforward. In particular, note that at the time of the firm's announcement (i.e., in the first period), the impact of the valuation effect is uncertain because it depends on the realization of the posterior parameter q_u. Since this effect operates on the valuations of second-period consumers, we are predominantly interested in how it affects the firm's pre-announced second-period price. To illustrate, we construct a paradigm in which we "switch off" the behavioral effect of SL by making customers myopic, but

maintain the valuation effect by retaining the influence of first-period reviews on second-period consumer valuations.

LEMMA 3. *Suppose that $\delta_c = 0$, and fix the first-period price at some arbitrary p_1. Then for any $k > 0$, $p_2^*|_{\gamma=k} > p_2^*|_{\gamma\to 0}$. Furthermore, $\pi_p^*|_{\gamma=k} > \pi_p^*|_{\gamma\to 0}$, where $\pi_p^* = \pi_p(p_1, p_2^*)$.*

All else being equal, the firm will choose a higher second-period price in the presence of SL, and extract higher overall expected profit. The rationale behind Lemma 3 is based on the symmetric nature of the uncertainty faced by the firm (i.e., q_u is viewed ex ante as a Normal random variable; see Lemma 1). For every favorable SL scenario (there exists a continuum of these), there exists a corresponding unfavorable "mirror" scenario that is equally probable. By increasing the second-period price, the firm is able to capitalize on highly favorable scenarios more effectively, while at the same time its profit in the corresponding highly unfavorable scenarios is at worst zero.

How do the behavioral and valuation effects combine? In terms of the optimal price plan, we have seen that the behavioral effect causes a decrease in p_1^* and an increase in p_2^*, while the valuation effect causes a further increase in p_2^*. The following special case of our model suggests that the overall impact of SL on the optimal price plan can be significant.

PROPOSITION 4. *Suppose $\delta_c = 1$. Then for any $\gamma > 0$, the optimal price plan satisfies $p_1^* < p_2^*$.*

Recall from Proposition 1 that in the absence of SL, an increasing price plan can never be optimal. Surprisingly, although Proposition 4 applies only for the extreme case of infinitely patient consumers, the optimality of an *increasing* price plan in the presence of SL is the rule rather than the exception. To illustrate the generality of this result, we present the numerical experiments of Figure 1, in which we vary the two main parameters in our model, namely, customers' degree of patience (δ_c) and the SL influence parameter (γ). Observe that the firm maintains a decreasing price path (which is optimal in the absence of SL) only when δ_c is very low or γ is extremely low (i.e., SL is absent or uninformative).

Recall that δ_c can be viewed as a measure of the opportunity-cost perceived by consumers of postponing their purchase to the second period. In practical settings, δ_c can be low (i.e., high perceived opportunity-cost), for example, when customers attach value to the "newness" of the product (e.g., being the first to watch a new Broadway show). In such cases, our analysis indicates that the firm finds it most profitable to charge a price-premium in the first period aimed towards extracting rents from high-valuation consumers, followed by a lower second-period price.

By contrast, when consumers are not highly impatient, the firm announces a low "introductory" price followed by a higher "regular" price. Here, we find that it is optimal for the firm to pre-announce a second-period *information-premium*; that is, to charge consumers for the privilege of
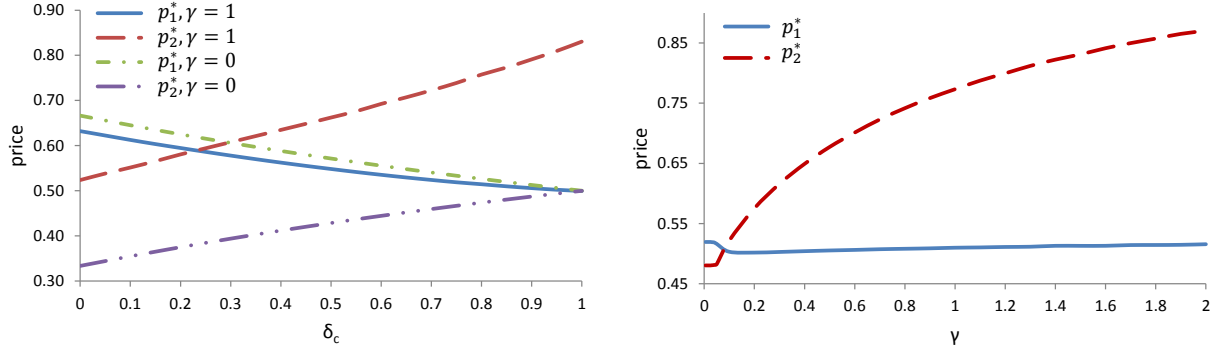
Figure 1 Optimal first- and second- period prices for different combinations of parameter values. Default parameter values: $\delta_c = 0.85$, $\sigma_p = 1$.

making a better-informed purchasing decision. This premium has two effects. First, it counter-balances consumers' increased willingness-to-wait in anticipation of product reviews, and shifts demand back to the first period. Second, from those consumers who choose to wait despite the high second-period price, the firm extracts high profit in cases of highly favorable SL scenarios. Crucially, notice that the first effect feeds forward and reinforces the second, in the sense that a larger number of first-period reviews (generated by shifting demand back to the first period) renders highly favorable SL scenarios ex ante more probable (by increasing the ex ante variability of q_u; see Lemma 1).[10]

Even though the firm can do its best to mitigate the negative effects of SL on strategic consumer behavior through its pricing policy, it remains unclear whether the overall impact of SL on expected firm profit is positive or negative. When consumers are myopic ($\delta_c = 0$), Lemma 4 is sufficient to establish that the SL process is ex ante beneficial for the firm. However, the picture is not as clear when consumers are strategic ($\delta_c > 0$). In particular, SL exacerbates the amount of strategic purchasing delays, which causes a relative decrease in firm profit. On the contrary, the valuation effect generates a relative increase in firm profit. Thus, the overall impact of SL depends on the relative magnitude of these two opposing effects. Proposition 5 takes consumer strategicness to the extreme.

PROPOSITION 5. *Suppose that $\delta_c = 1$. Then the firm achieves greater expected profit in the presence of SL ($\gamma > 0$) than it achieves in its absence ($\gamma \to 0$).*

Interestingly, even for the case in which consumers are infinitely patient, the presence of SL is ex ante beneficial for the firm. While the corresponding result is difficult to obtain analytically

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[10] Swinney (2011) illustrates than when consumers' preferences are revealed exogenously over time (i.e., as opposed to product quality being learned endogenously through SL), it is optimal for the firm to employ an increasing price plan so as to decrease strategic purchasing delays among uninformed consumers. The increasing price plan in our model also reduces strategic delays but, more importantly, serves the purpose of reinforcing the (endogenous) second-period valuation effect.

for more general cases of our model, since the positive valuation effect dominates the negative behavioral effect when $\delta_c = 1$, it is perhaps not surprising that the result generalizes across our numerical experiments (see Figure 2).
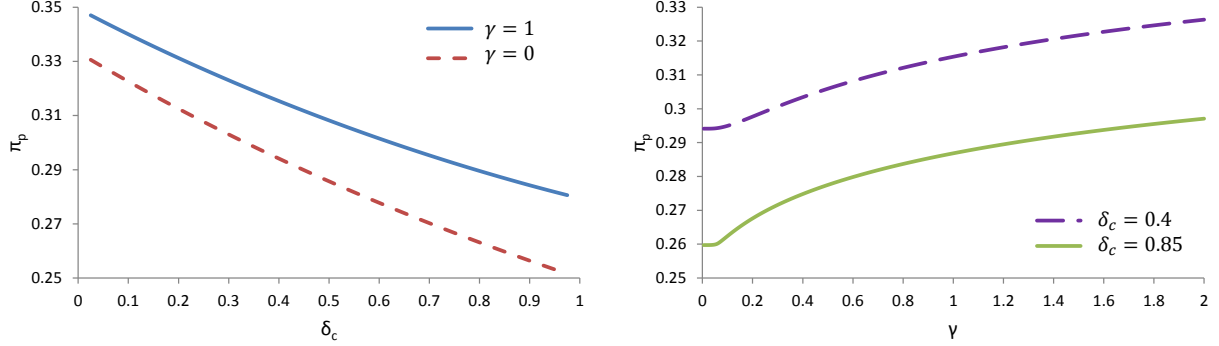

Figure 2   **Optimal pre-announced profit at different combinations of γ and δ_c. Parameter values: $\sigma_p = 1$.**

To conclude this section, it is instructive to consider how the result of Proposition 5 is affected if we restrict the firm to charge a constant price (i.e., by adding the constraint $p_1 = p_2$ to problem (5)). This issue is of particular relevance in settings in which fairness considerations are important for long-term firm-customer relationships (e.g., *The New York Times* 2007), or when implementing price changes is costly or impractical (see also Aviv and Pazgal (2008)) – in such settings, the firm may be reluctant to price intertemporally. We find that the result (i.e., that SL is beneficial for the firm in the presence of strategic consumer behavior) continues to hold even when the firm is restricted to maintaining a constant price (see Appendix, proof of Proposition 5).

5.3. Responsive Pricing

Now suppose that the firm does not commit to a second-period price from the beginning of the selling season. Under a responsive pricing strategy, the game between the firm and consumers is modified as follows: In the beginning of the first period, the firm sets the first-period price, p_1, and consumers make first-period purchasing decisions. In the beginning of the second-period, the firm and consumers observe the product reviews generated by first-period buyers and update their belief over product quality. The firm then sets the second-period price, p_2, and consumers remaining in the market make second-period purchasing decisions. The two-period stochastic game between the firm and consumers is analyzed in reverse chronological order; we seek a subgame-perfect equilibrium.

5.3.1. Second-Period Pricing and Purchasing Strategies
In order to analyze the second-period subgame, we temporarily assume that in the first period, for any first-period price p_1 chosen by the firm, customers adopt a threshold purchasing policy; the validity of this assumption is proven in the next section.

At the beginning of the second period and as a result of customers' first-period purchasing decisions, the firm faces a population of consumers of total mass $\bar{x}$ with idiosyncratic preference components x_i distributed uniformly $U[0, \bar{x}]$, for some $\bar{x}$ satisfying $0 \leq \bar{x} \leq 1$. Moreover, both the firm and consumers have access to the product reviews of first-period buyers, and arrive at a posterior mean q_u, according to Bayes' rule in (1). At an arbitrary second-period price p_2, customer i's expected utility from purchase in the second period is $E[u_{i2}] = \delta_c(x_i + q_u - p_2)$. The firm's profit in the second period is defined by

$$\pi_2(p_2) = p_2 \left[\min(\bar{x}, \bar{x} + q_u - p_2) \right]^+,$$

and the unique equilibrium of the second-period subgame is characterized in Proposition 6.

PROPOSITION 6. *Under responsive pricing, given any q_u and $\bar{x}$, there exists a unique equilibrium in the second-period subgame played between the firm and consumers. Specifically:*
- The firm's optimal second-period pricing policy is defined by

$$p_2^*(q_u, \bar{x}) = \begin{cases} q_u & \text{if } q_u > \bar{x}, \\ \frac{q_u + \bar{x}}{2} & \text{if } -\bar{x} < q_u \leq \bar{x}, \\ 0 & \text{if } q_u \leq -\bar{x}. \end{cases}$$

- Customer i purchases the product in the second period if and only if $p_2^*(q_u, \bar{x}) - q_u \leq x_i < \bar{x}$.

In the second period, customers simply choose to purchase the product if their expected utility from purchase (given what they have learned from product reviews and the firm's decision p_2^*) is non-negative. The firm's profit-maximizing p_2 depends on the outcome of SL, q_u, as well as customers' first-period purchasing decisions which specify $\bar{x}$ (that is, q_u and $\bar{x}$ jointly define the demand curve faced by the firm in the second period). If q_u is very high (a sign of high quality for firm and consumers), the firm finds it most profitable to choose the market-clearing price $p_2^* = q_u$. When q_u is at intermediate levels, the firm chooses a price at which only a fraction of consumers remaining in the market choose to adopt the product. Finally, when q_u is very low, the firm cannot extract profit at any positive price p_2, and therefore exits the market; this is signified by a second-period price of $p_2^* = 0$.

5.3.2. First-Period Pricing and Purchasing Strategies In the first period, the firm and consumers anticipate the effects of their actions on the equilibrium of the second-period subgame. However, note that since the realization of the posterior mean q_u is ex ante uncertain, the equilibrium in the second-period subgame is itself uncertain; unlike the benchmark case in §4.2, the firm and consumers in this case form rational *probabilistic* beliefs over the second-period equilibrium.

We first consider the consumer's first-period decision for any arbitrary price p_1 and subsequently address the firm's first-period pricing problem. Recall that customer i purchases the product in

the first period only if her expected utility from a first-period purchase is higher than that of a delayed purchasing decision.

THEOREM 2. *Under responsive pricing and any given first-period price p_1, there exists a unique optimal first-period purchasing strategy for consumers. Specifically, customer i purchases the product in the first period if and only if $x_i \geq \zeta(p_1)$, where*

$$\zeta(p_1) = \begin{cases} 1 & \text{if } p_1 > 1 - \frac{\delta_c}{2}, \\ \frac{p_1}{1 - \frac{\delta_c}{2}} & \text{if } p_1 \leq 1 - \frac{\delta_c}{2}. \end{cases}$$

Customers' first period adoption strategy under responsive pricing is reminiscent of that under pre-announced pricing. If the first-period price is relatively high ($p_1 > 1 - \frac{\delta_c}{2}$), all consumers delay their purchasing decision (adoption inertia) in anticipation of a significantly lower second-period price. If the first-period price is not too high ($p_1 \leq 1 - \frac{\delta_c}{2}$), customers with relatively higher valuations purchase in the first period, while the rest of the population delays the purchasing decision (case $\zeta = \frac{p_1}{1 - \frac{\delta_c}{2}}$).

Theorem 2 also suggests the following, quite remarkable phenomenon.

COROLLARY 2. *Under responsive pricing and for any given first-period price p_1, the first-period purchasing strategy adopted by forward-looking consumers is independent of the SL influence γ.*

Surprisingly, consumers make identical first-period purchasing decisions in the absence ($\gamma \to 0$) and in the presence of SL ($\gamma > 0$). When the firm maintains the flexibility to respond to the reviews of first-period buyers by adjusting the product's price, it is no longer the case that SL generates a relative increase in strategic purchasing delays in the first period; that is, *SL has no behavioral effect*, in contrast with the case of pre-announced pricing.[11]

More specifically, what occurs is that the presence of SL does not change the decisions of the forward-looking consumers, instead, it strengthens the incentives that drive these decisions. To illustrate, define the "value of SL" for customer i, $v_{sl}(x_i)$, as the difference between the customer's equilibrium expected utility from delaying her decision in the presence (i.e., for some $\gamma > 0$) and absence of SL ($\gamma \to 0$); that is, $v_{sl}(x_i) := E[u_{i2}^*]|_{\gamma > 0} - E[u_{i2}^*]|_{\gamma \to 0}$. It is straightforward to show that, at any p_1, we have $v_{sl}(x_i) > 0$ for $x_i \leq \zeta(p_1)$ (i.e., for customers who delay) and $v_{sl}(x_i) < 0$ for $x_i > \zeta(p_1)$ (i.e., for customers who buy early); Figure 3 provides a numerical illustration of this "rotational" effect. Interestingly, this implies that high-x_i customers have an even stronger preference for purchasing early in the presence of SL, for if they do not, the flexible second-period firm will leave them with relatively less expected surplus in the second period (as compared to that in the absence of SL).

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[11] Although YU et al. (2013b) study a setting similar to that considered in this section, the sharp characterization of consumers' first-period strategy in Theorem 2, and in particular how this strategy compares to the case of no SL (Proposition 2), appears to be novel; these characterizations are possible in our model owing to our different approach to modelling the review-generation and SL processes (see §3 and footnote 6).
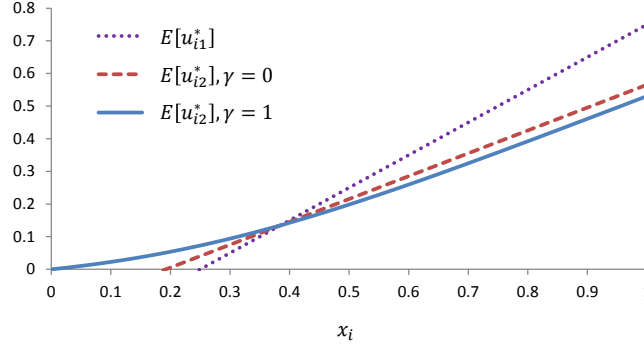
Figure 3 **Equilibrium expected utility from purchasing early, $E[u_{i1}^*]$, and delaying (with and without SL), $E[u_{i2}^*]$, for customer type x_i. Both in the presence of SL ($\gamma = 1$) and in its absence ($\gamma \to 0$), the indifferent consumer has $x_i = 0.39$. Consumers with $x_i > 0.39$ ($x_i < 0.39$) have relatively lower (higher) expected utility from delaying their decision in the presence of SL. Parameter values: $p_1 = 0.25$, $\delta_c = 0.7$, $\sigma_p = 1$.**

Consider next the firm's first-period pricing decision. While SL has no effect on the first-period purchasing decisions of strategic consumers, the same is not true for the firm's first-period pricing strategy. In the presence of SL and strategic consumers, the firm chooses p_1^* to maximize its overall expected profit, which is defined by

$$\pi_r(p_1) = p_1[1 - \zeta] + \int_{-\zeta}^{\zeta} \left(\frac{q_u + \zeta}{2}\right)^2 f(q_u; \zeta) dq_u + \int_{\zeta}^{+\infty} \zeta q_u f(q_u; \zeta) dq_u, \tag{6}$$

where the dependence of ζ on p_1 has been suppressed to simplify notation. As opposed to the case of pre-announced pricing, problem (6) accounts for the fact that in the second period, the firm will adjust the product's price in response to the posterior realization of q_u. According to the results of Theorem 2 and Proposition 6, any first-period price chosen by the firm generates a unique subgame-perfect equilibrium in the pricing-purchasing game. Although it is difficult to characterize the firm's optimal first-period price analytically, the following proposition reduces the problem to a simple one-dimensional optimization problem over a finite interval.

PROPOSITION 7. *Under responsive pricing, it can never be optimal for the firm to induce adoption inertia in the first period.*

Similarly as in the case of pre-announced pricing, Proposition 7 suggests that the firm will never induce adoption inertia under responsive pricing, and that the SL process will always be "active" in equilibrium. Furthermore, combined with Theorem 2, the result of Proposition 7 implies that an optimal first-period price p_1^* exists and satisfies $0 \leq p_1^* \leq 1 - \frac{\delta_c}{2}$.[12]

---

[12] This result contrasts with Jing (2011), who shows that when consumers learn their preferences through SL (and the firm faces no ex ante uncertainty), adoption inertia may arise in equilibrium since the firm may find it preferable to sell to an uninformed and therefore more homogeneous population in the second period. Since consumers in our model know their preferences ex ante (and therefore consumer heterogeneity is not affected by the SL process), such incentives do not arise.

Given that the firm will always avoid adoption inertia, how does the first-period price in the presence of SL compare to that in its absence? Observe in Figure 4 that the firm will introduce the product at a relatively lower price in the presence of SL. (This occurs in general in our experiments, unless γ is extremely high.) The rationale underlying this strategy is associated with the firm's preference for high ex ante variability in q_u (i.e., high variance in the pre-posterior distribution of q_u). To understand this preference, note that the firm's second-period profit is convex increasing in q_u (see (6)); this implies that, *ceteris paribus*, the firm's expected profit is increasing in the ex ante variability of q_u. Next, recall from Corollary 2 that the presence of SL does not change consumers' first-period adoption strategy. Therefore, the relatively lower introductory price is an attempt to reinforce the SL process (i.e., to increase the variance of the pre-posterior distribution of q_u: see Lemma 1) by increasing the number of product reviews generated in the first period. Of course, this tendency to lower p_1 is tempered by the firm's desire to (a) extract substantial profit in the first period and (b) keep a substantial portion of the consumer population in the market so that it may capitalize on its second-period pricing flexibility.
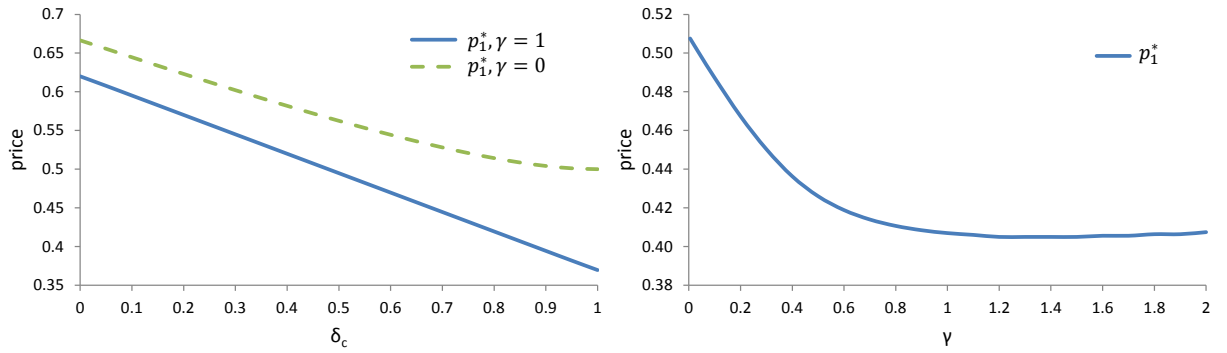

Figure 4   **Optimal first-period price in the presence and absence of social learning, for different combinations of parameter values. Default parameter values:** $\delta_c = 0.85$, $\sigma_p = \gamma = 1$.

Next, we consider the structure of the equilibrium price path. Recall that in the absence of SL ($\gamma \to 0$), the equilibrium price path is always decreasing (see Proposition 2). In the presence of SL, given p_1^*, the optimal second-period price p_2^* depends on the firm and consumers' posterior mean belief q_u (see Proposition 6 and replace $\bar{x}$ with $\zeta(p_1^*)$). Although customers remaining in the market in the second period have relatively lower preference components x_i than those who purchase in the first period, it is no longer the case that the price of the product will always be lower in the second period. In particular,

COROLLARY 3. *The ex ante probability of an increasing price path satisfies* $0 < P(p_1^* < p_2^*) < \frac{1}{2}$.

An increasing price path may indeed arise in equilibrium; however, a decreasing price path is ex ante more likely. To see why, note that if consumers' valuations did not change at all (i.e., as would be the case in the absence of SL), then the firm would charge a price lower than p_1^* in the second period (see Proposition 2). It follows that for p_2^* to be higher than p_1^* in the presence of SL, it must be the case that consumers' valuations have increased considerably in the second-period; given the ex ante symmetric nature of SL, the ex ante probability of this occurring cannot be greater than one half. This result suggests a contrast between the structure of the price path under responsive and pre-announced pricing: when the firm commits to a price path ex ante, this path will generally be increasing (unless customers are highly impatient); when the firm does not commit to a price path, it is more likely that prices will decline over time.

To conclude this section, we present the following result, which characterizes how the firm's expected profit is affected by changes in the SL influence (γ) as well as the ex ante quality uncertainty surrounding the product (σ_p, keeping γ fixed; see Lemma 1).

PROPOSITION 8. *Under responsive pricing, the firm's expected profit is strictly increasing in both the SL influence (γ), and in quality uncertainty (σ_p).*

The intuition underlying Proposition 8 is again based on the firm's preference for high ex ante variability in q_u, as discussed previously. We note that the beneficial effects of SL on expected profit when the firm adjusts prices dynamically have been established previously in the literature, but under the assumption that consumers are non-strategic (e.g., Bose et al. 2006, Ifrach et al. 2013). Proposition 8 generalizes this finding to the case of forward-looking consumers.[13]

6. Pre-Announced vs. Responsive Pricing

A recurring theme in the recent literature that considers strategic consumer behavior is the value of price-commitment for the firm. For instance, Aviv and Pazgal (2008) report that when customers are forward-looking (and in the absence of future rationing risk), pre-announced pricing is a more effective way of managing strategic consumer behavior than responsive pricing, in the sense that it allows the firm to extract higher overall profit (see "announced" and "contingent" pricing in their model). Our benchmark model replicates this prediction.

LEMMA 4. *In the absence of SL ($\gamma \to 0$) and for any $\delta_c \in (0,1)$, firm profit under pre-announced pricing is strictly higher than that under responsive pricing.*

---

[13] Note that the firm in our model is by assumption more patient than the consumer population (since it does not discount second-period profit). In the analysis of YU et al. (2013b), the authors also allow for the opposite case. Interestingly, they provide a numerical example in which they observe that when the consumer population is more patient than the firm, reviews that are relatively more informative may generate a relative decrease in expected firm profit. From the proof of Proposition 8 it becomes evident that such cases do not arise in our model, even if we allow for an arbitrary combination of firm and consumer patience. In other words, the result of the proposition does not depend on the fact that the firm discounts at a lower rate than consumers.

The question of interest in this section is whether price-commitment is preferred by the firm when SL is accounted for. Surprisingly, we observe that, in general, the *opposite* is true. Let us begin by first proving the sub-optimality (in terms of expected firm profit) of a pre-announced price plan in the presence of SL for the special case of myopic consumers.

LEMMA 5. *Suppose that $\delta_c = 0$. Then responsive pricing generates higher firm profit than pre-announced pricing for any $\gamma > 0$.*

When customers are myopic but second-period customers' valuations are affected by the reviews of their peers, it is always beneficial for the firm to maintain price-flexibility. To see why, notice that any arbitrary first-period price generates the same amount of first-period sales (and profit) under either pricing policy; however, under responsive pricing the firm can maximize its second-period profit by choosing the best price given the content of product reviews.

The sub-optimality of price-commitment in the presence of SL generalizes far beyond the special case of myopic consumers considered in Lemma 5. Although a comparison between the two policies is difficult to perform analytically for general cases of our model, our numerical study points to three main observations (see Figure 5): first, a responsive pricing policy is optimal in the vast majority of cases; second, the cases in which a pre-announced price plan is preferred are those that combine patient customers with weak SL influence (i.e., high δ_c and low γ); third, in those cases in which a pre-announced price plan is optimal, the increase in profit with respect to the optimal responsive price plan is much smaller (1.5% on average) than the corresponding increase in profit when a responsive price plan is optimal (8.7% on average).
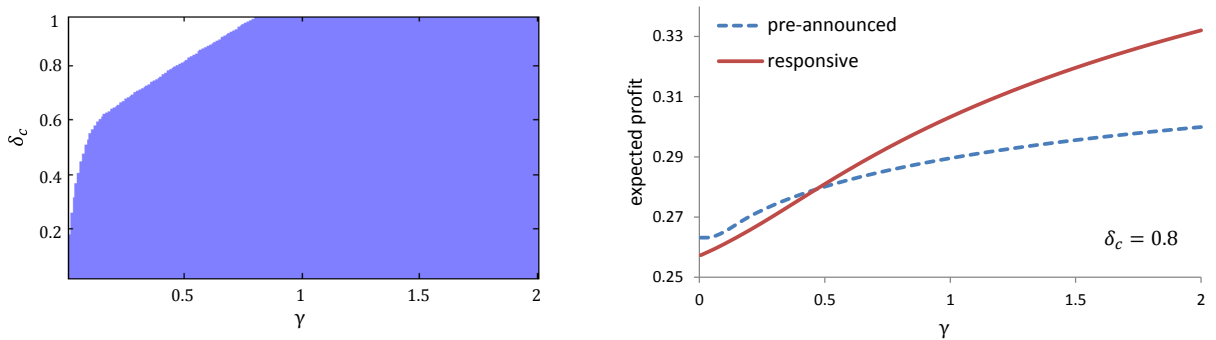

Figure 5   **Left: Region plot for the optimal class of dynamic pricing policy as a function of consumer patience (δ_c) and SL influence (γ); shaded regions mark the optimality of a responsive pricing policy; white regions mark the optimality of a pre-announced policy. Parameter values: $\sigma_p = 1$.**
Right: Expected firm profit under the optimal pre-announced and responsive pricing policies as a function of SL precision (γ), for the case of $\delta_c = 0.8$. Parameter values: $\sigma_p = 1$.

The rationale underlying these observations is as follows. The flexibility offered by a responsive pricing policy is significantly advantageous for the firm when the valuations of second-period consumers are likely to change significantly as a result of SL. *Ceteris paribus*, a significant change in consumer valuations occurs when the number of first-period reviews is large (high n_1) and/or when the influence of SL is strong (high γ). When the value of γ is low, the only way in which the firm can profit substantially from its second-period flexibility is if it generates a very large number of first-period sales/reviews. As customers become progressively more patient (i.e., δ_c increases) the firm is required to introduce the product at a progressively lower price in order to achieve the volume of reviews necessary to capitalize on its flexibility. It is for this reason that, when δ_c is high and γ is low, the firm prefers to employ the optimal pre-announced price path, rather than the optimal responsive price plan (which would entail either a large amount of sales at a very low price, or reduced effectiveness of pricing flexibility). Moreover, we note that even when the firm does prefer a pre-announced price plan, the advantage with respect to a responsive policy is generally very small and quickly disappears as γ increases from zero (see, e.g., the right-hand side plot of Figure 5).

Next, consider the customers' perspective. In the absence of SL, the firm's profit is maximized under a pre-announced price plan (Lemma 4), but it is straightforward to show that the opposite is true for consumer surplus.

LEMMA 6. *In the absence of SL ($\gamma \to 0$) and for any $\delta_c \in (0,1)$, consumer surplus is greater under responsive pricing that it is under pre-announced pricing.*

In the absence of SL, a planner seeking to maximize consumer surplus by simply choosing the class of policy to be employed by the firm would opt for a responsive price plan. In our numerical study, the result of Lemma 6 continues to hold when SL is accounted for (i.e., for all $\gamma > 0$). Combining this observation with Figure 5, we conclude that SL has the positive effect of aligning the preferences of the firm and consumers regarding which class of policy arises in equilibrium (this is true whenever responsive pricing is preferred by the firm). Clearly, this has the further implication that, among the two classes of policies available to the firm, the class chosen in equilibrium is generally the one that maximizes expected total welfare (i.e., the sum of expected firm profit and expected consumer surplus).

7. Conclusion

We have presented a stylized analysis of the effects of quality uncertainty and social learning on the purchasing behavior of strategic consumers, and the pricing decisions of a monopolist firm.

Recent research has highlighted that firms may neglect the ever-increasing sophistication of the modern-day consumer at their peril. Yet this research has itself neglected perhaps one of the most

important aspects of this sophistication: the ability of consumers to exchange experiences and learn from their peers. This paper demonstrates that pricing techniques which have come to constitute conventional wisdom may in fact be overturned by the increasing influence of SL on consumer decision-making. For instance, we have shown that price-commitment, which has been advocated as an effective way of managing strategic consumer behavior, is in fact suboptimal for the firm once SL is accounted for. This suggests that managers pricing products for which SL (i.e., product reviews) is known to be a significant driver of demand should be encouraged to price products dynamically in response to buyers' sentiments (e.g., media items on Amazon.com). Furthermore, this paradigm shift from pre-announced to responsive pricing constitutes a "win-win" situation – both the firm and the consumer population benefit from responsive prices.

Another main result of our analysis is that, even taking into account the negative effects of strategic consumer behavior, the SL process is one that should be endorsed and promoted by modern-day firms (in our model, the SL influence parameter γ and expected firm profit are positively related under either class of pricing policy). There are at least three dimensions along which firms can act to enhance the influence of SL on consumers' product evaluations and purchasing decisions. The first is to simply encourage the consumer population to pay more attention to buyer opinions; for instance, KIA and Ford motors have recently invested heavily in advertising campaigns aimed at promoting consumer attention to buyer reviews (see their "Reviews and Recommendations" and "Good Reviews" campaigns, respectively). The second is to provide the platform upon which consumers can communicate and exchange their product experiences; examples of high-profile firms that have actively facilitated the exchange of consumer experiences on their online spaces include Amazon.com, Dell Computers, and Apple. A third dimension is to increase the precision of buyer-generated reviews by asking consumers to rate products on multiple dimensions (e.g., see reviews on Hotels.com) rather than simply providing a uni-dimensional star rating.

A more subtle implication of our work is associated with firms' new product development (NPD) strategy. Specifically, we find that in the presence of SL, greater ex ante quality uncertainty is associated with higher expected firm profit. This result suggests that firms developing products for which SL is known to be significantly influential (e.g., high-tech electronics) should be encouraged to take risks and innovate with their new products (as opposed to incrementally improving previous product versions). When the resulting product proves to be a success, our results suggest that the SL process will be handsomely rewarding, while in cases of product failures, the corresponding downside is relatively less severe.

Finally, we have seen that when consumers can anticipate future prices perfectly (e.g., in cases of price-commitment), the SL process generates a relative increase in strategic purchasing delays; by contrast, this relative increase is not present when future prices are unpredictable. This may have

implications for researchers and firms attempting to estimate the effects of SL on firm profits (e.g., as in Moretti (2011)). Specifically, neglecting to account for consumers' increased strategicness may lead to over-estimates of the impact of SL since intertemporal demand shifts (i.e., from earlier to later in the selling season) may be misdiagnosed as demand generated through the SL process.

Appendix

Proof of Proposition 1 (Outline)

Using the expression for $\tau(p_1, p_2)$ in (2), it is straightforward to verify that for price plans satisfying $p_1 \leq p_2$ profit is bounded by $\pi \leq \frac{1}{2}$, and that the same is true for price plans satisfying both $p_1 > p_2$ and $p_1 - \delta_c p_2 > 1 - \delta_c$ simultaneously. Next, for price plans satisfying $p_1 > p_2$ and $p_1 - \delta_c p_2 \leq 1 - \delta_c$, the firm's problem is concave and the optimal price plan $\{p_1^*, p_2^*\}$ as quoted in the proposition can be derived via first order conditions, noting also that $\pi(p_1^*, p_2^*) > \frac{1}{2}$ for all $\delta_c \in (0, 1)$. The properties stated follow readily.

Proof of Proposition 2 (Outline)

Using the expression for $\chi(p_1)$ in (3) it follows that $\pi(p_1) = \frac{1}{4}$ for $p_1 > 1 - \frac{\delta_c}{2}$, because no sales occur in the first period and the optimal second-period price is $\frac{1}{2}$. Next, for $p_1 \leq 1 - \frac{\delta_c}{2}$ we have $\pi(p_1) = p_1(1 - ap_1) + \frac{a^2 p_1^2}{4} = p_1 - ap_1^2[1 - \frac{a}{4}]$ where $a = \frac{1}{1 - \frac{\delta_c}{2}}$ and $1 < a < 2$. The profit function is thus strictly concave in p_1 (in this region of prices) with a unique maximizer at $p_1^* = \frac{2}{4a - a^2}$. Finally note that $p_1^* = \frac{2}{4a - a^2} < \frac{1}{a} = 1 - \frac{\delta_c}{2}$ for all $a \in (1, 2)$ and the profit function is continuous; this implies that the global maximizer of the profit function is p_1^* and that at least some sales occur in the first period. The properties of the price path and profit follow readily.

Proof of Lemma 1

In the second period, if the product reviews of n_1 first-period buyers have a mean of R, then by Bayes' rule, the posterior belief over $\hat{q}$, denoted by $\tilde{q}_u$, is Normal with a mean of q_u, where

$$q_u = \frac{\sigma_q^2}{n_1 \sigma_p^2 + \sigma_q^2} q_p + \frac{n_1 \sigma_p^2}{n_1 \sigma_p^2 + \sigma_q^2} R$$
$$= \frac{1}{n_1 \gamma + 1} q_p + \frac{n_1 \gamma}{n_1 \gamma + 1} R,$$

where q_p is the mean of the prior belief and $\gamma = \frac{\sigma_p^2}{\sigma_q^2}$.

In the first period, the posterior mean belief q_u is viewed as a random variable (r.v.), since it depends on the unobservable realization of product quality $\hat{q}$, as well as the noise in first period reviews (i.e., it is subject to sampling error). Specifically, if the product's quality realization is $\hat{q}$, the sample mean of n_1 (i.i.d. Normal) reviews, R, follows $R \sim N(\hat{q}, \frac{\sigma_q^2}{n_1})$ so that $q_u \mid \hat{q} \sim N\left(\frac{\sigma_q^2}{n_1 \sigma_p^2 + \sigma_q^2} q_p + \frac{n_1 \gamma}{n_1 \gamma + 1} \hat{q}, \left(\frac{n_1 \gamma}{n_1 \gamma + 1}\right)^2 \frac{\sigma_q^2}{n_1}\right)$. Therefore, since $\hat{q}$ is an ex ante Normal r.v., $\hat{q} \sim N(q_p, \sigma_p^2)$,

$$E[q_u] = E(E[q_u \mid \hat{q}]) = E\left(\frac{\sigma_q^2}{n_1 \sigma_p^2 + \sigma_q^2} q_p + \frac{n_1 \gamma}{n_1 \gamma + 1} \hat{q}\right) = q_p, \text{ and}$$
$$\text{Var}[q_u] = E(\text{Var}[q_u \mid \hat{q}]) + \text{Var}(E[q_u \mid \hat{q}])$$
$$= \left(\frac{n_1 \gamma}{n_1 \gamma + 1}\right)^2 \left(\frac{\sigma_q^2}{n_1} + \sigma_p^2\right) = \left(\frac{n_1 \gamma}{n_1 \gamma + 1}\right)^2 \left(\frac{\sigma_q^2(n_1 \gamma + 1)}{n_1}\right) = \frac{n_1 \gamma}{n_1 \gamma + 1} \sigma_p^2.$$

To complete the proof, recall that we have normalized q_p to zero (see §3).

Proof of Lemma 2

Let $v_2(x_i, y) := \delta_c \int_{p_2-x_i}^{+\infty} (x_i + q_u - p_2)f(q_u; y)dq_u$ denote the second-period expected utility of customer x_i, conditional on $n_1 = 1 - y$ customers purchasing in the first period. We will show that $\frac{\partial v_2}{\partial y} < 0$. Rewrite $v_2(x_i, y) = \delta_c \int_{p_2-x_i}^{+\infty} (x_i + q_u - p_2)f(q_u; y)dq_u = \delta_c \int_{-\infty}^{+\infty} m_i(q_u)f(q_u; y)dq_u$, where $m_i(q_u) = 0$ if $q_u < p_2 - x_i$ and $m_i(q_u) = x_i + q_u - p_2$ otherwise. Note that $m_i(q_u)$ is non-negative, convex and increasing in q_u. It follows that the integral $\int_{-\infty}^{+\infty} m_i(q_u)f(q_u; y)dq_u$ is strictly increasing in the variance of the pre-posterior distribution of q_u. In turn this variance is decreasing in y. (Proceeding in a similar manner, it is straightforward to show also that $\frac{\partial v_2}{\partial \gamma} > 0$.)

Proof of Theorem 1

The proof proceeds in three steps. First, we show that any purchasing equilibrium must be characterized by a first-period threshold policy. Second, we establish the condition under which this threshold is such that no customers purchase in the first period (adoption inertia). Third, we show that when adoption inertia does not occur, the implicit equation which characterizes the equilibrium threshold has exactly one solution in the interval $[0, 1]$, and that this solution lies in the open interval $(p_1, 1)$.

Step 1. Consider any arbitrary price plan $\{p_1, p_2\}$. Define customer i's best response function given that (any) ψ customers choose to purchase the product in the first period by $b_i(\psi)$ and note that

$$b_i(\psi) = \begin{cases} 1 \ (buy \ now) & if \ \Delta_i(\psi) = E[u_{i1}] - E[u_{i2}] \geq 0 \\ 0 \ (defer \ decision) & if \ \Delta_i(\psi) = E[u_{i1}] - E[u_{i2}] < 0 \end{cases}$$

We establish strict monotonicity of $\Delta_i(\psi)$ in x_i, thereby proving that the equilibrium must admit a first-period threshold structure. We have

$$\Delta_i(\psi) = x_i - p_1 - \delta_c E\left[(x_i + q_u - p_2)^+\right]$$
$$= x_i - p_1 - \delta_c \int_{p_2-x_i}^{+\infty} (x_i + q_u - p_2)f(q_u; 1 - \psi)dq_u,$$

such that $q_u \sim N\left(0, \frac{\psi \sigma_p^4}{\psi \sigma_p^2 + \sigma_q^2}\right)$. The derivative with respect to x_i is given by

$$\frac{\partial \Delta_i}{\partial x_i} = 1 - \delta_c \int_{p_2-x_i}^{+\infty} f(q_u; 1 - \psi)dq_u$$
$$= 1 - \delta_c \bar{F}(p_2 - x_i; 1 - \psi)$$
$$= 1 - \delta_c \bar{\Phi}\left(\frac{p_2 - x_i}{\sigma(1 - \psi)}\right) > 0.$$

Since the above monotonicity holds for any arbitrary ψ customers, it follows that any pure-strategy equilibrium must admit a threshold structure.

Step 2. Given that any equilibrium must follow a threshold structure, we now identify the condition under which the first-period purchasing threshold is such that no customer purchases in the first period (i.e., $\theta(p_1, p_2) = 1$). To do so, we consider the highest x_i customer (i.e., $x_i = 1$). Her first-period expected utility from purchase is $1 - p_1$. If she delays, the threshold structure of the equilibrium implies that no customer buys in the first period, no reviews are generated, and her expected utility from purchasing in the second period is $\delta_c(1 - p_2)$. Thus, the condition for adoption inertia is simply $1 - p_1 < \delta_c(1 - p_2)$, or $p_1 - \delta_c p_2 > 1 - \delta_c$.

Step 3. If the above condition for adoption inertia does not hold, this implies that at least some customers will purchase the product in the first period. Denote customer i's expected utility from purchase in the first period by $v_1(x_i)$, $v_1(x_i) = x_i - p_1$, and customer i's expected utility from purchase in the second period, conditional on customers with $x_i \geq y$ choosing to purchase the product in the first period, by $v_2(x_i, y)$, $v_2(x_i, y) = \delta_c \int_{p_2 - x_i}^{+\infty} (x_i + q_u - p_2) f(q_u; y) dq_u$.

We will show that the indifference equation $v_1(y) = v_2(y, y)$ has exactly one solution, y^*, satisfying $y^* \in (p_1, 1)$. Notice first that $\frac{\partial v_1}{\partial y} = 1$. Next, rewrite $v_2(y, y) = \delta_c \int_{p_2 - y}^{+\infty} (y + q_u - p_2) f(q_u; y) dq_u = \delta_c (y - p_2) \Phi\left(\frac{y - p_2}{\sigma(y)}\right) + \delta_c \sigma(y) \phi\left(\frac{y - p_2}{\sigma(y)}\right) =: u(y)$. The derivative of $u(y)$ yields

$$
\begin{aligned}
\frac{\partial u}{\partial y} =& \delta_c (y - p_2) \phi\left(\frac{y - p_2}{\sigma(y)}\right) \frac{\partial}{\partial y}\left(\frac{y - p_2}{\sigma(y)}\right) + \delta_c \Phi\left(\frac{y - p_2}{\sigma(y)}\right) \\
& + \delta_c \sigma'(y) \phi\left(\frac{y - p_2}{\sigma(y)}\right) + \delta_c \sigma(y) \phi'\left(\frac{y - p_2}{\sigma(y)}\right) \frac{\partial}{\partial y}\left(\frac{y - p_2}{\sigma(y)}\right) \\
=& \delta_c \left(\Phi\left(\frac{y - p_2}{\sigma(y)}\right) + \sigma'(y) \phi\left(\frac{y - p_2}{\sigma(y)}\right) \right),
\end{aligned}
$$

and since $\sigma'(y) < 0$, we have $\frac{\partial u}{\partial y} < 1$. By comparing the derivatives of the two functions, it follows that $v_1(y) = v_2(y, y)$ can have at most one solution in the interval $y \in [0, 1]$. From Step 2, we know that $v_1(1) > v_2(1, 1)$. Moreover, note that $v_1(p_1) = 0$ and that $v_2(y, y) > 0$ for all $y \in [0, 1]$; therefore, we know that $v_1(p_1) < v_2(p_1, p_1)$. Combining the above, we deduce that a unique solution y^* exists and satisfies $y^* \in (p_1, 1)$.

Proof of Corollary 1

Note first that if $p_1 - \delta_c p_2 > 1 - \delta_c$ then no customer purchases in the first period, for any parameter values. If $p_1 - \delta_c p_2 \leq 1 - \delta_c$, then from the proof of Theorem 1, we have $\theta \in (p_1, 1)$ and the number of first-period buyers is $1 - \theta$. From Theorem, 1, this θ satisfies the equation $\theta - p_1 - u(\theta) = 0$ (†), where $u(\theta) = \delta_c \int_{p_2 - \theta}^{+\infty} (\theta + q_u - p_2) f(q_u; \theta) dq_u$. The properties stated in the proposition can be obtained via the implicit function theorem.

Taking the total derivative of (†) with respect to p_1, we have

$$
\frac{\partial \theta}{\partial p_1} - 1 - \frac{\partial u}{\partial \theta} \frac{\partial \theta}{\partial p_1} = \frac{\partial \theta}{\partial p_1}\left(1 - \frac{\partial u}{\partial \theta}\right) - 1 = 0.
$$

From the proof of Theorem 1 we know that $\frac{\partial u}{\partial \theta} < 1$ and therefore the above equation implies $\frac{\partial \theta}{\partial p_1} > 0$, which in turn means that the number of first-period buyers is decreasing in p_1. Next the total derivative with respect to p_2 yields

$$
\begin{aligned}
\frac{\partial \theta}{\partial p_2} - \left(\frac{\partial u}{\partial p_2} + \frac{\partial u}{\partial \theta} \frac{\partial \theta}{\partial p_2}\right) &= \frac{\partial \theta}{\partial p_2} - \left(-\delta_c \bar{F}(p_2 - \theta; \theta) + \frac{\partial u}{\partial \theta} \frac{\partial \theta}{\partial p_2}\right) \\
&= \frac{\partial \theta}{\partial p_2}\left(1 - \frac{\partial u}{\partial \theta}\right) + \delta_c \bar{F}(p_2 - \theta; \theta) = 0.
\end{aligned}
$$

Therefore $\frac{\partial \theta}{\partial p_2} < 0$, which implies that the number of first-period buyers is increasing in p_2.

As for the result pertaining to customers' degree of patience δ_c, we have

$$
\frac{\partial \theta}{\partial \delta_c} - \left(\frac{\partial u}{\partial \delta_c} + \frac{\partial u}{\partial \theta} \frac{\partial \theta}{\partial \delta_c}\right) = \frac{\partial \theta}{\partial \delta_c}\left(1 - \frac{\partial u}{\partial \theta}\right) - \frac{\partial u}{\partial \delta_c} = 0.
$$

Since $\frac{\partial u}{\partial \delta_c} > 0$, it follows that $\frac{\partial \theta}{\partial \delta_c} > 0$.

Finally, note that we may rewrite $u(\theta) = \delta_c(\theta - p_2)\Phi\left(\frac{\theta - p_2}{\sigma(\theta)}\right) + \delta_c\sigma(\theta)\phi\left(\frac{\theta - p_2}{\sigma(\theta)}\right)$, from which it follows that $\frac{\partial u}{\partial \sigma_p} = \delta_c\frac{\partial \sigma(\theta)}{\partial \sigma_p}\phi\left(\frac{\theta - p_2}{\sigma(\theta)}\right) > 0$, since $\sigma(\theta) = \sigma_p\sqrt{\frac{(1-\theta)\gamma}{(1-\theta)\gamma + 1}}$. Taking the total derivative of (†) with respect to σ_p, we have

$$\frac{\partial \theta}{\partial \sigma_p} - \left(\frac{\partial u}{\partial \sigma_p} + \frac{\partial u}{\partial \theta}\frac{\partial \theta}{\partial \sigma_p}\right) = \frac{\partial \theta}{\partial \sigma_p}\left(1 - \frac{\partial u}{\partial \theta}\right) - \delta_c\frac{\partial \sigma(\theta)}{\partial \sigma_p}\phi\left(\frac{\theta - p_2}{\sigma(\theta)}\right) = 0.$$

Therefore $\frac{\partial \theta}{\partial \sigma_p} > 0$, which implies that the number of first-period buyers is decreasing in σ_p. In a similar manner, it can also be verified that $\frac{\partial \theta}{\partial \sigma_q} < 0$, and since $\gamma = \frac{\sigma_p^2}{\sigma_q^2}$ the two latter results imply that $\frac{\partial \theta}{\partial \gamma} > 0$.

Proof of Proposition 3

Note that among all price plans which induce adoption inertia, those which achieve the highest profit have second-period price $p_2 = \frac{1}{2}$ and achieve total profit of $\pi_{in}^* = \frac{1}{4}$. An equivalent way for the firm to achieve this profit is by announcing a price plan $\{\frac{1}{2}, +\infty\}$, i.e., to announce market exit in the second period. Under both policies, exactly half of the population purchases at price $\frac{1}{2}$. To prove that none of these policies can be optimal, we will show that a specific price plan, which does not induce adoption inertia, achieves profit strictly greater than $\frac{1}{4}$. Since this price plan is not necessarily optimal for the firm, it follows that neither adoption inertia nor market exit can be optimal policies.

Consider the price plan $\{\frac{1}{2}, \frac{1}{2}\}$. In this case, first-period profit is $\pi_1 = \frac{1}{2}(1 - \theta)$ for some $\theta \in (\frac{1}{2}, 1)$ (see Theorem 1). Denoting second-period expected profit by π_2, we will show that $\pi_1 + \pi_2 > \frac{1}{4}$. Equivalently, we will show $\pi_2 = \frac{1}{2}E[s_2] > \frac{1}{2}(\theta - \frac{1}{2})$, i.e., $E[s_2] > (\theta - \frac{1}{2})$ where s_2 denotes second-period sales. Note that $q_u \sim N(0, \sigma(\theta))$ and that

$$s_2(q_u) = \begin{cases} 0 & \text{if } q_u \leq \frac{1}{2} - \theta, \\ \theta + q_u - \frac{1}{2} & \text{if } \frac{1}{2} - \theta < q_u \leq \frac{1}{2}, \\ \theta & \text{if } q_u > \frac{1}{2}. \end{cases}$$

Next, decompose $s_2(q_u)$ into $s_2(q_u) = s_a(q_u) + s_b(q_u)$, where

$$s_a(q_u) = \begin{cases} 0 & \text{if } q_u \leq \frac{1}{2} - \theta, \\ \theta + q_u - \frac{1}{2} & \text{if } \frac{1}{2} - \theta < q_u \leq \theta - \frac{1}{2}, \\ 2\theta - 1 & \text{if } q_u > \theta - \frac{1}{2}, \end{cases}$$

and

$$s_b(q_u) = \begin{cases} 0 & \text{if } q_u \leq \theta - \frac{1}{2}, \\ -\theta + q_u + \frac{1}{2} & \text{if } \theta - \frac{1}{2} < q_u \leq \frac{1}{2}, \\ 1 - \theta & \text{if } q_u > \frac{1}{2}. \end{cases}$$

We have

$$E[s_2] = \int_{-\infty}^{+\infty} s_2(q_u)f(q_u; \theta)dq_u = \int_{-\infty}^{+\infty}(s_a(q_u) + s_b(q_u))f(q_u; \theta)dq_u \tag{7}$$

$$= \int_{-\infty}^{+\infty} s_a(q_u)f(q_u; \theta)dq_u + \int_{-\infty}^{+\infty} s_b(q_u)f(q_u; \theta)dq_u. \tag{8}$$

The symmetry of the r.v. q_u around zero and the function $s_a(q_u)$ imply $\int_{-\infty}^{+\infty} s_a(q_u)f(q_u; \theta)dq_u = \frac{2\theta - 1}{2} = \theta - \frac{1}{2}$. Moreover, since $s_b(q_u)$ is a non-negative function of q_u (and positive for some values of q_u), it follows that $\int_{-\infty}^{+\infty} s_b(q_u)f(q_u; \theta)dq_u > 0$. Therefore, we have $E[s_2] > \theta - \frac{1}{2}$, and the proof is complete.

Proof of Lemma 3

We show the pricing and profit impact of the valuation effect in turn, starting from the pricing impact. For $\delta_c = 0$, we have $\theta(p_1, p_2) = p_1$ for all p_1, p_2. Consider some arbitrary p_1; denoting by p_2^* the optimal second period price we want to show that for $\gamma > 0$ we have $p_2^* > \frac{p_1}{2}$ (note that for $\gamma \to 0$ we have $p_2^* = \frac{p_1}{2}$ from Proposition 1). Expected profit is given by

$$\pi(p_2) = p_1(1 - p_1) + p_2 \int_{p_2 - p_1}^{p_2} (p_1 + q - p_2) f(q; p_1) dq + p_2 \int_{p_2}^{+\infty} p_1 f(q; p_1) dq.$$

The first-order necessary condition for p_2^* is

$$\frac{d\pi}{dp_2} = \int_{p_2 - p_1}^{p_2} (p_1 + q - p_2) f(q; p_1) dq + p_2 \left(- \int_{p_2 - p_1}^{p_2} f(q; p_1) dq + p_1 f(p_2; p_1) \right) + \int_{p_2}^{+\infty} p_1 f(q; p_1) dq - p_2 p_1 f(p_2; p_1)$$

$$= p_1 \int_{p_2 - p_1}^{+\infty} f(q; p_1) dq - 2p_2 \int_{p_2 - p_1}^{p_2} f(q; p_1) dq + \int_{p_2 - p_1}^{p_2} q f(q; p_1) dq = 0.$$

Notice that $\int_{p_2 - p_1}^{+\infty} f(q; p_1) dq > \int_{p_2 - p_1}^{p_2} f(q; p_1) dq > 0$ and $\int_{p_2 - p_1}^{p_2} q f(q; p_1) dq > 0$. Defining $a := \int_{p_2 - p_1}^{+\infty} f(q; p_1) dq$, $b := \int_{p_2 - p_1}^{p_2} f(q; p_1) dq$ and $c := \int_{p_2 - p_1}^{p_2} q f(q; p_1) dq$, we have

$$ap_1 - 2bp_2 + c = 0,$$

where $a > b > 0$. Next notice that for $p_2 = \frac{p_1}{2}$ (i.e., the optimal second-period price at $\gamma = 0$) we have $c = 0$. It follows that at $p_2 = \frac{p_1}{2}$, we have $\frac{d\pi}{dp_2} = (a - b)p_1 > 0$, which implies $p_2^* > \frac{p_1}{2}$.

Next, consider the impact of social learning on profit. Note that since we have fixed p_1 and customers are myopic, the firm's first-period profit is fixed and we need only consider differences in expected second-period profit. Assume that the firm announces $\hat{p}_2 = \frac{p_1}{2}$ irrespective of γ. Denoting second period profit at second-period price p_2 by $\pi_2(p_2)$, we will show that $E[\pi_2(\hat{p}_2)]|_{\gamma=k} = E[\pi_2(\hat{p}_2)]|_{\gamma\to 0}$ for any $k > 0$. Since $\hat{p}_2$ is optimal for $\gamma \to 0$ but suboptimal for $\gamma = k$, this implies that the firm achieves higher expected second-period profit, and therefore higher overall expected profit, in the presence of SL. The firm's second-period profit at $\hat{p}_2 = \frac{p_1}{2}$ as a function of q_u is

$$\pi_2(\hat{p}_2) = \begin{cases} 0 & \text{if } q_u \leq -\frac{p_1}{2}, \\ \left(\frac{p_1^2}{4} + q_u \frac{p_1}{2} \right) & \text{if } -\frac{p_1}{2} < q_u \leq \frac{p_1}{2}, \\ \frac{p_1^2}{2} & \text{if } q_u > \frac{p_1}{2}. \end{cases}$$

Now, note that $E[\pi_2(\hat{p}_2)]|_{\gamma\to 0} = \frac{p_1^2}{4}$ and consider the difference $E[\pi_2(\hat{p}_2)]|_{\gamma=k} - E[\pi_2(\hat{p}_2)]|_{\gamma\to 0}$.

$$E[\pi_2(\hat{p}_2)]|_{\gamma=k} - E[\pi_2(\hat{p}_2)]|_{\gamma\to 0} = \int_{-\frac{p_1}{2}}^{\frac{p_1}{2}} \left(\frac{p_1^2}{4} + q_u \frac{p_1}{2} \right) f(q_u; p_1) dq_u + \int_{\frac{p_1}{2}}^{+\infty} \frac{p_1^2}{2} f(q_u; p_1) dq_u - \frac{p_1^2}{4}. \quad (9)$$

Replacing $\frac{p_1^2}{4} = \int_{-\infty}^{+\infty} \frac{p_1^2}{4} f(q_u; p_1) dq_u$ and breaking up the integral accordingly, it is straightforward to show $E[\pi_2(\hat{p}_2)]|_{\gamma=k} - E[\pi_2(\hat{p}_2)]|_{\gamma\to 0} = 0$. Since the second-period price $\hat{p}_2 = \frac{p_1}{2}$ is suboptimal for any $\gamma > 0$, we conclude that $\pi^*|_{\gamma=k} > \pi^*|_{\gamma\to 0}$ for any $k > 0$.

Proof of Proposition 4

Using the result of Proposition 3, we may restrict our attention to policies $\{p_1, p_2\}$ which result in $\theta \in (p_1, 1)$. The indifference equation which connects the price plan to the first-period purchasing threshold, θ, is $\theta - p_1 = \delta_c \int_{p_2 - \theta}^{+\infty} (\theta + q_u - p_2) f(q_u; \theta)$ $(\star)$. From the proof of Lemma 2, we know that for any $\gamma > 0$, $\delta_c \int_{p_2 - \theta}^{+\infty} (\theta + q_u - p_2) f(q_u; \theta) > \delta_c (\theta - p_2)$; that is, the marginal customer's second-period expected utility in the presence of SL ($\gamma > 0$) is greater than that in its absence ($\gamma \to 0$). Therefore, $(\star)$ implies that the equilibrium price plan and θ satisfy $\theta - p_1 > \delta_c(\theta - p_2)$. At the extreme case of $\delta_c = 1$, the last inequality implies $p_2 > p_1$.

Proof of Proposition 5

Denote firm profit at prices $\{p_1, p_2\}$ in the absence of SL by $\pi_n(p_1, p_2)$. From Proposition 1 it follows that when $\delta_c = 1$, $\pi_n^* = \max_{p_1, p_2} \pi_n = \frac{1}{4}$. In the presence of social learning, the firm can achieve profit equal to π_n^* by inducing adoption inertia (see proof of Proposition 3). But from the result of Proposition 3, we know that adoption inertia is suboptimal for the firm for any $\gamma > 0$ and the result stated in the proposition follows. (Furthermore, from the proof of Proposition 3 we also know that if the firm chooses the price plan $p_1 = p_2 = \frac{1}{2}$ it achieves expected profit strictly higher than $\frac{1}{4}$. It follows that the result continues to hold even if the firm is restricted to announcing a constant price.)

Proof of Proposition 6

In the second period, the firm faces a mass of $\bar{x}$ customers with valuations uniformly distributed $U[q_u, q_u + \bar{x}]$. If $q_u \leq -\bar{x}$ then no customer will purchase the product in the second period at any non-negative price. In this case, the firm exits the market; this is denoted by an optimal price $p_2^* = 0$, and second-period profit is $\pi_2 = 0$. If $q_u > -\bar{x}$ then the firm's profit function is $\pi_2(p_2) = p_2 \min\{q_u + \bar{x} - p_2, \bar{x}\}$. Clearly, any price $p_2 < q_u$ cannot be optimal; we therefore restrict our attention to the function $\pi_2(p_2) = p_2(q_u + \bar{x} - p_2)$. If $q_u > \bar{x}$, the profit function is decreasing for $p_2 \geq q_u$, and we have $p_2^* = q_u$ with associated profits $\pi_2^* = q_u \bar{x}$. If $-\bar{x} < q_u \leq \bar{x}$, the profit function is increasing at $p_2 = q_u$ and concave, and we have $p_2^* = \frac{q_u + \bar{x}}{2}$ with associated profits $\pi_2^* = \left(\frac{q_u + \bar{x}}{2}\right)^2$. The customer's second-period purchasing decision is trivial: she purchases provided $x_i + q_u - p_2^*$ is non-negative.

Proof of Theorem 2

We first establish that any pure-strategy purchasing equilibrium must follow a threshold structure. Assume that (any) ψ customers purchase the product in the first period and that the firm's profit-maximizing price in the second period is some p_2. In the spirit of the proof of Theorem 1, at first-period price p_1, we have

$$\Delta_i(\psi) = x_i - p_1 - \delta_c E\left[(x_i + q_u - p_2)^+\right]$$
$$= x_i - p_1 - \delta_c \int_{p_2 - x_i}^{+\infty} (x_i + q_u - p_2) f(q_u; 1 - \psi) dq_u,$$

where $q_u \sim N\left(0, \frac{\psi \sigma_p^4}{\psi \sigma_p^2 + \sigma_q^2}\right)$. As in Theorem 1, we have $\frac{\partial \Delta_i}{\partial x_i} > 0$, which establishes that any equilibrium must follow a threshold structure.

Using the optimal second-period price charged by the firm given by Proposition 6, at any first-period price p_1 and assuming that customers with $x_i \geq \bar{x}$ purchase in the first period, we have for customer i

$$\Delta_i = x_i - p_1 - \delta_c \int_{\bar{x} - 2x_i}^{+\bar{x}} \left(x_i + q_u - \frac{q_u + \bar{x}}{2}\right) f(q_u; \bar{x}) dq_u - \delta_c \int_{+\bar{x}}^{+\infty} x_i f(q_u; \bar{x}) dq_u.$$

To find the equilibrium $\bar{x}$, we solve the system of equations $\Delta_i = 0$ and $x_i = \bar{x}$. Inserting the latter into the former

$$\Delta_i = \bar{x} - p_1 - \delta_c \int_{-\bar{x}}^{+\bar{x}} \left(\frac{\bar{x} + q_u}{2}\right) f(q_u; \bar{x}) dq_u - \delta_c \int_{+\bar{x}}^{+\infty} \bar{x} f(q_u; \bar{x}) dq_u$$
$$= \bar{x} - p_1 - \delta_c \frac{\bar{x}}{2} \left(\bar{F}(-\bar{x}; \bar{x}) - \bar{F}(\bar{x}; \bar{x})\right) - \delta_c \bar{x} \bar{F}(\bar{x}; \bar{x})$$
$$= \bar{x} - p_1 - \delta_c \frac{\bar{x}}{2} \left(\bar{F}(-\bar{x}; \bar{x}) + \bar{F}(\bar{x}; \bar{x})\right)$$
$$= \bar{x} - p_1 - \delta_c \frac{\bar{x}}{2} = 0,$$

which leads to $\bar{x} = \frac{p_1}{1 - \frac{\delta_c}{2}}$. Finally, note that if $\frac{p_1}{1 - \frac{\delta_c}{2}} > 1 = \max x_i$ all customers defer their purchasing decision.

Proof of Corollary 2

Follows directly by comparing the equilibrium outcome of Theorem 2 with equation (3).

Proof of Proposition 7

When $p_1 > 1 - \frac{\delta_c}{2}$ (i.e., in the case of adoption inertia), the firm's profit in the presence of social learning is identical to that in its absence, and equal to $\frac{1}{4}$. From Proposition 2 we know that in the benchmark case, the firm's optimal price, p_{1b}, satisfies $p_{1b} < 1 - \frac{\delta_c}{2}$ which implies $\pi(p_{1b})|_{\gamma \to 0} > \frac{1}{4}$. We will show that $\pi(p_{1b})|_{\gamma=k} > \pi(p_{1b})|_{\gamma \to 0}$ for any $k > 0$, which implies that adoption inertia can never be optimal for the firm in the presence of SL. Since $\zeta(p_1)$ of Theorem 2 is independent of γ, the firm's first-period profit is fixed, and we need only consider differences in expected second-period profit. The firm's second-period profit as a function of q_u is defined by

$$\pi_2(q_u) = \begin{cases} 0 & \text{if } q_u \leq -\zeta, \\ \left(\frac{q_u + \zeta}{2}\right)^2 & \text{if } -\zeta < q_u \leq \zeta, \\ q_u \zeta & \text{if } q_u > \zeta. \end{cases}$$

Note that $\pi_2(q_u)$ is continuous, non-negative, strictly increasing and convex in q_u. The firm's expected second-period profit is

$$E[\pi_2] = \int_{-\infty}^{+\infty} \pi_2(q_u) f(q_u; \zeta) dq_u.$$

Since the ex ante variance of q_u is is strictly increasing in γ, it follows that $E[\pi_2]|_{\gamma=k} > E[\pi_2]|_{\gamma \to 0}$ for any $k > 0$. Therefore, $\pi(p_{1b})|_{\gamma=k} > \pi(p_{1b})|_{\gamma \to 0}$. Since, p_{1b} is not necessarily optimal in the presence of SL but nevertheless achieves profit higher than inducing adoption inertia, it follows that adoption inertia can never be optimal for the firm.

Proof of Corollary 3

In Proposition 6, we may set $\bar{x} = \zeta(p_1)$ as per Theorem 2. Then, it follows that p_2^* is higher than p_1 when $\frac{q_u + \zeta(p_1)}{2} > p_1$. Since $\zeta(p_1) = \frac{p_1}{1 - \frac{\delta_c}{2}}$, this implies $q_u > \frac{1 - \delta_c}{1 - \frac{\delta_c}{2}} p_1$. The probability with which the latter occurs is $\bar{F}\left(\frac{1 - \delta_c}{1 - \frac{\delta_c}{2}} p_1; \zeta(p_1)\right)$ and since q_u is a zero-mean Normal r.v. this probability is strictly less than $\frac{1}{2}$ for any $p_1 > 0$ and strictly higher than zero.

Proof of Proposition 8

Consider any fixed p_1, such that adoption inertia does not occur in accordance with Proposition 7. Since $\zeta(p_1)$ is independent of both γ and σ_p, the firm's first-period profit is independent of both. From the proof of Proposition 7, we have that the firm's second-period profit $\pi_2(q_u)$ is continuous, non-negative, strictly increasing and convex in q_u, and the firm's expected second-period profit is

$$E[\pi_2] = \int_{-\infty}^{+\infty} \pi_2(q_u) f(q_u; \zeta) dq_u.$$

Recall that $f(\cdot; \zeta)$ is the density function of a zero-mean Normal random variable with standard deviation $\sigma(\zeta) = \sigma_p \sqrt{\frac{(1-\zeta)\gamma}{(1-\zeta)\gamma+1}}$. It follows that $\sigma(\zeta)$ is strictly increasing in both γ and σ_p. Therefore, $E[\pi_2]$ is strictly increasing in $\sigma(\zeta)$. Since the result holds for any arbitrary first-period price p_1, it must be the case that the firm's optimal expected profit is strictly increasing in γ, σ_p.

Proof of Lemma 4

Using the optimal prices stated in Propositions 1 and 2, we may calculate the difference in optimal firm profit under pre-announced and responsive pricing as $\Delta\pi_b^* = \pi_{bp}^* - \pi_{br}^* = \frac{\delta_c^2(1-\delta_c)}{4(9-2\delta_c^2-3\delta_c)}$. Thus, we have $\Delta\pi_b^* > 0$ for any $\delta_c \in (0,1)$.

Proof of Lemma 5

Assume $\delta_c = 0$. Let $\{p_1^*, p_2^*\}$ be the optimal pre-announced price plan. Consider a responsive price plan $\{p_1, p_2\}$ with $p_1 = p_1^*$ and $p_2 = p_2^*$ for $q_u \geq p_2^* - p_1^*$, and $p_2 = p_2^* - \epsilon$ for $q_u < p_2^* - p_1^*$, where ϵ is small and positive (this responsive price plan is not necessarily optimal, but suffices for the purposes of our argument). Notice that the two pricing policies achieve identical first-period profit, identical reviews and identical second-period profit for any realization of q_u such that $q_u \geq p_2^* - p_1^*$. However, the responsive price plan achieves higher second-period profit for at least some $q_u < p_2^* - p_1^*$: under all such scenarios the pre-announced price plan achieves zero second-period profit, while the responsive price plan achieves positive profit under at least some scenarios. Since scenarios under which the responsive price plan achieves higher second-period profit occur with a strictly positive probability (q_u is a Normal r.v.), it follows that the optimal responsive policy outperforms the optimal pre-announced policy for any $\gamma > 0$.

Proof of Lemma 6

Using the optimal prices charged by the firm under each pricing policy from Propositions 1 and 2, it is straightforward to write down the difference between consumer surplus under pre-announced pricing and under responsive pricing, $\Delta s_b = s_{bp} - s_{br}$, at any δ_c as

$$\Delta s_b = -\frac{\delta_c^2\left(\delta_c^3 + 14\delta_c^2 - 51\delta_c + 36\right)}{8\left(2\delta_c^2 + 3\delta_c - 9\right)^2}.$$

The above difference is negative for any $\delta_c \in (0,1)$, implying that consumer surplus is always greater under responsive pricing.

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