Yardstick Competition for Service Systems

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Yardstick competition is a regulatory scheme for local monopolists (e.g., hospitals), where the monopolist's reimbursement is linked to her performance relative to other equivalent monopolists. This regulatory scheme is known to work well in providing cost-reduction incentives and serves as the theoretical underpinning behind the hospital prospective reimbursement system used throughout the developed world. This paper uses a game-theoretic queueing model to investigate how yardstick competition performs in service systems (e.g., hospital emergency departments), where, in addition to incentivizing cost reduction, the regulator's goal is to provide incentives to reduce customers' waiting times. We first show that the form of cost-based yardstick competition used in practice results in inefficiently long waiting times and may also lead to inefficiently high costs. We then demonstrate how yardstick competition can be appropriately modified to achieve the dual goal of cost and waiting-time reduction. In particular, we show that full efficiency (*first-best*) can be restored if the regulator makes the providers' reimbursement contingent on their service rates and is also able to charge a provider-specific "toll" to consumers. If such a toll is not feasible, as may be the case in healthcare, we show that an alternative waiting-time-based version of yardstick competition can significantly improve system efficiency (*second-best*), and it is also easier to implement, as does not require the regulator to have detailed knowledge of the queueing discipline. We conclude with a detailed numerical analysis that provides insights on the practical implementation of yardstick competition in the healthcare industry.

Key words: Yardstick competition, hospital regulation, emergency care, game theory, queueing theory.
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1. Introduction

Services constitute a large part of the developed world economy, and in some cases, they operate as regulated monopolies. A case in point is the hospital industry, which in 2014 constituted 5.6% of the US economy, and is highly regulated by bodies such as the Centers for Medicare and Medicaid (CMS), which are responsible for approximately 45% of hospital reimbursement(CMS 2014).[1] Despite this, academic research on the regulation of service monopolies has not received as much attention as that of "production" monopolies, e.g., defense systems, water, and energy (Laffont and Tirole 1993, Roques and Savva 2009).

[1] In other developed world healthcare systems, such as the UK, hospitals are just as large as a part of the national economy and most of their reimbursement comes from a single government-funded payer that acts as the regulator.

This paper focuses on a specific regulatory scheme, yardstick competition, in which the regulator induces artificial competition between local monopolists by rewarding firms on the basis of their performance relative to each other. Yardstick competition has been shown to provide incentives for monopolists to minimize production costs (Shleifer 1985) and has been widely applied to the reimbursement of regulated utilities (e.g., electricity (Jamasb and Pollitt (2000)) and tertiary care (through CMS's Diagnosis-Related Group (DRG) prospective reimbursement in the US and equivalent schemes in other developed economies (Fetter 1991)). This paper shows that yardstick competition fails to incentivize investment in wait-time reduction in service systems. In fact, this may be a contributing factor to the undesirably long waiting times observed in some some service systems, such as hospital care (Government Accountability Office 2009), that are reimbursed through yardstick competition. To incentivize waiting-time reduction through capacity investment or process re-engineering, without compromising incentives for cost control, the yardstick competition scheme must be modified. This paper proposes one such modification.

What is yardstick competition? Historically, franchised monopolies had been subject to "cost-of-service" regulation, where the regulated firm's price is set equal to the (marginal) cost of production, along with a transfer payment, if needed, to ensure the monopolist breaks even. The fee-for-service reimbursement model, which had been used by CMS in reimbursing US-based hospitals up to 1983 is one such example (Mayes 2007). Besides simplicity, the advantage of this regulatory scheme is that it avoids the higher monopoly price and the associated welfare loss, whilst providing incentives for the firm to continue production. The disadvantage is that it does not provide incentives to minimize the cost of production.

A better alternative would be to dissociate the firm's price from the firm's cost of production, and instead set it equal to an exogenous benchmark. Setting this benchmark optimally, however, would require the regulator to know as much about the available cost-reduction technologies as the regulated firm. Yardstick competition gets around this difficulty by making use of the production cost of other equivalent monopolists to infer a firm's attainable costs, which will then serve as the exogenous benchmark. For example, the regulator could choose to reimburse the monopolist at the average production cost of all other monopolists. By doing so, the regulator forces the firms to engage in a cost-reduction competition, akin to a tournament, which, as shown in Shleifer (1985), has (under mild conditions) a unique symmetric Nash equilibrium that leads to first-best outcomes, i.e., it achieves the same cost-reduction investment as that chosen by the regulator under full information. Since the price is set through observable and verifiable benchmarks, this scheme can be implemented using accounting data without requiring symmetric information between the regulator and the regulated firms.

Yardstick competition in services. While the economic rationale behind yardstick competition is sound, the theory outlined above abstracts away one important aspect of service provision: waiting times. Effectively, it assumes that production is instantaneous, or equivalently, that consumers' cost of waiting is negligible. This assumption may fit product-based monopolies, but it is less realistic for service settings where long waiting times are costly and, as in the case of hospital Emergency Departments (EDs), even dangerous. It is, therefore, important that, in addition to cost reduction, the regulation of service systems should also incentivize wait-time reduction.

To do this we present a game theoretic queueing model of yardstick competition, where a regulator is responsible for multiple identical service providers that act as local monopolists. The regulator's objective is to maximize total welfare by dictating the price that customers are charged and any transfer payments made to the service providers. Service providers are assumed to be profit maximizers and, given the price and transfer payment set by the regulator, decide how much to invest in cost- and wait-time-reduction effort. Finally, customers are heterogeneous in their willingness to pay for the service leading to an endogenous demand function that is decreasing in both the price, as well as the waiting time they expect to incur. A crucial feature of our model, which explains why optimal regulation may be difficult, is that the providers' cost technology and the customer demand function are not known to the regulator. We note that extant models that ignore the cost of waiting (e.g., Shleifer 1985) are a special case of the model presented here.

Using this model, we show that in contrast to product-based monopolies, applying the standard cost-based yardstick competition scheme to service monopolies results in inefficiently long waiting times, and may even result to inefficiently high costs due to two types of inefficiencies. First, customers will over-join the service system compared to first-best, a result first noted in Naor (1969). This is due to a negative externality: a customer's decision to join the system will lead to an increase in the expected waiting time of others. Nevertheless, this increase does not feature in his personal calculation as to whether to join the system or not. Second, the cost-based yardstick competition fails to generate incentives for the service providers to increase capacity.

To resolve these two inefficiencies, i.e., incentivize optimal customer joining behavior and adequately incentivize waiting-time reduction, yardstick competition should be modified in two ways. First, the price that customers pay to access the service should be higher than the costs of service production estimated through the yardstick competition benchmarks. This higher price constitutes a form of "toll" and is equal to the displaced utility that a customer's joining decision generates (Naor 1969). Second, the monopolists' reimbursement should have a component that depends on their investment in wait-time reduction. More specifically, the monopolist's reimbursement should include a component that depends on the difference between her service rate and the average service rate of all other monopolists that serve as the benchmark. This modified yardstick competition

achieves first-best outcomes on both cost and waiting-time reduction and, just like the cost-based yardstick competition, can be implemented using accounting data on costs and waiting times, along with some information on the queueing discipline.

An essential feature of the yardstick competition scheme discussed above is that customers are charged a provider-specific fee for accessing the service. In many healthcare settings, this is not the case; care is either free at the point of access, or patients are charged a fixed insurance deductible. We show that if customers are charged an exogenous fee (which could be zero) then there is an inefficiency due to the suboptimal customer joining behavior. Nevertheless, there exists an alternative yardstick competition scheme that still achieves second-best, i.e., there is no additional loss of welfare due to underinvestment in either cost or waiting-time reduction. Furthermore, this scheme also attenuates the regulator's information requirements – the regulator needs to observe the customer waiting time and total provider costs, but does not need to know anything about the queueing discipline or how the costs are split between fixed and variable.

Finally, we present a series of extensions that explain how yardstick competition can be implemented in settings with multiple customer classes, time-varying arrival rates, more general cost structures and discuss how the regulator could adjust the yardstick competition scheme to account for heterogeneity between the local monopolists based on exogenous and observable characteristics. We also present a detailed numerical analysis that investigates the magnitude of the welfare loss associated with second-best outcomes compared to fist-best, and the dependance of the equilibrium outcome on the cost of waiting. One interesting finding is that, in contrast to equilibrium waiting times, total welfare is not very sensitive to the cost-of-waiting parameter. This suggests that the service provider may be able to use the cost-of-waiting parameter as a lever to shift the equilibrium outcome associated with the modified yardstick competition to one with lower waiting times at the expense of higher costs.

Yardstick competition and hospital reimbursement. One immediate application of the modified yardstick competition model proposed by this paper is the regulation of hospital EDs. First, hospitals act, to a large extent, as local monopolists, as patients do not (or cannot) shop around for hospital care (Thomson 1994, Wang et al. 2016). Second, hospitals in general, and EDs in particular, are comparable in the sense that the medical "technology," which is based on the existing understanding of biology, pharmacology, physiology etc., is not hospital specific and neither is the waiting-time reduction "technology." Furthermore, any differences between hospitals, such as labor costs, case mix, and size of catchment area, are readily observable and exogenous. These two reasons explain why yardstick competition has been widely adopted around the world

as a form of regulation for this industry.[2] Third, waiting time to receive emergency care is both costly and, in many cases, excessively long.[3]

This paper provides an explanation as to why there is such a systematic underinvestment in reduction of waiting times in emergency care. It also provides guidelines on how regulators could modify the reimbursement system already in use to provide incentives for waiting time reduction. More specifically, the second-best scheme that we propose is particularly useful, as it requires little additional information to implement (just average waiting time at the ED)[4] and achieves outcomes that cannot be improved upon without changing the way that patients are charged for accessing emergency care.

Beyond healthcare and hospital reimbursement, our work serves as an introduction to the notion of yardstick competition to the operations management community. More specifically, yardstick competition might be a useful tool for other service systems that operate as local monopolies (e.g., governmental agencies, such as the Department of Motor Vehicles and Social Security offices in the US; (former) quasi-state monopolies, such as the post office; airport or border security checkpoints), as well as within a service firm to incentivize better performance for individual servers.

2. Literature Review

The observation that relative-performance evaluations is a useful tool for setting incentives has been made by Holmstrom (1982), Nalebuff and Stiglitz (1983), and Shleifer (1985) in related

[2] The form of yardstick competition used in hospital reimbursement is based on the work of Professor Fetter and co-authors in the 1970-80s, which classifies acute patients into Diagnosis-Related Groups (DRGs) based on their diagnosis, existing complications and comorbidities, and patient-specific characteristics, e.g., age (see Fetter (1991)). The classification is done so that patient episodes within a group will require a homogeneous bundle of services and goods to be diagnosed and treated, and should therefore cost the same irrespective of where this treatment takes place. Hospitals are then reimbursed a fixed amount per patient that depends only on the patient's DRG. The amount is set to the average of the reported (and audited) cost of treating patients of the same DRG in other hospitals (subject to adjustments based on exogenous hospital characteristics). Since its introduction by CMS in the US in the 1980s, the system has been adopted by private insurance firms and healthcare payers throughout the developed world.

[3] According to the US Government Accountability Office, the most urgent patients who should have been seen in less than 1 minute waited for 28 minutes on average, and the emergent patients who are recommended to be seen in less than 14 minutes waited an average of 37 minutes (Government Accountability Office 2009). Similarly, ED wait times in England have risen by one third in November 2015 compared to November 2014 (Siddique 2016) and one out of 10 patients spend at least eight hours in Canadian EDs (Canadian Institute for Health Information 2012). Furthermore, delays in the ED have been associated with a number of adverse outcomes, such as patient dissatisfaction, higher rates of medical errors, higher mortality rates, and more patients leaving without receiving treatment (Batt and Terwiesch 2015).

[4] In fact, regulators around the world have started collecting waiting-time data. For example, Monitor, the UK hospital regulator, collects data on ED waiting times and has placed an ad-hoc target that at least 95% of patients have to be admitted or discharged within 4 hours of arriving to the ED with financial penalties for failure to comply (Campbell 2016). Similarly, CMS has started collecting data on ED waiting times, which are reported on the Hospital Compare website (CMS 2016b).

contexts: the first focuses on curbing free-riding in teams, the second on optimal risk-sharing, and the third on cost-cutting incentives for regulated firms. In fact, the term "yardstick competition" is used by Shleifer (1985) to describe this form of regulation. From a practical perspective, yardstick competition has been implemented in some industries (e.g., electricity (Jamasb and Pollitt 2000), water and sewage (Sawkins 1995)) and has been used to generate insights on the structure and fiscal policy of government in democratic countries (Besley and Case 1995, Bordignon et al. 2003).

Several extensions to the model of yardstick competition for regulation have been presented in the literature. For example, Laffont and Tirole (1993), p. 84-86 augment the model of yardstick competition to regulate firms whose costs are imperfectly correlated. Sobel (1999) examines the case where transfers are costly to show that yardstick competition may discourage investment. This setting has been examined further in Dalen (1998) and more recently Lefouili (2015). More relevant to our work are models that use yardstick competition to incentivize improvements in additional dimensions of performance, such as quality. Examples include Ellis and McGuire (1986), Pope (1989), Ma (1994), and Tangerås (2009), where the general finding is that quality is better served by more complicated forms of yardstick competition. Our work adds to this literature because unlike quality, i) waiting times are endogenous to customer behaviour and generate an externality on the customer side: a customer's decision to visit a service provider increases waiting times which places a negative externality on all other customers who join after her (see also Naor (1969)); ii) waiting times are governed by well-understood non-linear dynamics that need to be accounted for; iii) waiting times are often easier to quantify and less controversial to compare across providers than other quality measures (e.g., in-hospital mortality).

In addition to the literature on yardstick competition, this paper also contributes to the operations management (OM) literature that examines incentives and competition in queueing systems. Traditionally, queueing theory, which is well surveyed in Kleinrock (1975), has been concerned with the mathematical description and optimization of queueing systems without considering customer behavior or agency issues on behalf of firm management. Early attempts to include such economic considerations are well-reviewed by Hassin and Haviv (2003), while more recent work is presented in Hassin (2016). Our work brings together elements from i) the literature on strategic customer behavior in monopolistic queueing systems, which was first studied in Naor (1969) for observable queues and extended to unobservable queues by Edelson and Hilderbrand (1975) and multiclass queues by Mendelson and Whang (1990) and Afeche (2013); and ii) the literature on queueing games where service providers compete based on price and congestion (e.g., Cachon and Harker (2002), Cachon and Zhang (2006), Allon and Federgruen (2007) and Allon and Federgruen (2008)). Closest to our work are the studies of queuing games in the context of hospital/ED congestion. An early example is Lee and Cohen (1985) which studies a setting where agents (e.g., physicians)

direct customers to multiple service providers (e.g., hospitals) as a non-cooperative queuing game where congestion at each service provider plays the role of prices. They show that the equilibrium outcome will, in most cases, entail loss of efficiency. More recently, literature has examined resource pooling though ambulance diversion as a non-cooperative queueing game between competing EDs (see Deo and Gurvich (2011) and Do and Shunko (2016)). They show that there exist equilibria where diversions are not used optimally and propose alternative policies that are Pareto-improving. In contrast to this stream of literature, in our setting service providers do not compete directly. Instead, competition is induced by the regulator through the reimbursement mechanism. Furthermore, our work is complementary to the aforementioned papers on ambulance diversion, as it focuses on hospital reimbursement mechanisms that incentivize optimal investment in capacity, which, as a side-effect, may make the need to divert ambulances less prevalent.

Our work is also related to the OM literature on performance-based incentives in services in general (Akan et al. 2011, Bakshi et al. 2015, Hasija et al. 2008, Kim et al. 2007, Kim et al. 2010, Ren and Zhou 2008) and in healthcare specifically (So and Tang 2000, Jiang et al. 2012, Lee and Zenios 2012, Adida et al. 2016, Guo et al. 2016, Andritsos and Aflaki 2015, Jiang et al. 2016). The last two papers also consider direct competition between providers (i.e., a queueing game) in the presence of performance-based incentives. Our paper differs, as the performance-based incentives that we consider are not set exogenously by the regulator, but are the result of endogenous benchmarks. This is an important difference because it generates (indirect) competition between otherwise monopolistic providers and, as we show in this paper, may be easier to implement, as it places less onerous informational burden on the regulator.

3. Model Description

The model of this paper considers the interaction between three parties: the regulator, the service provider, and the customers. The regulator has $N \geq 2$ identical service providers (or firms) under his jurisdiction and has the ability to set the price that customers are charged and may also decide to award an additional transfer payment to the service providers, which may depend on any observable and verifiable quantity. Customers observe how much they need to pay for service and make a decision as to whether to request service, which is provided on a-first-come-first-served basis. As a result, customers may experience a costly wait, which we model explicitly using queueing theory. Finally, the service providers act as risk-neutral local monopolists. They observe the price and transfer payment set by the regulator and, given customer behavior, decide how much to invest in cost- and wait-time-reduction effort. We present the details of the decisions and payoffs of each of the three parties and discuss how our model applies to the hospital emergency care setting in §§3.1-3.3.

3.1. Service Environment, Customer Utility and Equilibrium Arrival Rate

We assume that within the catchment area of each service provider, there is a large population of customers who may experience a service need with an exogenous probability. The aggregate arrival rate of service needs may then be modeled as a Poisson process with rate Λ per unit time (even if customers are strategic, see Lariviere and Van Mieghem (2004)). Each customer with a service need makes a decision whether to visit the service provider to receive service on a first-come-first-served basis, or use their outside option which, without loss of generality, we assume has value of zero. In the case of emergency care, these assumptions reflect the case where patients have a single ED that they would consider visiting, either due to prohibitive transportation costs, informational frictions, or idiosyncratic preferences for a specific ED, e.g., the closest (Brown et al. 2015). In this case, Λ would reflect the number of patients per unit time that exhibit a symptom (e.g., chest pain) for which they would consider visiting the ED. Since the patients exhibit the same symptom, they would all be classified in the same triage category; therefore, first-come-first-served is a reasonable assumption. (We present an extension to multiple customer classes and non-stationary arrivals in §5.1). If patients choose not to visit the ED, their outside options could be to use primary care or not to seek treatment.

Each customer's utility from receiving service comprises of three components. The first is the benefit from the service r, which is net of any indirect costs associated with the service (e.g., net of transportation costs). We assume the value of service r to be heterogeneous across customers. The proportion of customers who value the service less than $x \geq 0$ is given by $\Theta(x)$. By definition, $\Theta(x)$ is non-negative and increasing. We also assume that its derivative, which we denote with θ, is strictly positive everywhere in $[0, \infty)$.[5] In the ED setting, the benefit r denotes the value that patients place on treatment and it is natural to assume that it heterogeneous across patients due to the variability in the severity of patients' conditions, which is present even within the same triage category. The second component of customers' utility is the price of the service, p. In the ED setting, this may reflect the co-payment for an ED visit, which may well be zero. The third component is the cost of waiting to receive service, which we assume to be t per unit time. In general, it reflects opportunity cost and in the ED setting in particular, it may also reflect the monetary value of the anxiety, pain, and inconvenience that patients might experience until they are diagnosed and/or treated. We assume heterogeneity in this cost to be less pronounced than that in the benefit from receiving the service and, for tractability purposes, we model this as homogeneous across customers.

[5] To avoid subtle technical questions and to facilitate game-theoretic analysis, we assume that all functions defined are twice differentiable.

More formally, the utility that each customer expects to receive from seeking service is given by $r - tW(\lambda, \mu) - p$, where $W(\lambda, \mu)$ denotes the expected waiting time given the number of customers arriving to the service provider λ and the actions of the service provider that result in increasing throughput, which are summarized by the variable μ (see also §3.2). Throughout, we assume that $W(\lambda, \mu)$ is increasing in λ and decreasing in μ. Any customer with positive utility will decide to seek service and the equilibrium arrival rate $\lambda(p, \mu)$ is given by the solution of the equation

$$\lambda(p, \mu) = \Lambda \bar{\Theta} \left(p + tW(\lambda(p, \mu), \mu) \right), \tag{1}$$

where $\bar{\Theta}(r) := 1 - \Theta(r)$ and $\lambda(p, \mu) < \mu$. If, for example, the service is provided in an $M/M/1$ queue, this equation reduces to

$$\lambda(p, \mu) = \Lambda \bar{\Theta} \left(p + \frac{t}{\mu - \lambda(p, \mu)} \right). \tag{2}$$

We note that the formulation above assumes that customers do not observe the actual waiting time when they make the decision to seek service. This is consistent with many practical settings including EDs where patients have little visibility on actual waiting times before they visit the ED (see Chapter 3 of Hassin and Haviv (2003) for an excellent review of the literature on unobservable queues and its applications, and Anand et al. (2011), Rajan et al. (2014) for more recent examples). Nevertheless, customers are assumed to have accurate beliefs about expected waiting times, W, which they may have formed trough repeated interactions with the service provider or word-of-mouth.

3.2. Service Provider's Profit and Actions

We next discuss the profit maximization problem of one service provider of the N identical service providers. The provider is assumed to know the customers' equilibrium arrival rate. For simplicity, and in order to generate results that are comparable with extant literature, we present a single-period model where the reimbursement mechanism, which consists of a customer price p and transfer payment T, is set by the regulator at the beginning of the period. The duration of this period is much longer than the average patient-interarrival time. Given the reimbursement mechanism and the customers' equilibrium arrival rate $\lambda(p, \mu)$ given in (1), the service provider's profit per unit time (thought the time period) is given by

$$\Pi(c, \mu | p, T) = (p - c)\lambda(p, \mu) - R(c, \mu) + T, \tag{3}$$

where c is the cost of providing service per customer and μ represents the level of effort that the service provider chooses to exert in order to reduce waiting time. The cost function $R(c, \mu)$ denotes the cost of all activities undertaken by the service provider to reduce the cost of providing service

to the level c and the cost of effort μ associated with reducing the waiting time. We assume that cost $R(c,\mu)$ is a fixed cost, at least in the short-run, and it is decreasing in the cost per customer c, increasing in effort μ, and it is jointly convex.

In the case of the ED, the single period of time may represent a year within which the regulator has committed not to make any further adjustments to the regulatory environment. The cost per customer c represents the overall cost of treating a patient with a specific condition and $R(c,\mu)$ denotes the cost (per unit time) of any interventions or process re-engineering that may yield a more cost-efficient process or increase in throughput. For instance, purchasing capital intensive new equipment that allows for more precise and faster patient treatment, employing more and better-qualified staff, and/or re-engineering processes (e.g., having patients triaged and diagnosed by more experienced health care providers (Saghafian et al. (2014)), can reduce the cost of treating patients and simultaneously increase the throughput (see Saghafian et al. (2015) for more on throughput improvements in EDs.) We extend this model to more general cost structures in §5.2. If the service is provided in an $M/M/1$ queue, the variable μ can be interpreted as the service rate (or service capacity) of the system and, for this reason, we will refer to μ as effort or capacity interchangeably.

At the beginning of the period, the provider chooses her optimal cost c and waiting time reduction effort μ by solving the profit maximization problem

$$\max_{c_o \geq c > 0, \mu \geq \mu_o > 0} \Pi(c,\mu|p,T). \tag{4}$$

We assume that the service provider must choose the cost per patient c from the interval $[0, c_o]$ and the capacity level μ from the interval $[\mu_o, \infty)$. The limits c_0, which can be arbitrarily large, and m_0, which can be arbitrarily small, can be thought of as the (exogenous) default cost and capacity decisions of the provider. The objective of profit maximization is not inconsistent with hospital care, see for example the discussion in Andritsos and Aflaki (2015).

Finally, we note that in our model there exist N providers who we assume are identical in terms of their profit function. We extend our analysis to heterogenous providers in §5.4.

3.3. Regulator's Welfare

The regulator has the authority to dictate the price p paid by customers, the transfer payment T received by the providers and may also choose to dictate the cost c and capacity μ set by the service provider. These are chosen at the beginning of the time horizon in order to maximize total welfare, which comprises the accumulated customer surplus and the profits of each of N service provider. The expression for the total welfare rate associated with one such service provider is given by

$$S(p,c,\mu) = \Lambda \int_{p+tW(\lambda,\mu)}^{\infty} (x - p - tW(\lambda,\mu))\, d\Theta(x) + (p-c)\lambda(p,\mu) - R(c,\mu), \tag{5}$$

where $\lambda(p,\mu)$ is given in (1). The first term in the expression above is the total consumer surplus per unit of time. The second and third terms together constitute the profit of the service provider, net of the transfer payment. We note that the transfer payment, T, does not appear in the welfare function as it is a payment within the system. Nevertheless, the transfer payment may be necessary to ensure that the service provider breaks even, i.e., $\Pi(c,\mu|p,T)\geq 0$, and would therefore continue to provide service. Under the $M/M/1$ assumption, the social welfare rate can be written as

$$S(p,c,\mu)=\Lambda\int_{p+\frac{t}{\mu-\lambda(p,\mu)}}^{\infty}\left(x-p-\frac{t}{\mu-\lambda(p,\mu)}\right)d\Theta(x)+(p-c)\lambda(p,\mu)-R(c,\mu). \tag{6}$$

where $\lambda(p,\mu)$ is given by (2).

In the ED setting, we assume that the role of the regulator is fulfilled by the main payer (e.g., CMS in the US or the national payer in other more centralized systems) whose objective is to maximize the sum of patient utility and hospital profits. Similar objectives have been used extensively in healthcare economics and healthcare operations management literature, (e.g., Andritsos and Tang (2015), Adida et al. (2016)).

3.4. First-Best Benchmark

We start the analysis by finding the welfare maximizing price, p, transfer payment, T, cost per customer, c, and capacity, μ assuming that the regulator has full information about all model parameters, including the cost technology $R(c,\mu)$ of the service provider and the equilibrium arrival rate $\lambda(p,\mu)$ of the customers. In this centralized setting, the regulator solves

$$\max_{p\geq 0, c_o\geq c>0, \mu\geq\mu_o>0, T} S(p,c,\mu) \tag{7}$$

$$\text{s.t. } \Pi(c,\mu|p,T)\geq 0. \tag{8}$$

We highlight that given any level of price, p, cost per customer, c, and capacity, μ, any transfer payment, T, above a threshold would satisfy the provider's break even constraint in (8). Here, we implicitly assume that the regulator prefers reimbursing the provider as little as possible while ensuring that (8) holds (see also Sobel 1999). We also note that due to the complicated (and endogenous) queuing dynamics, the welfare function might not always be concave. As usual in the literature of queueing games, we assume that first order conditions are necessary and sufficient for determining the unique solution to the regulator's welfare maximization problem. We present sufficient conditions for this to be the case in Appendix A. The next proposition presents the first-best solution.

PROPOSITION 1. *The unique welfare-maximizing (first-best) price, p^*, cost per customer, c^*, capacity, μ^*, and transfer payment, T^*, are given by*

$$\frac{\partial}{\partial c}R(c^*,\mu^*)=-\lambda^*, \tag{9}$$

$$\frac{\partial}{\partial \mu} R(c^*, \mu^*) = -t\lambda^* \frac{\partial}{\partial \mu} W(\lambda^*, \mu^*), \tag{10}$$

$$p^* = c^* + t\lambda^* \frac{\partial}{\partial \lambda} W(\lambda^*, \mu^*), \tag{11}$$

$$T^* = R(c^*, \mu^*) - t{\lambda^*}^2 \frac{\partial}{\partial \lambda} W(\lambda^*, \mu^*), \tag{12}$$

where $\lambda^* = \lambda(p^*, \mu^*)$ *is given by* (1). *In the* $M/M/1$ *case,* $-t\lambda^* \frac{\partial}{\partial \mu} W(\lambda^*, \mu^*) = t\lambda^* \frac{\partial}{\partial \lambda} W(\lambda^*, \mu^*) = \frac{t\lambda}{(\mu-\lambda)^2}$.

Proofs of all results are presented in Appendix B.

The solution to the regulator's problem makes intuitive sense. First, the transfer payment of (12) is such that the service provider breaks even. Second, the first-best service cost c^* given by (9) is set so that the marginal benefit from a reduction in the treatment cost across all customers seeking service ($\lambda \Delta c$) is equal to the marginal cost of cost reduction $\left(\frac{\partial R}{\partial c} \Delta c\right)$. Third, the first-best service capacity μ^*, given by (10), is set so that the the marginal cost of increasing capacity $\left(\frac{\partial R}{\partial \mu} \Delta \mu\right)$ is equal to the reduction in waiting time associated with the increase in capacity experienced by all customers who choose to seek service $\left(-t\lambda \frac{\partial W}{\partial \mu} \Delta \mu\right)$. Fourth, the first-best price p^*, given by (11), makes the customers who choose to seek service bear the cost of providing the service (c) plus an additional "toll" which is equal to the marginal externality cost incurred by their fellow customers due to the increase in the waiting time $\left(t\lambda \frac{\partial W}{\partial \lambda}\right)$.

We note that if the cost of waiting $t \to 0$, our results converge to those of earlier models where waiting is assumed not to be costly, e.g., Shleifer (1985). Comparing the setting presented in this paper with one where there are no waiting costs, we note one important difference. In this setting, customers are asked to pay more than the cost of providing service, i.e., $p > c$. This is a result similar to Naor (1969). The additional charge reflects the endogenous nature of waiting costs; that is, by joining the service provider, consumers make it more expensive for anyone else to join. They, therefore, have to be charged an additional "toll" to incentivize optimal joining behavior. As a consequence of this toll, the break-even transfer payment required is less than the investment cost, $R(c^*, \mu^*)$. Throughout the paper, we focus on the more interesting case where $\mu^* > \mu_o$.

4. Regulatory Schemes

To implement the welfare maximizing capacity, μ^*, cost per customer, c^*, and price, p^*, the regulator needs to have perfect knowledge of the cost function $R(c, \mu)$, and the customer equilibrium arrival rate $\lambda(p, \mu)$. In practice, regulated firms often have privileged information vis-a-vis the regulator (Armstrong and Porter 2007). As Weitzman (1978) puts it in the context of production "[...] there is no way the regulators can know beforehand exactly what it will cost to achieve a certain output level." In the hospital setting specifically, the number of conditions treated and the pace

of technological change associated with treatments, would make it difficult for the regulator to maintain an accurate understanding of the costs. Similarly, the regulator may be less able to estimate the distribution of customers' benefit from service $\Theta(.)$, a critical input into the calculation of the equilibrium arrival rate $\lambda(p,\mu)$, vis-a-vis the service provider who regularly interacts with the customers.

In contrast, the regulator may be able to observe through audited accounting data the cost c, investment cost R, and the actual number of customers served λ, along with the average waiting time W after the service provider chooses her capacity and marginal cost levels. For example, CMS already collects and audits the first three figures for all hospitals in the US and has recently started collecting the last. Similarly, in addition to costs, ED waiting times are also monitored by the UK hospital regulator. Motivated by this observation, we will present four regulatory regimes that do not assume knowledge of the cost function $R(c,\mu)$ or the customer equilibrium arrival rate function $\lambda(p,\mu)$. The first two, cost-of-service regulation and cost-based yardstick competition, have been implemented in practice, but their performance in a service setting, where waiting times are costly, has not been assessed before. The third and fourth regulatory regimes, which modify the cost-based yardstick competition, are, to the best of our knowledge, new. For each of the regulatory schemes that we introduce, we need to make specific assumptions about the providers' profit function to ensure sufficiency of first order conditions. As in the previous section, we present these sufficient conditions in Appendix A.

4.1. Cost-of-Service Regulation

Under "cost-of-service" regulation, the service provider is free to decide on the capacity μ, cost per customer c, and reimbursement (in the form of price and transfer payment) that is designed to cover the total cost of the service provider, while avoiding distortions associated with the monopolist price that the service provider would naturally be inclined to impose. More specifically, under this scheme, which is similar to the way that hospitals were reimbursed by CMS until 1983 (Mayes 2007), the regulator would audit the service provider to determine the costs c and $R(c,\mu)$ and would then impose a customer price $p=c$ and transfer payment $T=R(c,\mu)$. Clearly, this scheme cannot implement socially optimal investment because the service provider makes zero profit regardless of the capacity and cost-reduction effort that it makes and, therefore, has no incentives to invest in either. This result is also noted in Shleifer (1985).

4.2. Cost-Based Yardstick Competition

The reason as to why "cost-of-service" regulation fails to provide cost-reduction incentives is the dependence of the firm's reimbursement on its own chosen cost structure. An alternative regulatory scheme, which eliminates this dependence, has been proposed by Shleifer (1985). In his setting,

customers do not experience costly waiting times, and the proposed regulatory regime achieves socially optimal levels of cost-reduction effort without relying on the regulator knowing the cost function of the firm, $R(c,\mu)$. Since this scheme is similar to the DRG payment system implemented by CMS in hospital reimbursement, it is important to investigate if it can achieve socially desired outcomes in the case where consumers' delays are costly. Before we discuss this, we first explain how the scheme proposed by Shleifer (1985) works, which we will present after we define additional notation.

We follow the same notation as before but add a subscript i, which stands for the service provider index, $i=1,...,N$. For each firm i, $i=1,...,N$, define

$$\bar{c}_i = \frac{1}{N-1}\sum_{j\neq i} c_j \tag{13}$$

$$\bar{R}_i = \frac{1}{N-1}\sum_{j\neq i} R(c_j,\mu_j). \tag{14}$$

The regulator sets $p_i=\bar{c}_i$, the price that customers pay for service at provider i, and $T_i=\bar{R}_i$, the transfer payment thatthe service provider i receives from the regulator. Based on the price p_i and the capacity choice of service provider i which determines its expected wait time, customers seek service at provider i with the rate given in (1). Given the price, transfer payment, and arrival rate, the service providers engage in a simultaneous-move game with complete information, where each provider chooses her capacity, μ_i, and cost per customer, c_i, to maximize the payoff function given in (3), $i=1,...,N$. This is a game because the profit of each provider is linked to the actions of all other providers through the reimbursement mechanism. We present the equilibrium of this game below.

PROPOSITION 2 **(Shleifer 1985)**. *In the absence of costly waiting time ($t=0$), if the regulator sets firm i's price and transfer payments by* (13) *and* (14)*, the unique Nash equilibrium is for each firm i to choose $c_i=c^*$, $i=1,...,N$, $N\geq 2$. Also, all firms make zero profit in equilibrium.*

By implementing the regulatory scheme described above, the regulator forces the firms to engage in indirect competition to reduce costs. This is achieved by first, decoupling the reimbursement rate of the firm from the cost chosen by the firm, and second, setting it equal to the that of an exogenous industry-wide benchmark. In the absence of costly waiting time ($t=0$), the unique symmetric Nash equilibrium of this tournament-style competition generates first-best outcomes, i.e., it achieves the same cost-reduction investment as that chosen by the regulator under full information derived in §3.4. However, this scheme can be implemented using cost-accounting data, and therefore, does not require symmetric information between the regulator and the regulated firms. Furthermore, under this scheme all service providers achieve zero profits in equilibrium and, as a result, there

is no reason for the regulator to want to renegotiate any payments after investments have been made, thus alleviating any concerns for hold-up problems (Sobel 1999).

Since this scheme depends only on the cost of providing service (and not the level of capacity investment), we refer to this scheme as cost-based yardstick competition. We next investigate the performance of this scheme in the setting where customers are delay-sensitive.

PROPOSITION 3. *If customers experience costly waiting time ($t>0$) under the cost-based yardstick competition of Proposition 2, the firms' installed capacity is the minimum capacity level $\mu_o < \mu^*$ in all potential symmetric equilibria. If, in addition, $\frac{\partial^2 R(c,\mu)}{\partial \mu \partial c} \geq 0$ for all $0 < c \leq c_0$ and $\mu \geq \mu_o$, then this reimbursement scheme results in a unique symmetric equilibrium where firms choose higher cost compared to the first-best, c^*.*

This proposition shows that in the presence of costly waiting time, cost-based yardstick competition results in extreme underinvestment in capacity. This result, which holds irrespective of the detailed queueing discipline employed by the service provider, arises because in equilibrium the service provider has no incentive to increase capacity. Adding capacity investment is costly; however, the service provider does not receive any direct benefit from the associated reduction in waiting times (as her payment is not linked to capacity or waiting time) or indirect benefit (as although the reduction in waiting time will increase the equilibrium arrival rate, this will not increase the service providers profit, as the marginal profit for each additional customer is, in equilibrium, zero). Furthermore, this scheme suffers from an additional source of inefficiency – given the capacity and marginal cost-reduction investment, more customers choose to seek service than socially optimal (for *that* cost per customer and capacity level). This can be seen by noting that the first-best price (see Proposition 1) is greater than the marginal cost (see Proposition 3). In addition to underinvestment in capacity, Proposition 3 shows that if marginal cost reduction is cheaper for providers with higher capacities (i.e., $\frac{\partial^2 R(c,\mu)}{\partial \mu \partial c} \geq 0$), the cost-based yardstick competition leads to underinvestment in cost reduction as well (i.e., the cost per customer is greater than first-best).

Clearly, the cost-based yardstick competition falls short of achieving socially-desired outcomes in a setting with capacity-constrained firms and delay-sensitive customers. The systematic underinvestment in capacity that arises as an equilibrium result from this scheme may be a contributing factor in the long waiting times observed in accessing emergency healthcare throughout the developed world (see e.g., Government Accountability Office 2009, Campbell and Mason 2013). For the rest of this paper, we investigate whether this shortcoming of the cost-based yardstick competition can be improved by implementing alternate regulatory schemes.

4.3. Cost- and Capacity-Based Yardstick Competition: First-Best

We next propose a regulatory scheme that incentivizes service providers to take the first-best actions established in Proposition 1 when wait is costly, i.e., $t > 0$. Let λ_i and μ_i respectively denote the arrival rate and capacity of service provider i and define

$$\bar{\lambda}_i = \frac{1}{N-1}\sum_{j\neq i}\lambda_j \quad \text{and } \bar{\mu}_i = \frac{1}{N-1}\sum_{j\neq i}\mu_j. \tag{15}$$

for $i = 1, ..., N$. Consider the following payment scheme: each customer seeking service from service provider i is charged a price p_i, where

$$p_i = c_i + t\lambda_i \frac{\partial}{\partial \lambda} W(\lambda_i, \mu_i). \tag{16}$$

In addition, the regulator sets the transfer payment T_i to provider i as

$$T_i = (\bar{c}_i - c_i)\bar{\lambda}_i + t\bar{\lambda}_i \frac{\partial}{\partial \mu} W(\bar{\lambda}_i, \bar{\mu}_i)(\bar{\mu}_i - \mu_i) + \bar{R}_i - t\lambda_i^2 \frac{\partial}{\partial \lambda} W(\lambda_i, \mu_i). \tag{17}$$

Under this payment scheme, service provider i's objective function is

$$\Pi(c_i, \mu_i | p_i, T_i) = (\bar{c}_i - c_i)\bar{\lambda}_i + t\bar{\lambda}_i \frac{\partial}{\partial \mu} W(\bar{\lambda}_i, \bar{\mu}_i)(\bar{\mu}_i - \mu_i) - R(c_i, \mu_i) + \bar{R}_i. \tag{18}$$

As in the case of the cost-based yardstick competition, this payment scheme induces a simultaneous-move game between the providers. We investigate the equilibrium outcome of this game with the proposition below.

THEOREM 1. *If the regulator sets service provider i's price equal to p_i given in* (16) *and transfer payment equal to T_i given in* (17)*, then the unique symmetric Nash equilibrium is for each provider i to pick $c_i = c^*$ and $\mu_i = \mu^*$, for $i = 1, ..., N$. Also, all providers make zero profit in equilibrium.*

The regulatory scheme proposed in this section consists of a per-customer price, p_i, and a transfer payment, T_i, similar to that of the cost-based yardstick competition. Each now also depends on the capacity decision, μ_i, directly in addition to costs. The price p_i, which is equal to the cost of providing the service, c_i, plus the expected waiting-cost externality ($t\lambda_i \frac{\partial}{\partial \lambda} W(\lambda_i, \mu_i)$), serves the purpose of regulating the customers' joining behavior and, in equilibrium, is equal to the first-best price, p^*. Without it, customers would over-join compared to the socially optimal arrival levels as explained in §3.4. The transfer payment, T_i, coupled with the fee paid by each customer, serves to align the incentives of the providers' with the regulator's and, at the same time, ensures that the providers break even. The first term of the transfer payment ($(\bar{c}_i - c_i)\bar{\lambda}_i$), which is decreasing in the difference between the costs of provider i and the industry average, puts pressure on each provider to reduce costs to first-best levels. The second term ($t\bar{\lambda}_i \frac{\partial}{\partial \mu} W(\bar{\lambda}_i, \bar{\mu}_i)(\bar{\mu}_i - \mu_i)$), which is increasing

in the difference between the service rate of provider i and the rest of industry that serves as a benchmark $(\mu_i - \bar{\mu}_i)$, provides the right incentives for each provider to increase capacity to first-best levels. The final two terms serve to ensure that the providers break even in equilibrium, thus alleviating any concerns that the contracts may be renegotiated. Furthermore, in equilibrium, all but the last two terms in the transfer payment would simplify to zero; thus, the actual equilibrium payment would simplify to $\bar{R}_i - t\lambda_i^2 \frac{\partial}{\partial \lambda} W(\lambda_i, \mu_i)$, or, in the case of $M/M/1$ queueing discipline to $\bar{R}_i - \frac{t\lambda_i^2}{(\mu_i - \lambda_i)^2}$.

We note that the scheme proposed in this section achieves first-best without requiring the regulator to have symmetric information about either the cost function $R(c, \mu)$ or the customer equilibrium arrival rate function $\lambda(p, \mu)$. Nevertheless, we think that it would be difficult to implement in practice, especially in the case of hospital ED regulation. First, the mechanism proposed above requires customers to be charged a provider- and condition-specific fee to achieve socially optimal arrivals. This might be possible in certain industries; however, in most healthcare delivery systems, patients do not bear the cost of treatment directly. For example, in the UK healthcare is free of charge to all EU residents and health care is funded through taxes. Although in other healthcare systems, such as the US, patients may be required to pay a fee when they receive treatment (e.g., in the form of co-payments), this fee is not tied to the performance of the provider and does not depend on the patient's condition. Second, in order for the regulator to implement the yardstick competition mechanism proposed in this section, it is necessary to have some information about the queueing discipline at the providers' sides. This is needed in order to estimate the service capacity, μ, installed by each provider and the waiting time function and its derivatives with respect to the arrival rate, λ, and capacity, μ. We suspect that in the highly complex hospital ED setting, the queueing discipline would be hard to observe for the regulator. Third, the transfer payment, T_i, involved in the scheme above may well be negative in equilibrium. This happens if $\bar{R}_i < t\lambda_i^2 \frac{\partial}{\partial \lambda} W(\lambda_i, \mu_i)$. Although the break-even constraint ensures that the provider can afford to pay this fee, in a healthcare setting it may be politically difficult for the regulator to levy such a tax on hospitals.

Motivated by the first practical difficulty above, in the next section we provide an alternative scheme which does not charge the customers a provider- and condition-specific fee. Fortunately, as we show in the next section, this scheme has some additional advantages, such as being able to address the other two concerns.

We conclude this section by noting that given the assumption of joint convexity of the cost function $R(c, \mu)$, the proposed regulatory scheme leads to a unique symmetric equilibrium. Nevertheless, we cannot rule out the existence of asymmetric equilibria. This is a common problem in such settings, see e.g., Shleifer (1985). Furthermore, there also exist other regulatory schemes

that generate exactly the same symmetric equilibrium as the one described above. For example, in Appendix C we present one more complex regulatory scheme for which we can also rule out the existence of any asymmetric equilibria.

4.4. Free-at-the-Point of Care Yardstick Competition: Second-Best

To address the concern that it is often not feasible to charge customers directly in healthcare, in this section, we propose an alternative payment scheme that guarantees that the chosen actions of the service providers will maximize welfare in the case where customers are not charged directly for the service. For the rest of this section, we fix the price $p=0$ and drop if from the notation, e.g., we set $\lambda(\mu)=\lambda(0,\mu)$, with a slight abuse of notation. The analysis of this section would be almost identical if customers were charged a fixed fee, as in the case of patient co-payments for visiting EDs.

First, consider the objective function of the regulator $S(c,\mu)$ defined as in (6) with $p=0$. Under the assumptions laid out in Appendix A, the welfare function is such that the first order conditions are necessary and sufficient for determining the unique solution to the regulator's second-best problem. We present the second-best solution, which we denote with μ_o^* and c_o^* with the following proposition.

PROPOSITION 4. *The unique welfare-maximizing (second-best) capacity μ_o^*, cost per customer c_o^*, and transfer payment T_o^*, when price $p=0$ are given by*

$$\frac{\partial}{\partial c}R(c_o^*,\mu_o^*)=-\lambda(\mu_o^*), \tag{19}$$

$$\frac{\partial}{\partial \mu}R(c_o^*,\mu_o^*)=-t\lambda(\mu_o^*)\frac{d}{d\mu}W(\lambda(\mu_o^*),\mu_o^*)-c\lambda'(\mu_o^*), \tag{20}$$

$$T_o^*=R(c_o^*,\mu_o^*). \tag{21}$$

where $\frac{d}{d\mu}W(\lambda(\mu),\mu)=\frac{\partial}{\partial\lambda}W(\lambda(\mu),\mu)\lambda'(\mu)+\frac{\partial}{\partial\mu}W(\lambda(\mu),\mu)$. *In the $M/M/1$ case,* $\frac{d}{d\mu}W(\lambda(\mu),\mu)=\frac{\lambda'(\mu)-1}{(\mu-\lambda(\mu))^2}$.

We note that in the absence of a direct fee (i.e., $p=0$), consumer behavior is going to be inefficient – some customers with sufficiently low valuation who would have chosen not to visit the provider under first-best price p^* will now find it optimal to seek service. In fact, the only reason that not everyone seeks service is congestion – some potential customers find the cost of their (equilibrium) expected waiting time to outweigh the benefit from service.

For each service provider i, we define the average waiting time of all other service providers as

$$\bar{W}_i=\frac{1}{N-1}\sum_{j\neq i}W(\lambda(\mu_j),\mu_j),\ i=1,\ldots,N. \tag{22}$$

For notational simplicity, we set $W_i := W(\lambda(\mu_i), \mu_i)$. Consider the payment scheme where the regulator pays provider i a transfer payment equal to

$$T_i = t(\bar{W}_i - W_i)\bar{\lambda}_i + \bar{R}_i + \bar{c}_i\bar{\lambda}_i. \tag{23}$$

Under this payment scheme, service provider i's objective function is given by

$$\Pi(c_i, \mu_i | T_i) = -c\lambda(\mu_i) + t(\bar{W}_i - W_i)\bar{\lambda}_i - R(c_i, \mu_i) + \bar{R}_i + \bar{c}_i\bar{\lambda}_i. \tag{24}$$

The payment scheme defined above forces the service providers to engage in a simultaneous-move game whose equilibrium outcome we present below.

THEOREM 2. *If the regulator makes transfer payment T_i defined as in* (23) *to provider i, for $i = 1, \ldots, N$ and customers are not charged directly, the unique symmetric Nash equilibrium is for each provider i to pick $\mu_i = \mu_o^*$ and $c_i = c_o^*$ for $i = 1, \ldots, N$. Also, all providers make zero profit in equilibrium.*

The implication of Theorem 2 is that in the absence of a direct customer fee, yardstick competition is still useful. Although it cannot restore first-best (as there is no way to counter the inefficient joining behavior of customers), by implementing the scheme proposed above, the regulator can achieve the second-best outcome even though he has no information about the cost structure of the service providers $R(c, \mu)$, or the customer equilibrium arrival rate function $\lambda(p, \mu)$. The incentive to invest optimally (in the second-best sense) in capacity μ comes from the transfer payment, which is an increasing function in the difference between the industry benchmark waiting time, $\bar{W}_i$, and that chosen by the provider, W_i. This creates the tournament-style incentives that lead to a unique symmetric equilibrium where all providers invest optimally in capacity. Similarly, each provider has an incentive to invest optimally in cost reduction (again, in a second-best sense) as the payment scheme described above, which pays the provider a fee that is independent of her actions, makes the provider the residual claimant – the additional value generated by lower costs is fully appropriated by the provider.

Furthermore, the scheme proposed in this section is simpler than that proposed in §4.3, where customers are charged a direct fee, for three reasons. First, it requires no information on the service rate μ or the queueing discipline and the associated waiting time function $W(\lambda, \mu)$ and its derivatives. Instead, the only additional requirement compared to the simpler cost-based yardstick competition of §4.2 is that the regulator also monitors the average wait time for each provider. Second, the equilibrium transfer payment is equal to the total cost incurred by the service provider $(R_i + c_i\lambda_i)$, which, in contrast to the transfer payment of the first-best scheme of §4.3, is going to be non-negative. Third, it does not require that the regulator is able to separately observe the

cost of providing service c and the investment costs R. Instead, it suffices to observe the total cost incurred by each provider $R_i + c_i\lambda_i$ (see also Meran and Von Hirschhausen (2009)), which is a simpler task in many cases where variable and fixed costs are not easy to delineate (such as hospital care, see Dranove (1995), Freeman et al. (2016)). For these reasons, we expect that this scheme to be easier to implement in practice than the first-best scheme of §4.3. We note, however, that despite its simplicity this scheme requires that the regulator knows the patients' cost of waiting t, which may not always be the case. We investigate this dependence numerically in §6.

Furthermore, the simplicity of the second-best yardstick competition comes at a cost of efficiency. The loss of efficiency, which we also investigate numerically in §6, is due to the suboptimal customer joining behaviour, which this regulatory scheme does nothing to curtail. In that sense, this regulatory scheme treats waiting times as any other quality exogenous measure that the regulator might care about (e.g., hospital readmission rates (see Zhang et al. 2016) or adherence to best-practice protocols (see Gaynor (2004) for a literature review and background)) and augments the standard yardstick competition of §4.2 in order to provide sufficient incentives to invest optimally in improving this quality measure. Therefore, and perhaps not surprisingly, the scheme proposed in this section has some similarities to a scheme already in use by CMS to provide quality improvement incentives in dimensions other than costs (e.g., Hospital Value-Based Purchasing program (CMS 2016a) or the Hospital Readmission Reduction Program (Zhang et al. 2016)).

We conclude this section by noting that Theorem 2 does not rule out the existence of asymmetric equilibria. Nevertheless, we are able to show in Appendix D that when there are only two providers (i.e., $N = 2$), the symmetric equilibrium under the proposed scheme is indeed unique. Using this observation, we can then propose an alternative mechanism that does result in a unique equilibrium which leads to second-best outcomes. In this mechanism, providers are divided into two disjoint sets, and the average performance of one set is used to set a yardstick for the other and vice versa.

5. Extensions

In this section, we discuss how yardstick competition could be implemented under more general conditions than those of §4. Namely, we look at the case of multiple customer classes, time-varying arrival rates, more general cost structure, using tail-statistics instead of average waiting time, and provider heterogeneity. Although it is possible to present these extensions for the first-best yardstick competition of §4.3, for the most part, we have chosen to restrict attention to the simpler second-best yardstick competition of §4.4, which we believe to be of more practical relevance. We again consider the simultaneous-move game described in §4.2 and for the new payment schemes to lead to the socially-optimal equilibrium proposed in these extensions, we need to make similar assumptions to those that we made in §3 about the objective functions of the regulator and

providers. Specifically, we assume that the first order conditions are necessary and sufficient for determining the unique solution to the regulator's welfare maximization problem in each new model and that each provider's objective is concave with the new payment scheme for each extension.

5.1. Multiple Customer Classes and Time-Varying Arrival Rates

In most practical settings, there are multiple customer classes with different treatment needs utilizing the same limited resources, and there are settings where the arrival rates are time-variant. For example, in emergency care, patients are triaged into different levels based on their severity and arrival rates are much higher during the day than the evenings (Armony et al. 2015). The way that the payment mechanisms need to be modified to account for these two additional features are somewhat similar, so we focus on the extension for multiple customer classes and then explain how the time-variant arrivals can be handled in a similar manner.

Assume for simplicity that each provider caters to two different customer classes. We use the model presented in §3.1 for customer joining behavior, but we append a superscript j to denote the quantity associated with each customer class j, i.e. $\Lambda^{(j)}, \Theta^{(j)}$, and $t^{(j)}$ are all assumed to depend on the customer class. Because customers from both classes use the same resources, the average waiting time of each class not only depends on the service rate and the arrival rate for that class but also to those of the other class, as well as the priority policy. We assume that all providers follow the same priority policy and that the class a customer belongs to is observable to the provider. For example, in emergency care, patients are prioritized according to their severity levels and a similar triage method is used across hospital EDs (see McHugh et al. (2012) and Gilboy et al. (2011)).

The service rate of type i customers is denoted by $\mu^{(i)}$ and we assume that the provider can invest in increasing service rates and/or reducing cost per patient at a cost given by $R(c, \mu^{(1)}, \mu^{(2)})$. Let $W^{(j)}(\mu^{(1)}, \mu^{(2)}, \lambda^{(1)}, \lambda^{(2)})$ denote the expected waiting time for customer class j if the service and arrival rates of each class are given by $\mu^{(j)}$ and $\lambda^{(j)}$, $j = 1, 2$, where $\lambda^{(j)}$ satisfies (1) (with the average waiting time definition extended as described). We denote the expected wait by $W^{(j)}$ and in provider i by $W_i^{(j)}$ for class j for notational simplicity. The objective of the regulator with two customer classes is

$$S\left(c, \mu^{(1)}, \mu^{(2)}\right) = \sum_{j=1}^{2} \Lambda^{(j)} \int_{t^{(j)}W^{(j)}}^{M_r} \left(x - t^{(j)} W^{(j)}\right) d\Theta^{(j)}(x) - c\left(\lambda^{(1)} + \lambda^{(2)}\right) - R\left(c, \mu^{(1)}, \mu^{(2)}\right). \quad (25)$$

Let c^*, $\mu^{(1),*}$ and $\mu^{(2),*}$ denote the optimal solution to (25). The model can easily be extended to the case for which the cost c depends on the customer class as well. Set the transfer payment to provider i to

$$T_i = t^{(1)}\left(\bar{W}_i^{(1)} - W_i^{(1)}\right)\bar{\lambda}_i^{(1)} + t^{(2)}\left(\bar{W}_i^{(2)} - W_i^{(2)}\right)\bar{\lambda}_i^{(2)} + \bar{R}_i + \bar{c}_i\left(\bar{\lambda}_i^{(1)} + \bar{\lambda}_i^{(2)}\right), \quad (26)$$

where, similar to (15),

$$\bar{\lambda}_i^{(j)} = \frac{\sum_{k\neq i}\lambda_k^{(j)}}{N-1},\ \bar{W}_i^{(j)} = \frac{\sum_{k\neq i} W_k^{(j)}}{N-1} \text{ and } \bar{R}_i = \frac{\sum_{k\neq i} R\left(c_k, \mu_k^{(1)}, \mu_k^{(2)}\right)}{N-1},\ i=1,\ldots,N, \text{ and } j=1,2. \tag{27}$$

Then, in the unique symmetric equilibrium, each provider picks c^*, $\mu^{(1),*}$, and $\mu^{(2),*}$. The proof is very similar to that of Proposition 2 and, in the interest of brevity, is omitted. We note that the transfer payment in the case of multiple patient classes given in (26) is a rather straightforward extension to that of the single patient class. It consists of the *total* expected cost $\left(\bar{R}_i + \bar{c}_i\left(\bar{\lambda}_i^{(1)} + \bar{\lambda}_i^{(2)}\right)\right)$ of providing service across all customers (i.e., it is not necessary to know the exact breakdown of this cost across different customer classes), plus a customer-class-specific fee that is increasing in the difference between the industry-wide and the specific provider waiting time for that customer class. Extensions to more customer classes are also straightforward.

If arrival rates are time-varying, the proposed scheme can be modified similarly if the aggregate and the actual arrival rates (Λ and λ in our notation, respectively) are assumed to remain constant in non-overlapping intervals and also we assume that expected waiting times for any customers arriving in the same interval are the same. Although this approach ignores the transient behavior of the queues going from one interval to the other, if the service times are relatively short, it should yield accurate results. This approach is widely used in call centers, see Gans et al. (2003).

For simplicity assume that there are two intervals in one day during which the arrival rates are constant and the arrival rate only changes going from interval 1 to 2. We consider only one customer class and assume that customers do not choose strategically when to arrive. With a slight abuse of notation let $W^{(j)}(\mu^{(1)}, \mu^{(2)}, \lambda^{(1)}, \lambda^{(2)})$ denote the expected waiting time for customers arriving during period j if the service and arrival rates during each time interval are given by $\mu^{(j)}$ and $\lambda^{(j)}$, $j=1,2$. Then, if the regulator sets the transfer payment, as in (26) (with the averages defined over time intervals instead of customer classes), it can be shown that socially optimal choices for the providers is a unique symmetric Nash equilibrium.

In summary, in the presence of multiple customer classes and time-varying arrival rates, the regulator needs to augment the yardstick competition model of §4.4 by making the transfer payment contingent on the relative performance of the provider for each customer class and each time interval.

5.2. Extending the cost model

The model we used for the regulator's and the providers' objective can be extended to the case when the cost per customer also depends on the service rate, e.g., treating patients faster affects the cost per patient. We could do this by assuming that the marginal cost, c, is no longer a decision

variable itself, but is instead a nonnegative valued function, $c(e,\mu)$, of costly effort (denoted by e) and the service rate μ. Similarly, the total investment cost would also be the function $R(e,\mu)$. The mechanisms proposed in §§4.3–4.4 would still result in first- and second-best outcomes in equilibrium with the definitions of $\bar{c}_i$ and $\bar{R}_i$ are modified as follows

$$\bar{c}_i = \frac{1}{N-1}\sum_{j\neq i} c_j(e,\mu) \text{ and } \bar{R}_i = \frac{1}{N-1}\sum_{j\neq i} R(e_j,\mu_j)$$

in payment schemes (16)-(17) and (23).

5.3. Yardstick competition using tail statistics

An alternative to incentivizing service providers based on their average wait is to use the tail statistics of their wait time, e.g., the fractile of the wait time distribution. For example, in order to incentivize EDs to reduce their wait times, Monitor, the UK hospital regulator, mandates EDs to admit or discharge 95% of the patients within four hours of their arrival, and financially penalizes the hospitals that fail to reach this target (Campbell 2016). Since the welfare-maximizing performance target cannot be computed by the regulator without knowing the cost function $R(c,\mu)$ or the patient equilibrium arrival rate $\lambda(c,\mu)$, the yardstick competition that we propose in §§4.3–4.4 can be modified to achieve first- and second-best outcomes by using tail statistics of wait time.

To demonstrate, assume that the utility of a customer is given by $r - t\Sigma(\lambda,\mu) - p$ for a nonnegative function Σ (the model in §3.1 is a special case). For example, if the regulator believes that customers' utility depends on the probability of waiting more than four hours, then $\Sigma(\lambda,\mu) = \mathbb{E}[\mathbb{1}\{w(\lambda,\mu)\geq 4\}]$, where w is the (random) waiting time of a customer in steady state. Similarly, if the regulator believes that customers' utility depends on 95% fractile of waiting times, then $\Sigma(\lambda,\mu) = H^{-1}_{\lambda,\mu}(95\%)$, where $H_{\lambda,\mu}$ is the distribution of the waiting time in steady-state when arrival and service rates are λ and μ, respectively. Then, the proposed schemes still lead to first- and second-best outcomes in equilibrium if the function W is replaced by Σ in (16)–(17) or (22)–(24).

5.4. Heterogeneous hospitals

To implement the proposed regulatory schemes, it was assumed that the regulator was able to identify (at least pairs of) identical providers. In many real-world settings, such as hospital EDs, this might not be possible because hospitals may differ along multiple dimensions, e.g., the size of their respective catchment areas Λ, the distribution of customer benefits $\Theta(.)$ (e.g., due to case mix variation), and due to difference in the local labor markets giving rise to different costs of treatment, c, and investment cost, $R(c,\mu)$. Nevertheless, if the regulator is able to observe the characteristics that make the providers differ, the proposed schemes can be modified in a way similar to Shleifer (1985).

To illustrate, note that to implement the second-best yardstick competition, the regulator needs to be able to project the total cost, total number of customer arrivals, and average waiting time of each provider. Assume that each provider exhibits a different total cost $f_i(\mu, c, \delta) = c_i(\delta)\lambda_i(\delta) + R_i(\delta)$, arrival rate $\lambda(\mu, \delta)$, and waiting time $W_i(\mu, \delta)$, where δ is a vector containing all observable characteristics that make providers different. Also, assume that δ is not under the control of the providers. Then the regulator can use the information on the total costs, arrival and average waiting times of all other provides, along with the vector of observable characteristics, to predict the expected costs of each provider. For example, this could be achieved through a multivariate panel regression or indeed any other method (e.g., machine learning). If the predictive model is 100% accurate in predicting costs, and all the observable characteristics are correctly accounted for, the proposed scheme generates the socially optimal equilibrium. Obviously, as the explanatory power of the model used by the regulator degrades, so will the value of using yardstick competition.[6]

6. Numerical Investigation

In §4, we have argued that the second-best regulatory scheme would be easier to implement in healthcare settings such as the regulation of ED, as a) customers are typically not charged a provider-specific fee for accessing care, and b) it places a much lower informational burden on the provider. These advantages come at the cost of not achieving first-best level of investment in either wait-time reduction or cost reduction. In this section, we numerically investigate the efficiency loss associated with this second-best outcome. In addition, we also investigate the impact of error in the estimation of one critical parameter model parameter, the cost of waiting (t).

6.1. Model Parameters

Although the queueing model that we deploy is a stylized representation of reality, we have chosen a range of parameter values that match, as far as possible, the ED setting.

- We set the cost of waiting t equal to the median of US wages which is \$20, see Bureau of Labor Statistics (2011). For our study we vary this from \$5 to \$40 per hour, a range which contains more than 80% of the population's hourly wages.
- We assume that the distribution of patients' benefit from treatment follows the exponential distribution as given by

$$\Theta(x) = 1 - e^{-\alpha x}, \tag{28}$$

[6] An alternative scheme, which can be particularly useful as the unexplained heterogeneity between providers increases, is the modified yardstick competition proposed by Laffont and Tirole (1993), pp 84-86, where the regulator offers a menu of incentive-compatible yardstick-competition contracts that allows providers to self-select. Although this may be more complex to implement, it has the potential to further reduce the inefficiency associated with asymmetric information and heterogeneous providers. It cannot, however, restore socially optimal outcomes and the more efficient providers retain positive rents.

where x is benefit from service (in dollars) and $\alpha > 0$. We note that the exponential distribution gives rise to demand price elasticity which is equal to $-\alpha x$. To estimate the elasticity parameter α we use the fact that (i) at the average cost, US healthcare price-elasticity is estimated to be -0.17 (Ringel et al. 2002); and (ii) the average cost is approximately equal to \$180 – this is the sum of the average co-pay for an ED visit (estimated to be \$140 (CEB 2016)) and the average cost of waiting (which is given by multiplying the average ED waiting time of two hours, as reported in Batt and Terwiesch (2015), with the cost of waiting of \$20 per hour). This generates a base estimate of $\alpha = 9.5 \times 10^{-4}\$^{-1}$. We run our sensitivity analysis for α ranging from $5 \times 10^{-4}\$^{-1}$ to $20 \times 10^{-4}\$^{-1}$, which corresponds to price elasticity ranging from -0.09 to -0.36.

- To estimate the size of the total potential demand Λ (i.e., the demand if waiting times and price were both zero), we start from the observation that at current waiting times, average realized demand in the U.S. in 2011 was 44.5 visits per 100 persons per year (CMS (2011)). We also assume that these visits happen at a constant rate through the year and time of day and that the catchment area of the ED's is 250,000 people (see for example Williams et al. (2004)). This gives a base estimate for the actual demand, λ=13.9 patients per hour. At current average cost of \$180, the demand is given by $\lambda = \Lambda e^{-180 \times 9.5 \times 10^{-4}}$, which gives an estimate of $\Lambda = 15.1$ patients per hour. In our experiments, we consider the range between 50% to 150% of this base potential demand rate (corresponding to a catchment area from 125,000 to 375,000).
- The marginal cost of treating a patient at the ED is estimated to be c_0 =\$337 in Grannemann et al. (1986) and \$156 in Williams (1996) (both figures inflated to 2016 dollars). We set c_o =\$268 based on these cost figures that is slightly higher than the average \$246.
- To estimate the default ED capacity μ_0, we once again make use of the observation that waiting times are, on average, equal to two hours. If we assume that the queuing discipline at the ED can be approximated by an $M/M/1$ queueing system, then given the arrival rate of 12.7 patients per hour (see above), we can estimate that the average hospital is able to serve $\mu_0 = 13.2$ patients per hour. Since we are interested in equilibrium outcomes, in all of our numerical examples, we report the resulting equilibrium capacity μ even if it is less than μ_0.
- We use the following function for the cost of investment in capacity and cost reduction

$$R(c, \mu) = e^{\beta \mu} + \gamma (c_o - c)^2, \tag{29}$$

where $\gamma > 0$ and $\beta > 0$. The structure of the first part of the cost function is similar to that used in Grannemann et al. (1986) to estimate the average cost per hospital patient. Since the cost of capacity is largely personnel, we start from the estimated personnel cost per patient treated at the hospital, which is reported to be to \$110 (Williams (1996), inflated to 2016 prices). Since the

average service rate and the average arrival rates are $\mu_0 = 13.2$ and $\lambda = 12.7$ patients per hour, respectively, then we can estimate β by solving $e^{13.2\beta} = 12.7 \times 110$. This produces an estimate of 0.55 for β. In our experiments, we consider the range of values between 0.40 and 0.70, which corresponds to cost of capacity per patient between \$15 and \$800, respectively. Finally, we set $\gamma = 0.072$, which makes the cost of capacity equal to the total cost of providing care, if all parameters are set to base case. Because the investment in cost reduction is not the focus of this work, we do not perform a detailed sensitivity analysis on the marginal-cost-related parameters γ and c_o.

The parameter values chosen, as well as the range within which they are varied (if applicable), are displayed in Table 1. We confirm that for the chosen parameter values, total welfare and the providers' profit functions are concave and the optimal solutions are interior (unless otherwise stated). To maintain connection with reality, we assume that under second-best regulation the regulator imposes the \$140 fixed co-pay (see above). This amount is always lower than the optimal first-best price and our results remain qualitatively similar if the co-pay is reduced to zero.

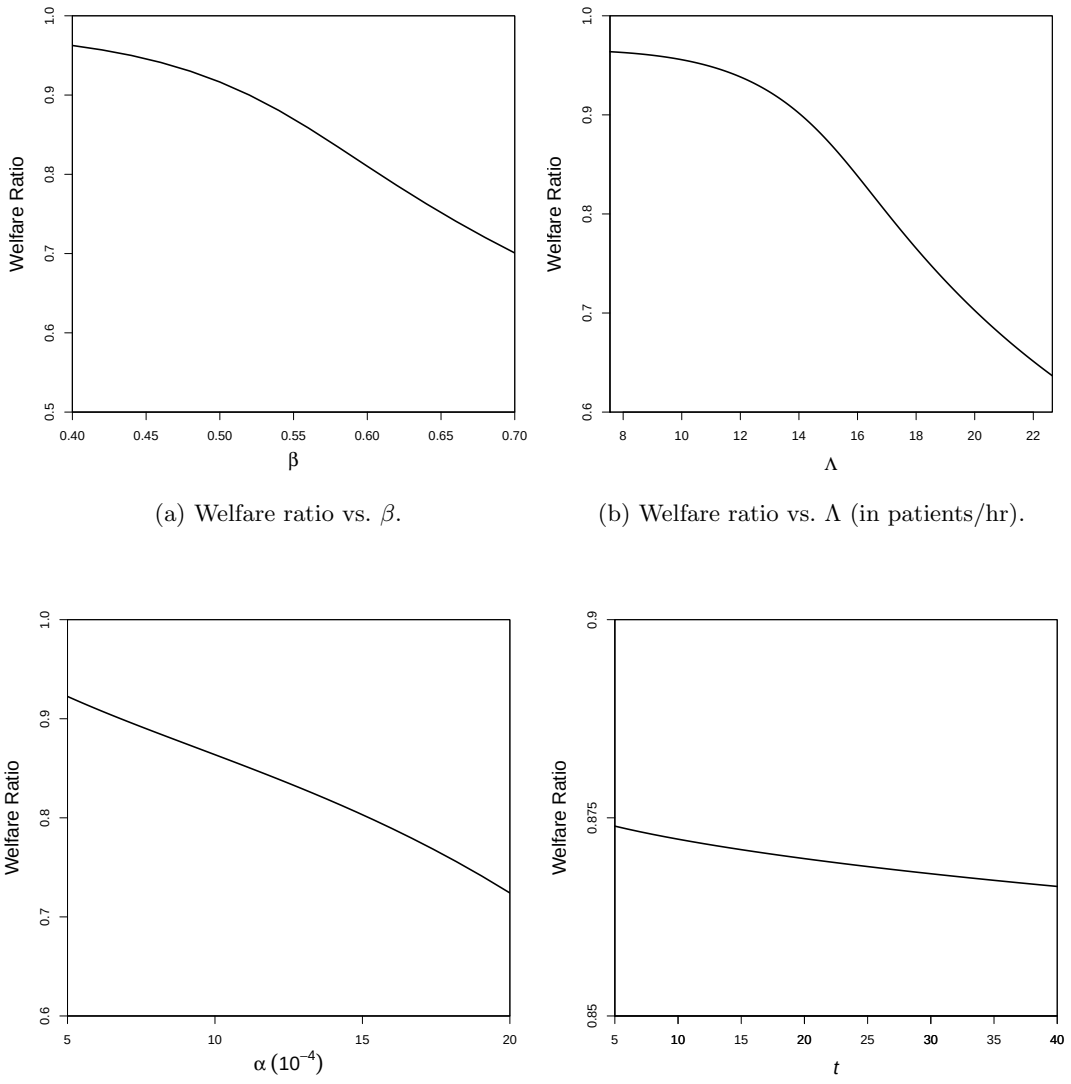
Parameter Description	Parameter	Base Estimate	Range
Size of catchment area	Λ	15.1 patient/hr	$[7.6, 21.7]$
Demand elasticity coefficient	α	$9.5 \times 10^{-4}\$^{-1}$	$[5, 20] \times 10^{-4}$
Cost of waiting	t	\$20/hr	$[5, 40]$
Capacity cost coefficient	β	0.55	$[0.40, 0.70]$
Cost-reduction coefficient	γ	0.072	N/A
Default cost per patient	c_o	\$268/patient	N/A

Table 1 Parameter estimates for numerical analysis. The investment cost function is assumed to be $R(c, \mu) = e^{\beta\mu} + \gamma(c_o - c)^2$ and the the cumulative distribution function of the patients' benefit from receiving treatment is $\Theta(x) = 1 - e^{-x/\alpha}$.

6.2. Welfare ratio

The loss of welfare associated with implementing second-best yardstick competition, where patients are charged a constant fee, compared to first-best, where patients are charged the welfare-maximizing fee, is presented in Figure 1. A value of 1 indicates that there is no welfare loss. For the parameters tested, we observe that the capacity cost coefficient, β, the size of catchment area, Λ, and the demand elasticity coefficient, α, have the most significant impact on welfare ratio. More specifically, as β, Λ or α increase the welfare ratio reduces to as low as 64%. The change in cost of wait per unit time, t, however, impacts welfare ratio to a much lesser extent. Hence, in situations with relatively large capacity cost, and/or large catchment areas, and/or elastic demand, the additional effort to determine the appropriate fee may be warranted.

We next investigate what drives the impact of each of the four parameters (β, Λ, α, and t) on the welfare ratio described above. We start with the capacity cost coefficient β. As β increases,



(a) Welfare ratio vs. β.

(b) Welfare ratio vs. Λ (in patients/hr).

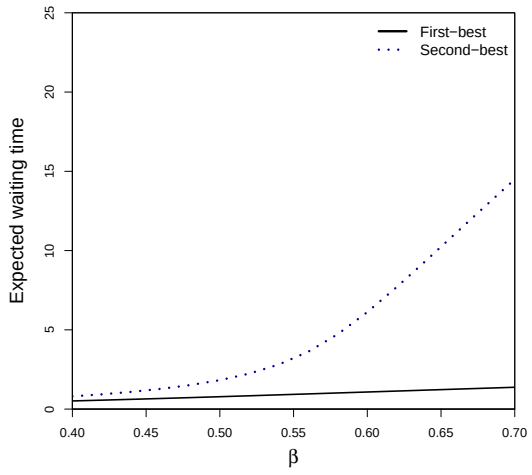
(c) Welfare ratio vs. α.

(d) Welfare ratio vs. t (in \$/hr).

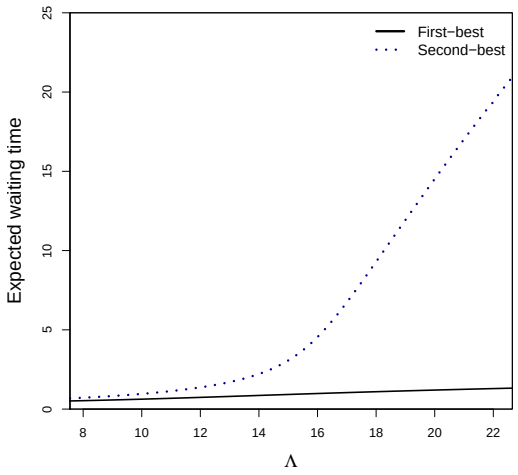
Figure 1 Ratio of second-best to first-best welfare vs. β, Λ, α and t.

capacity becomes more costly, therefore providers choose to operate at higher utilization levels (defined as the effective arrival rate divided by the capacity) under both first- and second-best regulation, resulting in average waiting times that are increasing in β – see Figure 2(a). We note that waiting times increase more for second-best as opposed to first-best regulation. This is due to the fact that customers over-join under second-best regulation, coupled with the fact that expected waiting times become more sensitive to increases in arrival rate in an $M/M/1$ queue as it becomes more congested. Hence, as β gets larger, the welfare loss associated with second-best regulation increases. We observe a similar phenomenon as Λ increases, see Figure 2(b).

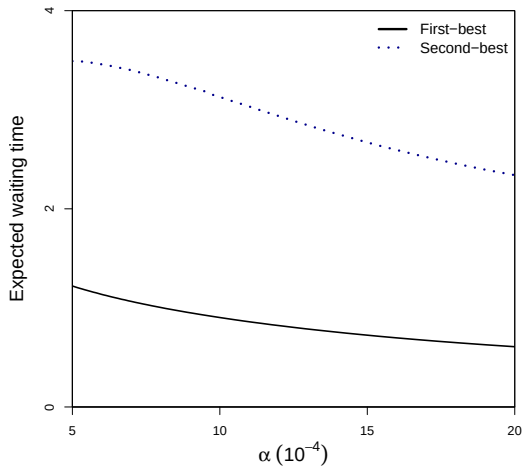
We next turn to the demand sensitivity parameter α. As α decreases, demand becomes less price/wait-time sensitive and so more patients are willing to visit the ED for any given price/wait time. Under first-best regulation, as α decreases, the regulator can react to the associated increase



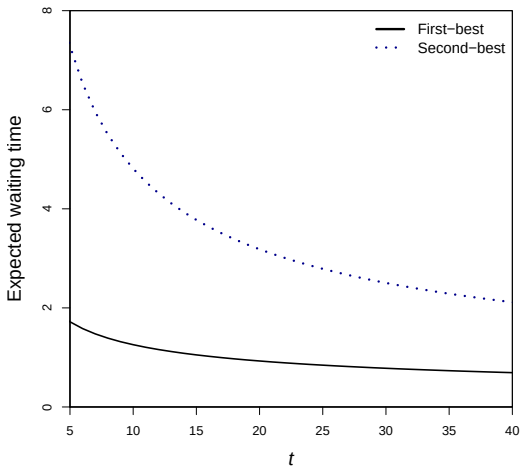
(a) Expected waiting time vs. β.



(b) Expected waiting time vs. Λ (in patients/hr).



(c) Expected waiting time vs. α.



(d) Expected waiting time vs. t (in \$/hr).

Figure 2 Expected waiting time (measured in hours) under first-best (FB) and second-best (SB) regulation vs. β, Λ, α and t.

in demand using two levers: i) increase capacity in order to serve the increased demand faster; ii) increase the price to curtail the increase in arrivals. In our numerical analysis, we find that the regulator will both increase capacity and the price as α decreases. Nevertheless, the increase in capacity under first-best regulation is not enough to reduce waiting times which will increase as α decreases, see Figure 2(c). In contrast, under second-best regulation, as demand becomes less price-sensitive (i.e., α decreases), the regulator only has the first lever available; he can increase capacity but cannot charge a higher price. Furthermore, increasing capacity is more effective in increasing social welfare as arrivals increase, i.e., when α is lower. As a result, we observe that as α decreases, under second-best regulation waiting times also increase, but the gap between the waiting times under first- and second-best regulation remains roughly constant, see Figure 2(c).

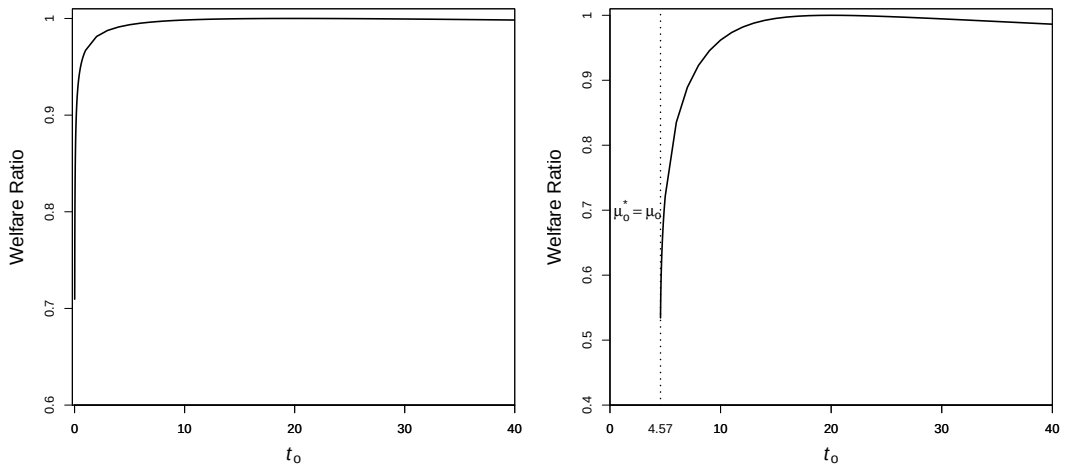
Naturally, since the gap in waiting times remains constant as α increases while the total welfare decreases as demand becomes more price-sensitive (i.e., α increases), the welfare loss associated with second-best compared to first-best regulation also increases in α, as observed in Figure 1.

We next examine the impact of the cost of waiting t on the welfare ratio. We note that it has a less pronounced impact on the welfare ratio than the other parameters (β, Λ, or α), as shown in Figure 1. To see why this is the case, note that if t is high, then under either first- or second-best regulation, the system operates at relatively low utilization, resulting in relatively low waiting times, see Figure 2(d). Hence, the over-joining behaviour observed under second-best regulation does not affect social welfare as much. If, on the other hand, t is low, under either type of regulation, the system will operate under high utilization, resulting in long waiting times, see again Figure 2(d). However, since the waiting time cost t is low, customers are less sensitive to delays and therefore social welfare is, again, not greatly affected by the inefficient over-joining behaviour under second-best regulation.

6.3. Impact of misestimating the cost of waiting

To implement the proposed payment schemes, the regulator needs to estimate the cost of waiting, t. We next test the impact of potential estimation errors on total social welfare using the following procedure. We assume that $t = \$20$ and that the regulator misestimates this cost and sets it equal to $t_o (\neq t)$ in (16)-(17) for first-best and in (23) for second-best regulation. We identify the equilibrium for each t_o ranging from \$0.1 to \$40 in increments of \$0.1 by solving the first-order conditions of the provider's objective and verifying that the provider's objective is maximised with these actions. We then compare the welfare in this equilibrium to the base case, i.e., when the regulator estimates t accurately. We present the welfare ratio as a function of t_o in Figures 3(a) and 3(b) for first- and second-best regulation, respectively. Under second-best regulation the providers exert no effort in capacity expansion in equilibrium for $t_o \leq \$4.57$ hence we plot the welfare ratio beyond this point only. In addition, again for second-best regulation, there exist two symmetric equilibria for $\$4.57 \leq t_o \leq \$5.10$ and so we only report the welfare ratio using the equilibrium that results in a lower welfare loss.

It is clear from from Figure 3 that estimation error in the cost of waiting t generates a loss of welfare. When t is underestimated, providers operate under high utilisation simply because they are not incentivized enough to cut wait times. When t is overestimated, the providers invest more in costly capacity which is underutilized. Nevertheless, there are two interesting observations. First, the welfare loss under first-best regulation is less sensitive to estimation errors than under second-best. Second, the impact of the estimation error is more substantial when t is underestimated (it reaches 70% and 54% in first- and second-best, respectively) compared to when it is overestimated



(a) First-best regulation. (b) Second-best regulation.

Figure 3 Ratio of realized to first-best (second-best) welfare vs. miss-estimated cost of waiting t_o. Actual cost of waiting is $= \$20$. Other parameters set to base case.

(it is above 99.8% and 98% in first- and second-best, respectively, when $t_o \geq t$).[7] We believe that this has to do with the fact that waiting times are convex in capacity – starting from the optimal capacity (set when t is estimated accurately), a small decrease in capacity (due to t being underestimated), will generate a greater loss of welfare due to a larger increase in waiting times than the welfare gain associated with the small reduction in waiting-times generated by a small increase in capacity (due to t being overestimated). Hence, the welfare loss is more substantial when t is underestimated than it is overestimated.

The fact that total welfare is not very sensitive to the actual cost-of-waiting assumed by the regulator points to the fact that social welfare is relatively flat around the the actual cost t. An overestimate in the cost of waiting will generate a welfare loss due to installing more capacity than optimal that will be almost fully compensated by an increase in welfare due to the associated reduction in waiting times and increase in demand. In light of the discussion above, the regulator may be able to use the waiting-time cost parameter t as a lever to influence waiting times; by choosing to implement a regulatory scheme with high t, the regulator shifts the equilibrium outcome towards lower waiting times, at the expense of higher hospital costs, without sacrificing much in terms of total welfare.

[7] We verify the robustness of this observation by generating 1000 random scenarios for Λ, α, and β using the ranges specified in Table 1. In all scenarios, we set $t = \$20$. As in the case above, we find the welfare loss under first- and second-best regulation if the regulator first underestimates t to $t_o = \$10$ and second, overestimates t to $t_0 = \$30$. In all the parameter combinations the welfare loss for first-best regulation was minimal; the average welfare loss was equal to 0.2%, with maximum loss equal to 0.3% for $t_o = \$10$ and it was equal to 0.05% with maximum loss equal to 0.1%, for $t_0 = \$30$. In the second-best (excluding three scenarios which did not have an equilibrium) for $t_o = \$10$, the average welfare loss was equal to 3.53% with maximum loss equal to 40.2% and for $t_o = \$30$, the average welfare loss was 0.4% with maximum loss equal to 3.6%. In addition, the welfare loss was lower for $t_o = \$30$ than for $t_o = \$10$ in all the scenarios under both first- and second-best regulation.

7. Conclusions

This paper investigates the use of yardstick competition, a regulatory scheme that creates cost-reduction incentives (Shleifer 1985), in service settings where, in addition to cost control, the regulator is also interested in incentivizing wait-time reduction. This scheme has proliferated in the regulation (and reimbursement) of hospitals (Fetter 1991). We find that the standard form of yardstick competition fails in this second dimension of performance. Perhaps this finding helps explain the persistently long waiting times experienced by patients in many healthcare systems throughout the world.

We also present two alternative schemes that fare better. The first scheme, which involves a provider-specific customer fee, achieves first-best investment in both cost and wait-time reduction, but is rather difficult to implement in practice – besides the customer fee being politically sensitive in healthcare setting, this scheme places a high informational burden on the regulator with respect to the queueing discipline. The second scheme, which assumes that the service is funded exclusively through transfer payments (e.g., taxes or insurance premia), may be easier to implement. In essence, this scheme modifies the transfer payment of the standard cost-based yardstick competition by adding a component which is decreasing in the difference between the average waiting times of each provider and that of an exogenous benchmark constructed by averaging the waiting time of all other providers. We also show how this scheme can be implemented in a relatively straightforward way in systems with multiple customer classes, time-varying arrival rates, more complex cost structures, and heterogeneous providers. The simplicity of this second regulatory scheme comes at a cost of efficiency as it no longer achieves first-best incentives. Nevertheless, as our numerical investigation illustrates, it is likely to be better than the status quo where waiting-time reduction is not incentivized.

We hope that this paper will contribute to the current debate on how to best incentivize investment in waiting-time reduction in healthcare, particularly in Emergency Departments where waiting times have been argued to be undesirably long. In fact, our paper provides a high-level guideline for regulators, such as CMS in the US and the NHS in the UK who have started monitoring ED waiting times, on how to use waiting-time information in the reimbursement formula. We believe that this is a promising alternative to top-down targets, such as the four-hour target that has been implemented in the UK for patients visiting EDs (see, e.g., Campbell (2016)).

Of course, the exact application may be complicated, especially by concerns about patient selection based on service times or system congestion. We believe this may not be a problem in practice, as was the case with the advent of cost-based yardstick competition which is not believed to have given rise to significant selection based on costs. Nevertheless, understanding and mitigating selection problems that arise in the presence of waiting-time yardstick competition is an issue that

future research should address. An additional limitation of this work is that all of the schemes proposed assume that the regulator knows the cost of customer waiting per unit time, t. This may not always be the case, but, as we show in our numerical investigation, total welfare is not sensitive to the precise value that it takes. In fact, one may view the waiting-time cost parameter t as a lever that can be used to influence waiting times; by choosing to implement a regulatory scheme with high t, the regulator shifts the equilibrium outcome towards lower waiting times at the expense of higher costs with little loss in overall welfare. Nevertheless, identifying a modified scheme that does not require this information may be a promising direction for further research. Finally, we note that our analysis assumes that the service providers are regional monopolists, i.e., they do not compete directly for customers. While the presence of direct competition would make the need for regulatory intervention less pronounced, it would nevertheless be of interest to examine the performance of the proposed scheme under (imperfect) competition.

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Appendix

A. Conditions

As is often the case in queueing games, we have made the assumption that first order conditions (FOC) are necessary and sufficient for both the regulator's and the individual provider's problems (see for example Mendelson and Whang (1990)). This assumption however may not always hold (Stidham 2009). For this reason, in this section, we present additional conditions which ensure that FOC are sufficient for the specific case of the $M/M/1$ queue. More specifically, we provide sufficient (but not necessary) conditions for

- FOC to be necessary and sufficient for determining the first- and second-best solutions of the regulator's welfare maximization problem of Propositions 1 and 4 (see Appendices A.1 and A.4, respectively).
- FOC to be necessary and sufficient for the provider's profit maximization problem under the payment schemes of Propositions 2, Theorems 1 and 2 (see Appendices A.2, A.3, and A.5, respectively).

Where applicable, we compare the conditions presented here to the closest extant literature (Shleifer 1985). We note that these conditions can all be interpreted as either assumptions about the cost function $R(c,\mu)$, which, loosely speaking, needs to be "sufficiently" convex for the objective functions to be well-behaved, or about the default capacity μ_0 (and default cost c_0), which needs to be sufficiently small (large) to allow for interior solutions. We also note that these conditions are sufficient but by no means necessary.

A.1. Assumptions for Proposition 1: Regulator's First-Best Solution

We present sufficient conditions for the FOC to be necessary and sufficient to obtain the regulators' optimal (first-best) actions. We begin by defining $\lambda^*(c,\mu)\in[0,\Lambda]$ by

$$\bar{\Theta}^{-1}\left(\frac{\lambda^*(c,\mu)}{\Lambda}\right)=\frac{t\mu}{(\mu-\lambda^*(c,\mu))^2}+c, \tag{30}$$

and $\mu^*(c)\geq\mu_o$ by

$$\frac{t\lambda^*(c,\mu^*(c))}{(\mu^*(c)-\lambda^*(c,\mu^*(c))^2}=\frac{\partial}{\partial\mu}R(c,\mu^*(c)) \tag{31}$$

for $c\in(0,c_o]$ and $\mu\geq\mu_o$. We show that $\lambda^*(c,\mu)$ and $\mu^*(c)$ are well-defined in the proof of Proposition 5 below. We next define Conditions 1–5 that are used in the next proposition.

Condition 1: $\frac{t\lambda^*(c,\mu)}{(\mu-\lambda^*(c,\mu))^2}$ is decreasing in μ for $\mu\geq\mu_0$ and $c\in(0,c_o]$,
Condition 2: $\frac{t\lambda^*(c,\mu_o)}{(\mu_o-\lambda^*(c,\mu_o))^2}-\frac{\partial R(c,\mu_o)}{\partial\mu}>0$ for $c\in(0,c_o]$,
Condition 3: $\left(-\lambda^*(c,\mu^*(c))-\frac{dR(c,\mu^*(c))}{dc}\right)$ is decreasing in c for $c\in(0,c_o]$,
Condition 4: $\lambda^*(0,\mu^*(0))<-\left(\frac{\partial R(0,\mu^*(0))}{\partial c}+\frac{\partial R(0,\mu^*(0))}{\partial\mu}\right)$,
Condition 5: $\lambda^*(c_o,\mu^*(c_o))>-\left(\frac{\partial R(c_o,\mu^*(c_o))}{\partial c}+\frac{\partial R(c_o,\mu^*(c_o))}{\partial\mu}\right)$.

PROPOSITION 5. *Conditions 1–5 are sufficient for the FOC to be necessary and sufficient for the regulator's welfare maximization problem to define the first-best solution.*

Conditions 2, 4 and 5 are the boundary conditions that guarantee that first-best outcomes are interior. Condition 1 ensures the concavity of welfare function in capacity μ for all $c \in (0, c_o]$ when customers join at rate $\lambda^*(c,\mu)$. Condition 3 is a necessary and sufficient condition for the concavity of welfare function in cost per customer c when customers join at rate $\lambda^*(c,\mu)$ and the provider chooses capacity level $\mu^*(c)$.

Conditions 3–5 are similar to the conditions in Shleifer (1985) (given in (44) and (45)) that guarantee that FOC are sufficient if waiting time is not costly. The only difference is that Shleifer (1985) inherently assumes that capacity is fixed at its default level, μ_o, and hence defines conditions for capacity level μ_o instead of $\mu^*(c)$.

Proof of Proposition 5: We proceed in three main steps.

Step i) First we show that for fixed $c \in [0, c_o]$ and $\mu \in [\mu_o, \infty)$ there exists a unique $p \geq 0$, denoted by $p^*(c,\mu)$, that maximizes $S(p,c,\mu)$, that is,

$$S(p^*(c,\mu),c,\mu) = \max_{p\geq 0} S(p,c,\mu).$$

Step ii) Then we show that for fixed $c \in [0, c_o]$, $\mu^*(c)$ defined in (31) is unique and maximizes $S(p^*(c,\mu),c,\mu)$, that is,

$$S(p^*(c,\mu^*(c)),c,\mu^*(c)) = \max_{0\leq \mu \leq c_o} S(p(c,\mu),c,\mu).$$

Step iii) Next we show that there exists a unique $c^* \in (0, c_o)$ that maximizes $S(p^*(c,\mu^*(c)),c,\mu^*(c))$, that is,

$$S(p^*(c,\mu^*(c^*)),c^*,\mu^*(c^*)) = \max_{0\leq c \leq c_o} S(p(c,\mu),c,\mu^*(c)).$$

We provide the details of these steps next.

Step i) To prove the existence and uniqueness of $p^*(c,\mu)$ we show that $S(p,c,\mu)$ is a concave function of p and $p^*(c,\mu)$ is not at the boundaries, i.e. $p^*(c,\mu) \in (0,\infty)$, for fixed $c \in [0, c_o]$ and $\mu \in [\mu_o, \infty)$. By Leibnitz rule

$$\frac{\partial}{\partial y}\left(\Lambda \int_{\bar{\Theta}^{-1}\left(\frac{y}{\Lambda}\right)}^{\infty} x d\Theta(x)\right) = -\Lambda \bar{\Theta}^{-1}\left(\frac{y}{\Lambda}\right)\theta\left(\bar{\Theta}^{-1}\left(\frac{y}{\Lambda}\right)\right)\frac{\partial \bar{\Theta}^{-1}\left(\frac{y}{\Lambda}\right)}{\partial y} = \bar{\Theta}^{-1}\left(\frac{y}{\Lambda}\right) \text{ for } y \in [0,\Lambda]. \quad (32)$$

By (6) we have

$$\frac{\partial}{\partial p} S(p,c,\mu) = f(p,c,\mu)\frac{\partial}{\partial p}\lambda(p,\mu), \quad (33)$$

where, by (32), f is given by

$$f(p,c,\mu) := \bar{\Theta}^{-1}\left(\frac{\lambda(p,\mu)}{\Lambda}\right) - \frac{t\mu}{(\mu-\lambda(p,\mu))^2} - c. \tag{34}$$

Let $v:[0,\Lambda]\to[0,\infty)$ be defined by

$$v(\lambda) = \bar{\Theta}^{-1}(\lambda/\Lambda). \tag{35}$$

By (2) we have

$$\frac{\partial}{\partial p}\lambda(p,\mu) < 0, \text{ for } p \geq 0 \text{ and } \mu \geq \mu_o. \tag{36}$$

Therefore, to show $S(p,c,\mu)$ is concave in p it is enough to show that $f(p,c,\mu)$ is increasing in p. By (34)

$$\frac{\partial}{\partial p} f(p,c,\mu) = \left(\nu'(\lambda(p,\mu)) - \frac{2t\mu}{(\mu-\lambda(p,\mu))^3}\right)\frac{\partial}{\partial p}\lambda(p,\mu). \tag{37}$$

(We use $'$ to denote the derivative of functions whose domain is one dimensional) We have $\nu'(y) < 0$ for all $y \in [0,\Lambda]$ because $\theta(x) \geq 0$ for $x \geq 0$, and that $\lambda(p,\mu) < \mu$ by (2). Hence $f(p,c,\mu)$ is increasing in p.

To show that $p^*(c,\mu) > 0$ and is unique we next show that $\frac{\partial}{\partial p}S(0,c,\mu) > 0$ and $\lim_{p\to\infty}\frac{\partial}{\partial p}S(p,c,\mu) < 0$. By (34)

$$f(0,c,\mu) = \nu\left(\lambda(0,\mu)\right) - \frac{t\mu}{(\mu-\lambda(0,\mu))^2} - c = \frac{t}{\mu-\lambda(0,\mu)} - \frac{t\mu}{(\mu-\lambda(0,\mu))^2} - c < 0, \tag{38}$$

where the second equality follows from (2), giving $\frac{\partial}{\partial p}S(0,c,\mu) > 0$. Also, since $\lim_{p\to\infty}\lambda(p,\mu) = 0$ for all $\mu \geq \mu_o$ by (2), we have

$$\lim_{p\to\infty} f(p,c,\mu) > 0, \tag{39}$$

giving $\lim_{p\to\infty}\frac{\partial}{\partial p}S(p,c,\mu) < 0$.

Therefore, for any $c \in [0,c_o]$ and $\mu \in [\mu_o,\infty)$, there exists a unique optimal $p^*(c,\mu) \in (0,\infty)$, which is obtained by

$$\frac{\partial}{\partial p}S(p^*(c,\mu),c,\mu) = \nu\left(\lambda(p^*(c,\mu),\mu)\right) - \frac{t\mu}{(\mu-\lambda(p^*(c,\mu),\mu))^2} - c = 0. \tag{40}$$

This also proves the existence and uniqueness of $\lambda^*(c,\mu)$ defined in (30).

Step ii) To prove that there exists a unique $\mu \in (\mu_o,\infty)$ that maximizes $S(p^*(c,\mu),c,\mu)$ for fixed $0 \leq c \leq c_o$, we show that (i) $S(p^*(c,\mu),c,\mu)$ is concave in μ. (ii) $\lim_{\mu\downarrow\mu_o}\frac{d}{d\mu}S(p^*(c,\mu),c,\mu) > 0$ and (iii) $\lim_{\mu\to\infty}\frac{d}{d\mu}S(p^*(c,\mu),c,\mu) < 0$. By (6) and (32),

$$\frac{d}{d\mu}S(p^*(c,\mu),c,\mu) = \left(\nu(\lambda^*(c,\mu)) - \frac{t\mu}{(\mu-\lambda^*(c,\mu))^2} - c\right)\frac{\partial}{\partial\mu}\lambda^*(c,\mu) + \frac{t\lambda^*(c,\mu)}{(\mu-\lambda^*(c,\mu))^2} - \frac{\partial R(c,\mu)}{\partial\mu}$$

$$= \frac{t\lambda^*(c,\mu)}{(\mu-\lambda^*(c,\mu))^2} - \frac{\partial R(c,\mu)}{\partial \mu}, \tag{41}$$

where (41) follows from (40). By Condition 1 and that $\frac{\partial R(c,\mu)}{\partial \mu}$ is increasing in μ by convexity of R, $\frac{d}{d\mu}S(p^*(c,\mu),c,\mu)$ is decreasing; and hence, $S(p^*(c,\mu),c,\mu)$ is concave in μ. By Condition 2 and (41), we have $\frac{d}{d\mu}S(p^*(c,\mu_o),c,\mu_o)>0$. Also, by (41), convexity of R and $\lambda^*(c,\mu)\leq\Lambda$ by (2), we get

$$\lim_{\mu\to\infty}\frac{d}{d\mu}S(p^*(c,\mu),c,\mu) = \lim_{\mu\to\infty} -\frac{\partial R(c,\mu)}{\partial \mu} < 0. \tag{42}$$

Thus, the optimal capacity $\mu^*(c)$ given any $c\in(0,c_o]$ can be obtained by (31).

Step iii) Finally, we show that there is a unique $c\in(0,c_o)$ that maximizes $S(p^*(c,\mu^*(c)),c,\mu^*(c))$. Let $S^*(c)=S(p^*(c,\mu^*(c)),c,\mu^*(c))$ for notational simplicity. By (6) and (32), we have

$$\begin{aligned}\frac{\partial S^*(c)}{\partial c} &= \left(\nu(\lambda^*(c,\mu^*(c))) - \frac{t\mu^*(c)}{(\mu^*(c)-\lambda^*(c,\mu^*(c)))^2} - c\right)\frac{d}{dc}\lambda^*(c,\mu^*(c)) \\ &\quad + \left(\frac{t\lambda^*(c,\mu^*(c))}{(\mu^*(c)-\lambda^*(c,\mu^*(c)))^2} - \frac{\partial R(c,\mu^*(c))}{\partial \mu}\right)\frac{\partial \mu^*(c)}{\partial c} - \lambda^*(c,\mu^*(c)) - \frac{dR(c,\mu^*(c))}{dc} \\ &= -\lambda^*(c,\mu^*(c)) - \frac{dR(c,\mu^*(c))}{dc}, \end{aligned} \tag{43}$$

where (43) follows from (30) and (31). By (43) and Condition 3, $\frac{dS^*(c)}{dc}$ is decreasing and hence $S^*(c)$ is strictly concave in c. In addition, by Conditions 4 and 5, we have $\frac{dS^*(0)}{dc}>0$ and $\frac{dS^*(c_o)}{dc}<0$. Thus, there exists a unique global maximum c^* of $S^*(c)$ and $(p^*(c^*,\mu^*(c^*)),\mu^*(c^*),c^*)$ is the unique optimizer of $S(c,p,\mu)$ and satisfies the FOC (10)–(12). Because (40), (41) and (43) have unique solutions, $(p^*(c^*,\mu^*(c^*)),\mu^*(c^*),c^*)$ is the unique point that satisfies the FOC. □

A.2. Assumptions for Proposition 2

We provide sufficient conditions that guarantee the FOCs of the provider's profit maximization problem to be necessary and sufficient under the payment scheme of Proposition 2. Since this is the problem studied in Shleifer (1985), these conditions were first presented in Shleifer (1985). Expressed in our notation, these are that $R(c,\mu_o)$ is convex in c and

$$\frac{\partial \lambda(c,\mu_o)}{\partial c} + \frac{\partial^2 R(c,\mu_o)}{\partial c^2} > 0, \tag{44}$$

along with the boundary conditions

$$\lambda(c_o,\mu_o) + \frac{\partial}{\partial c}R(c_o,\mu_o) > 0 \text{ and } \lambda(0,\mu_o) + \frac{\partial}{\partial c}R(0,\mu_o) < 0. \tag{45}$$

Further to Shleifer (1985), we need to also assume that (44) holds for all $\mu\geq\mu_0$. These conditions ensure that the objective function of the provider is concave and the solution is interior for all $\mu\geq\mu_0$.

A.3. Assumptions for Theorem 1

We provide sufficient conditions that guarantee the FOCs of the provider's profit maximization problem to be necessary and sufficient under the payment scheme of Theorem 1. The provider's profit is given by (18) and if R is strictly convex then Π is strictly concave. If in addition the optimal actions for the provider are not at the boundaries, then the FOC are necessary and sufficient to determine the provider's optimal actions. We next present sufficient conditions that guarantee that the maximum is attained at an interior point.

The following conditions imply that $c^* \in (0, c_o)$.

$$\frac{\partial}{\partial c} R(c_o, \mu) > -\lambda(c_o, \mu_o) \text{ and } \frac{\partial}{\partial c} R(0, \mu) < -\Lambda, \text{ for all } \mu \geq \mu_o. \tag{46}$$

This follows from (18) because R is convex in c and $\bar{\lambda}_i \in [\lambda(c_o, \mu_o), \Lambda]$ for all i.

To list conditions that are sufficient for $\mu^* \in (\mu_o, \infty)$ we first introduce some terminology. By (2) and (16) $\frac{t}{(\bar{\mu}_i - \bar{\lambda}_i)} \leq v(\lambda_{min})$ for all $i = 1, \ldots, N$. Hence there exists $K > 0$ such that

$$\frac{\bar{\lambda}_i t}{(\bar{\mu}_i - \bar{\lambda}_i)^2} < K.$$

In addition this implies that it is never optimal to set $\mu \geq M$ in any equilibrium for a provider by (18) for some $M > 0$. The following conditions are sufficient for μ^* to be an interior point. Assume that there exits μ_K such that $\frac{\partial}{\partial \mu} R(c, \mu) > K$ for $\mu \geq \mu_K$ and for any $c \in [0, c_o]$ and that

$$\frac{\partial}{\partial \mu} R(c, \mu_o) < \frac{t\lambda(c_o, \mu_o)}{\left(M - \lambda(c_o, \mu_o)\right)^2}, \text{ for all } c \in [0, c_o]. \tag{47}$$

The fact that $\mu^* \in (\mu_o, \infty)$ follows from these conditions because; i) R is convex in μ, ii) $\frac{t\lambda}{(\mu - \lambda(c,\mu))^2}$ is minimized at $\mu = \mu_o$ and $c = c_o$ and (iii) M is an upper bound for μ.

A.4. Assumptions for Proposition 4: Regulator's Second-Best Solution

We present sufficient conditions for the FOC to be necessary and sufficient to obtain the regulators' optimal (second-best) actions.

Let $\tilde{c}(\mu)$ be the solution of

$$\frac{\partial}{\partial c} R(\tilde{c}(\mu), \mu) = -\lambda(\mu) \text{ for } \mu \geq \mu_o. \tag{48}$$

We show that $\tilde{c}(\mu)$ is well-defined in the proof of Proposition 6 below. We first define Conditions 6–9 that are used in the next proposition.

Condition 6: $\lambda(\mu) < -\frac{\partial R(0,\mu)}{\partial c}$ for $\mu \geq \mu_o$,
Condition 7: $\lambda(\mu) > -\frac{\partial R(c_o,\mu)}{\partial c}$ for $\mu \geq \mu_o$,
Condition 8: $\left(\frac{t\lambda(\mu)}{(\mu - \lambda(\mu))^2}\left(1 - \frac{\partial \lambda(\mu)}{\partial \mu}\right) - \tilde{c}(\mu)\frac{\partial \lambda(\mu)}{\partial \mu} - \frac{d}{d\mu} R(\tilde{c}(\mu), \mu)\right)$ is decreasing for $\mu \geq \mu_o$,
Condition 9: $\left(\frac{t\lambda(\mu_o)}{(\mu_o - \lambda(\mu_o))^2}\left(1 - \frac{\partial \lambda(\mu_o)}{\partial \mu}\right) - \tilde{c}(\mu_o)\frac{\partial \lambda(\mu_o)}{\partial \mu}\right) > \frac{\partial}{\partial c} R(\tilde{c}(\mu_o), \mu_o) + \frac{\partial}{\partial \mu} R(\tilde{c}(\mu_o), \mu_o)$.

PROPOSITION 6. *Conditions 6–9 are sufficient for the FOC of the regulator's problem to be necessary and sufficient to obtain the unique second-best solution.*

Conditions 6, 7 and 9 are the boundary conditions that guarantee the second-best outcomes are interior. Condition 8 ensures that if $p=0$ then the welfare function is concave in capacity μ when the provider chooses the optimal marginal cost level $\tilde{c}(\mu)$ for all $\mu \geq \mu_o$. Sufficient conditions presented for the first-best (Conditions 1–5) and second-best (Conditions 6–9) solutions are quite different despite the fact that the second best setting can be considered as a special case of the first best. In fact only Condition 6 implies Condition 4 because $\lambda^*(0,\mu) \leq \lambda(\mu)$ by (30).

Proof of Proposition 6: The proof is similar to that of Proposition 5 in that we first show that for fixed μ there is a unique optimal cost per patient $\tilde{c}(\mu)$ and then we show there is a unique optimal μ.

If customers are not charged for service, i.e., $p=0$, given any $c \in (0, c_o]$ and $\mu \geq \mu_o$, the welfare function is $S(c,\mu)$, where S is as given in (6) for $p=0$ under the $M/M/1$ assumption. We start by showing that FOC suffices to obtain the second-best marginal cost (denoted by $\tilde{c}(\mu)$) for fixed $\mu \geq \mu_o$. We prove this by showing that $S(\cdot,\mu)$ is concave for fixed μ and $\tilde{c}(\mu)$ cannot be at the boundaries. By (6) (with $p=0$), we have

$$\frac{\partial S(c,\mu)}{\partial c} = -\lambda(\mu) - \frac{\partial R(c,\mu)}{\partial c}, \quad \frac{\partial^2 S(c,\mu)}{\partial c^2} = -\frac{\partial^2 R(c,\mu)}{\partial c^2}. \tag{49}$$

Because R is convex $S(\cdot,\mu)$ is concave for fixed μ. Since, in addition, $\frac{\partial S(0,\mu)}{\partial c} > 0$ and $\frac{\partial S(c_o,\mu)}{\partial c} < 0$ by Conditions 6 and 7, given any $\mu \geq \mu_o$ there exists a unique $\tilde{c}(\mu) \in (0, c_o)$ that is obtained by the FOC and it satisfies (48).

We next show that FOC suffices to obtain the maximum of $S(\tilde{c}(\mu),\mu)$ by showing that it is concave and the optimal is attained at an interior point. By (6) and (32), we have

$$\begin{aligned}
\frac{d}{d\mu} S(\tilde{c}(\mu),\mu) &= \left(\nu(\lambda(\mu)) - \frac{t}{\mu-\lambda(\mu)}\right)\frac{\partial \lambda(\mu)}{\partial \mu} - \left(\lambda(\mu) + \frac{\partial R(\tilde{c}(\mu),\mu)}{\partial c}\right)\frac{\partial \tilde{c}(\mu)}{\partial \mu} \\
&\quad + \frac{t\lambda(\mu)}{(\mu-\lambda(\mu))^2}\left(1 - \frac{\partial \lambda(\mu)}{\partial \mu}\right) - \tilde{c}(\mu)\frac{\partial \lambda(\mu)}{\partial \mu} - \frac{d}{d\mu} R(\tilde{c}(\mu),\mu) \\
&= \frac{t\lambda(\mu)}{(\mu-\lambda(\mu))^2}\left(1 - \frac{\partial \lambda(\mu)}{\partial \mu}\right) - \tilde{c}(\mu)\frac{\partial \lambda(\mu)}{\partial \mu} - \frac{d}{d\mu} R(\tilde{c}(\mu),\mu),
\end{aligned} \tag{50}$$

where (50) follows from (2) and (48). By (50) and Condition 8, $\frac{d}{d\mu} S(\tilde{c}(\mu),\mu)$ is decreasing in μ and hence $S(\tilde{c}(\mu),\mu)$ is concave. By (50) and Condition 9, $\frac{\partial}{\partial c} R(\tilde{c}(\mu_o),\mu_o) + \frac{\partial}{\partial \mu} R(\tilde{c}(\mu_o),\mu_o) > 0$. Also, because $\lim_{\mu\to\infty} \frac{t\lambda(\mu)}{(\mu-\lambda(\mu))^2} = 0$, $\frac{d\lambda(\mu)}{d\mu} \geq 0$ by (2) and $\tilde{c}(\mu) > 0$ as discussed above, we have $\lim_{\mu\to\infty} \frac{d}{d\mu} S(\tilde{c}(\mu),\mu) \leq \lim_{\mu\to\infty} -\frac{d}{d\mu} R(\tilde{c}(\mu),\mu) < 0$. Hence $\mu_o^* \in (\mu_o, \infty)$. Clearly $(\tilde{c}(\mu_o^*), \mu_o^*)$ maximizes S and is the unique solution that satisfies the FOC's under Conditions 6–9. $\square$

A.5. Assumptions for Theorem 2

We provide sufficient conditions that guarantee the FOC of the providers' profit maximization problem to be necessary and sufficient under the payment scheme of Theorem 2, which yields the profit function in (24).

We claim that if

i) $c\lambda(\mu)+R(c,\mu)$ is jointly convex in c and μ,

ii) ν is convex,

iii) $\frac{1}{\nu}$ is convex,

then $\Pi(c,\mu)$ in (24) is concave. Clearly, if $c\lambda(\mu)+R(c,\mu)$ is convex, it suffices to show that $W(\lambda(\mu),\mu)$ is convex to prove the the concavity of $\Pi(c,\mu)$. By (1)

$$\frac{d^2}{d\mu^2}W(\lambda(\mu),\mu)=(\lambda'(\mu))^2\nu''(\lambda(\mu))+\lambda''(\mu)\nu'(\lambda(\mu)). \tag{51}$$

Because $\nu'(\lambda(\mu))<0$ by definition it is enough to show that $\lambda''(\mu)<0$ to prove the concavity of W. We note that $\frac{1}{\nu}$ implies $\lambda''(\mu)<0$ because $\lambda(\mu)=\mu-\frac{t}{\nu(\lambda(\mu))}$ by (1). Hence, if the optimal is attained at an interior point (i)–(iii) guarantee that the FOC are necessary and sufficient to determine the provider's profit maximizing actions. We next provide sufficient conditions that imply that this is the case.

Similar to (46) assume that

$$\frac{\partial}{\partial c}R(c_o,\mu)>-\lambda(\mu_o) \text{ and } \frac{\partial}{\partial c}R(0,\mu)<-\Lambda, \text{ for all } \mu\geq\mu_o. \tag{52}$$

Conditions (52) imply that $c_o^*\in(0,c_o)$ by (24) because R is convex in c and $\lambda_i\in[\lambda(\mu_o),\Lambda]$ for all $i=1,\ldots,N$.

The following conditions imply that $\mu_o^*\in(\mu_o,\infty)$ by (24) because R is convex in μ. Assume that

$$\frac{\partial}{\partial\mu}R(c,\mu_o)<\left(-c-\lambda(\mu_o)v'(\lambda(\mu_o))\right)\lambda'(\mu_o) \tag{53}$$

and

$$\lim_{\tilde{\mu}\to\infty}\frac{\partial}{\partial\mu}R(c,\tilde{\mu})>\lim_{\tilde{\mu}\to\infty}-(\Lambda v'(\lambda(\tilde{\mu}))\lambda'(\tilde{\mu})) \tag{54}$$

for all $c\in[0,c_0]$, where v is defined as in (35).

B. Proofs of the results

Proof of Proposition 1: Under the assumption that FOC are necessary and sufficient to to obtain the first-best outcomes (see Appendix A for conditions that guarantee that this is the case), the first-best price, p^*, marginal cost, c^*, and capacity, μ^*, are the unique solutions to $\frac{\partial}{\partial p}S(p,c,\mu)=0$, $\frac{\partial}{\partial c}S(p,c,\mu)=0$ and $\frac{\partial}{\partial\mu}S(p,c,\mu)=0$, which, yield (9)–(11). The first-best transfer payment, T^*, is obtained by solving for $\Pi(c^*,\mu^*|p^*,T^*)=0$, which leads to (12) by (3). $\square$

Proof of Proposition 2: See proof of Proposition 1 in Shleifer (1985).

Proof of Proposition 3: For the regulatory scheme given in Proposition 1, by (3) provider i's profit function is

$$\Pi(c_i, \mu_i | p_i, T_i) = (\bar{c}_i - c_i)\lambda(\bar{c}_i, \mu_i) - R(c_i, \mu_i) + \bar{R}_i. \tag{55}$$

Because $\frac{\partial}{\partial \mu_i}\Pi(c_i, \mu_i | p_i, T_i) = (\bar{c}_i - c_i)\frac{\partial}{\partial \mu_i}\lambda(\bar{c}_i, \mu_i) - \frac{\partial}{\partial \mu_i}R(c_i, \mu_i)$, in any symmetric equilibrium where $\bar{c}_i = c_i$, we have $\frac{\partial}{\partial \mu_i}\Pi(c_i, \mu_i | p_i, T_i) < 0$ for all $\mu_i \geq \mu_o$. Thus, in all potential symmetric equilibria, all providers choose their default capacity level, μ_o.

Also, because $R(c, \mu_o)$ is convex $\Pi(c_i, \mu_o | p_i, T_i)$ is concave in c_i. By (44) and (45) provider i's optimal marginal cost is obtained by

$$\frac{\partial}{\partial c_i}\Pi(c_i, \mu_o | p_i, T_i) = -\lambda(\bar{c}_i, \mu_o) - \frac{\partial}{\partial c_i}R(c_i, \mu_o) = 0, \tag{56}$$

which holds at a unique $c_i = \bar{c}_i = \breve{c}$. Hence, there exists a unique symmetric equilibrium where all providers choose capacity level μ_o and marginal cost level $\breve{c}$ (and make zero profit).

We next show that $\breve{c} > c^*$ under the additional assumptions that $\frac{\partial^2 R(c,\mu)}{\partial c \partial \mu} \geq 0$ and (44) holds for all $\mu \geq \mu_0$. If $\frac{\partial^2 R(c,\mu)}{\partial c \partial \mu} \geq 0$, because $\lambda(p, \mu)$ is strictly increasing in μ by (1), $\frac{\partial}{\partial \mu}W(\lambda, \mu) < 0$ and $\frac{\partial}{\partial \lambda}W(\lambda, \mu) > 0$, $\left(\lambda(c, \mu) + \frac{\partial R(c,\mu)}{\partial c}\right)$ is strictly increasing in μ for $c \in (0, c_o]$. Thus, for $\mu^* > \mu_o$, we have

$$\lambda(\breve{c}, \mu^*) + \frac{\partial R(\breve{c}, \mu^*)}{\partial c} > \lambda(\breve{c}, \mu_o) + \frac{\partial R(\breve{c}, \mu_o)}{\partial c} = 0, \tag{57}$$

where the equality follows from the fact that $\breve{c}$ satisfies (56) by definition in the unique symmetric equilibrium. By (44) if $c^* \geq \breve{c}$, then

$$\lambda(c^*, \mu^*) + \frac{\partial R(c^*, \mu^*)}{\partial c} \geq \lambda(\breve{c}, \mu^*) + \frac{\partial R(\breve{c}, \mu^*)}{\partial c},$$

which, along with (57), leads to $\lambda(c^*, \mu^*) + \frac{\partial R(c^*, \mu^*)}{\partial c} > 0$. However, this contradicts the optimality of (c^*, μ^*) in the welfare maximization problem because (9) cannot hold. Thus $c^* < \breve{c}$. □

Proof of Theorem 1: Assume that the FOC are necessary and sufficient to obtain the optimal actions of each provider (see Appendix A.3 for sufficient conditions). If the regulator sets service provider i's price equal to p_i given in (16) and transfer payment equal to T_i given in (17), provider i's objective function is as given in (18) by (3). We next show that there is a unique symmetric equilibrium. Let $a_j = (\tilde{c}, \tilde{\mu})$ denote the action of provider j for all $j \neq i$ and let $\tilde{\lambda}$ denote the associated arrival rate that satisfies (1) with price set as in (16).

By (18) the FOC of Π for provider i are

$$\frac{\partial}{\partial c}\Pi(c_i, \mu_i) = -\tilde{\lambda} - \frac{\partial}{\partial c}R(c_i, \mu_i) = 0, \tag{58}$$

$$\frac{\partial}{\partial \mu}\Pi(c_i,\mu_i) = -t\frac{\partial}{\partial \mu}W(\tilde{\mu},\tilde{\lambda})\tilde{\lambda} - \frac{\partial}{\partial \mu}R(c_i,\mu_i) = 0. \tag{59}$$

If $\tilde{c} = c^*$ and $\tilde{\mu} = \mu^*$, then $\tilde{\lambda} = \lambda^*$ by (1) and (16). Because (9)–(11) have a unique solution, so do (58)–(59). In addition, because FOC are necessary and sufficient to obtain the optimal actions of each provider, (c^*, μ^*) is a Nash equilibrium. It is easy to check that providers make zero profit in equilibrium.

Now consider any other $\tilde{c}$ and $\tilde{\mu}$. In order for provider i to pick the same actions, $\tilde{c}$ and $\tilde{\mu}$ have to satisfy the FOC (58) and (59) because the FOC are necessary and sufficient to obtain the optimal actions of each provider. However, because S has a unique optimal solution and the FOC are sufficient, for $\tilde{c}$ and $\tilde{\mu}$ to be a solution to (58) and (59), they must satisfy $\tilde{c} = c^*$ and $\tilde{\mu} = \mu^*$. Hence, (c^*, μ^*) is the unique symmetric equilibrium. □

Proof of Theorem 2: Assume that the FOC are necessary and sufficient to obtain the optimal actions of each provider (see Appendix A.5 for sufficient conditions). Assume that the regulator pays the transfer payment T_i defined as in (23) to provider i, for $i = 1, \ldots, N$ and customers are not charged a toll. The proof of the result is similar to that of Theorem 1.

When patients are not charged a toll, the objective of the regulator is

$$S(c,\mu) = \Lambda \int_{tW(\lambda(\mu),\mu)}^{\infty} (x - tW(\lambda(\mu),\mu))\, d\Theta(x) - c\lambda(\mu) - R(c,\mu). \tag{60}$$

Hence, by (1) and (32), the FOC of $S(c,\mu)$ are given by (19) and (20). Because FOC are assumed to be necessary and sufficient, (19) and (20) have a unique solution, which is μ_o^* and c_o^*.

We next show that $\mu_i = \mu_o^*$ and $c_i = c_o^*$ for $i = 1, \ldots, N$ is an equilibrium under the scheme given in Theorem 2. Assume that each provider except provider i picks $(\tilde{c}, \tilde{\mu})$. Then, by (24) provider i's optimal actions satisfy

$$\frac{\partial}{\partial c}\Pi(c,\mu) = -\lambda(\mu) - \frac{\partial}{\partial c}R(c,\mu) = 0, \tag{61}$$

$$\frac{\partial}{\partial \mu}\Pi(c,\mu) = -c\lambda'(\mu) - t\tilde{\lambda}\frac{\partial}{\partial \mu}W(\lambda(\mu),\mu) - \frac{\partial}{\partial \mu}R(c,\mu) = 0, \tag{62}$$

because the FOCtions are necessary and sufficient to obtain the optimal actions of each provider. If $\tilde{c} = c_o^*$ and $\tilde{\mu} = \mu_o^*$, because (19) and (20) have a unique solution (c_o^*, μ_o^*), so do (61) and (62). Because (c_o^*, μ_o^*) is the solution to (19) and (20), (c_o^*, μ_o^*) is a symmetric equilibrium. It can easily be shown that providers make zero profit in this equilibrium. Uniqueness of the symmetric equilibrium follows as in the proof of Theorem 1. □

C. Alternative payment scheme for first best with a unique equilibrium

In this section we first present an alternative scheme that achieves first-best outcomes in the unique symmetric equilibrium. Then, we show that a modified version of this scheme leads to a unique equilibrium, i.e., the unique symmetric equilibrium that achieves first-best outcomes is the only equilibrium.

For $\lambda, \mu \geq 0$ let $\Psi_i : \mathbb{R}^2_+ \to \mathbb{R}$ be defined as

$$\Psi_i(\lambda, \mu) = \begin{cases} W(\lambda, \mu), \text{ if } \mu \geq \lambda, \\ \bar{W}_i + \frac{\bar{c}_i \lambda + \bar{R}_i}{t\lambda}, \text{ if } \mu < \lambda \end{cases}, \tag{63}$$

for given $\bar{W}_i, \bar{c}_i$ and $\bar{R}_i$ and also let the transfer payment, T_i, to provider i be

$$T_i = \bar{R}_i + (\bar{c}_i - c_i)\bar{\lambda}_i - t\lambda_i^2 \frac{\partial}{\partial \lambda} W(\lambda_i, \mu_i) - t\bar{\lambda}_i(\Psi_i(\bar{\lambda}_i, \mu_i) - \overline{W_i}). \tag{64}$$

If in addition the toll is set as in (16) the ith provider's objective becomes

$$\Pi(c_i, \mu_i | p_i, T_i) = (\bar{c}_i - c_i)\bar{\lambda}_i - t\bar{\lambda}_i(\Psi_i(\bar{\lambda}_i, \mu_i) - \overline{W_i}) - R(c_i, \mu_i) + \bar{R}_i. \tag{65}$$

Before we establish the equilibrium under this payment system we note that for a given $\bar{\lambda}_i$ provider i will never choose $\mu_i \leq \bar{\lambda}_i$ because by (63) it's profit will be negative and it can always set $c_i = \bar{c}_i$ and $\mu_i = \bar{\mu}_i$ to obtain zero profits. Therefore we assume without loss of generality that $\mu_i \geq \bar{\lambda}_i$. Also if $\mu_i \geq \bar{\lambda}_i$

$$\Pi(c_i, \mu_i | p_i, T_i) = (\bar{c}_i - c_i)\bar{\lambda}_i - t\bar{\lambda}_i(W(\bar{\lambda}_i, \mu_i) - \bar{W}_i) - R(c_i, \mu_i) + \bar{R}_i \tag{66}$$

by (63) and (24). For the rest of this section we assume that FOC are necessary and sufficient for determining the optimal actions of the provider and the waiting time function satisfies the following assumption.

Assumption 1. *We assume that* $\frac{\partial^2}{\partial \lambda^2} W(\lambda, \mu) > 0$, $\frac{\partial^2}{\partial \mu^2} W(\lambda, \mu) > 0$, *and* $\frac{\partial^2 W(\lambda, \mu)}{\partial \mu \partial \lambda} \leq 0$.

Proposition 7. *If the regulator sets service provider i's price equal to p_i given in* (16) *and transfer payment equal to T_i given in* (64)*, then the unique symmetric Nash equilibrium is for each provider i to pick $c_i = c^*$ and $\mu_i = \mu^*$, for $i = 1, ..., N$. Also, all providers make zero profit in equilibrium.*

Proof of Proposition 7: Provider i's optimal actions are obtained by

$$\frac{\partial}{\partial c_i} \Pi(c_i, \mu_i | p_i, T_i) = -\bar{\lambda}_i - \frac{\partial}{\partial c_i} R(c_i, \mu_i) = 0, \tag{67}$$

$$\frac{\partial}{\partial \mu_i} \Pi(c_i, \mu_i | p_i, T_i) = -\frac{\partial}{\partial \mu_i} R(c_i, \mu_i) - t\bar{\lambda}_i \frac{\partial}{\partial \mu_i} W(\bar{\lambda}_i, \mu_i) = 0. \tag{68}$$

Because first-best marginal cost, c^*, and capacity, μ^*, satisfy (9)–(11), the case where $c_i = c^*$ and $\mu_i = \mu^*$ for all $i = 1, \ldots, N$ is clearly an equilibrium. Also, similar to the proof of Theorem 1, there cannot exist a different symmetric solution because c^* and μ^* are the unique solution to (9)–(11). Hence, there exists a unique symmetric equilibrium that yields first-best outcomes. □

Next we show that if $N = 2$ there cannot be an asymmetric equilibrium.

PROPOSITION 8. *Assume that $N = 2$ and $\frac{\partial^2 R(c,\mu)}{\partial c \partial \mu} \geq 0$. Under the payment scheme in Proposition 7 there exists a unique equilibrium.*

We need the following result to prove Proposition 8.

PROPOSITION 9. *Consider the payment scheme in Proposition 7 and assume that $\frac{\partial^2 R(c,\mu)}{\partial c \partial \mu} \geq 0$ and $N = 2$. Let $(\tilde{c}_i, \tilde{\mu}_i)$ for $i = 1, 2$ denote an equilibrium and $\tilde{\lambda}_i = \lambda(p_i, \tilde{\mu}_i)$ be given by (1) where p_i is defined as in (16). If $\tilde{\lambda}_1 > \tilde{\lambda}_2 > 0$ then $\tilde{\mu}_1 > \tilde{\mu}_2$ and $\tilde{c}_1 < \tilde{c}_2$.*

Proof of Proposition 8: Similar to the proof of Proposition 7, we only consider $\mu_i \geq \bar{\lambda}_i$. By (3) if provider j choses action $(\tilde{c}_j, \tilde{\mu}_j)$ provider i's profit function is

$$\Pi(c_i, \mu_i | p_i, T_i) = (\tilde{c}_j - c_i)\tilde{\lambda}_j - R(c_i, \mu_i) + R(\tilde{c}_j, \tilde{\mu}_j) - t\tilde{\lambda}_j (W(\tilde{\lambda}_j, \mu_i) - W(\tilde{\lambda}_j, \tilde{\mu}_j)). \tag{69}$$

We next show that an asymmetric equilibrium does not exist when $N = 2$. Let $(\tilde{\mu}_i, \tilde{c}_i)$ denote the action chosen by provider i and $\tilde{\lambda}_i = \lambda(\tilde{p}_i, \tilde{\mu}_i)$ be given by (1) where p_i is defined as in (16)

Assume that $\tilde{\lambda}_1 = \tilde{\lambda}_2$. By (67) and (69)

$$\frac{\partial R(\tilde{c}_1, \tilde{\mu}_1)}{\partial c} = -\tilde{\lambda}_2 = -\tilde{\lambda}_1 = \frac{\partial R(\tilde{c}_2, \tilde{\mu}_2)}{\partial c}. \tag{70}$$

By convexity of R if $\tilde{\mu}_1 = \tilde{\mu}_2$, then $\tilde{c}_1 = \tilde{c}_2$. Hence, a potential asymmetric equilibrium with $\tilde{\lambda}_1 = \tilde{\lambda}_2$ must have $\tilde{\mu}_1 \neq \tilde{\mu}_2$. Without loss of generality we assume $\tilde{\mu}_1 < \tilde{\mu}_2$. By (1), $\tilde{\lambda}_1 = \tilde{\lambda}_2$ implies $p_1 < p_2$ because $W(\tilde{\lambda}_1, \tilde{\mu}_1) > W(\tilde{\lambda}_2, \tilde{\mu}_2)$ when $\tilde{\mu}_1 < \tilde{\mu}_2$ and $\tilde{\lambda}_1 = \tilde{\lambda}_2$. By (16), $p_1 < p_2$ implies $\tilde{c}_1 < \tilde{c}_2$ because

$$\frac{\partial}{\partial \lambda} W(\tilde{\lambda}_1, \tilde{\mu}_1) \geq \frac{\partial}{\partial \lambda} W(\tilde{\lambda}_2, \tilde{\mu}_2)$$

by Assumption 1. Because $\tilde{c}_1 < \tilde{c}_2$ and $\tilde{\mu}_1 < \tilde{\mu}_2$,

$$\frac{\partial R(\tilde{c}_1, \tilde{\mu}_1)}{\partial c} < \frac{\partial R(\tilde{c}_2, \tilde{\mu}_2)}{\partial c}$$

by convexity of R and $\frac{\partial^2 R(c,\mu)}{\partial c \partial \mu} \geq 0$. Because this contradicts (70), an asymmetric equilibrium with $\lambda_1 = \lambda_2$ does not exist.

Next we consider the case $\tilde{\lambda}_1 \neq \tilde{\lambda}_2$ and assume without loss of generality that $\tilde{\lambda}_1 > \tilde{\lambda}_2$. Then, by Proposition 9 we have $\tilde{\mu}_1 > \tilde{\mu}_2$. By Assumption 1, $\tilde{\lambda}_1 > \tilde{\lambda}_2$ and $\tilde{\mu}_1 > \tilde{\mu}_2$ imply

$$W(\tilde{\lambda}_1, \tilde{\mu}_2) - W(\tilde{\lambda}_1, \tilde{\mu}_1) > W(\tilde{\lambda}_2, \tilde{\mu}_2) - W(\tilde{\lambda}_2, \tilde{\mu}_1). \tag{71}$$

By (69), if $\Pi(\tilde{c}_1, \tilde{\mu}_1 | p_1, T_1) \geq 0$ then

$$\lambda_2 \Big(\tilde{c}_2 - \tilde{c}_1 + t(W(\tilde{\lambda}_2, \tilde{\mu}_2) - W(\tilde{\lambda}_2, \tilde{\mu}_1)) \Big) \geq R(\tilde{c}_1, \tilde{\mu}_1) - R(\tilde{c}_2, \tilde{\mu}_2).$$

This with (71), implies

$$\tilde{\lambda}_2 \Big(\tilde{c}_2 - \tilde{c}_1 + t(W(\tilde{\lambda}_1, \tilde{\mu}_2) - W(\tilde{\lambda}_1, \tilde{\mu}_1)) \Big) > R(\tilde{c}_1, \tilde{\mu}_1) - R(\tilde{c}_2, \tilde{\mu}_2). \tag{72}$$

Then because $\tilde{\lambda}_1 > \tilde{\lambda}_2$, (72) and (69) imply $\Pi(\tilde{c}_2, \tilde{\mu}_2 | p_2, T_2) < 0$. Hence, an asymmetric equilibrium where two providers experience different equilibrium arrival rates does not exist. □

Proof of Proposition 9: Let $(\tilde{c}_i, \tilde{\mu}_i)$ for $i = 1, 2$ denote an equilibrium and $\tilde{\lambda}_i = \lambda(p_i, \tilde{\mu}_i)$ be given by (1). Assume that $\tilde{\lambda}_1 > \tilde{\lambda}_2 > 0$. We first show that $\tilde{\mu}_1 > \tilde{\mu}_2$ by contradiction. We show below that

$$\frac{\partial R(\tilde{c}_1, \tilde{\mu}_1)}{\partial c} < \frac{\partial R(\tilde{c}_2, \tilde{\mu}_2)}{\partial c} \tag{73}$$

if $\tilde{\mu}_1 \leq \tilde{\mu}_2$. By (69), provider i's optimal action in equilibrium satisfies

$$\frac{\partial}{\partial c_i} \Pi(\tilde{c}_i, \tilde{\mu}_i | p_i, T_i) = -\tilde{\lambda}_j - \frac{\partial R(\tilde{c}_i, \tilde{\mu}_i)}{\partial c} = 0, \text{for } j \neq i.$$

Because $\tilde{\lambda}_1 > \tilde{\lambda}_2$, we have

$$\frac{\partial R(\tilde{c}_1, \tilde{\mu}_1)}{\partial c} = -\tilde{\lambda}_2 > -\tilde{\lambda}_1 = \frac{\partial R(\tilde{c}_2, \tilde{\mu}_2)}{\partial c},$$

which contradicts (73). Hence, if $\tilde{\lambda}_1 > \tilde{\lambda}_2$, $\tilde{\mu}_1 \leq \tilde{\mu}_2$ cannot hold.

Next we prove (73) if $\tilde{\mu}_1 \leq \tilde{\mu}_2$. If $\tilde{\lambda}_1 > \tilde{\lambda}_2$ and $\tilde{\mu}_1 \leq \tilde{\mu}_2$ then by (1)

$$(p_1 + tW(\tilde{\lambda}_1, \tilde{\mu}_1)) < (p_2 + tW(\tilde{\lambda}_2, \tilde{\mu}_2)), \tag{74}$$

where p_1 and p_2 are given in (16). Because $\tilde{\lambda}_1 > \tilde{\lambda}_2$ and $\tilde{\mu}_1 \leq \tilde{\mu}_2$, we have $W(\tilde{\lambda}_1, \tilde{\mu}_1) > W(\tilde{\lambda}_2, \tilde{\mu}_2)$. This implies $p_1 < p_2$ by (74). By (16), $p_1 < p_2$ implies

$$\tilde{c}_1 + t\tilde{\lambda}_1 \frac{\partial}{\partial \lambda} W(\tilde{\lambda}_1, \tilde{\mu}_1) < \tilde{c}_2 + t\tilde{\lambda}_2 \frac{\partial}{\partial \lambda} W(\tilde{\lambda}_2, \tilde{\mu}_2). \tag{75}$$

By Assumption 1 and that $\tilde{\lambda}_1 > \tilde{\lambda}_2$ and $\tilde{\mu}_1 \leq \tilde{\mu}_2$, we have $\frac{\partial W(\tilde{\lambda}_1, \tilde{\mu}_1)}{\partial \lambda} > \frac{\partial W(\tilde{\lambda}_2, \tilde{\mu}_2)}{\partial \lambda}$. This and $\tilde{\lambda}_1 > \tilde{\lambda}_2$ imply that if $p_1 < p_2$ then $\tilde{c}_1 < \tilde{c}_2$ by (75). Because $\tilde{\mu}_1 \leq \tilde{\mu}_2$ and $\tilde{c}_1 < \tilde{c}_2$ (73) holds by convexity of R and $\frac{\partial^2 R(c,\mu)}{\partial c \partial \mu} \geq 0$.

We now show that $\tilde{c}_1 < \tilde{c}_2$ if $\tilde{\lambda}_1 > \tilde{\lambda}_2$. By (69) provider i's action satisfies

$$\frac{\partial R(\tilde{c}_i, \tilde{\mu}_i)}{\partial \mu} = -t\tilde{\lambda}_j \frac{\partial}{\partial \mu} W(\tilde{\lambda}_j, \tilde{\mu}_i) \tag{76}$$

in any equilibrium. By Assumption 1, and the fact that $\tilde{\lambda}_1 > \tilde{\lambda}_2$ and $\tilde{\mu}_1 > \tilde{\mu}_2$, we have

$$\frac{\partial}{\partial \mu} W(\tilde{\lambda}_1, \tilde{\mu}_2) < \frac{\partial}{\partial \mu} W(\tilde{\lambda}_2, \tilde{\mu}_1).$$

Therefore

$$\left(-\tilde{\lambda}_1 \frac{\partial}{\partial \mu} W(\tilde{\lambda}_1, \tilde{\mu}_2)\right) > \left(-\tilde{\lambda}_2 \frac{\partial}{\partial \mu} W(\tilde{\lambda}_2, \tilde{\mu}_1)\right).$$

Thus by (76) we have

$$\frac{\partial R(\tilde{c}_2, \tilde{\mu}_2)}{\partial \mu} > \frac{\partial R(\tilde{c}_1, \tilde{\mu}_1)}{\partial \mu}. \tag{77}$$

Also, by the fact that $\mu_1 > \mu_2$ and R is convex

$$\frac{\partial R(\tilde{c}_2, \tilde{\mu}_1)}{\partial \mu} > \frac{\partial R(\tilde{c}_2, \tilde{\mu}_2)}{\partial \mu}. \tag{78}$$

By (77) and (78),

$$\frac{\partial R(\tilde{c}_2, \tilde{\mu}_1)}{\partial \mu} > \frac{\partial R(\tilde{c}_1, \tilde{\mu}_1)}{\partial \mu}.$$

This implies $\tilde{c}_2 > \tilde{c}_1$ by $\frac{\partial^2 R(c,\mu)}{\partial c \partial \mu} \geq 0$. $\square$

We finally consider the case with $N \geq 3$. Assume that providers are divided into two disjoint sets $\mathcal{A}_1$ and $\mathcal{A}_2$. For simplicity, assume that $\mathcal{A}_1 = \{1, 2, \ldots, n_1\}$ and $\mathcal{A}_2 = \{n_1 + 1, \ldots, N\}$, for $n_1 \geq 1$. For $i \in \mathcal{A}_1$, set

$$\bar{c}_i = \frac{1}{|\mathcal{A}_2|} \sum_{j \in \mathcal{A}_2} c_j,\ \bar{\lambda}_i = \frac{1}{|\mathcal{A}_2|} \sum_{j \in \mathcal{A}_2} \lambda_j,\ \bar{R}_i = \frac{1}{|\mathcal{A}_2|} \sum_{j \in \mathcal{A}_2} R_j,\ \bar{W}_i = \frac{1}{|\mathcal{A}_2|} \sum_{j \in \mathcal{A}_2} W_j,$$

and similarly for $i \in \mathcal{A}_2$ set

$$\bar{c}_i = \frac{1}{|\mathcal{A}_1|} \sum_{j \in \mathcal{A}_1} c_j,\ \bar{\lambda}_i = \frac{1}{|\mathcal{A}_1|} \sum_{j \in \mathcal{A}_1} \lambda_j,\ \bar{R}_i = \frac{1}{|\mathcal{A}_1|} \sum_{j \in \mathcal{A}_1} R_j,\ \bar{W}_i = \frac{1}{|\mathcal{A}_1|} \sum_{j \in \mathcal{A}_1} W_j.$$

Also set the transfer payment for hospital i equal to T_i defined as in (64) using $\bar{c}_i$, $\bar{\lambda}_i$, $\bar{W}_i$ and $\bar{R}_i$ as defined above. Effectively, we use providers in set $\mathcal{A}_2$ to set the "yardstick" for providers in set $\mathcal{A}_2$ and vice versa.

When providers are split into two sets $\mathcal{A}_1$ and $\mathcal{A}_2$, the providers in the same set would take the same actions since their profit functions are identical. Hence, the comparison of providers in two sets would reduce to the case of $N = 2$, and the unique symmetric equilibrium which yields the first-best outcomes for all providers would be the unique equilibrium by Proposition 8.

D. Alternative payment scheme for second best with a unique equilibrium

In this section we prove that under the proposed reimbursement scheme of §4.4 there cannot be an asymmetric equilibrium for $N = 2$. Then, we alter the proposed reimbursement scheme slightly to obtain a new scheme under which the welfare maximising actions are the unique equilibrium.

PROPOSITION 10. *Assume that there are only two providers (i.e., $N=2$) and the regulator pays the transfer payment T_i defined as in (23) to provider i for $i=1,2$. Then there is a unique Nash equilibrium.*

Proof of Proposition 10: By Theorem 2 the only symmetric equilibrium is where providers pick μ_o^* and c_o^*. We next prove that for $N=2$, this is the unique equilibrium. Assume not and let $a_1=(c_1,\mu_1)$ and $a_2=(c_2,\mu_2)$ denote another equilibrium with $a_1\neq a_2$. Under the proposed reimbursement scheme the profits of providers 1 and 2 are given by

$$\Pi(c_1,\mu_1)=-c_1\lambda(\mu_1)+t(W_2-W_1)\lambda(\mu_2)-R(c_1,\mu_1)+R(c_2,\mu_2)+c_2\lambda(\mu_2). \tag{79}$$

$$\Pi(c_2,\mu_2)=-c_2\lambda(\mu_2)+t(W_1-W_2)\lambda(\mu_2)-R(c_2,\mu_2)+R(c_1,\mu_1)+c_1\lambda(\mu_1). \tag{80}$$

In order for (a_1,a_2) be an equilibrium we must have $\Pi(c_1,\lambda_1)\geq 0$ and $\Pi(c_2,\lambda_2)\geq 0$ because if $a_2=a_1$, then $\Pi(c_2,\mu_2)=0$. We next show that this is not possible in an asymmetric equilibrium. Assume without loss of generality that $\lambda(\mu_1)>\lambda(\mu_2)$. This implies $W_1<W_2$ by (1), since $p=0$. By (79)

$$t(W_2-W_1)\lambda(\mu_2)\geq c_1\lambda(\mu_1)+R(c_1,\lambda(\mu_1))-R(c_2,\lambda(\mu_2))-c_2\lambda(\mu_2). \tag{81}$$

Hence

$$\begin{aligned}\Pi(c_2,\mu_2)&=-c_2\lambda(\mu_2)+t(W_1-W_2)\lambda(\mu_1)-R(c_2,\lambda(\mu_2))+R(c_1,\lambda(\mu_1))+c_1\lambda(\mu_1)\\&\overset{(a)}{<}-c_2\lambda(\mu_2)+t(W_1-W_2)\lambda(\mu_2)-R(c_2,\lambda(\mu_2))+R(c_1,\lambda(\mu_1))+c_1\lambda(\mu_1)\overset{(b)}{\leq}0,\end{aligned}$$

where (a) above follows from the fact that $W_1<W_2$ and $\lambda(\mu_1)>\lambda(\mu_2)$, and (b) follows from (81). This proves that (a_1,a_2) cannot be an equilibrium if $\lambda_1\neq\lambda_2$. □

A regulatory scheme with a unique equilibrium for $N\geq 3$: If $N\geq 3$, we can modify the reimbursement scheme so that the only equilibrium is the second best. Specifically, as in Appendix C we divide the providers into two disjoint sets $\mathcal{A}_1$ and $\mathcal{A}_2$. For simplicity assume that $\mathcal{A}_1=\{1,2,\ldots,n_1\}$ and $\mathcal{A}_2=\{n_1+1,\ldots,N\}$ for $n_1\geq 1$. For $i\in A_1$ set

$$\bar{W}_i=\frac{1}{|\mathcal{A}_2|}\sum_{j\in A_2}W_j \quad\text{and}\quad \bar{\lambda}_i=\frac{1}{|\mathcal{A}_2|}\sum_{j\in A_2}\lambda_j.$$

Similarly for $i\in A_2$ set

$$\bar{W}_i=\frac{1}{|\mathcal{A}_1|}\sum_{j\in A_1}W_j \quad\text{and}\quad \bar{\lambda}_i=\frac{1}{|\mathcal{A}_1|}\sum_{j\in A_1}\lambda_j.$$

Again, set the transfer payment for hospital i equal to T_i defined as in (23) using $\bar{W}_i$ and $\bar{\lambda}_i$ as defined above. Effectively, we use providers in set $\mathcal{A}_2$ to set the "yardstick" for providers in set

$\mathcal{A}_2$ and vice versa. The proof that this mechanism can only have a unique equilibrium where each provider picks socially optimal levels μ_o^* and c_o^* follows from a similar argument to that at the end of Appendix C using Theorem 2 and Proposition 10. In addition, we conjecture that for N large enough the original mechanism should also have a unique equilibrium. This follows from the fact that when N is large $\bar{W}_i \sim \bar{W}_j$ for all i, j. The proof outlined above for the modified mechanism might be used when this is case to prove the uniqueness of the symmetric equilibrium.