Social Learning from Early Buyer Reviews: Implications for New Product Launch

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We investigate the implications of social learning on a monopolist firm's pricing and inventory decisions. In our model, customers who purchase the product early in the selling season report their *ex-post* opinions of product quality through buyer reviews, while customers remaining in the market observe these reviews before making purchasing decisions. Because customers are heterogeneous in their preferences, inferring product quality from buyer reviews is challenging. We consider two modes of social learning: perfect (Bayesian) and imperfect (motivated by empirical evidence). Our analysis illustrates that, apart from extracting revenue, price influences both the *amount* of information made available to potential customers, as well as its *content*, thereby modulating the social learning process. We find that even though learning from buyer reviews is rational for individual customers, aggregate customer surplus typically *decreases* in the presence of social learning. Consistent with anecdotal evidence, we show that, when social learning is imperfect, the firm may *deliberately* induce an early supply shortage in order to achieve increased overall product adoption. In addition, we demonstrate that optimal inventory management may allow the firm to approximate dynamic pricing outcomes while charging a fixed price.

Key words: social learning; buyer reviews; pricing; product availability; OM/Marketing interface

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1. Introduction

Buyer reviews have become an integral component of the process by which customers make purchasing decisions in the post-Internet era (*The New York Times* 2003). Especially in cases of new and innovative products whose quality is difficult to assess before use (i.e., *experiential* products; e.g., high-tech consumer electronics, movies, books), potential customers seek relevant information by consulting the opinions of early buyers (Chevalier and Mayzlin 2006). As a result, the reviews generated by buyers in the early stages of a product's life-cycle play a decisive role in potential customers' purchasing decisions.[1] In this paper, we develop a stylized model of social learning from early buyer reviews, and use it to investigate the implications of social learning on firms' pricing

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[1] Dellarocas et al. (2007) find evidence that early buyer reviews are powerful predictors of subsequent movie sales, while in the same industry Moretti (2011) estimates that up to 32% of movie revenues can be attributed to social learning.

and inventory decisions.

We consider a profit-maximizing (risk-neutral) monopolist launching a new experiential product. The market consists of customers who are heterogeneous in their preferences and hold a common *ex-ante* belief over the product's unknown quality. Sales of the product occur over two periods. The first period is of short time-length and represents the product's *introductory phase*. During the introductory phase, customers who purchase (i.e., *early buyers*) experience the product and report their (truthful) opinions of product quality to the rest of the market (e.g., through online reviews). Product reviews in our model contain information which is useful for potential future buyers, but are also influenced by the idiosyncratic preferences of the buyers who generated them (e.g., Dellarocas et al. 2004, Hu et al. 2009, Li and Hitt 2008). That is, product quality in our model is, at least to some degree, *subjective*.

The second period is a stylized representation of the subsequent selling horizon. In the second period, customers remaining in the market observe the reviews of the early buyers, update their beliefs over product quality (social learning), and make purchasing decisions accordingly. Due to the partially subjective nature of product reviews, inferring product quality accurately is a challenging task for potential customers. We consider two modes of social learning. The first endows customers with all of the information and cognitive abilities necessary to apply Bayesian updating (*perfect social learning*). The second relaxes the informational and cognitive burden placed on customers, assuming instead that customers simply treat early buyer reviews as a representative sample of *ex-post* quality perceptions (*imperfect social learning*). The latter mode of social learning has been shown empirically to be an accurate representation of customer behavior in real-world settings (Li and Hitt 2008).

We first consider the firm's fixed-pricing problem in the presence of social learning. Our model illustrates that the firm's pricing decision affects both the *amount* of information made available to potential second-period buyers (i.e., a higher price generates a smaller number of buyer reviews), as well as the *content* of this information (i.e., the product's average rating exhibits *self-selection bias* which amplifies with price). We show how customers' differential interpretation of this information under perfect and imperfect social learning affects the firm's optimal pricing policy, and deduce that firm profit is non-decreasing in quality under both modes of learning. Our numerical experiments suggest that the presence of social learning generally increases firm profit (consistent with real-world firms' willingness to invest in online reviewing platforms). Moreover, even though social learning allows customers to make better-informed purchasing decisions (which may justify customers' propensity to review and learn from reviews), interestingly, we observe that aggregate customer surplus generally *decreases* in the presence of social learning.

Our investigation of the firm's optimal pricing policy uncovers an interesting phenomenon for the case of imperfect social learning. In particular, the firm's profit-maximizing price is often lower than the price which optimizes the social learning effect (i.e., maximizes the quality perceptions of customers remaining in the market). This occurs because although charging a higher price would generate a more favorable social learning effect, it would also render the product too expensive for many potential second-period buyers. We proceed to show that the firm may be able to relieve the pricing decision of its social-learning role, instead using its *early inventory* (i.e., product availability) decision to modulate the social learning process.

To illustrate how this can be achieved, we model the commonly observed phenomenon of waiting-line formation in the presence of supply shortages, with *endogenous customer arrivals* (i.e., when early demand outstrips supply, customers may choose to arrive earlier than the product launch and form a waiting line; e.g., Apple product launches, movie premieres). We find that, by deliberately restricting early supply, the firm is able to selectively sell its product to its highest valuation customers (see also Holt and Sherman (1982)). Effectively, this suggests that inventory management can be used by firms as an indirect method of "price discrimination", which, as we demonstrate, has the effect of generating an *increased* average product rating following first-period purchases. (A real-world example in support of our result is presented in Figure 1, in which three products with widely-publicized, early, short-term supply shortages are seen to have a significantly higher average rating in the early stages of the selling season, as compared to the long-term average rating.)
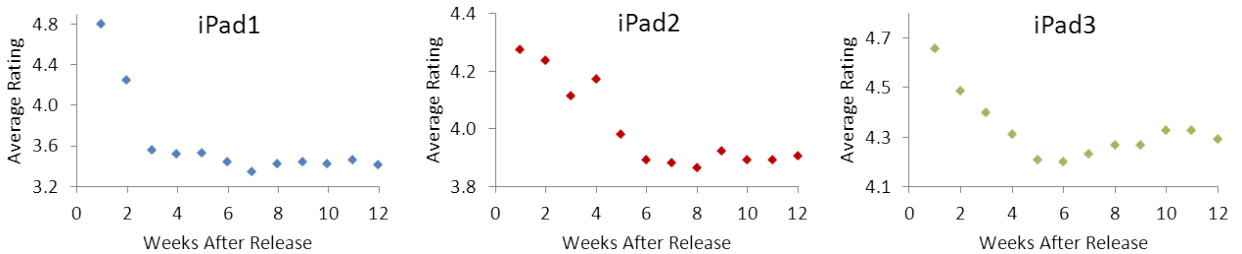

Figure 1 **Average rating (cumulative) for Apple iPad1, iPad2 and iPad3 following their introduction in March 2010, 2011 and 2012 respectively. The decreasing average rating suggests that early buyers rated the product more highly on average than subsequent buyers (data collected from Amazon.com via web crawler).**

Taking the waiting-line formation process into account, we allow the firm to choose product price and first-period availability simultaneously. We show that it is optimal for the firm to deliberately under-supply demand in the first period only if social learning is imperfect *and* product quality does not exceed an upper limit. In addition, we find that early supply shortages are significantly profitable only when product quality is not much lower than customers' prior expectations. Surprisingly, we observe that, in certain cases, customer surplus may in fact *increase* as a result of firm-induced supply shortages.

Finally, we compare the performance of fixed-pricing policies with that of dynamic pricing. Here, our numerical study indicates that fixed-pricing policies with strategic supply shortages (under imperfect social learning) not only provide the firm with a significant profit advantage over fixed-pricing policies with unrestricted supply, but also allow firms to *approximate* profit under dynamic pricing. As a result, when dynamic pricing in the early stages of the selling season is difficult to implement − as is the case in many real-world settings because of fairness considerations (e.g., *The New York Times* 2007) and the complexities associated with strategic customer behavior (e.g., Liu and van Ryzin 2008) − firms may instead be employing an effective alternative strategy in charging a fixed price and deliberately under-supplying early demand. This finding may help explain the high frequency of early stock-outs observed in practice for new and innovative experiential products (e.g., Apple iPads and iPods, Sony Playstation 2, Palm Pre, Nintendo Wii).[2]

2.   Related Literature

A large body of work in operations management considers the operational implications of firm learning. For instance, firms may plan their pricing and inventory in anticipation of demand learning (e.g., Fisher and Raman 1996, Besbes and Zeevi 2009), or modify their product assortment in response to learning customer preferences (e.g., Caro and Gallien 2007). By contrast, the focus of this paper is to investigate how firm decisions are affected by *customer-to-customer (social) learning.* Our work contributes primarily to the literature concerned with the operational implications of social learning on new product launch. Within this area of research, there are two alternative approaches to modeling the social learning process: *action-based* and *outcome-based.*

Action-based social learning typically considers homogeneous agents who receive private *iid* signals of product quality, and observe the actions of their predecessors before deciding whether to purchase the product. Under such assumptions, the seminal paper by Banerjee (1992) demonstrates that herding can be rational, but that rational herds may contain very little information. Bose et al. (2006) consider the dynamic-pricing problem faced by a monopolist in which price, apart from contributing to current revenues, serves to screen the information transmitted to potential future buyers. When the price of the product is fixed, Sgroi (2002) and Liu and Schiraldi (2012) find that firms may benefit from allowing a subset of customers to make observable purchasing decisions simultaneously, before the rest of the market (see also Bhalla (2013), in which the setting of Liu and Schiraldi (2012) is extended to incorporate dynamic pricing). The social learning process in our model differs substantially from that in the above papers. In our model, purchasing actions

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[2] In the words of Andrew McGuinness, the former European head of Apple's ad agency: "When we were planning the launch of the iPod [...] we had to manage to make sure we under-supplied the demand [...]. That's how we kept that kind of *cachet* for the iPod in its early years." (BBC documentary, http://www.youtube.com/watch?v=0Q-11cSNshc)

in themselves are not informative, because customers share identical *ex-ante* information (i.e., a public belief). By contrast, social learning occurs on the basis of buyers' post-purchase perceptions of product quality (our model is motivated by the prominence of online buyer review sites; this mode of social learning is particularly prevalent for products with experiential attributes, e.g., see Chevalier and Mayzlin (2006), Dellarocas et al. (2007)). In this respect, our work is more closely related to the outcome-base social learning literature, and in particular to word-of-mouth social learning.

In word-of-mouth social learning, agents typically learn from the *ex-post* payoffs of their predecessors, rather than actions which reflect their *ex-ante* information (e.g., Ellison and Fudenberg 1995). Banerjee and Fudenberg (2004) investigate how the social learning process is affected when agents observe the payoffs of biased samples from the population. While they consider samples generated according to exogenously specified sampling rules, in our paper, biased samples arise endogenously as a result of customer decision-making. Moreover, rather than on the social learning process itself, our focus is on the implications for firm policy. Ifrach et al. (2011) consider a monopolist's pricing problem in the presence of binary word-of-mouth social learning. We consider a similar problem at a high level, however, our approach to modeling review-generation and social learning from buyer reviews differs. In particular, we focus on the manifestation of preference heterogeneity in buyers' quality evaluations. Moreover, we consider both pricing as well as inventory implications.

A large stream of literature concerned with new product diffusion implicitly captures the effects of social learning through the stylized Bass diffusion model (Bass 1969). Operational models incorporating supply constraints include Ho et al. (2002), Ho et al. (2011), Kumar and Swaminathan (2003), and Shen et al. (2011). In contrast with the Bass diffusion model, our paper explicitly considers the micro-foundations and mechanics underlying the social learning process and, as a result, generates significantly different insights. Most strikingly, our model suggests that, in certain contexts, a smaller number of early adopters may lead to a more favorable social learning effect.

Our finding that early supply shortages may be optimal in the presence of social learning appears to be new in the literature; however, our work relates to a number of papers which illustrate the potential benefits of firm-induced limited product availability. The notion that limited availability renders products more desirable dates back to commodity theory (Brock 1968). The existing literature has identified a number of mechanisms through which firms may benefit by restricting product availability. Debo and Van Ryzin (2009) find that a firm can increase its profits through an imbalanced allocation of inventory to multiple retailers, as stock-outs in one retailer may lead to more sales in other retailing outlets. In our paper, it is not the event of a stock-out *per se* that increases sales, but the social learning outcome it generates. Stock and Balachander (2005)

develop a model in which product scarcity is used by the firm to signal product quality, however, the feasibility of this mechanism relies on the purchasing decisions of the early, informed customers being unobservable. Recent work on strategic customer behavior (Cachon and Swinney 2009, Liu and van Ryzin 2008) demonstrates that when prices are decreasing, firms may under-invest in inventory in order to create stock-out risk and induce more high-price purchases. In addition, when the product's price is fixed and customers incur holding and shortage costs, Glazer and Hassin (1986) and Anily and Hassin (2012) demonstrate that it may be optimal for a seller to restrict sales to certain instants in the selling horizon. Finally, DeGraba (1995) suggests that a monopolist may benefit from creating a supply shortage in the market in order to induce an early buying frenzy (see also Courty and Nasiry (2012) for a dynamic model of buying frenzies). Buying frenzies critically hinge on the firm's inability to commit to a fixed price and/or on customers' expectations of being rationed if purchase is delayed; neither of the two feature in our model. Moreover, none of the above papers consider social interactions between buyers and potential customers.

3.   Model Description

In this section we first provide a high-level overview of our model and then discuss its important elements individually.

3.1.   Overview

We consider a monopolist firm offering a new and innovative experiential product to a market of N potential buyers, over two periods. The first period is of a short time-length and represents the product's *introductory phase*, while the second period is an aggregated representation of the subsequent selling horizon. Customers are assumed to be risk-neutral, heterogeneous in their preferences over the product's *ex-ante* observable characteristics (e.g., brand, color) and homogeneously informed regarding the product's uncertain quality (e.g., owing to uniform exposure to product advertising). Customers whose *ex-ante* valuation is higher than price are willing to purchase the product in the first period.[3] All first-period buyers (i.e., *early buyers*) report their truthful perceptions of product quality to the rest of the market through buyer reviews (e.g., online). Importantly, buyer reviews in our model contain information which is useful for potential future buyers, but are also influenced by the idiosyncratic preferences of the customers who generated them (consistent with empirical evidence). That is, product quality in our model is, in part, *subjective*. In the second period, customers who have not yet purchased observe the product reviews of the early buyers, update their valuations for the product (*social learning*) and make purchasing decisions.

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[3] See §6.2 for a discussion pertaining to strategic purchasing delays, which do not feature in this paper but constitute an interesting avenue for future research.

The firm is risk-neutral and seeks to maximize its profit over the two selling periods, through its pricing and inventory decisions. For most of our analysis, we assume that the firm commits to a fixed price, while dynamic-pricing policies are considered subsequently as a model extension.[4]
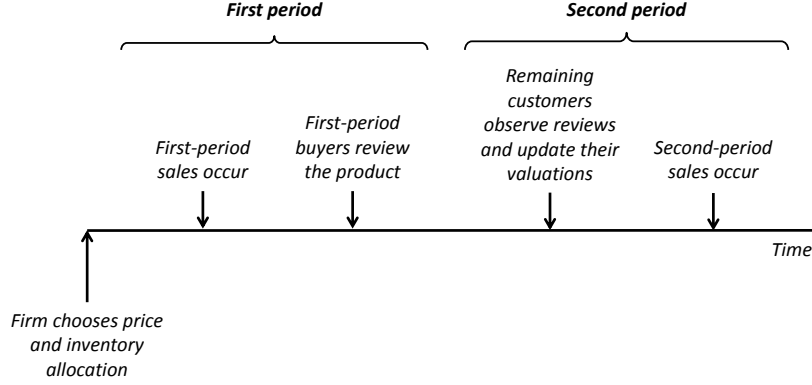

Figure 2   Sequence of events

3.2.   Customer Preferences and Quality Perceptions

Building on existing literature (e.g., Villas-Boas 2004, Li and Hitt 2008), customer i's preferences over the product are characterized by two components, (x_i, q_i). The value of element x_i reflects the customer's idiosyncratic preferences over the product's *ex-ante* observable characteristics (e.g., brand, color). The value of element q_i represents the product's "quality" for customer i. Customers learn their q_i component only after they purchase and experience the product. Importantly, product quality may be perceived differently by different customers. The wealth-equivalent net benefit of purchasing the product for customer i is defined by $u(x_i, q_i) = ax_i + q_i - p$, where p is the product's price and parameter a captures the relative importance of pre-purchase observable attributes versus post-purchase experienced attributes in customers' valuations for the product. We assume that the distribution of preference components, x_i, in the population is Normal, such that for a customer drawn at random we have $x_i \sim N(\bar{x}, \sigma_x^2)$ with $F(\cdot)$ denoting the corresponding distribution function (and $\bar{F}(\cdot) := 1 - F(\cdot)$).

Consistent with empirical evidence on products with experiential attributes, in our model, customer i's post-purchase perception of product quality, q_i, is subject to random noise, as well as a systematic dependence on her idiosyncratic preferences (e.g., Li and Hitt 2008, Dellarocas et al.

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[4] The transition between the first and second periods of our model occurs early in the selling season. As such, dynamic pricing in our model entails a price change shortly after the product is launched (i.e., *not* an end-of-season markdown). In practice, attempts by firms to implement dynamic pricing in the early stages of the selling season have had detrimental consequences, owing mostly to fairness considerations (e.g., *The New York Times* 2007); we note that *early* dynamic pricing strategies are, in any case, not generally observed for new and innovative durable products (e.g., Apple iPad).

2004). For instance, the same film may be evaluated *differently* by cinema-goers who have a preference for the same film genre (random noise) as well as by cinema-goers who differ in their genre preferences (idiosyncratic). Let the marginal distribution of post-purchase quality perceptions, q_i, be Normal, such that for a customer drawn at random we have $q_i \sim N(\hat{q}, \sigma_q^2)$, where $\hat{q}$ is the product's *mean quality*. The link between buyer preferences and *ex-post* quality perceptions may then be captured by allowing x_i and q_i to be correlated, with correlation parameter ρ. The conditional distribution of q_i given customer i's x_i component is

$$q_i \mid x_i = \hat{q} + \rho \frac{\sigma_q}{\sigma_x}(x_i - \bar{x}) + \epsilon_i, \tag{1}$$

where ϵ_i is zero-mean Gaussian noise of variance σ_ϵ^2, $\sigma_\epsilon^2 = \sigma_q^2(1 - \rho^2)$.

A few comments are warranted on the role of the correlation parameter ρ. If $\rho = 0$, customers' post-purchase perceptions of product quality are *independent* of their idiosyncratic preferences. In this case, *ex-post* quality perceptions are *iid* draws from a distribution centered around the product's mean quality, $\hat{q}$. However, if $\rho \neq 0$, customers' idiosyncratic preferences are related to their *ex-post* perceptions of product quality. The sign of ρ determines the nature of this relationship, while the magnitude determines its strength. When $\rho > 0$, customers with relatively higher x_i are likely to perceive higher *ex-post* product quality (for instance, Mac users may enjoy their experience with the new iPad more than PC users). Conversely, when $\rho < 0$, customers with relatively higher x_i are likely to perceive lower product quality (this case is less intuitive; for instance, Mac users may be more sensitive to software glitches in the new iPad than PC users).

Li and Hitt (2008) find evidence that preferences are positively linked with quality perceptions in approximately 70% of cases they consider; similar evidence is reported in Dellarocas et al. (2004). Taking these findings into account, in our analysis we choose to focus on cases of $\rho > 0$, however, our framework can readily be extended to accommodate cases of $\rho < 0$ (as well as the simpler case of $\rho = 0$).

3.3. Customer Valuations

At the beginning of the selling season, customers know their idiosyncratic preference component, x_i, and are homogeneously informed regarding the product's unobservable mean quality, $\hat{q}$. Specifically, we assume that customers hold a common belief over the value of $\hat{q}$, which is expressed as a random variable $\tilde{q}$, $\tilde{q} \sim N(q_p, \sigma_p^2)$. Accordingly, customers are endowed with a willingness to pay in the first period defined by

$$\begin{aligned} v_{i1} &= ax_i + E[q_i|x_i] \\ &= ax_i + q_p + \rho\sigma_q \frac{x_i - \bar{x}}{\sigma_x}. \end{aligned}$$

We denote first period demand for the product at price p by $D_1(p)$, where $D_1(p) = N \cdot P(v_{i1} \geq p)$. Following first-period purchases, we assume that all first-period buyers report their truthful perceptions of product quality to the rest of the market through buyer reviews.[5] Buyer reviews are observable to all customers remaining in the market (i.e., who did not purchase the product in the first period), who use the available reviews to update their beliefs over the product's mean quality. Let q_u denote customers' updated expectation of mean quality after observing the reviews of the first period buyers. That is, if customer i remains in the market in the second period, her updated willingness to pay is defined by

$$v_{i2} = a x_i + q_u + \rho \sigma_q \frac{x_i - \bar{x}}{\sigma_x}.$$

For simplicity, we refer to customer i's willingness to pay in the first (second) period as her first (second) period valuation for the product throughout.

3.4. Social Learning from Buyer Reviews

We now characterize the social learning process by which q_p is updated to q_u.[6] We consider and analyze two alternative approaches to social learning from buyer reviews. The first is a full-information Bayesian approach which endows customers with full knowledge of all model parameters, reviewer characteristics, as well as the relationship between preferences and quality perceptions; we refer to this approach as *perfect social learning* throughout. The second approach is an empirically motivated relaxation of the full-information case, and aims at investigating the implications of social learning under more realistic customer behavior; we refer to this approach as *imperfect social learning*.

Perfect Social Learning According to the relationship between customer preferences and *ex-post* quality perceptions in (1), buyer reviews are noisy and reflect, to an extent specified by the value of ρ, the preferences of the buyers who generate them. Under perfect social learning, customers account accurately for these two features of product reviews and apply Bayes' rule accordingly. The outcome of the social learning process (i.e., customers' updated expectation of mean quality) is described in Lemma 1.

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[5] A product review in our model reflects the individual buyer's *ex-post* perception of quality, and is not influenced by the reviews of other buyers (for a treatment of such cases see Besbes and Scarsini (2013)). Moreover, we do not allow the firm to engage in review-manipulation, which in any case constitutes a risky strategy (see Dellarocas (2006) for related work).

[6] In general, customers may learn quality both from buyer reviews, as well as from firm actions (e.g., pricing decisions). This paper abstracts away from the latter and focuses on the former, peer-to-peer mode of learning, which seems to be particularly influential for experiential products (e.g., Chevalier and Mayzlin 2006, Moretti 2011). See §6.1 for further discussion on the implications of this assumption.

LEMMA 1. *Let S denote the set of first-period reviewers (i.e., buyers) and let $n_s = |S|$, $\bar{x}_s = \frac{1}{n_s} \sum_{i \in S} x_i$ and $R_s = \frac{1}{n_s} \sum_{i \in S} q_i$. Under perfect social learning, customers' updated expectation of mean quality is given by the Bayesian update*

$$q_u = \frac{\sigma_q^2(1-\rho^2)}{n_s\sigma_p^2 + \sigma_q^2(1-\rho^2)} q_p + \frac{n_s\sigma_p^2}{n_s\sigma_p^2 + \sigma_q^2(1-\rho^2)} \left(R_s - \rho\sigma_q \frac{\bar{x}_s - \bar{x}}{\sigma_x} \right). \tag{2}$$

All proofs are provided in Appendix A. The social learning rule of Lemma 1 follows from the standard Gaussian updating model (Chamley 2004) and exhibits a number of intuitive features. Customers' updated quality expectation, q_u, is a linear interpolation between customers' prior, q_p, and the *preference-adjusted* average rating from first-period reviews, $R_s - \rho\frac{\sigma_q}{\sigma_x}(\bar{x}_s - \bar{x})$; the subtracted term corrects the observed rating, R_s, for the early buyers' idiosyncratic preferences. The relative weight assigned to q_p increases with the variance of individual buyer reviews, σ_p^2; the more noisy the reviews, the more conservative the quality update. The relative weight assigned to the adjusted average rating increases with (a) the number of reviewers, n_s, since a larger number of data points (i.e., reviews) renders the average rating more credible and (b) the variance of customers' prior, $\sigma_q^2(1-\rho^2)$, since greater prior uncertainty suggests greater susceptibility to persuasion from buyer reviews.

Imperfect Social Learning The paradigm described above provides a transparent starting point for our investigation into the process by which customers learn from buyer reviews. However, notice that, for social learning to be perfect, customers are required to know (a) the exact relationship between buyer preferences and quality perceptions, (b) the idiosyncratic preferences of the early buyers, and (c) how these preferences compare with the preferences of the average customer. Since this information is typically not readily available in real-world settings, customers may instead be engaging in a simplified form of social learning. In an empirical study with product reviews from *Amazon.com*, Li and Hitt (2008) propose that when faced with the task of utilizing product reviews, customers simply take the reviews of early buyers as a representative sample of post-purchase quality perceptions; their proposition is motivated by "the cost and difficulty for consumers to infer their valuation for a product based on reviews of other consumers." More specifically, Li and Hitt (2008) hypothesize that customers behave *as if* the value of the correlation parameter is $\rho = 0$. This hypothesis is subsequently verified empirically, with the authors pointing out a number of consequences for profit-maximizing firms; see Li and Hitt (2008) for details, as well as a thorough discussion of the described phenomenon.

Expanding on their model, we consider a second, simpler mode of social learning, which we refer to as imperfect social learning.[7] Under imperfect social learning, the update from q_p to q_u occurs as in Lemma 1, but under the simplifying assumption of $\rho = 0$,

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[7] More specifically, the model proposed in Li and Hitt (2008) may be viewed as a special case of our model with "uninformative" prior beliefs.

$$q_u = \frac{\sigma_q^2}{n_s \sigma_p^2 + \sigma_q^2} q_p + \frac{n_s \sigma_p^2}{n_s \sigma_p^2 + \sigma_q^2} R_s. \tag{3}$$

Learning rule (3) significantly alleviates the informational (as well as the computational) burden placed on customers by Lemma 1, since it does not require customers to know (and account for) the preferences of the early buyers. Notice that the assumption of $\rho = 0$ also entails individual buyer reviews being treated as relatively more noisy, as all variability in reviews is attributed to random noise (and not partly random noise and partly generated by heterogeneity in buyer preferences). This results in an increase in the relative weight placed by customers on their prior beliefs. Consequently, a natural feature of rule (3) is that, while customers do not account explicitly for the idiosyncratic preferences of the early buyers, they are more *conservative* in updating their beliefs based on the observed reviews. When ρ is small and/or early buyers constitute a fairly representative sample of the customer population, rule (3) allows customers to abstract away from the complexities associated with Lemma 1 with minimal consequences.

3.5. Firm's Problem

We assume that, through its market research, the firm knows the product's mean perceived quality, $\hat{q}$, possesses perfect demand information and a good understanding of review-generation and customer learning. Since the main focus of our model is to investigate the implications of social learning, we allow the firm to operate in the absence of any binding capacity constraints, so that it can produce and distribute any amount of the product in either selling period. All production costs are normalized to zero. The firm seeks to maximize expected profit over the two selling periods. We first investigate the firm's fixed-pricing problem in the presence of social learning. Motivated by our results, we next consider the firm's joint fixed-pricing and early-availability problem. Finally, we perform a comparison between fixed and dynamic-pricing policies. The mathematical formulations of the firm's problem under social learning are presented in the relevant analysis sections in §4.

Table 1   List of notation

Symbol	Description
p	Product's price (decision variable)
N	Market size
x_i	Preference component of customer i
$F(\cdot)$	Cumulative distribution function of x_i
q_i	Post-purchase quality perception (i.e., review) of customer i
$\hat{q}$	Product's mean quality
q_p	Mean of customers' prior belief over $\hat{q}$
σ_p^2	Variance of customers' prior belief over $\hat{q}$
v_{i1}	First-period (prior) valuation of customer i
$D_1(p)$	First-period demand at price p
q_u	Mean of customers' updated belief over $\hat{q}$
v_{i2}	Second-period (updated) valuation of customer i

4. Analysis
4.1. Optimal Pricing

In this section, we investigate the firm's pricing problem under the two modes of social learning described in §3.4. We begin by studying the relationship between the firm's pricing decision and the reviews generated in the first period. Next, for the case of perfect social learning, we highlight the trade-offs faced by the firm in choosing its price optimally, illustrate the influence of social learning on firm profit, and consider the implications of social learning from buyer reviews on economic welfare. We then examine how these insights are affected under more realistic customer behavior, namely, under imperfect social learning.

4.1.1. Buyer Reviews We begin our analysis of the firm's pricing problem by examining the relationship between product price and the reviews generated in the first period of our model. Let the product's price be p so that any customer with first-period valuation $v_{i1} \geq p$ purchases a unit in the first period and subsequently reviews the product. Recall that, in our model, heterogeneity in customer valuations is generated by heterogeneity in preferences, and that preferences are linked to quality perceptions through the correlation parameter ρ. Let $\bar{R}$ denote the product's expected rating following first-period purchases and let $h(\cdot)$ denote the hazard ratio $h(\cdot) := \frac{f(\cdot)}{\bar{F}(\cdot)}$.

LEMMA 2. *At any price p there exists a unique threshold $\tau(p)$,*

$$\tau(p) = \frac{p - q_p + \rho \frac{\sigma_q}{\sigma_x} \bar{x}}{a + \rho \frac{\sigma_q}{\sigma_x}}, \tag{4}$$

such that customer i purchases the product in the first period only if $x_i \geq \tau(p)$. The number of first-period buyers, $D_1(p)$, and the product's expected rating, $\bar{R}(p)$, are given by

$$D_1(p) = N\bar{F}(\tau(p)) \ and \ \bar{R}(p) = \hat{q} + \rho\sigma_q\sigma_x h(\tau(p)). \tag{5}$$

Moreover, $D_1(p)$ is strictly decreasing, while $\bar{R}(p)$ is strictly increasing in p.

The average rating, $\bar{R}$, communicates information regarding the product's mean quality, $\hat{q}$, but also exhibits *self-selection bias*. This self-selection bias arises as a consequence of early buyers' purchasing decisions − customers who choose to purchase the product in the first period have different preferences to customers remaining in the market, and this difference in preferences is reflected in their reviews (this phenomenon is often referred to in the empirical literature as "purchase bias"; e.g., see Hu et al. (2009)). A higher price leads to a smaller number of first-period buyers, but a higher product rating. The latter occurs because by increasing the product's price, customers with relatively lower x_i components are priced out of the market. It follows that product reviews are generated by customers with relatively higher x_i components, which results in an amplification of the bias term in (5).

Lemma 2 establishes that the firm's pricing decision affects both the *amount* of information made available to potential second-period buyers (i.e., the number of first-period reviews, D_1), and the *content* of this information (i.e., the product's average rating, $\bar{R}$). As we demonstrate next, the firm's optimal pricing policy depends on how customers interpret this information.

4.1.2. Perfect Social Learning We first assume that social learning occurs as described in Lemma 1; that is, customers hold full information and apply Bayes' rule accordingly. Let $\bar{q}_u(p) :=$ $E[q_u(p)]$ denote the updated expectation of mean quality held by customers after observing the first-period reviews, when the product's price is p.

LEMMA 3. *Under perfect social learning,*

$$\bar{q}_u(p) = q_p + \frac{D_1(p)\sigma_p^2}{D_1(p)\sigma_p^2 + \sigma_q^2(1-\rho^2)}\left(\hat{q} - q_p\right). \tag{6}$$

Moreover, if $\hat{q} < q_p$ ($\hat{q} > q_p$), then $\bar{q}_u$ is strictly increasing (decreasing) in p.

Under perfect social learning, what is ultimately extracted from individual reviews are noisy, but unbiased, indications of the product's mean quality, $\hat{q}$. Customers' updated expectation of mean quality, $\bar{q}_u$, is higher (lower) than their prior, q_p, only when $\hat{q} > q_p$ ($\hat{q} < q_p$). That is, under perfect social learning, customers' updated estimate $\bar{q}_u$ is always a more accurate reflection of $\hat{q}$ than their prior q_p.

The firm's pricing problem in the presence of social learning may be expressed as

$$\max_{p \geq 0} \quad \left(Np\int_{\tau(p)}^{\infty} f(x)dx + Np\int_{\min\left\{\tau(p), \tau(p) - \frac{\bar{q}_u(p) - q_p}{a + \rho\frac{\sigma_q}{\sigma_x}}\right\}}^{\tau(p)} f(x)dx\right). \tag{7}$$

The first and second terms correspond to first and second-period profit respectively.[8] By inspection of the above formulation, it is evident that for any second-period sales to occur in our model, it must be the case that the quality perception of customers who chose not to purchase in the first period (i.e., had valuations $v_{i1} < p$) is increased after observing the reviews of the first period buyers (i.e, so that $v_{i2} \geq p$ for at least some customers remaining in the market).

The firm's optimal pricing policy under perfect social learning is characterized in Theorem 1.

THEOREM 1. *Under perfect social learning:*
- *If the product's mean quality is lower than customers' prior expectation ($\hat{q} \leq q_p$), the firm's optimal profit is independent of $\hat{q}$ and there exists a unique optimal price, p_l^*, which satisfies*

$$p_l^* \cdot h(\tau(p_l^*)) = a + \rho\frac{\sigma_q}{\sigma_x}. \tag{8}$$

---

[8] Note that the expression for second period profit in formulation (7) is approximate, since $q_u(p)$ in the lower limit of the integral is an *ex-ante* Normal random variable of mean $\bar{q}_u(p)$ and variance less than $\frac{\sigma_q^2(1-\rho^2)}{N F(\tau(p))}$. However, the latter expression is of order $1/N$ and, as a result, the variance plays a negligible role in firm profit for any reasonable market size, N; it is therefore ignored in our analysis for tractability purposes.

- If the product's mean quality is higher than customers' prior expectation $(\hat{q} > q_p)$, the firm's optimal profit is strictly increasing in $\hat{q}$ and there exists an optimal price, p_h^*, which satisfies

$$p_h^* \cdot h\left(\tau(p_h^*) - \frac{\bar{q}_u(p_h^*) - q_p}{a + \rho\frac{\sigma_q}{\sigma_x}}\right) \cdot \left(1 + \frac{\lambda(p_h^*)}{a + \rho\frac{\sigma_q}{\sigma_x}} \cdot (\hat{q} - q_p)\right) = a + \rho\frac{\sigma_q}{\sigma_x}, \qquad (9)$$

$$\text{with } \lambda(p) = \frac{Nf(\tau(p))\sigma_p^2\sigma_q^2(1-\rho^2)}{[D_1(p)\sigma_p^2 + \sigma_q^2(1-\rho^2)]^2}.$$

A numerical illustration of the results of Theorem 1 is presented in Figure 3. We discuss the intuition underlying the two cases of the Theorem in turn. Consider first the case in which the product's mean quality is lower than customers' prior expectation $(\hat{q} \leq q_p)$. Lemma 3 suggests that for any non-zero number of first-period sales, $D_1(p)$, the effect of social learning will be negative — after observing the reviews of the early buyers, the valuations of customers remaining in the market will inevitably be decreased. Since these customers were unwilling to purchase the product in the first period, and social learning serves only to decrease their valuations, we conclude that no sales occur in the second period (note that this is true *irrespective* of the price chosen by the firm). When this is the case, the firm simply seeks to maximize its first-period profit by resolving the *classic pricing trade-off* between a smaller number of sales at a higher revenue-per-purchase and a larger number of sales at a lower revenue-per-purchase. In this respect, when true quality is lower than the market's prior expectation, the firm approaches its pricing decision as if it were operating *in the absence of social learning.*

The firm's problem is less straightforward when quality is higher than customers' prior expectation $(\hat{q} > q_p)$. In particular, the firm's pricing decision now plays a dual role. First, it extracts revenue; this role is associated with the classic trade-off as per the case discussed above. Second, it serves to modulate the social learning process, which has a direct impact on second-period profit. To see this, notice that in Lemma 3, the larger the number of first-period reviews (purchases), $D_1(p)$, the more customers' updated expectation of mean quality approaches the true value, $\hat{q}$. In turn, the number of first-period reviews is dictated by the firm's pricing decision, which implies that the social learning effect is stronger (in terms of increasing perceptions of quality in the market) when the product's price is relatively lower. Therefore, the firm's pricing decision in the presence of social learning must be viewed by the firm not only as a means for extracting rents, but also as an instrument for shaping future demand. Note that while Theorem 1 suggests that firm profit is monotone increasing in quality for high-quality products (i.e., $\hat{q} > q_p$), we observe that the optimal price may be either strictly increasing (e.g., Figure 3b) or unimodal (i.e., decreasing and then increasing) in quality; however, we note that the latter occurs only in cases of extreme parameter values (e.g., when customers' prior uncertainty, σ_p^2, is extremely small).

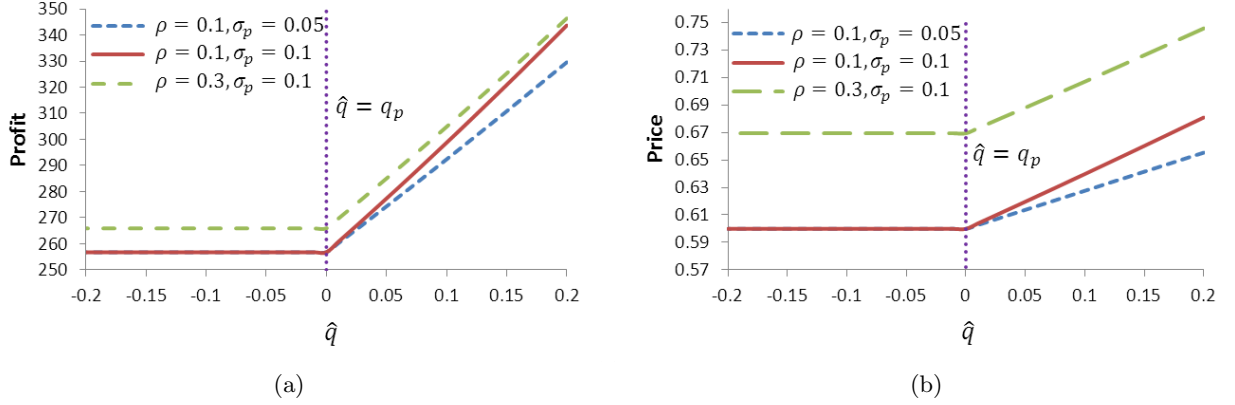(a)   (b)

Figure 3   **Optimal profit and price under perfect social learning, at different values of $\hat{q}$. Parameter values:**
$$\bar{x} = \sigma_x = \sigma_q = 0.5; \; q_p = 0; \; a = 1; \; N = 1000.$$

The following Corollary illustrates the relationship between firm profit and key drivers underlying the social learning process.

COROLLARY 1. *Under perfect social learning, when the product's mean quality is higher than customers' prior expectation ($\hat{q} > q_p$):*

- *Optimal profit is strictly increasing in customers' prior uncertainty over quality, σ_p^2.*
- *Optimal profit is strictly decreasing in the variability in post-purchase perceptions of quality, σ_q^2.*

In essence, the firm achieves higher profit when the social learning process is more *efficient* (i.e., when the quality expectations of customers remaining in the market converge towards the product's mean quality faster). When customers are relatively more uncertain over product quality (i.e., large σ_p^2), they tend to be influenced more by the content of buyer reviews (this is evident in (6)). As a result, at any fixed price (and therefore fixed set of first-period reviews) the firm is able to achieve a higher number of second-period sales through the effects of social learning. Similar intuition holds for the case of relatively lower variability in post-purchase quality perceptions, σ_q^2.

We conclude this section by discussing the implications of social learning from buyer reviews on economic welfare. We define total welfare, W, as the sum of firm profit, π, and aggregate customer surplus, S. Let ΔW denote the difference in total welfare generated by the presence of social learning.

COROLLARY 2. *Under perfect social learning:*

- *When the product's mean quality is lower than customers' prior expectation ($\hat{q} \leq q_p$), the difference in total welfare generated by social learning is $\Delta W = 0$.*
- *When the product's mean quality is higher than customers' prior expectation ($\hat{q} > q_p$), the difference in total welfare generated by social learning is positive, $\Delta W > 0$, if and only if*

$$p_h^* - p_l^* < \bar{q}_u(p_h^*) - q_p, \tag{10}$$

where p_h^* and p_l^* are given in Theorem 1.

Recall that for cases of $\hat{q} \leq q_p$, social learning (a) does not alter the purchasing decisions of customers remaining in the market in the second period and (b) does not influence the firm's pricing decision or profit. As a result, in this case the presence of social learning has no effect on total welfare or customer surplus. Conversely, when $\hat{q} > q_p$, the difference in total welfare generated by social learning is positive provided the number of total purchases increases, which is ensured by the condition in the Corollary. Figure 4a illustrates a typical example of our numerical experiments (see Appendix C for details).

A related question of interest is whether the firm, the customer population, or both benefit from the presence of social learning. First, note that from Theorem 1, it follows that the firm is (weakly) better off in the presence of social learning than it is in its absence. Next, from a customer's point of view, notice that learning from buyer reviews always leads to better-informed purchasing decisions. This is true under perfect social learning, since, as Lemma 3 suggests, customers' updated quality estimate, $\bar{q}_u$, is always a more accurate reflection of the product's mean quality, $\hat{q}$, than the prior, q_p. However, social learning also influences the firm's pricing decision. Interestingly, our numerical study indicates that the customer population, through its propensity to review and learn from reviews, is left with *less* surplus than it would receive in the absence of social learning (see Figure 4b and Appendix C). That is, for a given product price customers are better off by engaging in social learning; however, when the firm's ability to internalize the social learning process through its pricing decision is taken into account, we observe that customers suffer a net loss in surplus. Surprisingly, we conclude that while social learning from buyer reviews is generally considered to be beneficial for both firms and customers, it may in fact be *detrimental* for the latter.
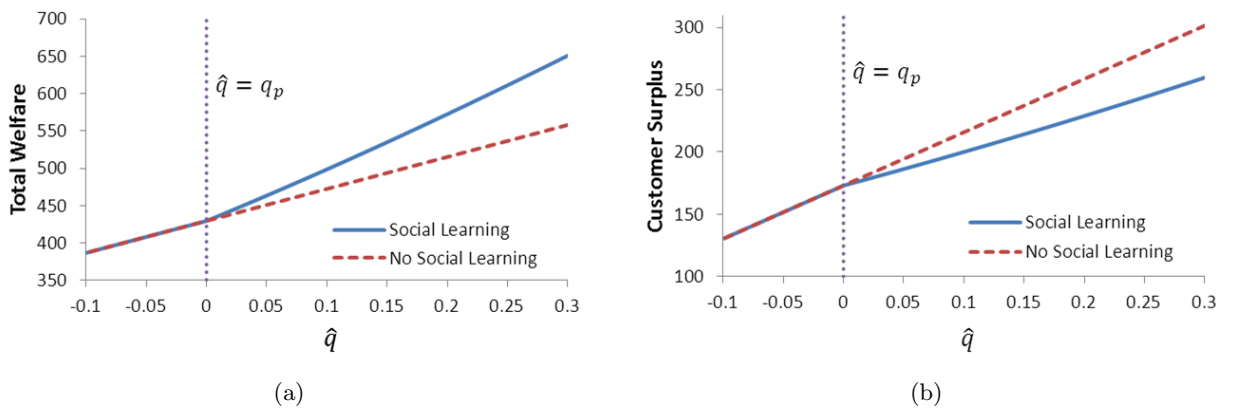

Figure 4   **Social welfare and customer surplus with and without (perfect) social learning, at different values of** $\hat{q}$**. Parameter values:** $\bar{x} = \sigma_x = \sigma_q = 0.5$; $q_p = 0$, $\sigma_p = 0.1$; $a = 1$; $N = 1000$; $\rho = 0.1$**.**

4.1.3. **Imperfect Social Learning** We next consider the firm's pricing problem under imperfect social learning. The outcome of the social learning process (i.e., customers' updated expectation of mean quality, $\bar{q}_u(p)$) is more intricate in this case, owing to customers' simplified approach to extracting information from buyer reviews.

LEMMA 4. *Under imperfect social learning,*

$$\bar{q}_u(p) = q_p + \frac{D_1(p)\sigma_p^2}{D_1(p)\sigma_p^2 + \sigma_q^2}\Big(\hat{q} + \rho\sigma_q\sigma_x h(\tau(p)) - q_p\Big). \tag{11}$$

Moreover, $\bar{q}_u$ is strictly unimodal with a maximum at $p = \tau^{-1}(\tau^*)$, where τ^* is the unique solution of

$$\tau = \bar{x} - \frac{(\hat{q} - q_p)\sigma_x}{\rho\sigma_q} + \frac{N\sigma_p^2}{\sigma_q^2}\int_\tau^{+\infty}(x - \tau)f(x)\,dx. \tag{12}$$

From the firm's perspective, Lemma 4 suggests a complicated interaction between the amount of information generated by first-period purchases and the content of this information. As was the case under perfect social learning, a relatively lower price generates a stronger effect by releasing more first-period reviews, $D_1(p)$. However, at the same time, the larger the number of reviews released, the lower the product's expected rating (this follows from Lemma 2), which tends to lower customers' updated perceptions of quality. Moreover, the non-monotonic nature of $\bar{q}_u$ suggests that the effect which dominates the social learning process (i.e., influences customers' updated expectation of quality the most) differs at different regions of the product's price.

Under imperfect social learning, the firm's problem is as stated in (7), with the difference that customers' update from q_p to $\bar{q}_u$ occurs according to Lemma 4. The implications of the imperfections of the social learning process on the firm's optimal pricing policy are summarized in Theorem 2.

THEOREM 2. *Under imperfect social learning there exists a threshold $\phi = q_p - \rho\sigma_q\sigma_x h(\tau(p_{li}^*)) < q_p$, such that:*

- *If the product's mean quality satisfies $\hat{q} \le \phi$, the firm's optimal profit is independent of $\hat{q}$ and there exists a unique optimal price, p_{li}^*, which satisfies*

$$p_{li}^* \cdot h(\tau(p_{li}^*)) = a + \rho\frac{\sigma_q}{\sigma_x}. \tag{13}$$

- *If the product's mean quality satisfies $\hat{q} > \phi$, the firm's optimal profit is strictly increasing in $\hat{q}$ and there exists an optimal price, p_{hi}^*, which satisfies*

$$p_{hi}^* \cdot h\left(\tau(p_{hi}^*) - \frac{\bar{q}_u(p_{hi}^*) - q_p}{a + \rho\frac{\sigma_q}{\sigma_x}}\right) \cdot \left(1 + \frac{\lambda(p_{hi}^*)}{a + \rho\frac{\sigma_q}{\sigma_x}} \cdot \left(\hat{q} - q_p + \rho\sigma_q \cdot \frac{\tau(p_{hi}^*) - \bar{x}}{\sigma_x}\right)\right.$$
$$\left. + \frac{\delta(p_{hi}^*)}{a + \rho\frac{\sigma_q}{\sigma_x}} \cdot \left(\rho\sigma_q \cdot \frac{\tau(p_{hi}^*) - \bar{x} - \sigma_x^2 h(\tau(p_{hi}^*))}{\sigma_x}\right)\right) = a + \rho\frac{\sigma_q}{\sigma_x}, \tag{14}$$

$$\text{with } \lambda(p) = \frac{N f(\tau(p)) \sigma_p^2 \sigma_q^2}{[D_1(p)\sigma_p^2 + \sigma_q^2]^2} \quad \text{and} \quad \delta(p) = \frac{N D_1(p) f(\tau(p)) \sigma_p^4}{[D_1(p)\sigma_p^2 + \sigma_q^2]^2}.$$

A numerical illustration of the results of Theorem 2 is presented in Figure 5. Theorem 2 appears to be qualitatively similar to Theorem 1, but there are two significant differences which are a direct consequence of the imperfect social learning process. First, note that the threshold value of quality above which social learning is beneficial for the firm, ϕ, is now *lower* than customers' prior expectation, q_p. This is important, because it suggests that the firm may benefit from the effects of social learning, even when the product under-performs with respect to market expectations. When the social learning process is imperfect, the firm is able to leverage the idiosyncratic preferences of its first-period buyers to compensate for lower-than-expected product quality. Note the dependence of ϕ on σ_q and ρ. Parameter σ_q captures the overall variability around $\hat{q}$ of post-purchase perceptions of product quality in the customer population. When this variability is relatively larger, high x_i customers are likely to generate higher product ratings. As a result, such reviews can be used to "mask" relatively lower product quality, which leads to a lower threshold ϕ. Next, ρ may be viewed as a measure of the self-selection bias in early reviews which is not explicitly accounted for by potential customers. For large ρ, customers misinterpret early reviews as being indicative of higher-than-expected quality, even though in reality quality may be significantly lower than their prior beliefs.

Second, the firm's pricing decision when quality is above the threshold ϕ, now involves an additional layer of complexity, as compared to the perfect social learning case (this is captured by the term multiplying $\delta(p)$ in (14)). Recall that in Theorem 1, the firm's pricing decision served to extract revenue, but also to modulate the social learning process, specifically, through the number of product reviews generated in the first period. Under imperfect social learning, Lemma 4 suggests that the firm must now consider the effect of its pricing decision not only on the amount of information generated, but also on its content; specifically, a lower price generates more reviews, but a higher price achieves more favorable content. The firm's optimal pricing decision therefore takes into account the interplay between two trade-offs: the classic *pricing* trade-off, whereby a higher price generates a larger number of sales but a lower revenue-per-purchase, and an *informational* trade-off whereby a lower price generates a larger number of buyer reviews but a lower product rating. Moreover, while firm profit increases in quality, we once again observe that the optimal price may be strictly increasing or non-monotone (for extreme parameter values) in quality. We also state the following Corollary, which corresponds to Corollary 1 of the perfect-learning case.

COROLLARY 3. *Under imperfect social learning, when the product's mean quality satisfies $\hat{q} > \phi$:*
- Optimal profit is strictly increasing in customers' prior uncertainty over quality, σ_p^2.

(a)   (b)
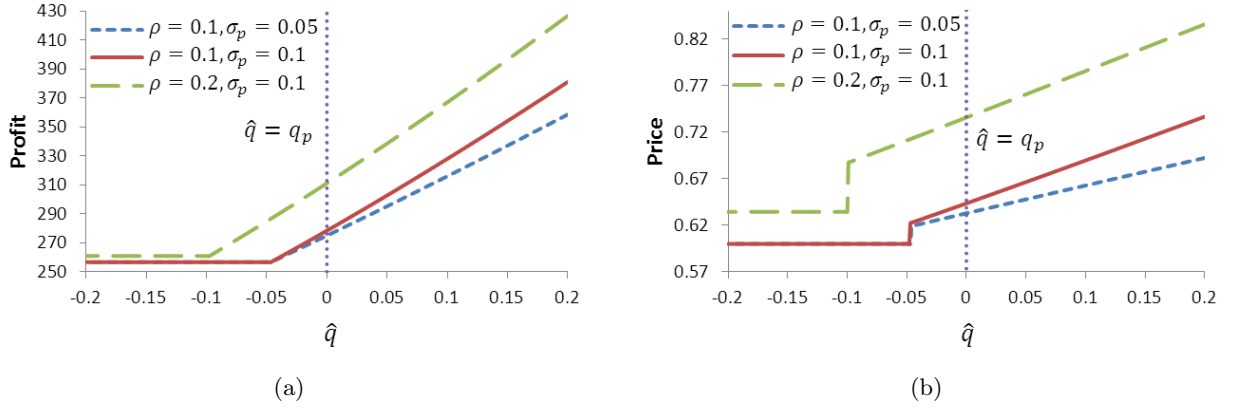
Figure 5   Optimal profit and price under imperfect social learning, at different values of $\hat{q}$. Parameter values: $\bar{x} = \sigma_x = \sigma_q = 0.5$; $q_p = 0$; $a = 1$; $N = 1000$.

The following result compares firm profit under perfect social learning with that under imperfect social learning.

PROPOSITION 1. *Let π_p^* denote optimal profit under perfect social learning and let π_i^* denote optimal profit under imperfect social learning.*

- *If the product's mean quality satisfies $\hat{q} \leq \phi$, profits under perfect and imperfect social learning are equal, $\pi_p^* = \pi_i^*$.*
- *If the product's mean quality satisfies $\phi < \hat{q} \leq q_p$, profit under imperfect social learning is strictly higher than under perfect social learning, $\pi_p^* < \pi_i^*$.*
- *If the product's mean quality satisfies $\hat{q} > q_p$, a sufficient condition for $\pi_p^* < \pi_i^*$ is*

$$Nf(\tau(p_h^*)) > \frac{\rho\sigma_q(\hat{q} - q_p)}{\sigma_x\sigma_p^2}.$$

The first two cases of Proposition 1 follow directly from Theorems 1 and 2. For products of high quality ($\hat{q} > q_p$), the Proposition provides a sufficient condition such that profit under imperfect social learning is strictly higher than under perfect social learning. In essence, the condition suggests that the latter result holds provided the market size, N, is sufficiently large. To see why this is the case, recall that under imperfect social learning customers are inherently more conservative, in the sense that they place, *ceteris paribus*, more relative weight on their prior belief. This effect dominates customers' updated beliefs only if the number of early product reviews is very small, which in turn occurs if the market size itself is small.

We conclude this section by considering the implications of imperfect social learning on total welfare, firm profit, and customer surplus. In our numerical experiments (see Appendix C), we observe that, unlike for the case of perfect social learning, imperfect social learning may generate a decrease in total welfare, when the product is of relatively low quality (e.g., see Figure 6a). According to Theorem 2, firm profit increases in the presence of social learning; therefore, the

decrease in welfare is attributed exclusively to a decrease in customer surplus (e.g., see Figure 6b). In general, customer surplus decreases more severely under imperfect social learning than under perfect social learning, for two reasons: first, because the firm generally charges a higher price in the presence of imperfect social learning; second, because some customers who choose to purchase the product in the second period in fact derive *negative* utility from their purchase; this occurs because the simplified approach to learning leads customers to over-estimate product quality based on the available reviews.
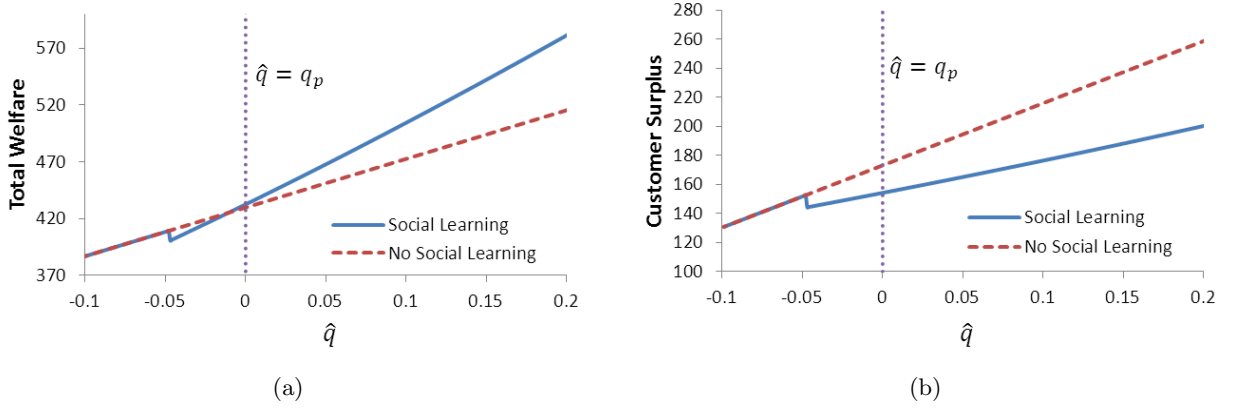

(a)   (b)

Figure 6   **Social welfare and customer surplus with and without (imperfect) social learning, at different values of $\hat{q}$. Parameter values:** $\bar{x} = \sigma_x = \sigma_q = 0.5$; $q_p = 0$, $\sigma_p = 0.1$; $a = 1$; $N = 1000$; $\rho = 0.1$.

4.2.   Optimal Pricing and First-Period Product Availability

Much of the analysis of Section §4.1 was concerned with highlighting the trade-offs faced by the firm under perfect and imperfect social learning. In particular, we have demonstrated that the firm's pricing decision plays a dual role: first, it extracts revenue; second, it modulates the social learning process. In general, these two roles are conflicting and the firm's profit-maximizing pricing decision is a compromise between the two. In particular, under imperfect social learning, the price at which customers' updated quality perceptions are maximized is often higher than the profit-maximizing price. In such cases, by charging the higher price the firm would achieve the best possible social learning effect (i.e., highest possible updated quality perception, $\bar{q}_u$), but at the same time it would render its product too expensive for many subsequent customers, making it a suboptimal decision from a profit-maximizing perspective.

In this section, we demonstrate that under certain conditions, the firm is able to relieve the pricing decision of its social learning role, relying instead on its *early inventory decision* to modulate the social learning process. In this way, the firm may enjoy the optimal social learning effect without compromising its pricing policy. The described notion relies on the firm's ability to use inventory

management as an indirect method of "price discrimination." More specifically, the results presented in this section rely on first-period rationing being *efficient*, meaning that customers holding relatively higher valuations for the product are more likely to secure a unit in the first period when demand exceeds supply (Denicolò and Garella 1999). When facilitated by an appropriate product allocation mechanism, efficient rationing arises naturally as a result of customer behavior. To illustrate, we consider the commonly observed phenomenon of product allocation through a waiting line, and *endogenize* customer decisions on whether and when to join the waiting line.[9]

4.2.1. First-Period Supply Shortages with Waiting Lines Let n_1 denote the number of units made available by the firm in the first period. Accordingly, if the firm chooses $n_1 < D_1(p)$, first-period demand exceeds supply and an early supply shortage occurs. Motivated by examples such as the launch of innovative and eagerly anticipated consumer electronics (e.g., Apple iPad) or movie premieres (e.g., *The Hunger Games*), consider a scenario of limited early availability in which customers may form a waiting-line prior to product launch and receive the product on a first-in-line, first-served basis (e.g., *Los Angeles Times* 2011, *CBS Los Angeles* 2012).[10] We consider the described setting with endogenous customer arrivals: A customer who chooses to join the waiting line relatively earlier has a higher chance of securing a unit, but must spend a larger amount of costly time in the waiting line. Our analysis is an extension of the work first presented in Holt and Sherman (1982); we provide a detailed description and analysis of the waiting-line game in Appendix B.

We first point out that all customers with $v_{i1} \geq p$ attempt to secure a unit in the first period. Note that this does not imply that all such customers incur a positive waiting-time cost in the first period (which can presumably be avoided by deferring purchase). Instead, since customers choose their own arrival times, they may simply choose to arrive at the time of product launch and check whether the product is available (i.e., incurring zero waiting-time cost). However, customers who are successful in securing a unit in the first period are only those who choose to join the waiting line the earliest. In turn, the composition of the waiting-line that forms in the first period is determined by the equilibrium process which specifies customer decisions on individual arrival times. Let w_i denote customer i's waiting cost per unit time and let v_{i1} and w_i be related through the mapping $w_i = w(v_{i1})$, where $w : \mathbb{R} \mapsto \mathbb{R}$ and $w(\cdot)$ is assumed to be differentiable across its domain.

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[9] The term "waiting line" is used here in accordance with relevant work in economics (e.g., Holt and Sherman 1982); a waiting line differs from the classic notion of a queue, in that customers in a waiting line are all served simultaneously at some pre-announced time (e.g., the time of product launch), rather than service occurring continuously.

[10] We implicitly assume that when $n_1 < D_1(p)$, interested customers are made aware of the early supply shortage. This assumption is justified by real-world situations in which customers are informed of supply shortages through firm announcements (*TechNewsWorld* 2006), media speculation (*The Washington Post* 2012) or prior purchasing experience with similar products.

PROPOSITION 2. *If $\eta = (v_{i1} - p)\frac{w'(v_{i1})}{w(v_{i1})} < 1 \ (> 1)$, then in equilibrium customers with $v_{i1} \geq p$ join the first-period waiting line in descending (ascending) order of their valuations.*

That is, if $\eta < 1$, customers holding relatively higher valuations for the product will choose to arrive relatively earlier. We note that this sufficient condition subsumes both the commonly assumed case of homogeneous waiting costs (e.g., Veeraraghavan and Debo 2011), as well as the case of higher valuation customers incurring relatively lower waiting costs, which appears to be consistent with anecdotal evidence that those who join the waiting line first are high-valuation, low-cost-of-waiting customers such as students (*UTV* 2012) or speculators.[11]

The significance of the result of Proposition 2 is that it allows the firm to use its early inventory decision as an *indirect method of price-discrimination*. More specifically, restricting supply effectively amounts to selling the product to the firm's highest valuation customers. Therefore, by setting $n_1 < D_1(p)$, the firm is able to replicate the social learning effect of charging a higher price in the first period. This notion is formalized in the following Lemma.

LEMMA 5. *Assume that $\eta < 1$ and let the product's price be p. If the firm makes n_1 units available in the first period, the product's expected rating from first-period reviews is given by*

$$\bar{R}(p, n_1) = \hat{q} + \rho\sigma_q\sigma_x h(c(p, n_1)), \tag{15}$$

where

$$c(p, n_1) = \begin{cases} \bar{F}^{-1}\left(\frac{n_1}{N}\right) & \text{if } 0 \leq n_1 < D_1(p) \\ \tau(p) & \text{if } n_1 \geq D_1(p). \end{cases} \tag{16}$$

Moreover, $\bar{R}$ is strictly decreasing in n_1 for $0 \leq n_1 \leq D_1(p)$.

Using expression (16) we may define $z := \tau^{-1}\left(\bar{F}^{-1}\left(\frac{n_1}{N}\right)\right)$, where $z > p$ for $0 \leq n_1 < D_1(p)$, as the actual price which would generate the social learning effect achieved by the firm's early-availability decision n_1. By restricting supply, the firm is able to amplify the self-selection bias in early reviews in analogous fashion to raising the product's price. However, note that the increased average rating is achieved at the expense of reduced review volume, n_1. Moreover, we stress that, for the firm to be able to trade off review volume for a higher product rating, it is required to facilitate early customer self-selection by putting in place a suitable product allocation mechanism (e.g., a waiting line). By contrast, it is straightforward to show that when availability is restricted to be less than demand in the first period, but each customer has an *equal* probability of obtaining the product

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[11] The presence of speculators in the market does not affect the applicability of our model, since speculators may be viewed as an additional channel through which units are allocated to high-valuation customers; see Su (2010) for related work.

(i.e. so that proportional rationing occurs in the first period), the number of first-period reviewers decreases, but the product's average rating $\bar{R}$ remains as described in (5).

We proceed to investigate the circumstances under which it is optimal for the firm to restrict early sales in order to achieve a higher average product rating. To simplify exposition, in the analysis that follows we assume that when early demand outstrips supply the product is allocated through a waiting-line and that $\eta < 1$, as required for the results in this section to hold.

4.2.2. Perfect Social Learning Under perfect social learning and firm policy $\{p, n_1\}$, customers' updated expectation of mean quality in the second period takes the following form.

LEMMA 6. *Under perfect social learning,*

$$\bar{q}_u(p, n_1) = q_p + \frac{n_1 \sigma_p^2}{n_1 \sigma_p^2 + \sigma_q^2(1 - \rho^2)} \left(\hat{q} - q_p\right). \tag{17}$$

Moreover, if $\hat{q} < q_p$ ($\hat{q} > q_p$), then $\bar{q}_u$ is strictly increasing (decreasing) in n_1 for $0 \leq n_1 \leq D_1(p)$.

The intuition underlying this result is similar to that of Lemma 1 and is thus omitted. The firm's joint pricing–availability problem in the presence of social learning may be expressed as

$$\max_{\substack{p \geq 0 \\ 0 \leq n_1 \leq D_1(p)}} \left(p n_1 + N p \int_{\min\left\{c(p,n_1), \tau(p) - \frac{\bar{q}_u(p,n_1) - q_p}{a + \rho \frac{\sigma_q}{\sigma_x}}\right\}}^{c(p,n_1)} f(x) dx \right), \tag{18}$$

where $c(p, n_1)$ is given in (16).[12] The first and second terms represent profit in the first and second periods, respectively. When social learning is perfect, the firm's optimal pricing–availability policy is stated in Theorem 3.

THEOREM 3. *Under perfect social learning, it can never be optimal for the firm to induce a first-period supply shortage. That is, the firm's optimal pricing policy p^* is characterized by Theorem 1 and the corresponding optimal inventory policy is $n_1^* = D_1(p^*)$.*

As described in §4.2.1, early shortages can be used by the firm to amplify the self-selection bias in early reviews. However, this amplification is clearly not advantageous if customers account perfectly for the bias in reviews. When the product's mean quality exceeds prior expectations ($\hat{q} > q_p$), irrespective of the firm's chosen price, the firm chooses to fulfill all first-period demand, since customers' updated expectation of quality, $\bar{q}_u$, is strictly increasing in n_1 (see Lemma 6). Conversely, when mean quality is below prior expectations ($\hat{q} < q_p$), $\bar{q}_u$ is strictly decreasing in n_1; however, it is *still* optimal for the firm to sell as many units as possible in the first period. To

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[12] We note here that n_1 is assumed to be large enough so that it can be treated as a continuous variable and that, without loss of generality, we map any inventory decision that fulfills all first-period demand (i.e. $n_1 \geq D_1(p)$) to the decision $n_1 = D_1(p)$, since such decisions leads to $D_1(p)$ sales. In addition, note that all second-period demand is trivially fulfilled, so that second-period inventory need not be considered as a decision variable.

see this, notice that, at any price, if the firm does not fulfill all first-period demand, it will lose customers who were willing to purchase in the first period, through the effects of social learning. Therefore, the firm sells as many units as it can in the first period and simply chooses the price which maximizes first-period profit, as is the case in Theorem 1.[13]

It is worth noting that the insight of Theorem 3, and in particular the generality of this result, seems difficult to transfer to real-world settings. Specifically, empirical evidence suggests that customers, *ceteris paribus*, tend to associate a higher average product rating with higher product quality (Chevalier and Mayzlin 2006). Therefore, the insight that the firm could improve the average rating by restricting early supply, but nonetheless *never* finds it optimal to do so, seems to be inconsistent with documented customer behavior. As we demonstrate in the next section, this result is overturned under the assumption of imperfect social learning. Furthermore, as we discuss in §6.3, the result of Theorem 3 may not hold (i.e., early supply shortages may indeed be optimal) if customers are *ex-ante* uncertain about *multiple* problem parameters (i.e., not just product quality), *even if* they are perfect social learners.

4.2.3. Imperfect Social Learning Under imperfect social learning and firm policy $\{p, n_1\}$, customers' updated expectation of mean quality in the second period takes the following form.

LEMMA 7. *Under imperfect social learning,*

$$\bar{q}_u(p, n_1) = q_p + \frac{n_1 \sigma_p^2}{n_1 \sigma_p^2 + \sigma_q^2} \Big(\hat{q} + \rho \sigma_q \sigma_x h(c(p, n_1)) - q_p \Big), \tag{19}$$

where $c(p, n_1)$ is given in (16). Moreover, $\bar{q}_u$ is strictly unimodal with a unique maximum at $n_1 = N\bar{F}(\tau^*)$, where τ^* is the solution of (12).

The result of Lemma 7 is analogous to the result of Lemma 4, where we may think of the firm's availability decision n_1 as taking the role of the pricing decision in (11), through $c(p, n_1)$. From the analysis in §4.2.1 we know that the product's average rating is decreasing in early availability — the fewer the units made available by the firm, the higher the product's average rating. However, a higher average rating is accompanied by reduced review volume, which reduces the relative weight placed by potential customers on the review content. Furthermore, the unimodal form of $\bar{q}_u$ suggests that there exists a minimum number of first-period sales, such that restricting availability beyond this point is detrimental for the firm, even though it results in a higher average rating.

The firm's problem remains as stated in (18), with the social learning process as described in Lemma 7. The firm's optimal pricing-availability policy is characterized in Theorem 4.

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[13] A profit-equivalent policy is for the firm to make no units available in the first period (so that no social learning occurs) and sell as many units as possible in the second period at the price stated in Theorem 1.

THEOREM 4. *Under imperfect social learning, there exists a quality threshold Q, such that:*

- *If the product's mean quality satisfies $\hat{q} < Q$, there exists a unique optimal pricing–availability policy which includes a first-period strategic supply shortage and is given by $\{p^*, n_1^*\} = \{\xi^*, N\bar{F}(\tau^*)\}$, where ξ^* satisfies*

$$\xi^* \cdot h\left(\frac{\xi^* - \bar{q}_u^* + \rho\frac{\sigma_q}{\sigma_x}\bar{x}}{a + \rho\frac{\sigma_q}{\sigma_x}}\right) = a + \rho\frac{\sigma_q}{\sigma_x}, \;\; \text{with } \bar{q}_u^* = \bar{q}_u(\xi^*, N\bar{F}(\tau^*)). \tag{20}$$

- *If the product's mean quality satisfies $\hat{q} \geq Q$, the firm's optimal policy does not include a strategic supply shortage and is given by $\{p^*, D_1(p^*)\}$, where p^* is characterized by Theorem 2.*

Theorem 4 establishes that early supply shortages are optimal for the firm under imperfect social learning, provided product quality does not exceed an upper threshold. The existence of an upper, rather than a lower, bound on product quality warrants further explanation; the result is best described using an example such as that of Figure 7. The figure compares firm profit when the first-period inventory allocation is restricted to fulfill all first-period demand, and when the firm has the option of restricting supply in the first period (in the optimal manner of Theorem 4). As suggested in the Theorem, the added value of the "shortage option" is zero when the product's mean quality is much higher than customers' prior expectations. Put simply, when product quality greatly exceeds prior expectations there is no need for the firm to manipulate the information available to potential customers; instead, it chooses to satisfy all demand in the first period and let "quality speak for itself." However, it is also evident from Figure 7 that the added value of the shortage option is approaching zero (from above) for low-quality products. The intuition here is as follows. When product quality is significantly lower than expected, only customers whose idiosyncratic x_i component is very high (e.g., who are very brand loyal) will generate customer reviews with a positive social learning influence. However, such customers tend to be limited in number and, as a result, do not have a significant impact on the quality perceptions of customers remaining in the market. For this reason, early shortages continue to be optimal but nonetheless fail to provide the firm with any significant profit advantage.

Next, we highlight the following properties of the firm's optimal policy.

COROLLARY 4. *If $\hat{q} < Q$ so that first-period supply shortages are optimal, then:*

- *p^* and n^* are strictly increasing in the product's mean quality, $\hat{q}$.*
- *p^* is strictly increasing, while n_1^* is strictly decreasing in customers' prior uncertainty over product quality, σ_p^2.*

Intuitively, the firm charges a relatively higher price for a product of higher quality, as well as when it faces customers who are more susceptible to persuasion from the reviews of early buyers
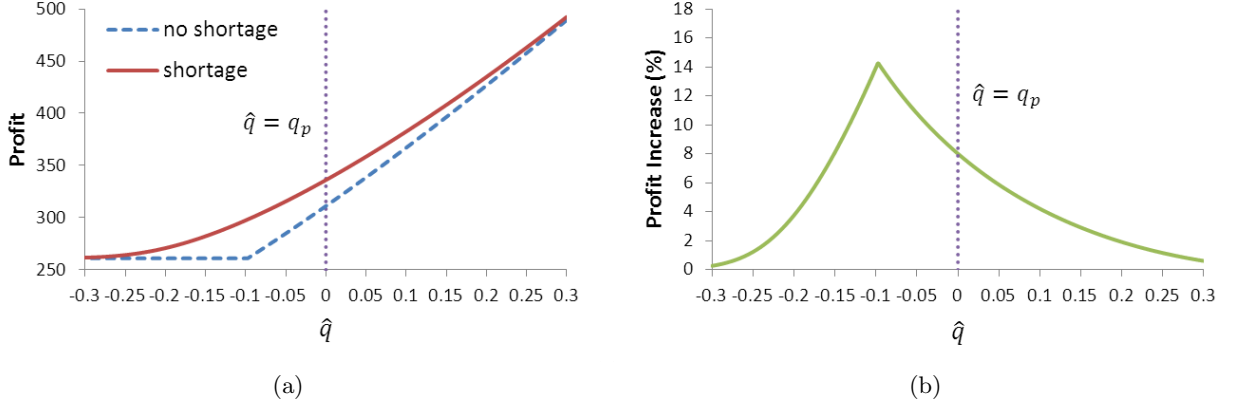
(a)   (b)

Figure 7   **(a) Optimal profit under imperfect social learning with and without early supply shortages; (b)**
Increase in profit from early supply shortage. Parameter values: $\bar{x} = \sigma_x = \sigma_q = 0.5$; $q_p = 0$, $\sigma_p = 0.1$; $a = 1$;
$N = 1000$; $\rho = 0.2$.

(i.e., who are *ex-ante* more uncertain of product quality). The firm's optimal early-availability decision is less intuitive. As product quality increases, the firm is able to achieve a high product rating accompanied by a relatively larger number of early reviews; in essence, the trade-off between review volume and content (described in Lemma 7) becomes progressively less severe. Moreover, as customers' uncertainty over quality increases, the number of first-period reviews becomes less important to potential customers, who tend to pay more attention to the product's average rating. This is reflected in the firm's early-availability decision, which focuses more on generating a higher product rating, rather than a larger number of reviews.

We conclude this section with a discussion of the implications of early supply shortages for total surplus and customer welfare. Recall that, under imperfect social learning, customers tend to over-estimate product quality. This phenomenon is amplified under early supply shortages, with the end result being increased overall product adoption. However, note that this increase in total sales (and therefore firm profit) is accompanied by a relatively larger number of purchases which result in negative post-purchase utility. Therefore, early supply shortages often lead to a decrease in total welfare (see Appendix C for numerical study). However, surprisingly, we also observe that early supply shortages may in fact generate an *increase* in customer surplus (e.g., see Figure 8b). In particular, early shortages may be accompanied by a lower price. As a result, even though a relatively larger number of customers make erroneous purchasing decisions in the presence of supply shortages, the majority of product buyers are left with increased surplus. In such cases, both firm profit, as well as customer surplus are increased in the presence of early supply shortages.

5.   Comparison with Dynamic Pricing

In the previous section, we demonstrated that under imperfect social learning, subject to certain conditions, the firm maximizes its profit by deliberately restricting supply in the first period. The
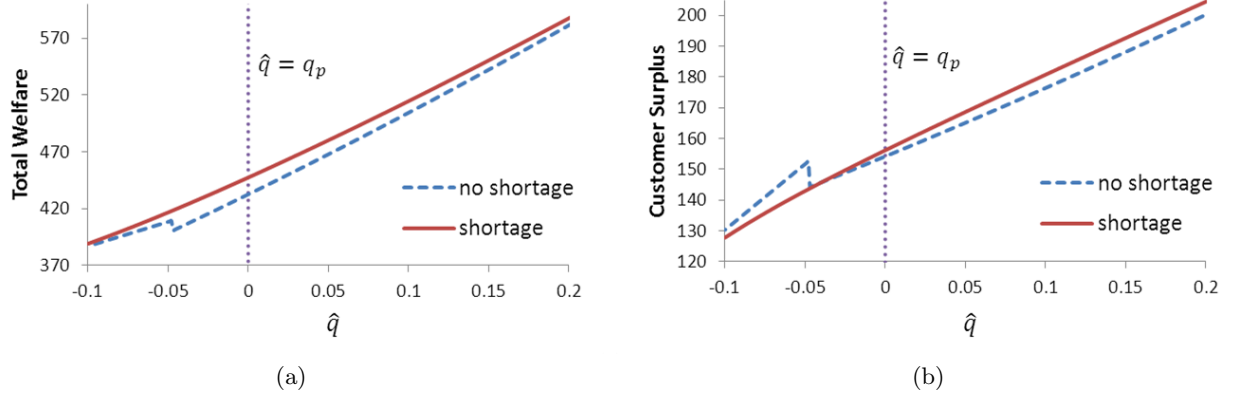
(a)   (b)

Figure 8 **Social welfare and customer surplus under imperfect social learning with and without early supply shortages, at different values of $\hat{q}$. Parameter values:** $\bar{x} = \sigma_x = \sigma_q = 0.5$; $q_p = 0$, $\sigma_p = 0.1$; $a = 1$; $N = 1000$; $\rho = 0.1$.

mechanism underlying the success of early supply shortages involves selection of the firm's highest prior valuation customers, whose reviews are likely to generate a more favorable social learning effect, than would the reviews resulting from unrestricted availability in the first period. In this respect, the firm's inventory policy may be viewed as an indirect method of price discrimination. It is then of interest to compare profits under this indirect method of price discrimination, and under direct price discrimination, i.e., dynamic pricing. In the analysis that follows, we assume that the market consists of myopic (i.e., non-forward-looking) customers. As a result, our estimates of profit under dynamic pricing are likely to be *over-estimates*, since they do not account for customers' propensity to delay their purchases in anticipation of a price reduction; nevertheless, the analysis is sufficient for the purposes of our comparison.

5.1. Perfect Social Learning

We consider first the firm's dynamic-pricing problem under perfect social learning, which may be expressed as follows.

$$\max_{\substack{p_1 \geq 0 \\ p_2 \geq 0}} \left(N p_1 \int_{\tau(p_1)}^{\infty} f(x)dx + N p_2 \int_{\min\{\tau(p_1), \tau(p_2) - \frac{\bar{q}_u(p_1) - q_p}{a + \rho \frac{\sigma_x}{\sigma_q}}\}}^{\tau(p_1)} f(x)dx \right). \tag{21}$$

In the above formulation, $\bar{q}_u(p_1)$ is specified as in Lemma 3. Note that n_1 is not considered as a decision variable in the above formulation since, from the result of Theorem 3, it is straightforward to verify that fulfilling all first-period demand is optimal. We may view the firm's first-period pricing decision as extracting a price premium from first period buyers, while at the same time controlling the amount of early information generated, in the manner described in §4.1.2. While it is difficult to obtain meaningful analytical expressions for the optimal dynamic-pricing policy, the following properties hold for the firm's optimal profit under perfect social learning.

PROPOSITION 3. *Under perfect social learning and dynamic pricing:*

- *Optimal profit is increasing in the product's mean quality, $\hat{q}$.*

- *When the product's mean quality, $\hat{q}$, is higher (lower) than customers' prior expectation, q_p, optimal profit is increasing (decreasing) in customers' prior uncertainty over mean quality, σ_p^2, and decreasing (increasing) in the variability in post-purchase perceptions of quality, σ_q^2.*

Unlike in the case of fixed pricing, Proposition 3 suggests that firm profit is now increasing in quality, $\hat{q}$, even when this is below customers' prior expectation, q_p. This occurs since, although the valuations of customers remaining in the market are decreased through social learning, the firm can now extract second-period profit by charging a lower price. However, as $\hat{q}$ decreases, social learning progressively limits the firm's ability to extract second-period profit. Moreover, the second point of Proposition 3 illustrates that when social learning is more influential (i.e., when σ_p^2 is large and/or σ_q^2 is small; see also Corollary 1), the firm benefits only when product quality is higher than customers' prior expectations.

We present the numerical results of Figure 9, in which the performance of the optimal fixed-pricing policy is compared to that of the optimal dynamic-pricing policy under perfect social learning. As the figure suggests, dynamic pricing is most advantageous when the product's mean quality is close to customers' prior expectations. When this is the case, we observe a significant gap between firm profit under dynamic and fixed pricing.
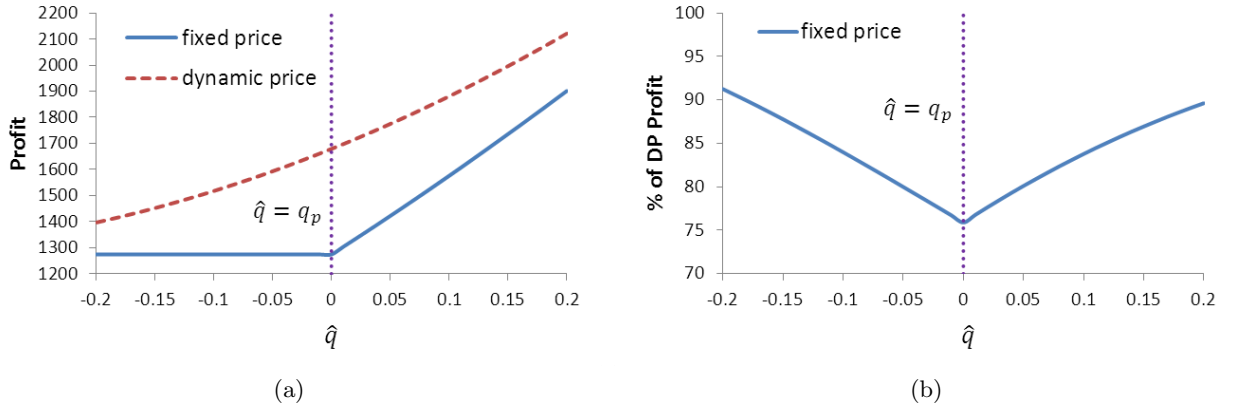

(a)   (b)

Figure 9   **Perfect social learning: (a) Optimal profit under fixed and dynamic pricing; (b) Percentage of dynamic pricing profit achieved through fixed pricing. Parameter values:** $\bar{x} = 0.5$, $\sigma_x = \sigma_q = 0.25$; $q_p = 0$, $\sigma_p = 0.1$; $a = 1$; $N = 5000$; $\rho = 0.2$.

5.2. Imperfect Social Learning

Next, we consider the firm's dynamic pricing problem under imperfect social learning. We may express the firm's problem as

$$\max_{\substack{p_1 \geq 0 \\ 0 \leq n_1 \leq D_1(p_1) \\ p_2 \geq 0}} \left(N p_1 \int_{c(p_1,n_1)}^{\infty} f(x)dx + N p_2 \int_{\min\{c(p_1,n_1),\tau(p_2) - \frac{\bar{q}u(p_1,n_1)-q_p}{a+\rho\frac{\sigma_x}{\sigma_q}}\}}^{c(p_1,n_1)} f(x)dx \right), \quad (22)$$

where $c(p_1, n_1)$ is given by (16) and $\bar{q}(p_1, n_1)$ by (19). We first establish the following result.

PROPOSITION 4. *Under dynamic pricing, it can never be optimal for the firm to deliberately restrict supply in the first period.*

The result of Proposition 4 implies that any solution to problem (22) must satisfy $n_1 = D_1(p_1)$. Recall that the incentive underlying a deliberate supply shortage in §4.2 was to ensure that only high-valuation customers obtain a unit in the first period. When the firm is free to price dynamically, it is strictly better off by charging a higher first-period price, rather than a lower first-period price combined with an early supply shortage. To see this, note that for any first-period price combined with a supply shortage, there exists a higher first-period price which achieves identical first period sales, but at a higher revenue-per-purchase.

The firm's optimal profit under imperfect social learning and dynamic pricing admits similar properties as those described in Proposition 3; we omit the corresponding discussion for brevity. Moreover, we provide the following result, which bounds the difference between profits achieved under fixed pricing combined with supply shortages and dynamic-pricing policies.

PROPOSITION 5. *Let π_s^* denote firm profit under the optimal fixed-pricing policy $\{p^*, n_1^*\}$, and define $\hat{p} := D_1^{-1}(n_1^*) \geq p^*$. Let π_d^* denote firm profit under an optimal dynamic pricing policy with $p_1^* \geq p_2^*$, where p_1 and p_2 denote first and second-period prices, respectively. Then the difference in profits between the two policies $\Delta \pi^* = \pi_d^* - \pi_s^*$ is bounded by*

$$(\hat{p} - p^*)n_1^* \leq \Delta \pi^* \leq (p_1^* - p_2^*)D_1(p_1^*). \tag{23}$$

The reasoning underlying the bounds in Proposition 5 is as follows. First, consider the lower bound. Under fixed pricing, firm profit is maximized at $\{p^*, n_1^*\}$. When employing dynamic pricing, the monopolist can at least charge $p_1 = \hat{p}$ and $p_2 = p^*$ and extract profit which exceeds its profit under fixed pricing by $(\hat{p} - p^*)n_1^*$; the excess profit is generated through the price premium attached to first period purchases. Next, consider the upper bound. Under dynamic pricing, the monopolist achieves maximum profits at $\{p_1^*, p_2^*\}$. When employing fixed pricing, the monopolist can at least set $p = p_2^*$ and $n_1 = D_1(p_1^*)$. In this way, the firm forgoes only the revenues extracted through the first period price premium under dynamic pricing, which is the expression of the upper bound.

Finally, Figure 10 compares profit under three policies: dynamic pricing (with unrestricted first-period availability), fixed pricing without early supply shortages, and fixed pricing with early supply shortages. Here, we observe that the comparison between fixed pricing without shortages and dynamic pricing is very similar to that observed under perfect social learning in Figure 9. However, surprisingly, our numerical experiments indicate that, not only do early supply shortages provide the firm with a profit advantage over unrestricted early supply, in the majority of cases

they allow the firm to closely mimic profits under dynamic pricing (see Appendix C for details). That is, we find that the majority of profit lost due to the firm's inability to price dynamically in the early stages of the selling season can in fact be retrieved through a fixed-pricing policy combined with an early supply shortage.
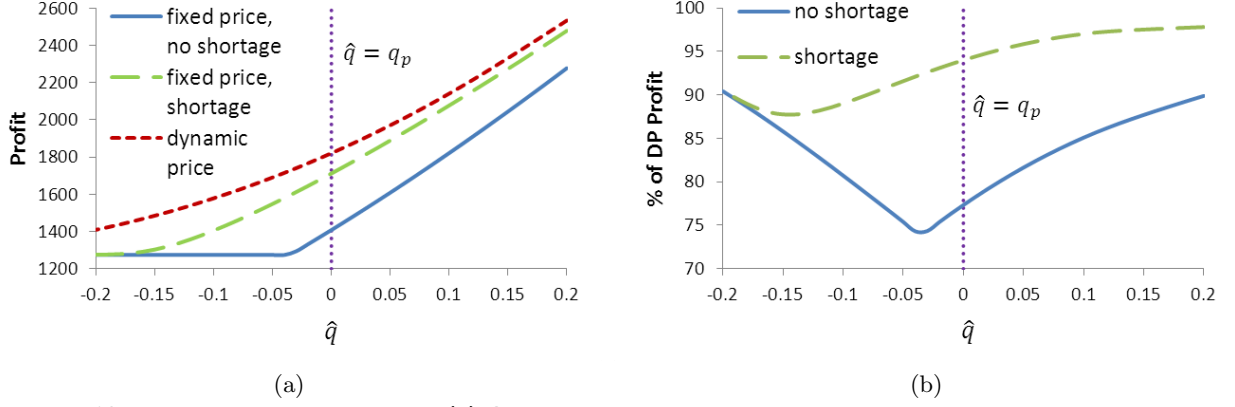

Figure 10   **Imperfect social learning: (a) Optimal profit under fixed pricing with and without early supply shortages, and dynamic pricing; (b) Percentage of dynamic pricing profit achieved through fixed pricing with and without early supply shortages. Parameter values:** $\bar{x} = 0.5$, $\sigma_x = \sigma_q = 0.25$; $q_p = 0$, $\sigma_p = 0.1$; $a = 1$; $N = 5000$; $\rho = 0.2$.

The above observation is all the more important when considering the potential detrimental effects of strategic customer behavior in the presence of dynamic pricing (e.g., Cachon and Swinney 2009, Liu and van Ryzin 2008, Mersereau and Zhang 2012), which is not accounted for in our analysis. With this in mind, we identify inventory management as an effective substitute for dynamic pricing, which also has the advantage of avoiding the effects of strategic customer behavior.

6.   Model Limitations and Future Work

In our study of the implications of social learning from early buyer reviews we have proceeded with a number of simplifying assumptions, aimed at isolating the desired insights in a complex setting. In this section, we highlight the limitations of our work and suggest potentially interesting avenues for future research.

6.1.   Learning Product Quality from Firm Actions

There is ample empirical evidence to suggest that customers utilize buyer reviews to infer product quality and make better-informed purchasing decisions (e.g., Chevalier and Mayzlin 2006); this specific mode of customer learning has been the focus of this paper. However, an alternative mode of customer learning, may be proposed based on the well-developed signalling literature; namely, learning product quality from firm actions (e.g., Milgrom and Roberts 1986, YU et al. 2013). Consider a model in which both modes of customer learning co-exist, there is a continuum

of firm/product quality types and customers are imperfectly informed. In a separating equilibrium, social learning is redundant (i.e., there is no need to learn from customer reviews), since quality is revealed perfectly by the firm's actions (see Mailath (1987) for a relevant analysis). However, in a pooling (or semi-pooling) equilibrium, social learning provides an additional source of information which can be utilized by customers to distinguish between quality types. While we have not explicitly accounted for signalling in this paper, our analysis remains relevant in the latter case and appears to be consistent with the prevalence of social learning from buyer reviews in real-world settings.

6.2. Delaying Purchase for Information

In the current paper, we have focused on the influence of *firm decisions* (i.e., pricing and inventory) on the social learning process. Another interesting and unexplored related topic is the influence of *strategic customer behavior* on the social learning process. Specifically, when dealing with new and innovative products, customers may choose to delay their purchasing decision until more information on product quality becomes available through product reviews (see also Acemoglu et al. 2013). If customers discount second-period utility, they face a trade-off between early consumption (in the first period) and a better-informed purchasing decision (in the second period). In the current analysis, we have implicitly assumed that the discount rate is sufficiently high so that customers do not strategically delay their purchasing decision. (This is consistent with anecdotal evidence of customers' eagerness to purchase new and innovative products early, e.g., see *Los Angeles Times* (2011).) Incorporating such behavior in the current model adds an additional layer of complexity in extracting insights, because, apart from firm decisions, such behavior has its own impact on the social learning process. To see this, note that if some customers choose to delay their purchase, both the amount (number of reviews) as well as the content (average product rating) of information generated in the first period is directly influenced; in turn, this should also be accounted for in customers' decision whether to strategically delay their purchasing decision. We note that the relevant equilibrium analysis is considerably complicated and warrants thorough treatment in isolation, before being embedded in a model with firm decisions; we therefore hope to addressed this topic in future work.

6.3. First-Period Supply Shortages under Alternative Model Assumptions

One of our goals in this paper has been to investigate the implications of social learning not only from a fully-rational perspective (perfect social learning), but also from a perspective which corresponds more closely to social learning in real-world settings (imperfect social learning). The optimality of early firm-induced supply shortages in the presence of imperfect social learning is an interesting phenomenon, which, as we discuss below, persists under alternative modeling assumptions.

- *Partial correction for early buyer preferences.* Under perfect social learning, potential customers account perfectly for the idiosyncratic preferences of early buyers and the influence of these preferences on buyer reviews, while under our model of imperfect social learning, customers learn as if preferences are uncorrelated with quality perceptions or, equivalently, as if any correlation has a negligible impact on the information conveyed through reviews. To verify that the shortage phenomenon persists even if customers correct *partially* for early-buyer preferences, we have conducted our analysis under an alternative mode of social learning which lies *between* perfect and imperfect, in which customers learn from reviews as if the value of ρ is ρ_b, where $0 < \rho_b < \rho$. The qualitative nature of our result on the optimality of early shortages is unaffected (proofs omitted for brevity).

- *Multi-dimensional (perfect) social learning.* The rationale behind restricting supply in the early stages of the selling season under imperfect social learning is to amplify the bias in the sample of reviews observed by potential customers, which in turn leads them to over-estimate product quality. The same rationale, that restricting supply leads customers to over-estimate product quality, may hold true even under *perfect social learning*; specifically, this may occur when customers are *ex-ante* uncertain over multiple model parameters (i.e., not just product quality), which they try to infer from buyer reviews. To illustrate, assume that customers employ Bayes' rule but hold probabilistic beliefs over both ρ and $\hat{q}$; early buyer reviews must then be used to update beliefs on both parameters simultaneously. In this case, for example, it may be difficult (i.e. take a very large sample) or even impossible (due to observationally equivalent states) to distinguish whether favorable reviews are generated by a combination of high $\hat{q}$ and low ρ, or high ρ and low $\hat{q}$. This implies that overcoming a potential bias in early buyer quality evaluations may not be possible for any finite number of reviews made available in the first period (see also Chamley (2004)). In Appendix D we provide a stylized numerical example which illustrates that customers may overestimate product quality even if they are perfect learners; a result which is consistent with Goeree et al. (2006).

6.4. Heterogeneously Informed Customers

We have assumed a customer population which is heterogeneous in preferences, but *ex-ante* homogeneously informed over product quality. That buyers are heterogeneous in their preferences and that this is reflected in their *ex-post* reviews is a well-documented fact in the empirical literature (e.g., Li and Hitt 2008). Moreover, while the assumption of homogeneous *ex-ante* information may be unrealistic in some settings, it allows us to focus on the main challenge faced by customers when engaging in (outcome-based) social learning from buyer reviews. Future work may formulate models in which customers are heterogeneous both in their preferences, as well as in their information endowments. However, we point out that under such an assumption, models would have to

incorporate both *action-based* as well as *outcome-based* social learning (see §2; to the best of our knowledge, this has not been attempted before in the literature). In particular, in contrast to the social learning model presented in this paper, purchasing decisions (actions) *in themselves* must now be treated as informative of product quality, apart from the *ex-post* reviews (outcomes) of the early buyers.

6.5. Multiple Periods

The current model has focused on operational decisions pertaining to the early stages of a product's life-cycle. In particular, we have partitioned the selling season into a first period of short time-length (introductory phase) and a second period which is a stylized representation of the subsequent selling horizon. We have done so in order to focus on the effects of social learning from *early* buyer reviews, which are presumably most influential in shaping the opinions of *ex-ante* uncertain customers remaining in the market (since these reviews constitute the first sources of information that become available after the product is launched). A more long-term view of the implications of social learning could subdivide the second period of our model into multiple subperiods, with customers learning from the reviews generated in each period. Under perfect social learning, as reviews accumulate in number, potential customers' expectation of product quality will steadily approach the product's true underlying quality. However, under imperfect social learning, the product's launch may be followed by periods in which product quality is over-estimated in the market, before expectations start to approach the product's true quality. A formal treatment of the multi-period setting may provide interesting insights on topics such as multi-period dynamic pricing and inventory control in the presence of social learning.

6.6. Sequential Customer Arrivals

We have assumed that the entire customer population enters the market before sales commence, and makes purchasing decisions in the first and second periods (e.g., as in Liu and van Ryzin (2008)). More realistically, we may assume that customers arrive over time (e.g., Su 2007). Sequential customer arrivals complicate the analysis but may enrich our model insights, in particular those pertaining to the launch of low-quality products. In the current model, because no new customers arrive in the second period, social learning for low-quality products has no detrimental effects for the firm, since customers who chose not to purchase in the first period simply do not change their mind in the second. However, if new customers enter the market in the second period − some of them endowed with an *ex-ante* willingness to pay that exceeds the product's price − and observe the available reviews, their purchasing decisions may be influenced, and this in turn should be taken into account by the firm at the beginning of the selling horizon.

7. Conclusion

This paper has examined the effects of social learning in the early stages of a product's life-cycle on a monopolist firm's pricing and inventory policy. We have presented a model of review-generation which incorporates preference heterogeneity in a manner consistent with empirical findings, and demonstrated that preference heterogeneity generates a non-trivial relationship between the firm's pricing decision and the information emitted from early buyer reviews. We have examined the implications of two modes of social learning from buyer reviews, namely, perfect (Bayesian) and imperfect (simplified) social learning.

For the case of perfect social learning we have shown that, in choosing price optimally, the firm considers both the revenue-generating role of price, as well as its informational role, namely, in controlling the amount of information released (number of buyer reviews) in the early stages of the selling season. We have further demonstrated that social learning from buyer reviews generally increases total welfare; however, this increase is completely absorbed by the firm at the expense of the customer population, which is left with decreased surplus. Surprisingly, we observe that even though customers are rational in learning from the reviews of others, they may have been better off by committing *not* to engage in social learning.

When social learning is imperfect, the informational role of price is more complicated and the firm is required to consider the impact of price both on the amount of information released, as well as the content (i.e., average product rating) of this information. We have shown that firms operating in the presence of imperfect social learning may have an incentive to deliberately induce early supply shortages, provided a suitable product allocation mechanism is in place (e.g., waiting lines). Interestingly, we observe that such early supply shortages allow the firm to approximate dynamic pricing outcomes while charging a fixed price. In this respect, we identify inventory management as a subtle and effective substitute for dynamic pricing, in cases in which the latter is difficult to implement (e.g., owing to fairness considerations and/or strategic customer behavior).

Our findings may also have implications beyond the scope of our model setting. For instance, the increase in firm profit associated with social learning from buyer reviews suggests firm investment in online reporting systems, as well as in maintaining the credibility of these systems. Moreover, our model indicates that firms may benefit from aiming their pre-launch advertising campaigns at customers who are predisposed to having a positive experience with their product. In terms of global product launches, firms which are aware of a significant pool of loyal customers in specific markets may wish to consider launching their product first in such favorable regions, in order to extract higher product ratings. Finally, our result on the optimality of early supply shortages suggests that early non-deliberate stock-outs may not be as detrimental as previously considered, provided the firm facilitates early customer self-selection to help drive future demand.

In conclusion, this paper demonstrates that social learning considerations may have a significant impact on how firms plan their operations and may further help explain phenomena that were previously less well-understood (e.g., in this paper, the success of early, short-term scarcity strategies). With the rise to prominence of social networking sites and the enabling role of the Internet in customer-to-customer communication, we believe that in many contexts the socially isolated customer paradigm has become a restrictive method of modeling customer decision-making. As such, incorporating social interactions between customers may enhance the relevance of future modeling work concerned with issues such as demand forecasting, innovation diffusion, product design, and marketing strategy, among others.

Appendix

A. Proofs

Proof of Lemma 1 Consider an individual buyer whose idiosyncratic component is x_i. The buyer's review is generated from a normal source $N(\hat{q} + \rho\frac{\sigma_q}{\sigma_x}(x_i - \bar{x}), \sigma_q^2(1-\rho^2))$. Since the customer's objective is to learn $\hat{q}$, he subtracts the deterministic bias term $\rho\frac{\sigma_q}{\sigma_x}(x_i - \bar{x})$ from the buyer's review, i.e. he "centers" the review around $\hat{q}$. Note that the variance (precision) of this signal is unaffected by x_i and therefore does not require any modification. Employing the standard Gaussian updating model (Chamley 2004), the Bayesian update rule is given by

$$q_u = q_p + \frac{\sigma_p^2}{\sigma_p^2 + \sigma_q^2(1-\rho^2)}\left(\left[q_i - \rho\frac{\sigma_q}{\sigma_x}(x_i - \bar{x})\right] - q_p\right), \tag{24}$$

where q_i is the buyer's review. The same procedure can be repeated, updating q_u sequentially after correcting each of the n_s available reviews. The end result of updating from n_s reviews is

$$q_u = q_p + \frac{n_s\sigma_p^2}{n_s\sigma_p^2 + \sigma_q^2(1-\rho^2)}\left(R_s - \rho\frac{\sigma_q}{\sigma_x}(\bar{x}_s - \bar{x}) - q_p\right), \tag{25}$$

where $R_s = \frac{1}{n_s}\sum_{i\in S} q_i$ is the observed average rating from the n_s reviews and $\bar{x}_s = \frac{1}{n_s}\sum_{i\in S} x_i$ is the average x_i of reviewers. The last expression can be rearranged to give the expression quoted in the Lemma.

Proof of Lemma 2 At price p, any customer with $v_{i1} = ax_i + q_p + \rho\frac{\sigma_q}{\sigma_x}(x_i - \bar{x}) \geq p$ obtains a unit. There are $N\bar{F}(\tau(p))$ such customers. Next, we have $\bar{R}(p) = E[q_i \mid x_i \geq \tau(p)]$ which, using (1), leads to

$$\bar{R}(p) = \hat{q} + \rho\frac{\sigma_q}{\sigma_x}\left(\frac{\int_{\tau(p)}^{+\infty} x f_1(x)\,dx}{\bar{F}(\tau(p))} - \bar{x}\right). \tag{26}$$

The result stated in the main text follows by making use of the following Lemma.

LEMMA 8. *Let $X \sim N(\bar{x}, \sigma_x)$ with $f(\cdot)$ and $F(\cdot)$ the corresponding density and distribution functions, respectively. Then $\frac{\int_z^{+\infty} x f(x)\,dx}{1-F(z)} = \bar{x} + \sigma_x^2 h(z)$, where $h(\cdot) = \frac{f(\cdot)}{1-F(\cdot)}$.*

Proof. Note that for the normal distribution $\frac{df}{dx} = -\left(\frac{x-\bar{x}}{\sigma_x^2}\right)f(x)$, therefore, $xf(x) = -\sigma_x^2\frac{df}{dx} + \bar{x}f(x)$. The result then follows by evaluating the integral $L(z) = \int_z^{+\infty}(x-z)f(x)\,dx$.

Finally, note that $\tau(p)$ is strictly increasing in p, so that $D_1(p)$ is strictly decreasing in p. Moreover, since $f(\cdot)$ is log-concave (normal) it admits a strictly increasing hazard ratio. It follows that $h(\tau(p))$ is strictly increasing in p and therefore $\bar{R}(p)$ is strictly increasing in p.

Proof of Lemma 3 Rewrite (2) as $q_u = q_p + \frac{n_s \sigma_p^2}{n_s \sigma_p^2 + \sigma_q^2(1-\rho^2)}\left(R_s - \rho\frac{\sigma_q}{\sigma_x}(\bar{x}_s - \bar{x}) - q_p\right)$. Now, $E[R_s] = \hat{q} + \rho\frac{\sigma_q}{\sigma_x}(\bar{x}_s - \bar{x})$ and $n_s = D_1(p)$, so that $\bar{q}_u = E[q_u] = q_p + \frac{D_1(p)\sigma_p^2}{D_1(p)\sigma_p^2 + \sigma_q^2(1-\rho^2)}(\hat{q} - q_p)$. Finally, note that $\frac{D_1(p)\sigma_p^2}{D_1(p)\sigma_p^2 + \sigma_q^2(1-\rho^2)}$ is strictly increasing in $D_1(p)$, which itself is strictly decreasing in p.

Proof of Theorem 1 Firm profits may be expressed as $\pi(p) = Np\max\left(\bar{F}(\tau(p)), \bar{F}\left(\tau(p) - \frac{\bar{q}_u(p) - q_p}{a + \rho\frac{\sigma_q}{\sigma_x}}\right)\right)$, where $\bar{F}(\tau(p)) \geq F\left(\tau(p) - \frac{\bar{q}_u(p) - q_p}{a + \rho\frac{\sigma_q}{\sigma_x}}\right)$ only if $\bar{q}_u(p) \leq q_p$ and no second period sales occur.

Case $\hat{q} \leq q_p$. From Lemma 3 we have that $\bar{q}_u \leq q_p$ for any price $p \in [0, +\infty)$. Therefore, in this case we have $\pi(p) = Np\bar{F}(\tau(p))$. The first order condition with respect to p yields $1 - \frac{p}{a + \rho\frac{\sigma_q}{\sigma_x}}\frac{f(\tau(p))}{\bar{F}(\tau(p))} = 0$. Note that the solution exists and is unique because F is a normal cdf and therefore admits a strictly increasing hazard ratio. Moreover, the optimal price and optimal profit are independent of $\hat{q}$, since $\tau(p)$ is independent of $\hat{q}$.

Case $\hat{q} > q_p$. From Lemma 3 we have that $\bar{q}_u > q_p$ for any price $p \in [0, +\infty)$. Therefore, in this case we have $\pi(p) = Np\bar{F}\left(\tau(p) - \frac{\bar{q}_u(p) - q_p}{a + \rho\frac{\sigma_q}{\sigma_x}}\right)$. The first order condition with respect to p yields $ph\left(\tau(p) - \frac{\bar{q}_u(p) - q_p}{a + \rho\frac{\sigma_q}{\sigma_x}}\right)\left(1 + \lambda(p)\cdot\frac{\hat{q} - q_p}{a + \rho\frac{\sigma_q}{\sigma_x}}\right) = a + \rho\frac{\sigma_q}{\sigma_x}$, where $\lambda(p)$ is as stated in the main text. Note that a solution exists since p and $h\left(\tau(p) - \frac{\bar{q}_u(p) - q_p}{a + \rho\frac{\sigma_q}{\sigma_x}}\right)$ are strictly increasing in p, and $1 + \lambda(p)\cdot\frac{\hat{q} - q_p}{a + \rho\frac{\sigma_q}{\sigma_x}} > 1 \ \forall \ p \in [0, +\infty)$. Finally, to see that firm profit is strictly increasing in $\hat{q}$, let $\hat{p} = \arg\max_p Np\bar{F}\left(\tau(p) - \frac{\bar{q}_u(p;\hat{q}) - q_p}{a + \rho\frac{\sigma_q}{\sigma_x}}\right)$ and note that $\bar{q}_u(\hat{p};\hat{q}) < \bar{q}_u(\hat{p};\hat{q} + \epsilon)$ for any $\epsilon > 0$. Therefore, $N\hat{p}\bar{F}\left(\tau(\hat{p}) - \frac{\bar{q}_u(\hat{p};\hat{q}) - q_p}{a + \rho\frac{\sigma_q}{\sigma_x}}\right) < N\hat{p}\bar{F}\left(\tau(\hat{p}) - \frac{\bar{q}_u(\hat{p};\hat{q} + \epsilon) - q_p}{a + \rho\frac{\sigma_q}{\sigma_x}}\right) \leq \max_p Np\bar{F}\left(\tau(p) - \frac{\bar{q}_u(p;\hat{q} + \epsilon) - q_p}{a + \rho\frac{\sigma_q}{\sigma_x}}\right)$ for any $\epsilon > 0$.

Proof of Corollary 1 Note first that from the firm's problem formulation in (7), it follows that first period profit is independent of the problem parameters listed in the Corollary. Consider an arbitrary price, $\hat{p}$, and let $\pi(\hat{p})$ denote firm profit at this price. Notice that (6) may be restated as

$$\bar{q}_u(\hat{p}) = q_p + \frac{1}{1 + \frac{\sigma_q^2(1-\rho^2)}{D_1(\hat{p})\sigma_p^2}}(\hat{q} - q_p),$$

and that $\bar{q}_u(\hat{p})$ is strictly increasing (strictly decreasing) in σ_p^2 (σ_q^2). From (7) it follows that second period profit is affected by changes in these parameters, and that total profit obeys $\frac{\partial\pi}{\partial\sigma_p^2} > 0$ and $\frac{\partial\pi}{\partial\sigma_q^2} < 0$. Since these properties hold at any arbitrary price, the result of the Corollary is evident.

Proof of Corollary 2 Define total welfare, W, as the sum of firm profit, π, and customer surplus, S, so that $W = \pi + S$. Under perfect social learning, we have $\pi^{SL} = Np_l^*\bar{F}(\tau(p_l^*))$ for $\hat{q} \leq q_p$ and $\pi^{SL} = Np_h^*\bar{F}(\tau(p_h^*) - \frac{\bar{q}_u(p_h^*) - q_p}{a + \rho\frac{\sigma_q}{\sigma_x}})$ for $\hat{q} > q_p$. In the absence of social learning, we have $\pi^N = Np_l^*\bar{F}(\tau(p_l^*))$. Next, under perfect social learning we have $S^{SL} = N \cdot \int_{\tau(p_l^*)}^{+\infty}(ax_i + q_i - p_l^*)f(x_i)dx_i$ for $\hat{q} \leq q_p$ and $S^{SL} = N \cdot \int_{\tau(p_h^*) - \frac{\bar{q}_u(p_h^*) - q_p}{a + \rho\frac{\sigma_q}{\sigma_x}}}^{+\infty}(ax_i + q_i - p_h^*)f(x_i)dx_i$ for $\hat{q} > q_p$. (Note that S^{SL} is always positive at p_h^* for perfect social learning.) In the absence of social learning, we have $S^N = N \cdot \int_{\tau(p_l^*)}^{+\infty}(ax_i + q_i - p_l^*)f(x_i)dx_i$. The result of the Corollary then follows by evaluating $\Delta W = W^{SL} - W^N = (S^{SL} + \pi^{SL}) - (S^N + \pi^N)$ and using (4).

Proof of Lemma 4 The first part of the Lemma follows similarly as for Lemma 3, using (3). For the unimodality result, note that $\frac{d\bar{q}_u}{dp} = \frac{\partial\bar{q}_u}{\partial\tau}\cdot\frac{\partial\tau}{\partial p} = \frac{1}{a + \rho_b\frac{\sigma_q}{\sigma_x}}\cdot\frac{\partial\bar{q}_u}{\partial\tau}$ and write (11) as

$$\bar{q}_u(\tau) = q_p + \frac{N\sigma_p^2}{N\bar{F}(\tau)\sigma_p^2 + \sigma_q^2}\left(\bar{F}(\tau)(\hat{q} - q_p) + \rho\frac{\sigma_q}{\sigma_x}\left(\int_\tau^{+\infty}xf_1(x)\,dx - \bar{F}(\tau)\bar{x}\right)\right). \tag{27}$$

Differentiating with respect to τ and noting that $\frac{\int_\tau^{+\infty} x f_1(x)\, dx}{\bar{F}(\tau)} = \tau + \mu(\tau)$ (see (29) below) we have after some manipulation

$$\frac{\partial \bar{q}_u}{\partial \tau} = \frac{N f(\tau) \sigma_p^2}{[N\bar{F}(\tau)\sigma_p^2 + \sigma_q^2]^2} \left(N\bar{F}(\tau)\sigma_p^2 \rho \frac{\sigma_q}{\sigma_x} \mu(\tau) - \sigma_q^2 \left[\hat{q} - q_p + \rho \frac{\sigma_q}{\sigma_x}(\tau - \bar{x}) \right] \right). \tag{28}$$

Note that in the last parenthesis, since $\rho > 0$, $\rho N\bar{F}(\tau)\mu(\tau)$ in the first term is strictly decreasing in τ while the second term is strictly increasing in τ. Therefore, we have $\frac{\partial \bar{q}_u}{\partial \tau} > 0$ for $\tau < \tau^*$, and $\frac{\partial \bar{q}_u}{\partial \tau} < 0$ otherwise, where τ^* is the unique solution of

$$\tau = \bar{x} - \frac{\sigma_x(\hat{q} - q_p)}{\sigma_q \rho} + \frac{N\bar{F}(\tau)\sigma_p^2 \mu(\tau)}{\sigma_q^2} \text{ and } \mu(\tau) = \frac{\int_\tau^\infty (x - \tau) f(x)\, dx}{\bar{F}(\tau)}. \tag{29}$$

The last equation can be simplified to give the expression quoted in the Lemma.

Proof of Theorem 2 Firm profit may be expressed as $\pi(p) = Np \max\left(\bar{F}(\tau(p)), \bar{F}\left(\tau(p) - \frac{\bar{q}_u(p) - q_p}{a + \rho \frac{\sigma_q}{\sigma_x}}\right) \right)$, where $\bar{F}(\tau(p)) \geq F\left(\tau(p) - \frac{\bar{q}_u(p) - q_p}{a + \rho \frac{\sigma_q}{\sigma_x}}\right)$ only if $\bar{q}_u(p) \leq q_p$ and no second period sales occur.

When $\hat{q} > q_p$, from Lemma 4 we have that $\bar{q}_u \geq q_p$ for any price $p \in [0, +\infty)$. Therefore, in this case we have $\pi(p) = Np\bar{F}\left(\tau(p) - \frac{\bar{q}_u(p) - q_p}{a + \rho \frac{\sigma_q}{\sigma_x}}\right)$. The first order condition with respect to p yields the expression stated in the second case of the Theorem in the main text. Note that at $p \to +\infty$ and $p = 0$ we have $\pi(p) = 0$, and that for any $p \in (0, +\infty)$ we have $\pi(p) > 0$ and $\pi(p)$ is continuously differentiable, so that a solution exists and satisfies the first order condition.

Next, consider $\hat{q} \leq q_p$ and note from (11) we have that at any price p, second period sales $(\bar{q}_u(p) > q_p)$ occur only if $\hat{q} > q_p - \rho \sigma_q \sigma_x h(\tau(p))$. From Theorem 1, we know that when no second period sales occur the optimal price is p_l^* given by the unique solution to $p h(\tau(p)) = a + \rho \frac{\sigma_q}{\sigma_x}$; therefore $p_{li}^* = p^* l$. However, we know that if $\hat{q} > q_p - \rho \sigma_q \sigma_x h(\tau(p_{li}^*)) = \phi$ then $\bar{q}_u(p_{li}^*) > q_p$. Therefore, we have for $\hat{q} > \phi$, $Np_{li}^* \bar{F}(\tau(p_{li}^*)) < Np_l^* \bar{F}(\tau(p_{li}^*) - \frac{\bar{q}_u(p_{li}^*) - q_p}{a + \rho \frac{\sigma_q}{\sigma_x}}) \leq \max_p Np\bar{F}(\tau(p) - \frac{\bar{q}_u(p) - q_p}{a + \rho_b \frac{\sigma_q}{\sigma_x}})$. It then follows that for $\hat{q} > \phi$ the optimal price is found similarly as in case $\hat{q} > q_p$ above, while otherwise the optimal price is p_{li}^* and no second period sales occur. Finally, to see that firm profit is strictly increasing in $\hat{q}$ for $\hat{q} > \phi$, let $\hat{p} = \arg\max_p Np\bar{F}(\tau(p) - \frac{\bar{q}_u(p) - q_p}{a + \rho \frac{\sigma_q}{\sigma_x}})$ and note that $N\hat{p}\bar{F}(\tau(\hat{p}) - \frac{\bar{q}_u(\hat{p};\hat{q}) - q_p}{a + \rho \frac{\sigma_q}{\sigma_x}}) < N\hat{p}\bar{F}(\tau(\hat{p}) - \frac{\bar{q}_u(\hat{p};\hat{q} + \epsilon) - q_p}{a + \rho \frac{\sigma_q}{\sigma_x}}) \leq \max_p Np\bar{F}(\tau(p) - \frac{\bar{q}_u(p;\hat{q} + \epsilon) - q_p}{a + \rho \frac{\sigma_q}{\sigma_x}})$ for any $\epsilon > 0$.

Proof of Corollary 3 Follows as per the proof of Corollary 1.

Proof of Proposition 1 Consider the three cases in turn. For $\hat{q} \leq \phi$ (note here that $\phi < q_p$ by Theorem 2), under both types of social learning, no additional sales are generated in the second period and the optimal price is p_l^* (recall $p_{li}^* = p_l^*$). Here, we have in both modes of social learning the same number of sales at the same price, resulting in $\pi_p^* = \pi_i^*$. Next consider case $\phi < \hat{q} \leq q_p$. Under perfect social learning profit is independent of quality at any $\hat{q} \leq q_p$. Under imperfect social learning we have profit independent of quality for $\hat{q} \leq \phi$ (equal to that under perfect social learning) and, from Theorem 2, strictly increasing profit for $\phi < \hat{q} \leq q_p$. We conclude that in this region of quality we have $\pi_p^* < \pi_i^*$.

Finally consider case $\hat{q} > q_p$. Denote customers' updated quality at price p under perfect (imperfect) social learning by $\bar{q}_{up}(p)$ $(\bar{q}_{ui}(p))$. Then a sufficient condition for $\pi_p^* < \pi_i^*$ is that $\bar{q}_{up}(p_h^*) < \bar{q}_{ui}(p_h^*)$, i.e., that the

updated quality estimate under imperfect social learning at price p_h^* is higher than that under perfect social learning, thereby generating higher sales at the same price. That is, we require

$$q_p + \frac{D_1(p_h^*)\sigma_p^2}{D_1(p_h^*)\sigma_p^2 + \sigma_q^2(1-\rho^2)}(\hat{q} - q_p) < q_p + \frac{D_1(p_h^*)\sigma_p^2}{D_1(p_h^*)\sigma_p^2 + \sigma_q^2}(\hat{q} + \rho\sigma_q\sigma_x h(\tau(p_h^*)) - q_p) \tag{30}$$

Rearranging, we have

$$\rho < \frac{[D_1(p_h^*)\sigma_p^2 + \sigma_q^2(1-\rho^2)]\sigma_x h(\tau(p_h^*))}{\sigma_q(\hat{q} - q_p)} \tag{31}$$

Next, note $D_1(p_h^*)\sigma_p^2 + \sigma_q^2(1-\rho^2) = \frac{D_1(p_h^*)(\hat{q}-q_p)\sigma_p^2}{\bar{q}_{up}(p_h^*)-q_p}$ from (6). Therefore, inequality (31) is satisfied if

$$\rho < \frac{D_1(p_h^*)\sigma_p^2\sigma_x h(\tau(p_h^*))}{(\bar{q}_{up}(p_h^*) - q_p)\sigma_q}. \tag{32}$$

Finally, since for any $\hat{q} > q_p$ we have $q_p < \bar{q}_u(p) < \hat{q} \; \forall p$, and noting that $D_1(p_h^*) = N\bar{F}(\tau(p_h^*))$ we have the sufficient condition on the market size N,

$$N > \frac{\rho\sigma_q(\hat{q} - q_p)}{\sigma_p^2\sigma_x f(\tau(p_h^*))}. \tag{33}$$

Proof of Proposition 2   See Appendix B.

Proof of Lemma 5   When $n_1 \geq D_1(p)$ units are made available, all customers with $v_{i1} \geq p$ obtain a unit, or equivalently all customers with $x_i \geq \tau(p)$. However, when $n_1 < D_1(p)$ units are available, only the n_1 highest v_{i1} (and therefore x_i) customers obtain the product. The threshold value of x_i in this case is $x_i \geq c(p, n_1)$, where $c(p, n_1)$ is as stated in the first case of the lemma. We then have $\bar{R}(p, n_1) = E[q_i \mid x_i \geq c]$ which, using (1), leads to (15). Finally, note that c is strictly decreasing in n_1 for $0 \leq n_1 \leq D_1(p)$ so that $\bar{R}$ is strictly increasing in n_1 for $0 \leq n_1 \leq D_1(p)$, because F is a normal cdf and therefore has an increasing hazard ratio.

Proof of Lemma 6   Follows as per Lemma 3.

Proof of Theorem 3   We consider cases of $\hat{q} > q_p$ and $\hat{q} \leq q_p$ in turn. For $\hat{q} > q_p$, we establish that any optimal solution must have $n_1^* = D_1(p^*)$, which in turn reduces the firm's problem to that considered in the second case of Theorem 1. Consider any arbitrary price $p \in [0, +\infty)$, and note that by inspection of (17) we know that $\bar{q}_u > q_p$ for any $n_1 > 0$. Next, notice from (18) that when $\bar{q}_u > q_p \; \forall n_1$ the firm's goal in choosing n_1 reduces to maximizing $\bar{q}_u$. From Lemma 6, we know that this occurs at the maximum possible n_1 which, at price p, is simply $D_1(p)$. Since this is true at any arbitrary price, it must hold at the profit-maximizing price, which can then be found by applying the second case of Theorem 1.

Now consider $\hat{q} \leq q_p$ and note that at any arbitrary price $p \in [0, +\infty)$, by inspection of (17), we know that $\bar{q}_u \leq q_p$ for any $n_1 > 0$. As a result, from (18) it is evident that no second period sales occur irrespective of the firm's choice of n_1. Therefore, the firm's problem in this case is equivalent to that considered in the first case of Theorem 1. Note here that a profit-equivalent approach is to set $n_1^* = 0$ in the first period, and sell the product only in the second period (at the price prescribed in Theorem 1).

Proof of Lemma 7   Follows directly from the proof of Lemma 4.

Proof of Theorem 4 The proof of the Theorem makes use of the following Proposition, which considers an exogenous price.

PROPOSITION 6. *Let the product's price be p. Under imperfect social learning, first period supply shortages are optimal if and only if $p < \bar{p}$, where $\bar{p} = \tau^{-1}(\tau^*)$. If $p < \bar{p}$, the unique optimal first period inventory allocation is $n_1^* = N\bar{F}(\tau^*)$, while if $p \geq \bar{p}$, it is optimal to fulfill all demand in the first period, $n_1^* = D_1(p)$.*

Proof. Note that by unimodality of $\bar{q}_u$ in Lemma 7, it follows that at $n_1 = N\bar{F}(\tau^*)$ we have $\bar{q}_u(p, n_1) > q_p$, since $\lim_{n \to 0} \bar{q}_u = q_p$. From (18), we have that, if $n_1 = N\bar{F}(\tau^*)$ is feasible at exogenously specified price p, then $n_1 = N\bar{F}(\tau^*)$ is optimal, since this is the inventory allocation which maximizes $\bar{q}_u$, $\bar{q}_u(p, n_1) > q_p$, and therefore maximizes overall profit. In turn, it is easy to verify that $n_1 = N\bar{F}(\tau^*)$ is feasible only when $p < \bar{p}$. Conversely, when $p \geq \bar{p}$ so that $n_1 = N\bar{F}(\tau^*)$ is not feasible, we know from unimodality of $\bar{q}_u$ that $\bar{q}_u$ is maximized at the largest possible n_1, which at price $p \geq \bar{p}$ is simply $n_1 = D_1(p)$, thus completing the proof.

From Proposition 6 we know that for $p < \bar{p}$, firm profits are maximized by setting $n_1^* = N\bar{F}(\tau^*)$, since $\bar{q}_u(p, n_1)$ is maximized at n_1^*. Let $\bar{q}_u^* = \bar{q}_u(p, n_1^*)$. For $p < \bar{p}$ the firm's problem is $\max_p Np\bar{F}(\frac{p - \bar{q}_u^* + \rho\frac{\sigma_q}{\sigma_x}\bar{x}}{a + \rho\frac{\sigma_q}{\sigma_x}})$, since any optimal solution must have $n_1 = N\bar{F}(\tau^*)$ and consequently, $\bar{q}_u = \bar{q}_u^*$. The unique solution to this problem is ξ, that is, among all prices $p \in [0, \bar{p}]$, ξ is optimal. Since we know that for $p > \bar{p}$ early shortages are not optimal, we must next evaluate whether profit at $\{\xi, n_1^*\}$ exceeds profit for all $\{p, D_1(p)\}$ for $p > \bar{p}$.

Note that since $\bar{q}$ is unimodal in n_1 it follows that for $p \geq \bar{p}$ we have $n_1^* = D_1(p)$ and $\bar{q}_u(p, D_1(p)) < \bar{q}_u^*$ because $D_1(p) < N\bar{F}(\tau^*)$. It follows that $\max_p Np\bar{F}(\frac{p - \bar{q}_u(p, D_1(p)) + \rho\frac{\sigma_q}{\sigma_x}\bar{x}}{a + \rho\frac{\sigma_q}{\sigma_x}}) < \max_p Np\bar{F}(\frac{p - \bar{q}_u^* + \rho\frac{\sigma_q}{\sigma_x}\bar{x}}{a + \rho\frac{\sigma_q}{\sigma_x}})$ for $p \geq \bar{p}$. Therefore, $Np\bar{F}(\frac{p - \bar{q}_u^* + \rho\frac{\sigma_q}{\sigma_x}\bar{x}}{a + \rho\frac{\sigma_q}{\sigma_x}})$ is equal to the firm's profit function for $p < \bar{p}$ and is an upper bound on firm profits for $p \geq \bar{p}$. It then follows that $\xi < \bar{p}$ is necessary and sufficient to guarantee that ξ is the globally optimal price. Conversely, if $\xi \geq \bar{p}$, then we have $p^* \in [\bar{p}, +\infty)$ and p^* can be found through the one-dimensional problem $\max_p Np\bar{F}(\frac{p - \bar{q}_u(p, D_1(p)) + \rho\frac{\sigma_q}{\sigma_x}\bar{x}}{a + \rho\frac{\sigma_q}{\sigma_x}})$ as is the case in Theorem 2.

We next show that the condition $\xi < \bar{p}$ holds true when $\hat{q} < Q$, for some threshold Q. First note that $\frac{\partial \bar{q}_u}{\partial \hat{q}} > 0$, and as a result $\frac{\partial \bar{q}_u^*}{\partial \hat{q}} > 0$. Therefore, we have that $h(\frac{p - \bar{q}_u^* + \rho\frac{\sigma_q}{\sigma_x}\bar{x}}{a + \rho\frac{\sigma_q}{\sigma_x}})$ is strictly decreasing in $\hat{q}$, which implies that ξ is strictly increasing in $\hat{q}$. Next, note that $\bar{p} = q_p + a\tau^* + \rho\frac{\sigma_q}{\sigma_x}(\tau^* - \bar{x})$. From (12), τ^* is the solution of

$$\tau - \bar{x} + \frac{\sigma_x(\hat{q} - q_p)}{\sigma_q(\rho - \rho_b)} - \frac{N\bar{F}(\tau)\sigma_p^2\mu(\tau)}{\sigma_q^2(1 - \rho_b^2)} = 0 \text{ where } \mu(\tau) = \frac{\int_\tau^\infty (x - \tau)f(x)\,dx}{\bar{F}(\tau)}. \tag{34}$$

Taking the total derivative with respect to $\hat{q}$ we have

$$\frac{\partial \tau}{\partial \hat{q}} + \frac{\sigma_x}{\sigma_q(\rho - \rho_b)} - \frac{\partial}{\partial \tau}\left(\frac{N\bar{F}(\tau)\sigma_p^2\mu(\tau)}{\sigma_q^2(1 - \rho_b^2)}\right)\frac{\partial \tau}{\partial \hat{q}} = 0 \tag{35}$$

Note that $\bar{F}(\tau) > 0$, $\frac{\partial \bar{F}}{\partial \tau} < 0$ and $\mu(\tau) > 0$, $\frac{\partial \mu}{\partial \tau} < 0$ ($\mu(\tau)$ is the "mean residual life" function and is decreasing in τ for the normal distribution). Therefore, we have $\frac{\partial \tau^*}{\partial \hat{q}} < 0$, which implies that $\bar{p}$ is strictly decreasing in $\hat{q}$. Since $\frac{\partial \xi}{\partial \hat{q}} > 0$ and $\frac{\partial \bar{p}}{\partial \hat{q}} < 0$, it follows that there exists Q such that $\xi < \bar{p}$ if and only if $\bar{q} < Q$.

Proof of Corollary 4 The properties of the optimal policy with respect to $\hat{q}$ follow directly from the properties of ξ and τ^* as proven in the proof of Theorem 4 above. The properties with respect to σ_p^2 follow by applying the implicit function theorem in a similar manner.

Proof of Proposition 3   We consider the two points of the Proposition in turn. First point: Let $\{p_1^*, p_2^*\}$ be the optimal dynamic pricing policy (i.e., solution to problem (21)) at quality $\hat{q}$, generating profit $\pi^*(\hat{q}) = \pi(p_1^*, p_2^*; \hat{q})$. There are two relevant cases. (a) $\tau(p_1^*) > \tau(p_2^*) - \frac{\bar{q}_u(p_1^*) - q_p}{a + \rho \frac{\sigma_q}{\sigma_x}}$; in this case, since $\frac{\partial \bar{q}_u}{\partial \hat{q}} > 0$, we have $\pi^*(\hat{q}) = \pi(p_1^*, p_2^*; \hat{q}) < \pi(p_1^*, p_2^*; \hat{q} + \epsilon) \leq \pi^*(\hat{q} + \epsilon)$ for any $\epsilon > 0$. (b) $\tau(p_1^*) \leq \tau(p_2^*) - \frac{\bar{q}_u(p_1^*) - q_p}{a + \rho \frac{\sigma_q}{\sigma_x}}$ (e.g., this occurs when $\hat{q}$ is extremely low); here we have $\pi^*(\hat{q}) = \pi^*(\hat{q} + \epsilon)$ and profit is independent of $\hat{q}$. Note that the possibility of case (b) results in profit being weakly, rather than strictly increasing, in quality.

Second point: When $\hat{q} > q_p$ ($\hat{q} < q_p$) we have $\frac{\partial \bar{q}_u}{\partial \sigma_p^2} > 0$ ($\frac{\partial \bar{q}_u}{\partial \sigma_p^2} < 0$) and $\frac{\partial \bar{q}_u}{\partial \sigma_q^2} < 0$ ($\frac{\partial \bar{q}_u}{\partial \sigma_q^2} > 0$). The result then follows as per the proof of the first point of the Proposition above.

Proof of Proposition 4   By contradiction. Assume the optimal policy (i.e., solution to problem (22)) ζ^*, $\zeta^* = \{p_1^*, n_1^*, p_2^*\}$ with $n_1^* < D_1(p_1^*)$. Profit under this policy is

$$\pi(p_1^*, n_1^*, p_2^*) = N p_1^* \int_{c(p_1^*, n_1^*)}^{\infty} f(x) dx + N p_2^* \int_{\min\{c(p_1^*, n_1^*), \tau(p_2^*) - \frac{\bar{q}_u(p_1^*, n_1^*) - q_p}{a + \rho \frac{\sigma_q}{\sigma_x}}\}}^{c(p_1^*, n_1^*)} f(x) dx, \tag{36}$$

Next, consider policy ψ, $\psi = \{\hat{p}_1, D_1(\hat{p}_1), p_2^*\}$ (i.e., dynamic pricing with unrestricted availability), where $\hat{p}_1$ is such that $\tau(\hat{p}_1) = c(p_1^*, n_1^*)$ (and therefore $D_1(\hat{p}_1) = n_1^*$), and note that this implies $\hat{p}_1 > p_1^*$. Profit under this policy is

$$\pi(\hat{p}_1, D_1(\hat{p}_1), p_2^*) = N \hat{p}_1 \int_{\tau(\hat{p}_1)}^{\infty} f(x) dx + N p_2^* \int_{\min\{\tau(\hat{p}_1), \tau(p_2^*) - \frac{\bar{q}_u(\hat{p}) - q_p}{a + \rho \frac{\sigma_q}{\sigma_x}}\}}^{\tau(\hat{p}_1)} f(x) dx, \tag{37}$$

$$> \pi(p_1^*, n_1^*, p_2^*), \tag{38}$$

where the last inequality holds because first period profit under policy ψ is strictly greater than under ζ^* (this is true because $\tau(\hat{p}_1) = c(p_1^*, n_1^*)$ and $\hat{p}_1 > p_1^*$), while second period profit is equal under the two policies (this is true because $\tau(\hat{p}_1) = c(p_1^*, n_1^*)$ and $\bar{q}_u(p_1^*, n_1^*) = \bar{q}_u(\hat{p})$). We conclude that ζ^* cannot be optimal, and that any dynamic pricing policy with restricted first period availability is strictly dominated by some dynamic pricing policy with unrestricted availability.

Proof of Proposition 5   Let $\pi_d(p_1, p_2)$ denote profits under dynamic pricing at $\{p_1, p_2\}$ and let $\pi_s(p, n_1)$ denote profits under fixed pricing at $\{p, n_1\}$. We have $\pi_d^* = N p_1^* \int_{\tau(p_1^*)}^{+\infty} f(x) dx + N p_2^* \int_{\min\{\tau(p_1^*), \tau(p_2^*) - \frac{\bar{q}_u(p_1^*) - q_p}{a + \rho \frac{\sigma_q}{\sigma_x}}\}}^{\tau(p_1^*)} f(x) dx$. Assume that the firm sets $p = p_2^*$ and $n_1 = D_1(p_1^*)$ in a fixed-pricing policy. Then it achieves profit $\pi_s = N p_2^* \int_{c(p_1^*, D_1(p_1^*))}^{+\infty} f(x) dx + N p_2^* \int_{\min\{c(p_1^*, D_1(p_1^*)), \tau(p_2^*) - \frac{\bar{q}_u(p_2^*, D_1(p_1^*)) - q_p}{a + \rho \frac{\sigma_q}{\sigma_x}}\}}^{c(p_1^*, D_1(p_1^*))} f(x) dx$. However, note that $c(p_1^*, D_1(p_1^*)) = \tau(p_1^*)$ and that $\bar{q}_u(p_2^*, D_1(p_1^*)) = \bar{q}_u(p_1^*)$. Therefore, $\pi_s(p_2^*, D_1(p_1^*)) = \pi_d^* - (p_1^* - p_2^*) D_1(p_1^*)$. Since $\pi_s^* \geq \pi_s(p_2^*, D_1(p_1^*))$, it follows that $\Delta \pi^* = \pi_d^* - \pi_s^* \leq (p_1^* - p_2^*) D_1(p_1^*)$, which is the upper bound of the Proposition.

Next, note that under the optimal fixed-pricing policy $\{p^*, n_1^*\}$ we have $\pi_s^* = N p^* \int_{c(p^*, n_1^*)}^{+\infty} f(x) dx + N p^* \int_{\min\{c(p^*, n_1^*), \tau(p^*) - \frac{\bar{q}_u(p^*, n_1^*) - q_p}{a + \rho \frac{\sigma_q}{\sigma_x}}\}}^{c(p^*, n_1^*)} f(x) dx$. Assume that the firm sets $p_1 = \hat{p}$ (as described in the Proposition) and $p_2 = p^*$. Then it achieves profit $\pi_d(\hat{p}, p^*) = N \hat{p} \int_{\tau(\hat{p})}^{+\infty} f(x) dx + N p^* \int_{\min\{\tau(\hat{p}), \tau(p^*) - \frac{\bar{q}_u(\hat{p}) - q_p}{a + \rho \frac{\sigma_q}{\sigma_x}}\}}^{\tau(\hat{p})} f(x) dx$. However, note that $\tau(\hat{p}) = c(p^*, n_1^*)$ and that $\bar{q}_u(\hat{p}) = \bar{q}_u(p^*, n_1^*)$. Therefore $\pi_d(\hat{p}, p^*) = \pi_s^* + (\hat{p} - p^*) n_1^*$. Finally, since $\pi_d^* \geq \pi_d(\hat{p}, p^*)$, it follows that $\Delta \pi^* = \pi_d^* - \pi_s^* \geq (\hat{p} - p^*) n_1^*$ which is the lower bound of the Proposition.

B. First-Period Waiting-Line Game

Much of the contents of this section is adapted from work in economics concerned with "rationing by waiting" (see Barzel (1974), Holt (1979) and Holt and Sherman (1982)). The approach taken is to view customers with $v_{i1} \geq p$ as participants in an auction, in which bids for the product are made in units of costly waiting-time. Both winners and losers in the auction pay the monetary-equivalent of their time-bids. For winners, this entails an extra cost over and above the product's monetary price, while for losers it entails a net loss.

Here, we state briefly the assumptions underlying the waiting-line game; see Holt (1979) for details. There are n agents (customers), competing for m prizes (units of the product) in the first period of our model. Agent i values purchase of the product at $u_i = v_i - p$, where $u_i \in (\underline{u}, \bar{u})$. Agents have the option of arriving earlier and forming a waiting-line. However, waiting time is costly at rate w_i for customer i, where we assume that u_i and w_i are related through $u_i = w(u_i)$. Customer payoffs in the event of a win, loss, or no participation (i.e., defer purchase to second period) are respectively

$$\Pi_i^w = u_i - w_i t_i - k w_i$$
$$\Pi_i^l = \gamma u_i - w_i t_i - k w_i$$
$$\Pi_i^n = \gamma u_i,$$

where k may represent the time required by customers to travel to the waiting line and t_i is the time in advance customer i *chooses* to arrive. For simplicity in exposition, we assume $k = 0$ (this has no significant bearing on our analysis). Moreover, γ represents a discount factor for second period purchases; we assume that $\gamma < 1$ so that customers prefer to consume the product sooner rather than later. Note that customers who fail to secure a unit in the first period are sure to secure a unit in the second period, as per the specification of our model. Without loss of generality, we express payoffs in units of time such that $\pi_i^w = \frac{\Pi_i^w}{w_i}$ etc.

$$\pi^w = a_i - t_i$$
$$\pi^l = \gamma a_i - t_i$$
$$\pi^d = \gamma a_i,$$

where $a_i = \frac{u_i}{w_i}$ is the "time value" of securing a unit in the first period for customer i. Since $u_i \in (\underline{u}, \bar{u})$, it follows that $a_i \in (\underline{a}, \bar{a})$.

The informational structure of the game is as follows. We assume that each customer knows his own a_i and believes that the a_i of his rivals are *iid* draws from the distribution function $G(\cdot)$, where $g(\cdot) > 0$ on some finite open interval. The information structure is symmetric.

The described auction can thus be analyzed as a non-cooperative game of incomplete information. According to Harsanyi (1967), a Nash equilibrium of a game of this type is characterized by a number $a^* \in [\underline{a}, \bar{a})$ and a strictly increasing function $t(a_i)$, $t(a_i) \geq 0$ for $a_i \in (a^*, \bar{a})$. The approach taken here is to assume that such a Nash equilibrium exists and to show by construction what it must be.

We first seek a^* which is the no participation cut-off value of a_i, i.e., customers with $a_i \in [\underline{a}, a^*)$ will defer their purchase to the second period. It is clear that individual i will win in the first period if his

chosen t_i exceeds the mth largest arrival chosen arrival time of his rivals. Since $t(a_i)$ is strictly increasing in equilibrium, this means that customer i wins if his a_i exceeds the mth largest a_i in the population. Let $f(\cdot)$ denote the density function of the order statistic of rank m among $n - 1$ independent draws from the distribution with density $g(\cdot)$. $F(a_i)$ is then the probability that customer i secures a unit in the first period. Therefore, the marginal customer a^* (i.e., who is indifferent between arriving at the time of product launch $t_i = 0$ and delaying purchase) is one for whom

$$F(a^*)(a^*) + [1 - F(a^*)](\gamma a^*) = \gamma a^*,$$

$$a^* = 0,$$

which implies that the marginal customer holds valuation for the product in the first period $v_i = p$.

Next, we seek $t(a_i)$ for $a_i \in (a^*, \bar{a})$. The more general result of equilibrium bidding strategies quoted in equation (9) in Holt (1979) can be adapted to our setting, yielding

$$t'(a_i) = (1 - \gamma) a_i f(a_i).$$

A specific $t(a_i)$ can then be found by using the boundary condition $\lim_{a_i \to a^*, a_i > a^*} t(a_i) = 0$. It can easily be verified that

$$t(a_i) = \int_{a^*}^{a_i} (1 - \gamma) x f(x) \, dx$$

satisfies the above conditions and verification that $t(a_i)$ generates a Nash equilibrium follows similarly as in the Appendix of Holt and Sherman (1982).

Proof of Proposition 2   Following the above discussion it is clear that as long as customers with higher v_{i1} are associated with higher a_i, customers arrive in equilibrium in descending order of their valuations, and *vice-versa*. We suppress subscript 1, and refer to first period valuations throughout. Let v_i and w_i be related through $w_i = w(v_i)$ and note that all first period customers have $v_i \geq p$. We have $a_i = \frac{v_i - p}{w(v_i)}$, and $\frac{da_i}{dv_i} = \frac{1}{w(v_i)} - \frac{(v_i - p)w'(v_i)}{w(v_i)^2}$. Customers with relatively higher v_i have relatively higher a_i and therefore arrive relatively earlier in equilibrium if $\frac{da_i}{dv_i} > 0$. This implies $\eta = \frac{(v_i - p)w'(v_i)}{w(v_i)} < 1$ as stated in the Proposition. Finally note that the latter condition is satisfied for homogeneous waiting costs per unit time, $w'(v_i) = 0$, as well as relatively higher valuation customers incurring relatively lower waiting costs $w'(v_i) < 0$.

C.   Numerical Experiments

We tested 1620 possible combinations of the following parameters (problem instances): $q_p = 0$, $\sigma_p \in \{0.05, 0.1, 0.15, 0.2\}$, $\hat{q} \in \{-0.2, -0.1, 0, 0.1, 0.2\}$, $\sigma_q \in \{0.2, 0.35, 0.5\}$, $\rho \in \{0.1, 0.2, 0.3\}$, $a = 1$, $N \in \{1000, 5000, 10000\}$, $\bar{x} = 0.5$, $\sigma_x \in \{0.2, 0.35, 0.5\}$. Below, we report results for the relevant sections of the main text (indicated in the parentheses).

Economic welfare with and without perfect social learning (§4.1.2)   Customer surplus increases (decreases) in 0% (40%) of instances, i.e., customer surplus decreases in 100% of instances with $\hat{q} > q_p$; firm profit increases (decreases) in 40% (0%) of instances. Total welfare increases (decreases) in 40% (0%) of instances.

Economic welfare with and without imperfect social learning (§4.1.3) Customer surplus increases (decreases) in 0% (63%) of instances, i.e., customer surplus decreases in 100% of instances with $\hat{q} > \phi$; firm profit increases (decreases) in 63% (0%) of instances. Total welfare increases (decreases) in 49% (14%) of instances.

Economic welfare with and without supply shortages under imperfect social learning (§4.2.3) Customer surplus increases (decreases) in 24% (76%) of instances; firm profit increases (decreases) in 88% (0%) of instances. Total welfare increases (decreases) in 40% (60%) of instances.

Fixed-pricing profit vs. dynamic-pricing profit under imperfect social learning (§5.2) The following table shows profit under fixed pricing with and without supply shortages, as a percentage of profit achieved under dynamic pricing. Values reported in the table indicate percentages of problem instances.

	% of dynamic-pricing profit			
	65-75	75-85	85-95	95-100
fixed pricing with shortages	5%	26%	38%	29%
fixed pricing without shortages	11%	52%	31%	4%

D. 2-D Perfect Social Learning Example

We use the following simplified version of our model to demonstrate that the principle underlying the optimality of early firm-induced shortages (i.e., that they lead customers to over-estimate quality) may persist under perfect social learning with multiple dimensions of uncertainty. In this example, we assume that potential customers are *ex-ante* uncertain about both ρ and $\hat{q}$, which they attempt to learn simultaneously from buyer reviews (all other model parameters are common knowledge). We model x_i, $\hat{q}$, q_i and ρ as taking on discrete values. There are three equiprobable x_i types in the population, $x_i \in X$, where $X = \{x_l, x_m, x_h\}$, and, for simplicity, the x_i of reviewers are observable as part of their review. We restrict $\hat{q}$ to two possible values, $\hat{q} \in \{l, h\}$, and likewise we assume $\rho \in \{-|k|, +|k|\}$. We define customers' prior beliefs over possible realizations of $\hat{q}$ as $\{p_2, p_3\}$ and over ρ as $\{p_0, p_1\}$. Therefore, $q_p = lp_2 + hp_3$ and $\rho_b = -|k|p_0 + |k|p_1$ are the prior expectations over $\hat{q}$ and ρ, respectively. Customer reviews q_i may take values $q_i \in Q$, where $Q = \{l^-, l, m, h, h^+\}$. To characterize the review-generating process we must specify the discrete set of probabilities

$$P_x(q_i \mid \hat{q}, \rho) \text{ for } x \in X, \ q_i \in Q, \ \hat{q} \in \{l, h\}, \ \rho \in \{-|k|, +|k|\}, \tag{39}$$

where the above is the probability that a customer of type x gives a review q_i when the true state of the world is $\{\hat{q}, \rho\}$ (note that we require $\sum_i P_x(q_i \mid \hat{q}, \rho) = 1 \ \forall x, \hat{q}, \rho$).

Bayes' rule dictates that after observing review q_i by a customer of (observable) type x_i, customers' updated joint belief over $\hat{q}$ and ρ is

$$P(\hat{q} = m, \rho = j \mid q_i, x_i) = \frac{P_{x_i}(q_i \mid \hat{q} = m, \rho = j) P(\hat{q} = m, \rho = j)}{\displaystyle\sum_{r \in \{l,h\}, t \in \{\rho^+, \rho^-\}} \Big(P_{x_i}(q_i \mid \hat{q} = r, \rho = t) P(\hat{q} = r, \rho = t) \Big)} \tag{40}$$

According to the above specification, we present the following example in which we have the true state of the world $\{\hat{q} = 2, \rho = +|k|\}$. Customer prior beliefs are given by $\{p_2 = 0.6, p_3 = 0.4\}$ and $\{p_0 = 0.5, p_1 = 0.5\}$. We

set $N = 90$, equally divided into the three customer types x_i. Moreover, for simplicity, we set $(x_l + x_h)/2 = x_m = 0$, leading to $k = 0.475$. A negative value for x_i is interpreted as an *ex-ante* idiosyncratic aversion for some product-attribute, which when coupled with a positive (negative) ρ gives rise to a bias that results, in expectation, in a lower (higher) than $\hat{q}$ review. The review generating-process is common knowledge and specified in the following table, for each x_i type.

Table 2   Review-generating process

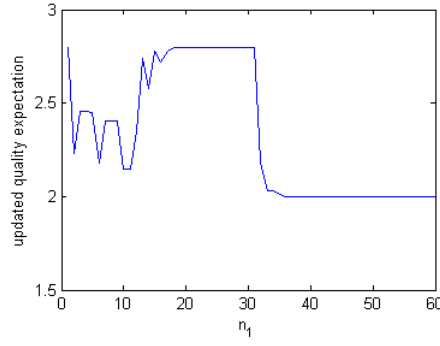|	$x_i = x_h$					$x_i = x_m$					$x_i = x_l$				
	$q_i=1$	$q_i=2$	$q_i=3$	$q_i=4$	$q_i=5$	$q_i=1$	$q_i=2$	$q_i=3$	$q_i=4$	$q_i=5$	$q_i=1$	$q_i=2$	$q_i=3$	$q_i=4$	$q_i=5$
$\hat{q}=2,\ \rho=+|k|$	0.15	0.175	0.35	0.175	0.15	0.325	0.35	0.175	0.1	0.05	0.675	0.175	0.1	0.04	0.01
$\hat{q}=4,\ \rho=+|k|$	0.01	0.04	0.1	0.175	0.675	0.05	0.1	0.175	0.35	0.325	0.15	0.175	0.35	0.175	0.15
$\hat{q}=2,\ \rho=-|k|$	0.675	0.175	0.1	0.04	0.01	0.325	0.35	0.175	0.1	0.05	0.15	0.175	0.35	0.175	0.15
$\hat{q}=4,\ \rho=-|k|$	0.15	0.175	0.35	0.175	0.15	0.05	0.1	0.175	0.35	0.325	0.01	0.04	0.1	0.175	0.675

Figure 11   Customers' posterior expectation over $\hat{q}$, after observing reviews from the n_1 highest prior valuation customers.

Figure 11 presents a typical plot of customers' updated expectation of product quality, $\hat{q}$, after observing reviews from the n_1 highest valuation customers (i.e., we assume a waiting-line allocation mechanism, as in the main paper). When the firm sells the product *only* to its highest x_i customers ($n_1 = 30$), customers' updated quality expectation is at a higher value (note that this value *over-estimates* product quality) than if the firm sells to its highest as well as medium x_i customers, which is the basic principle underlying the optimality of firm-induced supply shortages in our paper. Finally, we note that the above observation is further consistent with the social learning finding in Goeree et al. (2006) for a heterogeneous customer population, according to which "posterior beliefs may be concentrated on a wrong state for a long time, shifting suddenly to the correct state".

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