

# A Social-Learning Perspective on Inventory Decisions

Yiannos Papanastasiou · Nitin Bakshi · Nicos Savva

London Business School, Regent's Park, London NW1 4SA, UK

yiangosp@london.edu · nbakshi@london.edu · nsavva@london.edu

This paper highlights the role of inventory management in modulating social learning outcomes. A monopolist firm introduces a new experiential product of uncertain quality to a fixed population of consumers with heterogeneous preferences. Consumers who purchase the product early in the selling season report their ex post opinions of product quality; consumers remaining in the market observe these reports and update their beliefs over quality (via an empirically-motivated quasi-Bayesian learning rule) before deciding whether to purchase. Our analysis focuses on the firm's early quantity decision. We illustrate why, taking into account the interaction between its decisions and the social learning process, the firm may find it beneficial to deliberately under-supply the early demand for its product. We show that such a strategy is profitable provided excess demand is rationed *efficiently* and the product's quality is not too high relative to customers' prior expectations. We find that, in this setting, the impact of supply shortages on total welfare is generally positive; interestingly, shortages are in many cases beneficial not only for the firm (in terms of profit), but also for the consumer population (in terms of consumer surplus). Finally, we demonstrate that in the presence of social learning, optimizing inventory decisions may allow firms to approximate dynamic pricing profits while charging a fixed a price.

*History:* March 10, 2014

---

## 1. Introduction

The recent rise in popularity of online platforms hosting buyer-generated product reviews marks a fundamental change in consumers' approach to making purchasing decisions. While consumers previously relied on the firm's promotional efforts to learn about a new product's features and performance, they now consult the opinions of their peers, which are viewed as a more reliable source of product information (*Nielsen* 2012). This *social learning* (SL) process is particularly potent for experiential products (i.e., products whose consumption value is difficult to assess before use; e.g., media items and technological products), with an abundance of empirical evidence underlining the importance of peer reviews in shaping consumers' perceptions of product quality (e.g., Chevalier and Mayzlin 2006). Indeed, the impact of buyers' opinions on product adoption can be dramatic: in the movie industry, Dellarocas et al. (2007) find that early consumer reviews are instrumental in forecasting a new movie's long-term sales trajectory, while Moretti (2011) estimates that as much as 32% of box-office revenues can be attributed to SL.

Given its substantial implications for firm profit, it is a matter of strategic importance for firms to understand how their actions interact with the SL process and, if possible, to find ways to steer the process to their advantage. In recognition of this new challenge, a growing body of academic literature investigates how basic operational decisions interact with the SL process, and should perhaps be re-evaluated on the basis of this interaction. For instance, pricing decisions, which have traditionally been viewed in the light of extracting rents from consumers, are now being re-examined to better understand their role in the SL process (e.g., Ifrach et al. 2011, 2013). In a similar spirit, this paper is a first attempt to examine the SL dimension of inventory decisions. Traditionally, inventory decisions have been viewed in the light of balancing overage and underage costs; in this paper, we focus instead on the less well-understood role of inventory decisions in modulating SL outcomes. In particular, motivated by the commonly-observed phenomenon of early stock-outs of experiential products (e.g., Apple iPad, cinema tickets), we focus on quantity decisions pertaining to the early stages of a product's life-cycle.

We consider a stylized model of a monopolist firm selling a new experiential product to a fixed population of consumers. Consumers are heterogeneous in their preferences and hold a common *ex ante* belief over the product's unknown quality. Sales of the product occur over two periods and each consumer purchases at most one unit of the product. The first period is of short time-length and represents the product's *launch phase*. During the launch phase, customers who purchase a unit experience the product and report their opinions of product quality to the rest of the market (e.g., through online reviews). In the second period, consumers remaining in the market observe the reviews of the launch-phase buyers before deciding whether to purchase. The firm is assumed to have knowledge of the product's quality, and seeks to maximize its overall profit by choosing the product's price (which is constant across the two selling periods) and the launch-phase quantity.<sup>1</sup>

The focal point of our model is the SL process. For experiential products, an empirically-documented feature of consumer reviews is that individual buyers' opinions are shaped by both the product's intrinsic attributes, but also by the buyers' idiosyncratic tastes (e.g., some moviegoers may like the starring actor more than others). As a result, it may be difficult for potential buyers to accurately infer the product's quality by observing the reviews of their peers, especially if information on the reviewers' preferences is not readily available (e.g., as is typically the case in online settings). When faced with the task of learning from online consumer reviews, Li and Hitt (2008) establish empirically that consumers tend to ignore this feature of product reviews,

<sup>1</sup> The fixed-price assumption reflects settings in which price changes *early* in the selling season are not commonplace, not least for fairness considerations (Gilbert and Klemperer 2000). For example, cinema theaters typically charge a standard ticket price across time, while Apple on one occasion attempted to charge a price-premium in the launch phase of a new product, but, due to consumer backlash, was later forced to refund early purchases (*The New York Times* 2007).

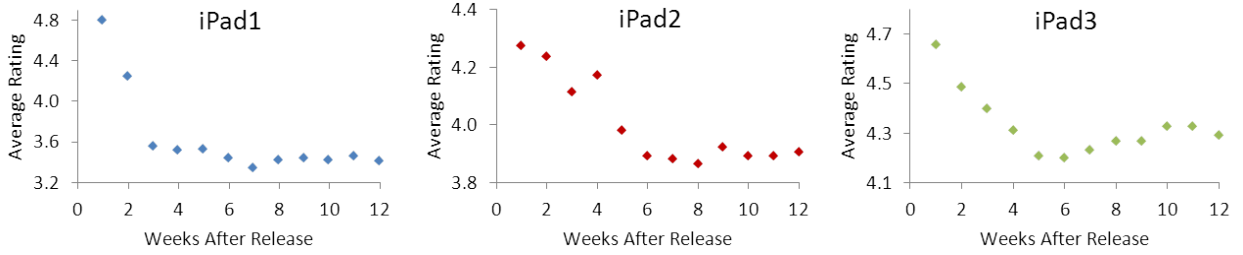
essentially engaging in SL *as if* their own preferences are, on average, aligned with those of the reviewers. To incorporate this behavioral dimension of SL into our model, we propose a quasi-Bayesian SL rule (see Camerer et al. (2003) and Rabin (2013) for a description of quasi-Bayesian models). Starting from Bayes' rule, this approach departs from fully-rational learning in a minimal and controlled manner, and is parameterized such that fully-rational learning (i.e., Bayes' rule) is nested as a special case. When learning is not fully rational, the heterogeneity in consumers' idiosyncratic preferences may give rise to biased SL outcomes.

Our model illustrates that the firm may be able to leverage such biases by deliberately under-supplying the launch-phase demand for its product. The rationale is as follows. Restricting product availability in the launch phase induces competition among consumers for the limited supply. Adopting the framework proposed by Holt and Sherman (1982), we demonstrate that (under certain intuitive conditions) the end result of this competition is *efficient rationing* of demand: consumers who secure a unit are those who value the product the highest. In terms of launch-phase revenue, it makes no difference for the firm *which* consumers receive a unit in the launch-phase; however, if launch-phase rationing is efficient, then the firm's second-period revenue may be increased through the effects of SL. Specifically, consumers who value the product highly *ex ante* may also be more likely to produce favorable reviews, by merit of their idiosyncratic preferences; in turn, a higher average rating may influence subsequent consumers' purchasing decisions and drive wider overall product adoption.

To illustrate the applicability of this rationale in a practical setting, consider the case of Apple product launches. Apple has created a culture of launch-phase stock-outs, thereby inducing its consumers into competition for units in the early stages of a product's life-cycle. The commonly observed-phenomenon of queue-formation leading up to product launches (e.g., *The Los Angeles Times* 2011) is symptomatic of this competition: the earlier a consumer joins the queue, the higher her chance of securing a unit. At the same time, the anecdotal evidence of Figure 1 suggests that early supply shortages have been accompanied by a higher average rating upon product introduction, as compared to the long-run average rating.<sup>2</sup> A similar figure is presented by Dellarocas et al. (2004) for the blockbuster movie "Spider-Man", using consumer ratings from Yahoo! Movies and IMDB. We do not suggest that this anecdotal evidence confirms our theory, however, it does appear to be consistent with the mechanism we propose.

<sup>2</sup> We make no claim that Apple deliberately induces supply shortages when launching new products; however, we point out that a high-ranking employee has admitted to such a strategy on at least one occasion:

"When we were planning the launch of the iPod across Europe, one of the important things we had to manage was to make sure we under-supplied the demand, so that we'd only roll it out almost in response to cities crying out for those iPods to become available – that's how we kept that kind of 'cachet' for the iPod in its early years. And we'd use extensive data research to understand what the relative strength of doing that in Rome versus Madrid would be." Andrew McGuinness (former European head of Apple's advertising agency), from the BBC documentary "Steve Jobs: Billion Dollar Hippy."



**Figure 1** Average rating (cumulative) for Apple iPad1, iPad2 and iPad3 following their introduction in March 2010, 2011 and 2012 respectively. The decreasing average rating suggests that early buyers rated the product more highly on average than subsequent buyers (data collected from Amazon.com via web crawler).

Expanding on the described rationale we show that, from a SL perspective, there exists a unique optimal number of launch-phase sales which optimally segments the market into higher-valuation “reviewers” and lower-valuation “learners.” This segmentation resolves a trade-off between a larger volume of reviews (achieved by selling more units in the launch-phase) and a higher average product rating (achieved by selling fewer units): while it is desirable for the firm to achieve a high average rating, in order to substantially influence prospective buyers’ opinions it is also necessary to generate a high enough volume of launch-phase reviews. From the perspective of profit-maximization in the presence of SL, the firm’s optimal pricing-and-quantity policy takes on a threshold structure. Specifically, we find that there exists a threshold on product quality such that: (i) if quality lies below this threshold, the firm maximizes profit by setting a relatively low price and restricting supply in the launch phase to the quantity which induces the optimal learning segmentation described above; (ii) if quality lies above the threshold, the firm maximizes profit by setting a higher price and ensuring ample product availability in the launch phase.

In this setting, we find that deliberate supply shortages increase total welfare. This is not ex ante obvious, because the firm’s decision to restrict supply generally leads consumers to overestimate product quality (a biased SL outcome), and therefore results in ex post negative experiences for some consumers (i.e., net losses in utility). We find that the firm’s gain in profit more than compensates for any losses in consumer surplus. More interestingly, we also observe that supply shortages in many cases constitute a “win-win” situation, resulting in an increase not only in the firm’s profit, but also in the consumers’ surplus; counter-intuitively, although preference heterogeneity in the consumer population is the main source of biases in the SL process, the consumer population is more likely to benefit from supply shortages when this heterogeneity is high.

Finally, we perform a comparison between fixed- and dynamic-pricing policies. Here, we first illustrate that early supply shortages can never be optimal when the firm has the ability to charge a different price during the launch-phase. We then perform a comparison between three types of

firm policy: (a) dynamic pricing, (b) fixed pricing with ample launch-phase availability, and (c) fixed-pricing with restricted launch-phase availability. We find that the revenue gap between fixed and dynamic pricing can be substantial, but is significantly reduced when the firm optimizes the launch-phase quantity decision. In many cases, by restricting supply in the launch phase, the firm is able to enjoy most of the benefits of dynamic pricing while charging a fixed price.

In summary, this paper makes three main contributions. First, we develop a modeling framework to investigate the role of quantity decisions in modulating the SL process, which in recent times has become a dominant force in shaping consumers' purchasing decisions. Second, using this framework, we propose a plausible rationale for the commonly-observed occurrence of launch-phase stock-outs for new experiential products. It is instructive to note that although alternative explanations for the profitability of launch-phase supply shortages do exist in the literature (e.g., "buying frenzies"; see DeGraba 1995), such explanations typically rely on dynamic-pricing arguments; by contrast, the mechanism described in this paper applies to settings in which price changes early in the selling season are not commonplace. Third, we demonstrate that, in settings characterized by SL, supply shortages may be beneficial for both the firm and the consumer population; furthermore, for the firm, restricting the launch-phase quantity may serve as a subtle and effective substitute for dynamic pricing.

## 2. Literature Review

This paper contributes to a growing stream of literature which investigates how social interactions between consumers interact with firms' operational decisions. Such interactions may take various forms. Hu et al. (2013) study a newsvendor setting in which two substitutable products are sold to consumers whose adoption decisions are directly influenced by those of their predecessors. Hu and Wang (2013) and Candogan et al. (2012) analyze pricing strategies when consumers experience positive consumption externalities in a social network. Tereyağoglu and Veeraraghavan (2012) consider the case of conspicuous consumers who use their purchases in order to advertise their social status. Veeraraghavan and Debo (2009) investigate how newly-arriving customers' choice of service-provider is influenced by the length of the queue (i.e., the choices of their peers) at each provider. Quite distinctly, the dimension of social interaction considered in this paper is that of peer-to-peer consumer learning from product reviews. Our work is therefore closely related to the *social learning* (SL) literature.

In the SL literature, there are two prevailing approaches to modeling peer-to-peer learning: *action-based* and *outcome-based*.<sup>3</sup> Action-based SL typically considers homogeneous agents who

<sup>3</sup> The majority of this work assumes that the full history of actions/outcomes is observable to the next decision-maker. For work that considers more complicated social networks see Acemoglu et al. (2011), Acemoglu et al. (2013), and references therein.

receive private *iid* signals regarding the true state of the world (e.g., product quality), and observe the actions of their predecessors (e.g., adoption decisions) before deciding on their own action. In such settings, the seminal papers by Banerjee (1992) and Bikhchandani et al. (1992) demonstrate that herding can be rational, but that rational herds may contain very little information. While earlier work on action-based SL focused on studying the properties of the learning process in the absence of strategic firm considerations, more recent work has incorporated firms' efforts to modulate the SL process through their decisions. Bose et al. (2006) consider a dynamic-pricing problem in which the firm uses price as a tool to screen the information transmitted to potential future buyers. For a fixed price, Liu and Schiraldi (2012) find that a sequential (rather than a simultaneous) product launch may benefit firms, since allowing a subset of markets to make observable purchasing decisions before the rest may trigger an adoption herd (see also Bhalla (2013), in which the setting of Liu and Schiraldi (2012) is extended to incorporate dynamic pricing). The SL process in our model differs substantially from that in the above purchasing actions *in themselves* are not informative, because customers share identical ex ante information (i.e., a public prior belief over quality). By contrast, SL occurs on the basis of buyers' post-purchase perceptions of product quality, which have become particularly visible and influential in the post-Internet era. In this respect, our work is more closely related to the outcome-base SL literature, and more specifically to word-of-mouth SL.

In word-of-mouth SL, agents typically learn from the ex post outcomes of their predecessors, rather than actions which reflect their ex ante information. A distinguishing theme in the word-of-mouth literature is the attempt to model peer-to-peer learning under cognitive limitations which are commonly encountered in real-world settings. For instance, it may be too cumbersome for agents to incorporate all available information in their decision process, but Ellison and Fudenberg (1993) show that even if consumers use fairly naive learning rules (i.e., "rules-of-thumb"), socially efficient outcomes may occur in the long run. Moreover, it may be costly or impossible for agents to gather all information relevant to their decision, but Ellison and Fudenberg (1995) find that socially efficient outcomes tend to occur when agents sample the experiences of only a few others. To capture cases in which agents may be likely to observe the experiences of a biased sample of predecessors (e.g., dissatisfied consumers may be more vocal than satisfied consumers), Banerjee and Fudenberg (2004) investigate how the learning process is affected under a range of exogenously specified sampling rules. The quasi-Bayesian model of SL developed in this paper also recognizes the challenges encountered by consumers in the learning process, however, our focus is more on how these challenges affect firm policy, rather than the learning process itself.

The recent rise in popularity of online reviewing platforms (e.g., epinions.com, Amazon.com) has brought issues pertaining to "electronic word-of-mouth" to the forefront of SL research. Besbes and

Scarsini (2013) investigate the informativeness of online product ratings when consumers' reports are based not only on their personal experience, but also on how this compares to the product's existing ratings. Given the ever-increasing empirical evidence on the association between buyer reviews and firm profits (e.g., Chevalier and Mayzlin 2006), a growing (but still scant) body of work studies how firms can *modulate* SL outcomes in online settings through their operational decisions. Ifrach et al. (2011) consider monopoly pricing when buyers report whether their experience was positive or negative, and subsequent customers learn from these reports according to an intuitive non-Bayesian rule (see also Bergemann and Välimäki (1997) who analyze optimal price paths in a duopolistic market). Li and Hitt (2010) investigate dynamic pricing when buyers' reviews are influenced by the price paid and subsequent consumers do not know the product's price history. Papanastasiou and Savva (2013) and YU et al. (2013) analyze dynamic-pricing policies when consumers may strategically delay their purchase in anticipation of product reviews. While the majority of existing research motivated by SL in online settings focuses on the interaction between pricing decisions and the SL process, our focus is instead on the modulating role of inventory management, with a specific focus on early quantity decisions; to the best of our knowledge, the association between quantities and SL outcomes is novel in the literature.

### 3. Model Description

We consider a monopolist firm offering a new experiential product to a fixed population of consumers of total mass  $N$ . The product is sold over two periods at a constant price  $p$ , and each consumer purchases at most one unit of the product throughout the selling season. The first period is of short time-length and represents the product's launch phase, while the second period is an aggregated representation of the subsequent selling horizon.

#### The Consumers

Customer  $i$ 's gross utility from consuming the product comprises two components,  $x_i$  and  $q_i$  (e.g., Villas-Boas 2004, Li and Hitt 2008). Component  $x_i$  represents utility derived from product features which are observable before purchase (e.g., product brand, film genre) and is known to the consumer ex ante. We assume that the distribution of  $x_i$  components in the population is Normal,  $N(\bar{x}, \sigma_x^2)$ , with density function denoted by  $f(\cdot)$ , distribution function  $F(\cdot)$  and  $\bar{F}(\cdot) := 1 - F(\cdot)$ . Component  $q_i$  represents utility derived from attributes which are unobservable before purchase (e.g., product usability, actors' performance) and is referred to generically as the product's quality for customer  $i$ ;  $q_i$  is ex ante unknown and is learned by the consumer only after purchasing and experiencing the product. We assume that the distribution of ex post quality perceptions is Normal  $N(\hat{q}, \sigma_q^2)$ , where  $\sigma_q$  captures the overall degree of heterogeneity in consumers' ex post evaluations of the product

and  $\hat{q}$  is the product's mean quality, which is *unobservable* to the consumers. The net utility from consuming the product for customer  $i$  in either period is defined simply by  $x_i + q_i - p$ .<sup>4</sup>

In the first period (i.e., the launch phase), all consumers hold a common prior belief over  $\hat{q}$ , which may be shaped, for example, by the firm's advertising efforts, media coverage and pre-release expert reviews.<sup>5</sup> This belief is expressed in our model through the Normal random variable  $\tilde{q}_p$ ,  $\tilde{q}_p \sim N(q_p, \sigma_p^2)$ ; in our analysis, we normalize  $q_p = 0$  without loss of generality. We assume that consumers are willing to purchase a unit provided their expected utility from doing so is positive. For simplicity in exposition of our results, we do not explicitly model strategic purchasing delays aimed at acquiring more information about the product (e.g., as in Papanastasiou and Savva (2013)). This approach is reasonable for the case of highly-anticipated products; furthermore, we note that accounting for such delays would have no significant bearing on our model insights (see §7.1). Consumers who purchase a unit in the first period report their ex post opinion of product quality  $q_i$  to the rest of the market through product reviews.<sup>6</sup>

In the second period, consumers remaining in the market observe the available reviews and update their belief over the product's mean quality from  $\tilde{q}_p$  to  $\tilde{q}_u$ , via the SL process described in the next section. Consumers whose expected utility from purchase is positive given their updated belief over product quality purchase a unit in the second period. Note that the described model implicitly assumes that all consumers in the market are aware of the product's existence from the beginning of the horizon (this assumption is most suitable for high-profile products) – our model focuses on the informative role of SL (reviews provide information on product performance), as opposed to its role in raising product awareness which features in product diffusion models (Bass 1969).

## Product Reviews and Social Learning

**Review-Generating Process** Each buyer's review consists of her ex post perception of product quality  $q_i$ . Consistent with empirical evidence (e.g., Dellarocas et al. 2004, Hu et al. 2009, Li and Hitt 2008), we suppose that customer  $i$ 's ex post perception of product quality is subject to random noise, but also to a systematic dependence on her idiosyncratic preferences. To illustrate, let  $x_i$  denote a customer's idiosyncratic preference for a specific film genre. Intuitively, the same film

<sup>4</sup> Since the first period of our model is short, we assume for simplicity that second-period utility is not discounted.

<sup>5</sup> Dellarocas et al. (2007) find that the correlation between the content of expert and consumer reviews is low, indicating that the two are viewed by consumers as complementary sources of information. The authors also find that consumer reviews produced shortly after the introduction of a new product are instrumental in predicting its long-term sales trajectory; this supports our focus on consumer reviews produced during the product's launch phase.

<sup>6</sup> Alternatively, we may assume that consumers report their ex post utility; the two approaches are equivalent in our model provided the distribution of consumer preferences is known to the consumers. We note also that it makes no qualitative difference whether all consumers report their experiences or whether each purchase generates a report with some fixed probability.



may be evaluated *differently* by cinema-goers who share an identical preference for the film’s genre (random noise), but also by cinema-goers who differ in their genre preferences (idiosyncratic). To capture this feature of product reviews, we assume that an individual’s ex post opinion of product quality  $q_i$  is correlated with her idiosyncratic preference component  $x_i$ , with correlation parameter  $\rho$ . In this case, customer  $i$ ’s review follows

$$q_i | x_i = \hat{q} + \rho \frac{\sigma_q}{\sigma_x} (x_i - \bar{x}) + \epsilon_i, \quad (1)$$

where  $\epsilon_i$  is zero-mean Gaussian noise of variance  $\sigma_\epsilon^2$ ,  $\sigma_\epsilon^2 = \sigma_q^2(1 - \rho^2)$ .<sup>7</sup> When  $\rho = 0$ , customer  $i$ ’s review is simply a random draw from the marginal distribution of ex post quality perceptions  $N(\hat{q}, \sigma_q^2)$ . By contrast, when  $\rho \neq 0$ , an individual’s review is a biased draw from  $N(\hat{q}, \sigma_q^2)$ , with the bias depending on the customer’s idiosyncratic preference component  $x_i$ . The sign of  $\rho$  determines the nature of the bias, while the magnitude determines its strength. When  $\rho > 0$ , customers with higher-than-average  $x_i$  are likely to perceive higher-than-average product quality (for instance, Mac users may enjoy their experience with the new iPad more than PC users). Conversely, when  $\rho < 0$ , customers with higher-than-average  $x_i$  are likely to perceive lower-than-average product quality (for instance, Mac users may be more sensitive to software glitches in the new iPad than PC users). Different consumers may react to their experience with a product in different ways, but Li and Hitt (2008) find that preferences are, on average, positively linked with quality perceptions (this is true in approximately 70% of cases they consider); similar evidence is reported in Dellarocas et al. (2004). Taking these findings into account, our analysis focuses on a positive correlation between preferences and ex post quality perceptions (in §7.2 we discuss alternative assumptions for the correlation parameter  $\rho$ ).

The above feature of product reviews clearly poses a challenge for consumers attempting to learn product quality from the reviews of their peers, because although these reviews contain information on  $\hat{q}$ , they also reflect opinions which are, to some degree, specific to the reviewers’ tastes. The rational approach to learning in this setting would be to “de-bias” each individual review by accounting for the preferences of the reviewer, replace the set of biased reviews with its unbiased counterpart, and then perform the belief update from  $\tilde{q}_p$  to  $\tilde{q}_u$  by applying Bayes’ rule (see Appendix C for description of this procedure).<sup>8</sup> Specifically, after observing a mass of  $n_1$  reviews, generated from buyers whose average idiosyncratic preference component is  $\bar{x}_1$ , and whose average

<sup>7</sup> Our choice of Normal marginal distributions for the consumers’ preferences and ex post quality perceptions is made for analytical convenience in incorporating correlation between the two.

<sup>8</sup> The choice of a Normal prior  $\tilde{q}_p$  permits use of the standard Gaussian updating model which is analytically tractable (e.g., see Chamley (2004)).

ex post opinion of product quality is  $R$ , the Bayesian posterior belief is Normal  $\tilde{q}_u \sim N(q_u, \sigma_u^2)$ , with

$$q_u = \frac{n_1 \sigma_p^2}{n_1 \sigma_p^2 + \sigma_q^2 (1 - \rho^2)} \left( R - \rho \sigma_q \frac{\bar{x}_1 - \bar{x}}{\sigma_x} \right) \quad \text{and} \quad \sigma_u^2 = \frac{\sigma_p^2 \sigma_q^2 (1 - \rho^2)}{n_1 \sigma_p^2 + \sigma_q^2 (1 - \rho^2)}.$$

In reality, the above process may be practically inconvenient, or indeed impossible, for two reasons: first, it requires a potentially large amount of cognitive effort; second, it requires perfect information on each reviewer's characteristics and preferences. Especially in online settings, which are characterized by large numbers of essentially anonymous reviews, both are likely to interfere with consumers' ability and/or capacity to learn in a fully-rational manner.

**A Quasi-Bayesian Model of Social Learning** Given these difficulties, how do consumers approach the learning process in real-world settings? Li and Hitt (2008) investigate this question empirically. They find that consumers take the simple approach of engaging in learning *as if*  $\rho = 0$ , essentially treating each review as representative of their own preferences. Whether this approach is the result of a conscious decision to avoid cognitive costs, a heuristic way of dealing with the lack of sufficient information, or simply an erroneous judgement of the review-generating process (or indeed any combination of the three) is unclear. Rather than investigating the causes of this behavior, our goal is to embed this finding into a theoretical model of SL and investigate its implications for the firm and the consumers.

Traditionally, behavioral biases in the SL process have been studied via intuitive heuristic learning rules (proposed as alternatives to Bayes' rule), which are typically custom-designed for the specific setting of interest. More recently, an alternative approach, termed *quasi-Bayesian*, has been gaining traction – Camerer et al. (2003) remark that they expect “the quasi-Bayesian view will quickly become the standard way of translating the cognitive psychology of judgement into a tractable alternative to Bayes' rule,” while Rabin (2013) discusses its advantages in detail and illustrates the approach through examples from the recent literature.

The main idea behind quasi-Bayesian learning models is to depart from Bayes' rule in a controlled manner, and only in the direction of the specific behavioral bias of interest. For example, consumers may have an erroneous understanding of the stochastic process which generates the outcomes they observe, but will otherwise behave as fully-rational agents and apply Bayes' rule in an internally consistent manner (see “warped-model Bayesians” in Rabin (2013); e.g., Barberis et al. (1998), Rabin (2002)). More generally, quasi-Bayesian models are designed to retain all the desirable features of their Bayesian counterparts which are not the specific focus of the behavioral modification. Furthermore, models are typically parameterized to nest Bayes' rule as a special case, allowing for a clear comparison between predictions of the fully-rational and behavioral learning models.

The quasi-Bayesian approach seems particularly suitable for our SL setting. In our model, we suppose that consumers, for one reason or another, operate under an erroneous internal model of the stochastic process which generates ex post quality perceptions and reviews (as suggested in Li and Hitt (2008)), but are otherwise fully rational agents. After observing a mass of  $n_1$  reviews, generated from buyers whose average idiosyncratic preference component is  $\bar{x}_1$ , and whose average ex post opinion of product quality is  $R$ , consumers in our model update their belief from  $\tilde{q}_p$  to  $\tilde{q}_u \sim N(q_u, \sigma_u^2)$ , where the posterior mean belief is given by

$$q_u = \frac{n_1 \sigma_p^2}{n_1 \sigma_p^2 + \sigma_q^2 (1 - \nu^2)} \left( R - \nu \sigma_q \frac{\bar{x}_1 - \bar{x}}{\sigma_x} \right).^9 \quad (2)$$

The above quasi-Bayesian learning rule nests both Bayesian updating (case  $\nu = \rho$ ) as well as the finding of Li and Hitt (2008) (case  $\nu = 0$ ). For the sake of brevity, in our analysis we will restrict  $\nu = 0$  and consider values of  $\rho$  in the region  $\rho \geq 0$ , with the understanding that cases of  $\nu = \rho > 0$  (i.e., Bayesian learning) are qualitatively equivalent to the case  $\nu = \rho = 0$ , and that all results presented for cases  $0 = \nu < \rho$  hold, again in a qualitative sense, for any  $0 < \nu < \rho$ . In this way, attention is focused on the simple learning rule

$$q_u = \frac{n_1 \sigma_p^2}{n_1 \sigma_p^2 + \sigma_q^2} R, \quad (3)$$

which retains a number of attractive Bayesian features. The updated mean belief is a weighted average between the prior mean  $q_p$  (recall  $q_p = 0$ ) and the average rating from product reviews  $R$ . The weight placed on  $R$  increases with the number of reviews  $n_1$  (a larger number of reviews renders the average rating more influential), with customers' prior uncertainty over quality  $\sigma_p^2$  (consumers are more susceptible to reviews when the ex ante quality uncertainty is large), and decreases with the noise in ex post product evaluations  $\sigma_q^2$  (noise in reviews renders SL less persuasive).

### The Firm

The firm is assumed to have knowledge of all model parameters through its market research, and seeks to maximize its total expected profit. To focus attention on the SL dimension of inventory decisions, we assume that the firm can produce and distribute any quantity of the product in either selling period and incurs zero marginal costs of production.

At the beginning of the selling season, the firm chooses the price of the product  $p$ , which remains unchanged across the two selling periods (dynamic pricing is considered subsequently in §6). Furthermore, at the beginning of each period the firm decides on the quantity of the product released during that period. The second-period quantity decision is trivial, since the firm will simply choose

<sup>9</sup> The posterior variance is given by  $\sigma_u^2 = \frac{\sigma_p^2 \sigma_q^2 (1 - \nu^2)}{n_1 \sigma_p^2 + \sigma_q^2 (1 - \nu^2)}$ , but is not important for our analysis because consumers are risk-neutral.

to fulfill all second-period demand (i.e., to ensure ample inventory). However, the first-period quantity decision is not as straightforward, owing to the interaction between product availability and SL outcomes – this interaction is the focus of our analysis. The described inventory setup is reminiscent of practical settings in which supply may be limited upon introduction of a new product, but product availability is virtually guaranteed for consumers at some subsequent point in the selling horizon (e.g., cinema screenings, Apple products).

Finally, an important assumption in our analysis is that the firm cannot signal product quality to the consumers through its actions. The justification for this approach in our model is twofold. First, it serves to isolate the effects of peer-to-peer consumer learning, which is the focus of our analysis (we note that the no-signaling assumption is commonly employed in existing SL literature; e.g., Sgroi (2002), Ifrach et al. (2011)). Second, this approach is supported by empirical studies of consumer behavior in settings characterized by SL. For example, Brown et al. (2012, 2013) find that consumers in the movie industry do not make quality inferences based on observable firm actions; moreover, the fact that consumer ratings have been shown to be an important driver of product adoption (e.g., Chevalier and Mayzlin 2006) lends further support since, had consumers been able to infer product quality by observing the firm’s actions, then the SL process would have been redundant.

## 4. Analysis

### 4.1. Preliminaries

According to the SL rule in (3), the number of reviews generated in the first period and the average rating of these reviews are sufficient statistics to characterize the *outcome* of the SL process, by which we mean the posterior mean belief  $q_u$  of consumers remaining in the market in the second period. As for the number of reviews  $n_1$ , this is by assumption equal to the number (i.e., mass) of first-period purchases. As for the average rating  $R$ , if first-period buyers have an average preference component of  $\bar{x}_1$ , then the average rating of their reviews follows

$$R = \hat{q} + \rho\sigma_q \frac{\bar{x}_1 - \bar{x}}{\sigma_x} + \epsilon_R, \quad (4)$$

where  $\epsilon_R$  denotes zero-mean Gaussian noise (i.e., sampling error). The firm’s problem may be expressed as

$$\max_{p, n_1 \leq D_1(p)} p(n_1 + E_R[D_2(p, n_1, R)]), \quad (5)$$

where  $D_1(p) := N\bar{F}(p)$  denotes first-period demand for the product at price  $p$  (note that any decision  $n_1 > D_1(p)$  may be mapped to the decision  $n_1 = D_1(p)$  without loss of generality, since any amount of inventory in excess of  $D_1(p)$  is leftover in the first period) and  $D_2$  is the number of

units demanded and sold in the second period, which depends on the SL outcome through  $n_1$  and  $R$  (the functional form of  $D_2$  is made precise in the subsequent sections' proofs). Because problem (5) is analytically intractable, in our analysis we approximate the firm's problem by

$$\max_{p, n_1 \leq D_1(p)} p(n_1 + D_2(p, n_1, \bar{R})), \quad (6)$$

where  $\bar{R} := E[R]$ ; in other words, we solve the firm's problem ignoring the random error term  $\epsilon_R$  in (4). As will become evident in our analysis, the firm's decisions are associated with a *systematic* relationship between  $n_1$  and  $R$  (rather than the effects of random sampling error), which is retained by the above simplification. Furthermore, in Appendix B we conduct extensive numerical experiments and demonstrate that the optimal policies and profit functions differ only marginally between problems (5) and (6) for all model parameters considered.

#### 4.2. Optimal Launch Quantities

As a first step in our analysis, we examine the role of the launch-quantity decision  $n_1$  in isolation, by solving the firm's problem for an arbitrary and exogenously specified product price  $p$ . The case of an exogenous price is practically relevant for cases in which the new product's price is constrained by previous versions of the product or consumers' price expectations (e.g., cinema-ticket prices do not tend to differ across movies) – even if the firm cannot price freely, the quantity of the product made available in the launch phase remains a decision variable (e.g., cinema theaters may choose how many of their screens to commit to the opening weekend of a specific movie).<sup>10</sup> As illustrated in this section, a key input to the optimal launch quantity is the mechanism employed by the firm to ration any excess demand in the first period.

**Proportional Rationing** It is natural to begin our discussion from the simple case of *proportional rationing* in the launch phase (e.g., as occurs when firms allocate units via a lottery system; see *ZDNet* (2012)). Under proportional rationing, each consumer willing to purchase the product in the first period has an equal chance of securing a unit. When this is the case, our first result suggests that the firm maximizes its profit by ensuring ample first-period product availability.

**LEMMA 1.** *For any price  $p$  and any  $\rho > 0$ , if first-period rationing is proportional then it is optimal for the firm to sell as many units as possible in the launch phase, that is,  $n_1^* = D_1(p)$ .*

All proofs are provided in Appendix A. In accordance with conventional wisdom, the more the firm sells in the first period, the higher its overall profit. The sub-optimality of any decision  $n_1 < D_1(p)$  for the firm is explained as follows. Under proportional rationing, the average preference component

<sup>10</sup> In the cinema-theater example, note that the decision  $n_1$  in our model represents “seats” rather than “screens.” The feasible set of  $n_1$  can be adjusted to represent screens (i.e., batches of seats); such an adjustment would have no qualitative bearing on our model insights.

of first-period buyers ( $\bar{x}_1$  in (4)) is fixed and independent of  $n_1$ . As a result, the average rating  $\bar{R}$  is also fixed and independent of  $n_1$  ( $\bar{R} = \hat{q} + \rho\sigma_q\sigma_x h(p)$ , where  $h(\cdot) := \frac{f(\cdot)}{1-F(\cdot)}$ ). In this case, the decision  $n_1$  modulates only the weight placed by consumers on  $\bar{R}$  when updating their beliefs (see (3)).

Depending on  $\hat{q}$  and the value of  $\rho$ , the SL process may either increase or decrease the valuations of consumers remaining in the market; however, in both cases it is optimal for the firm to sell as many units as possible in the first period. If  $\bar{R}(p) > q_p = 0$ , then the firm sells as many units as possible in the first period, because a larger number of “good” reviews will generate larger demand in the second period by increasing consumers’ posterior belief over quality. By contrast, if  $\bar{R}(p) < 0$ , then reviews are “bad” for the firm, because they decrease consumers’ perception of quality. Nevertheless, it is still optimal for the firm to sell as many units as possible in the first period, for if it does not, it loses sales in the second period through the effects of SL.

While the assumption of proportional rationing is simple and intuitive, we argue that the practical settings in which it arises are in fact limited, because proportional rationing entails some form of randomized product allocation. As we argue next, perhaps a more practically relevant assumption is that of efficient rationing.

**Efficient Rationing** Under *efficient rationing*, the product is allocated to consumers according to their idiosyncratic valuations, with priority given to high-valuation consumers (e.g., Denicolò and Garella 1999). Interestingly, efficient rationing arises naturally in many settings, as a result of competition among consumers for limited supply.

To illustrate, consider the commonly observed phenomenon of waiting-line formation, whereby consumers may “queue-up” in advance of product launch and receive a unit on a first-in-line first-served basis at the time of product launch (e.g., Apple product launches; *The Los Angeles Times* (2011)).<sup>11</sup> In this setting, customers who choose to join the waiting-line relatively earlier have a higher chance of securing a unit, but must spend a larger amount of costly time in the waiting line. As demonstrated by Holt and Sherman (1982), the composition of the waiting-line that forms is the equilibrium outcome of a game played between consumers, and depends on the relationship between consumers’ valuations for the product and their waiting cost per unit time. Let  $w_i$  denote customer  $i$ ’s waiting cost per unit time and let  $x_i$  and  $w_i$  be related through the mapping  $w_i = w(x_i)$ , where  $w : \mathbb{R} \mapsto \mathbb{R}$  and  $w(\cdot)$  is assumed to be differentiable across its domain. Adapting the analysis of Holt and Sherman (1982) to our setting (see Online Supplement) we state the following result.

<sup>11</sup> Consumers may become aware of short supply through firm announcements (*TechNewsWorld* 2006), media speculation (*The Washington Post* 2012), or prior experience with similar products.

LEMMA 2. If  $\eta = \frac{(x_i - p)w'(x_i)}{w(x_i)} < 1$ , then in equilibrium customers with  $x_i \geq p$  join the first-period waiting line in descending order of their  $x_i$  components.

Under the sufficient condition of Lemma 2, customers holding relatively higher ex ante valuations for the product will choose to queue-up relatively earlier; in this case, the composition of the waiting-line results in efficient rationing. We note that the sufficient condition of Lemma 2 subsumes both the commonly assumed case of homogeneous waiting costs, as well as the case of higher valuation customers incurring relatively lower waiting costs, which appears to be consistent with anecdotal evidence that those who join the waiting line first are high-valuation, low-cost-of-waiting customers such as students (UTV 2012) or speculators.<sup>12</sup>

The fact that competition among consumers results in efficient rationing makes no difference in terms of the firm's first-period revenues, but has a significant impact on the SL process. In particular, efficient rationing implies that the number of units made available in the first period interacts with the product's average rating  $\bar{R}$ : if the available number of units in the first period is  $n_1$  (where  $n_1 \leq D_1(p)$ ), this implies that reviews will be produced by the  $n_1$  highest-valuation consumers in the market. In turn, this means that  $n_1$  and  $\bar{R}$  are inversely related, since decreasing  $n_1$  results in a higher average preference component of first-period buyers (i.e.,  $\bar{x}_1$  in (4)). Defining

$$\bar{q}_u(n_1) = \frac{n_1 \sigma_p^2}{n_1 \sigma_p^2 + \sigma_q^2} \bar{R}(n_1)$$

as the posterior mean belief of consumer remaining in the market conditional on  $n_1$  units being sold in the first period, it is evident that the volume of reviews interacts with the average rating in a non-trivial way to shape consumers' updated perception of quality. Under an exogenous price  $p$ , the firm's optimal launch-quantity decision is characterized in Proposition 1.

PROPOSITION 1. For any  $\rho > 0$ , let  $\tau^*$  be the unique solution to the implicit equation

$$\tau = \bar{x} - \frac{\sigma_x \hat{q}}{\sigma_q \rho} + \frac{N \sigma_p^2}{\sigma_q^2} \int_{\tau}^{\infty} (x - \tau) f(x) dx. \quad (7)$$

Under efficient first-period rationing:

- If  $p < \tau^*$ , the unique optimal launch quantity is  $n_1^* = N \bar{F}(\tau^*) < D_1(p)$ .
- If  $p \geq \tau^*$ , the unique optimal launch quantity is  $n_1^* = D_1(p)$ .

The rationale underlying Proposition 1 is as follows. For an exogenous price, the firm's overall profit increases with  $\bar{q}_u$ . In the proposition's proof, we show that  $\bar{q}_u$  is unimodal in  $n_1$ : too few first-period sales generate a high average rating, but too few reviews to have a significant impact on consumers'

<sup>12</sup> The presence of speculators in the market does not affect the applicability of our model, since speculators may be viewed as an additional channel through which units are allocated to high-valuation customers; see Su (2010) for related work.

perceptions of quality; too many first-period sales generate an impactful but low average rating. The unimodal property of  $\bar{q}_u$  means that, from a SL perspective, there exists for the firm a unique optimal segmentation of the market into higher-valuation *reviewers* and lower-valuation *learners*. This segmentation strikes a balance between the volume and content of launch-phase reviews, and depends on setting characteristics such as the product's quality  $\hat{q}$  and consumers susceptibility to persuasion  $\sigma_p^2$ , among others. When the product's price is below the threshold  $\tau^*$ , the firm can achieve this segmentation through the effects of efficient rationing, by restricting the number of units made available in the first period to  $N\bar{F}(\tau^*)$  – this implies a deliberate under-supply of first-period demand for the product. By contrast, when the product's price is above the threshold  $\tau^*$  the firm is unable to sell to all desired reviewers in the first period, because some of them are initially unwilling to purchase owing to the product's high price; the best feasible approach in this case is to fulfill all demand in the first period.

## 5. Pricing and Launch Quantity

We now consider the more general problem of optimizing both the product's price and the launch-phase quantity simultaneously. Henceforth, we restrict our attention to efficient rationing, since the case of proportional rationing is both less interesting and arguably less practically relevant. The result of Proposition 1 can be further leveraged to gain insight into the firm's optimal pricing-and-quantity policy. Given a set of product and consumer characteristics, the firm may choose to employ one of two types of policy. The first type entails relatively lower prices combined with a restricted number of units made available in the launch phase (cases  $p < \tau^*$  in Proposition 1). The second type employs relatively higher prices combined with unlimited first-period product availability (cases  $p > \tau^*$  in Proposition 1). While the first type achieves a better SL outcome than the second, the second type generates higher revenue-per-purchase than the first. The following result characterizes the firm's optimal policy.

**PROPOSITION 2.** *For any  $\rho > 0$ , there exists a quality threshold  $Q(N, \bar{x}, \sigma_x, \sigma_p, \sigma_q)$ , such that:*

- *If the product's quality satisfies  $\hat{q} < Q$ , then the unique optimal pricing-quantity policy is described by  $\{p^*, n_1^*\} = \{\xi, m\}$ , where  $\xi$  satisfies  $\xi h(\xi - \bar{q}_u(m)) = 1$  and  $m = N\bar{F}(\tau^*) < D_1(\xi)$ .*
- *If the product's quality satisfies  $\hat{q} \geq Q$ , then the optimal pricing-quantity policy is described by  $\{p^*, n_1^*\} = \{\zeta, D_1(\zeta)\}$ , where  $\zeta = \arg \max p\bar{F}(p - \bar{q}_u(D_1(p)))$ .*

When product quality is below the threshold  $Q$ , the firm's chosen policy is oriented towards optimizing the SL process. The firm sets a price which is relatively affordable ( $p^* < \tau^*$ ), however, it does not allow all consumers who wish to purchase a unit in the launch phase to do so; instead, it uses its quantity decision to optimally segment the market into reviewers and learners. With the help of

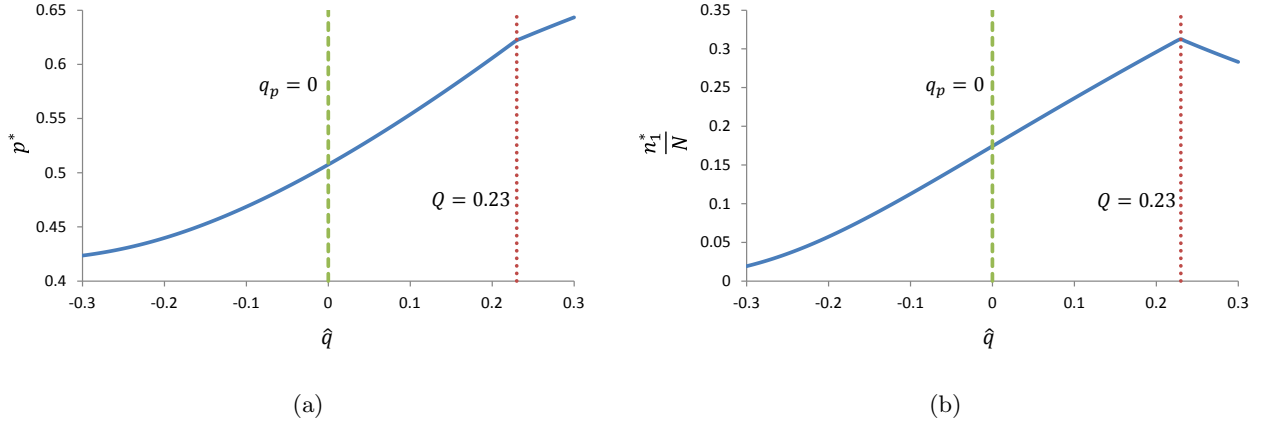


the optimized SL process, it then achieves a high volume of sales in the second period. By contrast, when quality is above the threshold  $Q$ , the firm takes a very different approach. It sets a high price ( $p^* > \tau^*$ ) so that only a small number of consumers are willing to purchase in the first period, and ensures ample product availability so that all such consumers obtain a unit in the launch phase. The firm then relies on the product's inherently high quality to drive second-period sales, rather than an optimized SL process (which would require commitment to lower revenue-per-purchase) – the result is a moderate number of high-revenue purchases.

The value of the threshold  $Q$  is difficult to characterize analytically; numerically, we observe that this is in general significantly higher than consumers' prior expectation of quality  $q_p$  (see Figure 2 for a typical example). Assuming that the product's true quality  $\hat{q}$  is drawn (by nature) from the distribution of consumers' prior belief, in most scenarios of product quality a restricted product launch will be beneficial for the firm from a SL perspective. For the remainder of this section, we focus our attention on the properties of restricted launch policies, starting with the following result.

**PROPOSITION 3.** *For  $\hat{q} < Q$ : (i) the optimal price  $p^*$  is strictly increasing in the product's quality  $\hat{q}$ , the correlation coefficient  $\rho$ , and customers' prior uncertainty  $\sigma_p$ ; (ii) the optimal launch quantity  $n_1^*$  is strictly increasing in  $\hat{q}$ , strictly decreasing in  $\sigma_p$ , and strictly increasing (decreasing) in  $\rho$  for  $\hat{q} \geq 0$  ( $\hat{q} < 0$ ); (iii) the firm's optimal profit  $\pi^*$  is strictly increasing in  $\hat{q}$ ,  $\rho$ ,  $\sigma_p$ .*

Intuitively, the firm will charge a higher price for products of higher quality, products for which high-valuation consumers are highly biased in their opinions, and products for which subsequent consumers are more uncertain of quality, and therefore more susceptible to the opinions of the early buyers. The properties of the optimal launch quantity are less intuitive, and are associated with the optimal segmentation of the market described in §4.2. For a fixed  $n_1$ , an increase in either  $\hat{q}$  or  $\rho$  results in an increased average rating  $\bar{R}$  (see (4)). Interestingly, increases in the two parameters call for different types of adjustment in the firm's launch quantity: as  $\hat{q}$  increases, the firm always prefers to increase the launch quantity so that consumers place more weight on the average rating when performing their update; by contrast, as  $\rho$  increases, the firm's adjustment of the launch quantity is contingent on the product's quality: if quality is higher than consumers' prior expectations, then the firm increases the launch quantity; if quality is lower than consumers' prior expectations, then the firm reduces the launch quantity with the goal of extracting more favorable review content  $\bar{R}$ . Finally, the more uncertain consumers are about the product's quality, the fewer the units that are made available by the firm in the launch phase; in this case, just a small number of reviews is enough to significantly influence consumers' opinions of the product, and the firm therefore chooses to focus on review content rather than volume.



**Figure 2** Optimal pricing-quantity policies at different value of product quality  $\hat{q}$ : (a) Optimal price  $p^*$  (b) Optimal launch quantity (normalized by total market size)  $\frac{n_1^*}{N}$ . Parameter values:  $N = 1000$ ,  $\bar{x} = 0.5$ ,  $\sigma_x = 0.25$ ,  $q_p = 0$ ,  $\sigma_p = 0.1$ ,  $\sigma_q = 1$ ,  $\rho = 0.15$ .

Proposition 3 also allows for a comparison of the firm's optimal policy and profit in general cases of our model setting against two benchmarks cases. The first is one in which consumers do not engage in SL; in our model, this case is operationally equivalent to the case  $\sigma_p \rightarrow 0$ , because this entails that consumers' updated mean belief in the second period is equal to the prior mean belief (see (3)). The second is one in which consumers' post-purchase opinions are unbiased (i.e., are not influenced by their preferences); this corresponds to the case  $\rho = 0$  in our general model (note that this benchmark is also qualitatively equivalent to the case in which reviews remain biased, but consumer learning is fully-Bayesian, i.e.,  $\nu = \rho > 0$  in (2)). In comparison with both benchmark cases, we find that the firm in the general case of our model (a) sets a higher price, (b) makes fewer units available in the launch phase, and (c) achieves higher overall profit.

More generally, we point out that the firm's profit in the absence of SL relies exclusively on the belief with which consumers enter the market. By contrast, in the presence of SL the firm's profit depends less on this prior belief, and more on the SL process. This observation marks a more general shift in the activities of the modern-day firm: while in times past firms focused more on promotional efforts aimed towards influence consumers' beliefs (i.e., consumers' prior when there is no SL in our model), there is now increasing investment in promoting and optimizing the SL process (e.g., KIA Motors have recently launched an advertising campaign aimed towards promoting the reviews of satisfied customers; see "Kia Reviews and Recommendations").

To conclude this section, we consider the implications of induced supply shortages for consumer surplus and total welfare. By restricting supply in the launch phase, the firm leverages the idiosyncratic preferences of early buyers to generate a more favorable SL effect, thus increasing overall product adoption and profit. Importantly, supply shortages amplify the bias in review content (this manifests as a higher average rating), which translates into erroneous customer decisions in

the second period: had the information contained in product reviews been unbiased, then fewer consumers would have chosen to purchase in the second period. Presumably, policies involving supply shortages therefore result in a decrease in the consumers' surplus; furthermore, if the loss in consumer surplus is larger than the firm's gain in profit, then supply shortages will be total-welfare-decreasing.

To investigate, we compare consumer surplus and total welfare when the firm employs the policy described in Proposition 2 versus that when the firm chooses the product's price, but satisfies all first-period demand (note that the chosen prices and quantities will generally differ between the two policy types). In Table 1, we present the difference in total welfare ( $\Delta W$ ) and consumer surplus ( $\Delta S$ ) between the two policy types, for various combinations of our model parameters; a positive difference indicates that  $W$  or  $S$  is higher under policies involving supply shortages. There are two main observations.

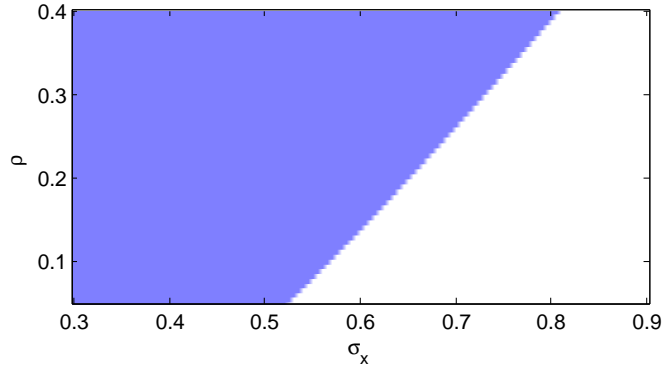
First, the impact of policies involving supply shortages on total welfare is positive (i.e.,  $\Delta W > 0$ ) under all parameter combinations. This implies that any losses in consumer surplus, which occur as a result of the firm's decision to manipulate the SL process, are more than compensated for by the increase in the firm's profit. Second, surprisingly, we observe that supply shortages in many cases constitute a "win-win" situation; that is, shortages often generate an increase not only in the firm's profit, but also in the consumers' surplus (this is increasingly the case as product quality increases). A closer look (see Figure 3) reveals that shortages are beneficial for both the firm and the consumers when the correlation  $\rho$  is low and the heterogeneity in consumers' preference  $\sigma_x$  is high. When  $\rho$  is low, the SL rule employed by consumers approaches Bayesian updating, therefore resulting in fewer erroneous decisions and fewer ex post negative outcomes. But even when  $\rho$  is relatively high, we observe that consumers may be better off under shortage policies provided they are sufficiently heterogeneous in their preferences – this is unexpected, since preference heterogeneity is a main contributor to the bias in the SL process.

The reasoning here is as follows. When the firm fulfills all demand in the launch phase, it typically chooses a relatively high price: the pricing decision represents a compromise between achieving a "good" SL effect (by charging a high price and therefore selling to relatively high-valuation consumers in the first period) and keeping the price low enough to drive a satisfactory amount of overall sales. By contrast, when the firm optimizes the launch-phase quantity decision, the pricing decision is decoupled from the SL process: the firm modulates the SL outcome through the quantity decision, and sets the price at a lower level which favours wider product adoption. In the latter case, although supply shortages amplify the bias in the review content, at the same time they may leave consumers who purchase the product with higher, on average, surplus, despite the fact that some consumers experience ex post negative outcomes. The consumers population is therefore

$\hat{q}$	-0.15	-0.1	-0.05	0	0.05	0.1	0.15
$\Delta W > 0$	100%	100%	100%	100%	100%	100%	100%
$\Delta S > 0$	3%	10%	22%	37%	48%	60%	72%

**Table 1** Impact of supply shortages on Total Welfare and Consumer Surplus:  $\Delta W > 0$  represents an increase in Total Welfare,  $\Delta S > 0$  represents an increase in Consumer Surplus. Scenario parameters:

$N \in \{1000, 3000, 5000, 7000\}$ ;  $q_p = 0$ ,  $\sigma_p = 0.1$ ,  $\hat{q} \in \{-0.15, -0.1, -0.05, 0, 0.05, 0.1, 0.15\}$ ;  $\sigma_x \in \{0.4, 0.6, 0.8\}$ ,  $\sigma_q = 0.6$ ,  $\bar{x} = 0.5$ ;  $\rho \in \{0.1, 0.15, 0.2, 0.25, 0.3\}$ .



**Figure 3** Impact of supply shortages on Consumer Surplus: shaded regions mark  $\Delta S < 0$ , white regions mark  $\Delta S > 0$ . Parameter values:  $N = 3000$ ,  $q_p = 0$ ,  $\sigma_p = 0.1$ ,  $\hat{q} = 0$ ,  $\sigma_q = 0.6$ ,  $\bar{x} = 0.5$ .

more likely to benefit from supply shortages when the difference in price between the two policies is higher. As Figure 3 suggests, this occurs when consumers preferences are more heterogeneous, because in this case consumers' valuations for the product are more "spread out."

## 6. Dynamic Pricing

Provided rationing is efficient in the first period and product quality is not too high, we have shown that it is optimal for the firm to choose a relatively low price accompanied with restricted first-period supply (i.e.,  $n_1^* < D_1(p^*)$ ). By restricting supply to the optimal quantity described in Proposition 2, the firm achieves the optimal balance between the volume of product reviews (controlled via the number of sales) and the content of these reviews (controlled by supplying its highest-valuation consumers first, via efficient rationing mechanisms). We point out that this balance can also be achieved by attaching a price-premium to first-period purchases since, in this case, the consumers from which the firm wishes to extract reviews will be the only ones who are willing to purchase a unit in the first period. When the product's price is fixed, the first-period decision to restrict supply may therefore be viewed as a method of *indirect price-discrimination*.

Viewed in this light, it is evident that when the firm is able to price-discriminate directly, launch-phase supply shortages cease to be optimal.<sup>13</sup> Denoting first-period price by  $p_1$  and second-period price by  $p_2$ , Proposition 4 formalizes this observation.<sup>14</sup>

**PROPOSITION 4.** *Let  $\{p_1^*, p_2^*\}$  be the optimal dynamic-price plan and let  $n_{1d}^*$  be the corresponding optimal launch-phase quantity. Then  $n_{1d}^* = D_1(p_1^*)$ .*

Consider any arbitrary price-plan  $\{p_1, p_2\}$  accompanied by launch-phase quantity  $n_{1d}$  satisfying  $n_{1d} < D_1(p_1)$  (i.e., a shortage of supply). Any such policy cannot be optimal, since the firm would be strictly better off by simply raising the first-period price to  $\hat{p}_1$ , such that  $D_1(\hat{p}_1) = n_{1d}$ . To see why, note that in this way the firm sells the same number of units in the launch phase and to the same consumers, therefore achieving the same SL effect and the same second-period revenues; however, the increased first-period price results in higher revenue-per-purchase in the first period. In general, under dynamic pricing, any policy which involves a deliberate supply shortage in the launch phase is strictly dominated by another with higher first-period price and unrestricted first-period availability.

The result of Proposition 4 is particularly interesting when compared against existing theories regarding the optimality of early firm-induced supply shortages. For instance, DeGraba (1995) and Courty and Nasiry (2012) describe how a monopolist may create an early supply shortage so as to induce a “buying frenzy” among consumers, while Denicolò and Garella (1999) argue that early shortages can be optimal provided rationing is not efficient.<sup>15</sup> Crucially, such theories have dynamic pricing as a requirement and do not apply to settings in which the product’s price remains unchanged after the launch phase. By contrast, supply shortages in our setting are optimal *only* under a constant price. The mechanism underlying the profitability of shortages in our paper is, of course, fundamentally different from those in existing theories, which do not consider SL among consumers; nevertheless, our results complement the existing literature in the sense that they advocate the optimality of early supply shortages in distinctly different settings, in which fixed pricing is commonplace.

We next turn our attention to how the firm’s profit under this indirect method of price-discrimination (i.e., fixed price with restricted launch-phase supply) compares with that under

<sup>13</sup> In many practical settings, dynamic pricing in the early stages of the selling season may be difficult to implement, for example, due to fairness considerations (Gilbert and Klemperer (2000); see also *The New York Times* (2007)).

<sup>14</sup> Keeping in line with the rest of our analysis, we assume that when the firm employs dynamic pricing consumers do not strategically delay their purchase in anticipation of a markdown; this does not affect the result of Proposition 4 and allows for a fair comparison between fixed- and dynamic-pricing policies.

<sup>15</sup> As opposed to early shortages, other papers have looked at the beneficial effects of product scarcity in the latter stages of the selling season (e.g., Liu and van Ryzin (2008), Stock and Balachander (2005)).

direct price-discrimination (i.e., dynamic pricing). Since maintaining a fixed price is simply a special case of dynamic pricing, from Proposition 4 it follows that the firm's profit under dynamic pricing is strictly higher than that under fixed pricing, even when the firm optimizes the SL effect through its launch-quantity decision. More specifically, the difference in firm profit when it employs fixed versus dynamic pricing is characterized in Proposition 5.

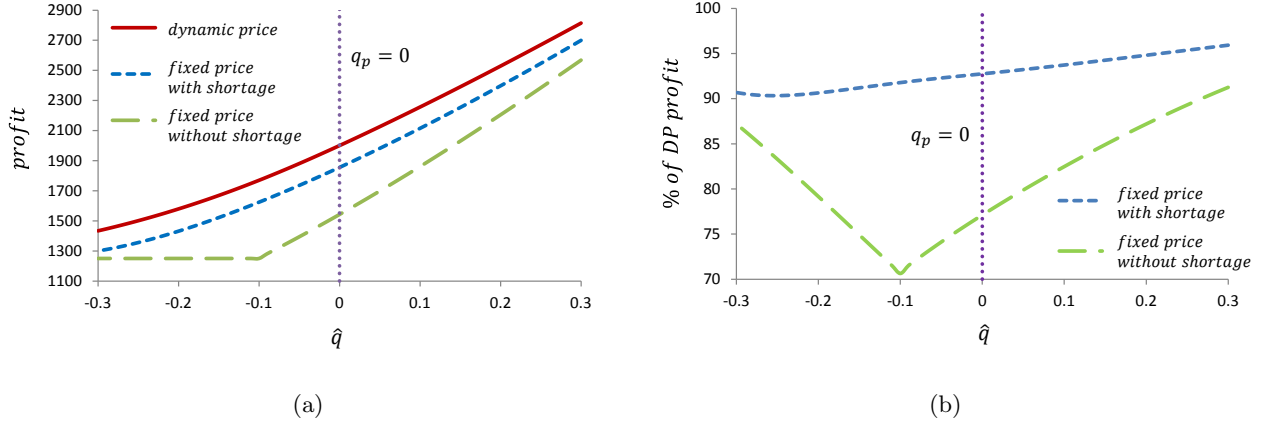
**PROPOSITION 5.** *Let  $\pi_s^*$  denote firm profit under the optimal fixed-pricing policy  $\{p^*, n_1^*\}$  with  $n_1^* \leq D_1(p^*)$ , and let  $\pi_d^*$  denote firm profit under the optimal dynamic pricing policy  $\{p_1^*, p_2^*\}$  with  $p_1^* \geq p_2^*$ . Then the difference in profits between the two policies  $\Delta\pi^* = \pi_d^* - \pi_s^*$  is bounded by*

$$(\tau^* - p^*)n_1^* \leq \Delta\pi^* \leq (p_1^* - p_2^*)D_1(p_1^*), \quad (8)$$

where  $\tau^*$  is the solution to (7).

The intuition for the bounds in Proposition 5 is as follows. First, consider the lower bound. Under fixed pricing, firm profit is maximized at  $\{p^*, n_1^*\}$ . When employing dynamic pricing, the monopolist can at least charge  $p_1 = \tau^*$  and  $p_2 = p^*$  and extract profit which exceeds its profit under fixed pricing by  $(\tau^* - p^*)n_1^*$ ; the excess profit is generated through the price-premium attached to first period purchases. Next, consider the upper bound. Under dynamic pricing, the monopolist achieves maximum profit at  $\{p_1^*, p_2^*\}$  (recall from Proposition 4 that the optimal quantity decision is  $n_{1d}^* = D_1(p_1^*)$ ). When employing fixed pricing, the monopolist can at least set  $p = p_2^*$  and  $n_1 = D_1(p_1^*)$ . In this way, the firm forgoes only the revenues extracted through the first period price premium under dynamic pricing, which is the expression of the upper bound.

To complement the result of Proposition 5, we conducted numerical experiments to examine the effectiveness of optimizing the launch-phase quantity decision. Our numerical experiments indicate that not only do early supply shortages provide the firm with a profit advantage over policies with unrestricted supply, in many cases they allow the firm to retrieve most of the profit lost due to the firm's inability to price dynamically in the launch-phase; an example is presented in Figure 4. Across all combinations of parameters listed in Table 1, we find that the profit gap between dynamic pricing and fixed pricing *without* shortages is 23% on average with a minimum gap of 11% (maximum 35%), while the gap between dynamic pricing and fixed pricing *with* supply shortages is 13% on average with a minimum of 3% (maximum 24%). Overall, we observe that policies involving supply shortages generate substantial value for the firm when product quality is relatively close to customers' prior expectations (e.g., see Figure 4b).



**Figure 4** (a) Firm profit under different policies (b) Percentage of dynamic-pricing profit achieved under fixed-pricing policies. Parameter values:  $N = 5000$ ,  $\bar{x} = 0.5$ ,  $\sigma_x = 0.4$ ,  $q_p = 0$ ,  $\sigma_p = 0.1$ ,  $\sigma_q = 0.6$ ,  $\rho = 0.2$ .

## 7. Discussion

### 7.1. Strategic Purchasing Delays

When the product's price is fixed across time, consumers have no monetary incentive to strategically delay their purchase in the first period.<sup>16</sup> However, provided consumers are sufficiently patient, strategic delays may occur for informational reasons; that is, some consumers with a positive expected utility from purchase in the first period may choose to delay their purchasing decision in anticipation of the information contained in the reviews of their peers (see Papanastasiou and Savva (2013)). Here, we illustrate that such consumer behavior has no significant bearing on our analysis.

Let us first make the concept of strategic waiting more concrete, by introducing a parameter  $\delta_c$  ( $0 \leq \delta_c \leq 1$ ), which represents consumers' patience or "strategicness" – consumers are myopic when  $\delta_c = 0$  and become more strategic as  $\delta_c$  increases (Cachon and Swinney 2011). Parameter  $\delta_c$  enters consumers' second-period utility as a multiplicative discount factor, and each consumer seeks to maximize her expected utility through her purchasing decisions. The analysis of Papanastasiou and Savva (2013) indicates that strategic consumers will follow a threshold policy in the first period, whereby consumers purchase in the first period provided  $x_i > \theta(p) > p$ ; that is, consumers whose expected utility is relatively high (low) in the first period choose to purchase immediately (delay purchase). Intuitively, the threshold  $\theta(p)$  is strictly increasing in price  $p$  and in consumers' patience  $\delta_c$ .

Next, consider how the result of Proposition 1 is affected by strategic purchasing delays. If the price of the product satisfies  $p > \tau^*$ , in the absence of strategic delays the firm prefers to sell as many units as possible in the first period. This remains the case in the presence of delays, the

<sup>16</sup> For strategic consumer behavior in dynamic-pricing settings see, for example, Su and Zhang (2008) and Elmaghraby et al. (2008).

difference being that the firm will now be able to sell only  $D_1(\theta(p))$ , as opposed to  $D_1(p)$  units, in the first period. On the other hand, if price satisfies  $p \leq \tau^*$  then in the absence of strategic delays it is optimal for the firm to sell only  $D_1(\tau^*)$  units in the first period. When consumers are strategic, two possible cases arise: either (i)  $D_1(\theta(p)) > D_1(\tau^*)$ , in which case the firm continues to under-supply the launch-phase demand (which is now equal to  $D_1(\theta(p))$  as opposed to  $D_1(p)$ ) and makes only  $D_1(\tau^*)$  units available in the first period, or (ii)  $D_1(\theta(p)) \leq D_1(\tau^*)$ , in which case the firm sells as many units as possible in the first period, namely,  $D_1(\theta(p))$ . Case (i) applies to relatively lower values of price, while case (ii) applies to values closer to (but still lower than)  $\tau^*$ . The end result is simple: in the presence of strategic purchasing delays, there exists again a threshold price below which the firm deliberately under-supplies early demand, say  $\psi^*$ , but this threshold will be lower than  $\tau^*$  as a result of strategic consumer behavior (i.e., for  $\delta_c > 0$ , we have  $\psi^* < \tau^*$ ). Furthermore, when  $p < \psi^*$  the optimal first-period quantity remains  $D_1(\tau^*)$ , as was the case without strategic delays – in terms of the formal statement of Proposition 1, the only thing that changes is the threshold on the product's price. Since Proposition 1 is the foundation for our subsequent results, it follows that our main results and model insights are qualitatively unchanged by the presence of strategic purchasing delays.

## 7.2. Quality-Dependent Correlation $\rho$

In our main analysis, we have focused on a positive correlation  $\rho > 0$  between consumer preferences and ex post quality perceptions (Dellarocas et al. 2004, Li and Hitt 2008). A plausible alternative assumption would be to assume that the value of the correlation parameter depends on how the consumers' ex post experiences compare with their ex ante expectations. One way of implementing this assumption is to make  $\rho$  a function of the comparison between the product's true mean quality  $\hat{q}$  and the consumers' prior expectation  $q_p$ . Consider the following simple case:

$$\rho = \begin{cases} +k & \text{if } \hat{q} \geq q_p \\ -k & \text{if } \hat{q} < q_p, \end{cases}$$

for some positive constant  $k$ . That is, if  $\hat{q} \geq q_p$  then the product's quality exceeds consumers' expectations and reviews are positively biased, while if  $\hat{q} < q_p$  then quality is below expectations and reviews are negatively biased. Under this structure for  $\rho$ , our main analysis holds unchanged for cases of  $\hat{q} \geq q_p$  (i.e., we can simply apply  $\rho = k$  in our existing results). When  $\hat{q} < q_p$ , consumers' opinions will be negatively biased. In this case, it is straightforward to show that the firm will choose the single-period optimal price, fulfill all demand in the first period, and achieve zero sales in the second period.



## 8. Conclusion

Motivated by empirical evidence from consumer learning in online settings, as well as anecdotal observations regarding the impact of early short-term supply shortages on consumer reviews, this paper is a first attempt to highlight the potential role of inventory decisions in modulating SL outcomes.

We have shown that it can indeed be beneficial for the firm (in terms of profits and long-term product adoption) to deliberately under-supply the early demand for its product. We have shown that, surprisingly, even though supply shortages may be viewed as an attempt by the firm to manipulate the SL process, in many cases they result in “win-win” situations, whereby both the firm and the consumer population are left better off in terms of profit and surplus, respectively. Moreover, contrary to other existing theories regarding the optimality of early supply shortages for the firm (e.g, buying frenzies), the mechanism identified in this paper applies to settings in which the product’s price remains constant across time, consistent with observations from practice. In many cases, by optimizing the launch-phase quantity decision, we have observed that the firm may be able to enjoy most of the profit-benefits of pricing dynamically while charging a fixed price. In this respect, inventory management can be viewed as a subtle and effective substitute for dynamic pricing, in cases in which the latter is difficult to implement (e.g., owing to fairness considerations).

Our findings have implications that extend beyond the scope of our model setting. For instance, the increase in firm profit associated with SL from buyer reviews suggests firm investment in online reporting systems, as well as in maintaining the credibility of these systems. Furthermore, our model indicates that firms may benefit from aiming their pre-launch advertising campaigns at customers who are predisposed to having a positive experience with their product. In terms of global product launches, firms which are aware of a significant pool of loyal customers in specific markets may wish to consider launching their product first in such favorable regions, in order to extract higher product ratings. Finally, our result on the optimality of early supply shortages suggests that early non-deliberate stock-outs may not be as detrimental as previously considered, provided the firm facilitates efficient rationing of demand through appropriate rationing mechanisms (e.g., waiting-lines, loyalty programs).

## Appendix

### A. Proofs

#### Proof of Lemma 1

Let  $\bar{q}_u$  denote the posterior mean belief, conditional on  $n_1$  reviews generated in the first period with an average rating of  $\bar{R}$ . Under proportional rationing, we have  $\bar{q}_u = \frac{n_1 \sigma_p^2}{n_1 \sigma_p^2 + \sigma_q^2} \bar{R} = \frac{n_1 \sigma_p^2}{n_1 \sigma_p^2 + \sigma_q^2} (\hat{q} + \sigma_q \sigma_x h(p))$ , that is,  $\bar{R}$  is independent of  $n_1$ . If  $\hat{q} + \sigma_q \sigma_x h(p) > 0$  then  $\bar{q}_u$  is positive and increasing in  $n_1$ , and the firm’s

problem is  $\max_{n_1 \leq D_1(p)} n_1 p + (N\bar{F}(p - \bar{q}_u) - n_1)p = \max_{n_1 \leq D_1(p)} N\bar{F}(p - \bar{q}_u)p$  (note that in this case  $D_2 = N\bar{F}(p - \bar{q}_u) - n_1 \geq 0$ ). The firm's profit is therefore increasing in  $\bar{q}_u$ , which itself is increasing in  $n_1$ ; the optimal launch quantity is  $n_1^* = D_1(p)$ .

Next, if  $\hat{q} + \sigma_q \sigma_x h(p) < 0$  then  $\bar{q}_u$  is negative and decreasing in  $n_1$ . Therefore, the number of overall sales are bounded from above at  $N\bar{F}(p)$ ; these sales are achieved by selling to all customers willing to purchase in the first period; this implies  $n_1^* = D_1(p)$ . Note that a profit-equivalent policy is to sell no units in the first period, and sell  $D_1(p)$  units in the second period; any other  $n_1$  such that  $n_1 \in (0, D_1(p))$  is suboptimal.

### Proof of Lemma 2

See online supplement.

### Proof of Proposition 1

Under efficient rationing, the firm's problem may be stated as  $\max_{n_1 \leq D_1(p)} n_1 p + \max\{(N\bar{F}(p - \bar{q}_u) - n_1), 0\}p$  where the two terms correspond to first- and second-period profit respectively (note  $D_2 = \max\{(N\bar{F}(p - \bar{q}_u) - n_1), 0\}$ ). Equivalently, the firm's problem is  $\max_{n_1 \leq D_1(p)} \max\{N\bar{F}(p - \bar{q}_u), n_1\}p$ . Because the firm always has the option to choose  $n_1 = 0$  so that  $\bar{q}_u = q_p = 0$  and achieve  $N\bar{F}(p)$  sales in the second period, it suffices to consider the problem  $\max_{n_1 \leq D_1(p)} N\bar{F}(p - \bar{q}_u)p$  and, since  $\bar{F}(p - \bar{q}_u)$  is increasing in  $\bar{q}_u$ , the optimization problem simplifies to  $\max_{n_1 \leq D_1(p)} \bar{q}_u$ . We next consider the properties of the function  $\bar{q}_u(n_1)$ . We have  $\bar{q}_u(n_1) = \frac{n_1 \sigma_p^2}{n_1 \sigma_p^2 + \sigma_q^2} \bar{R}(n_1)$ , and

$$\bar{R}(n_1) = \hat{q} + \rho \sigma_q \frac{E[x_i | x_i > \tau(n_1)] - \bar{x}}{\sigma_x},$$

where  $\tau(n_1)$  is a given by  $P(x_i \geq \tau) = \bar{F}(\tau) = \frac{n_1}{N}$ . Using this expression we have

$$\begin{aligned} \bar{q}_u(\tau) &= \frac{N\bar{F}(\tau)\sigma_p^2}{N\bar{F}(\tau)\sigma_p^2 + \sigma_q^2} \left[ \hat{q} + \rho \sigma_q \frac{\int_{\tau}^{+\infty} x f_1(x) dx}{\bar{F}(\tau)} - \bar{x} \right] \\ &= \frac{N\sigma_p^2}{N\bar{F}(\tau)\sigma_p^2 + \sigma_q^2} \left[ \bar{F}(\tau)\hat{q} + \rho \sigma_q \frac{\int_{\tau}^{+\infty} x f_1(x) dx - \bar{F}(\tau)\bar{x}}{\sigma_x} \right]. \end{aligned}$$

Differentiating with respect to  $\tau$ , we have (after some manipulation)

$$\frac{\partial \bar{q}_u}{\partial \tau} = \frac{N f(\tau) \sigma_p^2}{[N\bar{F}(\tau)\sigma_p^2 + \sigma_q^2]^2} \left( N\bar{F}(\tau)\sigma_p^2 \rho \frac{\sigma_q}{\sigma_x} \mu(\tau) - \sigma_q^2 \left[ \hat{q} + \rho \frac{\sigma_q}{\sigma_x} (\tau - \bar{x}) \right] \right),$$

where  $\mu(\tau) = \frac{\int_{\tau}^{+\infty} (x - \tau) f(x) dx}{\bar{F}(\tau)}$ . Note that in the last parenthesis, for any  $\rho > 0$ ,  $\rho N\bar{F}(\tau)\mu(\tau)$  in the first term is positive and strictly decreasing in  $\tau$  (because  $\mu(\tau)$  is the “mean residual life” of the Normal distribution which is a strictly decreasing quantity) while the second term increases linearly in  $\tau$ . Therefore, we have  $\frac{\partial \bar{q}_u}{\partial \tau} > 0$  for  $\tau < \tau^*$ , and  $\frac{\partial \bar{q}_u}{\partial \tau} < 0$  otherwise, where  $\tau^*$  is the unique solution of

$$\tau = \bar{x} - \frac{\sigma_x \hat{q}}{\sigma_q \rho} + \frac{N\bar{F}(\tau)\sigma_p^2 \mu(\tau)}{\sigma_q^2}. \quad (9)$$

Therefore,  $\bar{q}_u(\tau)$  is unimodal with a unique maximum at  $\tau^*$ ; equivalently,  $\bar{q}_u(n_1)$  is unimodal with a unique maximum at  $n_1^* = N\bar{F}(\tau^*)$ .

As a result, when  $N\bar{F}(\tau^*) \in [0, D_1(p)]$  the optimal launch-quantity is  $n_1^* = N\bar{F}(\tau^*)$ . However, when  $N\bar{F}(\tau^*) > D_1(p)$ , then by unimodality of  $\bar{q}_u$  the optimal inventory quantity decision is  $n_1^* = D_1(p)$  (i.e.,  $n_1$  as large as possible). Clearly, the latter holds when  $p > \tau^*$ , while the former holds when  $p < \tau^*$ .

### Proof of Proposition 2

From Proposition 1, we know that if the firm chooses a price which satisfies  $p < \tau$ , it is optimal to choose availability  $n_1 = D_1(\tau) < D_1(p)$ . By contrast, if the firm chooses  $p \geq \tau$ , then it is optimal to choose availability  $n_1 = D_1(p)$ . To prove the result, we will compare the best possible policy with price  $p < \tau$  against the best possible policy with price  $p \geq \tau$ .

Consider first policies with  $p < \tau$ . Any such policy is accompanied by availability  $n_1^* = D_1(\tau)$ ; let  $\bar{q}_u^* = \bar{q}_u(n_1^*)$ . The firm's pricing problem (within the class of policies with  $p < \tau$ ) is  $\max_{p < \tau} Np\bar{F}(p - \bar{q}_u^*)$ . The unique solution to this problem is  $\xi$  as stated in the main text. Next consider policies with  $p > \tau$ . Here, any policy is accompanied by availability  $n_1 = D_1(p)$ . The firm's pricing problem (within the class of policies with  $p \geq \tau$ ) is  $\max_{p \geq \tau} Np\bar{F}(p - \bar{q}_u(D_1(p)))$ .

Now, note that by unimodality of  $\bar{q}_u$  in the proof of Proposition 1, we have that  $\max_{p \geq \tau} Np\bar{F}(p - \bar{q}_u(D_1(p))) < \max_{p \geq \tau} Np\bar{F}(p - \bar{q}_u^*)$ , because  $\bar{q}_u(n_1) < \bar{q}_u^*$  for any  $n_1 < D_1(\tau)$  and for  $p \geq \tau$  we have that  $n_1$  can be at most  $D_1(p)$ , which is less than  $D_1(\tau)$ . Therefore, a necessary and sufficient condition for policy  $\{\xi, n_1^*\}$  to be globally optimal is that  $\xi < \tau$ , because  $\xi = \arg \max_p Np\bar{F}(p - \bar{q}_u^*)$ . Conversely, if  $\xi \geq \tau$ , then we have  $p^* \in [\tau, +\infty)$  and  $p^*$  is found through the one-dimensional problem  $\max_p Np\bar{F}(p - \bar{q}_u(D_1(p)))$ .

We next show that  $\xi < \tau$  holds true provided  $\hat{q} < Q$ , for some threshold  $Q$  which is a function of our model parameters. First note that  $\frac{\partial \bar{q}_u}{\partial \hat{q}} > 0$ , and therefore  $\frac{\partial \bar{q}_u^*}{\partial \hat{q}} > 0$ . It follows that the normal hazard ratio  $h(p - \bar{q}_u)$  is strictly decreasing in  $\hat{q}$ , which in turn implies that  $\xi$  is strictly increasing in  $\hat{q}$ . Next, taking the total derivative of (9) with respect to  $\hat{q}$  we have

$$\frac{\partial \tau}{\partial \hat{q}} + \frac{\sigma_x}{\sigma_q \rho} - \frac{\partial}{\partial \tau} \left( \frac{N\bar{F}(\tau)\sigma_p^2\mu(\tau)}{\sigma_q^2} \right) \frac{\partial \tau}{\partial \hat{q}} = 0$$

Note that  $\bar{F}(\tau) > 0$ ,  $\frac{\partial \bar{F}}{\partial \tau} < 0$  and  $\mu(\tau) > 0$ ,  $\frac{\partial \mu}{\partial \tau} < 0$  ( $\mu(\tau)$  is the “mean residual life” function and is decreasing in  $\tau$  for the normal distribution). Therefore, we have  $\frac{\partial \tau}{\partial \hat{q}} < 0$ . Since  $\frac{\partial \xi}{\partial \hat{q}} > 0$  and  $\frac{\partial \tau}{\partial \hat{q}} < 0$ , it follows that there exists  $Q$  such that  $\xi < \tau$  if  $\hat{q} < Q$ .

### Proof of Proposition 3

We establish first the properties of the optimal quantity decision  $n_1^* = N\bar{F}(\tau)$ , by considering the properties of the threshold  $\tau$ , which is given by the implicit equation  $\tau - \bar{x} + \frac{\sigma_x \hat{q}}{\sigma_q \rho} - \frac{N\bar{F}(\tau)\sigma_p^2\mu(\tau)}{\sigma_q^2} = 0$ . Define  $M = \frac{\sigma_x \hat{q}}{\sigma_q \rho}$  and  $K(\tau) = \frac{N\bar{F}(\tau)\sigma_p^2\mu(\tau)}{\sigma_q^2}$  such that  $\tau$  satisfies

$$\tau - \bar{x} + M - K(\tau) = 0,$$

and note that  $K(\tau) > 0$  and  $K'(\tau) < 0$  (see proof of Proposition 2). Taking the total derivative with respect to  $\hat{q}$ , we have

$$\frac{\partial \tau}{\partial \hat{q}} + \frac{\partial M}{\partial \hat{q}} - \frac{\partial K}{\partial \tau} \frac{\partial \tau}{\partial \hat{q}} = 0.$$

Since  $\frac{\partial M}{\partial \hat{q}} > 0$ , we have  $\frac{\partial \tau}{\partial \hat{q}} < 0$  and it follows that  $\frac{\partial n_1^*}{\partial \hat{q}} > 0$ . Next, consider the total derivative with respect to  $\rho$ .

$$\frac{\partial \tau}{\partial \rho} + \frac{\partial M}{\partial \rho} - \frac{\partial K}{\partial \tau} \frac{\partial \tau}{\partial \rho} = 0.$$

Here,  $\frac{\partial M}{\partial \rho} < 0$  ( $> 0$ ) for  $\hat{q} > 0$  ( $< 0$ ) and therefore  $\frac{\partial n_1^*}{\partial \rho} < 0$  ( $> 0$ ) for  $\hat{q} > 0$  ( $< 0$ ). Next, consider the total derivative with respect to  $\sigma_p^2$ .

$$\frac{\partial \tau}{\partial \sigma_p^2} - \left( \frac{\partial K}{\partial \sigma_p^2} + \frac{\partial K}{\partial \tau} \frac{\partial \tau}{\partial \sigma_p^2} \right) = 0.$$

Therefore,  $\frac{\partial n_1^*}{\partial \sigma_p^2} < 0$ .

We now consider the properties of the optimal price  $p^*$ . Recall that this is implicitly defined via

$$ph(p - \bar{q}_u^*) - 1 = 0,$$

where  $\bar{q}_u^* = \bar{q}_u(n_1^*)$  and it is straightforward that  $\frac{\partial \bar{q}_u^*}{\partial \hat{q}} > 0$ ,  $\frac{\partial \bar{q}_u^*}{\partial \rho} > 0$  and  $\frac{\partial \bar{q}_u^*}{\partial \sigma_p^2} > 0$ . Consider the total derivative with respect to  $\rho$ .

$$\left( h(p - \bar{q}_u^*) + p \frac{\partial h(p - \bar{q}_u^*)}{\partial p} \right) \frac{\partial p}{\partial \rho} + \left( p \frac{\partial h(p - \bar{q}_u^*)}{\partial \bar{q}_u^*} \right) \frac{\partial \bar{q}_u^*}{\partial \rho} = 0.$$

Since  $h(\cdot)$  is the hazard ratio of a Normal distribution, we have  $\frac{\partial h(p - \bar{q}_u^*)}{\partial p} > 0$  and  $\frac{\partial h(p - \bar{q}_u^*)}{\partial \bar{q}_u^*} < 0$ . Therefore,  $\frac{\partial p}{\partial \rho} > 0$ . In a similar manner it can be shown that  $\frac{\partial p}{\partial \hat{q}} > 0$  and  $\frac{\partial p}{\partial \sigma_p^2} > 0$ .

Finally, note that the firm's profit can be expressed as  $\pi = Np^* \bar{F}(p^* - q_u^*)$ . The properties of the firm's optimal profit follow readily from the properties of  $q_u^*$  as stated above.

#### Proof of Proposition 4

We will show the desired result by contradiction. Assume an optimal policy  $\zeta^* = \{p_1^*, n_1^*, p_2^*\}$  with  $n_1^* < D_1(p_1^*)$ .

The profit under this policy is

$$\pi(p_1^*, n_1^*, p_2^*) = p_1^* n_1^* + \pi_2(n_1^*, p_2^*),$$

where  $p_1^* n_1^*$  is the first period profit while  $\pi_2(n_1^*, p_2^*)$  is the second period profit. Next, consider policy  $\psi$ ,  $\psi = \{\hat{p}_1, D_1(\hat{p}_1), p_2^*\}$  (i.e., dynamic pricing with unrestricted availability), where  $\hat{p}_1$  is such that  $D_1(\hat{p}_1) = n_1^*$ . Note that the latter implies  $\hat{p}_1 > p_1^*$ . Profit under this policy is

$$\begin{aligned} \pi(\hat{p}_1, D_1(\hat{p}_1), p_2^*) &= \pi(\hat{p}_1, n_1^*, p_2^*) \\ &= \hat{p}_1 n_1^* + \pi_2(n_1^*, p_2^*) \\ &> p_1^* n_1^* + \pi_2(n_1^*, p_2^*) \end{aligned}$$

where the last inequality holds because first-period profit under policy  $\psi$  is strictly greater than under  $\zeta^*$ , owing to a price-premium attached to first-period sales. Therefore  $\zeta^*$  cannot be optimal.

**Proof of Proposition 5** Let  $\pi_d(p_1, p_2)$  denote profits under dynamic pricing at  $\{p_1, p_2\}$  and let  $\pi_s(p, n_1)$  denote profits under fixed pricing at  $\{p, n_1\}$ . We have  $\pi_d^* = Np_1^* \int_{p_1^*}^{+\infty} f(x)dx + Np_2^* \int_{\min\{p_1^*, p_2^* - \bar{q}_u(D_1(p_1^*))\}}^{p_1^*} f(x)dx$ . Assume that the firm sets  $p = p_2^*$  and  $n_1 = D_1(p_1^*)$  in a fixed-pricing policy. Then it achieves profit  $\pi_s = Np_2^* \int_{p_1^*}^{+\infty} f(x)dx + Np_2^* \int_{\min\{p_1^*, p_2^* - \bar{q}_u(D_1(p_1^*))\}}^{p_1^*} f(x)dx$ . Therefore,  $\pi_s(p_2^*, D_1(p_1^*)) = \pi_d^* - (p_1^* - p_2^*)D_1(p_1^*)$ . Since  $\pi_s^* \geq \pi_s(p_2^*, D_1(p_1^*))$ , it follows that  $\Delta\pi^* = \pi_d^* - \pi_s^* \leq (p_1^* - p_2^*)D_1(p_1^*)$ , which is the upper bound of the Proposition.

Next, note that under the optimal fixed-pricing policy  $\{p^*, n_1^*\}$  we have  $\pi_s^* = p^* n_1^* + Np_2^* \int_{\min\{\tau, p^* - \bar{q}_u(n_1^*)\}}^{\tau} f(x)dx$ . Under dynamic pricing, suppose that the firm sets  $p_1 = \tau$  such that  $D_1(p_1) = n_1^*$  and  $p_2 = p^*$ . Then it achieves profit  $\pi_d(\tau, p^*) = \tau n_1^* + Np_2^* \int_{\min\{\tau, p^* - \bar{q}_u(n_1^*)\}}^{\tau} f(x)dx$ . Therefore  $\pi_d(\tau, p^*) = \pi_s^* + (\tau - p^*)n_1^*$ . Finally, since  $\pi_d^* \geq \pi_d(\tau, p^*)$ , it follows that  $\Delta\pi^* = \pi_d^* - \pi_s^* \geq (\tau - p^*)n_1^*$  which is the lower bound of the Proposition.

## B. Numerical Investigation of Profit Function Approximation

For the case of efficient rationing, we compare the solution and optimal profit obtained analytically for problem (6) with the corresponding values obtained numerically for problem (5), for the 420 scenarios used in Table 1:  $N \in \{1000, 3000, 5000, 7000\}$ ;  $q_p = 0$ ,  $\sigma_p = 0.1$ ,  $\hat{q} \in \{-0.15, -0.1, -0.05, 0, 0.05, 0.1, 0.15\}$ ;  $\sigma_x \in \{0.4, 0.6, 0.8\}$ ,  $\sigma_q = 0.6$ ,  $\bar{x} = 0.5$ ;  $\rho \in \{0.1, 0.15, 0.2, 0.25, 0.3\}$ . We report summary statistics of this comparison in the following table (the maximum errors were observed under scenarios of low product quality, i.e., for  $\hat{q} = -0.15$ ).

	average error	max error	% of scenarios with < 1% error
Optimal profit $\pi^*$	0.01%	0.63%	100%
Optimal price $p^*$	0.16%	3.14%	97.4%
Optimal segmentation threshold $\tau^*$ (note $n_1^* = D_1(\tau^*)$ )	0.11%	4.8%	96.7%

Finally, we also note that for the case of proportional rationing, the result of Lemma 1 was found to hold across all parameter combinations.

## C. Fully-Bayesian Social Learning

We describe here the procedure by which individual product reviews can be de-biased by rational agents when information on reviewer preferences is readily available. Consider an individual buyer whose idiosyncratic component is  $x_i$ . According to (1), the buyer's review is generated from a Normal source  $N(\hat{q} + \rho \frac{\sigma_q}{\sigma_x}(x_i - \bar{x}), \sigma_q^2(1 - \rho^2))$ . Since our objective is to learn  $\hat{q}$ , we must first subtract the deterministic bias term  $\rho \frac{\sigma_q}{\sigma_x}(x_i - \bar{x})$  from the buyer's review, thus "centering" the review around  $\hat{q}$ . Next, employing the standard Gaussian updating model (Chamley 2004), the Bayesian posterior mean (after observing a single review generated by a customer with idiosyncratic preference component  $x_i$ ) is given by

$$q_u = \frac{\sigma_p^2}{\sigma_p^2 + \sigma_q^2(1 - \rho^2)} \left( q_i - \rho \frac{\sigma_q}{\sigma_x}(x_i - \bar{x}) \right), \quad (10)$$

where  $q_i$  is the buyer's review. If a set  $S$  of reviews is available with  $|S| = n_s$ , the same procedure can be repeated, updating  $q_u$  sequentially after de-biasing each review. The end result is

$$q_u = \frac{n_s \sigma_p^2}{n_s \sigma_p^2 + \sigma_q^2(1 - \rho^2)} \left( \frac{1}{n_s} \sum_{i \in S} q_i - \rho \frac{\sigma_q}{\sigma_x} \left( \frac{1}{n_s} \sum_{i \in S} x_i - \bar{x} \right) \right) \quad (11)$$

$$= \frac{n_s \sigma_p^2}{n_s \sigma_p^2 + \sigma_q^2(1 - \rho^2)} \left( R_s - \rho \frac{\sigma_q}{\sigma_x} (\bar{x}_s - \bar{x}) \right), \quad (12)$$

with  $R_s = \frac{1}{n_s} \sum_{i \in S} q_i$  and  $\bar{x}_s = \frac{1}{n_s} \sum_{i \in S} x_i$ .

## References

- Acemoglu, D., K. Bimpikis, A. Ozdaglar. 2013. Dynamics of information exchange in endogenous social networks. *Theoretical Economics*, forthcoming .
- Acemoglu, D., M. Dahleh, I. Lobel, A. Ozdaglar. 2011. Bayesian learning in social networks. *The Review of Economic Studies* **78**(4) 1201–1236.
- Banerjee, A., D. Fudenberg. 2004. Word-of-mouth learning. *Games and Economic Behavior* **46**(1) 1–22.

- Banerjee, A.V. 1992. A simple model of herd behavior. *The Quarterly Journal of Economics* **107**(3) 797–817.
- Barberis, N., A. Shleifer, R. Vishny. 1998. A model of investor sentiment. *Journal of financial economics* **49**(3) 307–343.
- Barzel, Y. 1974. A theory of rationing by waiting. *Journal of Law and Economics* **17**(1) 73–95.
- Bass, F. 1969. A new product growth model for product diffusion. *Management Science* **15**(5) 215–227.
- Bergemann, D., J. Välimäki. 1997. Market diffusion with two-sided learning. *RAND Journal of Economics* **28**(4) 773–795.
- Besbes, O., M. Scarsini. 2013. On information distortions in online ratings. *Working paper, Columbia University*.
- Bhalla, M. 2013. Waterfall versus sprinkler product launch strategy: Influencing the herd. *The Journal of Industrial Economics* **61**(1) 138–165.
- Bikhchandani, S., D. Hirshleifer, I. Welch. 1992. A theory of fads, fashion, custom, and cultural change as informational cascades. *Journal of political Economy* 992–1026.
- Bose, S., G. Orosel, M. Ottaviani, L. Vesterlund. 2006. Dynamic monopoly pricing and herding. *The RAND Journal of Economics* **37**(4) 910–928.
- Brown, A., C. Camerer, D. Lovallo. 2012. To review or not to review? Limited strategic thinking at the movie box office. *American Economic Journal: Microeconomics* **4**(2) 1–26.
- Brown, A., C. Camerer, D. Lovallo. 2013. Estimating structural models of equilibrium and cognitive hierarchy thinking in the field: The case of withheld movie critic reviews. *Management Science* **59**(3) 733–747.
- Cachon, G., R. Swinney. 2011. The value of fast fashion: Quick response, enhanced design, and strategic consumer behavior. *Management Science* **57**(4) 778–795.
- Camerer, C. F., G. Loewenstein, M. Rabin. 2003. *Advances in behavioral economics*. Princeton University Press.
- Candogan, O., K. Bimpikis, A. Ozdaglar. 2012. Optimal pricing in networks with externalities. *Operations Research* **60**(4) 883–905.
- Chamley, C.P. 2004. *Rational herds: Economic models of social learning*. Cambridge University Press.
- Chevalier, J.A., D. Mayzlin. 2006. The effect of word of mouth on sales: Online book reviews. *Journal of Marketing Research* **43**(3) 345–354.
- Courty, P., J. Nasiry. 2012. Product launches and buying frenzies: A dynamic perspective. *Working Paper, University of Victoria*.
- DeGraba, P. 1995. Buying frenzies and seller-induced excess demand. *The RAND Journal of Economics* **26**(2) 331–342.
- Dellarocas, C., X. Zhang, N. Awad. 2004. Exploring the value of online reviews to organizations: Implications for revenue forecasting and planning. *Proc. 24th International Conference on Information Systems*.

- Dellarocas, C., X. Zhang, N. Awad. 2007. Exploring the value of online product reviews in forecasting sales: The case of motion pictures. *Journal of Interactive Marketing* **21**(4) 23–45.
- Denicolò, V., P.G. Garella. 1999. Rationing in a durable goods monopoly. *The RAND Journal of Economics* **30**(1) 44–55.
- Ellison, G., D. Fudenberg. 1993. Rules of thumb for social learning. *Journal of Political Economy* **101**(4) 612–643.
- Ellison, G., D. Fudenberg. 1995. Word-of-mouth communication and social learning. *The Quarterly Journal of Economics* **110**(1) 93–125.
- Elmaghraby, W., A. Gülcü, P. Keskinocak. 2008. Designing optimal preannounced markdowns in the presence of rational customers with multiunit demands. *Manufacturing & Service Operations Management* **10**(1) 126–148.
- Gilbert, Richard J, Paul Klemperer. 2000. An equilibrium theory of rationing. *RAND Journal of Economics* **31**(1) 1–21.
- Harsanyi, John C. 1967. Games with incomplete information played by” bayesian” players, i-iii part i. the basic model. *Management Science* **14**(3) 159–182.
- Holt, C.A. 1979. A theory of signalling auctions. Tech. rep., Center for Economic Research, Department of Economics, University of Minnesota.
- Holt, C.A., R. Sherman. 1982. Waiting-line auctions. *Journal of Political Economy* **90**(2) 280–94.
- Hu, M., j. Milner, J. Wu. 2013. Liking and following and the newsvendor: Operations and marketing policies under social influence. *Rotman School of Management, Working Paper* .
- Hu, M., Z. Wang. 2013. The double-edged sword of social interactions: Revenue management of network goods under demand uncertainty. *Rotman School of Management, Working Paper* .
- Hu, N., J. Zhang, P.A. Pavlou. 2009. Overcoming the j-shaped distribution of product reviews. *Communications of the ACM* **52**(10) 144–147.
- Ifrach, B., C. Maglaras, M. Scarsini. 2013. Bayesian social learning from consumer reviews. *Working Paper, Columbia University* .
- Ifrach, B., M. Scarsini, C. Maglaras. 2011. Monopoly pricing in the presence of social learning. *Working Paper, Columbia University* .
- Li, X., L. Hitt. 2010. Price effects in online product reviews: An analytical model and empirical analysis. *Management Information Systems Quarterly* **34**(4) 809–831.
- Li, X., L.M. Hitt. 2008. Self-selection and information role of online product reviews. *Information Systems Research* **19**(4) 456–474.
- Liu, Q., G.J. van Ryzin. 2008. Strategic capacity rationing to induce early purchases. *Management Science* **54**(6) 1115–1131.

- Liu, T., P. Schiraldi. 2012. New product launch: herd seeking or herd preventing? *Economic Theory* **51**(3) 627–648.
- Moretti, E. 2011. Social learning and peer effects in consumption: Evidence from movie sales. *The Review of Economic Studies* **78**(1) 356–393.
- Nielsen. 2012. Consumer trust in online, social and mobile advertising grows. (Oct. 4th).
- Papanastasiou, Y., N. Savva. 2013. Dynamic pricing in the presence of social learning and strategic consumers. *Working paper, London Business School*.
- Rabin, M. 2002. Inference by believers in the law of small numbers. *The Quarterly Journal of Economics* **117**(3) 775–816.
- Rabin, M. 2013. Incorporating limited rationality into economics. *Journal of Economic Literature* **51**(2) 528–543.
- Sgroi, D. 2002. Optimizing information in the herd: Guinea pigs, profits, and welfare. *Games and Economic Behavior* **39**(1) 137–166.
- Stock, A., S. Balachander. 2005. The making of a “hot product”: A signaling explanation of marketers’ scarcity strategy. *Management Science* **51**(8) 1181–1192.
- Su, X. 2010. Optimal pricing with speculators and strategic consumers. *Management Science* **56**(1) 25–40.
- Su, X., F. Zhang. 2008. Strategic customer behavior, commitment, and supply chain performance. *Management Science* **54**(10) 1759–1773.
- TechNewsWorld. 2006. Nintendo announces Wii lineup, supply shortages. (Nov. 1st).
- Tereyağoglu, N., S. Veeraraghavan. 2012. Selling to conspicuous consumers: Pricing, production, and sourcing decisions. *Management Science* **58**(12) 2168–2189.
- The Los Angeles Times. 2011. Apple fans camp out to get new iPhone 4s. (Oct. 15th).
- The New York Times. 2007. iPhone owners crying foul over price cut. (Sept. 7th).
- The Washington Post. 2012. New iPad expected to break sales records. (Mar. 15th).
- UTV. 2012. Derry student among first to buy iPad 3. (Mar. 16th).
- Veeraraghavan, Senthil, Laurens Debo. 2009. Joining longer queues: Information externalities in queue choice. *Manufacturing & Service Operations Management* **11**(4) 543–562.
- Villas-Boas, J.M. 2004. Consumer learning, brand loyalty, and competition. *Marketing Science* **23**(1) 134–145.
- YU, M., L. Debo, R. Kapuscinski. 2013. Strategic waiting for consumer-generated quality information: Dynamic pricing of new experience goods. *Working Paper, University of Chicago*.
- ZDNet. 2012. Apple adopts lottery system for iPhone 5 sales in Hong Kong. (Sep 24th).



## ONLINE SUPPLEMENT

Much of the contents of this Online Supplement is adapted from work in economics concerned with “rationing by waiting” (see Barzel (1974), Holt (1979) and Holt and Sherman (1982)). The approach taken is to view customers with  $x_i \geq p$  as participants in an auction, in which bids for the product are made in units of costly waiting-time. Both winners and losers in the auction pay the monetary-equivalent of their time-bids. For winners, this entails an extra cost over and above the product’s monetary price, while for losers it entails a net loss.

Here, we state briefly the assumptions underlying the waiting-line game; see Holt (1979) for details. There are  $n$  agents (customers), competing for  $m$  prizes (units of the product) in the first period of our model. Agent  $i$  values purchase of the product at  $u_i = x_i - p$ , where  $u_i \in (\underline{u}, \bar{u})$ . Agents have the option of arriving earlier and forming a waiting-line. However, waiting time is costly at rate  $w_i$  for customer  $i$ , where we assume that  $u_i$  and  $w_i$  are related through  $u_i = w(u_i)$ . Customer payoffs in the event of a win, loss, or no participation (i.e., defer purchase to second period) are respectively

$$\Pi_i^w = u_i - w_i t_i - k w_i$$

$$\Pi_i^l = \gamma u_i - w_i t_i - k w_i$$

$$\Pi_i^n = \gamma u_i,$$

where  $k$  may represent the time required by customers to travel to the waiting line and  $t_i$  is the time in advance customer  $i$  chooses to arrive. For simplicity in exposition, we assume  $k = 0$  (this has no significant bearing on our analysis). Moreover,  $\gamma$  represents a discount factor for second period purchases; we assume that  $\gamma < 1$  so that customers prefer to consume the product sooner rather than later. Note that customers who fail to secure a unit in the first period are sure to secure a unit in the second period, as per the specification of our model. Without loss of generality, we express payoffs in units of time such that  $\pi_i^w = \frac{\Pi_i^w}{w_i}$  etc. Then

$$\pi_i^w = a_i - t_i$$

$$\pi_i^l = \gamma a_i - t_i$$

$$\pi_i^d = \gamma a_i,$$

where  $a_i = \frac{u_i}{w_i}$  is the “time value” of securing a unit in the first period for customer  $i$ . Since  $u_i \in (\underline{u}, \bar{u})$ , it follows that  $a_i \in (\underline{a}, \bar{a})$ .

The informational structure of the game is as follows. We assume that each customer knows his own  $a_i$  and believes that the  $a_i$  of his rivals are *iid* draws from the distribution function  $G(\cdot)$ , where  $g(\cdot) > 0$  on some finite open interval. The information structure is symmetric.

The described auction can thus be analyzed as a non-cooperative game of incomplete information. According to Harsanyi (1967), a Nash equilibrium of a game of this type is characterized by a number  $a^* \in [\underline{a}, \bar{a})$  and a strictly increasing function  $t(a_i)$ ,  $t(a_i) \geq 0$  for  $a_i \in (a^*, \bar{a})$ . The approach taken here is to assume that such a Nash equilibrium exists and to show by construction what it must be.

We first seek  $a^*$  which is the no participation cut-off value of  $a_i$ , i.e., customers with  $a_i \in [\underline{a}, a^*)$  will defer their purchase to the second period. It is clear that individual  $i$  will win in the first period if his chosen  $t_i$  exceeds the  $m$ th largest chosen arrival time of his rivals. Since  $t(a_i)$  is strictly increasing in equilibrium, this means that customer  $i$  wins if his  $a_i$  exceeds the  $m$ th largest  $a_i$  in the population. Let  $f(\cdot)$  denote the density function of the order statistic of rank  $m$  among  $n - 1$  independent draws from the distribution with density  $g(\cdot)$ .  $F(a_i)$  is then the probability that customer  $i$  secures a unit in the first period. Therefore, the marginal customer  $a^*$  (i.e., who is indifferent between arriving at the time of product launch  $t_i = 0$  and delaying purchase) is one for whom

$$F(a^*)(a^*) + [1 - F(a^*)](\gamma a^*) = \gamma a^*,$$

$$a^* = 0,$$

which implies that the marginal customer has  $x_i = p$ .

Next, we seek  $t(a_i)$  for  $a_i \in (a^*, \bar{a})$ . The more general result of equilibrium bidding strategies given by equation (9) in Holt (1979) can be adapted to our setting, yielding

$$t'(a_i) = (1 - \gamma)a_i f(a_i).$$

A specific  $t(a_i)$  can then be found by using the boundary condition  $\lim_{a_i \rightarrow a^*, a_i > a^*} t(a_i) = 0$ . It can be verified that

$$t(a_i) = \int_{a^*}^{a_i} (1 - \gamma)x f(x) dx$$

satisfies the above conditions and verification that  $t(a_i)$  generates a Nash equilibrium follows similarly as in the Appendix of Holt and Sherman (1982).

**Proof of Lemma 2** Following the above discussion it is clear that as long as customers with higher  $x_i$  are associated with higher  $a_i$ , customers arrive in equilibrium in descending order of their valuations, and *vice-versa*. Let  $x_i$  and  $w_i$  be related through  $w_i = w(x_i)$  and note that all first period customers have  $x_i \geq p$ . We have  $a_i = \frac{x_i - p}{w(x_i)}$ , and  $\frac{da_i}{dx_i} = \frac{1}{w(x_i)} - \frac{(x_i - p)w'(x_i)}{w(x_i)^2}$ . Customers with relatively higher  $x_i$  have relatively higher  $a_i$  and therefore arrive relatively earlier in equilibrium if  $\frac{da_i}{dx_i} > 0$ . This implies  $\eta = \frac{(x_i - p)w'(x_i)}{w(x_i)} < 1$  as stated in the Proposition. Finally note that the latter condition is satisfied for homogeneous waiting costs per unit time (case  $w'(x_i) = 0$ ) as well as relatively higher-valuation customers incurring relatively lower waiting costs (case  $w'(x_i) < 0$ ).