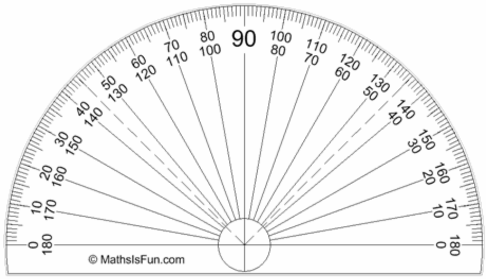
Chapter one

Motion in a Circle

Preliminary

(1) Degrees measure system

Each degree (1°) ° $\frac{1}{360^\circ}$ Of a complete revolution about the vertex



A Full Circle is 360°

Half a circle is 180° (called a Straight Angle)

Quarter of a circle is 90° (called a Right Angle)

We often measure degrees using a protractor

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(2) Radian measure system

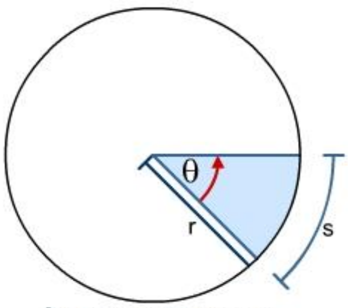
(One radian is the measure of a central angle θ that subtended by an arc equal in length to the radius r of the circle)*.*[Define: June 08, Nov 07]

$\theta^{rad} = \frac{\text{length of arc}}{\text{radius of circle}} = \frac{S}{r}$

So the angle in radians in a complete circle would be

$\theta^{rad} = \frac{\text{Circumference of the circle}}{\text{radius of the circle}} = \frac{2\pi r}{r} = 2\pi$

1 Radian is about 57.2958 degrees.



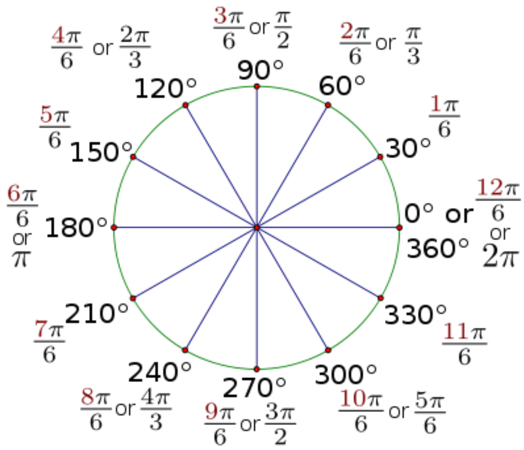
θ is measured in radians

(3) Relation between the degree measure and the radian measure

Because 2π radians corresponding to one complete revolution

Degrees and radians are related by the equations

$$\theta^{rad} = \frac{\pi}{180^\circ}\theta^\circ$$



Examples

a) 135° (Convert to radian)

$$=135^\circ \times \frac{\pi}{180} = \frac{3\pi}{4}$$

b) $-\frac{\pi}{2}$ (Convert to degree)

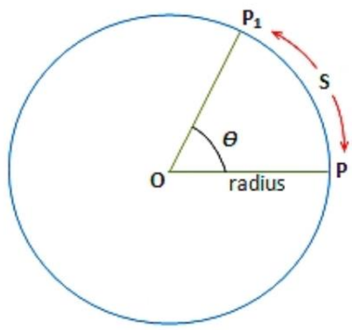
$$= -\frac{\pi}{2} \times \frac{180}{\pi} = -90^\circ$$

In circular motion

It is convenient to measure angles in radians rather than degrees

(4) Some Physical definitions

Suppose a particle travels from position (A) to position (B) along an arc (ΔS) in a time interval (Δt) as in the opposite figure.



(a) Angular Displacement $(\Delta\theta^{rad})$

$$\boxed{\Delta\theta^{rad} = \theta_f - \theta_i} \qquad (1)$$

Where: $p_1 = \theta_f$

$P = \theta_i$

(b) Angular velocity (ω)

- The angular speed is the angle swept out by the radius per second
- The angular velocity is the angular speed in a given direction

$$\boxed{\omega = \frac{\Delta\theta}{\Delta t} \qquad (rad\ s^{-1})} \qquad (2)$$

∵ At a specific point (A), the steady speed of the particle (V)

is given by $V = \frac{\Delta S}{\Delta t}$

, the radian angle $\Delta\theta = \frac{\Delta S}{r}$

And the arc $\Delta S = r\Delta\theta = r\omega\Delta t$

$$\therefore \boxed{V = \omega r} \qquad (3)$$

(c) Frequency (f)

It is the number of rotations in a given time

$$\boxed{f = \frac{\text{Number of Rotations}}{\text{Time taken}} \qquad (Hz)} \qquad (4)$$

(d) Period (T)

It is the time taken for one complete revolution

$$\boxed{T = \frac{1}{f} \qquad (Sec)} \qquad (5)$$

From Equations **(2)** and **(5)** we conclude that $\omega = \frac{2\pi}{T}$

Example (1):

An old record player spins records at 45 rpm (revolutions per minute). For a point on the circumference (radius = 10cm) calculate

a) The frequency

b) The angular speed in rad s^{-1}.

Solution:

- $45\,\text{rpm} = \frac{45}{60} = 0.75$ Revolutions per second

$\therefore f = 0.75\ \text{Hz}$

- $\omega = \frac{2\pi}{T} = 2\pi f = 2 \times 3.14 \times 0.75 = 4.7\ \text{rad s}^{-1}$

Example (2):

The minute hand on a watch is 6.40 mm long. calculate:

a) Its frequency

b) Its angular speed

c) The speed of its free end

Solution:

a) Frequency $f = \frac{1}{t} = \frac{1}{60 \times 60} = 2.78 \times 10^{-4}\ \text{Hz}$

b) Angular Speed $\omega = 2\pi f = 2 \times \pi \times 2.78 \times 10^{-4} = 1.75 \times 10^{-3}\ \text{rad s}^{-1}$

c) Speed $V = r\omega = 6.40 \times 10^{-3} \times 1.75 \times 10^{-3} = 1.12 \times 10^{-5}\ \text{ms}^{-1}$

According to Newton's First law

"Every body continues in its state of rest, or of uniform motion in a straight line, unless compelled to change that state by a net force."

In the opposite figure (1)

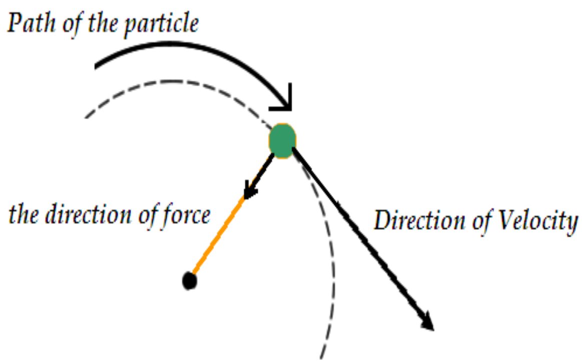
Fig(1)

The car is said to be moving with uniform motion if the resultant force (due to the applied force and the frictional force) equals zero.

Circular Motion

When an object moves in a circle at a constant speed its velocity (which is a vector) is changing not due to the change in the magnitude of the velocity but because their direction changes. A change in velocitymeans that the object is accelerating. This acceleration is towards the centre ofthe circle.

It is called the centripetal acceleration



For this acceleration there must be a resultant force

It is called the centripetal force

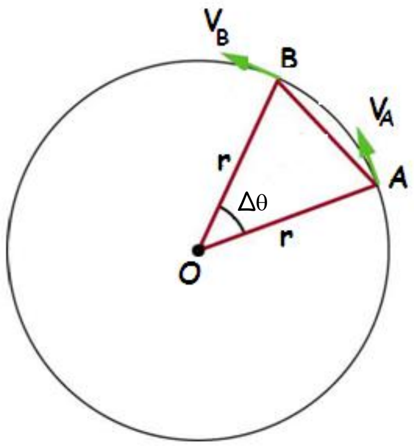
Note that

(1) The centripetal force acts towards the centre of the circle at right angle to the instantaneous velocity of the moving particle.

(2) Without the centripetal force, an object cannot travel in circular motion. In fact, if the forces are balanced, then an object continues in motion in a straight line at constant speed.

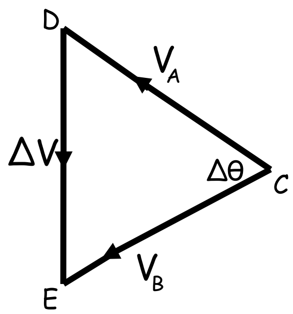
➢ Derivation of expression for centripetal acceleration

The opposite figure shows an object has travelled at constant Speed (V) in a circular path from A to B in time Δt where the velocity at A is V_A and the velocity at B is V_B. Both V_A and V_B are Vectors.



$$\therefore\ \Delta V = V_A + V_B$$

In Δ AOB consider the angle $\Delta\theta$ to be small that the arc AB may be approximated to a straight line. Then using the similar triangles OAB and CDE,



$$\therefore \frac{DE}{CD} = \frac{AB}{OA} \longrightarrow \frac{\Delta V}{\Delta t} = \frac{\Delta s}{r}$$

$$\therefore \qquad \Delta V = \Delta s \frac{V_A}{r}$$

Dividing both sides by ΔT

$$\frac{\Delta V}{\Delta T} = \frac{\Delta s}{\Delta T} \times \frac{V_A}{r} \qquad (1)$$

As $a = \dfrac{\Delta V}{\Delta T}$ and $V = V_A = V_B = \dfrac{\Delta s}{\Delta T}$

then equation (1) changes to

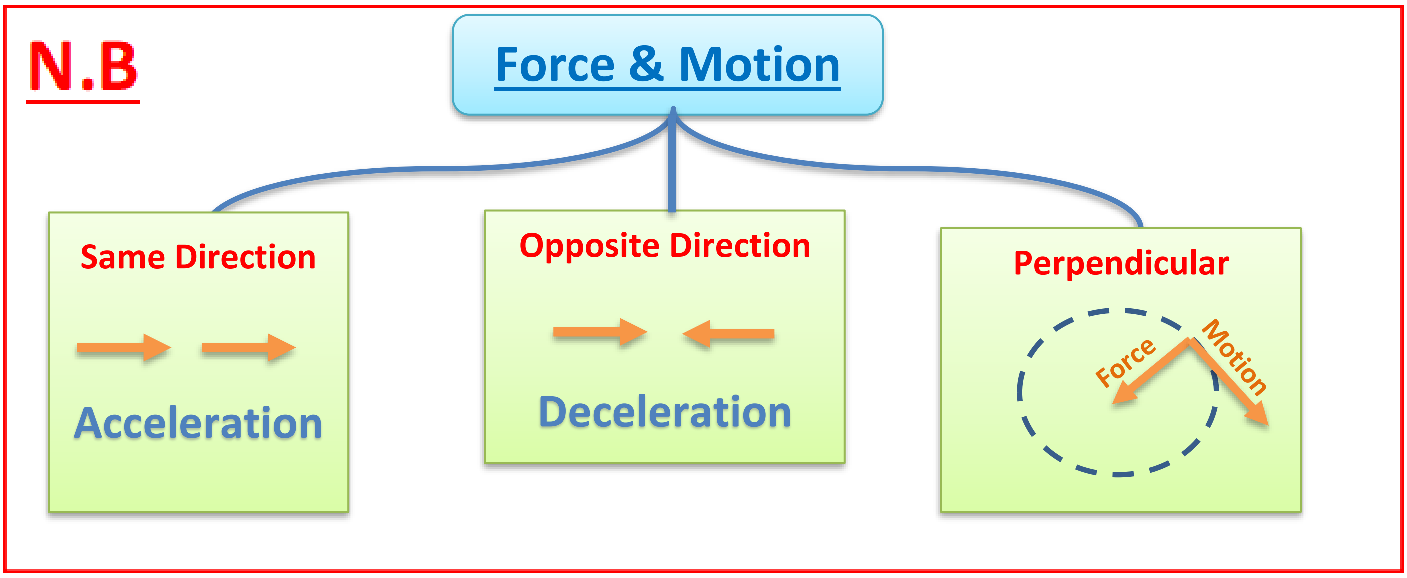
$$a = \frac{V^2}{r}$$

This expression can be written in terms of angular speed ω ,since $V = r\omega$

$$\therefore\ \text{Centripetal acceleration} = \frac{V^2}{r} = r\omega^2$$

and since $F = ma$

$$\therefore\ \text{Centripetal force} = \frac{mV^2}{r} = m\omega^2 r$$



S = linear displacement

θ = angular displacement

V = linear velocity

ω = angular velocity

$\omega = \theta/t$

$V = s/t$

$V=2\pi r/t$ and $\omega=2\pi/t = 2\pi f$

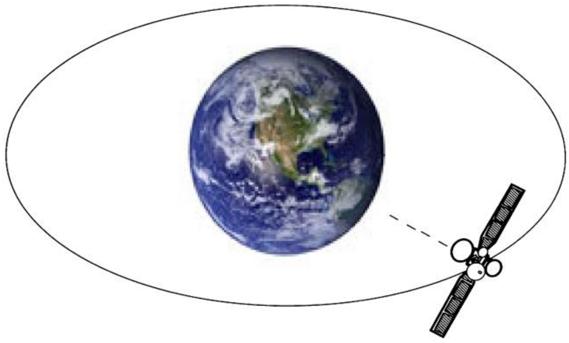
$V=\omega r$

$A=v^2/r$

➢ Example of Circular Motion

(1) The satellite in the earth orbit

A satellite in the earth orbit experiences gravitational attraction towards the centre of the earth. This attractive force provides the centripetal force and causes the satellite to accelerate towards the centre of the earth, and so it moves in a circle.



In a similar way, the moon is kept in orbit by the gravitational pull of the earth.

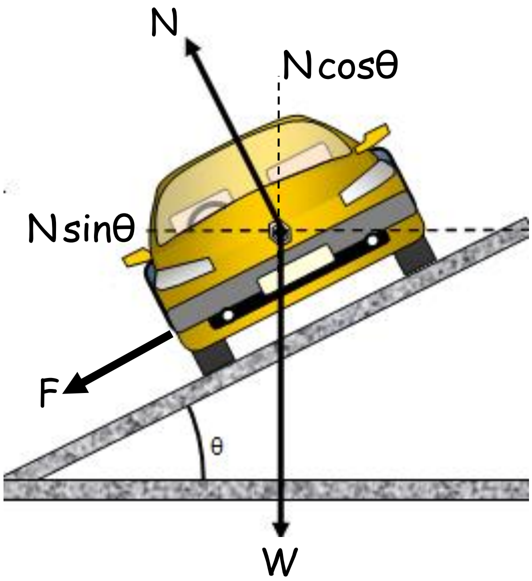
(2) A car on a banked track

The car in the opposite figure, the road provides two forces

1. the normal contact force (N)has a vertical component of N balances the car's weight. Therefore:

Vertically $N\cos\theta = mg$

2. The force of friction (F) between the tires and the road surface



The horizontal component of normal reaction and force of friction both have component that is needed to make the car go round the curve

Horizontally $N\sin\theta + F\cos\theta = \dfrac{mv^2}{r}$

Where r is the radius of the circular corner

V is the car's speed.

*Explain:

Many roads are banked for greater road safety

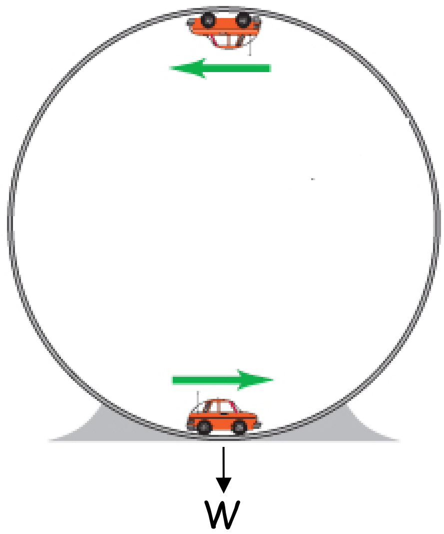
(3) Moving in vertical circles.

In this case you must have resultant force acting towards the centre of the circle

Consider a person moving round a vertical circle at speeded V, two main forces acting on the rider

1. The weight (W) which is unchangeable.
2. The reaction (R) that the seat exerts on the rider, it varies in size as the car goes round the circle.

➢ At the bottom of the ride



The reaction force from the seat must provide a centripetal force to overcome, so the contact force must be bigger than the weight.

$$R_B - W = \frac{mv^2}{r}$$

➢ At the top of the ride

Both the weight and the reaction force act downwards towards the centre of the circle, together they provide a centripetal force.

$$R_T + W = \frac{mv^2}{r}$$

➢ At the side

The reaction force alone provides a centripetal force; the weight has no component toward the centre of the circle.

$$R_s = \frac{mv^2}{r}$$

That is why you feel yourself being pushed into the seat more at the bottom .because the seats pushes up on you with a large force.

Remarks

(1) If a body of mass m is attached to a chord or an elastic string of length L, then the force exerted on this mass is given by

$$F = -kx$$

To return to its original length

This is called Hook's law

K is the string's constant

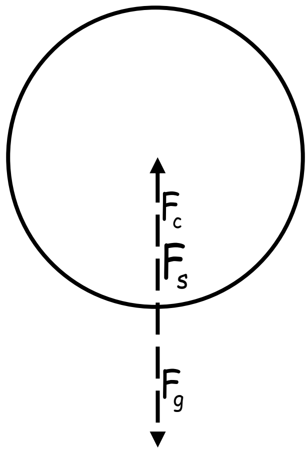
X is the position of the mass relative to the equilibrium position

1. Now, if the body is rotated about a point in a circular path of radius r

$$\therefore F_c + F_s = F_g$$

$$\therefore F_c + F_s = mg$$

$$\therefore m\omega^2 r - kx = mg$$



(Pendulum in vertical circular motion):

(2) In case of a body placed on a rotating plate, to find the maximum number of rotations to remain the body on the plate

The fractional force = the centripetal force

(3) The factors affecting the free fall acceleration of a planet

(a) The free fall acceleration of the planet

(b) The density of the planet

(4) On solving problems, Masses of the planets, moons, stars are considered to be constant.

N.B:

If you encounter a question like this: [June 20 V1, Q 1(b)]

A binary star system consists of two stars S1 and S2, each in a circular orbit. The orbit of each star in the system has a period of rotation T. Observations of the binary star from Earth are represented in Fig. 1.1.

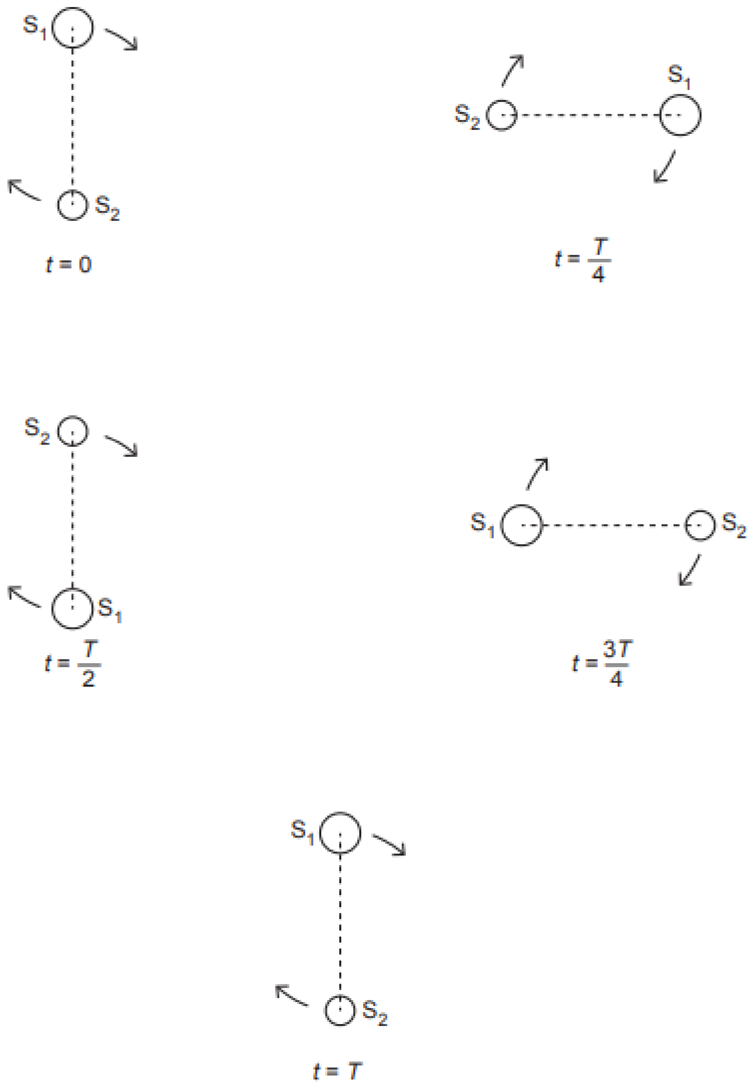


Fig. 1.1 (not to scale)

-Observed from Earth, the angular separation of the centres of S1 and S2 is 1.2×10^{-5} rad. The distance of the binary star system from Earth is 1.5×10^{17}m.

-Show that the separation d of the centres of S1 and S2 is 1.8×10^{12}m.

You could answer with:

Arc length = $r\theta$

$d = 1.5 \times 10^{17} \times 1.2 \times 10^{-5} = 1.8 \times 10^{12}$m

Chapter Two

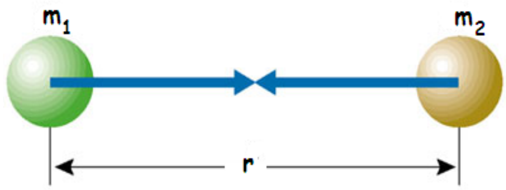
Gravitational fields

Isaac Newton was trying to find a way to explain why objects fell towards the centre of the Earth instead of simply staying put. He began to link the falling of an apple, with the "falling" of the Moon towards the Earth, and came up with his ***law of gravitation**. [*Define November 2009]

It states that:

[Two point masses attract each other with a force that is proportional to the product of their masses and inversely proportional to the square of their separation]

According to Newton, all masses create a gravitational field in the space around them. This field gives rise to a force on any object having mass placed in this field, where the strength of the field decrease as the separation between the two masses increases.



This is called the inverse square law

Using the above, Newton suggested that the force of attraction (F) between two masses m_1 and m_2 separated by a distance r by the relation

$$F \propto \frac{m_1 \times m_2}{r^2} \longrightarrow F = -G\frac{m_1 \times m_2}{r^2}$$

$$\boxed{F = -G\frac{m_1 m_2}{r^2}}$$

Where: G is a constant of proportionality known as the gravitational constant

Although, Newton wrote the equation for point masses we can use this law with real objects by assuming that the mass of each object is concentrated at its centre of mass. The distance r is measured between the centers of mass.

The minus sign represents fact that the force is attractive. The radial distance r is measured outwards from the attracting body; the force F acts in the opposite direction, and so F is negative.

Notes:

(1) The inverse square law is not a special condition for the gravitational field, it is true for anything which is a point source, such a light from a point or the amount of radiation emitted.

(2) Gravitational force is a mutual attraction .you attracts the earth with the same force that the earth attracts you!
However, this force has more effect on you because you are much lighter than the earth.

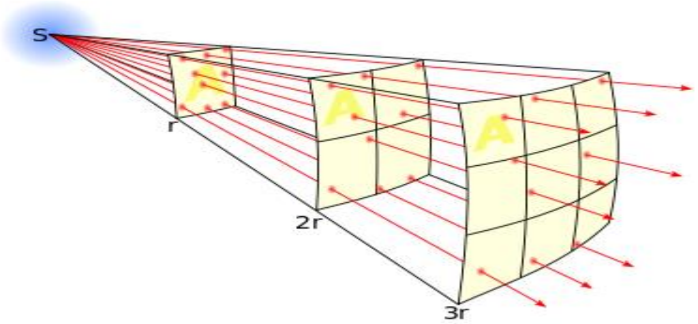
(3) G has a value of $6.67\text{x}10^{-11}$ N m^2Kg^{-2}.this is a very small number!
So the gravitational forces are very weak unless we are looking at the objects with enormous mass such as stars and planets.

(4) According to the inverse square law
$\therefore$ If the distance doubled, the force will be reduced to one-quarter from its original value as the field lines are spread out over four times the surface area

$$\therefore 2\times r \longrightarrow \frac{1}{4}\times F$$

$$3\times r \longrightarrow \frac{1}{9}\times F$$

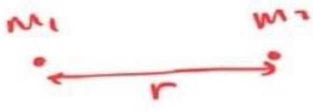
$$10\times r \longrightarrow \frac{1}{100}\times F$$



Care that:

$$F_{grav.} \propto m_1 \times m_2$$

$$F_{grav.} \propto \frac{1}{r^2}$$



$$F_{grav.} \propto \frac{m_1 \times m_2}{r^2}$$

$$F_{grav.} = \text{Cons}\ \frac{m_1 \times m_2}{r^2}$$

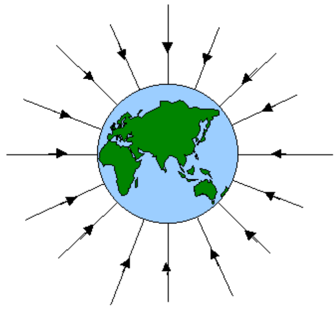
$$F_{grav.} = -G\,\frac{m_1 \times m_2}{r^2}$$

given

G = grav con. 6.67×10^{-11} N…

(−) → Attraction

We use the gravitational field lines to show the gravitational field around an object of mass.
For spherical objects such as the Earth the field is radial. All the field lines points towards the centre of mass (the centre of the sphere)
At the surface; field lines are almost parallel & equally spaced so gravitational field is uniform.



The field lines tell us

1. The direction of the field, this is the direction in which a mass in the field would be pulled.
2. The spacing of the lines tells us about the strength of the field so as we move away from the object (i.e. the Earth) the field lines get further apart as the field gets weaker.

Important

many questions require you to equalize between F_c and F_g

m

M

$$F_g = F_c$$

$$mg = GmM/r^2 = mv^2/r = m\omega^2 r$$

$(v=wr)$

$2\pi/T$

$$g = GM/r^2$$

$$V=\sqrt{(GM/R)}$$

$$\cancel{m}g = \frac{G\cancel{m}M}{r^2}$$

$$\frac{G\cancel{m}M}{r^{\cancel{2}}} = \frac{\cancel{m}v^2}{\cancel{r}}$$

$$V = \sqrt{\frac{GM}{r}}$$

❖ Gravitational field Strength (g)

[Define: May 2009]

The gravitational field strength at a point is the force per unit mass acting on a small mass placed at that point.

Suppose an attractive force (F) on a mass (m) caused by the mass (M)

$$\because F = -G\frac{Mm}{r^2}$$

$$\because g = \frac{F}{m}$$

$$\therefore g = -G\frac{M}{r^2}$$

Where: F is the force acting on a mass m in the field

The gravitational field strength (g) is a vector quantity

,it is measures in $N\ Kg^{-1}$,

- The field strength g is not a constant; it depends on the mass M of the body causing the field (i.e. Earth) and the distance r from the centre
- The field strength g also has units ms^{-2} it is an acceleration and it is also called (the acceleration of free fall)
- At the earth surface $g=9.8\ N\ Kg^{-1}$, in fact this value is varies slightly from place to place.

Example (1)

Find the mass of the earth given that its radius is 6400 km and g at the surface is $9.81\ N\ kg^{-1}$. ($G=6.67\times10^{-11}\ m^2kg^{-2}$).

Solution

$r=6400km=6400\times10^3m$

$$g = G\frac{M}{r^2} \rightarrow M = \frac{g\times r^2}{G} = \frac{9.81\times(6400\times10^3)^2}{6.67\times10^{-11}} = 6.02\times10^{24}Kg(3\,s.f.)$$

Example (2)

The radius of the earth is 6.4×10^6m, the gravitational field strength at its surface is 9.8 N Kg^{-1} and mass of the earth is 6.0×10^{24}kg.

(a) The radius of the moon's orbit about the earth is 3.8×10^8m. Calculate the strength of the Earth's gravitational field at this distance.

(b) The mass of the moon is 7.4×10^{22}kg.calculate the gravitational attraction between the earth and the moon.

(Gravitational constant $G=6.7 \times 10^{-11} Nm^2kg^{-2}$)

Solution

(a) $$g = G\frac{M}{r^2} = \frac{6.7 \times 10^{-11} \times 6.0 \times 10^{24}}{(3.8 \times 10^8)^2} = 2.8 \times 10^{-3} NKg^{-1}$$

(b) $$F = G\frac{Mm}{r^2} = mg = 7.4 \times 10^{22} \times 2.8 \times 10^{-3} = 2.1 \times 10^{20} N$$

Example (3)

Find the gravitational field strength at the surface and 1.0×10^{-11}mabove the surface of a star of radius 1.2×10^6m.

The mass of the star is 1.5×10^{30}kg and $G=6.67 \times 10^{-11} Nm^2kg^{-2}$.

Solution

(a) <u>At the surface</u>

$$g = G\frac{M}{r^2} = \frac{6.7 \times 10^{-11} \times 1.5 \times 10^{30}}{(1.2 \times 10^6)^2} = 6.9 \times 10^7 NKg^{-1} (2 s.f)$$

(b) <u>At 1.0×10^{-11}mabove the surface of a star of radius 1.2×10^6m</u>

(Remember that r is always measured from the centre of the star)

$$g = G\frac{M}{r^2} = \frac{6.7 \times 10^{-11} \times 1.5 \times 10^{30}}{(1.2 \times 10^6 + 1.0 \times 10^6)^2} = 2.1 \times 10^7 NKg^{-1} (2 s.f)$$

❖ Gravitational Potential (ϕ)

The gravitational potential at a point in a field is defined as the work done in bringing unit mass from infinity to the point.

$$\phi = -\frac{GM}{r}$$

Where, ϕ is the gravitational potential of the field.

The unit of (ϕ) is J Kg^{-1} (Joules per kilogram)

- The gravitational potential at a point tells us the potential energy of each kilogram of mass at that point
- If you lift an object from the ground , its The gravitational potential energy increases , as the work done on it increases

The object's change in the potential energy = $mg\Delta h$

Where, Δh is the change in its height

From this relation we concluded that

1. ϕ=0 when Δh=0 (the object is on the surface o the Earth)
2. Increasing Δh (by lifting the object)

- As we lift an object (increasing Δh) the magnitude of ϕ increases and eventually reaches zero at infinite distance this is cannot be happened except the gravitational potential is always negative.

When an object moves from one point in a field to another it produces a work against gravity leading to an energy change (ΔE) or work done

$$\Delta E = \Delta\phi \times m$$

Where: ΔE is the work done by the object (its unit is Joule)

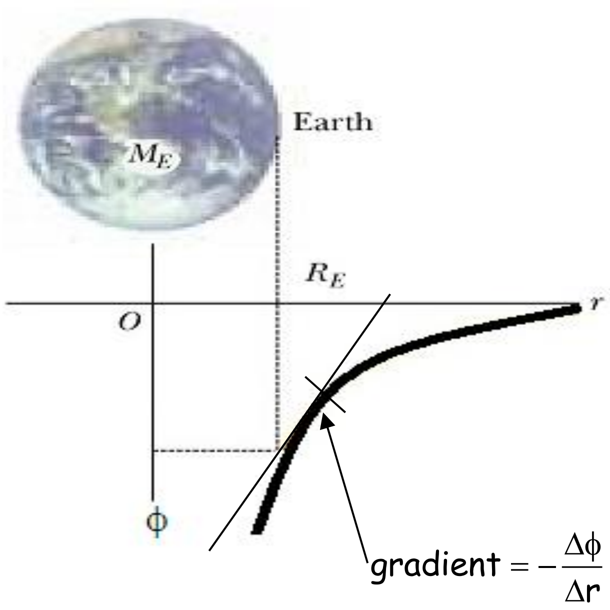
M is the mass of the object.

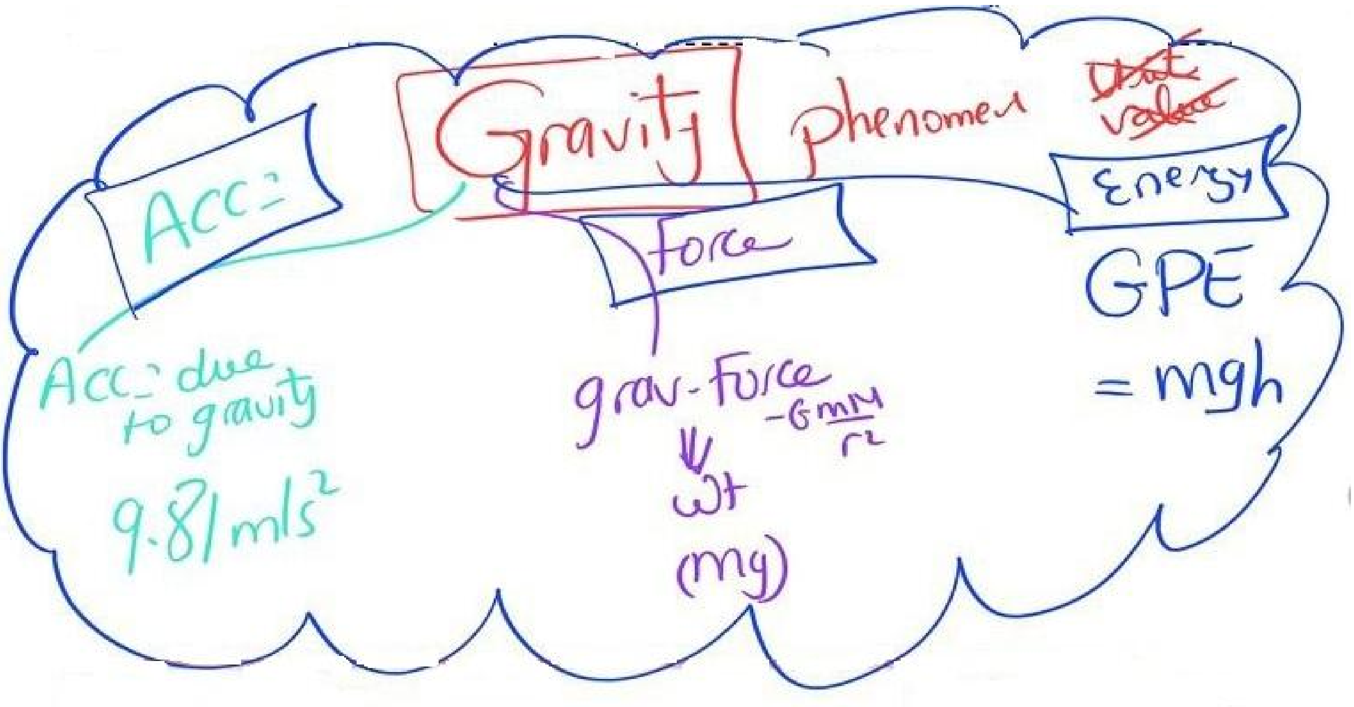
$\Delta\phi$ Is the change in gravitational potential (change in energy per kilogram)

The gravitational Potential (ϕ) and field Strength (g)

We can notice that the gradient gradually decreases and the value of the gradient at any point gives the value of the gravitational field strength at that distance

$$\boxed{g = -\frac{\Delta\phi}{\Delta r}}$$





gravity → Phenomena (No Value / No Unit)

- **Acceleration due to gravity** g=9.8 m/s^2.
- **grave. force** F_g =wt=mg.
- **gravitational potential energy(gpe)**

Example (1)

Calculate the energy change when a rocket of mass 50000 Kg moves from the surface to a height of $3.50x10^6$ m above the Earth. The Earth has a mass of $6.00x10^{24}$ kg and a radius of $6.40x10^6$m .($G=6.67x10^{-11}Nm^2Kg^{-2}$)

Solution

- At the surface of the Earth

$$\phi_1 = -\frac{GM}{r} = -\frac{6.7\times10^{-11}\times6.00\times10^{24}}{6.40\times10^{6}} = -6.25\times10^{7}\,JKg^{-1}$$

- At a height of $3.50x10^6$ m above the Earth

[The distance (r) from the centre of mass=($6.4x10^6+3.50x10^6$)m]

$$\phi_2 = -\frac{GM}{r} = -\frac{6.7\times10^{-11}\times6.00\times10^{24}}{(6.40\times10^{6}+3.50\times10^{6})} = -4.04\times10^{7}\,JKg^{-1}$$

- The change in the potential

$$\therefore \Delta\phi = \phi_2 - \phi_1$$
$$= (-4.04\times10^{7}) - (-6.25\times10^{7})$$
$$= 2.21\times10^{7}\ J(3\,s.f)$$

- The total energy change ΔE

$$\Delta E = \Delta\phi\times m$$
$$= 2.21\times10^{7}\times 50\,000$$
$$= 1.11\times10^{12}\ J\,(3\,s.f)$$

Orbiting Under Gravity

Most planets in the solar system have orbits which are nearly circular.
Consider a planet of mass m in circular orbit about the sun of mass M .if the radius of the orbit is r.

The gravitational Force F_{Grav} between the sun and planet is

$$\boxed{\therefore F_{grav} = \frac{GMm}{r^2}}$$

From Newton's law of gravitation

The centripetal force F_{circ} is Given By

$$\boxed{\therefore F_{Circ} = \frac{mv^2}{r}}$$

Where v is the linear speed of the planet

During Circular Motion

$$F_{grav} = F_{Circ}$$

$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

And the speed of any object (Satellite) moving in a circular motion could be calculated from the relation

$$\boxed{v^2 = \frac{GM}{r}} \qquad (1)$$

From Equation (1) we can observe that

1. In order for a satellite to maintain a particular orbit , it must travel as the correct speed v
2. The mass of the satellite m has cancelled out. The implication of this is that all satellites whatever their masses will travel at the same speed v in a particular orbit.
3. The closer the satellite is to the Earth, the faster it must travel.
4. If it travels too slowly, it will fall down on Earth
5. If it travels too quickly, it moves into a higher orbit
6. Relation between velocity of body in orbit and its radius in that exact same orbit:

The period T of the planet in its orbit is the time required for the planet to travel a distance $2\pi r$. it is moving at speed v, so

$$Fg = Fc$$

$$G\cancel{m}M/r^2 = \cancel{m}w^2r \qquad w=2\pi/T \longrightarrow w^2=4\pi^2/T^2$$

$$GM/r^3 = 4\pi^2/T^2$$

$$T^2 = 4\pi^2r^3 / GM$$

$$T^2 \alpha\ r^3$$

Equation (2) States that

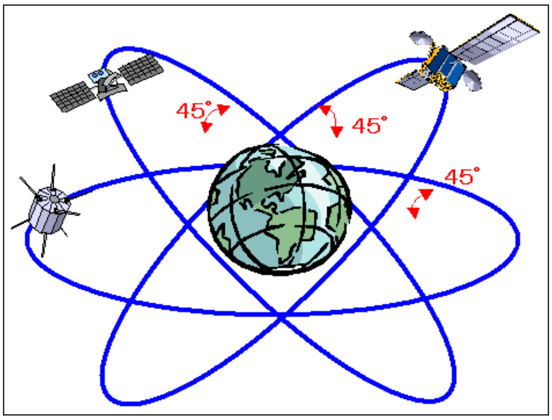
> For planets or satellites describing circular orbits about the same central body, the square of the period is proportional to the cube of the radius of the orbit.

This relation is known as Kepler's Third law of planetary motion.
In fact: the orbits of the planets are not circular but elliptical, this fact recognized by Kepler., as it

Geostationary orbits

(Earth view from geostationary satellite):

It is a special type o orbits in which the satellite is positioned so that, the satellite remains above a fixed point on the Earth's surface, all satellites on this orbit have the same period of rotation as the Earth(T=24 Hours) and they are widely used for telecommunications transmissions and for television broadcasting.



Geostationary Satellite

1) **At the equatorial plane.**
2) **Same time of rotation 24h.**
3) **Same rotation as earth (west → east).**

Example (1)

For a geostationary satellite, calculate:

(a) The height above the Earth surface,

(b) The speed in orbit.

($G=6.67\text{x}10^{-11}$ N kg^{-2}, $r_{Earth}=6.38\text{x}10^{6}$m and $M_{Earth}=5.98\text{x}10^{24}$kg)

Solution

(a) The period of satellite is 24 hours=$5.98\text{x}10^{4}$s.

$$\because T^2 = \frac{4\pi^2}{GM} r^3$$

$$\therefore r^3 = \frac{GMT^2}{4\pi^2}$$

$$= \frac{6.67\times10^{-11}\times5.98\times10^{24}\times(8.64\times10^{4})^2}{4\times\pi^2}$$

$$= 4.23\times10^{22}\,m^3$$

$$r = \sqrt[3]{4.23\times10^{22}} = 4.23\times10^{7}\,m$$

The distance above the Earth's surface is

$$\Delta r = 4.23\times10^{7} - 6.4\times10^{6}$$

$$= 3.59\times10^{7}\,m$$

(b)

$$\because v = \frac{2\pi r}{T}$$

$$\therefore v = \frac{2\pi\times4.23\times10^{7}}{8.64\times10^{4}} = 3070\,m\,s^{-1}$$

Example (2)

Calculate the period of a satellite in orbit 500 Km above the Earth.

($G=6.67\times10^{-11}$ N kg^{-2}, $r_{Earth}=6.38\times10^{6}$m and $M_{Earth}=6\times10^{24}$kg)

Solution

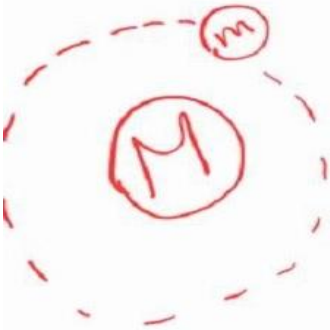
$$\because T^2 = \frac{4\pi^2}{GM}r^3$$

$$\therefore T = \sqrt{\left(\frac{4\pi^2(6.4\times10^6+5.0\times10^5)^3}{6.67\times10^{-11}\times6.0\times10^{24}}\right)}$$

$$= 5.69\times10^3 S$$

In order to Make the orbit geostationary, we have to make the period of the satellite equal to the period of the Earth (24 hrs) by positioning the satellite far enough From the Earth.

Care that:



$$\underset{\sim}{F_g} = F_c$$

$$\underset{\sim}{mg} = \boxed{-\frac{GmM}{r^2} = \frac{mv^2}{r}} = m\omega^2 r$$

$$-\frac{G\cancel{m}M}{r^{\cancel{2}}} = \frac{\cancel{m}v^2}{\cancel{r}}$$

$$\sqrt{\cancel{-}\frac{GM}{r}} = V$$

The analogy between Gravitational fields and Electric fields

Similarities	*Differences*
the variation of force with distance follows the inverse square law $F\alpha\frac{1}{r^2}$	Gravitational fields can only produce forces of attraction Electric fields can produce attraction and repulsion
the potential is inversely proportional with the distance $\phi\alpha\frac{1}{r}$	Objects can be shielded fr electric field, but there is no way to shield an object from a gravitational field
Both exert a force from a distance, with no contact	Electric fields only act upon charged masses. Gravitational fields act upon all masses

Remarks

(1) **The Kinetic energy of a body is given by**

$$K.E = \frac{1}{2}mv^2$$

For a satellite $V^2 = G\frac{M}{r}$

Where M is the Mass of the Earth

(2) **One Adventure of Launching satellites from the equator in the direction of the rotation of the earth** is getting some boost in velocity from the earth rotation by orbiting in the same direction.

(3) **When the total Energy of a satellite**

$$E = -G\frac{Mm}{r} + G\frac{Mm}{2r} = -G\frac{Mm}{2r}$$

(4) **The Change in K.E of a satellite**

$$\Delta K.E = \frac{1}{2}m\Delta v^2 = \frac{1}{2}m(v_1^2 - v_2^2)$$

(5) **To determine the mass of a planet we assume that**

(a) No energy dissipated due to friction with the atmosphere.

(b) Rockets must be outside atmosphere.

(c) It is not influenced by another planet.

(6) The satellite wouldn't be in its geostationary orbit when its period not equal 24hrs.

(7) For a point outside the sphere, the mass of the sphere may be considered to be a point mass at its centre.

(8) If the gravitational fields of 2 planets are in opposite directions, there's a point between them that has no field or the field strength in it equals zero.

(9) Why near the Earth's surface g is approx. constant?

*Lines are radial

*Earth has a large radius

*Parallel lines so constant field strength

*So constant acceleration of free fall "g"

(10) Why for small changes in height near Earth's surface, gravitational potential is approx. constant?

*Height change is much smaller than the Earth's radius

*ϴ α 1/R, R is approx. constant so ϴ approx. constant

(11) If speed is constant $F_g = F_c \longrightarrow$ *Most of the quer*

If speed changing $GPE = KE \longrightarrow$ *M18/41/No1*

N.B:

If you encounter a question like this: [Nov 18 V1-V2, Q 1(a)(ii)]

Suggest why, for small changes in height near the Earth's surface, gravitational potential/gravitational field strength is approximately constant.

You could answer with:

-Near Earth's surface change in height is much smaller than the radius **OR** height much less than radius. **(1 mark)**
-Gravitational potential is inversely proportional to the radius of the Earth and radius is approximately constant (so potential approximately constant). **(1-2 marks)**

If you encounter a question like this: [Nov 18 V2, Q1 1(c)]

The planet has radius **R** equal to 3.4×10^3 km and mean density 4.0×10^3 kgm^{-3}. Calculate the acceleration of free fall at a height R above its surface.

You could answer by:
-Calculating the volume using the radius of the planet and then calculating the mass of the planet using the newly found volume and the given density **(1 mark).** Then use mass found and radius given in equation to calculate free fall/field strength.

$M = (\frac{4}{3} \times \pi R^3) \times \rho$ $\qquad$ $M= (\frac{4}{3} \times\pi\times (3.4\times10^3 \times10^3)^3) \times4\times10^3= 6.6\times10^{23}$

$g = GM/(2R)^2$

$$g= \frac{6.67\times10^{-11}\times6.585^{23}}{(2\times3.4\times10^6)^2} = 0.95 \text{ m s}^{-2}$$

If you encounter a question like this: [March 18 V2, Q 1(b)(ii)]

The Moon may be considered to be a uniform sphere that is isolated in space. It has radius 1.74×10^3 km and mass 7.35×10^{22} kg.

A satellite is in a circular orbit about the Moon at a height of 320km above its surface. Calculate the time for the satellite to complete one orbit of the Moon.

You could answer with:

$F_c = F_g$

$m\,\omega^2 r = \frac{GMm}{r^2}$ and $\omega = \frac{2\pi}{T}$

$$T^2 = \frac{4\pi^2 r^3}{GM}$$

$$T^2 = \frac{(1.74 \times 10^6 + 320 \times 10^3)^3 \times 4\pi^2}{(6.67 \times 10^{-11} \times 7.35 \times 10^{22})}$$

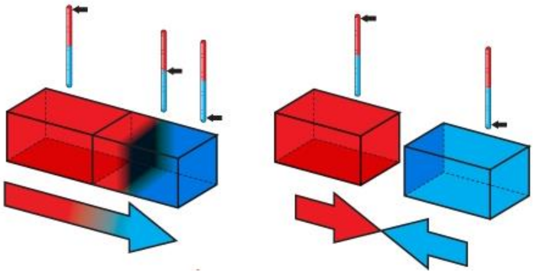
$T^2 = 7.04 \times 10^7$

T = **8400 s** (8390)

Chapter Three

Temperature

Thermal Equilibrium



For two objects A and B in contact, if heat flow from A to B, then a is at higher temperature than B.
So, when the heat flow from A to B is zero then the two objects are said to be in thermal Equilibrium and at the same temperature.

The meaning of temperature

Temperature is a degree of hotness.

Measuring temperature of an object using thermometer depends mainly on a physical property which is called the thermometric property

The thermometric property is some property of a material that varies continuously with the temperature

Examples

1. The length of column of mercury in a glass capillary tube.
The thermometric substance is mercury
The thermometric property is the expansion of mercury inside the tube when heated.
2. The resistance of a coil of platinum wire.
The thermometric substance is platinum.
The thermometric property is increasing the resistance of the wire by increasing temperature.
3. The voltage of metal in thermocouple.
The thermometric substance is two different metals.
The thermometric property is the change in voltage due to change in temperature.

Temperature scales

Celsius scale

It is based on the properties of water. It takes two fixed points, the melting point of pure ice and the boiling point of pure water, and divides the range between them into 100 equal intervals.

- Disadvantages of Celsius scale

There is nothing special about these two fixed points, both change if the pressure changes or if the water is impure.

The thermodynamic scale (Kelvin scale)

It is based on the relation between pressure P, Volume V and Temperature T.

As $\frac{PV}{T}$ = constant

We use this property to define the thermodynamic scale of temperature.
by plotting a relation between Pressure and temperature at constant volume gas, there seems to be a natural zero of temperature which is called the absolute zero or (0 Kelvin)

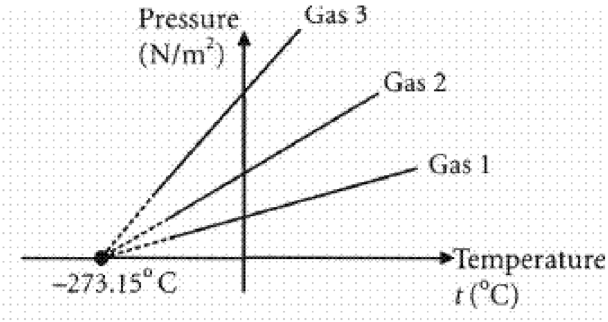


Figure 11.1

This is used as one of the fixed points of the thermodynamic scale. The upper fixed point is taken as the triple point of water (the temperature at which ice, water and water vapour are in equilibrium).
At the absolute zero (0K) the substances will have minimum internal energy!
Experimental data showed that the absolute zero= -273.15°C.

- Advantages of Kelvin scale

It is related to the average kinetic energy per particle and not to any thermodynamic property.

- Celsius temperature (θ) and Kelvin temperature (T)

$$\theta(^{\circ}C)=T(K)-273.15$$

Temperature: measure of average kinetic energy of molecules

- Celsius scale is based on the properties of pure water and has 2 fixed points:
  $0^{\circ}C$ (melting point of pure ice)
  $100^{\circ}C$ (boiling point of pure water)

- Those 2 fixed points can change with pressure changes or if the water is impure.

Absolute zero (0 K): fixed point of Kelvin scale
Temperature at which it's impossible to remove any more energy from mater
Hence, it's the temperature where all substances have minimum internal energy

Thermodynamic "Kelvin" scale does not depend on a property of substance

Based on the fact that the average kinetic energy of all substances is the same at a particular thermodynamic temperature

- Fixed points of a Kelvin scale :
  O K
  $273.16\ K = 0.01\ ^{\circ}C$ (temp at which water, ice and water vapor can co-exist)
  "triplet point of water"

Thermometers

In Choosing thermometer for a particular application needs a number of aspects to include accuracy, sensitivity and the range of temperatures it is able to measure.
In some cases thermometers should have

1. High response for varying temperature
2. Small sensitive part that does not absorb much heat or change the temperature of the object during measurements.

Examples

(1) The liquid in glass type

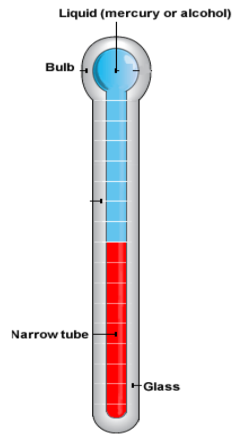
Principle: the expansion of the liquid with increasing temperature.

Range: mercury in glass (covers the range from -40°C to 350°C)

Ethanol in glass (covers the range from -120°C to 80°C)

Advantages: convenient and sensitive.

Disadvantages: fragile and not suitable for small objects



(2) Resistance thermometers

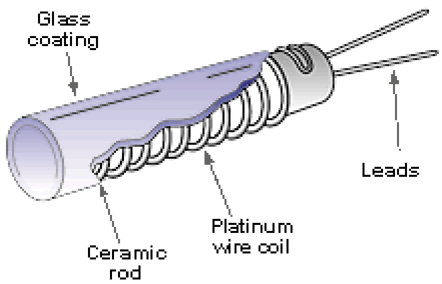
(i) Platinum resistance thermometer

Principle: increasing the resistance with increasing temperature.

Range: covers the range from -260°C to 1700°C

Advantages: wide range, accurate and sensitive.

Disadvantages:



a. the variation of resistance of a metal wire with temperature Is not exactly linear so the thermometer needs to be calibrated at a number of standard temperatures. However over a small range of temperature, the variation of resistance is linear.

b. Not suitable for small objects

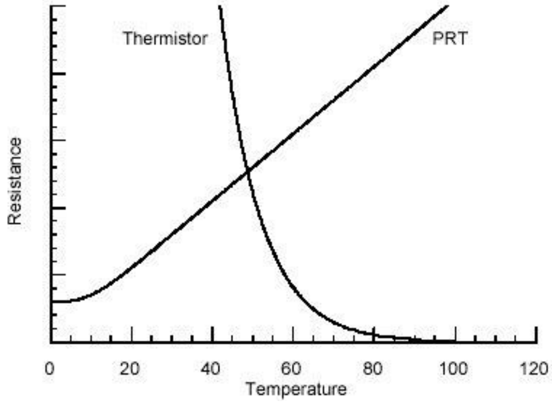
c. Slow response

(ii) Thermistor

Principle: decreasing the resistance with increasing temperature.

Range: covers the range from 250°K to 450°K

Advantages: suitable for small objects, can be linked to another circuit or computer and very sensitive (because of the rapid change in resistance).



Disadvantages:

a. the variation of resistance of a metal wire with temperature

Is not exactly linear so the thermometer needs to be calibrated at a number of standard temperatures. (Limited range)

b. not very accurate

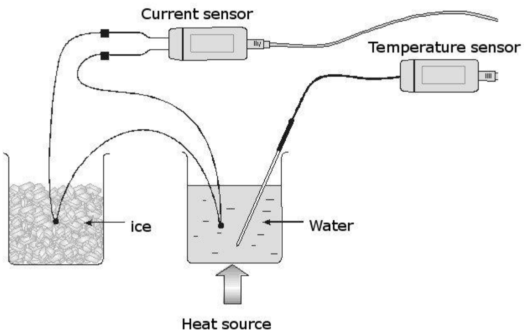
c. Slow response

(iii) Thermocouple

Principle: depends on the **thermoelectric effect**. When the junction of two different conductors is at different temperatures, an E.M.F (Voltage) is developed

Range: covers the range from 80°K to 1400°K

Advantages: Wide range, small, quick response, can be linked to another circuit or computer and can measure the temperature of small objects.



Disadvantages:

Less accurate than constant volume gas and platinum resistance thermometers.

video showcasing the thermocouple:

Remarks

(1) To find the steady temperature (Final temperature) for a material , use The idea of heat lost by material = heat gained by the thermometer

$$m_1C_1(T_i - T_F) = m_2C_2(T_i - T_F)$$

Where T_i: is the initial temperature
T_F: is the final temperature

(2) Keep in mind the range of temperature measured by each thermometer which are

(a) For Liquid thermometer

Hg-in glass **-40°C → 350°C**

Al-oH-in glass **-120°C → 80°C**

(b) **Resistance thermometers**

-260°C → 1700°C

(3) The boiling point of any material is constant and it is not affected by the thermometer bulb energy.

(4) <u>In any Heater</u>

Electrical Energy consumed in heater=Heat gained by liquid

$$\text{Power} \times \text{Time} = mc\Delta\theta$$

The mass per time (Flow rate)

$$\frac{m}{t} = \frac{\text{Power}}{C\Delta\theta} \qquad gm/sec \text{ or } Kg/sec$$

Chapter Four

Thermal Properties of materials

Specific Heat Capacity

When a solid, a liquid or a gas is heated, its temperature rises. Plotting a graph of thermal (Heat) energy (ΔQ) supplied against temperature rise ($\Delta\theta$) we find that for a particular substance:

$$\Delta Q \propto \Delta\theta$$

Also, another relation between the heat energy (ΔQ) and the mass (m) of the substance:

$$\Delta Q \propto m$$

Combining the two relations

$$\boxed{\Delta Q = m\,C\,\Delta\theta}$$

The thermal energy (J) =mass (Kg) x Specific heat capacity x Temperature change

C is the constant of proportionality known as the Specific heat capacity of the substance.

Specific Heat capacity is the quantity of heat required to raise the temperature of unit mass of the substance by one degree.

The SI unit of specific heat capacity is J Kg^{-1} K^{-1}

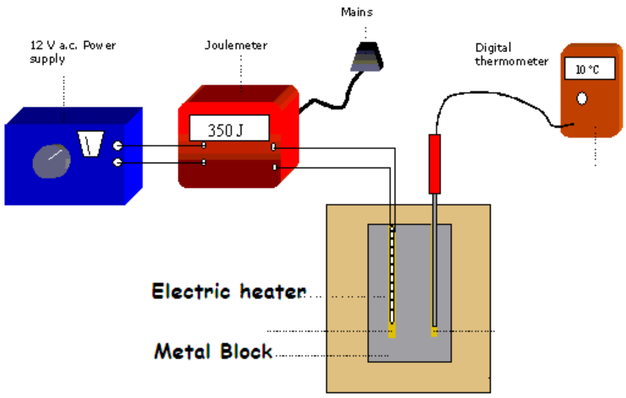
Remarks

1- The specific heat capacity depends on the substance; it is different for different materials.

2- The word specific means per unit mass.

Measuring Specific Heat Capacity (Electrical Method)

(1) For solids



- Procedure.

1- The mass m of the block is found, and its initial temperature θ_1

2- The block is lagged with expanded polystyrene.

3- A suitable steady current switched on as a stop-clock is started.

4- The voltmeter, and ammeter readings V and I are noted.

5- When the temperature has risen by about 10K, the current is stopped and the time t taken for which it passed

6- The highest reading θ_2 on the thermometer is noted.

- Calculations

Assuming that no energy loss occurs we have:

Electrical energy supplied by heater= Heat received by block

$$ItV = mC(\theta_2 - \theta_1)$$

Hence

$$\boxed{C = \frac{ItV}{m(\theta_2 - \theta_1)}}$$

Remarks

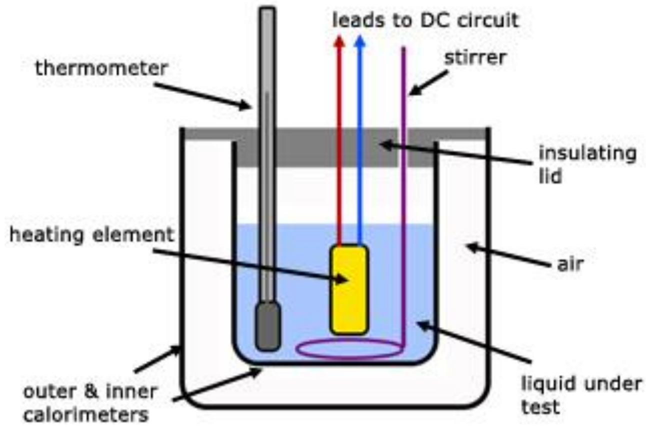
i. If I in amperes, t in seconds, Vin volts, m IN g, θ_1 and θ_2 in K, then C in J $g^{-1}K^{-1}$.

ii. The small amount received by the thermometer and the heater has been neglected.

(2) For liquids

- Procedure.

It is similar to that for solids except that the liquid is stirred continuously during the heating.



m is the mass of liquid.

C its specific heat capacity.

m_c the mass of the calorimeter and stirrer.

C_c is the specific heat capacity of the calorimeter and stirrer

- Calculations

Assuming that no energy loss occurs, we have:

$$\begin{pmatrix}\text{Energy supplied}\\ \text{by heater}\end{pmatrix} = \begin{pmatrix}\text{Energy received}\\ \text{by Liquid}\end{pmatrix} + \begin{pmatrix}\text{Energy received}\\ \text{by Calorimeter and stirrer}\end{pmatrix}$$

$$\begin{aligned} Itv &= mC(\theta_2 - \theta_1) + m_c C_c(\theta_2 - \theta_1) \\ &= (mC + m_c C_c)(\theta_2 - \theta_1) \end{aligned}$$

If C_c is known, c could be calculated

Sources Of errors in the experiment

(1) It is desirable to have a relatively low rate of heating so that energy spread throughout the block.

(2) Thermal insulation of the material is very important so that no energy escape happened,

(3) Cooling the block below the room temperature before beginning to heat it is vital so that as temperature rises past room temperature heat loses will be zero in principle. Because no temperature difference between the block and its surrounding.

Specific Latent heat

At times when the substance is changing phase (ice to water or water to steam), heat energy is being supplied without any change of temperature. Because the heat energy does not change the temperature of the substance, it is said to be latent (i.e. Hidden), the latent heat required to melt (fuse) a solid is known as Latent Heat of fusion.

to calculate the energy needed for a change of state,we use:

$$\boxed{\Delta Q = m\,L}$$

The thermal energy (J) =mass (Kg) x Specific latent heat capacity (J kg^{-1})

L is the constant of proportionality known as the Specific Latent heat of the substance.

Latent Heat of fusion is the Quantity of heat energy required to convert unit mass of solid to liquid without any change in temperature

The SI unit of specific heat capacity is J Kg^{-1}

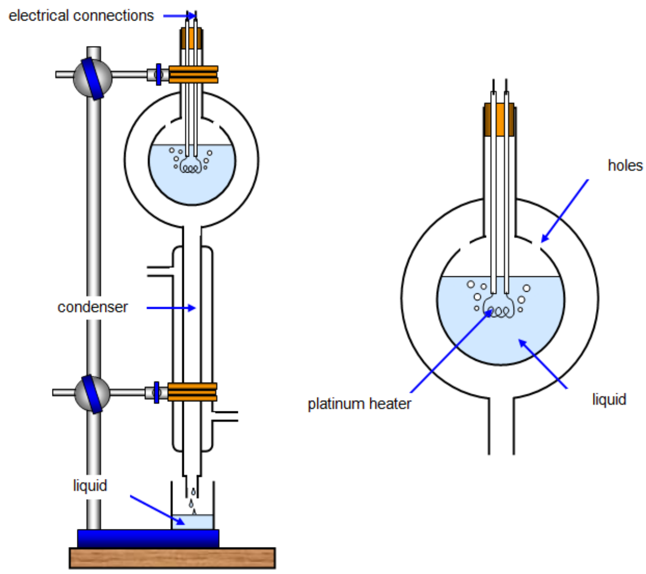
Materials generally have two latent heats, one for melting (or fusion), L_f, and one for boiling (or vaporization), L_v. (the external atmospheric pressure at which latent heat is measured must be specified, since it affects the amount of energy required to change the state of a substance.) in most cases $L_v > L_f$ because the large volume change which occurs on vaporization of a liquid.

The specific latent heat of fusion is the heat energy needed to change 1 kg of the material in its solid state at its melting point to 1 kg of the material in its liquid state, and that released when 1 kg of the liquid changes to 1 kg of solid.

The specific latent heat of vaporization of a liquid is the heat energy needed to change 1kg of the material in its liquid state at its boiling point to 1 kg of the material in its gaseous state, and that released when 1 kg of vapor changes to 1 kg of liquid.

Measuring Specific latent heat (Electrical Method)

A value can be found for L_v by a continuous flow method using the apparatus of fig.



The liquid is heated electrically by a coil carrying a steady current I and having a potential difference V across it.

Vapour passes down the inner tube of a condenser where it is changed back to liquid by cold water flowing through the outer tube.

After the liquid has been boiling for some time it becomes surrounded by a jacket of vapour at its boiling point, and a steady state is reached when the rate of vaporization equals the rate of condensation.

All the electrical energy supplied is then used to supply latent heat to the liquid (and none of it is used to raise its temperature) and to make good use of any heat loss from the jacket.

If mass **m** of liquid is now collected in time **t** from the condenser, we have:

$$\boxed{ItV = mL_V + h}$$

Where L_v is the specific latent heat of vaporization of the liquid.

h is the heat lost from the jacket in time **t** .

Remarks

(1) Note that if a piece of material is thrown in a liquid in a beaker or a vessel.

Then

(a) The quantity of heat lost by the piece of material= the heat gained by the beaker of vessel.

(b) The heat will transfer till the thermal equilibrium takes place and the mixture has a final temperature θ_f

(2) In case of Ice, to convert it into water

The energy required = the latent heat+ melting energy

(3) The work done on the system=P(V_2-V_1)

(4) To get the binding energy per molecule

$$\text{Binding energy per molecule}=\frac{\text{Total Energy}(\Delta U)}{\text{Number of molecules}(N_A)}$$

(5) When boiling water

i. The separation between molecules increases.

ii. The bond between molecules are broken

iii. A work produced against the atmosphere during expansion.

(6) The liquid is boiling at constant rate if the mass evaporated at constant rate or produced bubbles at constant rate.

(7) The temperature couldn't measure the amount of thermal energy because

(a) It shows whether the body in thermal equilibrium or not (I.e. Heat transfer from High to low)

(b) On melting or boiling, no change in temperature happened.

N.B

If you encounter a question like this: [March 20 V2, Q2(b)]

A large container of volume 85m^3 is filled with 110kg of an ideal gas. The pressure of the gas is 1.0×10^5Pa at temperature T.

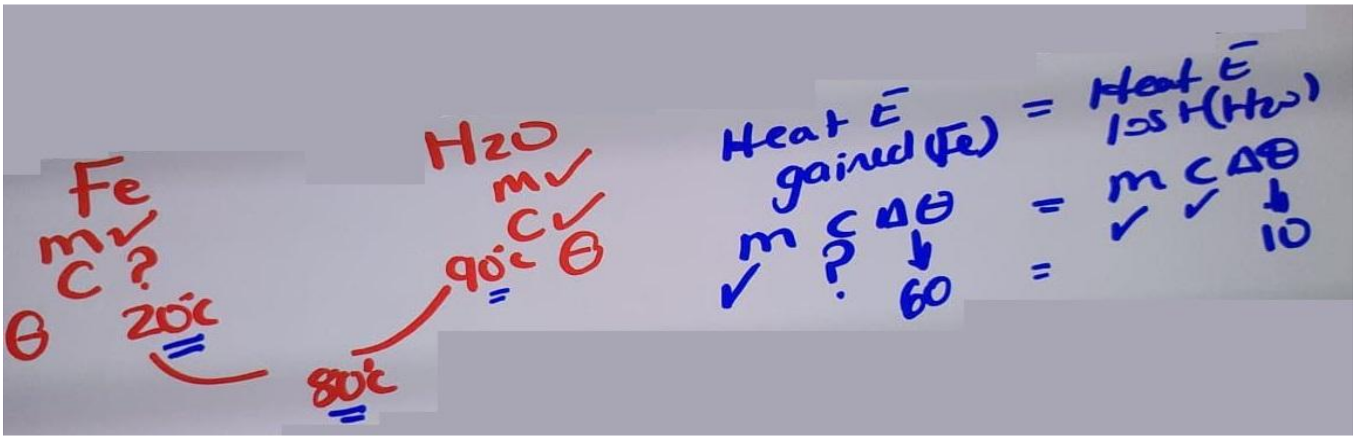
The mass of 1.0 mol of the gas is 32g

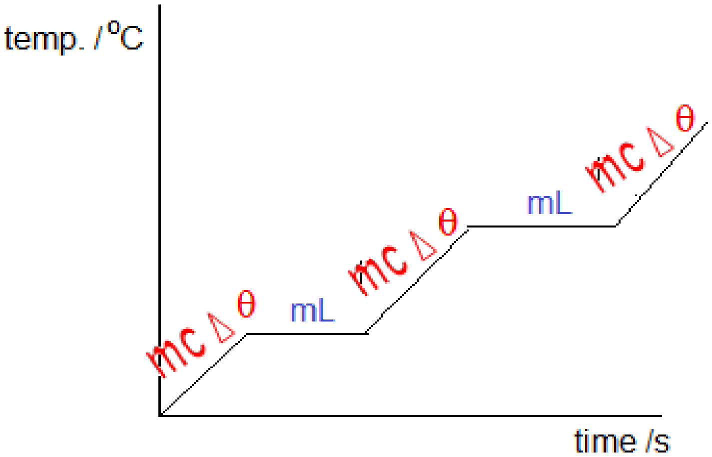
The temperature of the gas is increased to 350K (initially 300K) at constant volume. The specific heat capacity of the gas for this change is 0.66J kg^{-1}K^{-1}.

Calculate the energy supplied to the gas by heating.

You could answer with:

$\Delta Q = mc\Delta\Theta = 110\times0.66\times50 =$ **3600 J** **(2 marks)**





If you encounter a question like this: [March 21 V2, Q2(c)(ii)]

A heater supplies energy at a constant rate to 0.045 kg of a substance. The variation with time of the temperature of the substance is shown in Fig. 3.1. The substance is perfectly insulated from its surroundings.

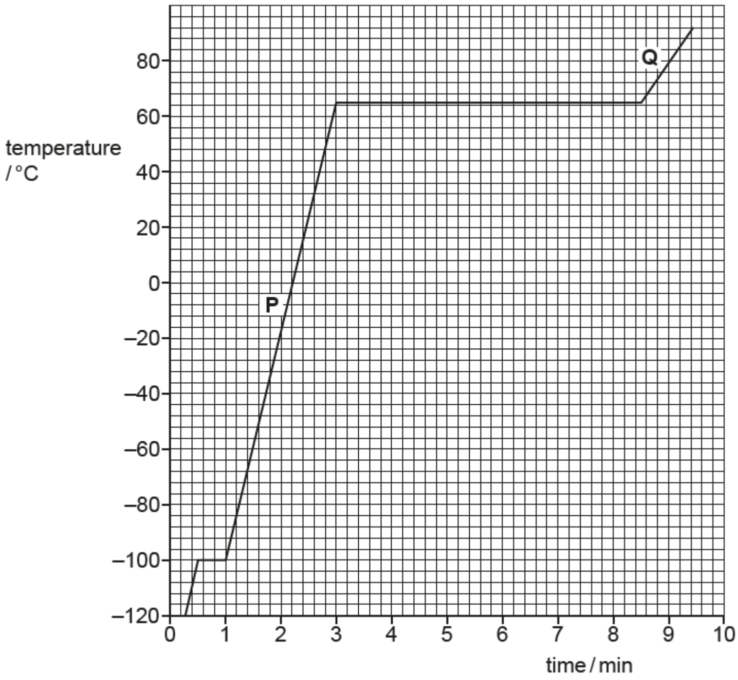


Fig. 3.1

The power of the heater is 150 W.

Use data from Fig. 3.1 to calculate, in kJ kg^{-1}, the specific latent heat of vaporisation L of the substance.

You could answer with:

Q=mL t=330s (from Figure 3.1) **(1 mark)**

Q= P×t = mL

150×330=0.045×L **(1 mark)**

L=1.1×10^6 **(1 mark)**

Chapter Five

Ideal Gases

One of the aims of physics is to describe and explain the behavior of various systems. For a system containing gas it is difficult to describe what happens to each component of the system without knowing the properties of its molecules. The Difficulty is that the number of molecules is **very large** ($1m^3$ of atmospheric air contains about 3×10^{25}Molecule!!!), its movement is **Random** and its volume is **very small**; there is no practical method of determining the position and velocity of every single molecule at a given time. We can make **statistical calculations** that tell us roughly how a collection of millions of molecules would behave.

The properties of gases could be described by two ways

Macroscopic Scale
The large scale

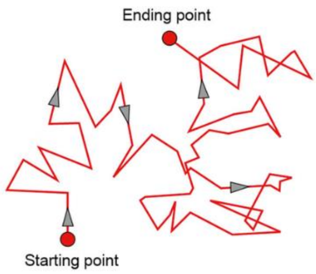
- Pressure (P)
- Temperature (T)
- Volume (V)

Microscopic Scale
The small scale

- Velocity (U_x)
- Acceleration (a_x)
- Position (x)

The kinetic theory

The Brownian Motion



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Motion

It is the evidence for the movement of molecules. Is named by Robert Brown who observed (1827) tiny pollen grains suspended in water under the microscope, He saw that the grains were always in a jerky, haphazard irregular motion. The effect is independent of all external factors and ascribed to the thermal motion of the molecules of the fluid. These molecules are in constant irregular motion with a velocity proportional to the square root of the temperature.

N.B : (q ⟶ K.E , w ⟶ P.E)

❖ **P.E+ K.E in a :**

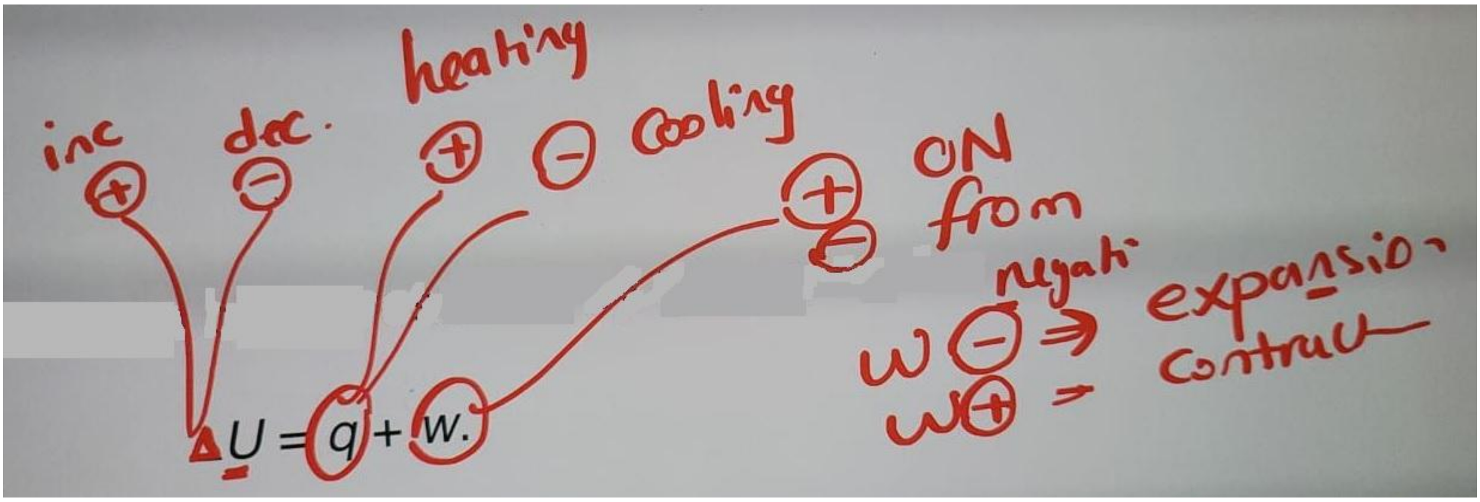
Body ⟶ mechanical energy
Molecule ⟶ internal energy [$\Delta U = q + w$]

❖ **Balance = thermal equilibrium (Same temperature).**

❖ **Hot object** ⟶ **Heat E** ⟶ **Cold object**

Note that:

Gas
- **Expand** ⟶ **-Work**
- **Compressed** ⟶ **+Work**



Note that:

If he said same volume

Work = ρ $\Delta volume$ ⟶ Work = 0

If you encounter a question like this: [June 19 V2, Q2(a)(i),(ii)]

(i) The first law of thermodynamics may be expressed in the form

$\Delta U = q + w$.

State, for a system, what is meant by:

1. **+q**
2. **+w**

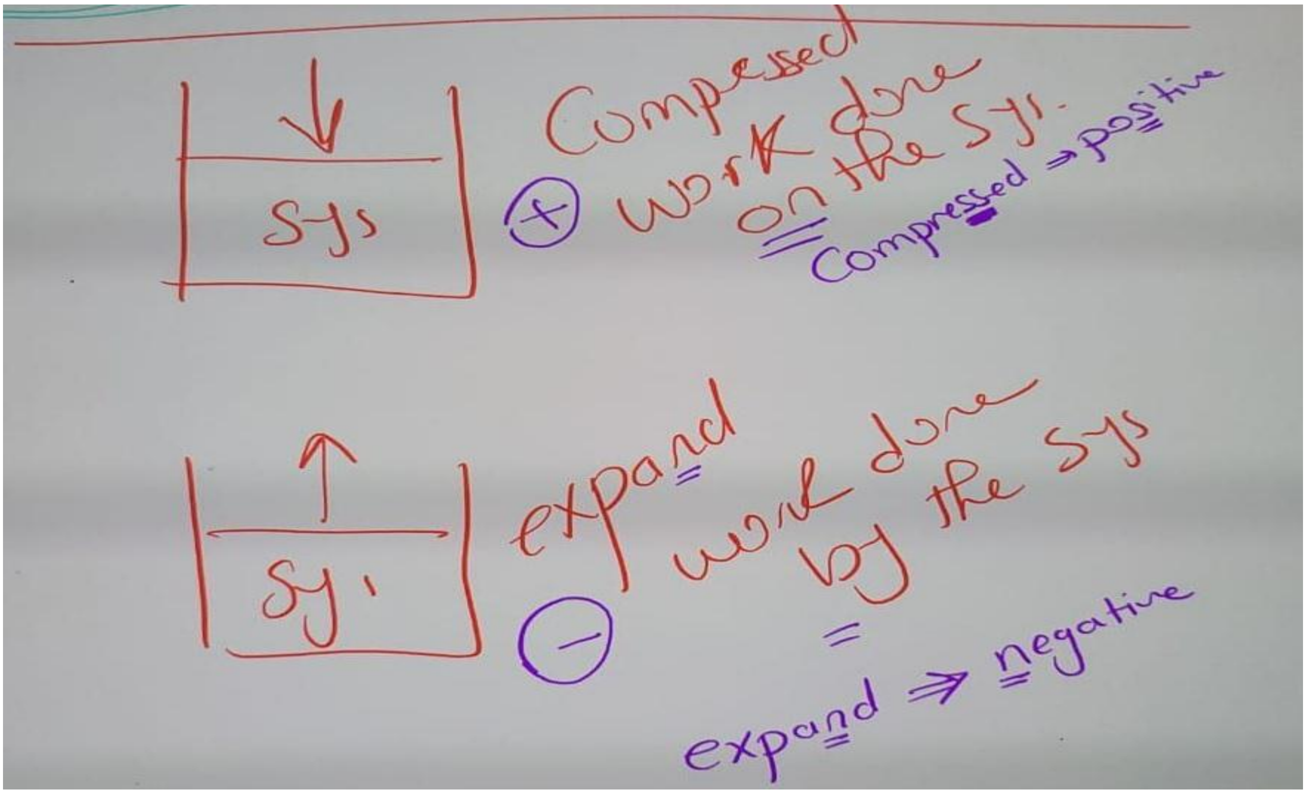
(ii) State what is represented by a negative value of ΔU.

You could answer with:

(i)1. energy transferred **to** the system by heating **(1 mark)**

2. work done **on** the system **(1 mark)**

(ii) **decrease** in internal energy



Ideal Gas Law

(fixed volume/amount of gas – ideal gas)

It is defined as one in which
(1) All collisions between atoms or molecules are perfectly elastic.
(2) There are no intermolecular attractive forces.
(3) One can visualize it as a collection of perfectly hard spheres which collide but do not interact with each other.
(4) All the internal energy is in the form of kinetic energy and any change in internal energy is accompanied by a change in temperature.
An ideal Gas can be characterized by three state variables: pressure (p), volume (V) and absolute temperature (T). The relationship between them Expressed as:

$$\boxed{PV=nRT}$$

It is called **the ideal gas equation** or **the equation of state**
Where
P =the pressure of the gas (Nm^{-2})
V=the volume of the gas (m^3)
n=the number of moles of gas (mol)
R=the universal molar gas constant (=8.31 $JK^{-1}mol^{-1}$)
T=the temperature of the gas in Kelvin ($T_K=T_{^\circ C}+273$)
This law stands in relation to gases in the same way that Newton's Second Law stands in relation to dynamics

The temperature, T, is directly proportional to volume

- The pressure is the result of molecular collisions with the walls of the container.
- Temperature might be measured in °C, but in practice it is more useful to use the thermodynamics Kelvin scale.
- Mass is measured in moles.

The mole of substance is defined as
The amount of that substance which contains the same number of particles as there are in 0.012 kg of carbon 12.A mole of any substance contains a standard number of particle $6.02\times10^{23}mol^{-1}$ (it is known as Avogadro constant)

Solved Problems

Example (1)

A helium gas cylinder is $0.200m^3$ in volume contains 50.0 mol of gas at room temperature of 293k.find the pressure in the cylinder (you can assume that the helium is an ideal gas)

Solution

$\because PV = nRT$

$\therefore P \times 0.200 = 50 \times 8.31 \times 293 = 121700J$

$$P = \frac{121700}{0.200} = 6.09 \times 10^5 P_a$$

Example (2)

Calculate the volume occupied by one mole of an ideal gas at room temperature (20^oC) and pressure ($1.013x10^5p_a$)

Solution

$T(k) = T^0(C) + 273^0$

$= 20^0 + 273^0 = 293^0$

$\because PV = nRT$

$\therefore 1.103 \times 10^5 \times V = 1 \times 8.31 \times 293^0$

$V = 0.0240m^3 = 2.40 \times 10^{-2} m^3$

Example (3)

A car tyre contains $0.020m^3$ of air at 27^0C and at a pressure of $3.0x10^5P_a$.calculate the mass of the air in the tyre (Molar mass of air=28.8 g mol^{-1})

Solution

$\because PV = nRT$

$\therefore 30 \times 10^5 \times 0.02 = n \times 8.31 \times 300^0 \longrightarrow n = 2.41 mol$

$mass = number\,of\,moles \times molar\,mass = 2.41 \times 28.8 = 69.4g$

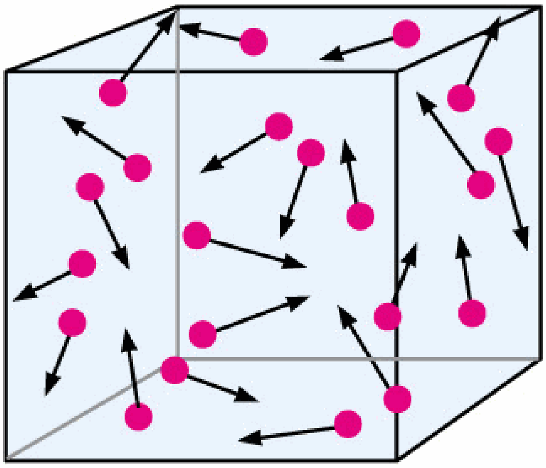
Modeling Gases –The kinetic Theory

The ideal gas equation is an empirical relationship (based on the experimental results), it gives a good description of gases in many different situations.
However, it does not Explain why gases behaves in this way.
So, a new Model introduced to explain this behavior and links the microscopic properties of particles to macroscopic properties of a gas called

The kinetic Theory of Gases

The assumptions of kinetic theory of ideal gases are

(1) All molecules behave as identical, hard, perfectly elastic spheres.
[a small cube of air can have as many as 10^{20} molecules and the kinetic energy of the molecules cannot be lost]



(2) The volume of the molecules in negligible compared with the volume of the containing vessel.
[When a liquid boils to become a gas, its particles move further from each other]

(3) There are no forces of attraction or repulsion between molecules.
[The particles travel in straight lines between collisions]

(4) There are many molecules, all moving randomly.

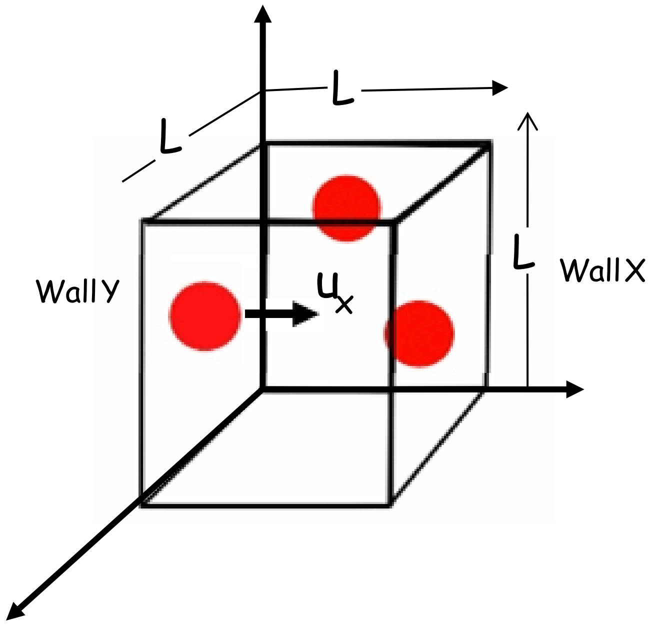
Note that:

No of moles (n) =

- Total mass / mass of one mole
- Total volume of gas / Volume of one mole
- Total No of molecules / No of molecules in one mole (NA)

Care:-
n = *No of moles*
N = *No of molecules*
N_A = *Avogadro's No*

Suppose a gas enclosed in cubical container of side **L**, let each molecule of the gas have mass **m**.



Consider initially a single molecule which is moving towards wall **X**, and suppose that its X-component of velocity is U_x

This molecule will have an x-component of momentum mU_x towards the wall.

The molecule will eventually reverse the direction of its momentum by colliding with the wall. Since the collision will be elastic, it will rebound with the same speed so that its momentum will now be $-mU_x$, the change in the X-component of momentum is therefore $2mU_x$.

The molecule has to travel a distance 2L (from X to Y and back to X) before it next collides with wall X, the time for such a trip is $\frac{2L}{U_x}$.

Therefore, the molecule's rate of change of momentum due to collision with X will be:

$$\frac{2mU_x}{\frac{2L}{U_x}} = \frac{mU_x^2}{L}$$

By Newton's second law, rate of change of momentum is equal to the force, and therefore $\frac{mU_x^2}{L}$ is the force exerted on the molecule by the wall.

By Newton's Third law, the molecules exert an equal but oppositely directed force on the wall, and therefore:

$$\boxed{\text{Force on X} = \frac{mU_x^2}{L}}$$

$$\text{Force per unit area on } X = \frac{mU_x^2}{L} \Big/ L^2$$

Therefore:

$$\text{Pressure on } X = \frac{mU_x^2}{L} \Big/ L^3$$

If there are N molecules in the container and their X-components of velocity are U_1, U_2,,......U_N
Then the total pressure P on the wall X will be given by:

$$P = \frac{m}{L^3}(U_1^2 + U_2^2 + \ldots\ldots + U_N^2)$$

Therefore

$$P = \frac{m}{L^3} N\overline{U^2} \qquad \text{OR} \qquad P = \frac{m}{V} N\overline{U^2}$$

Where, $\overline{U^2}$ is the mean square velocity in the x-direction.

Since **mN** is the total mass of gas in the container, $\frac{mN}{L^3}$ is the Density ρ of the gas

Therefore $$P = \rho\overline{U^2}$$

For a molecule moving with velocity C in three dimensions, the relation between C and C_X, C_Y and C_z

Where $C^2 = c_x^2 + c_y^2 + c_z^2$

And the average value of the components in the x-direction will be the same as for those in the y-direction or the Z-direction

$$\langle c_x^2 \rangle = \langle c_y^2 \rangle = \langle c_z^2 \rangle \qquad \text{and} \quad \langle c_x^2 \rangle = \frac{1}{3}\langle C^2 \rangle$$

Therefore

$$P = \frac{1}{3}\frac{mN}{V}\overline{C^2} = \frac{1}{3}\frac{mN}{V}\langle C^2 \rangle$$

The Relation Between the average Kinetic energy and temperature

From the previous Equation

$$PV = \frac{1}{3}mN\overline{C^2} \qquad (1)$$

The ideal gas equation for n moles of a gas of volume v and pressure P is

PV=nRT (2)

From (1) and (2)

$$\frac{1}{3}mN\overline{C^2} = nRT$$

$$\therefore \frac{2}{3}N(\frac{1}{2}m\overline{C^2}) = nRT$$

$$\therefore \frac{1}{2}m\overline{C^2} = \frac{3}{2}\frac{nRT}{N}$$

Since the average kinetic energy of a molecule is

$$\langle E_k \rangle = \frac{1}{3}Nm\langle C^2 \rangle$$

And $\frac{N}{n}$ is the number of molecules per mole (N_A Avogadro's Constant)

Therefore

$$\langle E_k \rangle = \frac{3}{2}\frac{R}{N_A}T$$

But $\frac{R}{N_A}$ is constant (Boltzmann Constant $K=1.38\times10^{-23}JK^{-1}$)

Therefore

$$\boxed{\langle E_k \rangle = \frac{3}{2}KT}$$

Since $\langle C^2 \rangle = \frac{3kT}{m}$

$$\boxed{\sqrt{\langle C^2 \rangle} = \sqrt{\frac{3KT}{m}}}$$

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Prove that $P = \frac{1}{3}\rho V^2$

density ρ, V vel.

$P = \frac{1}{3}\frac{Nm}{V}\langle c^2\rangle$

V volume, $\langle c^2\rangle$ Mean sq. speed

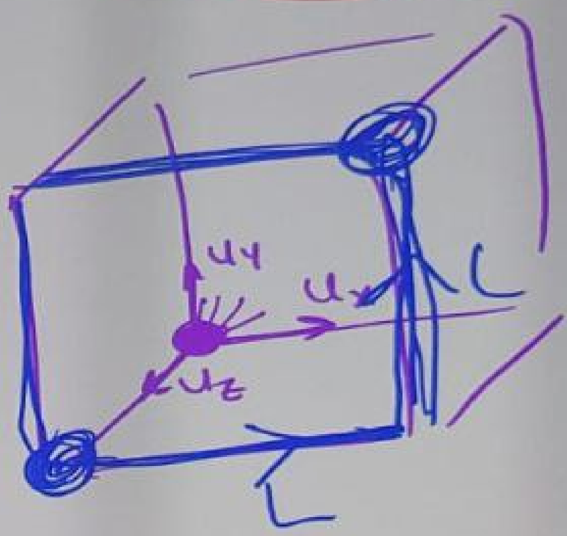
$u_x = u_y = u_z$

$u_x^2 + u_y^2 + u_z^2 = u^2_{total}$

$u_x^2 = \frac{1}{3}u^2$

- Momentum before col $= mu_x$
- after col $= -mu_x$
- change in Momentum $= 2mu_x$

$t = \frac{d}{v} = \frac{2L}{u_x}$



$F = \frac{\Delta mv}{\Delta t} = \frac{\cancel{2}mu_x}{\frac{\cancel{2}L}{u_x}}$

$F_{\text{one molecule}} = \frac{mu_x^2}{L}$

$F_{Gas} = \frac{Nmu_x^2}{L}$

$P = \frac{F}{A} = \frac{Nmu_x^2}{L^2\, L} = \frac{Nm\,u_x^2}{Vol}$

$= \frac{1}{3}\frac{Nm}{Vol}u^2$

$\frac{1}{3}\rho\langle c^2\rangle$

Mean sq. speed

prove that

$E_K = \frac{1}{2}mv^2 = \frac{3}{2}KT$

$= \frac{1}{2}m\langle c^2\rangle = 3/2\,KT$

$P = 1/3\,\rho\langle c^2\rangle$

$P = 1/3\,\frac{Nm}{V}\langle c^2\rangle$

$3PV = Nm\langle c^2\rangle$

$3nRT = Nm\langle c^2\rangle$

$3\,\frac{R}{N_A}T = \cancel{\frac{N_A}{N_A}}\,m\langle c^2\rangle$

$\times\frac{1}{2} \quad 3KT = m\langle c^2\rangle \times 1/2$

$E_K = 3/2\,KT = \frac{1}{2}m\langle c^2\rangle$

$PV = nRT$

For one mole

$n = 1 \quad N = N_A$

$K = \frac{R}{N_A}$

$\ast$ PE $\Rightarrow$ E due to the position

Grav. PE — h — gpe = mgh

PE:
- Chemical PE
- Elastic PE

Solved Problems

Example (1)

A mole of air molecules at room temperature and a pressure of $1.0x10^5P_a$ has a volume of $0.024\ m^3$.How fast do these air molecules move?(the average mass of an air molecule is $4.8x10^{-26}Kg$)

[$N_A=6.0x10^{23}$molecules mol^{-1}]

Solution

$$PV=\frac{1}{3}NmC^2$$

$$1.0\times10^5\times0.024=\frac{6.0\times10^{23}\times4.8\times10^{-26}\times C^2}{3}$$

$$C^2=\frac{2400}{0.0096}=250000$$

$$C=\sqrt{250000}=500ms^{-1}(2s.f)$$

Example (2)

At normal room temperature and pressure, the density of air is $1.2\ kg\ m^{-3}$, and the speed of the molecules is $500\ ms^{-1}$.what is the pressure?

Solution

$$P=\frac{1}{3}\rho C^2$$

$$=\frac{1.2\times(500)^2}{3}=100000P_a(2\,s.f)$$

Example (3)

Calculate the mean kinetic energy of a gas molecule at 288k.

[The Boltzmann constant K is $1.38x10^{-23}JK$]

Solution

$$\langle E_k\rangle=\frac{3}{2}KT$$

$$=\frac{3}{2}\times1.38\times10^{-23}\times288$$

$$=5.96\times10^{-21}J\,(3\,s.f)$$

Remarks

(1) For a gas molecule to escape from the earth gravity

$$\frac{3}{2}KT = \frac{1}{2}mV^2$$

$$3KT = mV^2$$

$$F = G\frac{m^2}{r}$$

Where T is the temperature of the atmosphere.

(2) To find the average separation between 2 atoms use

$$d = \sqrt[3]{\frac{v}{6.02 \times 10^{23}}}$$

(3) To find the gravitational force between 2 atoms

$$F = G\frac{m^2}{r}$$

Where $m_1 = m_2 = m$

(4) To find the number of atoms in any volume, get the first the number of moles n then

$$N = n \times 6.023 \times 10^{23}$$

(5) C_{RMS} depends on the temperature

N.B

If you encounter a question like this: [June 20 V1, Q2(a)]

State what is meant by the internal energy of a system.

You could answer with:

-total potential energy and kinetic energy of molecules/atoms **(1 mark)**
-in random motion **(1 mark)**

If you encounter a question like this: [June 20 V1, Q2(b)]

By reference to intermolecular forces, explain why the change in internal energy of an ideal gas is equal to the change in total kinetic energy of its molecules.

You could answer with:

-there are no intermolecular forces in an ideal gas **(1 mark)**
- no potential energy [so change in kinetic energy is change in internal energy] **(1mark)**

If you encounter a question like this: [June 20 V1, Q2(c)]

State and explain the change, if any, in the internal energy of a solid metal ball as it falls under gravity in a vacuum.

You could answer with:

-random potential energy of molecules does not change **(1 mark)**
-random kinetic energy of molecules does not change**(1 mark)**
-so internal energy does not change**(1 mark)**

OR

-decrease in total potential energy = gain in total kinetic energy**(1 mark)**
-no external energy supplied**(1 mark)**
-so internal energy does not change**(1 mark)**

OR

-no compression of ball so no work done on the ball**(1 mark)**
-no resistive forces so no heating of the ball**(1 mark)**
-so internal energy does not change**(1 mark)**

OR

-no change of state so potential energy of molecules unchanged**(1 mark)**
-no temperature rise so kinetic energy of molecules unchanged**(1 mark)**
-so internal energy does not change**(1 mark)**

If you encounter a question like this: [June 20 V2, Q2(b)]

An ideal gas has volume, pressure and temperature as shown in Fig. 2.1.

volume $6.0 \times 10^{-3}\,m^3$
pressure $3.0 \times 10^5\,Pa$
temperature 17 °C

Fig. 2.1

The mass of the gas is **20.7 g**.

Calculate the mass of one molecule of the gas.

You could answer with:

$pV = NkT$ $T = 17 + 273 = 290\,K$ **(1 mark)**

$$N = \frac{(3.0 \times 10^5 \times 6.0 \times 10^{-3})}{(1.38 \times 10^{-23} \times 290)} = 4.5 \times 10^{23}$$ **(2 marks)**

$$\text{mass} = \frac{20.7}{(4.5 \times 10^{23})} = 4.6 \times 10^{-23}\,g$$ **(1 mark)**

If you encounter a question like this:

At a pressure of 1.05×10^5 Pa and a temperature of 27°C, 1.00 mol of helium gas has a volume of $0.0240\ m^3$. The mass of 1.00 mol of helium gas, assumed to be an ideal gas, is 4.00g.

(i) Calculate the root-mean-square (r.m.s.) speed of an atom of helium gas for a temperature of 27°C.

(ii) Using your answer in **(i)**, calculate the r.m.s. speed of the atoms at 177°C.

You could answer with:

$pV = \frac{1}{3} Nm\langle c^2 \rangle$ $\quad 1.05 \times 10^5 \times 0.0240 = \frac{1}{3} \times 4.00 \times 10^{-3} \times \langle c^2 \rangle$ **(1 mark)**

$\langle c^2 \rangle = 1.87 \times 10^6$ **(1 mark)**

$c_{r.m.s.} = 1.37 \times 10^3\ m\ s^{-1}$ **(1 mark)**

OR

$\frac{1}{2}m\langle c^2 \rangle = (3 / 2)\ KT$

$0.5 \times (4.00 \times 10^{-3} / 6.02 \times 10^{23}) \times \langle c^2 \rangle = 1.5 \times 1.38 \times 10^{-23} \times 300$ **(1 mark)**

$\langle c^2 \rangle = 1.87 \times 10^6$ **(1 mark)**

$c_{r.m.s.} = 1.37 \times 10^3\ m\ s^{-1}$ **(1 mark)**

OR

$nRT = \frac{1}{3} Nm\langle c^2 \rangle$

$1.00 \times 8.31 \times 300 = \frac{1}{3} \times 4.00 \times 10^{-3} \times \langle c^2 \rangle$ **(1 mark)**

$\langle c^2 \rangle = 1.87 \times 10^6$ **(1 mark)**

$c_{r.m.s.} = 1.37 \times 10^3\ m\ s^{-1}$ **(1 mark)**

If you encounter a question like this: [June 19 V2, Q2(b)]

An ideal gas, sealed in a container, undergoes the cycle of changes shown in Fig. 2.1.

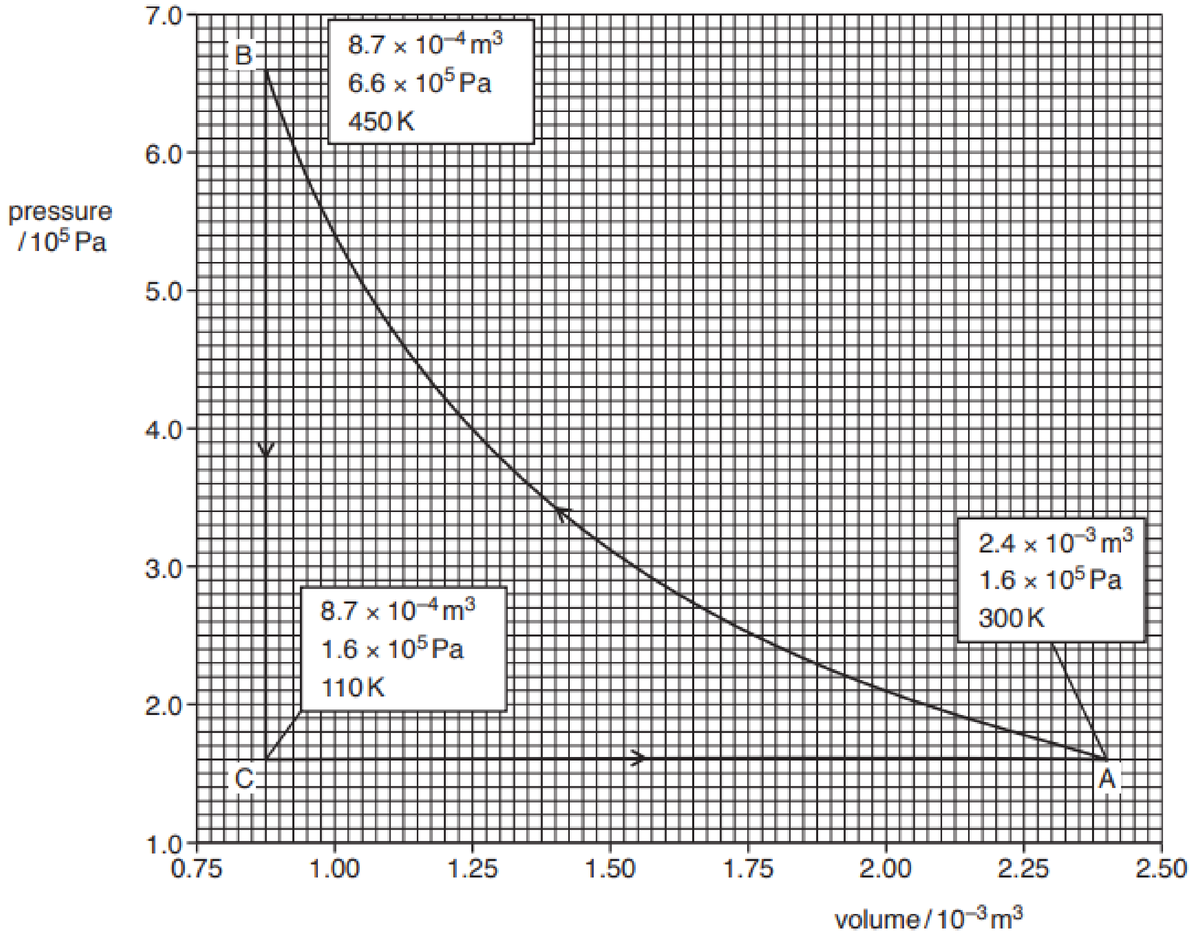


Fig. 2.1

At point A, the gas has volume 2.4×10^{-3}m^3, pressure 1.6×10^5Pa and temperature 300K.

The gas is compressed suddenly so that no thermal energy enters or leaves the gas during the compression. The amount of work done is 480J so that, at point B, the gas has volume 8.7×10^{-4}m^3, pressure 6.6×10^5Pa and temperature 450K.

The gas is now cooled at constant volume so that, between points B and C, 1100J of thermal energy is transferred. At point C, the gas has pressure 1.6 × 10^5Pa and temperature 110K.

Finally, the gas is returned to point A.

(i) State and explain the total change in internal energy of the gas for one complete cycle ABCA.

(ii) Calculate the external work done on the gas during the expansion from point C to point A.

(iii) Complete Fig. 2.2 for the changes from: **1**. point ***A*** to point **B**

2. point **B** to point ***C***

3. point ***C*** to point ***A***.

change	$+q$/J	$+w$/J	ΔU/J
A → B	....	....	....
B → C	....	....	....
C → A	....	....	....

Fig. 2.2

You could answer with:

(i) no change (in internal energy) **(1 mark)**

(because) no change in temperature **(1 mark)**

(ii) work done = p ΔV

= (–)1.6 × 10^5 × (2.4 – 0.87) × 10^{-3} = (–)240 J **(2 marks)**

(iii) from **a** to **b**:

+q = 0, because no change in temperature

+w = 480 (given)

$\Delta U = 480$

From **b** to **c**:

+q = -1100(given)

+w = 0, because no change in volume

$\Delta U = -1100$

From **c** to **a**:

$\Delta U + 480 - 1100 = 0$

$\Delta U = 620$

+w= -240(from (ii))

+q = 620+240

+q = 860

change	$+q$/J	$+w$/J	ΔU/J
A → B	0	480	480
B → C	-1100	0	-1100
C → A	860	-240	620

Fig. 2.2

Chapter six

Oscillation

An object oscillates when it moves back and forth repeatedly, on either side of an equilibrium position.

When the object oscillates with constant time period, we say that it is moving with simple harmonic motion **(SHM)**

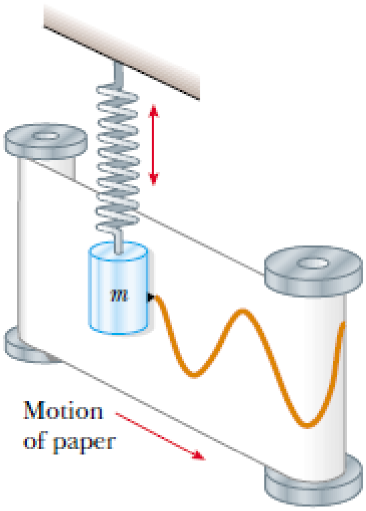
Examples of oscillations

- A guitar string.
- The motion of a child on a swing.
- The movement of a pendulum.
- The beating of a heart.

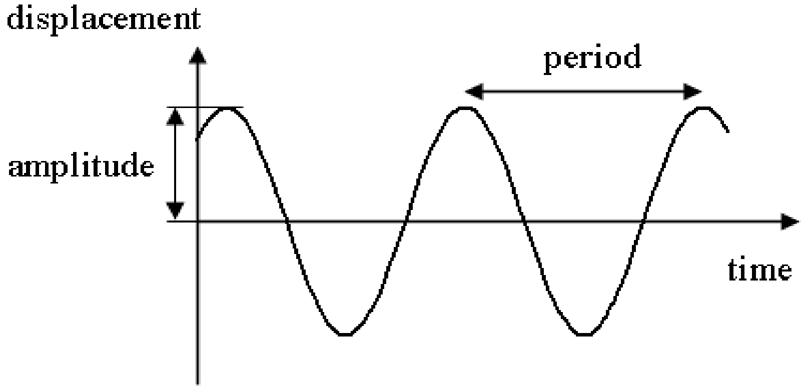
One way to represent the simple harmonic motion of an object is to draw graphs of displacement-time graph the shape of this graph is a sin curve and the motion is described as Sinusoidal

An experimental arrangement that exhibits simple harmonic motion is illustrated in the opposite figure. An object oscillating vertically on a spring has a pen attached to it.

While the object oscillating a sheet of paper is moved perpendicular to the direction of motion of the spring, and the pen traces out the cosine curve



Let us investigate further mathematical description of simple harmonic motion.



Frequency (f)

The number of oscillations per unit time

Units of frequency: $HZ = S^{-1}$

The period (T)

The time taken for one complete oscillation.

$$\text{Period} = \frac{1}{\text{Frequency}} \Rightarrow T = \frac{1}{f}$$

The displacement

The distance from the equilibrium position

It is a vector quantity

The amplitude (x_o)

The maximum displacement from the equilibrium position

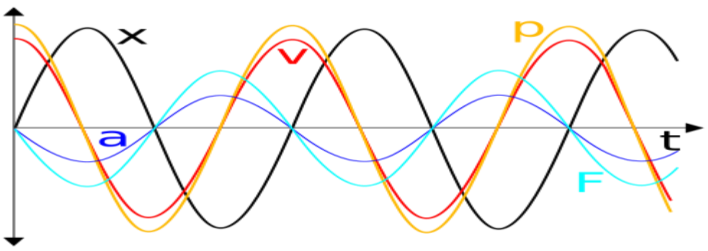
It is a scalar quantity

The Phase difference (φ)

The measure of how "in step" different particles are. If they are moving together, they are said to be in phase. If not, they are said to be out of phase.

FOR EXAMPLE

The phase difference between more than one oscillation is shown



Simple Harmonic motion

Here are some examples of simple harmonic motion

(1) Vibration of a pendulum

(2) Vibration of a string about its equilibrium position

(3) When a pure sound wave travels through air, the molecules of the air vibrate with S.H.M.

(4) When an alternating current flows in a wire, the electrons in the wire vibrate with S.H.M.

(5) The atoms that make up a molecule vibrate with S.H.M.

Requirements of Simple harmonic motion

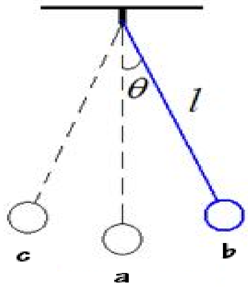
(1) An oscillating Object.

(2) An equilibrium position.

(3) A restoring force that act to return the object to its equilibrium position

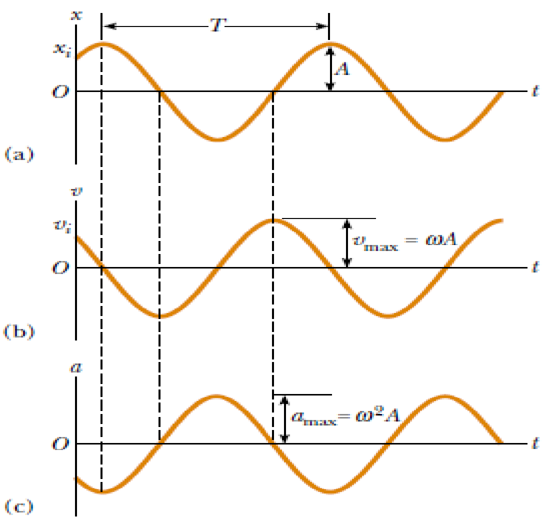
Graphical representation of simple Harmonic motion

We already know that the graph of displacement against time is sinusoidal. This is the first graph:



At t=0, the displacement is zero, So timing started as the swing passed through the equilibrium position **a**

The velocity is the rate of change of displacement ($v = \frac{\Delta x}{\Delta t}$).



Therefore the velocity at any point is equal to the gradient of the displacement-time graph.

By looking at the displacement-time graph, the gradient falls to zero as the swing reaches maximum amplitude, so the velocity is greatest at the centre of the motion

In the second graph (b), it shows the velocity-time graph for the motion. The forward motion of the swing is taken as positive. The backward motion is taken as negative.

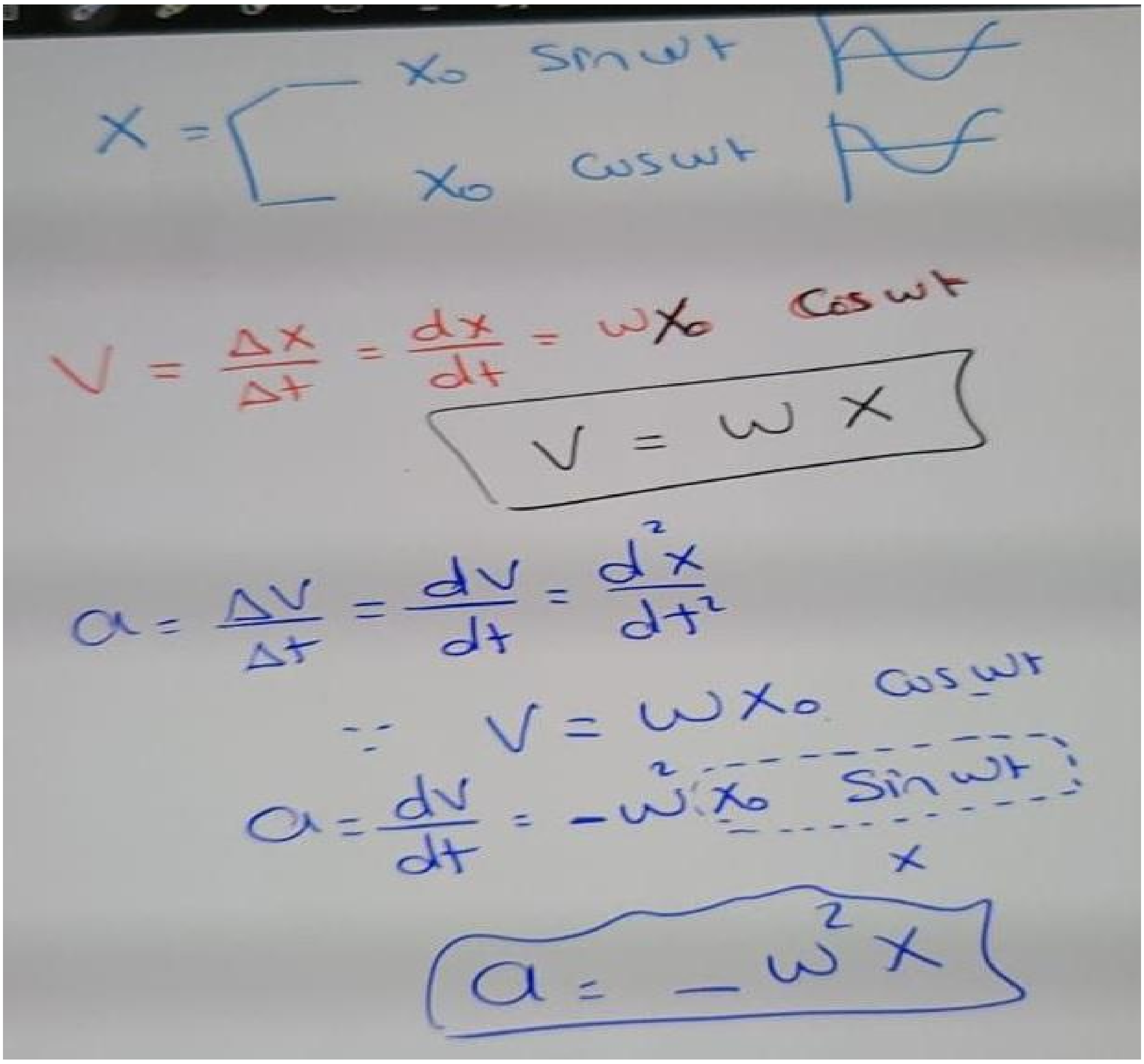
The acceleration: is the rate of change of velocity $\left(a = \frac{\Delta v}{\Delta t}\right)$

So, acceleration is the gradient of the velocity-time graph.

By looking at the velocity- time graph, the gradient is steepest when the velocity is zero, at the maximum amplitude. The acceleration increase as the displacement increases.

In the third graph (c), it shows the acceleration-time graph for the motion. Acceleration to the right is positive and to the left is negative.

Notice that when the displacement is positive, the acceleration is negative as the swing moves forwards from (**a** to **b)** and back again, the displacement is positive. The acceleration towards the centre is to the left. This is the negative direction, and as the swing moves forward from (**a** to **c)** and back again, the displacement is now negative. The acceleration towards the centre is now to the right and so the acceleration is positive.



$V = \omega x$ (for a Cetain Disp)

$V = \omega x_0$ (For the amplitude)

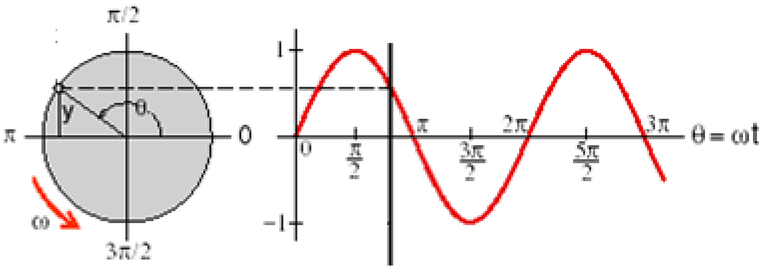
$V = \omega \sqrt{x_0^2 - x^2}$ For a vib. body

$KE = \frac{1}{2} m v^2 = \frac{1}{2} m \omega^2 (x_0^2 - x^2)$

<u>vib.</u>

Frequency and angular frequency

We can think of a complete oscillation of an oscillator or a cycle of S.H.M. as being represented by 2π radians. (This is similar to a complete cycle of circular motion where an object moves round through 2π radians),



The phase of the oscillation changes by 2π radians during one oscillation. Hence, if there are f oscillations in unit time, there must be 2πf radians in unit time. This quantity is the angular frequency of S.H.M and it is represented by the symbol ω.

Where $$\omega = 2\pi f$$

Since $f = \frac{1}{T}$

$$\omega = \frac{2\pi}{T}$$

N.B

Not that for a particular system the period of simple pendulum is independent of the amplitude

For example, if the amplitude of oscillation of a simple pendulum is decreased, its average speed decreases and there is no change in the time it takes to complete an oscillation.

The Equation of Simple Harmonic Motion

If a body vibrating with simple harmonic motion, its motion can be described by an equation of the form:

$$\text{Acceleration} = \frac{d^2x}{dt^2} = -\omega^2 x$$

Where $\frac{d^2x}{dt^2}$ = the acceleration of the body (ms^{-1})

X = the displacement of the body from its equilibrium position (m)

ω^2 = A positive constant (s^{-2}).

The **minus sign (-)** in the equation ensures that the acceleration is always directed towards the equilibrium position, as required. By solving this equation mathematically, the displacement-time relation could be expressed as

$X = x_o \sin \omega t$ **OR** $X = x_o \cos \omega t$

We use (sin) if the object started from the equilibrium position at t=0 and (cos) if it started from the maximum displacement at t=0

And the velocity **v** will equal the differential of the displacement **x** with respect to time **t**

$$v = \frac{dx}{dt} = x_o \omega \cos \omega t$$

And the acceleration **a** will equal the differential of the velocity **v** with respect to time **t**

$$a = \frac{dv}{dt} = -x_o \omega^2 \sin \omega t$$

Now x=Asin ωt, so:

$$a = -\omega^2 x$$

This equation shows that **a** is proportional to **x**

Definition

A body executes simple harmonic motion if its acceleration is directly proportional to its displacement from its equilibrium position, and is always directed towards the equilibrium position

The velocity of an oscillator varies as it moves back and forth. It has its greatest speed when it passes through the equilibrium position in the middle of the oscillation.

I we take **t=o** when the oscillator passes through the middle of the oscillation with its greatest speed **V_o** then we can represent the changing velocity as an equation

$$\boxed{V = V_o \cos \omega t}$$

But $\cos \omega t = \pm\sqrt{1-\sin^2 \omega t}$

Then

$$V = \pm X_o \omega \sqrt{1 - \sin^2 \omega t}$$

$$V = \pm \omega \sqrt{X^2_o - X^2_o \sin^2 \omega t}$$

$\therefore$

$$\boxed{V = \pm \omega \sqrt{X^2_o - X^2}}$$

At maximum speed when x=0

$$\boxed{V_o = \omega X_o}$$

According to this equation $\mathbf{V \propto X_o}$ and $\mathbf{V \propto \omega}$

So that the maximum speed is proportional to frequency and displacement.

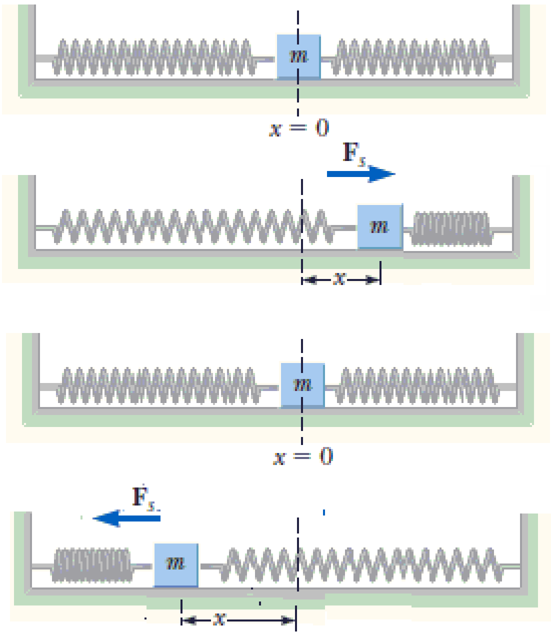
The period of S.H.M is independent of the amplitude

A greater amplitude means that the oscillator has to travel a greater distance in the same time-hence it has a greater speed.

Energy changes in simple Harmonic motion

During simple Harmonic motion, there is a constant interchange of energy between the Kinetic and potential forms, and if the system does no work against resistive forces (i.e. is undamped) its total energy is constant.

When the mass is pulled to one side, one spring is compressed and the other is stretched. The springs store elastic potential energy. When the mass is released, it moves back towards the equilibrium position. Once the mass has passed the equilibrium position, its kinetic energy decreases and the energy is transferred back to the springs.



The Kinetic energy $= \frac{1}{2} m\omega^2 (x_o^2 - x^2)$

$\because$ The average force $= \frac{1}{2} m\omega^2 x$

$\therefore$ Work done = average force x displacement in direction of force

$$= (\frac{1}{2} m\omega^2 x) x = \frac{1}{2} m\omega^2 x^2$$

The potential energy= $= \frac{1}{2} m\omega^2 x^2$

The total energy =Kinetic energy + potential energy

$$= \frac{1}{2} m\omega^2 (x_o^2 - x^2) + \frac{1}{2} m\omega^2 x^2$$

The total energy $= = \frac{1}{2} m\omega^2 x_o^2$

F = ma

Fav = ½ ma

= ½ mw²X

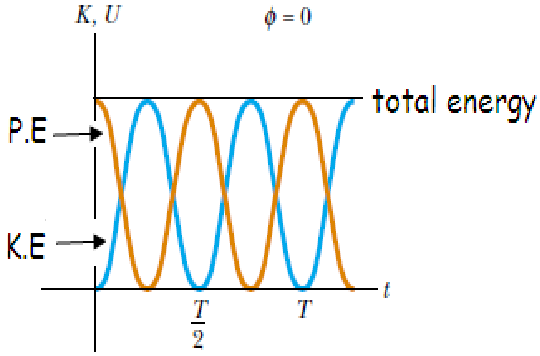
PE=Work=FX

=½ mw²XX

=½ mw²X²

The total energy is constant and does not depend on **X** and is directly proportional to the product of

i. The mass
ii. The square of the frequency
iii. The square of the amplitude



the first graph shows the variation of the kinetic energy and potential energy with time.

the second graph shows the variation of the kinetic energy and potential energy with displacement.

When the displacement is zero

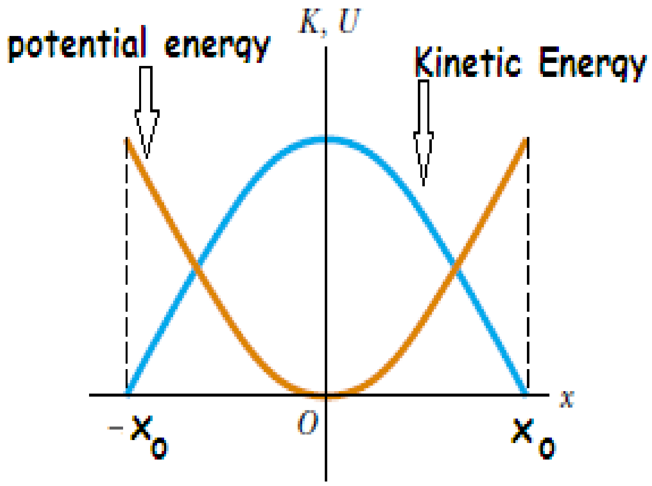
The kinetic energy is maximum

The potential energy is minimum

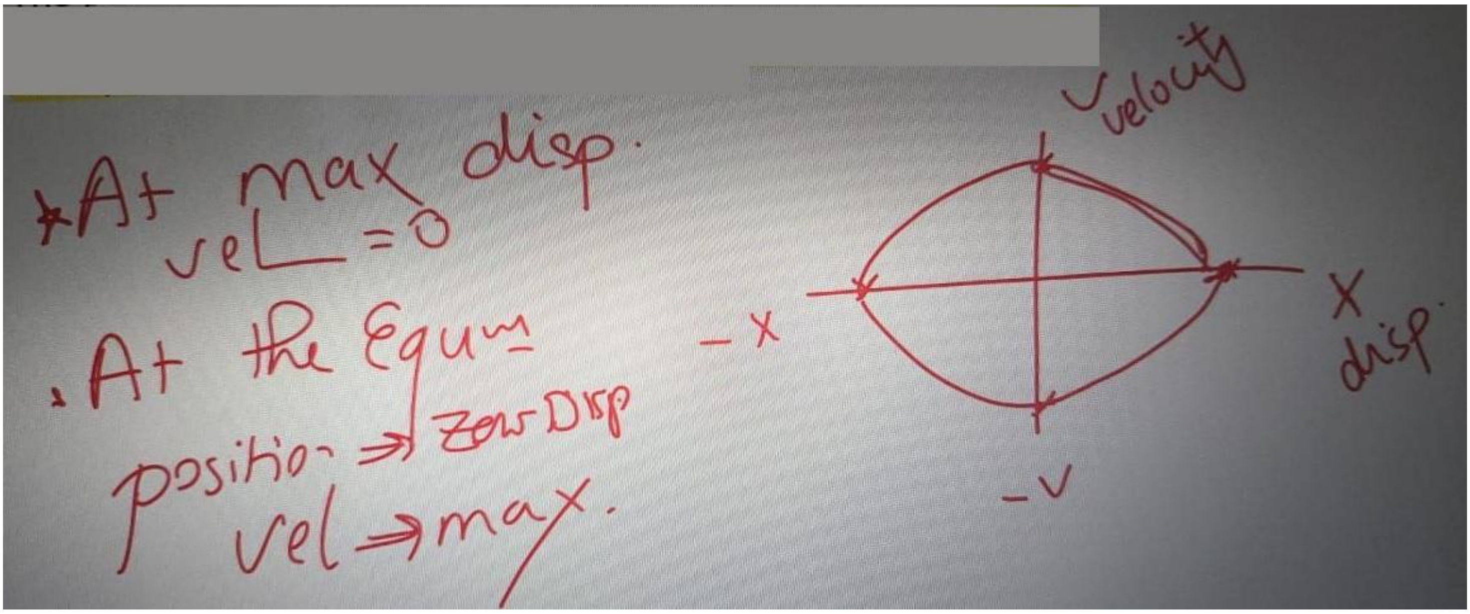
When the displacement is $\pm X_o$

The kinetic energy is minimum

The potential energy is maximum

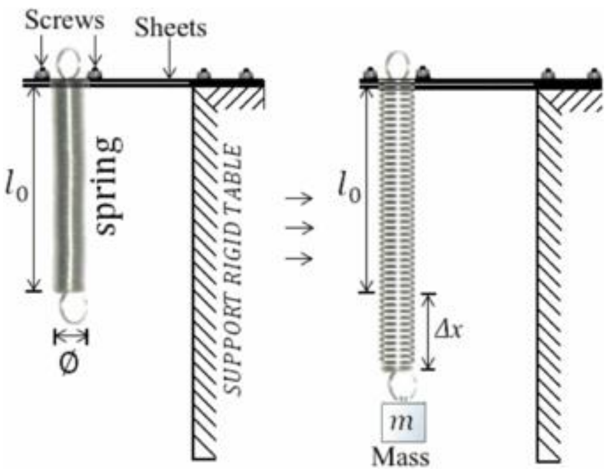


At any point on this graph the total energy has the same value.



Examples of Simple Harmonic Motion

(1) Mass on a helical spring



This sort of spring is called **helical** spring because it has the shape of helix

The weight **mg** of the mass is balanced by the tension **t** in the spring. When the spring is extended by an amount Δx, there is an additional upward force in the spring given by:

$$\boxed{F = -K\Delta x}$$

Where K is constant known as **the spring constant (Nm^{-1})**

The spring constant is a measure of stiffness of the spring. A stiff spring has a large value of **k**; a more flexible spring has a smaller value of **k**, and for the same force, would have a large extension than one with a large spring constant.

When the mass is released, the restoring force **F** pulls the mass towards the equilibrium position. (The minus sign in the expression for F shows the direction of this force). The restoring force is proportional to the displacement. This means that the acceleration of the mass is proportional to the displacement from the equilibrium position and is directed towards the equilibrium position; this is the condition for simple harmonic.

$$F=ma=-k\Delta x \qquad \Rightarrow a=-\frac{k}{m}\Delta x$$

By comparison with the equation

$a=-\omega^2x$

therefore $\omega^2=\frac{k}{m}$ or $\omega=\sqrt{\frac{k}{m}}$

but $\omega=2\pi f$

$$F=\frac{\omega}{2\pi}=\frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

$$T=\frac{1}{f}=2\pi\sqrt{\frac{m}{k}}$$

For oscillation to be simple harmonic, the spring must obey Hook's law throughout that is the extensions must not exceed the limit of proportionality. Furthermore, for large amplitude oscillations, the spring may become slack. Ideally, the spring would have no mass of the spring

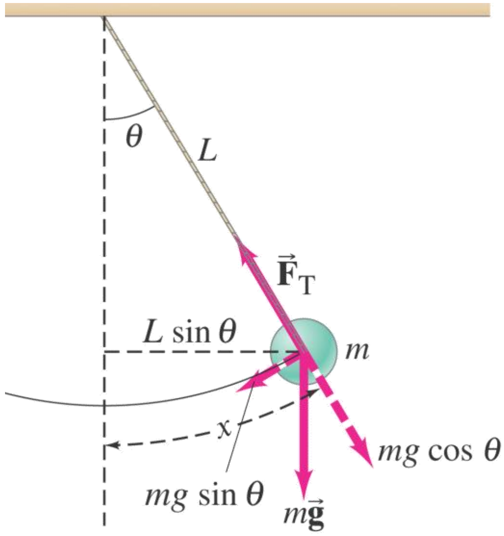
This example of Simple Harmonic Motion is particularly useful in modeling the vibrations of molecules. A molecule containing two atoms oscillates as it the atoms were connected by a tiny spring.

The spring constant depends on

(1) The type of bonding between the atoms

(2) The frequency of oscillation of the molecules

(2) The simple pendulum



A simple pendulum is a point mass **m** on a light, inelastic string.

In real experiment, we use a pendulum bob of finite size. When the bob is pulled aside through an angle θ and then released, there will be restoring force acting in the direction of equilibrium position.

Because the simple pendulum moves in the arc of a circle, the displacement will be an angular displacement θ rather than the linear displacement **x** we have been using so far.

The two forces on the bob are its weight mg and tension T in the string.

The component of the weight along the direction of the string (mg cos θ) is equal to the tension (T)

The component of weight at right angles to the direction of the string (mgsin θ) is equal to the restoring force (F)

The restoring force depends on sin θ. As θ increases, the restoring fore is not proportional to the displacement (in this case θ), and so the motion is oscillatory but not simple harmonic. However, the situation is different if the angle θ is kept small where θ is proportional to sin θ (θ^{rad}=sin θ)

This means that for small amplitude oscillations, the pendulum bob oscillates with simple harmonic motion.
For a pendulum of length **L** (**L** is the distance between the centre of mass of the pendulum bob and the point of suspension)

$$T = 2\pi\sqrt{\frac{L}{g}}\ ,\omega = \sqrt{\frac{g}{L}}\ \text{and}\ F = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$$

Where **g** is the acceleration of free fall (using Simple pendulum the free fall acceleration could be determined)

Solved problems

Example (1)

The displacement **x** at time **t** of a particle moving in simple harmonic motion is given by X=0.25 Cos 7.5t, where **x** is in meters and **t** is in seconds

(a) Use the equation to find the amplitude, frequency and period for the motion.
(b) Find the displacement when t=0.50s.

Answer

(a) By Comparing the equation with $x=x_o \cos\omega t$.

The amplitude x_o=0.25m and the angular frequency ω=7.5 rad^{-1}

$\because$ ω=2πf

$$\therefore f=\frac{\omega}{2\pi}=\frac{7.5}{2\pi}=1.2\,Hz$$
$$,T=\frac{1}{f}=\frac{1}{1.2}=0.84s$$

(b) X=0.25 Cos 7.5t
When t=0
X=0.25 Cos (7.5x0.25) =-0.20 m

Example (2)

A pluck guitar string vibrates at 258 Hz with amplitude of 2.0 mm. the vibration is timed from when the string moves through the centre of its oscillation, assuming the motion is S.H.M, find the displacement of the string after 0.02 s.

Answer

Since timing starts at the centre,

$$x = A\sin 2\pi ft$$
$$= (2.0\times10^{-3})\times\sin(2\pi\times258\times0.02)$$
$$= 1.7\times10^{-3}\text{m}$$

Example (3)

A baby is bounces up and down with time period of 1.2s and amplitude of 90mm. the motion can be assumed to be S.H.M, calculate:

(a) The frequency of the bounces.

(b) The baby's maximum velocity

Answer

(a) $F = \frac{1}{T} = \frac{1}{1.2} = 0.83\text{Hz}(2\,s.f)$

(b) Maximum Velocity $V_{max} = \pm 2\pi fA$

$$= \pm 2\pi\times0.83\times0.090 = \pm 0.47\text{ms}^{-1}$$

Example (4)

A helical spring damped at one end and hangs vertically. It extends by 10 cm; when a mass of 50 g is hung from its free end calculate:

(a) The spring constant of the spring

(b) The period of small amplitude oscillation of the mass.

Answer

(a) $K = \frac{F}{\Delta x} = \frac{50 \times 10^{-3} \times 9.81}{10 \times 10^{-2}} = 4.9 Nm^{-1}$

(b) $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{50 \times 10^{-3}}{4.9}} = 0.63 s$

Example (5)

A particle of mass 60g oscillates in S.H.M with angular frequency 6.3 rad s^{-1} and amplitude 15 mm. calculate:

(a) The total energy

(b) The kinetic and potential energies at half -amplitude

Answer

(a) $E_{tot} = \frac{1}{2} m\omega^2 A^2 = \frac{1}{2}(60 \times 10^{-3}) \times (6.3)^2 (15 \times 10^{-3})^2 = 2.7 \times 10^{-4} J$

(b) $E_k = \frac{1}{2} m\omega^2 (A^2 - x^2) = \frac{1}{2}(60 \times 10^{-3}) \times (6.3)^2 [(15 \times 10^{-3})^2 - (7.5 \times 10^{-3})^2] = 2.0 \times 10^{-4} J$

$E_k = \frac{1}{2} m\omega^2 x^2 = \frac{1}{2}(60 \times 10^{-3}) \times (6.3)^2 (7.5 \times 10^{-3})^2 = 0.7 \times 10^{-4} J$

Example (6)

A steel strip, clamped at one end, vibrates with frequency of 20Hz and an amplitude o 5 mm at the free end, where a small mass of 2g is positioned. Find:

(a) The velocity of the end when passing through the zero position.

(b) The acceleration at maximum displacement.

(c) The maximum kinetic energy of the mass.

Answer

(a) When the end of the strip passes through the zero position Y=0, the speed is a maximum V_m

$\because \omega = 2\pi f = 2\pi \times 20, \quad A = 0.005m$

$\therefore V_m = \omega A = 2\pi \times 20 \times 0.005 = 0.628 ms^{-1}$

(b) $a = -\omega^2 A = -(2\pi \times 20)^2 \times 0.005 = 79 ms^{-2}$

(c) $K.E = \frac{1}{2} m V_m^2 = \frac{1}{2} \times (2 \times 10^{-3}) \times (0.628)^2 = 3.9 \times 10^{-4} J$

Example (7)

A pendulum is pulled through a small angle to one side so that its horizontal displacement is 0.10m, it is released, and oscillates with a period of 2.0s. What is its displacement after?

(a) 0.5s after it's released

(b) 1.3s after it's released

Answer

$$X = A\,Cos[(\frac{2\pi}{T})t]$$

$$= 0.10 \times Cos[(\frac{2\pi}{2.0})t]$$

(a) $x = 0.10 \times Cos[(\frac{2\pi}{2.0})0.5] = 0.10 \times Cos(\frac{\pi}{2}) = 0$

The pendulum is at the centre of its motion, which is what we would expect since $t = \frac{T}{4}$.

(b) $x = 0.10 \times Cos[(\frac{2\pi}{2.0})1.3] = 0.10 \times Cos(1.3\pi) = -0.059cm$

The pendulum is 5.9cm from the centre of its motion, the negative sign shows that it is on the opposite side of the swing to where it was released

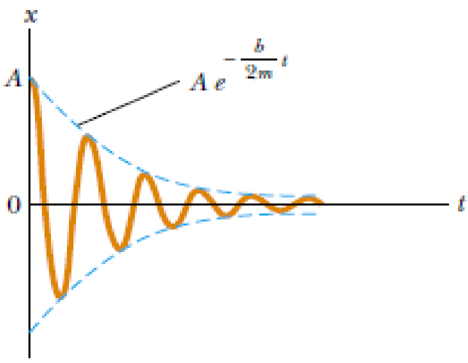
Free and Damped Oscillations

A particle is said to be undergoing **free oscillations** when the only force acting on it is the restoring force.

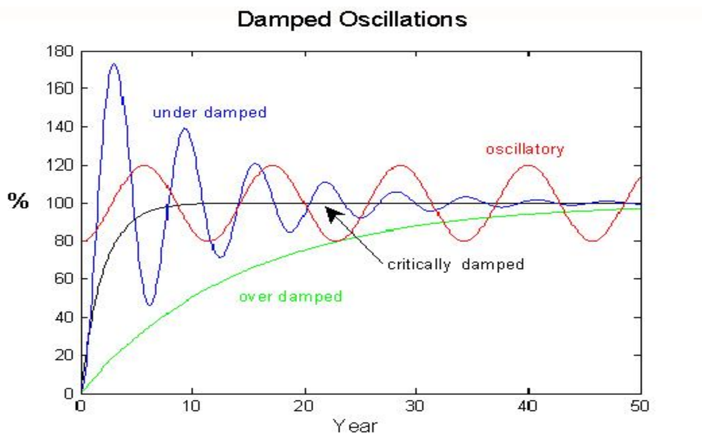
There are no forces to dissipate energy and so the oscillations have constant amplitude. Total energy remains constant. This is the situation we have been considering so far. Simple harmonic oscillations are free oscillations.

In real situations, however, frictional and other resistive forces cause the oscillator's energy to be dissipated, and the energy is converted eventually into heat energy. The oscillations are said to be **damped.**

The total energy of the oscillator decreases with time. The damping is said to be **light** when the amplitude of the oscillations decreases gradually with time. The decrease in amplitude is exponential with time and the period of the oscillation is slightly greater than that of the corresponding free oscillation.



Heavy damping causes the oscillations to die away more quickly. If the damping is increased further, then the system reaches critical damping point. Here the displacement decreases to zero in shortest time without any oscillation



Any further increase in damping produces **over damping.** The displacement decreases to zero in a longer time than for critical damping

Critical damping : minimum amount of damping required to return oscillation to its equilibrium position without oscillating.

Under damping : results in unwanted oscillations.

Over damping : slower return to equilibrium.

Different types of damping:

Heavy Damping

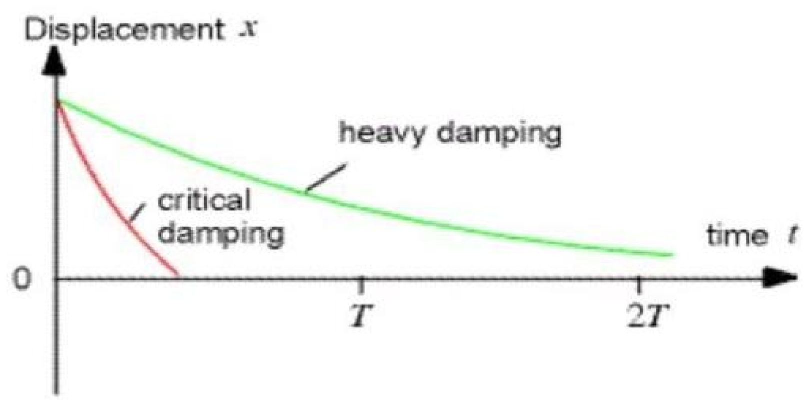
- The system returns to the equilibrium position very slowly, without any oscillation. Heavy damping occurs when the resistive forces exceed those of critical damping.

Critical Damping is important so as to prevent a large number of oscillations and there being too long a time when the system cannot respond to further disturbances.

- Instruments such as balances and electrical meters are critically damped so that the pointer moves quickly to the correct position without oscillating.
- The shock absorbers on a car critically damp the suspension of the vehicle and so resist the setting up of vibration wich could make control difficult or cause damage.

Critical Damping

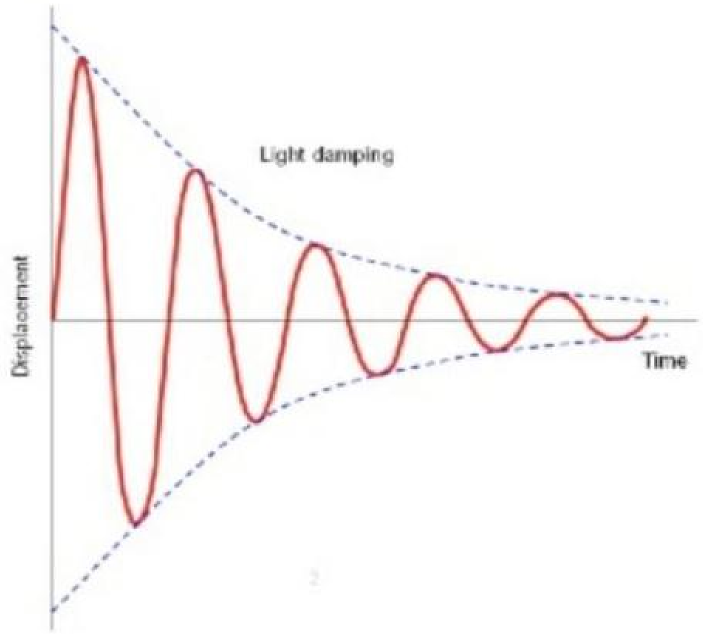
- The system returns to its equilibrium position in the shortest possible time without any oscillation.



Critical and heavy damping

Light damping

- Defined oscillations are observed, but the amplitude of oscillation is reduced gradually with time.



Light Damping

light damping :amplitude decreases, peak broadens, frequency at which resonance occurs decreases

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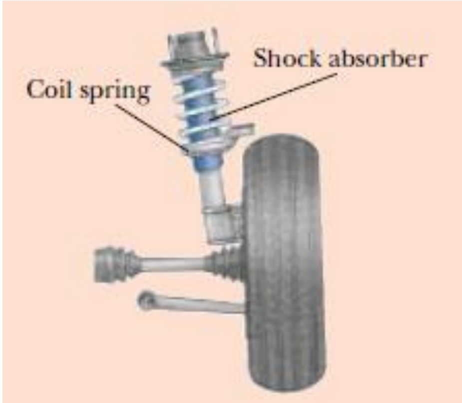


Damping is often useful in an oscillating system.

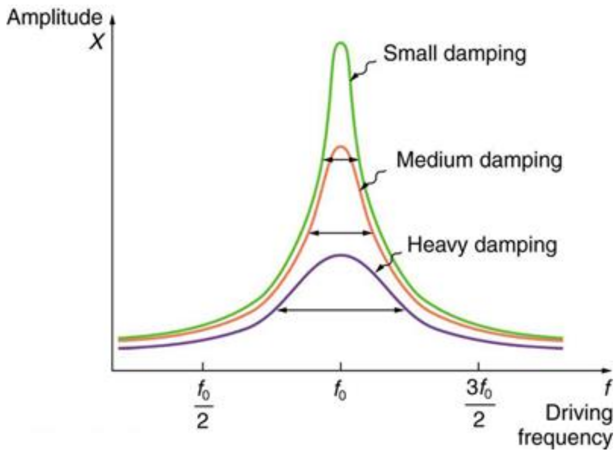
For example, Vehicles have springs between the wheels and the frame to give a smoother and more comfortable ride. If there was no damping, a vehicle would move up and down for some time after hitting a bump in the road

---

Forced Oscillations and resonance

Vibrating objects may have periodic forces acting on them. These periodic forces will make the object vibrate at the frequency of the applied force. rather than at the natural frequency of the system. The object is then said to be undergoing **forced vibrations**.

In the opposite figure, as the frequency of the vibrator is gradually increased from zero, the mass begins to oscillate. At first the amplitude of the oscillation is small, but it increases with increasing frequency. When the driving frequency equals the natural frequency of oscillation of the mass-spring system, the amplitude of the oscillation reaches maximum. The frequency at which this occurs is called the **resonant frequency** and **resonance** is said to occur. If the driving frequency is increased further, the amplitude of oscillation of the mass decreases.



The variation with the driving frequency **f** of the amplitude **A** of the vibration of the mass is illustrated in the opposite graph, the graph is called a **resonance curve**.

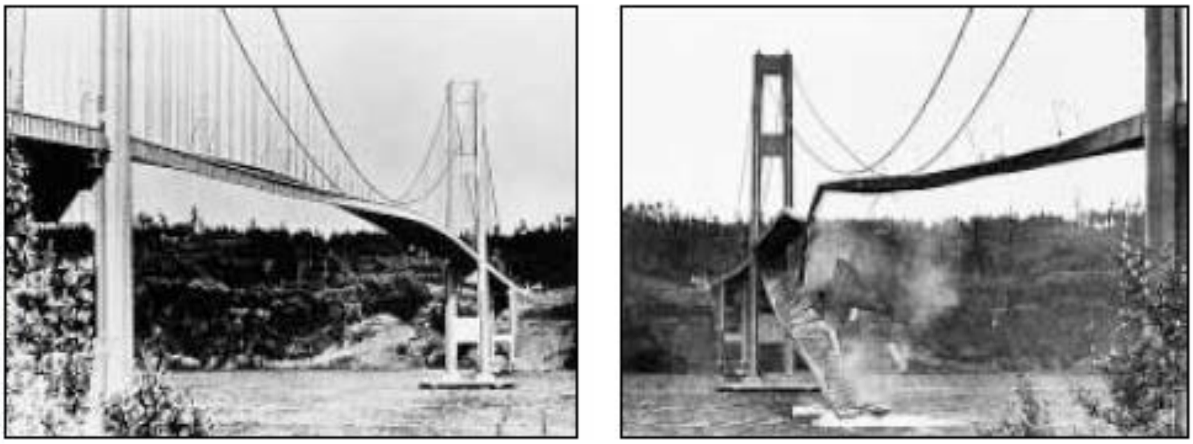
Resonance Occurs when the natural frequency of vibration of an object is equal to the driving frequency, giving maximum amplitude

From the experiment it can be seen that, as the degree of damping increases:

(1) The amplitude of oscillation at all frequencies is reduced
(2) The frequency at maximum amplitude shifts gradually towards lower frequencies.
(3) The peak becomes flatter.

Damping is used where resonance could be a problem. A good example is the damping of buildings during earthquakes. The foundations are designed to absorb energy. This stops the amplitude of the building's oscillations reaching dangerous levels when an earthquake wave arrives.

UPI / Bettmann Newsphotos



(a) (b)

(a) In 1940 turbulent winds set up torsional vibrations in the Tacoma Narrows Bridge, causing it to oscillate at a frequency near one of the natural frequencies of the bridge structure. (b) Once established, this resonance condition led to the bridge's collapse.

Video showcasing Tacoma bridge incident:

A system in resonance:

Its natural frequency = driving frequency

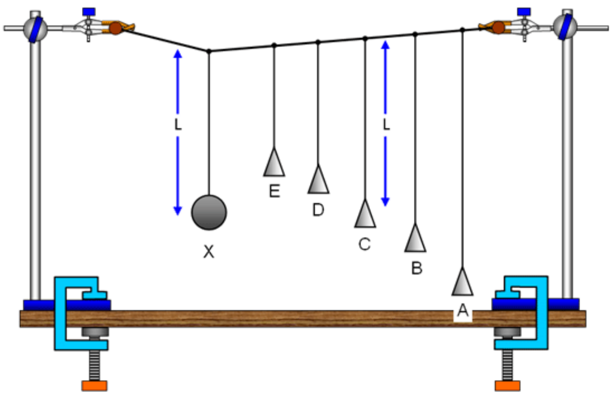
Amplitude is maximum

Absorbs greatest possible energy from driver

Examples of resonance

(a) Barton's pendulums

It is used to demonstrate resonance and effects of damping. The apparatus consists of a set of light pendulums, made from paper cones and a more massive pendulum (the driver). All supported on a taut spring, the lighter pendulum has different lengths, but one has the same length as the driver. This has the same natural frequency and vibrates with the largest frequency of all the pendulums



(b) Pushing a child on a swing

We push at the same frequency as the natural frequency of oscillation of the swing and child so that the amplitude of the motion increases.

(c) Musical instruments

In resonance to amplify the sound produces

Video showing Barton's pendulums in real life :

Using resonance

Resonance is not confined to mechanical systems, it's made use of in:

(1) Microwave cooking

The microwaves used have a frequency that matches the natural frequency of vibration of water molecules. The water molecules in food are forced to vibrate and they absorb the energy of the microwave radiation, the water gets hotter and the absorbed energy spreads through the food and heats it.

(2) Magnetic resonance imaging (MRI)

It is increasingly used in medicine to produce images showing aspects of a patient's internal organs. Radio waves having a range of frequencies are used, and particular frequencies are absorbed by particular atomic nuclei. The frequency absorbed depends on the type of nucleus and on its surroundings

(3) Radio and TV waves

The aerial picks up signals of many different frequencies from many transmitters, the tuner can be adjusted to resonate at the frequency of the transmitting station you are interested in , and the circuit produces a large-amplitude signal for this frequency only.

Video showcasing how microwave ovens work. Watch from **0:30** to **1:30**:

Remarks

(1) $\because a = -\dfrac{A\rho g}{m} y$

and $a = -\omega^2 x$

$$\because \omega^2 = \frac{A\rho g}{m} \Rightarrow \omega = \sqrt{\frac{A\rho g}{m}}$$

$$\therefore 2\pi f = \sqrt{\frac{A\rho g}{m}}$$

$$\therefore f = \frac{1}{2\pi}\sqrt{\frac{A\rho g}{m}}$$

(2) Decreasing the amplitude of the sin or Cos curve indicates that the oscillation is damped.

(3) At maximum displacement, Speed equal to zero and K.E is zero.

(4) To find the periodic time from the graph

$$T = \frac{\text{Total Time}}{\text{Number of Waves}}$$

(5) Damping means decreasing in amplitude due to force acting on an object in opposite direction.

N.B

If you encounter a question like this: [June 19 V2, Q3(c)]

A spring is hung vertically from a fixed point. A mass **M** is hung from the other end of the spring, as illustrated in **Fig. 3.1**.

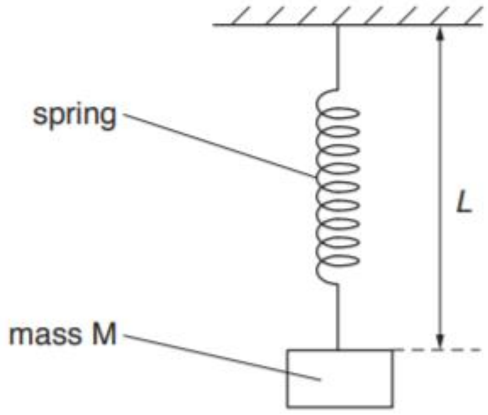


Fig. 3.1

(c) The mass **M** is now suspended from two springs, each identical to that in **Fig. 3.1**, as shown in **Fig. 3.3**.

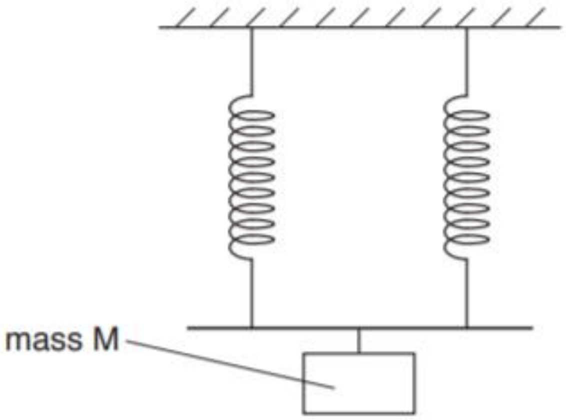


Fig. 3.3

Suggest and explain the change, if any, in the period of oscillation of the mass. A numerical answer is not required.

You could answer with:

-period is shorter/lower **(1 mark)**

AND

-larger spring constant **(1 mark)**

OR

-(restoring) force is greater **(1 mark)**

OR 1

-acceleration is greater **(1 mark)**

OR

-greater energy/maximum speed **(1 mark)**

If you encounter a question like this: [March 21 V2, Q4(a)]

The defining equation of simple harmonic motion is

$$a = -\omega^2 x.$$

State the significance of the minus (–) sign in the equation.

You could answer with:

-Acceleration/restoring force is always in the opposite direction of the displacement/towards the equilibrium position. **(1 mark)**

If you encounter a question like this: [March 20 V2, Q3(a)]

A body undergoes simple harmonic motion.
The variation with displacement **x** of its velocity **v** is shown in **Fig. 3.1**.

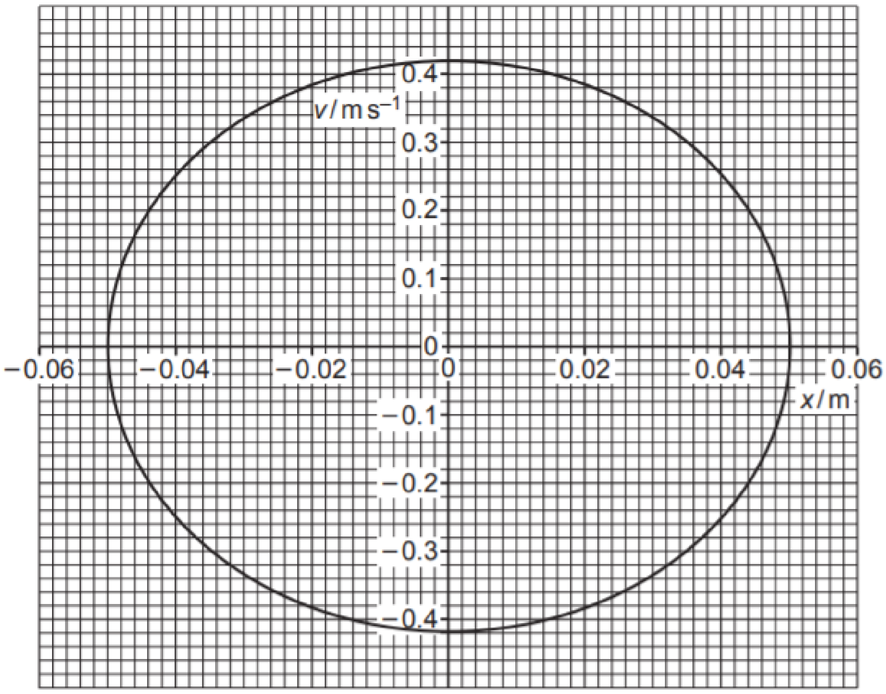


Fig. 3.1

(i) State the amplitude x_o of the oscillations.

(ii) Calculate the period **T** of the oscillations.

(iii) On Fig. 3.1, label with a **P** a point where the body has **maximum** potential energy

You could answer with:

(i) 0.050 m [find this value on graph] (1 mark)

(ii) $v_o = \omega x_o$ and $T = \frac{2\pi}{\omega}$, so $\omega = \frac{v_o}{x_o}$,

therefore: $T = \frac{2\pi}{0.42/0.050} = 0.75$ s

(iii) Mark where the graph meets the displacement axis (x-axis) on either side of the velocity axis (y-axis), as this is when the object has minimum speed/minimum kinetic energy, so it has the maximum potential energy.

If you encounter a question like this: [June 20 V2, Q4(c)]

Some moisture collects on the surface of the dish so that the motion of the ball becomes lightly damped. On the axes of Fig. 4.2, draw a line to show the lightly damped motion of the ball for the first 5.0s after the release of the ball.

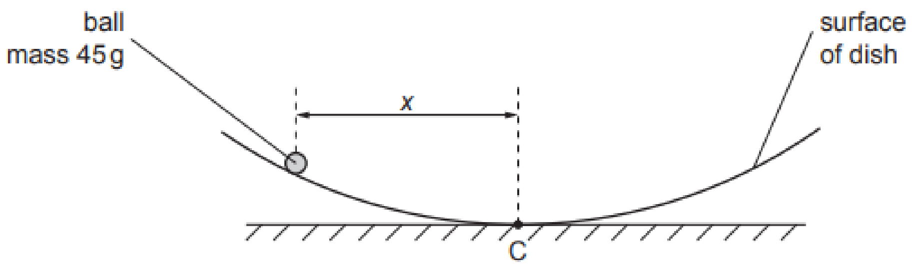


Fig. 4.1

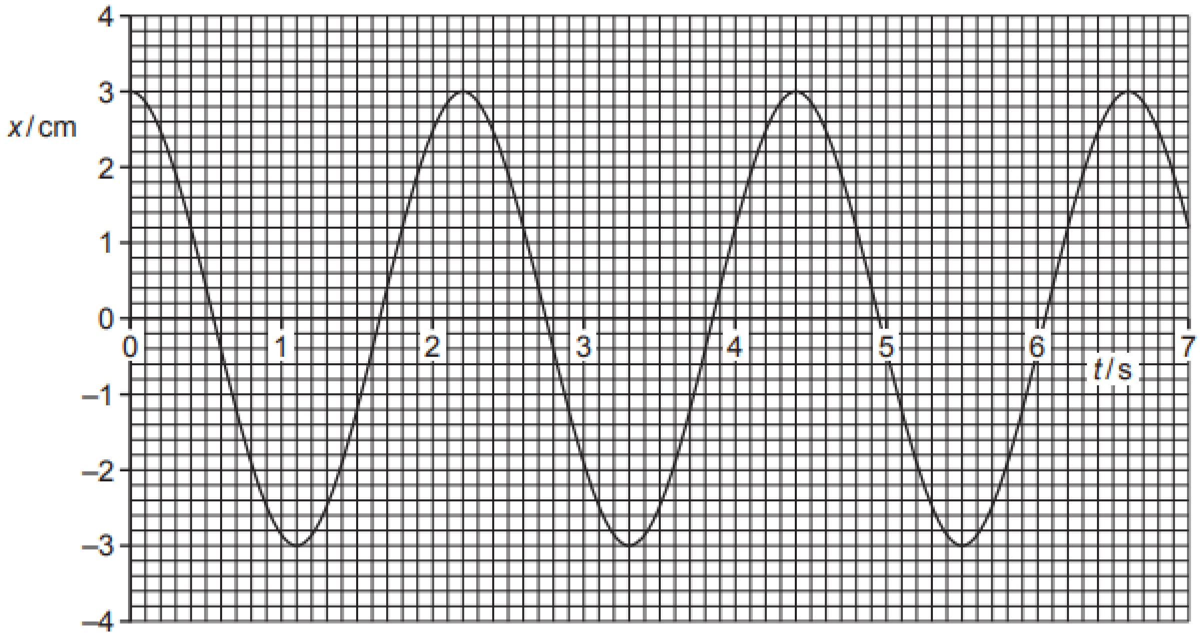


Fig. 4.2

You could answer with:

smooth wave starting at 3.0 cm when t = 0 **(1 mark)**

positions of peaks and troughs show same period (or slightly longer) **(1 mark)**

each peak and trough at lower amplitude than the previous one **(1 mark)**

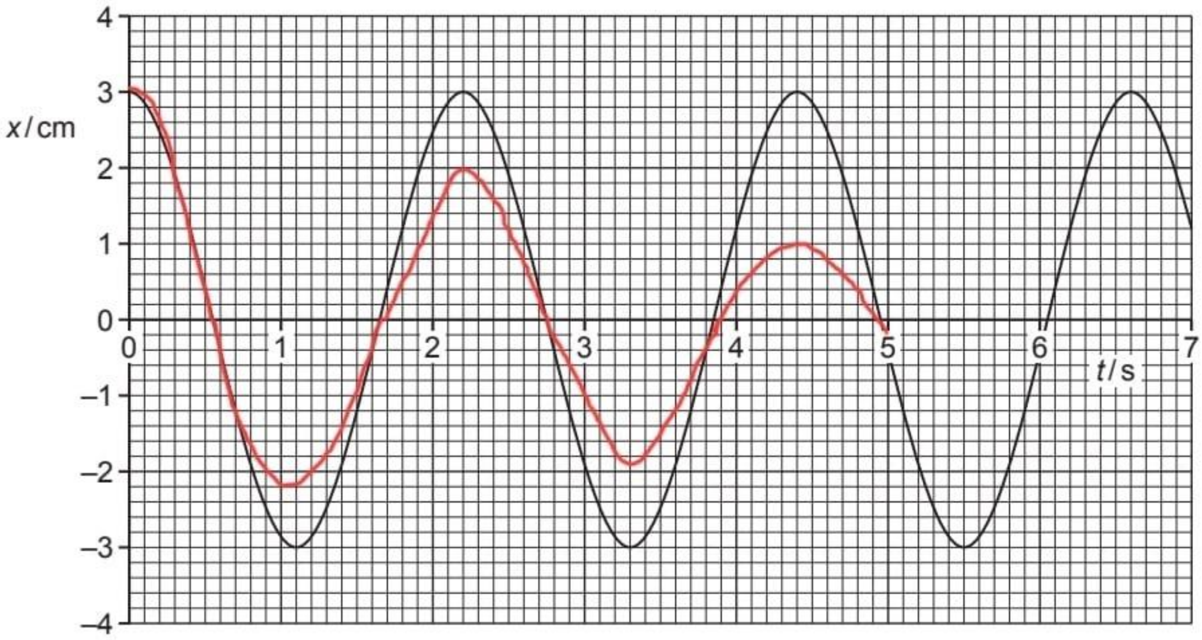


Fig. 4.2

Chapter Seven

Electric fields

Coulomb's Law

The magnitude of the force (F) between two electrically charged bodies, with their separation (r) is inversely proportional to r^2, and directly proportional to the product of their charges (Q_1 and Q_2).

Thus

$$F = K\frac{Q_1 Q_2}{r^2}$$

The value of the constant of the proportionality **K**, depends on the medium in which the charges are situated. The particular property of the medium, which determines the value of **K** according to **the permittivity** of the medium **∈** of the medium.

$$F = \frac{1}{4\pi\epsilon}\frac{Q_1 Q_2}{r^2}\ c$$

Where

F: Force (N)

Q_1 and **Q_2** = Charge (C)

r = Separation (m)

∈= permittivity in farad per metre (Fm^{-1})

[Note that: 1 farad is equivalent to columb per volt]

In case of the free space, the permittivity of free space (vacuum) is denoted by

$$\epsilon_o = 8.854\times10^{-12}\,Fm^{-1} \quad \text{and} \quad 1/4\pi\epsilon_o = 8.988\times10^{9}\,mF^{-1}$$

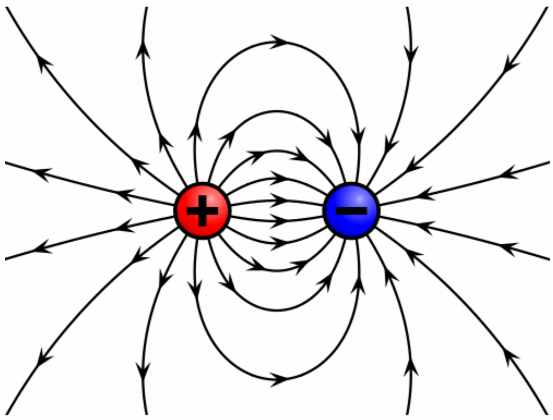
The permittivity of air at STP is $1.005\,\epsilon_o \approx \epsilon_o$

An electric field exists in a region, if electrical forces are exerted on charged bodies in that region.

Electric fields are invisible, but they can be represented by electric lines of force, the direction of the electric field is defined as the direction in which the positive charge would move if it were free to do so, so the lines of force can be drawn with arrows that go from positive **to** negative.

For any electric field:

- The lines of force start on a positive charge and end on a negative charge.
- The lines of force are smooth curves which never touch or cross.
- The strength of the electric field is indicated by the closeness of lines [the closer they are, the stronger the field]



The electric field intensity or electric field strength (E) at a point

Is defined as the force exerted by the field on a unit positive charge placed at that point (unit=NC^{-1} =Vm^{-1})

Field intensity due to a point charge

Since the electric field intensity is force per unit charge
Therefore, the electric field intensity is given by

$$\boxed{E = \frac{F}{Q_2}}$$

Therefore

$$\boxed{E = \frac{1}{4\pi\epsilon}\frac{Q_1}{r^2}}$$

Electrical potential

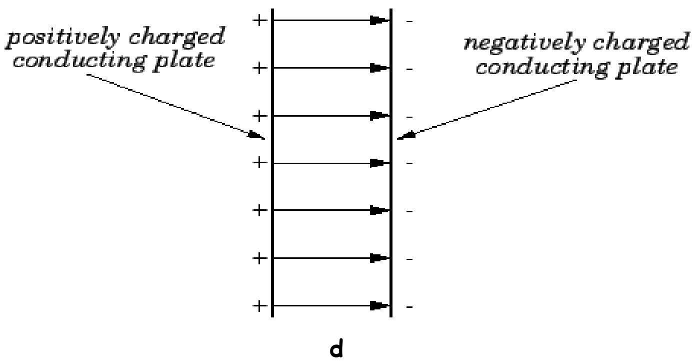
In order to move a charge from one point in an electric field to another, work has to be done on the charge. The two points must, therefore, have some property associated with them, which is different at the two points.
The property is called [Electric potential]. The concept is analogous to that of gravitational potential.
If two points have different electric potentials, then the potential energy of a charge changes as a result of moving from one point to the other. The potential is a property of the field; the potential energy depends on both the field and the size of the charge.

The potential at a point in an electric field

Is defined as the work done in bringing a unit positive charge from infinity to the point [unit is Volt(V)]

The potential due to a point charge

In the opposite figure, the two parallel plates are a distance **d** apart with potential difference **V** between them, a charge **+Q** in the uniform field between the plates has a force **F** acting on it. To move the charge towards the positive plates would require work to be done on the charge.



Work is given by: $W = F \times d$

And from the definition of potential difference: $W = V \times Q$

THEN

$$\boxed{\frac{F}{Q} = \frac{V}{d}}$$

But $\boxed{E = \frac{F}{Q}}$

THEN

$$E = \frac{V}{d}$$

Movement of electrons in an electric field:

From 5:45

Remarks:

- The potential at a point due to a number of point charges is the algebraic sum of the (separate) potentials due to each charge.
- It is clear from the equation that all points, which are equidistant from a point charge, at the same potential.

The electric potential difference between two points

Is defined as the work done in moving a unit positive charge from the point at the lower potential to that at higher potential

Volt

Is defined by the work done causing one coulomb of electric charge to flow between two points in one joule

The relation between potential energy and electric potential

We defined the electric field as the force per unit positive charge

Similarity, the electric potential is defined as the potential energy (**PE_A**) per unit positive charge.

$$V_A = \frac{PE_A}{Q}$$

We already know that only differences in potential energy are measurable.

So, in electrical problems, it is convenient to take the Earth's potential as zero, especially if part of the circuit is earthed.

The potential at a point in an electric field

Is defined as the work done in bringing unit positive charge from infinity to the point.

The field strength at a point

is equal to the negative of the potential gradient at that point

$$V = \frac{KQ}{r} = \frac{Q}{4\pi\epsilon_o r}$$

Comparing Electric and gravitational fields

Electric and gravitational field are quite analogous in many aspects

Gravitational Field	Electric Field
➢ The force is felt by objects with mass. ➢ The force is proportional to the size of the mass ➢ The force is inversely proportional to the square of the separation of the masses ➢ There is a gravitational field round a mass ➢ The force is always attractive. ➢ The constant of proportionality is G. G always has the value $6.67\times 10^{-11}\ Nm^{-2}kg^{-2}$.	➢ The force is felt by objects with Charge. ➢ The force is proportional to the size of the charge. ➢ The force is inversely proportional to the square of the separation of the charges. ➢ There is an electric field round a charge. ➢ The force is always attractive or repulsive ➢ The constant of proportionality is K. [K depends on the material] For air: $K = 9.0\times 10^{9}\ Nm^{-2}C^{-2}$.

$$F = K\frac{Q_1Q_2}{r^2} \xrightarrow{\div Q} E = K\frac{Q}{r^2} \xrightarrow{\div r} V = K\frac{Q}{r}$$

N.B

If you encounter a question like this: [Nov 17 V2, Q6(a)]

(a) For any point outside a spherical conductor, the charge on the sphere may be considered to act as a point charge at its centre. By reference to electric field lines, explain this.

You could answer with:

electric field lines are radial/normal to surface (of sphere) **(1 mark)**

electric field lines appear to originate from centre (of sphere) **(1 mark)**

If you encounter a question like this: [Nov 17 V2, Q6(b)]

(b) An isolated spherical conductor has charge **q**, as shown in Fig. 6.1.

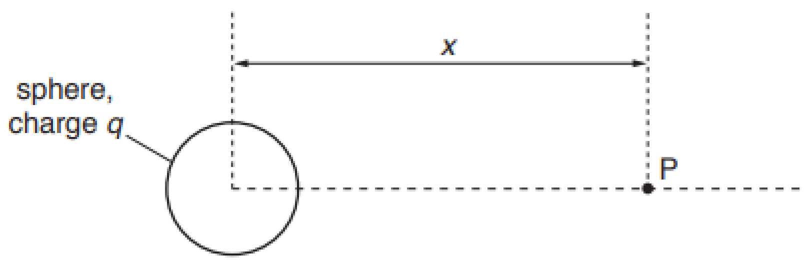


Fig. 6.1

Point **P** is a movable point that, at any one time, is a distance **x** from the centre of the sphere.

The variation with distance **x** of the electric potential **V** at point **P** due to the charge on the sphere is shown in Fig. 6.2.

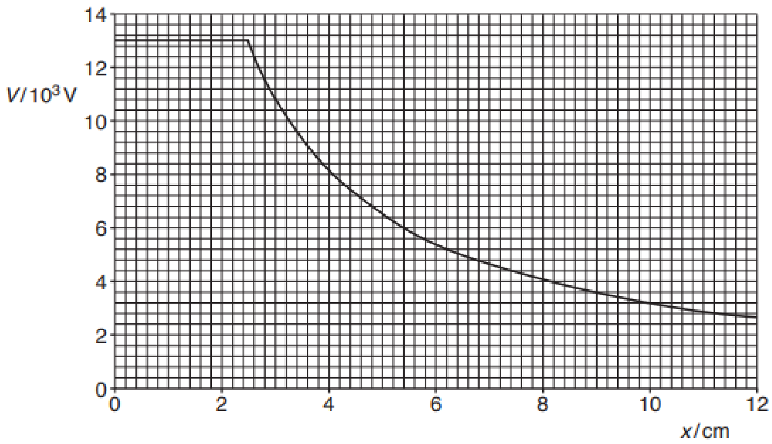


Fig. 6.2

Use Fig. 6.2 to determine

(i) the electric field strength E at point P where x = 6.0cm,

(ii) the radius R of the sphere. Explain your answer.

You could answer with:

(i)-tangent drawn at x = 6.0cm and gradient calculation attempted **(1 mark)**

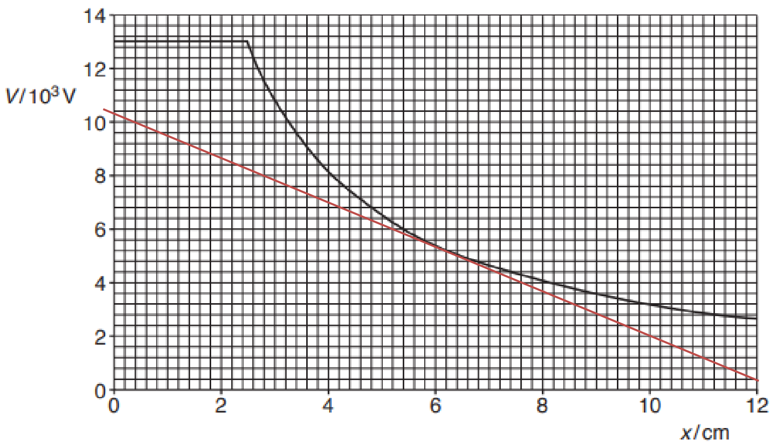


Fig. 6.2

-E = 9.0×10^4N C^{-1} **(2 marks)**

OR

-correct pair of values of **V** and **x** read from curved part of graph and substituted into $V = q/4\pi\varepsilon_0 x$ **(1 mark)**

- $q = 3.6 \times 10^{-8}C$ **(1 mark)**

-(then $E = q/4\pi\varepsilon_0 x^2$ and x = 6cm gives) $E = 9.0 \times 10^{4} N\,C^{-1}$ **(1 mark)**

OR

-($E = q/4\pi\varepsilon_0 x^2$ and $V = q/4\pi\varepsilon_0 x$ and so) $E = V/x$ **(1 mark)**

-$E = 5.4 \times 10^{3} / 0.060$ **(1 mark)**

$= 9.0 \times 104 N\,C^{-1}$ **(1 mark)**

(ii) (R =) 2.5cm **(1 mark)**

-potential inside a conductor is constant **OR** field strength inside a conductor zero (so gradient is zero) **(1 mark)**

If you encounter a question like this: [June 18 V1, Q6(a)]

State what is meant by electric field strength

You could answer with:

-force per unit charge **(1 mark)**

If you encounter a question like this: [June 18 V1, Q6(b)]

An isolated metal sphere A of radius 26cm is positively charged. Sphere A is shown in Fig. 6.1.

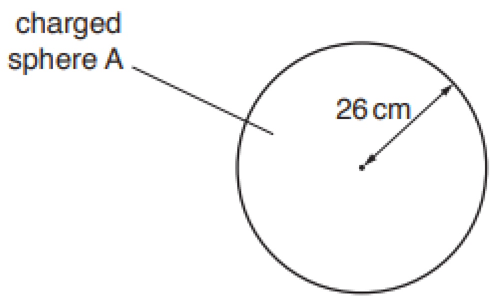


Fig. 6.1

Electrical breakdown (a spark) occurs when the electric field strength at the surface of the sphere exceeds $2.0 \times 10^4 Vm^{-1}$.
Calculate the maximum charge Q that can be stored on the sphere.

You could answer with:

-E = Q/(4 π ε0r 2) **(1 mark)**

$2.0 \times 10^4 = Q / (4 \pi \times 8.85 \times 10^{-12} \times 0.262)$

charge = 1.5×10^{-7} C **(1 mark)**

If you encounter a question like this: [June 18 V1, Q6(c)]

A second isolated metal sphere B, also with charge +Q, has a radius of 52cm

Calculate the additional charge, in terms of Q, that may be stored on this sphere before electrical breakdown occurs.

You could answer with:

charge (= Q [52/26]2) = 4Q **(1 mark)**

additional charge = 3Q **(1 mark)**

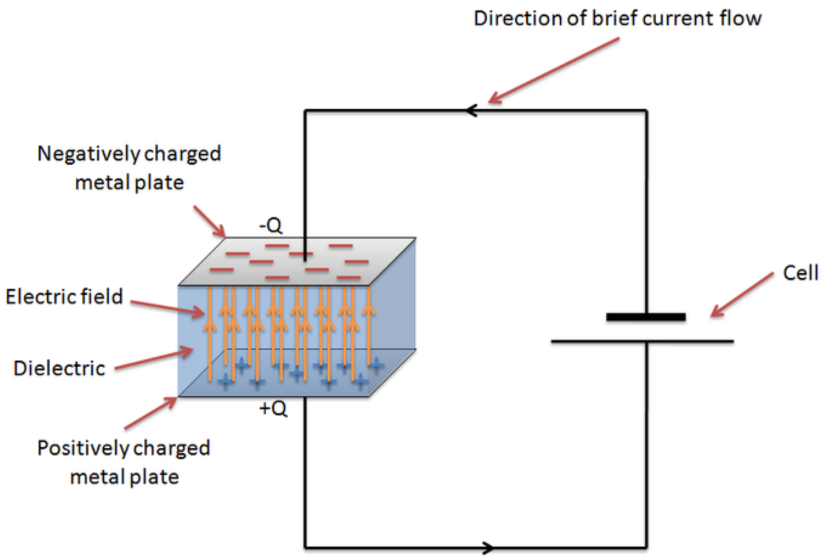
Chapter Eight

Capacitance

Capacitors store charge and energy.

A capacitor consists of two parallel conducting plates separated by an insulator (A dielectric material).

When it is connected to a voltage, supply charge (**Q**) flows onto the capacitor plates until the potential difference across them (**V**) is the same as that of the supply.



It can be seen that the charge **Q** is related to the potential **V** by

$$Q \propto V$$

Hence

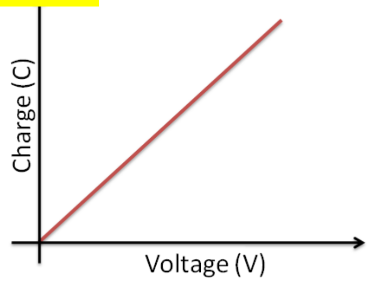
$$\boxed{Q = CV}$$

Where **C** is a constant which depends on the size of the conductor. **C** is known as the capacitance of the conductor.

Capacitance: is the ratio of charge to potential of the conductor

: is the charge **Q** on **one** plate required to cause unit potential difference, **V**, across both plates of capacitor

$$C = \frac{Q}{V}$$



unit of capacitance is Farad

"1 Farad is the capacitance of a conductor, which has potential difference of 1 volt when it carries a charge of 1 coulomb".

Farad=Coulomb/Volt

- In Electronic circuits the useful range used is [10^{-12}F-10^{-3}F]
- Capacitors usually have their values marked in picofarads(pF=10^{-12}F) or microfarads(μF=10^{-6}F)

Energy stored in a capacitor

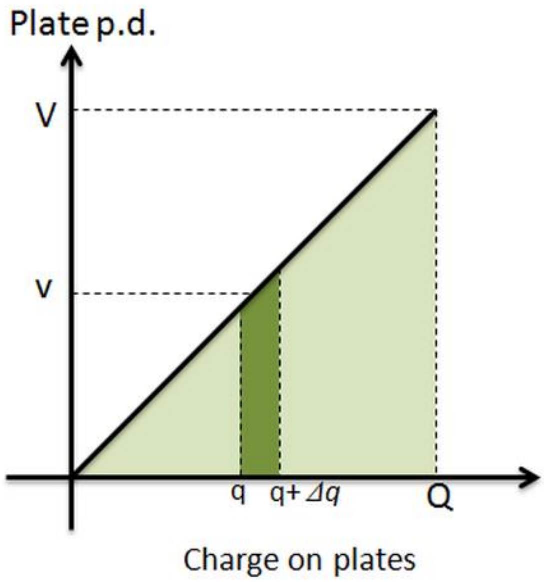
Electrical potential energy is stored when a capacitor is charged.
The energy stored is equal to the work done to force extra charge Δq.
From the definition of potential difference
The work done is the product of potential and charge:

$$W(\text{or } E_p)=QV$$

Then with a small amount of charge Δq

$$\Delta E=V\ \Delta q$$

is equal to the shaded area under the line between **q** and **q+Δq**
for the whole process



The energy transferred from a battery when a capacitor is charged is given by the area under the graph line when charge is plotted against potential difference
The area under the line represents a right-angled triangle. Thus,

$$\boxed{E_p=\frac{1}{2}QV}$$

Since **Q=CV**

Then

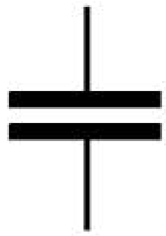
$$\boxed{E_p=\frac{1}{2}CV^2=\frac{Q^2}{2C}}$$

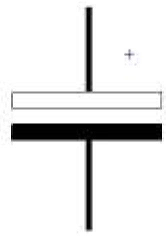
Example (1)
A 2000 μF capacitor is charged to a p.d of 10 V. Calculate the energy stored by the capacitor.
Solution
C=2000 μF , V=10v

$$E_p=\frac{1}{2}CV^2$$
$$=\frac{1}{2}\times 2000\times 10^{-6}\times (10^2)^2$$
$$=0.10\,J$$

Factors affecting capacitance

The capacitance of an air-filled capacitor can be increased by putting an insulating material, such as mica or waxed paper, between the plates. The material between the plates is called the dielectric. In a type of capacitor known as an electrolytic capacitor





The capacitance is directly proportional to the area A of the plates, and inversely proportional to the distance d between them

$$C \propto \frac{A}{d}$$

For a capacitor with air or vacuum between the plates, the constant of the proportionality is the permittivity of free space

$$\boxed{C = \frac{\epsilon_0 A}{d}}$$

C is measured in farad

A is measured in m^2

d is measured in m

Then ϵ_0 is measured in $\mathbf{Fm^{-1}}$

If a dielectric used to increase the capacitance, then another quantity called the relative permittivity ϵ_r of the dielectric increases.

$$\boxed{C = \frac{\epsilon_0 \epsilon_r A}{d}}$$

The relative permittivity is the capacitance of a parallel-plate capacitor with the dielectric between the plates divided by the capacitance of the same capacitor with vacuum between the plates

- It is a ratio and has no units

Capacitors in series and parallel

Circuits often contain combination of two or more capacitors. Like resistors, they can be connected in series or parallel

(a) Capacitors in parallel

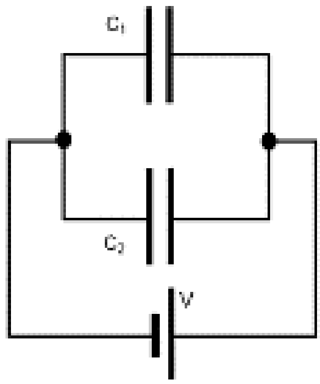
The diagram shows two capacitors connected in parallel

Here are 3 facts that you should know for this circuit

- The potential difference across each capacitor is the same and equal to v.
- The total charge Q is the sum of the charges on each capacitor

  $Q=Q_1+Q_2$
- The combined capacitance C is the sum of the two capacitances

$$\boxed{C=C_1+C_2}$$



To prove this formula

Suppose the opposite circuit

We know $Q=Q_1+Q_2$

But $Q=CV$

Then $Q_1=C_1V$ and $Q_2=C_2V$

By substitution

$CV= C_1V+ C_2V$

By cancelling the V's

$$C=C_1+C_2$$

Example (1)

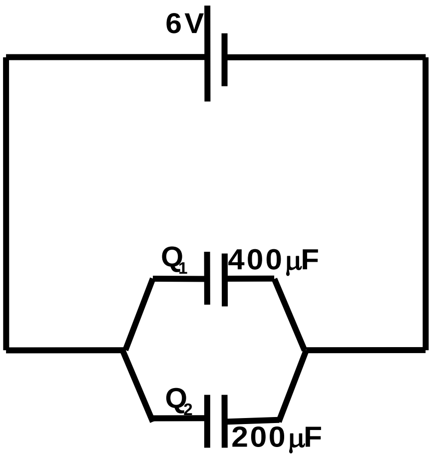
Calculate the charge on each capacitor in this diagram

Solution

Using $Q=CV$

$Q_1=C_1V=400\times6=$ **2400μC**

$Q_2=C_2V=200\times6=$ **1200μC**



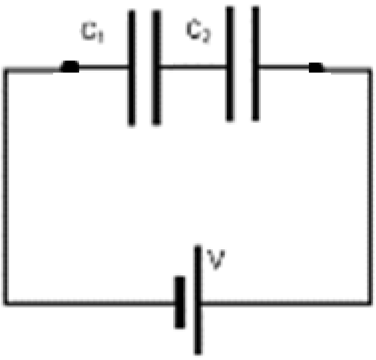
(a) Capacitors in Series

The diagram shows two capacitors connected in series
Here are 3 facts that you should know for this circuit

- The charge on each capacitor is the same and equal to Q.
- The supply potential difference V is shared between the two capacitor
  $V=V_1+V_2$
- The combined capacitance C is the sum of the two capacitances

$$\boxed{\frac{1}{C}=\frac{1}{C_1}+\frac{1}{C_2}}$$

To prove this formula
Suppose the opposite circuit
We know $V=V_1+V_2$



But $V=\frac{Q}{C}$

Then $V_1=\frac{Q}{C_1}$ and $V_2=\frac{Q}{C_2}$

By substitution

$$\frac{Q}{C}=\frac{Q}{C_1}+\frac{Q}{C_2}=C_1V+C_2V$$

By cancelling The Q's

$$\frac{1}{C}=\frac{1}{C_1}+\frac{1}{C_2}$$

Example (2)

A 3.0 μF and a 5.0 μF capacitor are connected in series with a 12 V battery.

Solution

a. Find the equivalent capacitance.

$$\frac{1}{C_S} = \frac{1}{3.0\,\mu F} + \frac{1}{5.0\,\mu F}$$

$$\frac{1}{C_S} = \frac{5+3}{15\,\mu F}$$

$$C_S = \frac{15\,\mu F}{8} \longrightarrow \boxed{C_S = 1.9\,\mu F}$$

b. Find the charge on each capacitor.

$Q = C_S V$

$Q = (1.9\ \mu F)(12\ V)$

$Q = 23\ \mu C$

c. Find the potential drop (or voltage) across each capacitor.

$$V_1 = \frac{q}{C_1};\ V_1 = \frac{23\,\mu C}{3.0\,\mu F};\quad V_1 = 7.7\ V$$

$$V_2 = \frac{q}{C_2};\ V_2 = \frac{23\,\mu C}{5.0\,\mu F};\quad V_1 = 4.6\ V$$

• Capacitance = $\frac{\text{charge}}{\text{Voltage (Pd)}}$ (1)

$\frac{\text{charge of one plate (1)}}{\text{Pd bet. 2 plates (1)}}$

Define Capacitance ✓

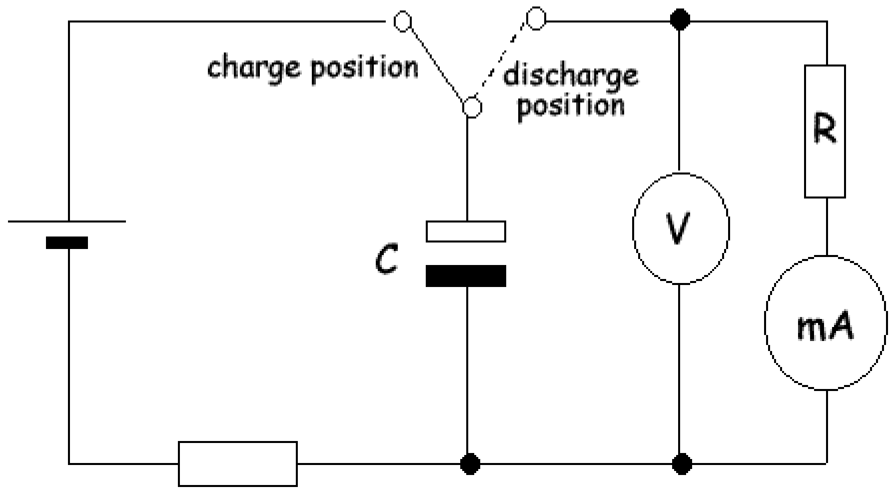
Capacitor store (E) not charge

Bec charge upper plate = (-ve) charge lower plate

So Total charge = Zero

Charging and discharging capacitors

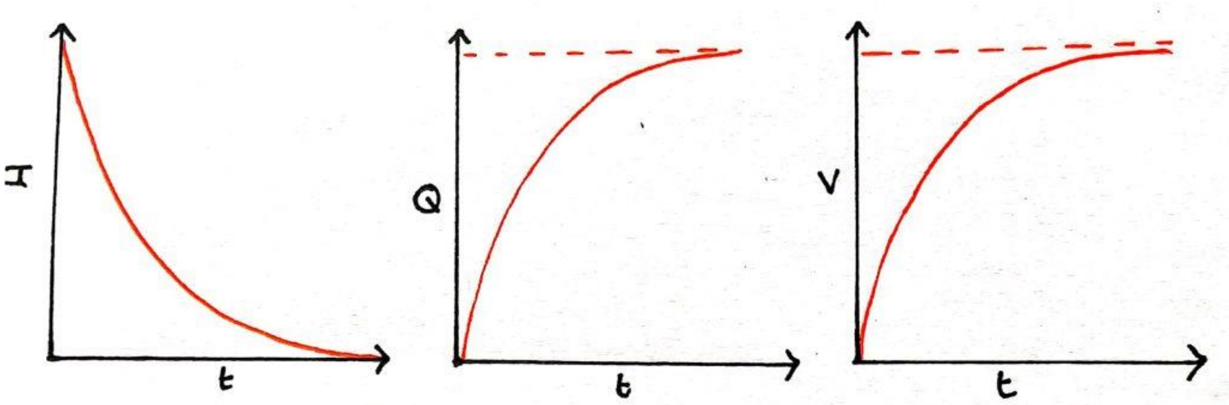
A capacitor is charged or discharged when it is connected to a d.c energy source (battery or rectified a.c input) while also connected to a second parallel circuit for discharge and a resistor to control the charging/discharging time and intensity as to not damage the capacitor.



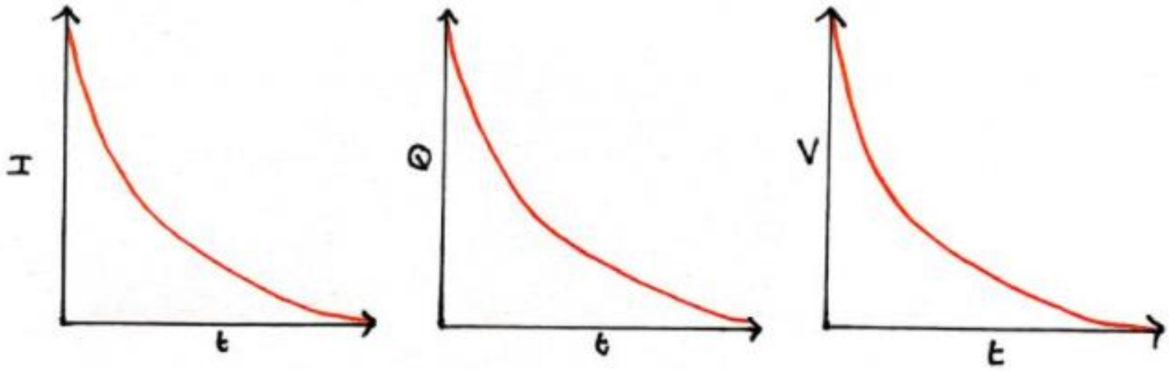
Graphs of charge and discharge:

While charging, the current **I** in the capacitor starts at the maximum value and then gradually decreases as more charge is stored on the capacitor plates. The same thing happens while discharging as more charge **Q** leaves the plates. The potential difference **V** across the capacitor increases while charging as it is directly proportional to the charge **Q**.

Charging:



Discharging:



Equation for the time taken to discharge:

We can derive that the time taken for a capacitor to discharge is dependent on the resistance of the circuit and the capacitance of the capacitor. This is shown by the equation: **$\tau = RC$** **(very important)** where the Greek letter **τ** (tau) is the constant for time.

Equations for the decrease in charge, p.d, current:

There are equations that describe the "decay" of charge, current and p.d across the capacitor when it discharges. These decay equations are very similar as p.d is directly proportional to charge and also to current:

$$I = I_0 \exp\left(-\frac{t}{RC}\right)$$

$$V = V_0 \exp\left(-\frac{t}{RC}\right)$$

$$Q = Q_0 \exp\left(-\frac{t}{RC}\right)$$

What happens if we change the resistance in the circuit?

There will be no change in the initial potential difference across the capacitor, but the initial current through the resistor will be changed. Increased resistance will mean decreased current, so charge flows off the capacitor plates more slowly and the capacitor will take longer to discharge. Of course, this means decreasing the resistance will cause the capacitor to discharge more quickly.

What happens if we increase the capacitance of the capacitor?

The initial p.d. across the capacitor is also unchanged. So, with an unchanged resistance, the initial current will be unchanged. However, there will be more charge on the capacitor and so it will take longer to discharge.

N.B

If you encounter a question like this: [Nov 20 V2, Q6(a),(b)]

(a) (i) Define the capacitance of a parallel plate capacitor

(ii) State three functions of capacitors in electrical circuits

1.

2.

3.

(b) A student has available **three** capacitors, each of capacitance **12μF**.

Draw diagrams, one in each case, to show how the student connects the capacitors to give a combined capacitance between the terminals of:

(i) 18μF

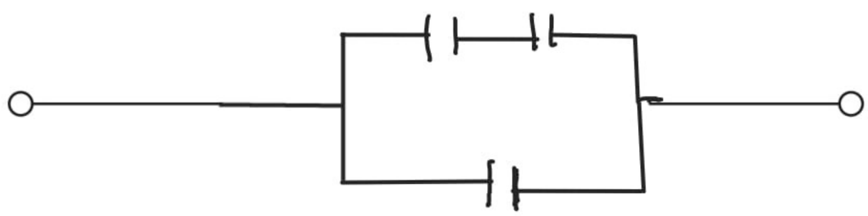
(ii) 8μF

You could answer with:

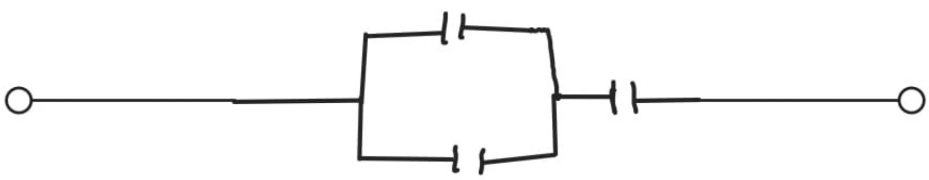
(a)(i) charge on one plate per unit potential difference between both plates
(2 mark)

(ii) any three from:
•smoothing
•timing/(time) delaying
•tuning
•oscillator
•blocking d.c. (direct current)
•surge protection
•temporary power supply
(each **1 mark**)

(b)(i)



(ii)



If you encounter a question like this: [Nov 20 V1, Q(b)]

A student has available **four** capacitors, each of capacitance **24μF**. The capacitors are connected as shown in **Fig. 6.1**.

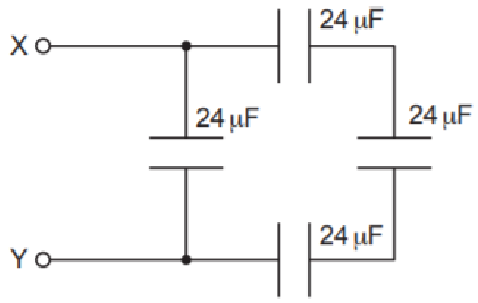


Fig. 6.1

Calculate the combined capacitance between the terminals X and Y.

You could answer with:

$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$ for capacitors in series,

therefore three capacitors in **series** have a combined capacitance of **8μF** **(1 mark)**, and since total capacitance for capacitors in parallel is simply $C = C_1 + C_2$,

24μF+8μF = 32μF (1 mark)

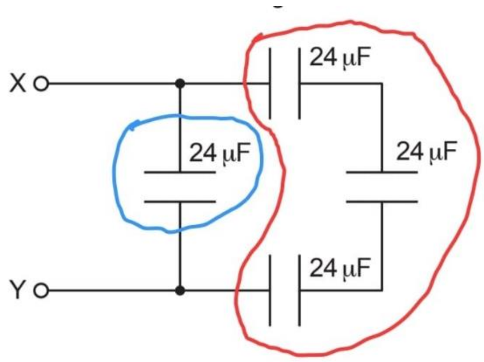


Fig. 6.1

If you encounter a question like this: [June 19 V1, Q6(b)]

Three uncharged capacitors of capacitances C_1, C_2 and C_3 are connected in series with a battery of electromotive force (e.m.f.) **E** and a switch, as shown in **Fig. 6.1**.

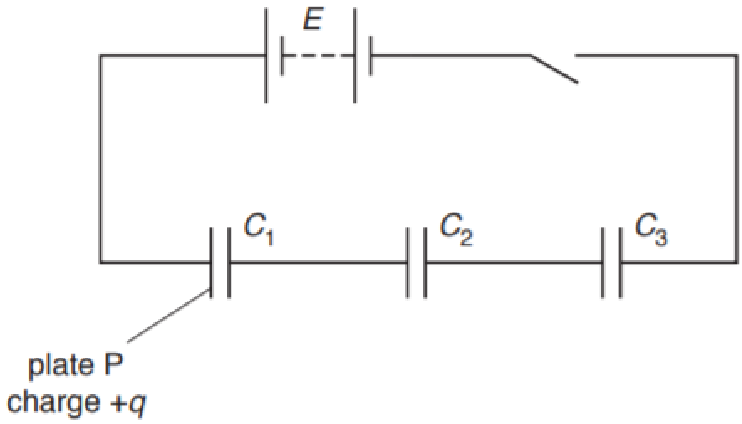


Fig. 6.1

When the switch is closed, there is a charge **+q** on plate **P** of the capacitor of capacitance C_1. Show that the combined capacitance **C** of the three capacitors is given by the expression

$$\frac{1}{C}=\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}.$$

You could answer with:

$V = V_1 + V_2$ and $V = \frac{Q}{C}$ therefore

$$\frac{Q}{\text{Total } C}=\frac{Q}{C_1}+\frac{Q}{C_2}+\frac{Q}{C_3}$$

and **Q** is constant at all capacitors, so

$$\frac{1}{\text{Total } C}=\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}$$

If you encounter a question like this:

The potential difference across the plates of a capacitor of capacitance 500 μF is 240 V. The capacitor is connected across the terminals of a 600 Ω resistor.
Find the time taken for the current to fall to 0.10 A.

You could answer with:

$I_0 = \frac{V}{R} = \frac{240}{600}$

$I_0 = 0.40$ A

$\tau = RC$

$\tau = 600 \times 500 \times 10^{-6}$

$\tau = 0.30$ s

$I = I_0 \exp(-\frac{t}{RC})$

$0.10 = 0.40 \exp(-\frac{t}{0.30})$

$0.25 = \exp(-\frac{t}{0.30})$

Take ln both sides

$\ln 0.25 = -\frac{t}{0.30}$

$t = 0.41$ s

If you encounter a question like this:

Explain what is meant by the time constant of a circuit containing capacitance and resistance.

You could answer with:

It's the product of the capacitance of capacitor and the total resistance of the circuit, it's also equal to the time taken for the capacitor to fully discharge

If you encounter a question like this: [Specimen 22, Q(12)(a)]

A sinusoidal alternating potential difference (p.d.) from a supply is rectified using a single diode.The variation with time *t* of the rectified potential difference *V* is shown in Fig. 5.1.

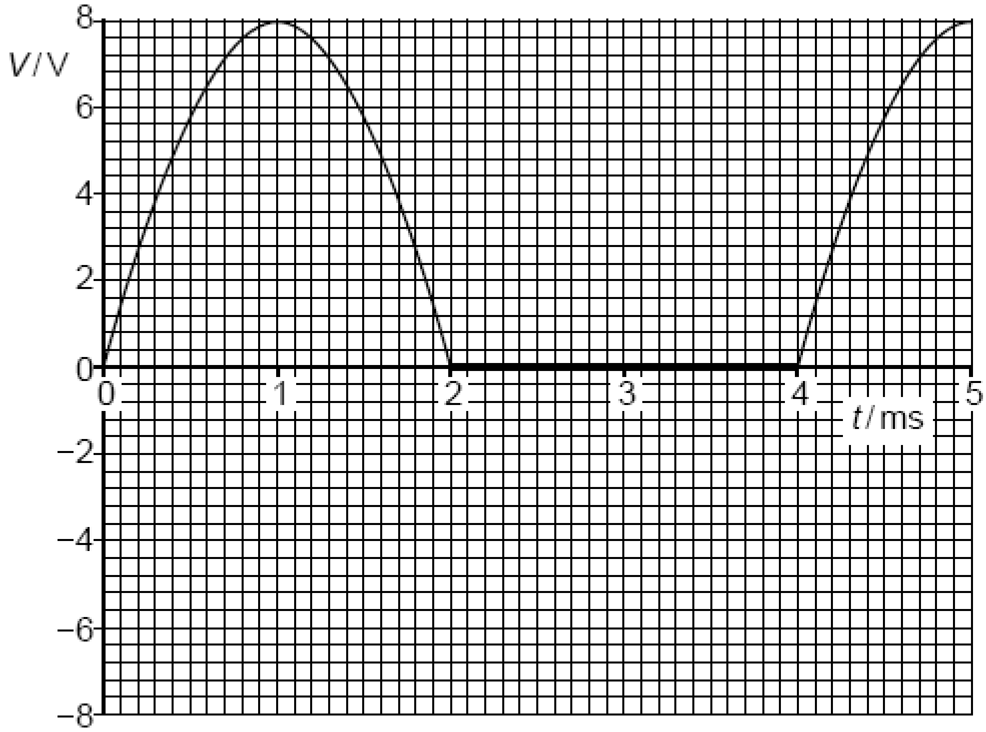


Fig. 5.1

The alternating potential difference is rectified and smoothed using the circuit in Fig. 5.2.

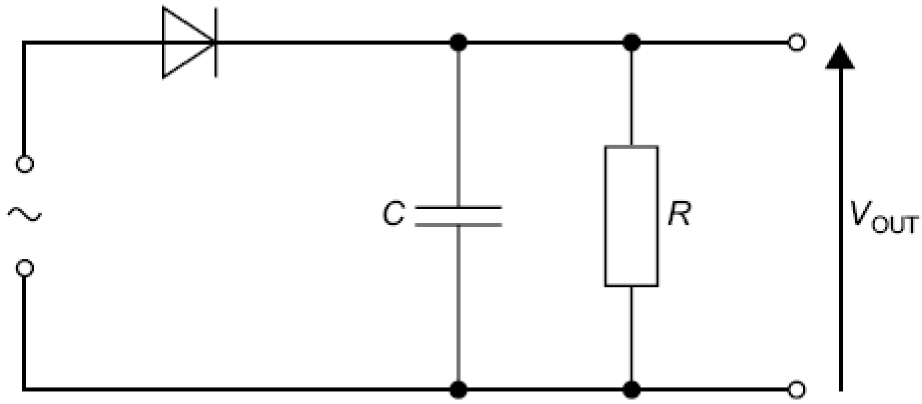


Fig. 5.2

The capacitor has capacitance **C** of 85 **μF** and the resistor has resistance **R**.

The effect of the capacitor and the resistor is to produce a smoothed output potential difference V_{OUT}. The difference between maximum and minimum values of V_{OUT} is 2.0 **V**.

(i) On Fig. 5.1, draw a line to show V_{OUT} between times **t** = 1.0 ms and **t** = 5.0 ms.

(ii) Determine the time, in s, for which the capacitor is discharging between times **t** = 1.0 ms and **t** = 5.0 ms.

(iii) Use your answers in **(b)(i)** and **(b)(ii)** to calculate the resistance **R**.

You could answer with:

(i)

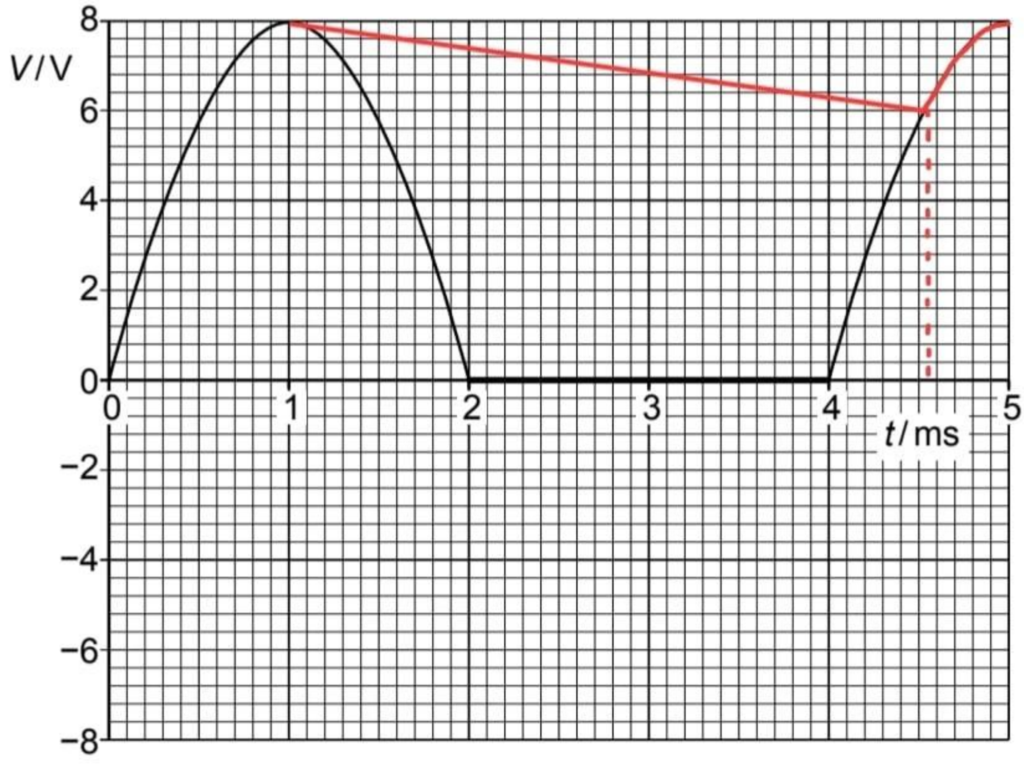


Fig. 5.1

(ii) $t = (4.6 - 1)\times10^{-3} = 3.6\times10^{-3}$ s

(iii) $\tau = RC$

$3.6 = R\times85\times10^{-6}$

$R = \dfrac{3.6\times10^{-3}}{85\times10^{-6}}$

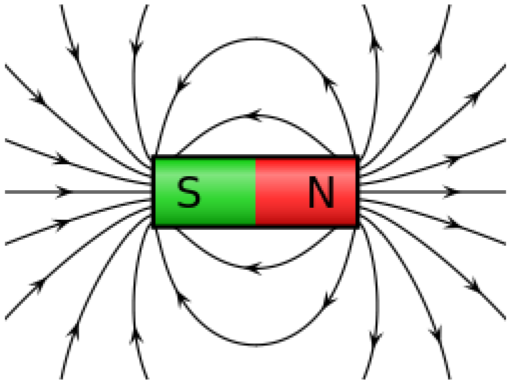
$R = 42$

Chapter Nine

Electromagnetism

Magnetic fields

The magnetic field of the magnet is the region around a magnet where magnetic effects can be experienced.

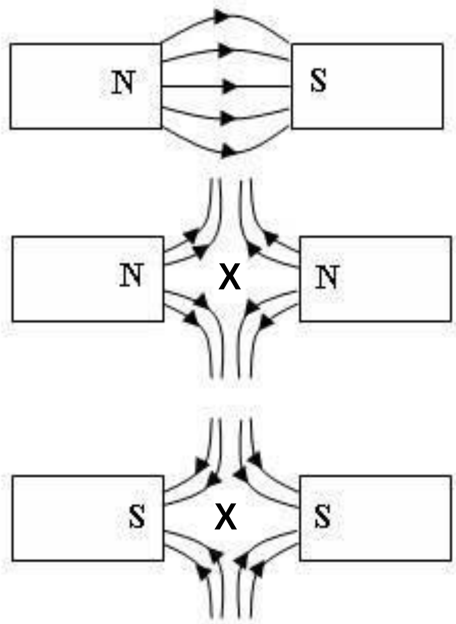


- The magnetic field is not visible but they may be represented by **magnetic field lines**. (or magnetic flux)
- The magnitude and the direction of the magnetic field can be represented by its magnetic flux density (**B**)
- **The direction** of the magnetic flux density at a point is that of the tangent to the field line at the point
- **The magnitude** of the flux density is proportional to the number of field lines per unit area.
- Although the magnetic field has been drawn it two dimensions, the actual magnetic field is three dimensional.
- The magnetic field lines start at a north pole and end at a south pole
- The magnetic field lines are smooth curves which never touch or cross
- The strength of the magnetic field is indicated by the distance between the lines-closer lines means a stronger field.

If you have two bar magnets the magnetic field appears as shown

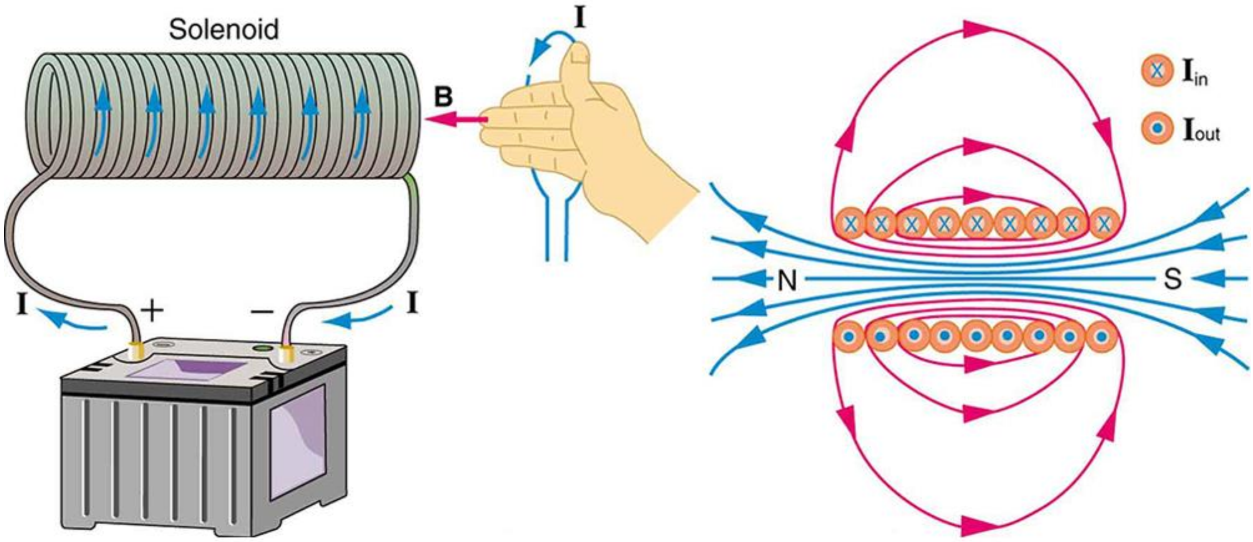
From the opposite figure we can notice that

The magnetic field due to one magnet opposes that due to the other .The magnetic field lines cannot cross and consequently there is a point X known as (Neutral point), where there is no resultant magnetic field because the two fields are equal in magnitude but opposite in direction.



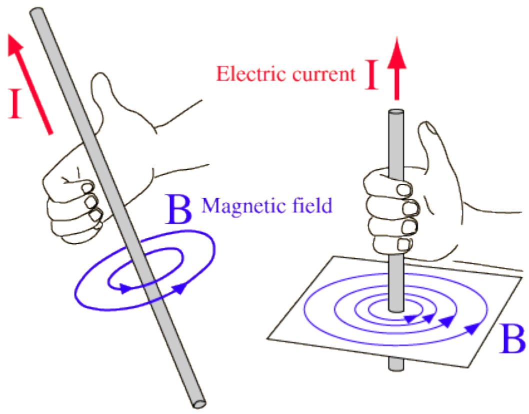
Magnetic fields and electric current

In 1890 Orested discovered that
"a wire carrying an electric current has an associated magnetic field"
The size and shape of the magnetic field depends on the size of the current and the shape of the conductor through which the current is travelling
Using the solenoid illustrated in the next figure



The magnetic field due to a straight wire appears as a series of concentric circles centered on the wire and the direction of the field can be found by using the right –hand grip rule

Grasp the wire using the right hand with the thumb pointing in the direction of the current, the figures then point in the direction of the field

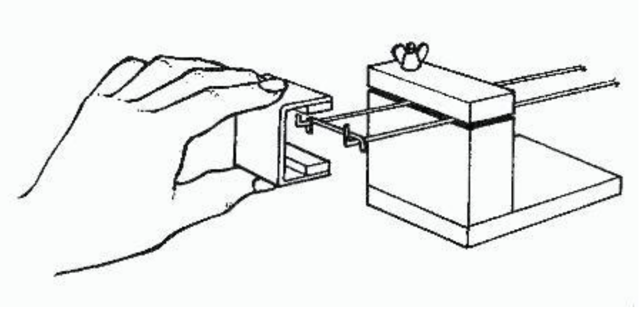


The force on a conductor in a magnetic field

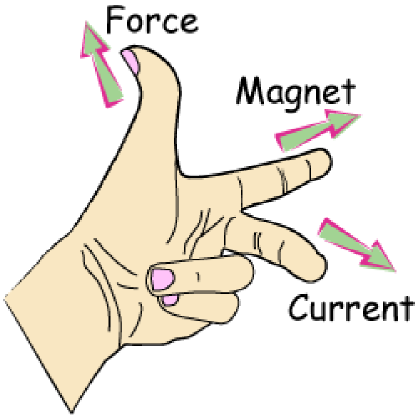
A current –carrying wire is surrounded by a magnetic field. This magnetic field interacts with an external magnetic field, giving rise to a force on the conductor

In the opposite figure

The magnets create a uniform field, as soon as the current in the copper rod is switched on; the rod starts to roll, showing that a force is acting on it.

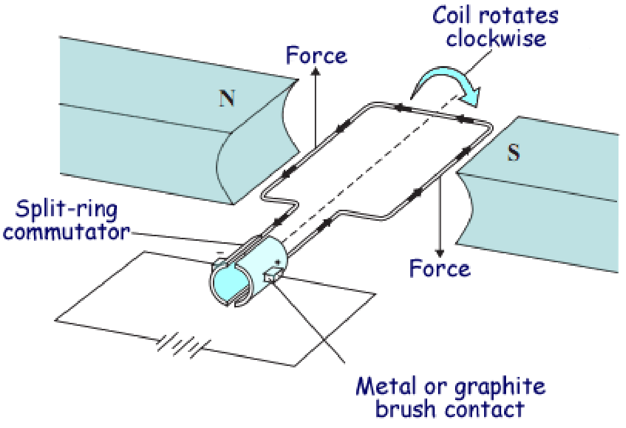


An experiment shows that the force is always perpendicular to the plane, which contains both the current and the external field at the site of the conductor. The direction o the force can be found by using Fleming's left rule "if the first and second fingers and the thumb of the left hand are placed comfortably, at right angles to each other, with the first finger, pointing in the direction of the field, and the second finger pointing in the direction of the current, then the thumb points in the direction of the force (the direction which motion take place)"



We can explain this force by thinking about the magnetic fields of the magnets and the current carrying the conductor; these fields combine and interact to produce the force on the rod

The production of this force is known as [the motor effect]



The strength of the magnetic field is known as the magnetic flux density (you can imagine this quantity by represent the number of magnetic field lines passing through a region per unit area)

The magnetic flux density is greater close to the bars of the magnets and gets smaller as you move away from it

It defined as

$$\boxed{B = \frac{F}{I \times L}}$$

B is the magnetic flux density [measured in **tesla**]
F is the force on the conductor [measured in **Newton**]
I is the current in the conductor [measured in **Ampere**]
L is the length of the conductor [measured in **metre**]

A magnetic flux density of 1 T produces a force of 1N on each meter of wire carrying a current 1A at 90° to the field

Example (1)

The two wires beside are in a uniform magnetic field of 0.25T. Both wires are 0.50m long and carry a current of 4.0 A, calculate the size of the force on each wire.

(a) The wire is at 90° to the field
(b) The wire is at 30° to the field

Solution

B=0.25T L=0.50m I=4.0A F=??

(a) The wire is at 90° to the field

$$B = \frac{F}{I \times L}$$

$$\therefore F = I \times L \times B$$

$$= 4.0 \times 0.50 \times 0.25 = 0.50\text{N}$$

(b) The wire is at 30° to the field

$$B\sin\theta = \frac{F}{I \times L}$$

$$\therefore F = I \times L \times B\sin\theta$$

$$= 4.0 \times 0.50 \times 0.25 \times \sin 30^\circ = 0.25\text{N}$$

The force on a charged particle in a magnetic field

The magnitude of the force

$$\boxed{F = B \times Q \times V \times Sin\theta}$$

Where

F= the force on the particle (**N**)
B= the magnitude of the magnetic flux density of the field (**T**)
Q= the charge on the particle (**C**)
V= the magnitude of the velocity of the particle (**ms^{-1}**)

From the last equation one can notice that a magnetic field cannot exert a force on a stationary charged particle.

> One tesla is the uniform magnetic flux density, which, acting normally to a long straight wire carrying a current of 1 Ampere, cause a force per unit length of 1 Nm^{-1} on the conductor.

- ### The direction on the force

The force on positively charged particle is the same as that on a conductor, which is carrying a current in the same direction as that, in which the particle is moving. It follows that the direction of the force can be found by using Fleming's left-rule.

Thus, if the particle shown in fig is positively charged, the force acting on it is directed perpendicularly into the paper. A negatively charged particle feels a force in the opposite direction.

Example (2)

An electron moving at $6.0x10^{5}$ ms^{-1} passes perpendicularly through a magnetic field of $2.0x10^{-2}$T. The charge of the electron is $6.1x10^{-19}$C.what is the force on the electron

Solution

$$F = BQV$$

$$F = 2.0 \times 10^{-2} \times 1.6 \times 10^{-19} \times 6.0 \times 10^{5}$$

$$= 1.9 \times 10^{-15} N (2s.f)$$

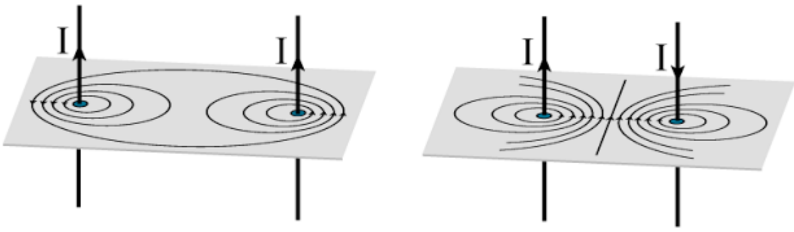
The force between two current carrying conductors

If two current-carrying conductors are close together, then each is in the magnetic field of the other and therefore, each experiences a force.

In the opposite Fig

X and Y are two infinitely long parallel conductors carrying currents I_1 and I_2 respectively.

The conductors are in vacuum and their separation is a. the magnitude of the magnetic flux density B, at any point P, on Y due to the current in X is given by



Catapult field produced by 2 straight curretn carrying conductors

$$F \propto \frac{I_1 \times I_2}{a}$$

By Fleming's left-rule, the force in the wire is directed towards X as shown; similar reasoning reveals that X is subject to a force of the same magnitude. The force on X is in the opposite direction to that on Y because the field along X is directed out of the paper.

Thus, the forces are such that the wires attract each other. It can be shown at the current are in opposite directions the wires repel each other. Thus

Like currents attract, unlike currents repel

Couple on a coil in magnetic field

Current carrying coils in magnetic fields are essential components of electric motors and meters of various kinds.

Consider a rectangular coil PQRS of N turns pivoted so that it can rotate about a vertical axis YY' which right angles to a uniform magnetic field of flux density B, (fig a).

Let the normal to the plane of the coil make an angle θ with the field, (fig B)

When the current I flows in the coil each side experiences a force(since all make some angle with B), acting perpendicularly to the plane containing the side and the direction of the field. The forces on the top and bottom (horizontal) sides are parallel to YY' and for the current direction shown, they lengthen the coil. the forces on the vertical sides, each of length L, are equal and opposite and have value F where:

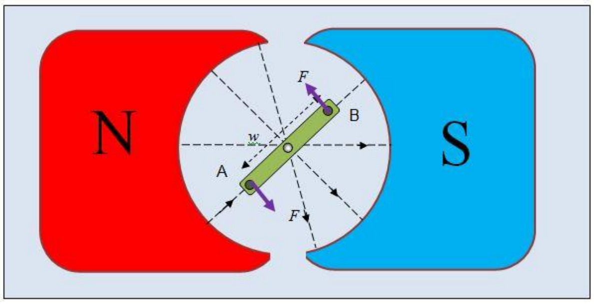
$$F = B \times I \times L \times N$$

Whatever the position of the coil, its vertical sides are right angles to B and so F remains constant. The forces continue a couple whose torque C is given by

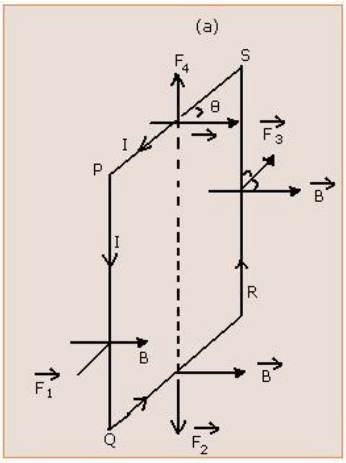
C=one force x perpendicular distance between lines of action of the forces
$=F \times PT = F \times b \sin PQT$ [b=width of coil]
$=Fb \sin \theta$
$=BIAN \sin \theta$

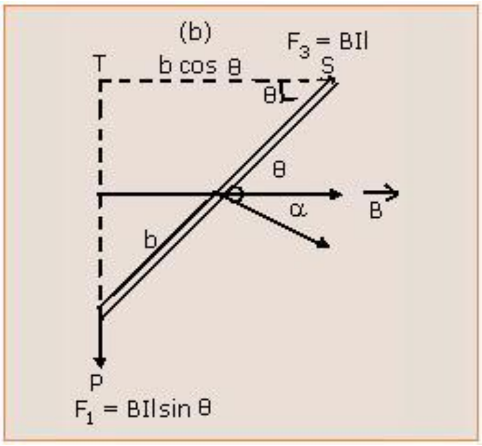
$$\boxed{C = BIAN \sin \theta}$$

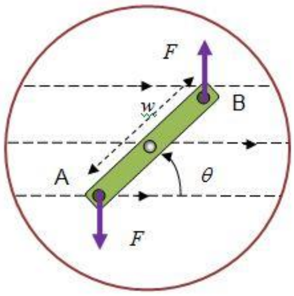
Where **A** area of face of coil=Lxb
The couple cause angular acceleration of the coil which rotates until its plane is perpendicular to the field (θ =0) and then C=0.



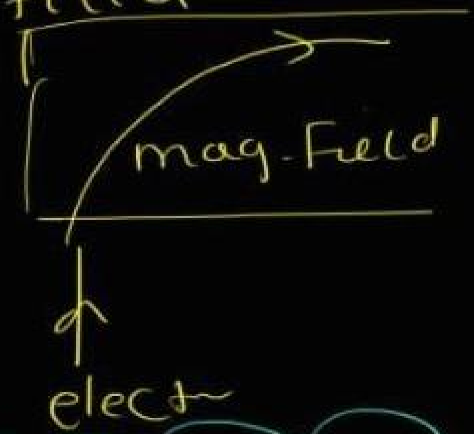
A circular coil carrying current I can be regarded as consisting of a large number of tiny rectangular coils, each with current I flowing in the same direction as in the circular coil.







* An electron in a magnetic field
(charge)



VVVVVI

The force acting on a charge in a mag. field = Centripetal Force

$$B\,Q\,\cancel{v} = \frac{m v^{\cancel{2}}}{r}$$

Care → $\frac{mv^2}{r} = m\omega^2 r$

$$\omega = 2\pi f = \frac{2\pi}{T}$$

* $r = \frac{d}{2}$ → diameter

* $BIL = BQv = mg = \frac{mv^2}{r} = m\omega^2 r$

Prove that Couple (C) / Torque (τ) $= BIAN \sin\theta$

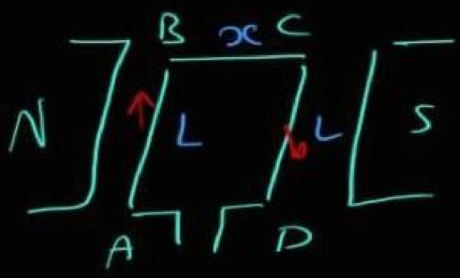
$\because F = BIL\sin\theta$

$\tau = F \times \perp d$

$= BI\boxed{L \times x}\sin\theta$

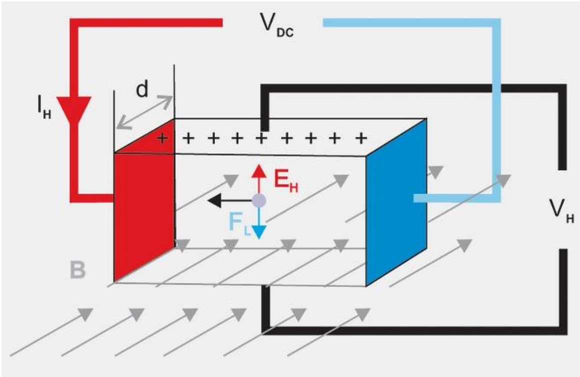
$= BI\boxed{A}\sin\theta$ For one turn

$= BIAN\sin\theta$ For (N) turn



Hall voltage:

the production of a potential difference across a conductor carrying an electric current when a magnetic field is applied in a direction perpendicular to that of the current flow



$F_B = F_E$

$Bqv = qE$

$Bv = \frac{V_H}{d}$

$V_H = dBv$

$v = \frac{I}{nAq}$

$V_H = d\frac{BI}{nAq}$

$t = \frac{A}{d}$

$$\boxed{V_H = \frac{BI}{ntq}}$$

Where:

F_B is the magnetic force acting on the material (N)

F_E is the electric force of the electrons passing through the conductor(N)

B is the magnetic flux density (T)

q is the charge of the current (C)

v is the velocity of the electrons passing through the conductor (ms^{-1})

d is the width of the conductor (m)

E is the electric field strength (NC^{-1})

T is thickness of the conductor (m)

n is the number of electrons in the conductor (m^{-3})

V_H is the hall voltage (V)

visualization of hall voltage:

N.B

If you encounter a question like this: [Nov 20 V1, Q8(a)]

A slice of a conducting material has its face QRLK normal to a uniform magnetic field of flux density B, as illustrated in Fig. 8.1.

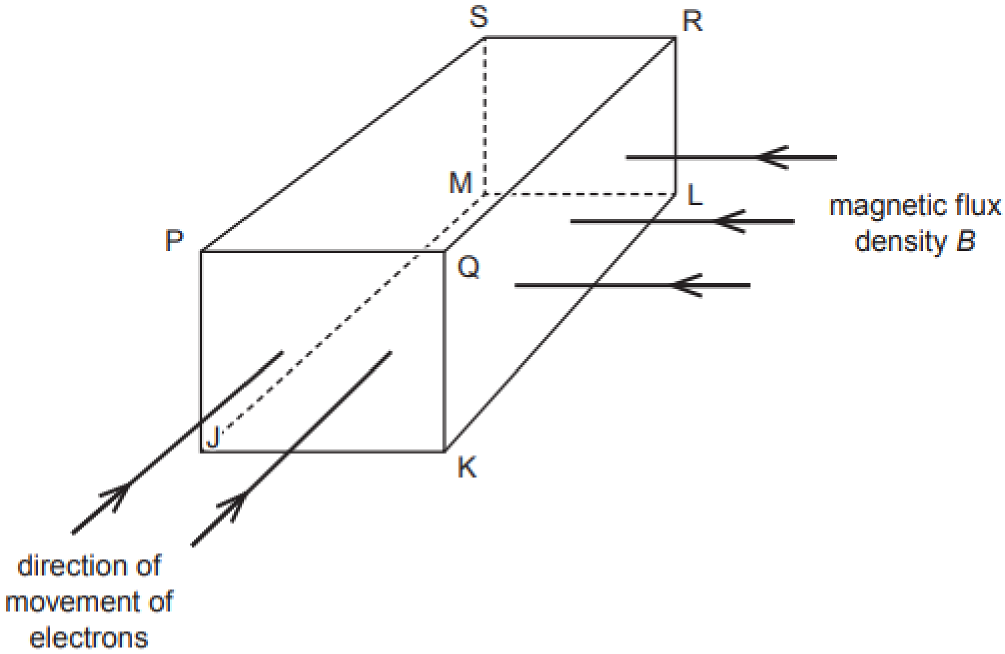


Fig. 8.1

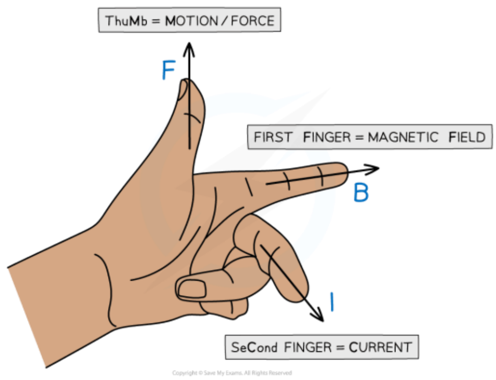
Electrons enter the slice travelling perpendicular to face PQKJ.

(a) For the free electrons moving in the slice:

(i) state the direction of the force on an electron due to movement of the electron in the magnetic field

(ii) identify the faces, using the letters on Fig. 8.1, between which a potential difference is developed.

You could answer with:



(i) Using Fleming's left hand rule we can deduce that the direction of the force is downwards

(ii) faces PQRS and JKLM

If you encounter a question like this: [Nov 20 V1, Q8(b)]

(b) Explain why the potential difference in (a)(ii) reaches a maximum value.

You could answer with:

-(as charge separates) an electric field is created (between opposite faces) **(1 mark)**
-(maximum value is reached when) electric force (on electron) is equal and opposite to magnetic force (on electron) **(1 mark)**

If you encounter a question like this: [Nov 20 V1, Q8(c)]

The number of free electrons per unit volume in the slice of material is $1.3 \times 10^{29} m^{-3}$. The thickness PQ of the slice is 0.10mm. The magnetic flux density B is 4.6×10^{-3}T.
Calculate the potential difference across the slice for a current of 6.3×10^{-4}A.

You could answer with:

$V_H = \dfrac{BI}{ntq}$

$= (4.6 \times 10^{-3} \times 6.3 \times 10^{-4}) / (1.3 \times 10^{29} \times 0.10 \times 10^{-3} \times 1.60 \times 10^{-19})$ **(1 mark)**

$= 1.4 \times 10^{-12}$ V **(1 mark)**

If you encounter a question like this: [Nov 20 V1, Q8(d)]

The slice in (c) is a metal.
By reference to your answer in (c), suggest why Hall probes are usually made using semiconductors rather than metals.

You could answer with:

-semiconductors have a (much) smaller value for n **(1 mark)**
- V_H for semiconductors is (much) larger so more easily measured **(1 mark)**

If you encounter a question like this: [June 19 V2, Q8(a)]

An electron is travelling in a vacuum at a speed of $3.4 \times 10^7 ms^{-1}$. The electron enters a region of uniform magnetic field of flux density 3.2mT, as illustrated in Fig. 8.1.

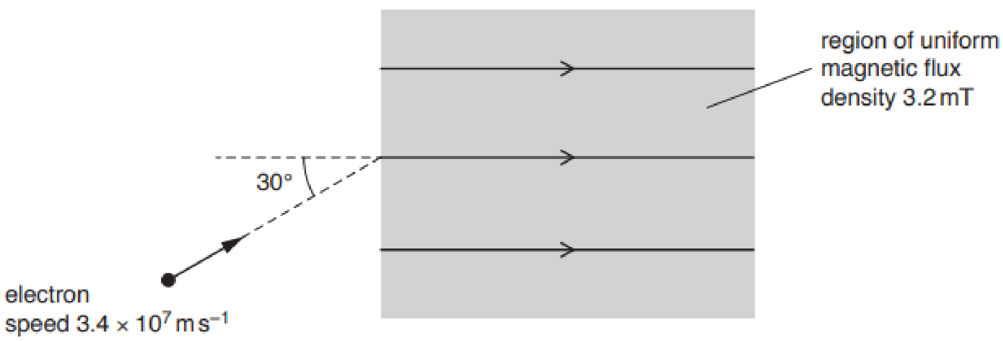


Fig. 8.1

The initial direction of the electron is at an angle of 30° to the direction of the magnetic field.
When the electron enters the magnetic field, the component of its velocity v_N normal to the direction of the magnetic field causes the electron to begin to follow a circular path.
Calculate:
(i) v_N

(ii) the radius of this circular path

You could answer with:

(i) $v_N = 3.4 \times 10^7 \times \sin 30° = 1.7 \times 10^7$ m s^{-1}

(ii) $\frac{mv^2}{r} = bqv \longrightarrow r = \frac{mv}{bq}$ **(1 mark)**
$r = (9.11 \times 10^{-31} \times 1.7 \times 10^7) / (3.2 \times 10^{-3} \times 1.60 \times 10^{-19})$ **(1 mark)**
= 0.030 m **(1 mark)**

If you encounter a question like this: [June 19 V2, Q8(b)]

State the magnitude of the force, if any, on the electron in the magnetic field due to the component of its velocity along the direction of the field.

You could answer with:

Zero (because velocity has no effect on the force

If you encounter a question like this: [June 19 V2, Q8(c)]

Use information from (a) and (b) to describe the resultant path of the electron in the magnetic field.

You could answer with:

helix/coil

If you encounter a question like this: [Nov 19 V1, Q8(a)]

A long straight vertical wire carries a current I. The wire passes through a horizontal card EFGH, as shown in Fig. 8.1 and Fig. 8.2.

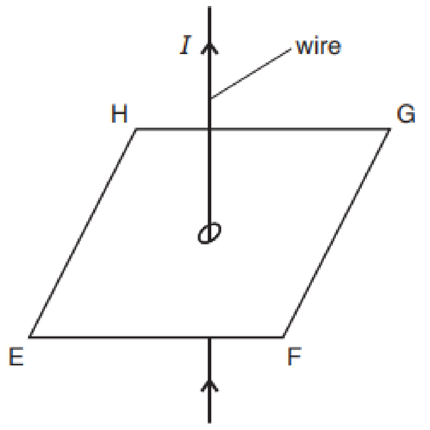


Fig. 8.1

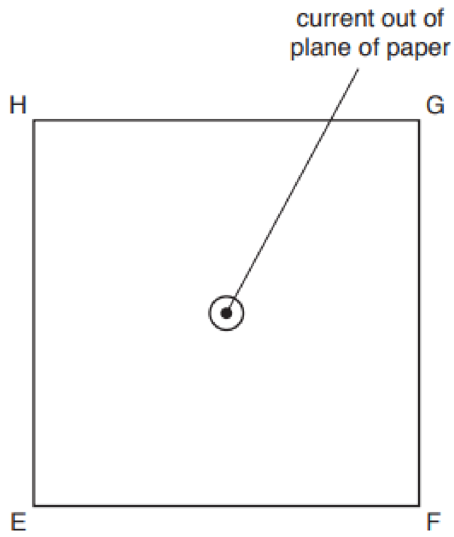


Fig. 8.2 (view from above)

On Fig. 8.2, draw the pattern of the magnetic field produced by the current-carrying wire on the plane EFGH

You could answer with:

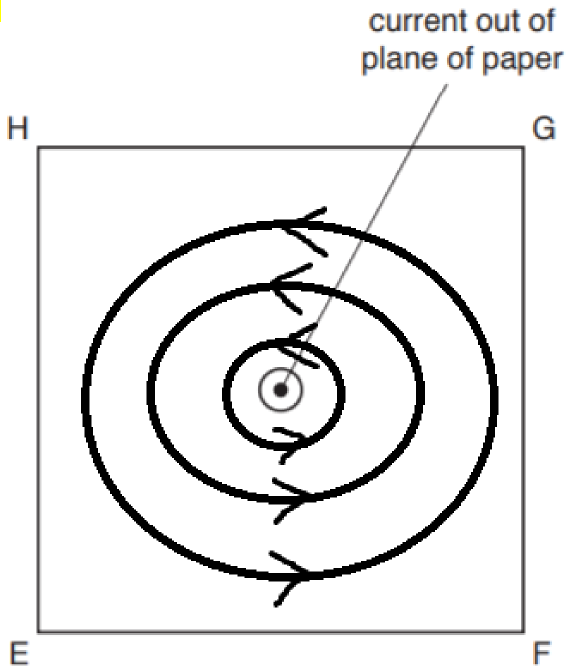


Fig. 8.2 (view from above)

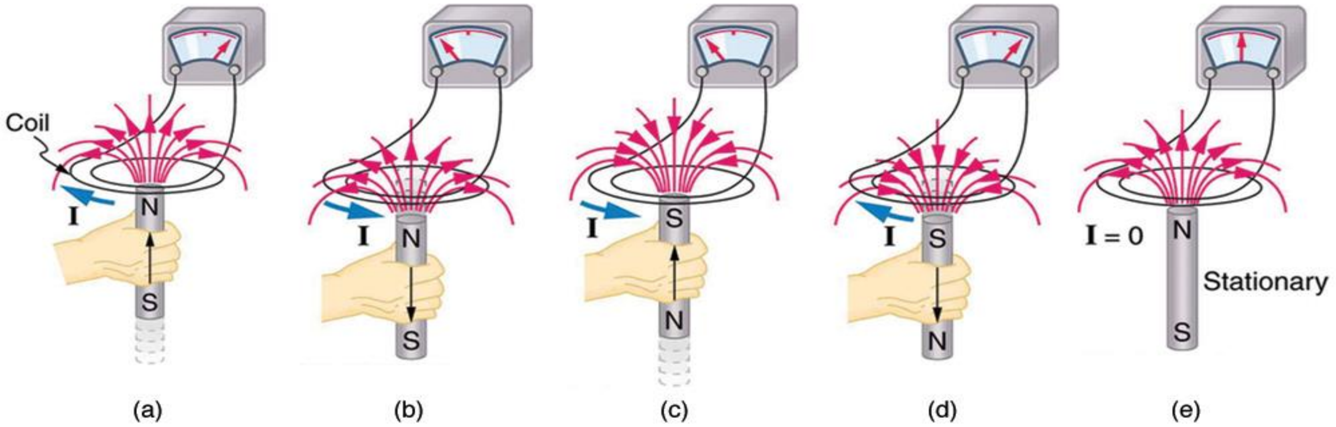
Chapter Ten

Electromagnetic Induction

Flux density and flux linkage

An EMF is induced in a coil in a magnetic field whenever the flux (ϕ) through the coil changes

Now you we will show you some ways of achieving this



The effect is called Electromagnetic induction and if the coil forms part of a closed circuit, the induced EMF causes a current to flow in the circuit.

Magnetic flux density B

Is the number of lines of magnetic force per unit area of an area at right angles to the lines.

Magnetic flux ϕ

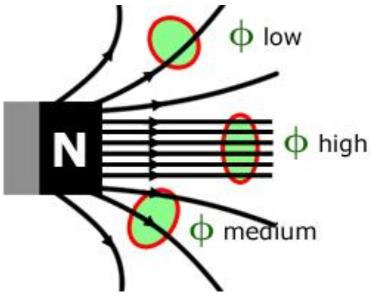
Is the total number of magnetic field lines.

It is the product of the magnetic flux density and the area normal to the lines of flux.

$$\text{Magneticflux}(\phi) = \text{Magneticflux}(\mathbf{B}) \times \text{Area}(\mathbf{A})$$

If the magnetic field of flux density **B** makes an angle **θ** with an area **A**, the magnetic flux **ϕ** given by the expression

$$\phi = B \times A \sin\theta$$



- The unit of magnetic flux is Weber (Wb).

One Weber is equal to one Tesla metre-squared (Tm^2)

The Weber

Is the magnetic flux if a flux density of one Tesla passes perpendicularly through an area of one square metre.

For a coil with cross sectional area A and N turns, each turn of the coil has a flux ϕ linking it, where $\phi = BA$

the total flux linkage of the coil=Nϕ=BAN

A change in flux linkage is given the symbol $\Delta\phi$. The flux linking a coil may change for two reasons:

1. The flux density **B** changes
2. The area A linked by the flux changes.

Example (1)

How much flux is linking a 200-turn coil of area 0.1 m^2, when it placed at 90^0 to a magnetic field of flux density 2.5×10^{-3}T?

Answer

total flux linkage=Nϕ=NBA

$= 200\times2.5\times10^{-3}\times0.1=0.05\text{Wb}$

Example (2)

A coil of 100 turns and area 0.2 m^2 is at 90^0 to flux density, which decreases from 0.5T to 0.2 T what is the change in flux linkage of the coil?

Answer

change in flux linkage=N$\Delta\phi$=NΔBA

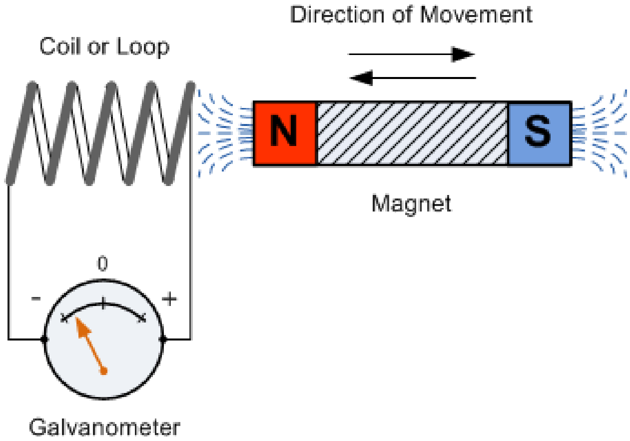
$= 100\times(0.5-0.2)\times0.2=6.0\text{Wb}$

If he said, I is reversed

$$emf_{ind} = -N\frac{\Delta BA \times 2}{\Delta t}$$

Electromagnetic induction

It is the phenomenon where EMF is induced when the magnetic flux linkage of a conductor changes



Experiments shows that the magnitude of the EMF depends on

1. **The rate at which the flux through the coil changes**
2. **The number of turns of the coil N**

An induced current will produce wherever the direction of cutting across the magnetic field lines

A detailed investigation of electromagnetic induction leads to two laws:-

1- [Faraday's law]

The magnitude of EMF in a circuit is directly proportional to the rate of change flux linkage or the rate of cutting of magnetic flux

2- [Lenz 's law]

The direction of the induced EMF is such that the current, which causes to flow (or would flow in a closed circuit) opposes the change, which is producing it.

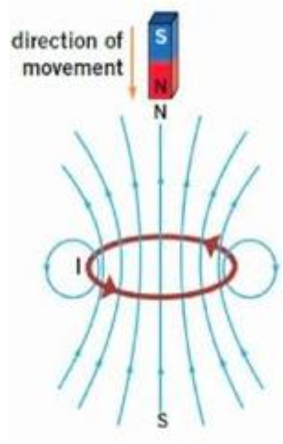
The two laws can be expressed by the equation:

$$E=-\frac{d}{dt}(N\phi)$$

Where:

E= the induced EMF in volts

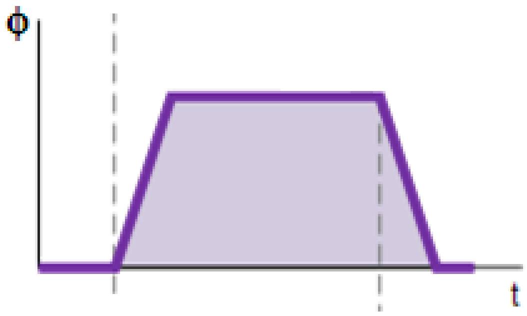
$\frac{d}{dt}(N\phi)$ = the rate of change of flux-linkage in Webers per second.



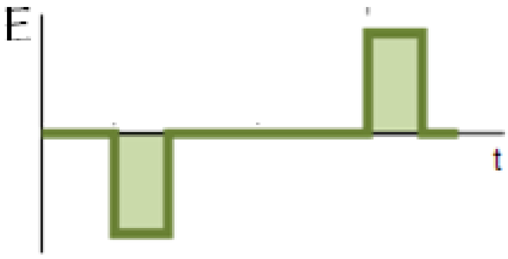
- The minus sign in the equation takes account of Lenz's law, the induced current flows in such a sense as to create a flux in the opposite direction to that in which, the external flux has increased.

To illustrate the use of the equation $E = -\frac{d}{dt}(N\phi)$

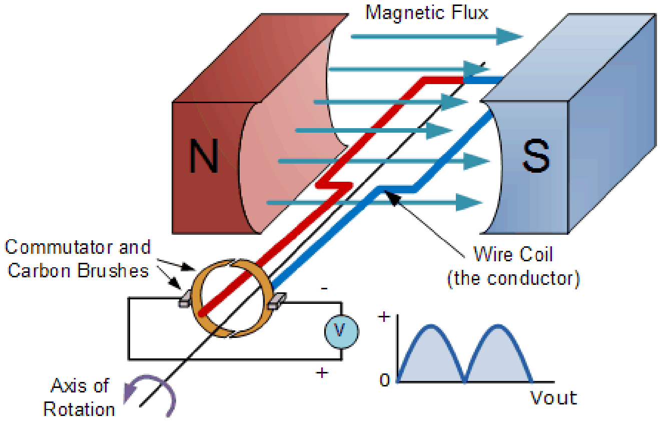
Consider the case where ϕ changes as shown:



The induced E will have the following variation



The A.C Generator



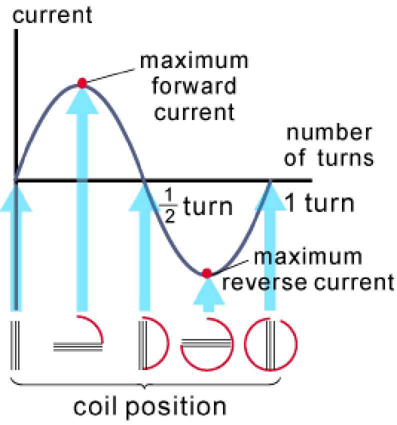
A generator converts kinetic energy into electrical energy; it consists of a rectangular coil that rotates in a magnetic field.

Each end of the coil is connected to a slip-ring, the slip-rings rotate with the coil and press against stationary carbon brushes Each side of the coil always makes contact with the same brush.

What happens if you rotate the coil at a steady rate?

As it turns, the flux linkage of the coil constantly changes, and the pointer on the ammeter swings from side to side showing that an alternating current (A.C) is induced.

The graph shows how the size of the induced e.m.f changes as the coil rotates once in the magnetic field, where one complete revolution of the coil gives one cycle of A.C.



How can we increase the size of the peak in e.m.f?

We must increase the rate of change of flux linkage of the coil as it spins in the magnetic field by;-

1. Increasing the number of turns **N**.
2. Increasing the cross-sectional area ***A***.
3. Increasing the strength of the magnetic field **B**
4. Increasing the frequency of rotation **F**

Then the value of the peak e.m.f can be calculated by:

$$E_0=2\pi FBAN$$

A.C generator animation showcasing how it works in a circuit:

N.B

If you encounter a question like this: [March 21 V2, Q9(a)]

(a) Define magnetic flux linkage.

You could answer with:

-flux density × number of turns × area perpendicular to a magnetic field **(2 marks)**

If you encounter a question like this: [March 21 V2, Q9(b)]

(b) A solenoid of diameter **6.0 cm** and **540** turns is placed in a uniform magnetic field as shown in **Fig. 9.1** .

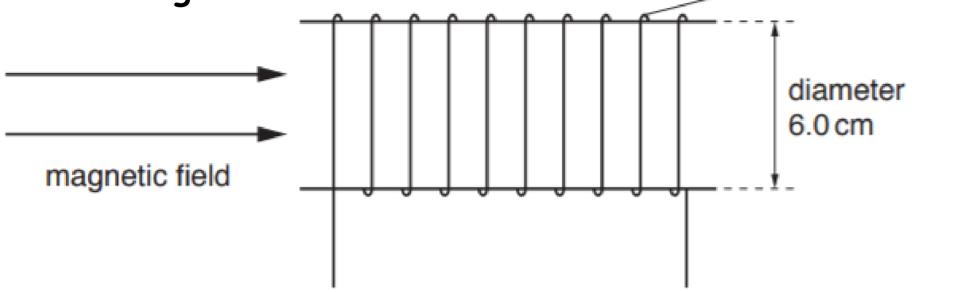


Fig. 9.1

The variation with time **t** of the magnetic flux density is shown in **Fig. 9.2** .

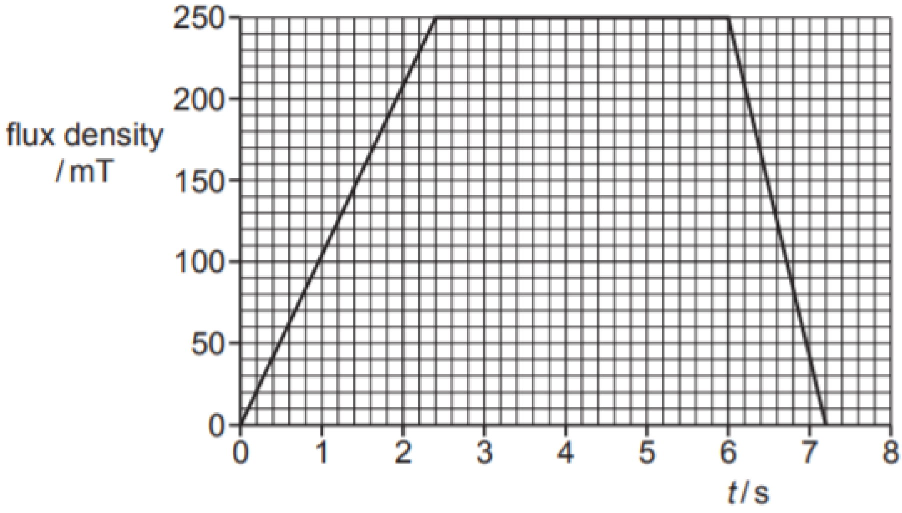


Fig. 9.2

Calculate the maximum magnitude of the induced electromotive force (e.m.f.) in the solenoid.

You could answer with:

t = **1.2 s** (from the graph) **(1 mark)**

$$\text{e.m.f} = (-)\,\frac{\Delta\Phi}{\Delta t} = (-)\,\frac{\Delta BAN}{\Delta t} = \frac{0.250\times\pi\times0.03^{2}\times540}{1.2} = 0.32\text{V}$$ **(2 marks)**

If you encounter a question like this: [March 21 V2, Q9(c)]

(c) A thin copper sheet **X** is supported on a rigid rod so that it hangs between the poles of a magnet as shown in **Fig. 9.3** .

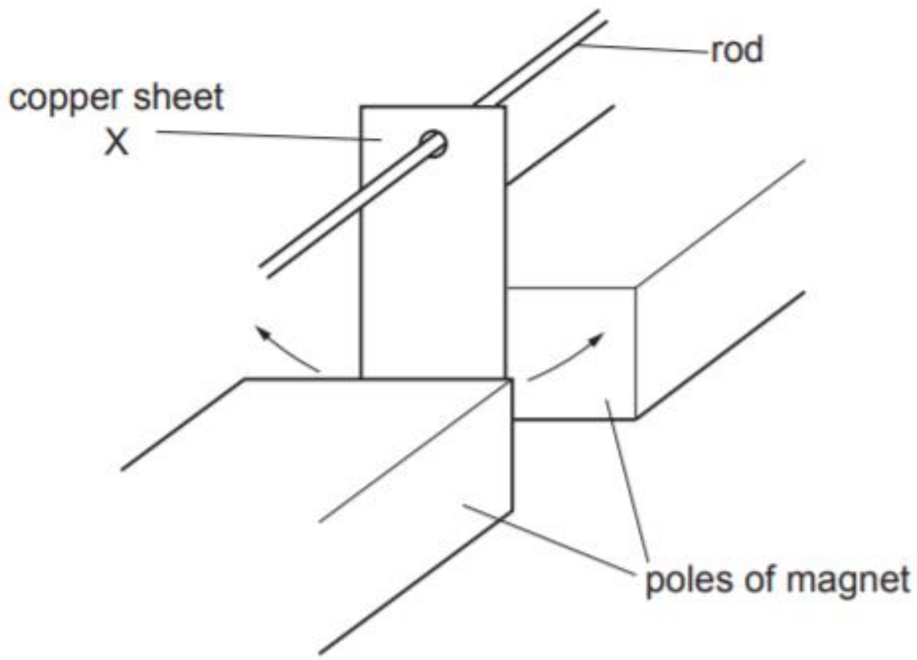


Fig. 9.3

Sheet **X** is displaced to one side and then released so that it oscillates. A motion sensor is used to record the displacement of **X**.

A second thin copper sheet **Y** replaces sheet **X**. Sheet **Y** has the same overall dimensions as **X** but is cut into the shape shown in **Fig. 9.4** .

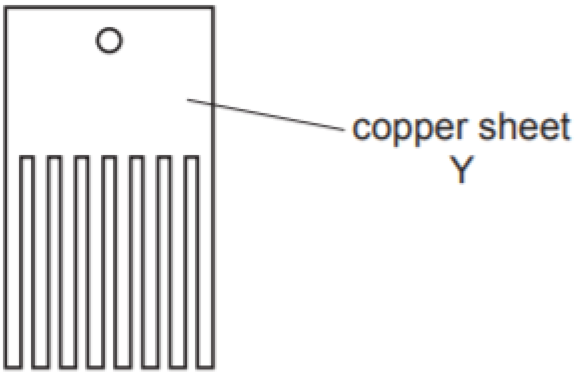


Fig. 9.4

The motion sensor is again used to record the displacement.

The graph in **Fig. 9.5** shows the variation with time **t** of the displacement ***s*** of each copper sheet.

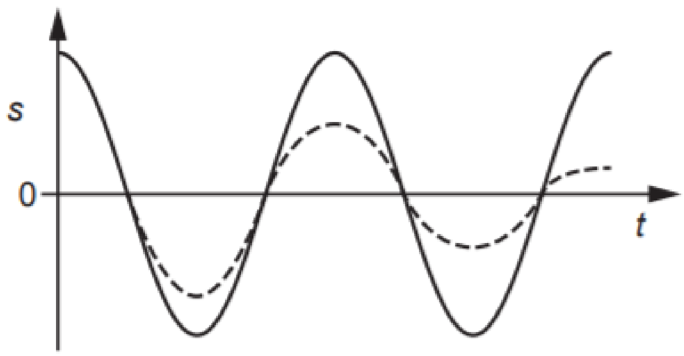


Fig. 9.5

(i) State the name of the phenomenon illustrated by the gradual reduction in the amplitude of the dashed line.

(ii) Deduce which copper sheet is represented by the dashed line. Explain your answer using the principles of electromagnetic induction.

You could answer with:

(i) light damping **(1 mark)**

(i)

- sheet cuts magnetic flux and causes an induced e.m.f **(1 mark)**

- induced e.m.f produces eddy currents in sheet **(1 mark)**

- currents in sheet cause resistive force /currents in sheet dissipate energy **(1 mark)**

- smaller currents in **Y** /larger currents in **X**, so dashed line is **X** **(1 mark)**

If you encounter a question like this: [June 20 V1, Q9(a)(i)]

(a) A coil of wire is situated in a uniform magnetic field of flux density **B**. The coil has diameter **3.6 cm** and consists of **350** turns of wire, as illustrated in **Fig. 9.1** .

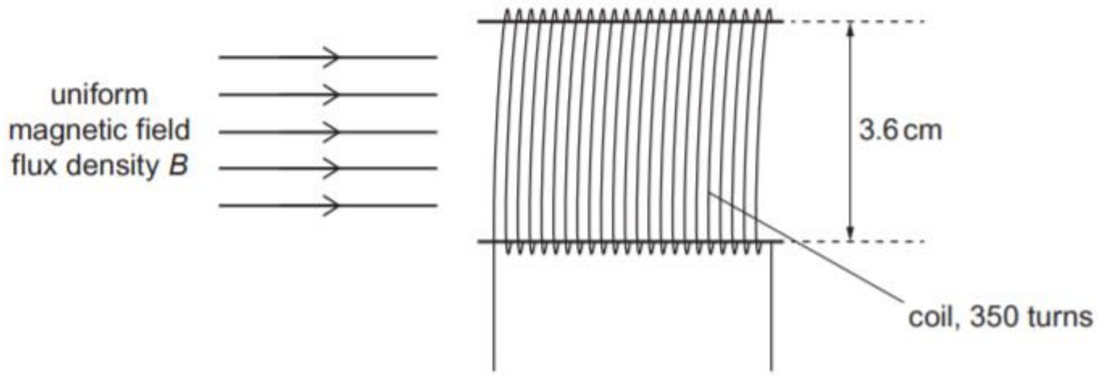


Fig. 9.1

The variation with time **t** of **B** is shown in **Fig. 9.2** .

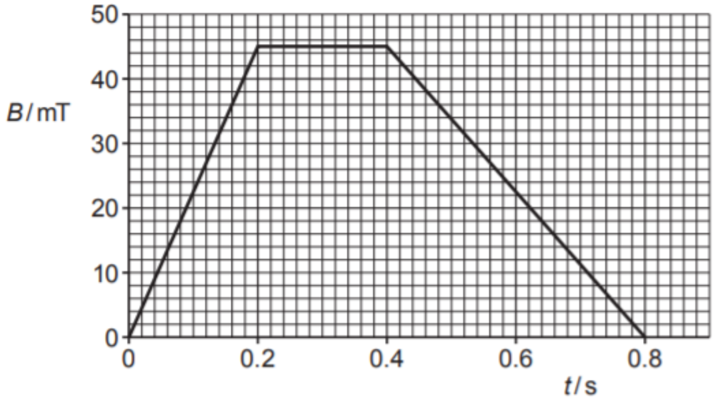


Fig. 9.2

(i) Show that, for the time **t = 0** to time **t = 0.20s**, the electromotive force (e.m.f.) induced in the coil is **0.080V**.

You could answer with:

e.m.f. = $(-)\dfrac{\Delta\Phi}{\Delta t} = (-)\dfrac{\Delta BAN}{\Delta t}$ **(1 mark)**

$= 45 \times 10^{-3} \times \pi \times (1.8 \times 10^{-2})^2 \times 350 / 0.20 =$ **0.080 V (1 mark)**

If you encounter a question like this: [June 20 V1, Q9(a)(ii)]

(ii) On the axes of **Fig. 9.3**, show the variation with time **t** of the induced e.m.f. **E** for time **t = 0** to time **t = 0.80s**.

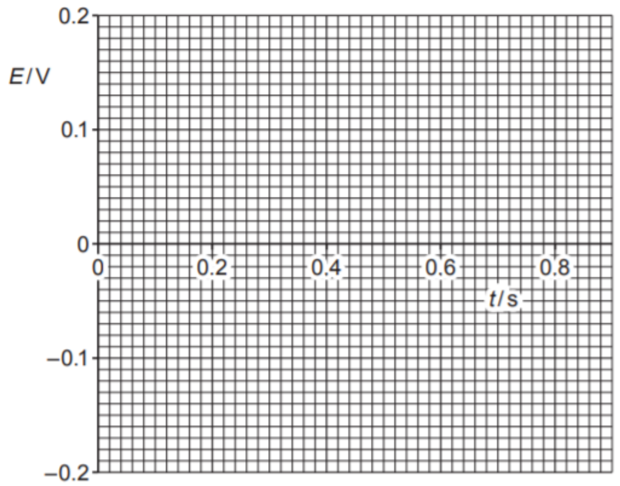


Fig. 9.3

You could answer with:

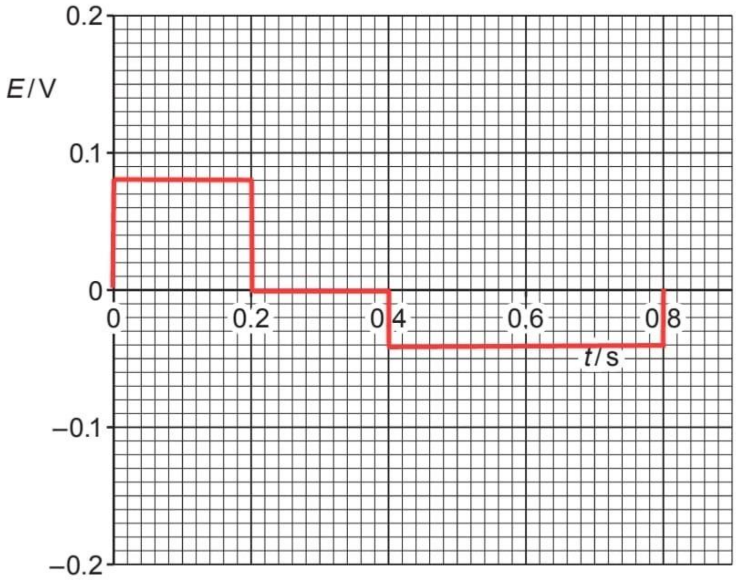


Fig. 9.3

Chapter Eleven

Alternating currents

If the polarity of an E.M.F changes with time, it is known as an alternating E.M.F. The current that such an E.M.F causes to flow repeatedly changes its direction and known as alternating current **(A.C)**

Some important definitions

Frequency (**f**): the number of cycles per second.
The unit of frequency is Hertz (**Hz**).
Angular frequency (ω): this is equivalent to angular velocity in circular motion.
It is measured in rad s^{-1}
In simple harmonic oscillation it has no direct physical meaning and it is defined by:

$$\boxed{\omega = 2\pi f}$$

The most commonly encountered type of alternating E.M.F varies Sinusoidally with time like that generated by the mains (D.C) and represented by

$$\boxed{E = E_o \sin(\omega t)}$$

$$\boxed{E = E_o \sin(2\pi f t)}$$

Where:
E= the value of the EMF at time t (**V**)
E₀= the peak value (the maximum value of E)
The corresponding alternating current I is given in terms of its maximum value $\mathbf{I_o}$

$$\boxed{I = I_O \sin(\omega t)}$$

$$\boxed{I = I_O \sin(2\pi f t)}$$

NOTE THAT:
The current has the same frequency as the E.M.F that produces it.

Root mean square values

It is the alternating current flowing in a circuit that produces the same heating effect as a direct current flowing in the same circuit.

In general, the effective value of an alternating current is equal to that of the direct current, which results in the same expenditure of energy under the same conditions.

The energy, **W**, supplied in time **t**, by an alternating current **I** flowing through a resistance **R** is equal to the product of **t** and the average value of I^2R.

$$W=(I^2R)_{average}\,t$$

Since **R** is constant

$$W=(I^2)_{average}\,Rt$$

If the effective value of **I** is denoted by I_{RMS}, then **W** is equal to the energy supplied by a steady current of magnitude I_{RMS} flowing through a resistance for the time **t**.

$$W=(I^2)_{RMS}\,Rt$$

$$\text{Power} = \frac{E}{t} = VI = I^2R = \frac{V^2}{R}$$

$$P_{loss} = \frac{{V_{rms}}^2}{R} = \frac{{V_o}^2}{2R}$$

***Effective value of AC:* the value of the dc which produces the same amount of heat E.**

Thus, the effective value (Denoted by I_{RMS} is the root mean square value)

* DC ⇒ Current (same direction)
measure it using the ammeter

but AC ⇒ current change direct ~~Ammeter~~

effective value of AC ⇒ value of dc which produces same amount of heat E

* $I_{RMS} = I_{eff} = \frac{I_{max}}{\sqrt{2}} = I_{max} \times 0.707$

$I_{max} = I_0$

ind emf

- AV. → $= -N\frac{\Delta \overset{BA}{\cancel{\phi}}}{\Delta t}$
- inst. → $NBA\omega \sin\underset{\underset{2\pi f t}{\omega t}}{\theta}$
- max → $NBA\omega = E_0$

$E_{inst.} = NBA\omega \sin\theta$

$E_{inst.} = E_{max} \sin\theta$

- effective (RMS) → $0.707\, E_0$ OR $\frac{E_0}{\sqrt{2}}$

$\boxed{E_{max} = E_0}$

ind emf E

max: $E_0 = NBA\cancel{\omega}$, $\omega \to 2\pi f$, $\pi \to 22/7$

AV: $-N\dfrac{\Delta \cancel{\phi} BA}{\Delta t}$

eff. / RMS: $\dfrac{E_{max}}{\sqrt{2}} = 0.707\,E_0$

$$E_{max} = E_0$$

$$\div R \Longrightarrow I$$

$$I_{inst} = I_0 \sin \cancel{\theta}\ \omega t$$

$\omega \to 2\pi f$, $\pi \to 180^\circ$

$$E_{inst} = E_0 \sin \cancel{\theta}\ \omega t$$

$\omega \to 2\pi f$, $\pi \to 180^\circ$

$$P_{loss} = V_{eff} I_{eff} = I_{eff}^2 R = \frac{V_{eff}^2}{R}$$

* $V_{eff} = V_{RMS} = \frac{V_0}{\sqrt{2}} = V_0 \times 0.707$
* $I_{eff} = I_{RMS} = \frac{I_0}{\sqrt{2}} = I_0 \times 0.707$
* $\cancel{R_{eff}}$

$$\boxed{\text{Heat } E_{loss} = V_{eff} I_{eff} t = I_{eff}^2 R t = \frac{V_{eff}^2 t}{R}}$$

$\boxed{V \ \& \ I}$

Directly — inv.

$V = I R$, $R \rightarrow$ cons.

(General)

$V = \frac{P}{I}$, $P \rightarrow$ cons.

(Transform)

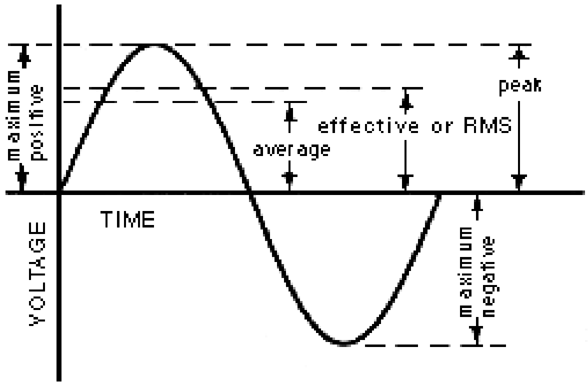
Sinusoidal A.C

It can be shown that:

$$I_{RMS}=\frac{I_o}{\sqrt{2}}=0.707I_o$$

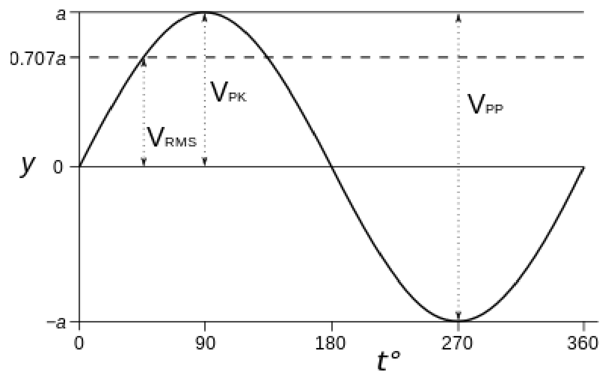
And

$$E_{RMS}=\frac{E_o}{\sqrt{2}}=0.707E_o$$



To be more accurate:

$$R.m.s.Value=\frac{Peakvalue}{\sqrt{2}}$$



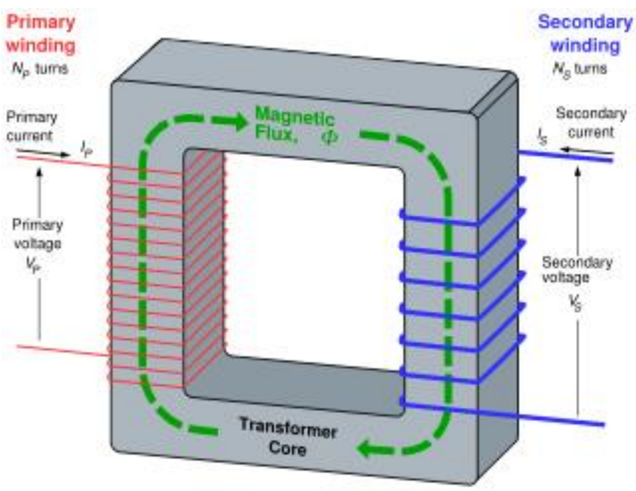
The Transformer

A transformer changes the value of an alternating voltage. For an ideal transformer, no energy is lost and so we can write

$$\frac{\text{Secondary voltage}}{\text{Primary voltage}}=\frac{\text{Number of turns on the secondary coil}}{\text{Number of turns on the primary coil}}$$

OR

$$\frac{V_S}{V_P}=\frac{N_s}{N_p}$$



In an ideal transformer, no energy is lost and so we can write:

$$Power_{primary}=Power_{secondary}$$

$$V_pI_p=V_sI_s$$

Now we can combine the two equations to give

$$\frac{V_S}{V_P}=\frac{I_p}{I_s}=\frac{N_s}{N_p} \longrightarrow \frac{V_1}{V_2}=\frac{I_2}{I_1}=\frac{N_1}{N_2}$$

Types of transformers

(1) Step up transformer:

Transformer increases the A.C voltage, because the secondary coil has more turns than the primary coil

(2) Step down transformer:

Transformer decreases the A.C voltage, because the secondary coil has fewer turns than the primary coil

---

Example (1)

A step-up transformer has a primary coil with 100 turns. It transforms the mains voltage of 230V A.C to 11500 V A.C

a. How many turns must be there on the secondary coil?

b. When the current in the secondary coil is 0.10A, what is the current in the primary coil?

Solution:

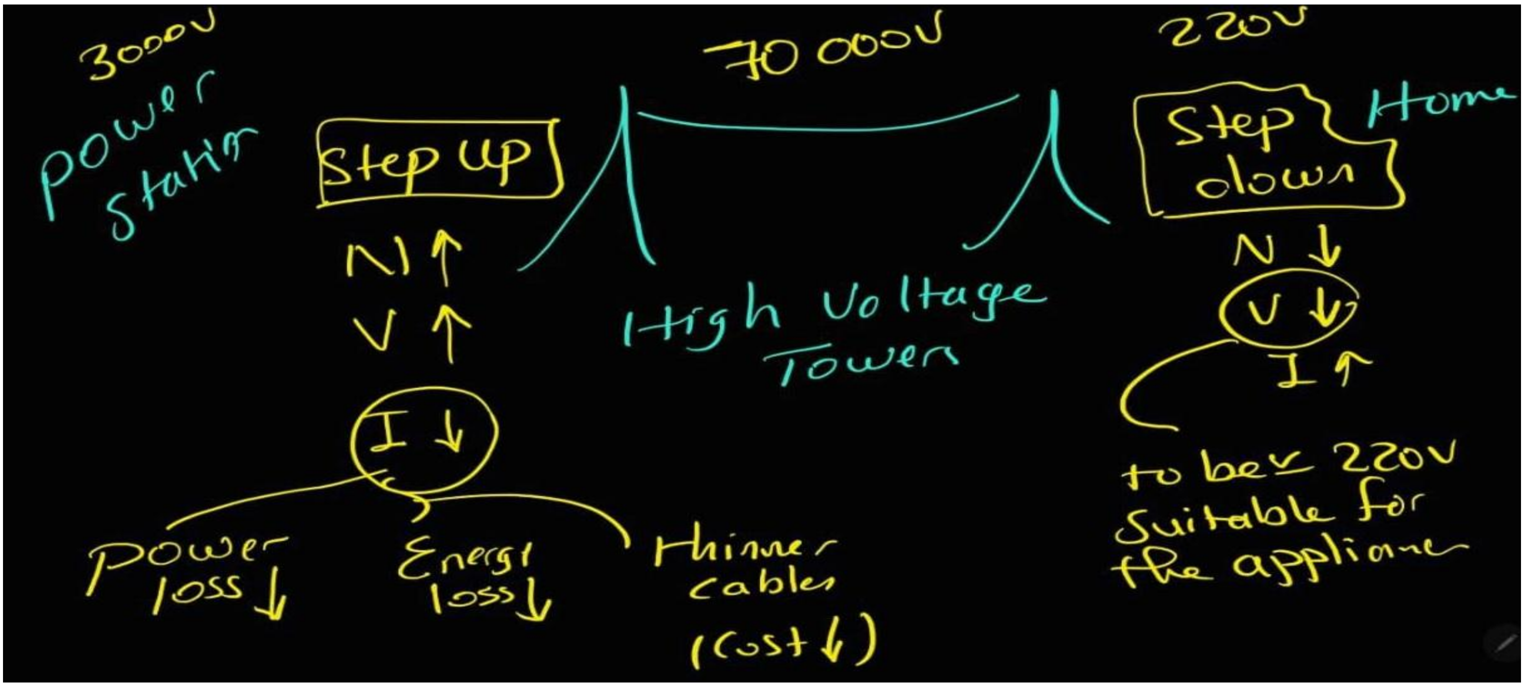
(a) $\frac{V_S}{V_p} = \frac{N_S}{N_p}$

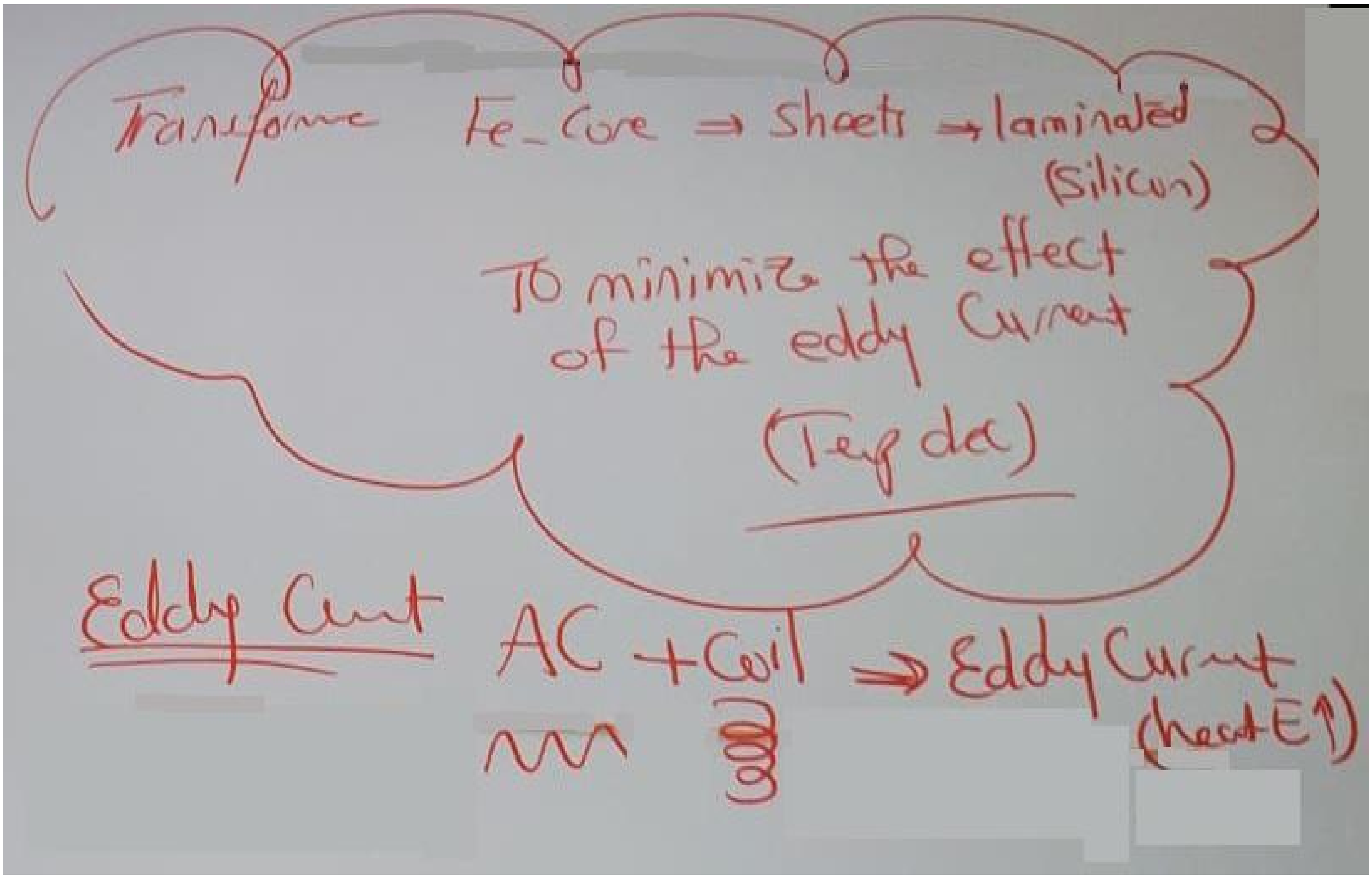
$\frac{11500}{230} = \frac{N_S}{100}$

$N_s = \frac{11500 \times 100}{230} = 5000 \text{turns}$

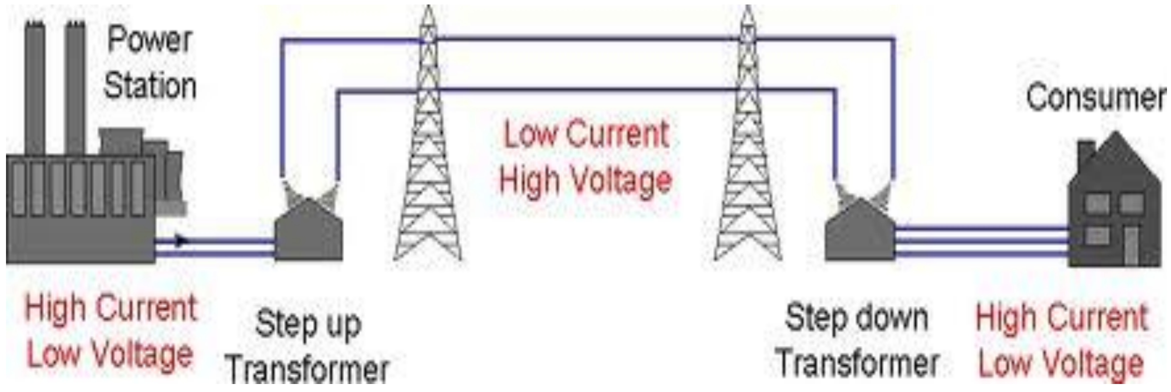
(b) the secondary voltage is 50 times the primary voltage and so the secondary current must be $\frac{1}{50}^{th}$ of the primary current.

$\therefore I_p = 50 \times 0.1 = 5A$




Transforme Fe_Core ⇒ Sheets → laminated (Silicon)
To minimize the effect of the eddy Current (Temp dec)
Eddy Current
AC + Coil ⇒ Eddy Current (heat E↑)

Transmission of electrical current



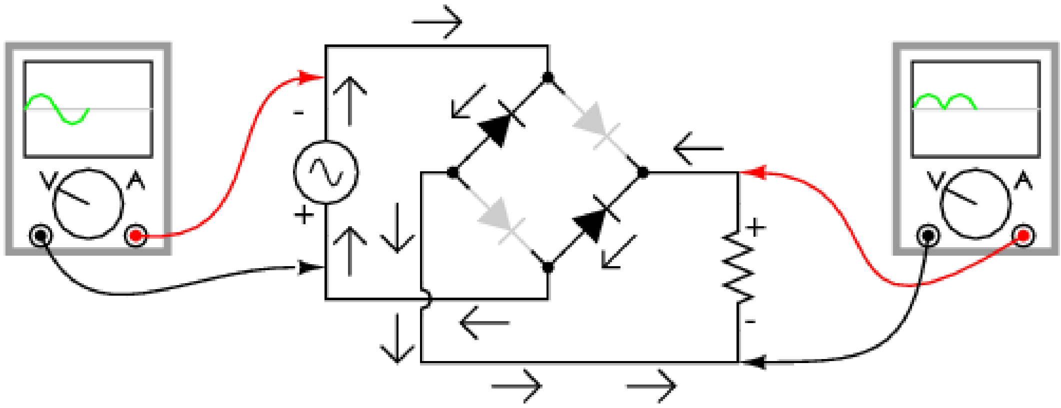
The national grid transmits large amounts of electrical energy each second, from the power stations to the consumers, often over large distances.

Since Power = Current x Voltage, we could use high voltage and low current, because the low current minimizes the power loss. Transformers at each end of the system step the voltage up (leaving the power station) and then down (when it reaches the consumer).

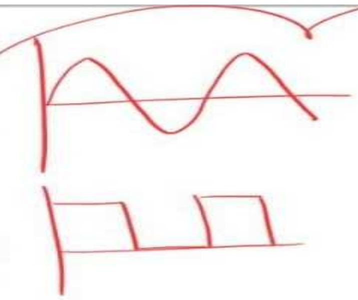
Rectification

Alternating currents can be converted to direct currents (rectified), by making use of devices which conduct appreciable amounts of current in one direction only. Such devices are called **Rectifiers and include**

1. **Thermo-ionic diodes**
2. **Semiconductor diodes**



A rectifier is said to be forward-biased when it is connected to a supply in such a way that it connects 2 positive or negative junctions. If connected with one positive and one negative junction, the rectifier is reversed-biased.



$V_{rms} = \frac{V_0}{\sqrt{2}}$

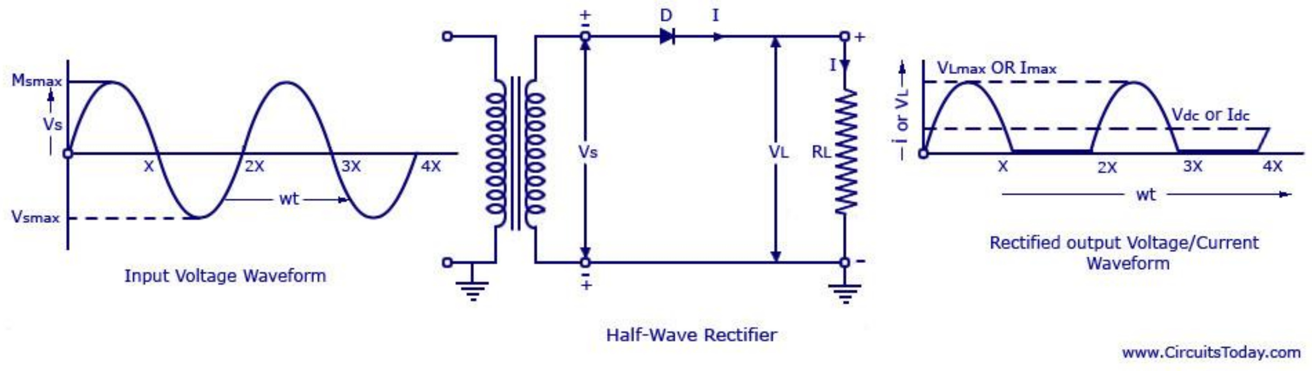
$V_{rms} = V_0$

J21/42/ No 10

Half wave rectification

The rectifier conducts only during one half of the cycle, which makes **X** positive. Although the output is pulsating, it is unidirectional (direct current)

Note: the output is exactly sinusoidal, only if the positive section of the **I-V** curve of the rectifier is linear.

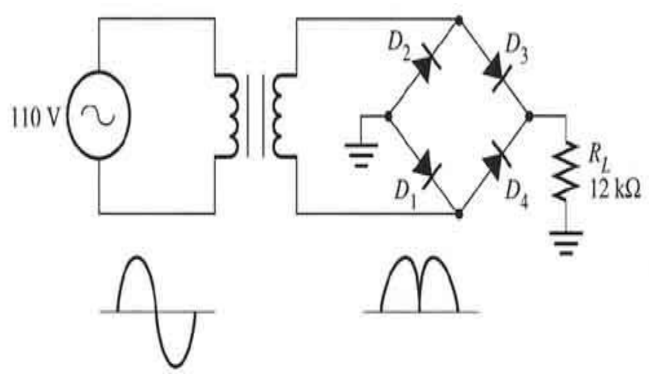


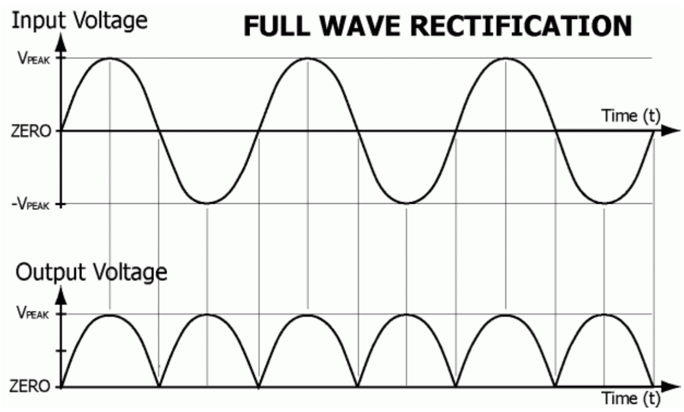
Full wave rectification

This can be achieved by using an arrangement of four rectifiers known as a bridge rectifier.

The output for the bridge circuit will have its positive D.C voltage terminal at the point where the cathode of diode 3 and diode 4 are connected, and the negative point of the circuit will be where the anode of diode 1 and diode 2 are connected. This point is also grounded.

When A.C voltage is applied to the four-diode full-wave bridge rectifier, the positive half of the sine wave will be rectified by diodes 1 and 3. The negative half of the sine wave is rectified by diodes 2 and 4.

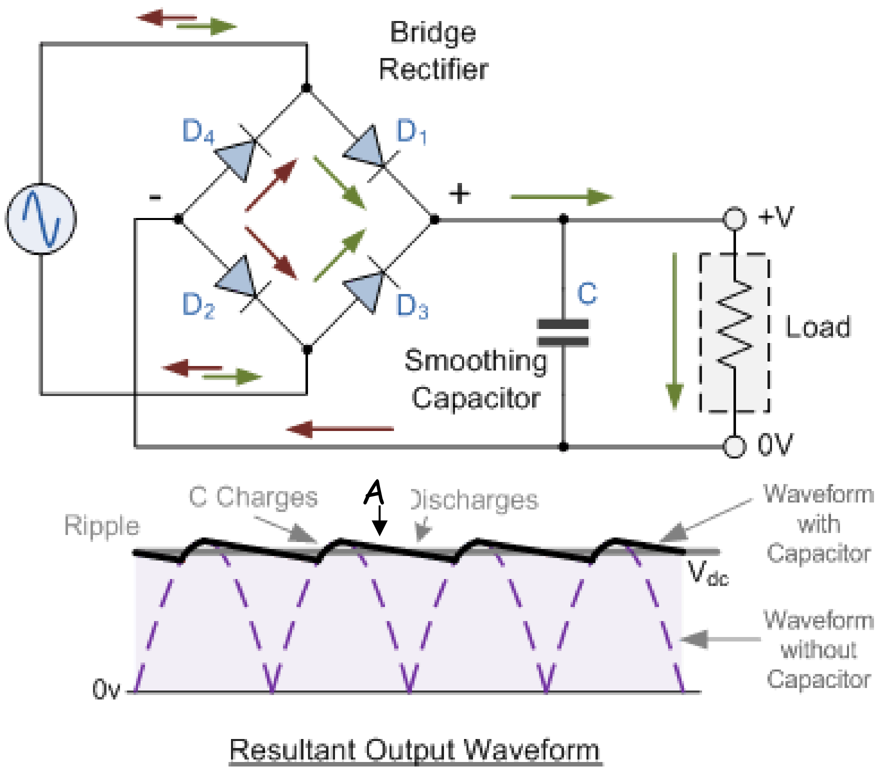




Smoothing

The pulsating output produced by both half wave and full wave rectifiers can be made steadier (smoothed) by putting a suitable capacitor in parallel with the load.

The smoothed outputs for the full-wave situations are shown



At a point such as A, the P.D across the load has just reached its maximum value. If the capacitor were not present, the PD would start to fall to zero along the broken curve.

However, as soon as the P.D would start to fall it becomes less than that across the capacitor and the capacitor starts to discharge through the load.

Since the charging process causing plate X to be positive, the discharge drives current through the load in the same direction as it flowed during charging.

If the time constant of the capacitor-load combination is suitably large, the P.D across the load falls by only a small amount before it starts to rise again

N.B

If you encounter a question like this: [June 16, Q11]

The variation with time **t** of the sinusoidal current **I** in a resistor of resistance **450 Ω** is shown in **Fig. 11.1** .

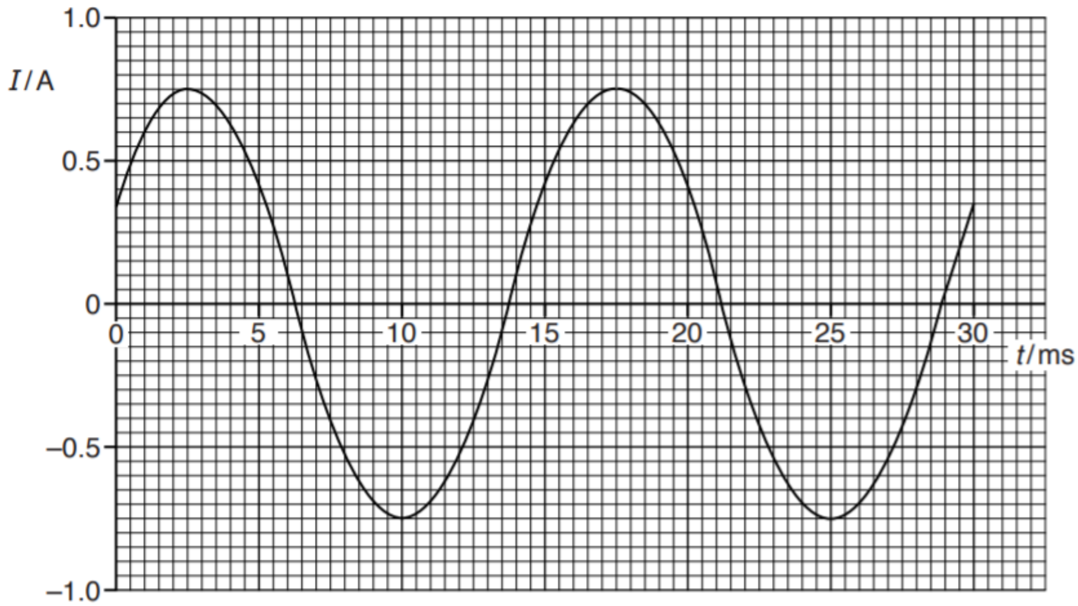


Fig. 11.1

Use data from **Fig. 11.1** to determine, for the time **t = 0** to **t = 30 ms**,

(a) the frequency of the current,

(b) the mean current,

(c) the root-mean-square (r.m.s.) current,

(d) the energy dissipated by the resistor.

You could answer with:

(a) T = (from graph) **(1 mark)**

frequency (= 1 / T) = **67 Hz (1 mark)**

(b) zero/0 **(1 mark)**

(c) $I_{r.m.s.} = I_0 / \sqrt{2}$ **(1 mark)**

$= 0.53$ A **(1 mark)**

(d) energy = $I_{r.m.s.}^{\,2} \times R \times t$ / $\frac{1}{2} I_0^{\,2} \times R \times t$

OR **(1 mark)**

power = $I_{r.m.s.}^{\,2} \times R$ and energy = power × t

energy = $0.53^2 \times 450 \times 30 \times 10^{-3}$ **(1 mark)**

= **3.8 J (1 mark)**

If you encounter a question like this: [June 17 V1, Q9(a)]

A simple transformer is illustrated in **Fig. 9.1**.

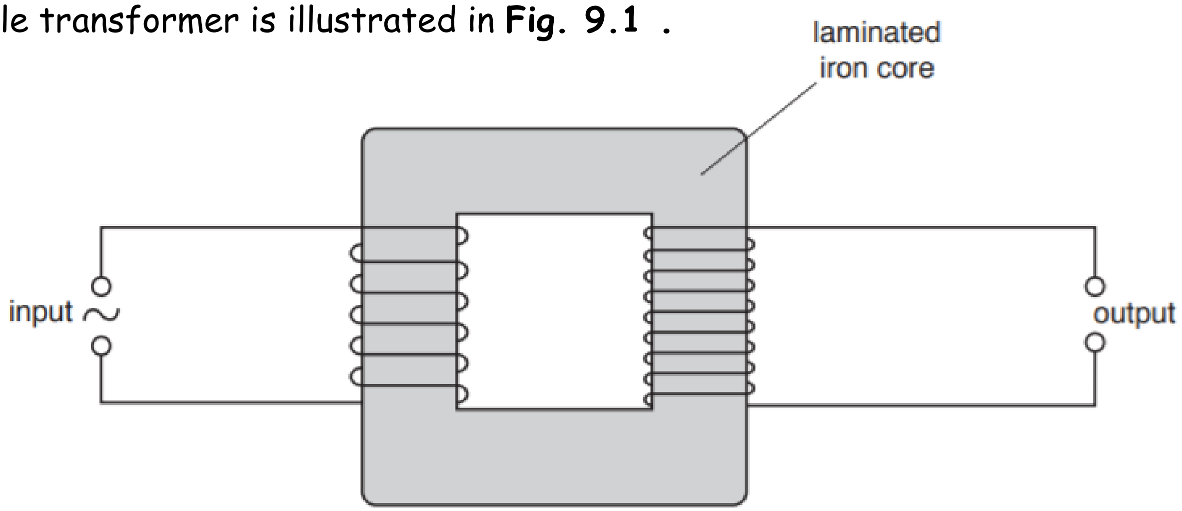


Fig. 9.1

(a) (i) State why the transformer has an iron core, rather than having no core.

(ii) Explain why the core is laminated

You could answer with:

(i) core reduces loss of (magnetic) flux linkage/improves flux linkage **(1 mark)**

(ii)
-reduces size of eddy currents in core **(1 mark)**
- so that heating of core is reduced **(1 mark)**

If you encounter a question like this: [June 17 V1, Q9(b)]

By reference to the action of a transformer, explain why the input to the transformer is an alternating voltage, rather than a constant voltage.

You could answer with:

-alternating voltage gives rise to changing magnetic flux in core **(1 mark)**
-changing flux links the secondary coil **(1 mark)**
-induced e.m.f. in secondary coil only when flux is changing/cut **(1 mark)**

Chapter Twelve

Part (1)

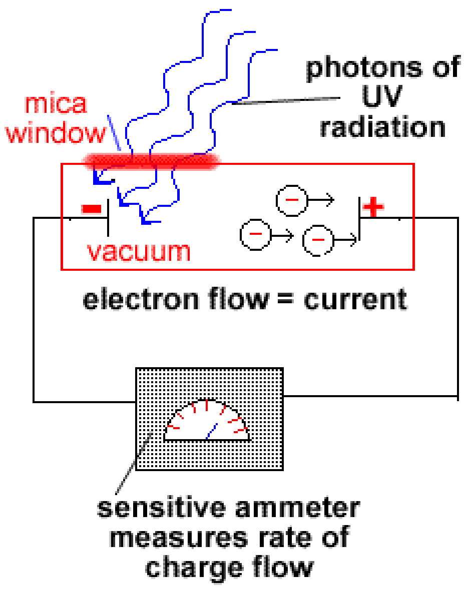
Quantum Mechanics

Quantum physics is the study of quanta. A quantum is, to quote, "The smallest possible, and therefore indivisible, unit of a given quantity or quantifiable phenomenon". The quantum of light is the photon. We are not describing it as a particle or a wave, as such, but as a lump of energy which behaves like a particle and a wave in some cases.
We are saying that the photon is the smallest part of light which could be measured, given perfect equipment.
A photon is an elementary particle
It is also the carrier of all electromagnetic radiation

The Photoelectric effect

Refers to the emission of electrons from a cold metal surface when electromagnetic radiation of sufficiently high frequency falls on it.

What happened when Electromagnetic radiation falls on a metal?



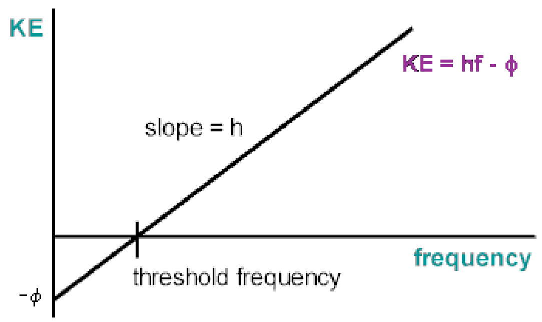
1. No electrons are emitted if the frequency of the light is below a minimum frequency {called the **threshold frequency**}, regardless of the intensity of light.
2. Rate of electron emission {ie photoelectric current} is proportional to the light intensity.
3. Emitted electrons have a range of kinetic energy, ranging from zero to a certain maximum value. Increasing the frequency increases the kinetic energies of the emitted electrons and in particular, increases the maximum kinetic energy.

This maximum kinetic energy depends only on the frequency and the metal used {ϕ}; the intensity has no effect on the kinetic energy of the electrons.

4. Emission of electrons begins <u>instantaneously</u> {i.e. no time lag between emission & illumination} even if the intensity is very low
   The last 2 observations couldn't be explained by wave theory of light instead they provide evidence for the particulate/particle nature of electromagnetic radiation.
   In 1905 Einestien introduced the explanation as follow

1. Light is quanta of energies which are called Photons each photon has an energy of E=hf
2. For each material there is an energy needed to free the electrons from its surface which is called Work function:

$$\phi = hf_0$$

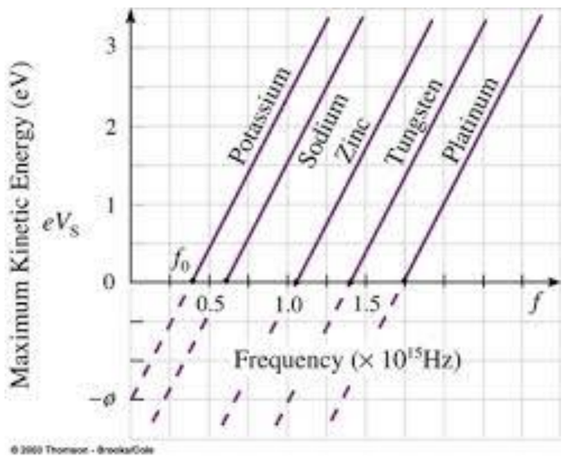


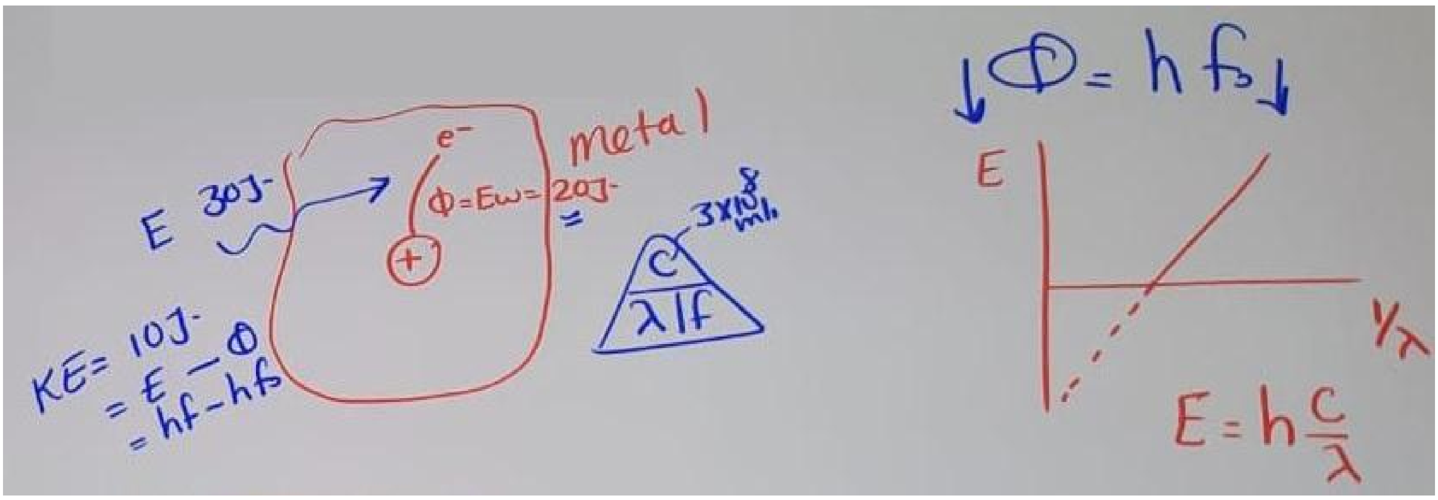
Where f_0 is the critical frequency (HZ)
h is the blank's constant (Joule.sec)

3. If a photon of Energy **E=hf** incident on a metal surface and this energy is equals **ϕ** then the photon can free an electron.
4. If the photon energy **E=hf** exceeds the work function **ϕ** then the freed electron will carry the energy difference between the two energies as a kinetic energy

$$E = \frac{1}{2}mv^2 = hf - hf_0$$

5. If the photon energy is less than the work function **ϕ**, the electrons will not emit at all.





Example (2)

Light of frequency 6.7×10^{14} Hz shines on clean Caesium metal. What is the maximum kinetic energy of electron emitted?

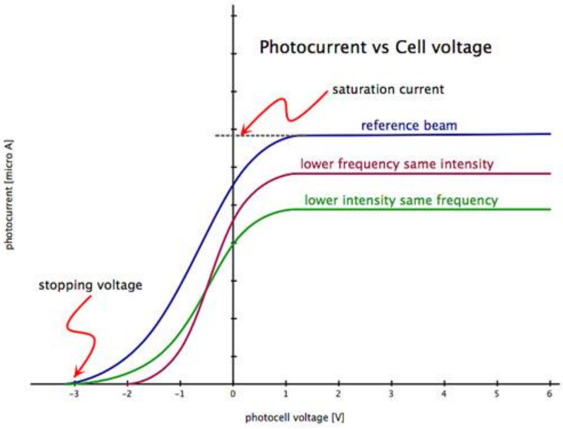
Solution

For Caesium $\phi=3.43\times10^{-19}$ J

$h=6.63\times10^{-34}$ J.s

$$E_k=hf-\phi$$

$$=6.63\times10^{-34}\times6.7\times10^{14}-3.43\times10^{19}$$

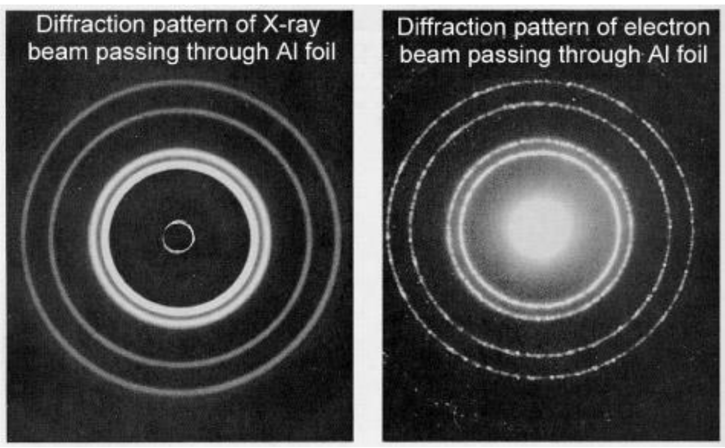


Threshold frequency is the minimum frequency of the Em radiation required to eject an electron from a metal surface. {This is because the electrons are held back by the attractive forces of the positive nuclei in the metal.}

Work function of a metal is the minimum energy required to eject an electron from a metal surface

Wave-particle Duality

If a beam of electrons is directed at a thin metal foil, a diffraction happened (Exactly like the diffraction happened due to Light waves) , then now one can say that the electrons have a wave behavior , the wave length of the radiation should be very small , about the size of the atom , so the separation due to the diffraction grating for electrons would have to be small to about the size of the atom.



In 1924 the French physicist (Louis de Broglie) suggested that [all moving particles have a wave –like nature. Based upon the quantum theory and Einstein's theory of relativity he suggested that the momentum P of a particle and its associated wavelength are related by the relation

$$\boxed{\lambda = \frac{h}{P}}$$

λ is called De Broglie Wavelength

Example (3)

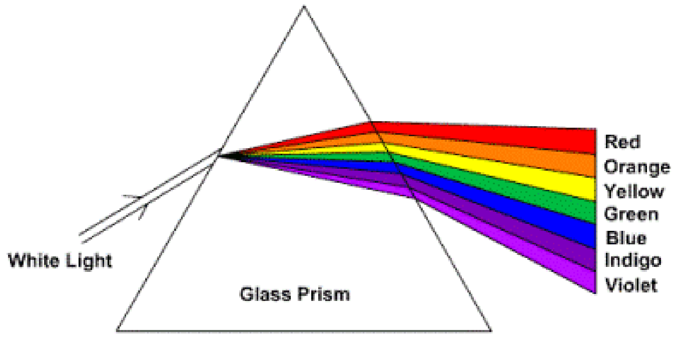
Calculate the de Broglie wavelength of an electon travelling with a speed of $1.0\times10^{7}ms^{-1}$ (Blank's constant $h=6.6\times10^{-34}$ J and the electron mass $m_e=9.1\times10^{-34}$kg)

Solution

$$\lambda = \frac{h}{P} = \frac{h}{mv} = \frac{6.6\times10^{-34}}{9.1\times10^{-31}\times1\times10^{7}} = 7.3\times10^{-11}m$$

Emission spectra

1. Continuous spectra

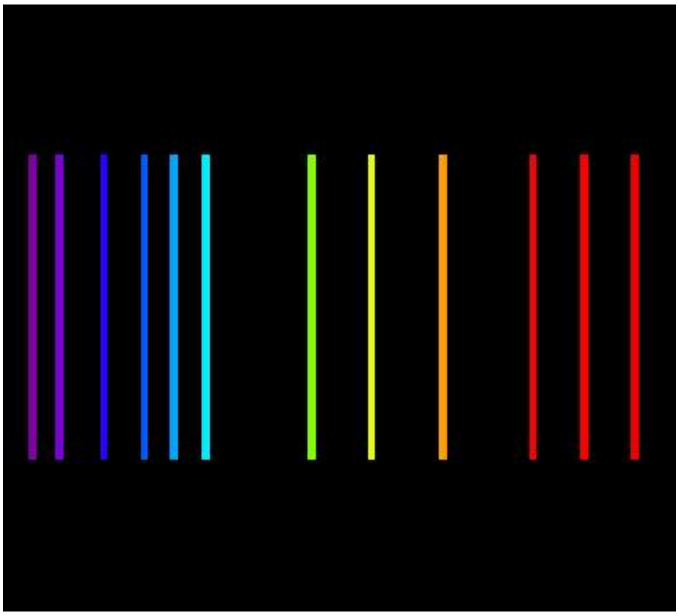


If white light from a tungsten filament lamp is passed through a prism, the light is dispersed into its component colors; the band of different colors is called

[Continues spectrum]

Since this spectrum has been produced by the emission of light from the tungsten filament lamp, it is referred as an emission spectrum

2. line spectra

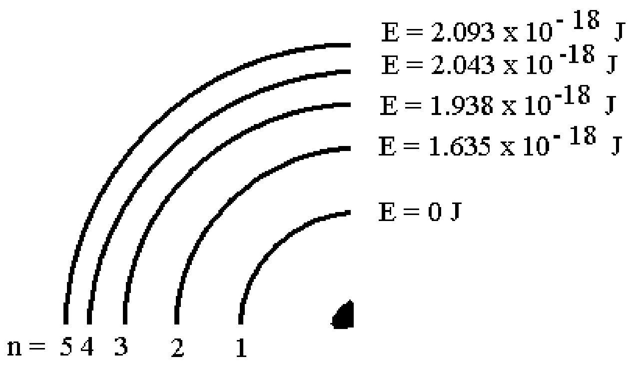


a discharge tube in a transparent tube containing a gas at low pressure, when high potential difference is placed across two electrodes in the tube, light is emitted and by examination the light by diffraction grating it shows the emission spectrum is no longer continuous but consists of a number of bright lines.

It is called [line spectrum]

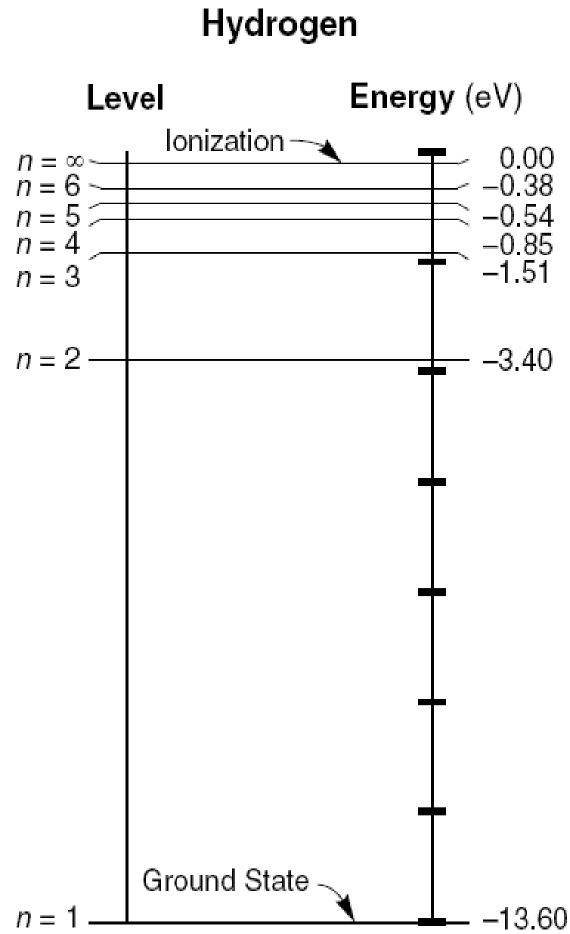
It consists of a number of separate colors, the wavelength corresponding to the lines of the spectrum are characteristic of the gas which is in the charge tube.

Electron energy levels in atoms



To explain how lie spectra are produced we need to understand how electron is atoms behave, electrons in an atom can have only certain specific energies, these energies are called **The electron energy levels** of the atom

Normally electrons occupy the lowest energy levels available, under these conditions the atom and its electrons are said to be in the **Ground state**, if the electron absorbs energy (by heating or by collision with another electron) it may be promoted to higher energy level where the energy absorbed is exactly equal to the difference in energy of the two levels, under this condition the atom is described as being in an **excited state**



Energy Levels for the Hydrogen Atom

An excited atom is unstable, after a short time, the excited electron will return to a lower level, to achieve this the electron must lose energy. It does so by emitting a photon of electromagnetic radiation of energy hf given by

$$\boxed{hf = E_2 - E_1}$$

Where E_2 is the energy of the higher level

Where E_1 is the energy of the lower level

Using the relation between the speed of light ***c*** , wavelength λ and frequency **f**

Then

$$\boxed{\lambda = \frac{hc}{\Delta E}}$$

ΔE is called the electron transition

One can say that the downward transition corresponds to photon emission and an upward transition to photon absorption

Example (4)

A hydrogen atom has its electron in the energy level at -1.51 ev. It absorbs a photon, which promotes the electron to -0.85 ev level , what is the wavelength of this photon?

[The electronic charge e=1.6x10^{-19}C
The speed of light C=3x10^{8}m/s
Blank's constant h=6.6x10^{-34} J]

Solution

$$\Delta E = E_2 - E_1$$

$$= -0.85 - (-1.51) = 0.66\,ev$$

$$0.66\,ev \equiv 0.66 \times 1.6 \times 10^{-19} \equiv 1.1 \times 10^{-19}\,J$$

$$\because \Delta E = hf$$

$$\therefore 1.1 \times 10^{-19} = 6.6 \times 10^{-34} \times f$$

$$\therefore f = 1.6 \times 10^{14}\,Hz$$

$$\because \lambda = \frac{c}{f} = \frac{3 \times 10^{8}}{1.6 \times 10^{14}} = 1.9 \times 10^{-6}\,m$$

N.B

If you encounter a question like this: [Nov 20 V1, Q(11)]

A photon of wavelength **540nm** collides with an isolated stationary electron, as illustrated in **Fig. 11.1**.

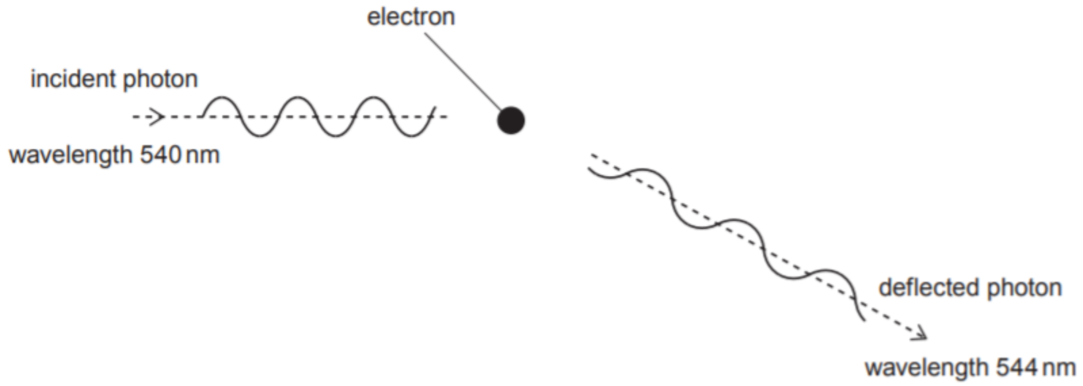


Fig. 11.1

The photon is deflected elastically by the electron.
The wavelength of the deflected photon is **544nm**.
(a)(i) State what is meant by a *photon*.

(ii) On Fig. 11.1, draw an arrow to indicate the approximate direction of motion of the deflected electron.

(b) Calculate:

(i) the momentum of the deflected photon

(ii) the energy transferred to the deflected electron.

(c) Another photon of wavelength 540nm collides with an isolated stationary electron.
Explain why it is not possible for the deflected photon to have a wavelength less

than 540nm.

You could answer with:

(a)(i)

- quantum/packet of energy **(1 mark)**

-of electromagnetic radiation **(1 mark)**

(ii) (1 mark)

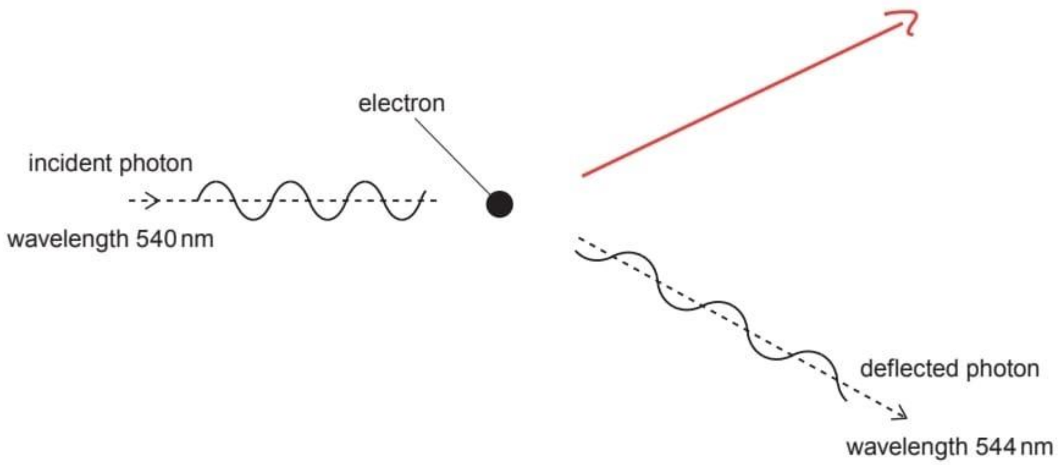


Fig. 11.1

(b)(i)

$\lambda = \frac{h}{p}$ **(1 mark)**

p = (6.63 × 10^{-34}) / (544 × 10^{-9}) = **1.22 × 10^{-27} N s (1 mark)**

(ii)

energy = $\frac{hc}{\lambda}$ **(1 mark)**

$= 6.63 \times 10^{-34} \times 3.00 \times 10^{8} \times (540^{-1} - 544^{-1}) \times 10^{9}$ **$= 2.7 \times 10^{-21}$ J (1 mark)**

(c)

-smaller wavelength corresponds to greater photon energy **(1 mark)**

- any one from: **(1 mark)**

• (deflected) photon loses energy (so not possible)

• (deflected) photon would need to gain energy (so not possible)

• electron would need to lose energy (so not possible)

• initially electron energy is zero (so not possible)

If you encounter a question like this: [Spec. 22, Q(8)]

(a) Describe two phenomena associated with the photoelectric effect that **cannot** be explained using a wave theory of light.

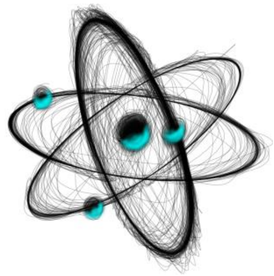
You could answer with:

1-The intensity of incident electromagnetic wave has no relation to the kinetic energy of the emitted electrons. **(1 mark)**

2- The emission of electrons begins instantaneously. **(1 mark)**

Part (2)

Nuclear physics



Mass Defect

At nuclear level, the masses we deal are so small that it would be clumsy to measure them in Kilogram, instead we measure the masses of nuclei and nucleons in atomic mass units (**U**)

One atomic unit (**1U**) is defined as being equal to one twelfth of the mass of a carbon 12 atom. 1u is equal to 1.66×10^{-27}Kg.

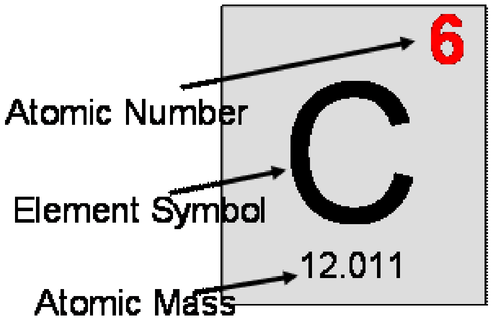
Using this scale of measurement [to six decimal places]

Proton mass m_p=1.007276U

Neutron mass m_n=1.008665U

electron mass m_e=0.00549U

Because all atoms and nuclei are made up of protons , neutrons and electrons we should be able to use these figures to calculate the mass of any atom or nucleus.



For example

The mass of a helium-4 nucleus [consists of two protons and two neutrons], should be:

$$(2\times1.007276)+(2\times1.008665)=4.031882U$$

However, the actual mass of a helium nucleus is 4.001508U.

This difference between the expected mass and the actual mass of a nucleus is called the **Mass defect** of the nucleus.

And in case of Helium-4 nucleus the mass defect is:

$$4.031882U-4.001508=0.030074U$$

The mass defect of a nucleus is the difference between the total mass of the separate nucleons and the combined mass of the nucleons

Mass- Energy Equivalence

In 1905 Albert Einstein suggested that there is equivalence between mass and energy which could be expressed as

$$\boxed{E=mc^2}$$

Where **E**: measured in Joule

C: is the speed of light measured in metre/second

Using this relation we can calculate that 1 kg of matter is equivalent to:

$$1.0\times(3.0\times10^{8})^{2}=9.0\times10^{16}\,J$$

Also for the mass defect of the helium atom 0.030074U is equivalent to:

$$0.030074\times1.66\times10^{-27}\times(3.00\times10^{8})^{2}=4.54\times10^{-12}\ J$$

A more convenient energy unit is the mega electronvolt (MeV)

One mega electron volt is the energy gained by one electron when it is accelerated through a potential difference of one million volts.

Since electrical energy= charge x potential difference

And the electron charge= $1.6\times10^{-19}C$

Then

$$1\ MeV= 1.6\times10^{-19}\times1.00\times10^{6}$$

Or

$$1\ MeV=1.6\times10^{-13}\ Joule$$

So the energy equivalent to mass defect of Helium nucleus will be:

$$4.54\times10^{-12}/1.6\times10^{-13}=28.4MeV$$

So if the mass measured in **U** and Energy in **MeV**

Then:

$$\boxed{1U\equiv931MeV}$$

Define

Binding Energy

1 mark

It is the energy needed to separate a nucleus into its individual protons and neutrons To infinity

1 mark

Or

It is the energy equivalent of the mass defect of a nucleus

This energy is equivalent to the mass difference

$E=mc^2$ 1u = 934 Mev

Example (1)

Calculate the binding energy in MeV, of a carbon-14 nucleus with a mass defect of 0.109736U

Solution

Using the equivalent 1U=931 MeV

then 0.109736U=102 MeV

the binding energy = mass defect=102 MeV

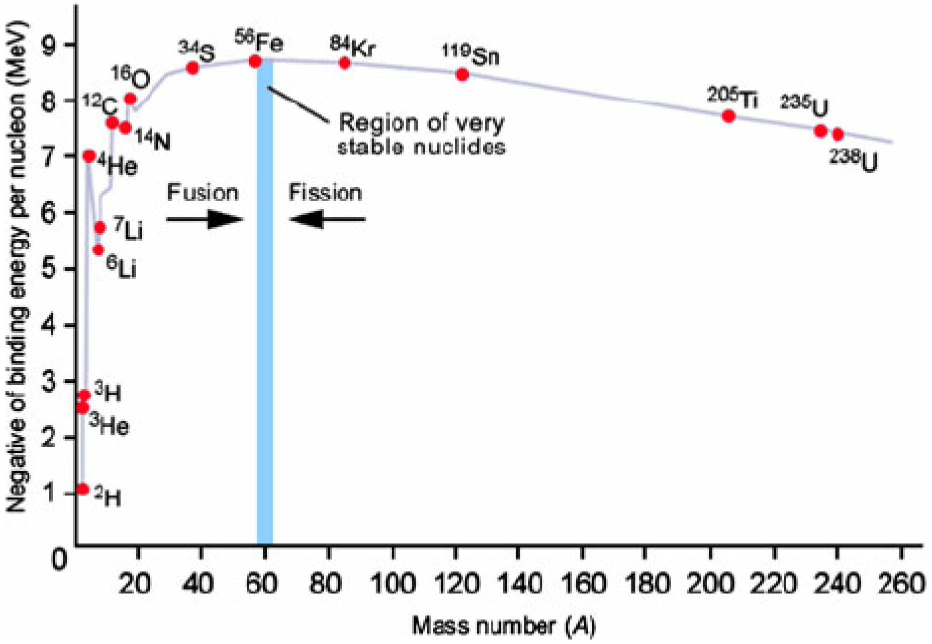
Binding energy per nucleon:

Is the total energy needed to completely separate all the nucleons in a nucleus divided by the number of nucleons in the nucleus.

The higher the binding energy per nucleon, the more stable the nucleus and so more energy is needed to pull it a part.

the opposite figure shows the variation with nucleon number of the binding energy per nucleon for different nuclides

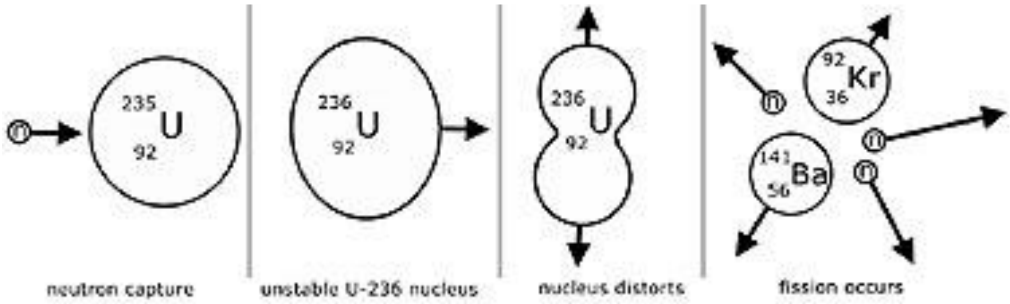
the most stable nuclides are those with the highest binding energy per nucleon [all elements before Iron], all these elements may combine or fuse in a very high temperature and pressure in a process called [nuclear fusion]. Heavy nuclei on the graph when bombarded



with neutrons may break into two smaller nuclei again with larger binding energy per nucleon values in a process called [nuclear fission]
When nuclear fusion or fission takes place, the nucleon numbers of the involved nuclei involved change, a higher binding energy per nucleons is achieved and this is accompanied by a release of energy.

Nuclear Fission

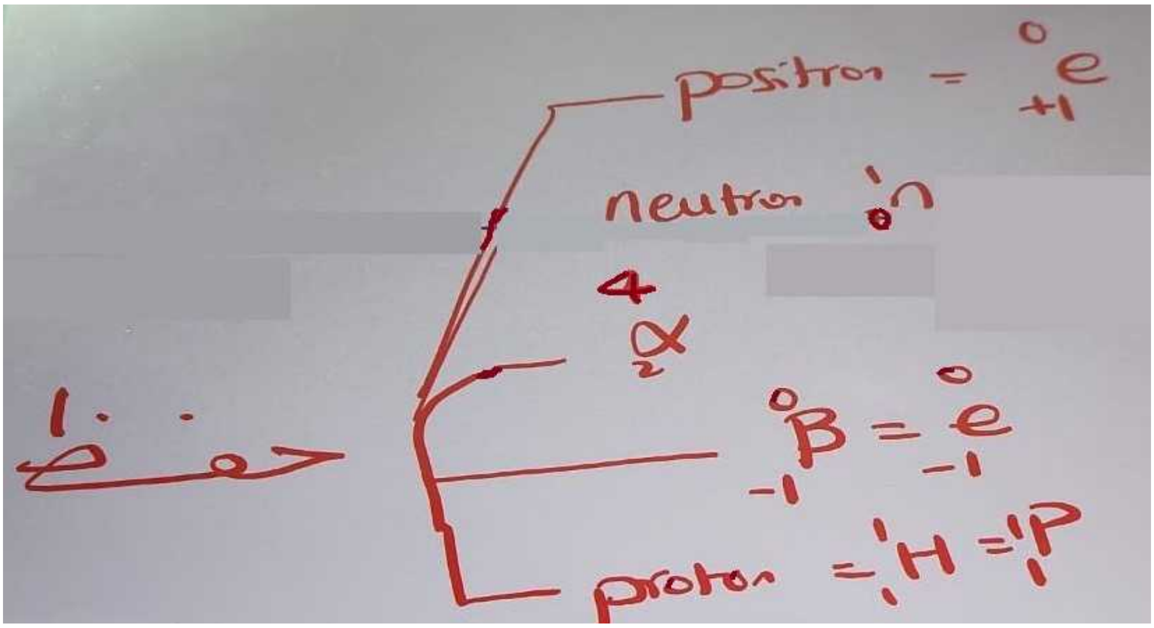
The splitting of a heavy atomic nucleus into two smaller nuclei by capture of a slow neutron.

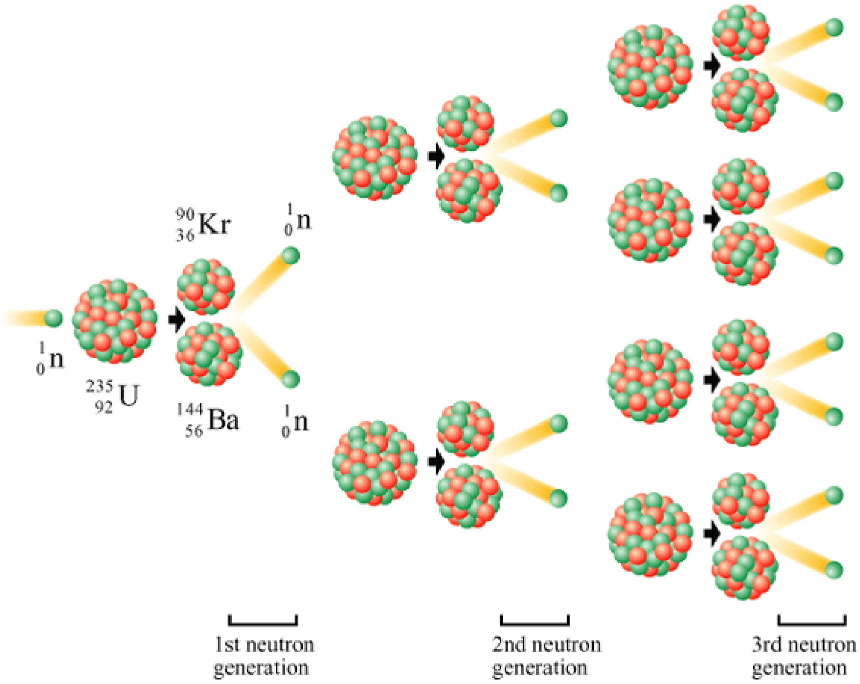


The equation describes the fission of uranium-235 by a slow neutron into barium and krypton nuclei, with the emission of three fast neutrons.

$$^{235}_{92}U + ^{1}_{0}n \rightarrow ^{236}_{92}U \rightarrow ^{141}_{56}Ba + ^{92}_{36}Kr + 3^{1}_{0}n$$

In order to start nuclear fission, one nucleus must be made to split apart. This is achieved by getting the nucleus to absorb a slow-moving neutron. When the nucleus splits, it releases energy, two components, and possibly some more neutrons.

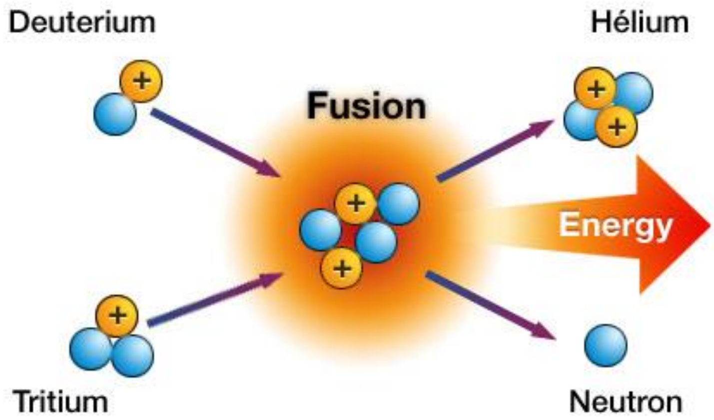




If at least one neutron is released, then a chain reaction occurs. This neutron goes on to make another nucleus unstable, which splits, and produces more neutrons, and so on. If this chain reaction is uncontrolled, a massive amount of energy is released very fast. This is an atomic explosion, which is used in nuclear bombs. In order to use nuclear fission in a power station, the number of neutrons released must be controlled by inserting a substance such as boron into the reactor, which absorbs the neutrons, preventing them from going on to make more nuclei split.

Nuclear Fusion

Nuclear fusion is the joining together of atomic nuclei to form a larger nucleus, and possibly some other products, including energy. It occurs naturally in stars, where hydrogen is fused together into larger isotopes of hydrogen and then into helium, releasing energy along the way.



The increase in binding energy per nucleon are much larger for fusion than fission reaction because the graph increases more steeply for light nuclei. So fusion gives out more energy per nucleon involved in reaction than fission.

Advantages of fusion than fission

1. Greater power output per kilogram
2. The raw materials are cheap and readily available
3. No radioactive elements are produced directly
4. Irradiation by the neutrons leads to radioactivity in the reactor materials but these have relatively short half life lives and only need to be stored safely for a short time

Radioactive Decay

1. The decay constant (λ)

The decay constant λ of a radio-nuclide is the probability that an individual nucleus will decay within a unit time.
The value of λ is a constant for any particular nuclide.

2. Activity (A)

The activity **A** of a source is the number of its nuclei that decays per unit time. Activity can be measured in decays per hour, etc.…
The SI unit of activity is the **Becquerel (Bq).**

1**Bq** in an activity of 1 decay per second 1**Bq**=1s^{-1}

The activity of a sample must depend on the decay constant λ of the nuclide. The greater the probability of decay, the more nuclei that decay in unit time.

Activity **A**(s^{-1})=Decay constant λ (s^{-1}) x Number of undecayed nuclei (**N**)

Or

$$\boxed{A=\lambda N}$$

Example (1)

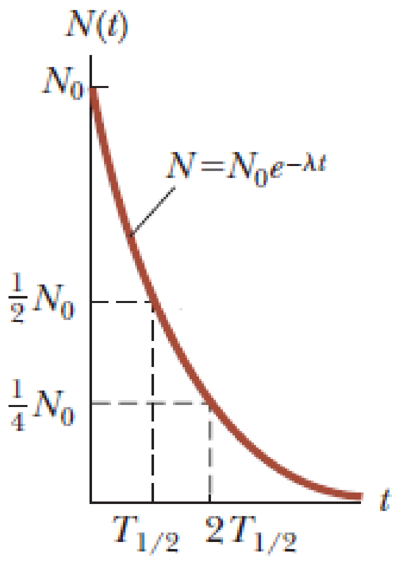
A sample of a radio-nuclide initially contains 200000 nuclei. its decay constant is 0.30 s^{-1}, what is the initial activity?

Solution

$\because A=\lambda N$

$\therefore A=0.3\times 200\,000=60\,000\,s^{-1}$

3. Exponential Decay



The graph shows the exponential decay curve.
It takes a time $t_{\frac{1}{2}}$ for the number of nuclei to fall from N_0 to $\frac{1}{2}N_0$. It takes the same time for the number of undecayed nuclei to fall from $\frac{1}{2}N_0$ to $\frac{1}{4}N_0$, and from $\frac{1}{4}N_0$ to $\frac{1}{8}N_0$. It takes a time $t_{\frac{1}{2}}$ is called the Half life of the radioactive nuclide.
It is the time taken for half the radioactive nuclei to decay.
The half lives of different radioactive nuclides vary widely, from fractions of a second up to millions of years
This graph is an exponential decay curve because the number of nuclei always falls by the same fraction in the same time.
The equation for this curve is

$$\boxed{N=N_o e^{-\lambda t}}$$

That where
N= the number of nuclei at time **t**
N_o=the original number of nuclei
λ = the decay constant

4. the decay constant and half life time

The higher the probability of decay, the more rapidly the nuclide decays, and so the shorter its half life.

In fact

$$t_{\frac{1}{2}}=\frac{\ln 2}{\lambda}=\frac{0.693}{\lambda}$$

Derivation

By Rearranging the formula $N=N_0e^{-\lambda t}$

$\therefore e^{\lambda t}=\frac{N_0}{N}$

When $t=t_{\frac{1}{2}}$ then $\frac{N}{N_{\frac{1}{2}}}=e^{\lambda t_{\frac{1}{2}}}$

$\therefore 2=e^{\lambda t_{\frac{1}{2}}}$

By taking Ln both sides

$$\ln 2=\ln e^{\lambda t_{\frac{1}{2}}}$$

$$t_{\frac{1}{2}}=\frac{\ln 2}{\lambda}=\frac{0.693}{\lambda}$$

Example (2)

A radio nuclide has a half life of 55.0 s. initially a sample of the nuclide contains 5000 nuclei.

(a) What is the decay constant for this nuclide?

(b) How many nuclei remain undecayed after 200s?

Solution

(a) $t_{\frac{1}{2}}=\frac{\ln 2}{\lambda}$

$\therefore 55=\frac{\ln 2}{\lambda}\longrightarrow\lambda=\frac{\ln 2}{55}=1.26\times10^{-2}\,s^{-1}$

(b) $N=N_oe^{-\lambda t}$

$\therefore N=5000\times e^{-1.26\times10^{-2}\times200}=5000\times e^{-2.52}=$

$\therefore N=400$ nuclei

N.B

If you encounter a question like this: [June 20 V1, Q(11)(a)]

An electron, at rest, has mass me and charge −q
A positron is a particle that, at rest, has mass me and charge +q.
A positron interacts with an electron. The electron and the positron may be considered to be at rest.
The outcome of this interaction is that the electron and the positron become two gamma-ray (γ-ray) photons, each having the same energy
Calculate, for one of the γ-ray photons:

(i) the photon energy, in J

(ii) its momentum

You could answer with:

(i) $E = mc^2$ **(1 mark)**
$= 9.11 \times 10^{-31} \times (3.0 \times 10^8)^2 = 8.2 \times 10^{-14}$ J **(1 mark)**

(ii) $p = h / \lambda$ and $E = hc / \lambda \longrightarrow E = pc$ **(1 mark)**
$p = (8.2 \times 10^{-14}) / (3.0 \times 10^8) = 2.7 \times 10^{-22}$ N s

---

If you encounter a question like this: [June 20 V1, Q(11)(b)]

State and explain the direction, relative to each other, in which the γ-ray photons are emitted.

You could answer with:

total momentum (before and after interaction) is zero **or** momentum must be conserved (in the interaction) **or** momentum of the photons must be equal and opposite **(1 mark)** (photons emitted in) opposite directions **(1 mark)**

If you encounter a question like this: [March 21 V2, Q(12)(a)]

Radioactive decay is both spontaneous and random
State what is meant by:

1. spontaneous decay

2. random decay

You could answer with:

1. not affected by external factors **(1 mark)**

2. cannot predict when a (particular) nucleus will decay or cannot predict which nucleus will decay (next) **(1 mark)**

If you encounter a question like this: [Nov 20 V1, Q(12)(a)]

State what is meant by:

(i) radioactive

(ii) decay constant.

You could answer with:

(i) unstable nucleus **(1 mark)**
emits ionising radiation **or** decays spontaneously **(1 mark)**

(ii) probability of decay (of a nucleus) **(1 mark)**
per unit time **(1 mark)**

If you encounter a question like this: [March 21 V2, Q(12)(b)]

Strontium-90 (90 38Sr) is an unstable nuclide.
The activity of a sample of 1.0×10^{-9} kg of strontium-90 is 5.2MBq
(i) Determine the decay constant λ of strontium-90.

(ii) The activity of the sample after a time of 1.0 half lives is found to be greater than the expected 2.6MBq.
Suggest a possible reason for this.

You could answer with:

(i) Number of atoms = $\dfrac{1.0\times10^{9}}{90\times1.66\times10^{-27}}$ = 6.693×10^{15} **(1 mark)**

$A=\lambda N$

$\lambda=\dfrac{5.2\times10^{6}}{6.693\times10^{15}}$ **(1 mark)**

$\lambda=7.8\ 10^{-10}\ s^{-1}$ **(1 mark)**

(ii) daughter nucleus is unstable daughter nucleus is unstable **(1 mark)**

Medical Physics

Production and use of X-ray

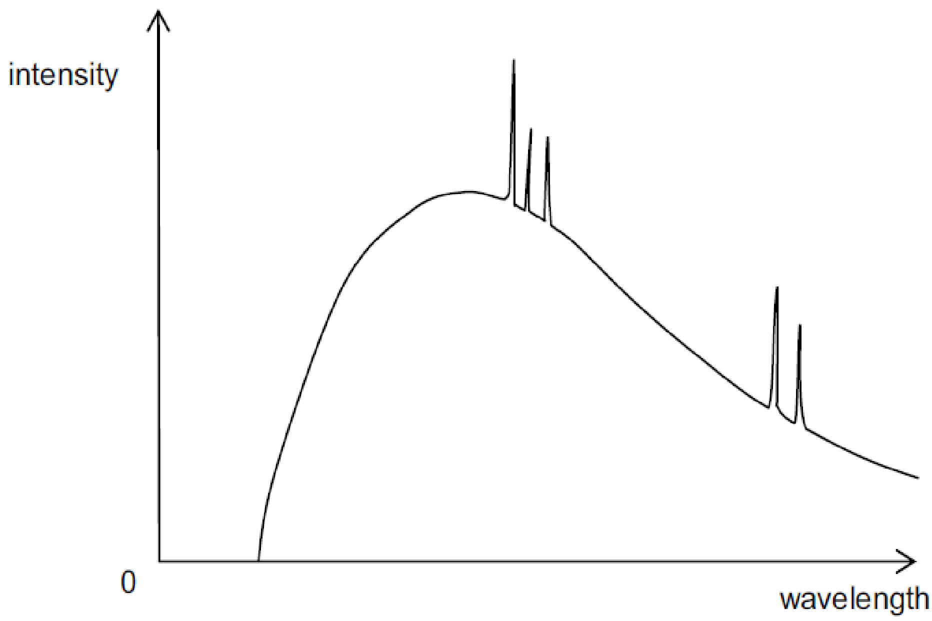
(1) Non-invasive techniques in medicine:

- Modern diagnostic techniques have concerned on using externally placed devices to obtain information from underneath the skin by using (X-rays, ultrasonic waves and magnetic resonance imaging). These techniques are called non-invasive techniques.

- **Non-invasive techniques:** method of probing deep into the body without cutting, avoiding the risk of trauma and infection. This includes wide range of non-invasive technique, such as X-rays, ultrasonic waves and MRI.

(2) Principles of production of X-rays:

- X-rays are produced by bombarding metal targets with high speeds. A spectrum of x-ray is shown in this figure



➢ **There are 2 types of x-ray spectrum :**

a) continuous spectrum

b) Line spectrum

➢ **Continuous spectrum:**

This type of x-ray spectrum is called Bremsstrahlung. When high speed electrons strike the metal target, large accelerations of different values are produced emitting a continuous spectrum of X-ray.

There is a cut-off wavelength where the whole energy of the electron is converted into energy of one photon.

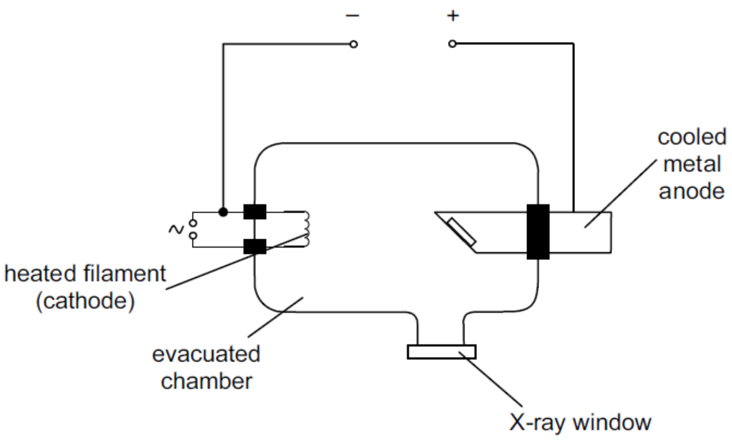
Kinetic energy of the electron = **ev** = **hc/λ**

Where **h** is the Planck's constant

➢ **Line Spectrum:**

When an energetic electron falls on the target anode, it gives part of its energy to an inner electron. The atomic electron jumps to a higher energy level according to the amount of energy gained. The vacancy left by the electron is immediately filled by an outer electron emitting X-ray photons having energy equal to the energy difference between the two levels.

(3) The main feature of modern X-ray tube:



- Electrons are emitted from the heated cathode. The electrons are accerlated through a large potential difference (20kv to 100kv) before bombarding the metal anode.

- The majority of the electron energy is transferred to thermal energy, making the anode to become hot, so as a way of cooling, it is made to rotate to distribute the heat over large area. Another way is to pass a current of water in tubes surrounding the anode.

- **Increasing the tube current** (that is controlled by the heater current of the cathode) will increase the number of the electrons emitted, this increases the number of the photons produced, and so the intensity of the X-rays will also increase.

- **Increasing the tube voltage between the cathode and the anode** will increase the energy of the electrons emitted, producing X-ray beam with higher energies and so the X-ray beam becomes harder and penetrating. The soft low energy X-rays are then removed from the beam by passing the beam through an aluminium sheet that absorbs the low energy photons.

Hardness of X-ray: Amount of penetration of X-ray beam as voltage between a node & cathode inc. penetration inc. , hardness inc. March 2016

(4) Attenuation of X-ray beam:

- **Attenuation:** Exponential reduction in the intensity of the X-ray beam when X-ray photons interact with matter and are removed from the beam.

- If Io is the initial intensity and I in the transmitted intensity, then:

$$I=I_0e^{-\mu x}$$

where μ is a constant for the medium that is dependent on photon energy. The unit of μ is mm^{-1} or cm^{-1} or m^{-1}. μ is referred to as the linear absorption coefficient or linear attenuation coefficient

➢ **Half value thickness:** It is the thickness of the material which reduces the transmitted intensity to half its initial value.

$$x_{1/2} \times \mu = \ln 2$$

(5) How to produce X-ray images:

➢ An X-ray beam is allowed to penetrate the body and fall on a photographic film. Different types of tissues produce different attenuations to the beam. An X-ray beam is formed as shadows on the photographic films.

➢ **Sharpness of the X-ray images:** This is determined by the clear distinction of boundaries between different organs in the image

➢ The sharpness of the image can be increased by:

a) By reducing the focal spot (the area of the target anode) (Fig 2.3)

b) By reducing the size of the aperture, through which the X-ray beam passes after leaving the tube. (Fig 2.4)

c) By using a lead grid in front of the photographic film, to absorb the X-ray photons. (Fig 2.5)

d) By reducing the distance between the patient and the film.

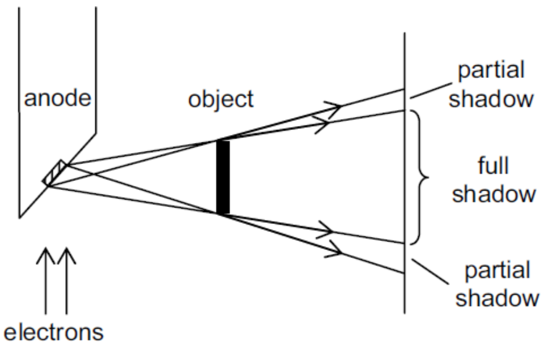


Fig 2.3

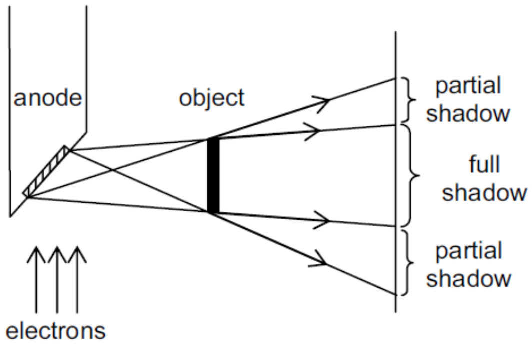
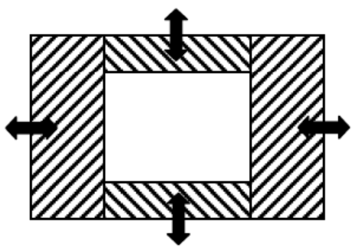


Fig 2.4



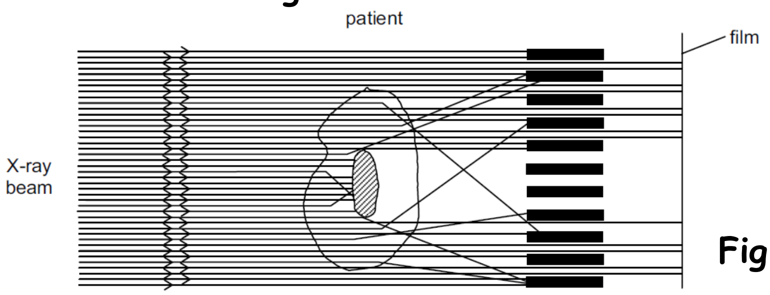


Fig 2.5

➤ **Contrast of the X-ray images**: This is the difference in the degree of blackening between dark and bright parts of the image.

➤ The contrast of the X-ray image can be increased by:

a) By using a contrast medium
b) By increasing the exposure time
c) By using a lead grid to remove the scattered photons
d) By backing the photographic film with a fluorescent material

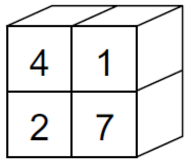
(6) Computed tomography (CT scan)

➤ The images produced from the X-ray are flat and does not give any impression of the depth. Tomography is a technique by which an image of a slice or plane of an object may be obtained.

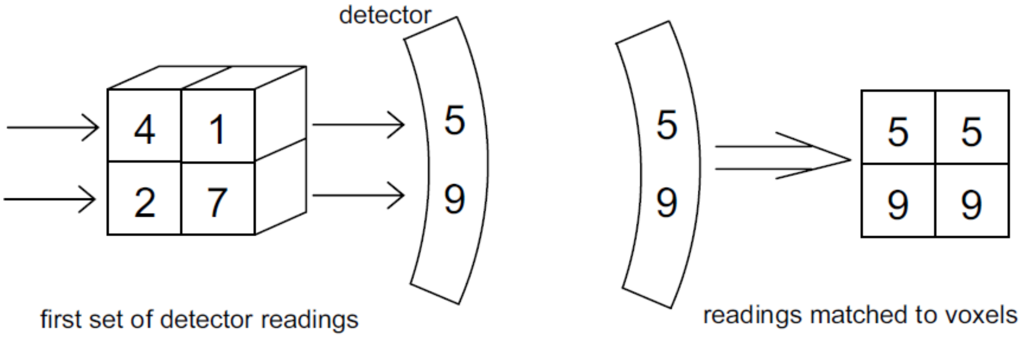
➤ Images of successive slices can be combined to form a three dimensional images, that can be rotated and viewed from many angles.

Construction of the CT scan images:

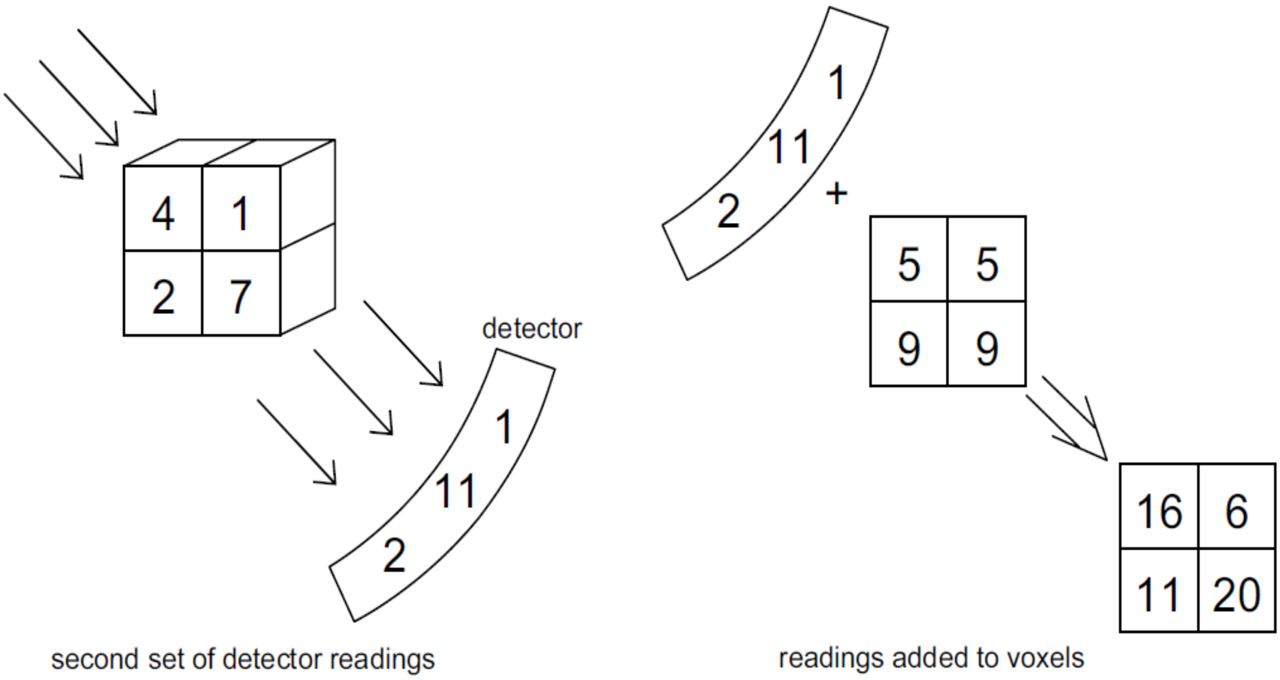
- The aim of the CT scan is to make an image of a slice through the organ from measurements made about its axis.
- The slice through the organ is divided up into small units called voxels. The image of each voxel would have a particular intensity called pixel. The pixels are built up from the measurements of the X-ray intensity made along a series of different directions around the slice of the organ.
- Suppose a section consists of four voxels with intensities shown in the figure, the number on each voxel is the pixel intensity that is to be reproduced.



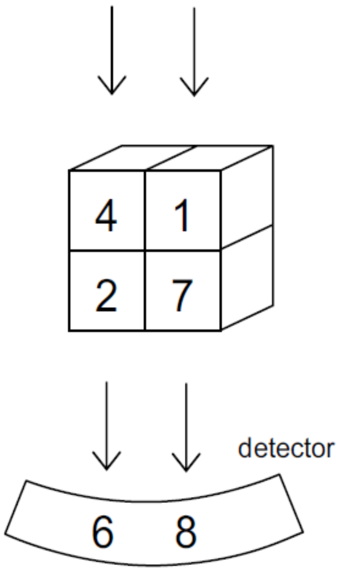
- If a beam of X-ray is directed from the left, then detectors will give readings of 5 and 9. This allow the four voxels to be reconstructed, as shown in the figure.



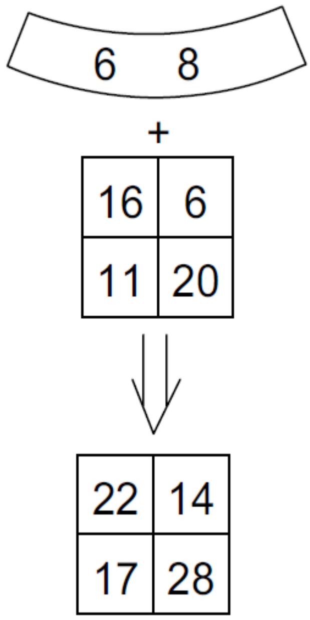
• The X-ray tube and detectors are now rotated through 45° and new detector readings are found, as shown in the figure.



• The procedure is repeated after rotating the X-ray tube and the detectors through a further 45°. The result is shown in the figure

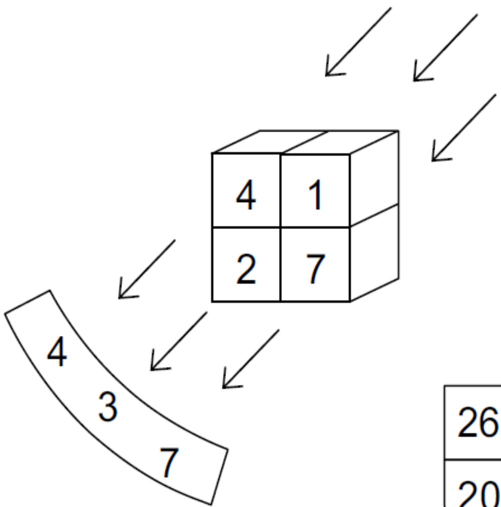


third set of detector readings

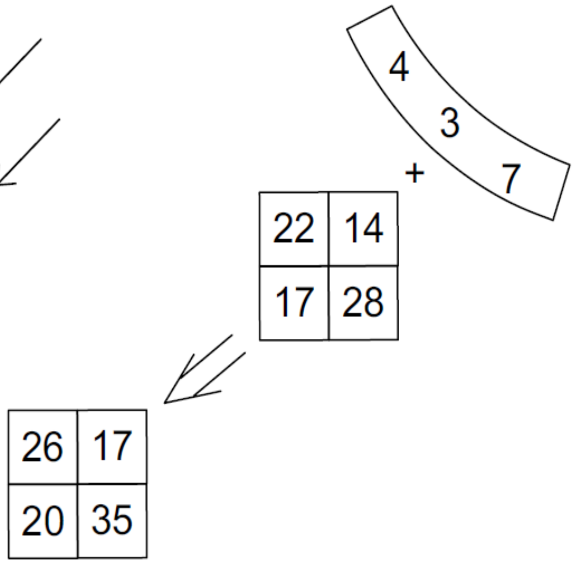


readings added to voxels

- The final images are taken after rotating the X-ray tube and the detectors through a further 45°. The result is shown in figure.

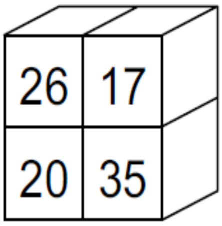


fourth set of detector readings



readings added to voxels

- This is the final pattern produced.



- In order to obtain the original pattern of pixels, two operations must be performed.

a) The background intensity must be removed. The background intensity is the total of each set of detector readings.

b) After deducting the background, the result must be divided by three to allow for the duplication of the views of the section

- The pattern of pixels for the section emerges.

26	17
20	35

deduct 14 →

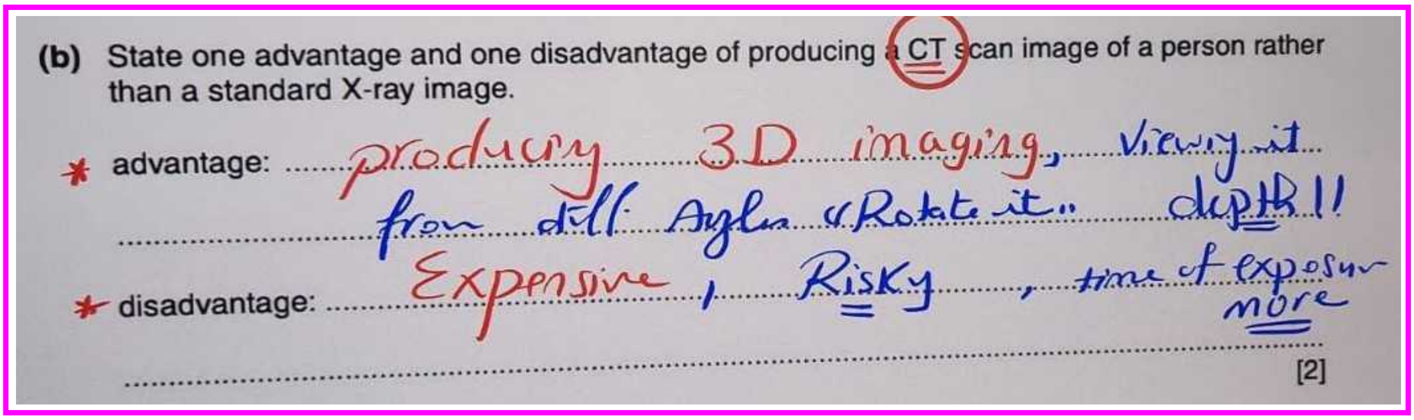
12	3
6	21

divide by 3 →

4	1
2	7

➢ **How the images produced from the CT scan differs from the X-ray images?**

- CT scan constructs the image of each slice of an organ. The image of a slice is built up of many small pixels, each viewed from many different angles.
- This procedure is repeated for many different slices to build up the image of the organ.
- X-ray images are flat, 2D images, while the images from CT scan are 3D images.
- The collection of data to construct 3D images require powerful computer to store and process large amount of data.
- The final image can then be rotated and viewed from many other angles.


(b) State one advantage and one disadvantage of producing a CT scan image of a person rather than a standard X-ray image.

* advantage: producing 3D imaging, viewing it from diff Angles & Rotate it.. depth!!

* disadvantage: Expensive, Risky, time of exposure more

[2]

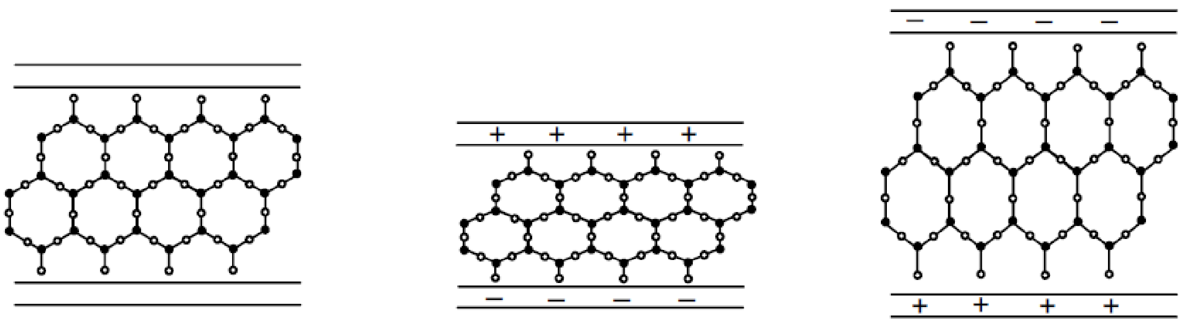
(7) Generation and detection of Ultrasonic waves:

➢ **Ultrasound waves:** These are the longitudinal waves that has a frequency that is well beyond the upper limit of human hearing >20KHZ

➢ Ultrasonic waves can be produced using a piezoelectric transducer. The basis of this is a piezoelectric crystal such as quartz. Two opposite sides of the crystals are coated with thin layers of silver to act as electrical contacts. Quartz has a complex structure made up of large number of repeating tetrahedral silicate units as shown in the figures.

➢ When the crystals are unstressed, the centres of charge of the positive and the negative ions in the lattice of the piezoelectric crystals coincide, so their effects are neutralized

➢ If a constant voltage is applied across the electrodes. The positive silicon ions are attracted towards the cathode and the negative oxygen ions are attracted towards the anode. This causes distortion of the silicate units. Depending on the polarity of the applied voltage, the crystal becomes thicker or thinner as a result of altered charge distribution



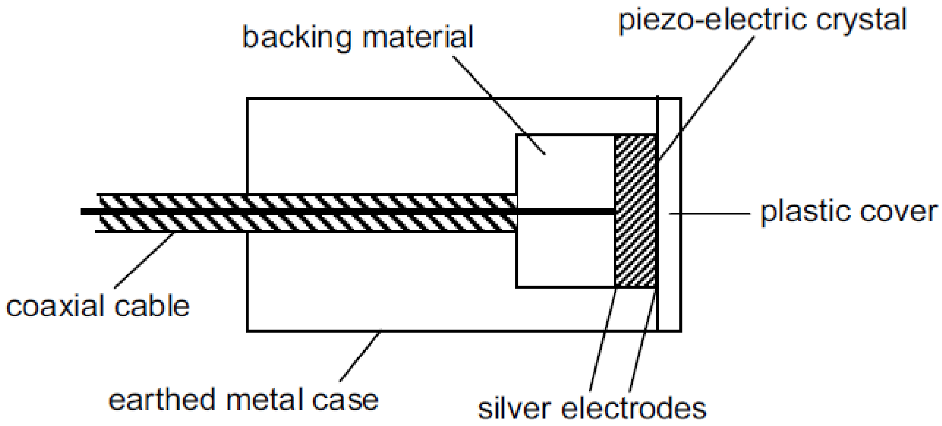
(a) unstressed (b) compressed (c) extended

➢ **Ultrasonic transducer can act as a generator of US waves:**

An alternating voltage is applied to the silver electrodes; this will cause the piezoelectric crystals to set up mechanical vibrations by compressing and expanding. If the frequency of the applied voltage is the same as the natural frequency of vibration of the crystal, resonance occurs and the vibrations have the maximum amplitude and thus Ultrasonic waves are produced.

➢ **Ultrasonic transducer can act as a receiver of US waves:**
When an ultrasonic wave is incident on an unstressed piezoelectric crystal, the pressure variations alter the positions of positive and negative ions within the crystal. This induces opposite charges on the silver electrodes, producing a potential difference between them. This varying potential difference can then be amplified and processed.

➢ A simplified diagram for the piezoelectric transducer and receiver.



(8) The meaning of specific acoustic impedance and Its importance to the intensity reflection coefficient at a boundary.

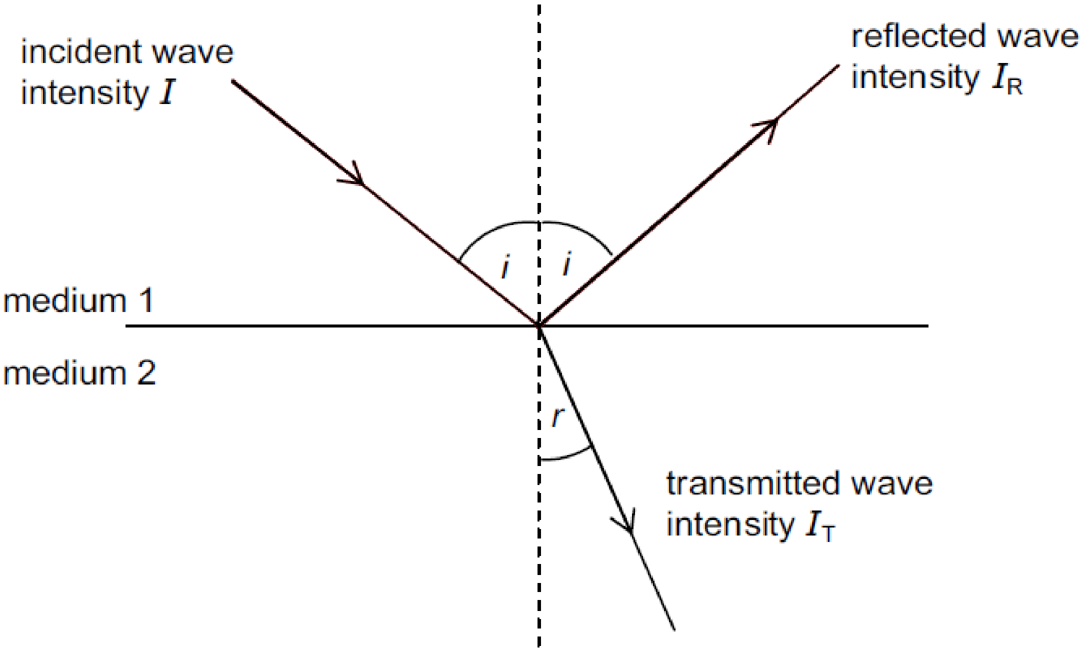
➢ **Specific acoustic impedance (Z):** This is the characteristic property of any medium. It is equal to the product of:

Speed of Ultrasound in the medium * Density of the medium

➢ These are some approximate values for specific acoustic impedances:

medium	$Z = \rho c$ / kg m^{-2} s^{-1}
air	430
quartz	1.52×10^7
water	1.50×10^6
blood	1.59×10^6
fat	1.38×10^6
muscle	1.70×10^6
soft tissue	1.63×10^6
bone	$(5.6 – 7.8) \times 10^6$

➢ Ultrasound obeys the same laws of reflection and refraction at boundaries as audible sound and light. When an ultrasound wave meets the boundary between two media, some of the wave energy is reflected and some is transmitted, as illustrated in Fig. 2.19.



➢ For an incident intensity I, reflected intensity I(R) and transmitted intensity I(T), then from energy considerations,

$$I = I(R) + I(T)$$

➢ The relative magnitudes of the reflected and transmitted intensities depend not only on the angle of incidence but also on the two media themselves.

➢ The intensity reflection coefficient (α): It is the ratio of the sound intensity reflected from the boundary (IR) to the sound intensity incident on the boundary (I). The value of α depends on the acoustic impedances of the two media.

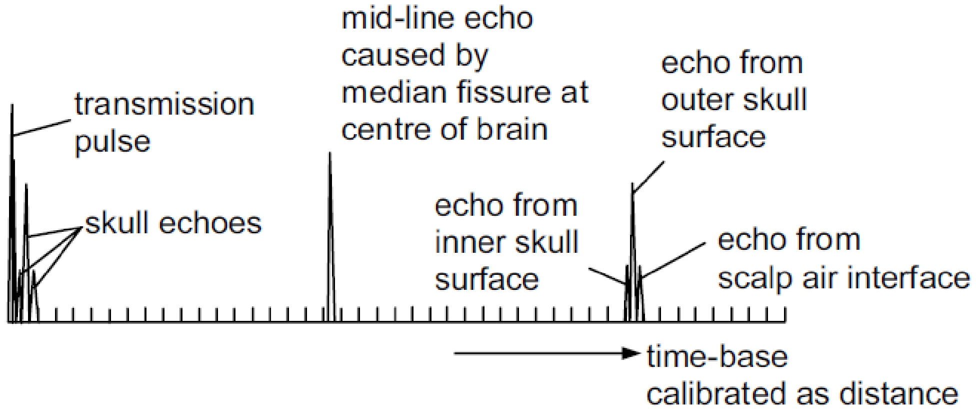
$$\frac{I_R}{I} = \frac{(Z_2 - Z_1)^2}{(Z_2 + Z_1)^2}$$

(9) The use of ultrasonic to obtain diagnostic information about the internal structure.

➢ A layer of gel is placed between the probe and the body using a coupling medium.
When the gel is used, most of the wave will be transmitted at the gel-skin boundary, when the wave enters or leave the body.

➢ There are two types of scans used in ultrasonic waves:

- A-scan (Amplitude scan): A graph of the strength of the reflected signal against time appears on the CRO. The graph appears as a set of line peaks. The height of the peak shows the nature of the reflective surface, while the time delay can find the depth of the reflecting boundaries. A-scan can make precise measurements on the brain as shown in the figure.



- B-scan (Brightness scan): The brightness of a spot of light on the screen is proportional on the strength of the reflected signal. The Ultrasound probe of the B-scan consists of a series of small crystals. Each reflective pulse is shown as a bright spot in the correct orientation of the crystal on the screen of a CRO. The image produced is 2 Dimensional images.

➢ **The advantages of Ultrasonic scanning:**

- Not at risk of any health hazards when using low power ultrasound, unlike with X-rays that cause multiple health problems when exposed to it.
- The equipment is portable and simple to use.
- With high frequency of ultrasound, smaller features within the body can be identified.
- Modern technologies allow low intensity echoes to be detected, and as a result, boundaries between the soft tissues, as well as between hard and soft tissues can be detected.

(10) Positron Emission Tomography (PET Scan)

Positron Emission Tomography (or PET scan for short) is another essential diagnostic tool in modern medicine. It is used to diagnose, monitor and help treat many illnesses including cancer, heart disease, gastrointestinal disorders and to observe brain function.

This technology relies on radioactive substances called tracers (sometimes called radiotracers. The radiation from this tracer is used to produce an image of the body using a computer.

(11) Tracers

Tracers that emit positrons (β^+) are injected into the patient's veins. This means that unlike X-Rays and CT scans, PET scans look at the patient from the inside, not the outside. There are many tracers that are used in PET scans. The most common type is a glucose-based molecule with a radioactive fluorine-18 nuclide. As different tissues absorb glucose at different rates, the rate of decay of the fluorine-18 nuclide changes from tissue to tissue. For example, cancer cells absorb glucose at a higher rate than surrounding tissue, so they appear brighter in the final image. This can be used to detect and diagnose diseases early, increasing the chances of a successful treatment.

(12) Production of gamma rays in PET scans

Positrons produced by tracers travel a very short distance inside the patient's body (significantly less than a millimetre) before it encounters an electron. After encountering an electron in the tissue an incident known as annihilation occurs. It's when a particle encounters its anti-particle and their mass becomes pure energy released in the form of two γ-rays that move opposite to each other.

It's important to note that during the annihilation process, both mass-energy and momentum are conserved. The initial kinetic energy of the positron is small/negligible compared to their rest mass-energy, hence, the γ-ray photons have specific energy and specific frequency that are determined, solely, by the mass-energy of both the positron and the electron

The energy of the γ-rays is given by:

$$\boxed{E=hf}$$

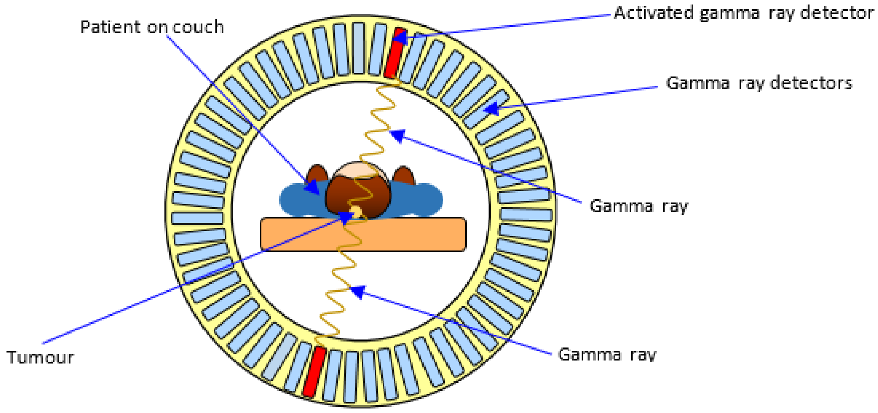
(13) Detection of gamma rays in PET scans

The patient is placed on a bed with a series of rings of detectors around him in a donut shape, that detect γ-rays falling on them. The bed is moved through the detectors so that a series of images (slices) through the patient are made in a similar manner to those made by a CT scan.

Video demonstrating how PET scans work on a molecular level:

(14) Reconstruction of the image

The γ-rays produced by annihilation travel in straight line in opposite directions to strike the detectors (as shown in the image). The path that the γ-rays take in both directions is called a 'line of response'. The signals from the detectors are sent to complex computers to be analysed and converted into a digital image. The concentration of the tracer at any given point is determined by the number of γ-ray photons being detected. The higher the number of photons- and therefore concentration of tracer- the brighter the spot on the image produced by the computer.



Positron emission tomography (PET) scanner

Differnces and similarities between CT scans and PET scans

Similarities	***Differences***
Both are non-invasive medical techniques	PET scans inject traces in the patient's body while CT scans don't
Both take different slices of the body	PET scans use γ-rays while CT scans use X-rays
Both use computers to construct the final image	PET scans look at the body from the inside, unlike the CT scan which looks at the body from the outside
Both use electromagnetic waves to penetrate the body	PET scans require positron emission, while CT scans require high speed electrons

Definitions of part (1)

1) **Attenuation of X-rays:** Exponential reduction of the intensity of X-ray beam when X-ray photons interact with matter and are removed from the beam.

2) **Half value thickness:** Thickness of the material which reduces the transmitted intensity to half its initial value.

3) **Sharpness of X-ray:** This is determined by the clear distinction of boundaries between different organs in the image.

4) **Contrast of X-ray images:** Difference in the degree of blackening between dark and bright parts of the image.

5) **CT- Scan:** These allow 3D images to be recorded, displayed and rotated to be viewed from all the angles.

6) **Ultrasound waves:** These are longitudinal pressure waves that have the frequency well-beyond the upper limit of human hearing.

Specific acoustic impedance: Characteristic property of any medium. It is given as the product of the speed of ultrasound in the medium and the density of the medium.

N.B

If you encounter a question like this: [Nov 20 V1, Q(10)]

(a) Outline briefly the principles of computed tomography (CT scanning).

(b) One section of a model designed to illustrate CT scanning is divided into four voxels. The pixel numbers **K**, **L**, **M** and **N** of the voxels are shown in **Fig. 10.1**.

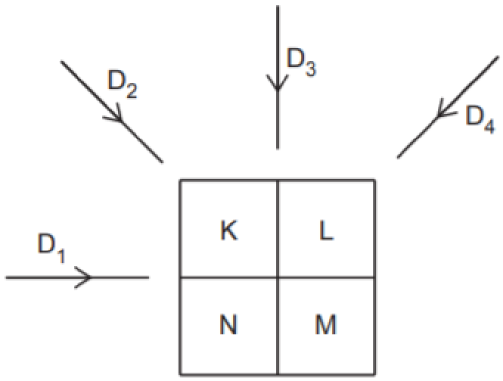


Fig. 10.1

The section is viewed, in turn, from four different directions **D1**, **D2**, **D3** and **D4**, as shown in **Fig. 10.1**.
The detector readings for each direction are noted and these are summed to give the values shown in **Fig. 10.2**.

42	45
51	30

Fig. 10.2

The background reading is **24**.
Determine the pixel numbers **K**, **L**, **M** and **N** shown in **Fig. 10.1**.

You could answer with:

(a)

- X-rays are used **(1 mark)**

-section (of object) is scanned **(1 mark)**

-scans/images taken at many angles/directions **OR** images of each section are 2 dimensional **(1 mark)**

-(images of (many)) sections are combined **(1 mark)**

-(to give) 3-dimensional image of (whole) structure **(1 mark)**

(b)

K = 6

L = 7

M = 2

N = 9

3 marks: all four correct

2 marks: three correct and one incorrect or all correct with two numbers transposed

1 mark: two correct and two incorrect

If you encounter a question like this: [March 21 V2, Q(11)(a)]

Electrons are accelerated through a potential difference of **15kV**. The electrons collide with a metal target and a spectrum of X-rays is produced.

(i) Explain why a continuous spectrum of energies of X-ray photons is produced.

(ii) Calculate the wavelength of the highest energy X-ray photon produced.

You could answer with:

(i)

- electrons decelerate (on hitting target) so X-ray photons produced **(1 mark)**
- with a range of decelerations **(1 mark)**
- and the photon energy depends on (magnitude of) deceleration **(1 mark)**

(ii) $eV = \frac{hc}{\lambda}$ **(1 mark)**

$\lambda = \frac{6.63 \times 10^{-34} \times 3.0 \times 10^{8}}{1.6 \times 10^{-19} \times 15000}$ **(1 mark)**

$= 8.3 \times 10^{-11}$ m **(1 mark)**

If you encounter a question like this: [March 21 V2, Q(11)(b)]

A beam of X-rays has an initial intensity **Io**. The beam is directed into some body tissue. After passing through a thickness **x** of tissue the intensity is **I**. The graph in **Fig. 11.1** shows the variation with **x** of **ln (I/Io)**.

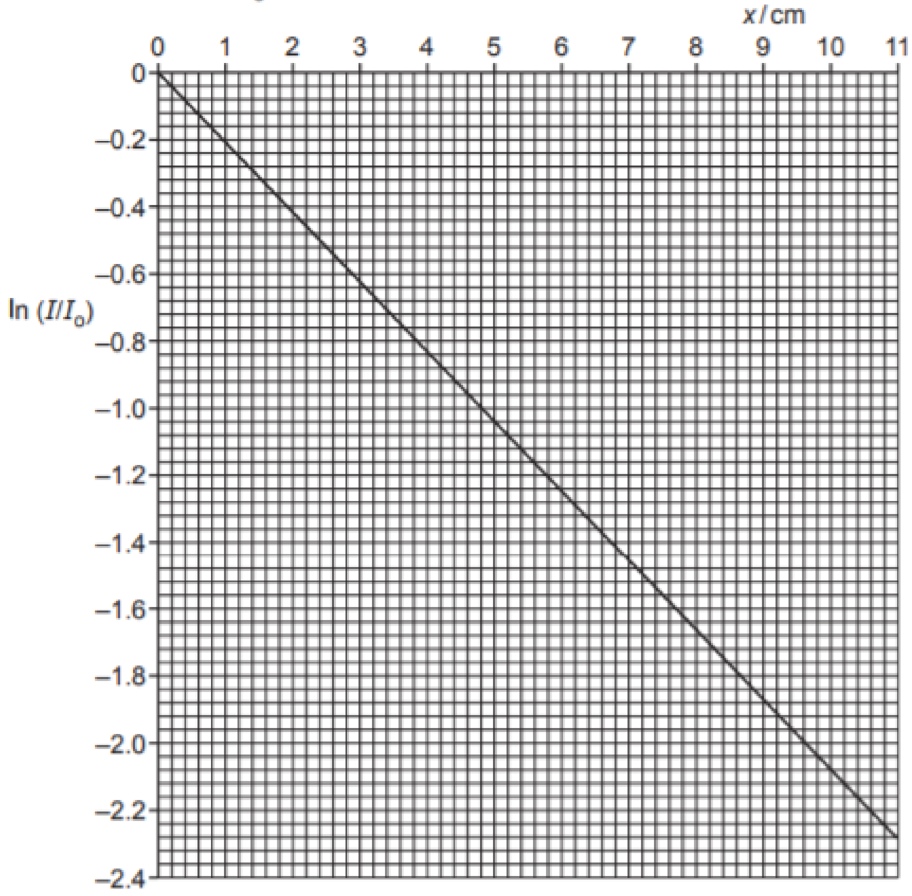


Fig. 11.1

(i) Determine the linear attenuation (absorption) coefficient μ for this beam of X-rays in the tissue.

(ii) Determine the thickness of tissue that the X-ray beam must pass through so that the intensity of the beam is reduced to **5.0%** of its initial value.

You could answer with:

(i) (calculate gradient using any 2 points on the graph, preferably as far away from each other as possible)

$\mu = -$ gradient OR $\ln(I / I_o) = -\mu x$ **(1 mark)**

$= 0.21\ cm^{-1}$ **(1 mark)**

(ii)

$\ln(0.05) = -\mu x$ **(1 mark)**

$x = \dfrac{\ln(0.05)}{-\mu}$ = (around) **14 cm** **(1 mark)**

If you encounter a question like this: **(VERY IMPORTANT!!!)**

(a)(i) With reference to PET scanning, explain the meaning of the term tracer

(ii) Explain what is meant by the term line of response and how it is used to identify the precise site of cancerous tissue.

(b)(i) With reference to **PET** scanning, explain what is meant by an annihilation event.

(ii) Name the important quantities which are conserved in an annihilation event.

You could answer with:

(a)(i) Radioactive substance that emit positrons/ β^+ particles and are injected into the body for PET scans. **(3 marks)**

(ii) Line on which both γ-rays produced in an annihilation event travel and strike the detectors, but in opposite directions. The point where many lines of response meet/intersect is the location of cancerous tissue. **(2 marks)**

(b)(i) Annihilation event is when a positron released/emitted from tracer travels a certain distance and meets an electron, which is its anti-particle. Annihilation occurs and both masses become pure energy in the form of two γ-rays the move apart in opposite directions. **(3 marks)**

(ii) – mass-energy
- momentum **(3 marks)**

If you encounter a question like this: [Spec. 22 Q(11)(b),(c)]

(b) Explain how the particles emitted from the body during a **PET** scan are created from positrons.

(c) Positrons can be artificially created by a process in the laboratory that is the reverse of the process in **(b).** This process creates both a positron and an electron moving at the same speed in opposite directions.
Suggest why two of the particles in **(b)** are needed to create one positron.

You could answer with:

(b) The positrons travel a small distance in the body before colliding with an electron, its anti-particle, in a tissue. Both particles will annihilate and their mass will become pure energy in the form of two γ-rays moving in opposite directions, while momentum is always conserved. **(3 marks)**

(c) The mass-energy of only one of these particles would not be equivalent to the mass-energy of a positron as the mass-energy of one γ-ray is very low.

If you encounter a question like this: **(VERY IMPORTANT!!!)**

Outline the theory of the **PET** scanner.

You could answer with:

(a) A small amount of tracer/radiotracer is injected into a vein, travels around the body and is absorbed by organs and tissues. The tracer emits positrons/ β^+ particles. The positrons travel a small distance in the body before colliding with an electron, its anti-particle, in a tissue. Both particles will annihilate and their mass will become pure energy in the form of two γ-rays moving in opposite directions, while momentum is always conserved. (The patient is placed on bed with multiple detector rings, in a donut shape.) The γ-rays are detected by the rings of detectors and the number of rays determines the concentration of a tracer in any tissue. The signals from the detectors are send to complex computers to be analysed and converted into an image. Tissues with high concentration will appear as bright spots on the image. **(5 marks)**

Chapter 13

Astronomy and Cosmology

Luminosity:

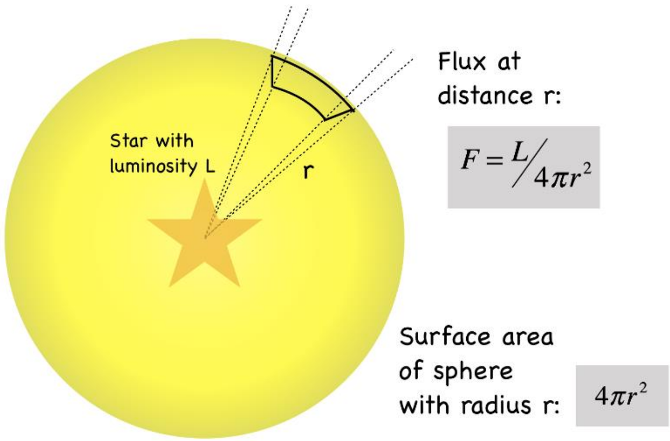
In astronomy luminosity of a star is defined as the **total** radiant energy emitted per unit time. In SI units, luminosity is measured in W or Js^{-1}.
The sun is considered to be the nearest star relative to earth, and it's been determined that the sun's luminosity (solar luminosity or $L_\odot$) is equal to 3.83×10^{26} W.

Standard candles:

In astronomy a standard candle is an astronomical object of known luminosity. Astronomers can determine the distance of a standard candle by measuring the intensity of electromagnetic radiation.

Since distances are vast in space, scientists have created a new way to measure distances in space called the light year.
1 ly = speed of light in vacuum x one year in space
1 ly = 3.00×10^{8} x 365 x 24 x 3600 = 9.5×10^{15}

Radiant flux intensity *F*: This is defined as the radiant power passing **normally** through a surface per unit area. (this equation obeys the inverse square law):



$$\boxed{F = \frac{L}{4\pi d^2}}$$

For a given star, luminosity **L** is constant and 4π is a constant, that means that the radiant flux intensity ***F*** obeys an inverse square law with distance **d**. That means that when doubling the distance from the centre of the star (**2d**) ***F*** will decrease by a factor of **4**.

By rearranging the equation above, we can use standard candles whose luminosities are know to calculate distances in space.

Wien's displacement law:

The color of stars **changes** based on the surface temperature of said stars. The hotter the star the more blueish-white it is, and the cooler the star the redder it appears, therefore Wien's law states that the higher the temperature, the shorter the wavelength at the peak (maximum) intensity, and the greater the intensity of electromagnetic radiation at each wavelength

$$\boxed{\lambda_{max} \times T = 2.9\times10^{-3}\,\text{m (constant)}}$$

$$\boxed{\lambda_{max} \propto \frac{1}{T}}$$

The Stefan-Boltzmann Law:

The luminosity of any star does not only depend on the surface temperature of the star, but also on the physical size, which in itself depends on the radius of star. There have been documented cases of super red giant stars (like KY Cygni) with a relatively low surface temperature (3500K of KY Cygni compared to the 5800K of our Sun), that have very high luminosities (200 000 times the luminosity of the Sun in the case of KY Cygni). This is because these stars are extremely large. A Slovenian physicist by the name of Josef Stefan created an expression for luminosity:

$$\boxed{L = 4\pi\sigma r^2 T^4}$$

where **σ** is a constant known as the Stefan-Boltzmann constant. The experimental value of **σ** is $\mathbf{5.67 \times 10^{-8}\ W\ m^{-2}\ K^{-4}}$

Deriving Stellar Radii from Wien's displacement law and the Stefan-Boltzmann Law:

The radius of a star can be calculated by using both Wien's displacement law and the Stefan-Boltzmann law by:

1) The use of Wiens's displacement law to determine the temperature **T** of the star ($T = \frac{2.9\times10^{-3}\,\text{m}}{\lambda_{max}}$). This requires the calculation of λ_{max} first, then comparing it to a known star's temperature (like the Sun).

2) The use of the Stefan-Boltzmann law to finally calculate the radius, using the luminosity that can be determined from the radiant flux intensity **F** equation.

Example:

The surface temperature of the Sun is **5800 K** and wavelength of light at peak intensity is **500 nm**. The wavelength at peak intensity for Sirius-B (a white dwarf star) is **120 nm**. The luminosity of this star is **0.056 times** that of the Sun. The luminosity of the Sun is **3.83×10^{26} W**.
Calculate the radius of Sirius-B.

1) Use Wien's displacement law to calculate the temperature of Sirius-B

$$\lambda_{max} \times T = 2.9 \times 10^{-3} \text{m (constant)}$$

$$\frac{\lambda_{max\,Sun}}{\lambda_{max\,Sirius}} = \frac{T_{Sirius}}{T_{Sun}}$$

$$\frac{500}{120} = \frac{T}{5800}$$

Therefore T_{Sirius} = 24 167 K ≈ 24 200 K.

2) We will use this number in the Stefan-Boltzmann law to calculate the radius of Sirius-B:

$$L = 4\pi\sigma r^2 T^4$$

$$r = \sqrt{\frac{L}{4\pi\sigma T^4}}$$

$$r = \sqrt{\frac{0.056 \times 3.83 \times 10^{26}}{4\pi \times 5.67 \times 10^{-8} \times 24167^4}}$$

$$\mathbf{r = 9.4 \times 10^6}$$

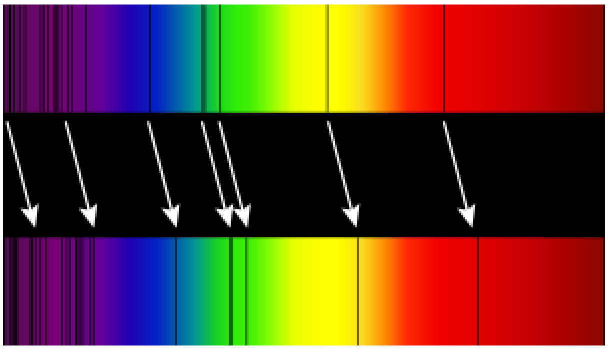
The expanding universe:

There is a fundamental idea that the universe was born from an extremely hot and dense state about 13.8 Billion years ago. This is referred to as the Big Bang theory. Multiple scientists came to this conclusion after observing a phenomenon known as redshift

Redshift:

Due to the constant expansion of the universe which began with the Big Bang. An effect similar to the doppler effect (the observed wavelengths of sound was longer for a receding source and shorter for an approaching source) the same happens with electromagnetic waves. The observed wavelengths of **all** spectral lines from distant galaxies are longer than the ones observed in laboratories This is known as redshift.

Absorption lines from the sun



Absorption lines from galaxies 1 Billion light years away

As observed in the figure above: The absorption lines from far galaxies are all shifted to longer wavelengths (redshifted)

video demonstrating redshift effect:

The fractional increase in the wavelength depends on the speed *v* of the source (galaxy).
For galaxies moving with speeds far less than the speed of light in a vacuum *c* we can use the relationship

$$\boxed{\frac{\Delta\lambda}{\lambda} \approx \frac{\Delta f}{f} \approx \frac{v}{c}}$$

Where λ is the wavelength of the electromagnetic waves from the surface of the source, Δλ is the change in the wavelength, *f* is the frequency of the electromagnetic waves from the source, Δ *f* is the change in frequency, v is the recession speed of the source and c is the speed of light in vacuum.

Hubble's law:

Hubble's law states that the recession speed ***v*** of a **galaxy** is directly proportional to its distance ***d*** from us.
Therefore

$$\boxed{v \propto d}$$

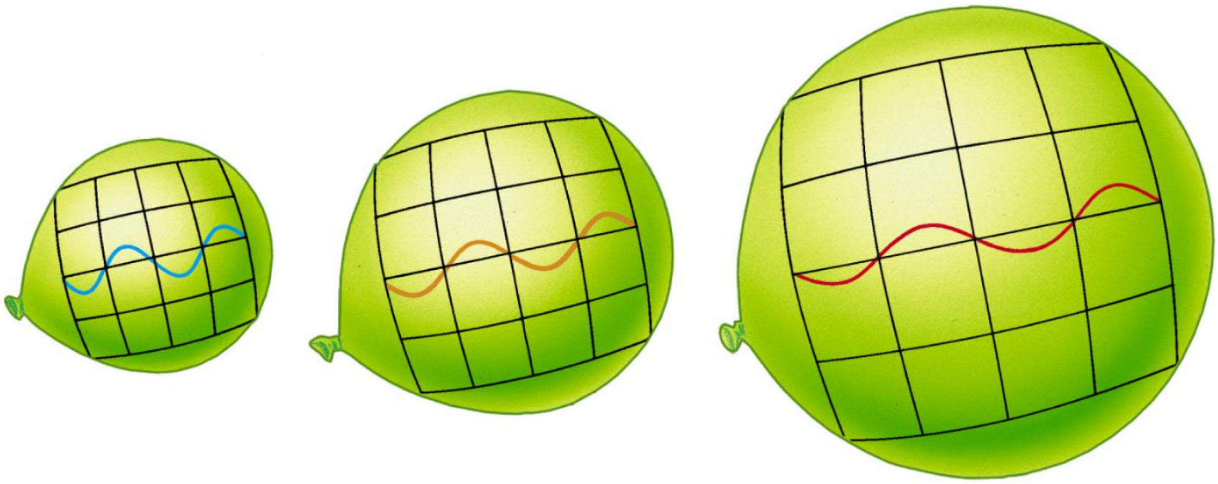
or

$$\boxed{v = H_0 d}$$

Where ***v*** is the recession speed, ***d*** is the distance of the galaxy and H_0 is the Hubble constant which is equal to $\mathbf{2.4 \times 10^{-18}\ s^{-1}}$

Big Bang theory:

The universe can be modelled after an expanding balloon with dots on its surface representing the stars and galaxies. As the balloon expands, the distance between the dots increases and so does the size of the balloon



video showcasing the expanding balloon model:

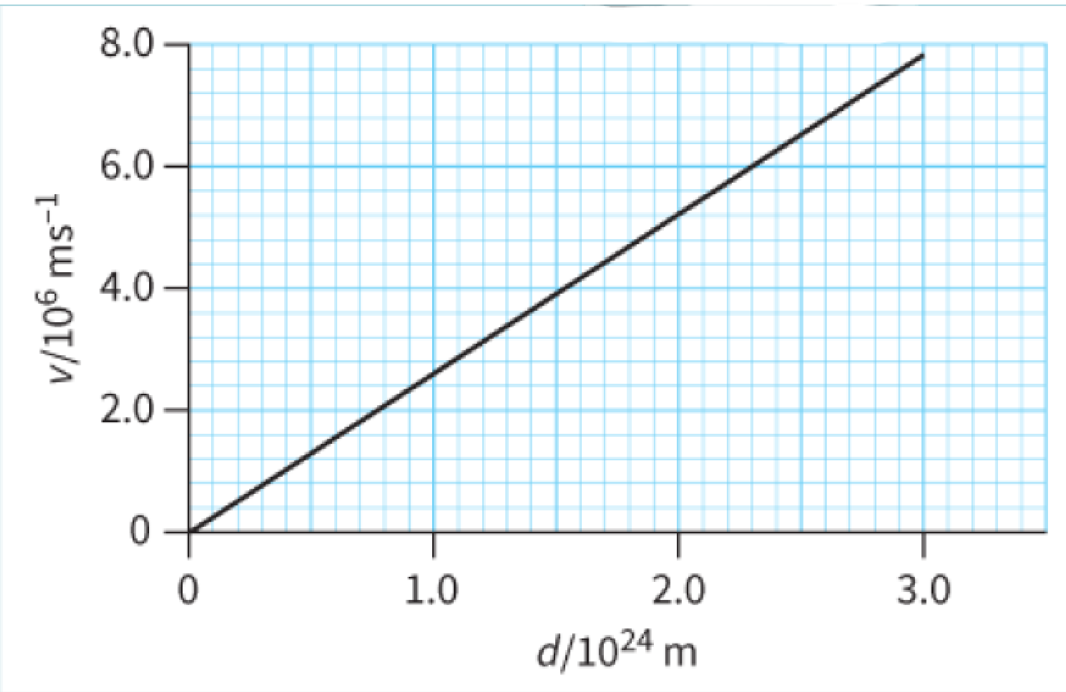
Hubble's law built the foundation on which the modern model of the Big Bang was created. Seeing as light has finite speed we can only assume that by looking deeper and further into space we are looking more and more into the past. The further back in time we go, the faster the galaxies are receding from each other. With this we can only assume that the universe must have a beginning…the Big Bang

N.B

If you encounter a question like this:

(a) State Hubble's law

(b) The recession speed **v** against distance **d** graph for some galaxies is shown



Determine the Hubble constant from this graph. Explain your answer.

(c) The Big Bang occurred some 14 billion years ago

1 year ≈ 3.15×10^7 s

Estimate the farthest distance we can observe. Explain your answer.

You could answer with:

(a) $v = H_0 d$ **(1 mark)**

(b) Gradient is the value of Hubble's constant.
Take any two points from the graph (as far as possible from each other) and calculate the gradient

$$G = \frac{y_2 - y_1}{x_2 - x_1}$$

The answer should be around 2.4×10^{-18} ($\pm 0.4 \times 10^{-18}$ accepted) **(3 marks)**

(c) Calculate number of seconds in 14 billion years

$$\mathbf{3.15 \times 10^{7} \times 14 \times 10^{9} = 4.9 \times 10^{17}\ s}$$

Speed of light = 3.0×10^{8}

distance = speed × time = $3.0 \times 10^{8} \times 14 \times 10^{9}$ = $\mathbf{1.47 \times 10^{26}}$ m **(3 marks)**

If you encounter a question like this:

a) Define the luminosity of a star

b) A red giant is a star bigger than our Sun. Explain how the surface of a red giant star can be cooler than the Sun, yet have a luminosity much greater than the Sun

c) An astronomer has determined the surface temperature of a white dwarf star to be 7800 K and its radius as 8.5×10^6 m. Calculate the luminosity of this star

d) Estimate the surface temperature of a star with $\lambda_{max} = 400 \pm 10$ nm. In your answer, include the absolute uncertainty

You could answer with:

a) The luminosity of a star is defined as the total radiant energy per unit time **(1 mark)**

b) because luminosity doesn't just depend on the surface temperature of the star, but also on the radius of the star, that's why a star bigger than the sun can have a lower surface temperature but a higher luminosity **(2 marks)**

c) $L = 4\pi\sigma r^2 T^4$

$$L = 4\pi\sigma \times 5.67\times10^{-8}\times(8.5\times10^6)^2\times7800^4$$

$L = 1.91\times10^{23}$ **(3 marks)**

d) $$T = \frac{2.9\times10^{-3}}{\lambda_{max}} = \frac{2.9\times10^{-3}}{400\times10^{-9}} = 7250$$

$$\text{uncertainty (\%)} = \frac{10}{400}\times100 = 2.5\%$$

uncertainty = 7250×2.5% = 181.25

T = 7250±181.25 **(4 marks)**

If you encounter a question like this:

(a) Define radiant flux intensity.

(b) State the relationship between radiant flux intensity F and distance d from the centre of a star

(c) Neptune is the farthest known planet from the Sun in the Solar System. Its distance from the Sun is **30** times greater than the distance of the Earth from the Sun. The radiant flux intensity from the Sun at the Earth is **1400 Wm^{-2}**.

A space probe is close to Neptune.

Calculate the maximum radiant power received by an instrument of cross-sectional area **1.0 cm^2** on this space probe.

You could answer with:

(a) Radiant power passing normally through a surface per unit area. **(1 mark)**

(b) Radiant flux intensity is inversely proportional to the square of the distance. **(1 mark)**

(c) F_{Sun} = 1400 Wm^{-2} at distance **d**.

$F \propto \frac{1}{d^2}$, therefore $F_{Neptune}$ is $\frac{1}{30^2}$ times F_{Sun}

$$\frac{1}{900} \times 1400 = 1.56\ Wm^{-2}$$

Convert meters to centimeters and the final answer will **1.56×10^{-4} W**
(3 marks)

If you encounter a question like this: [Specimen 22, Q(12)(a)]

A star has a luminosity that is known to be 4.8×10^{29} W. A scientist observing this star finds that the radiant flux intensity of light received on Earth from the star is 2.6 nW m^{-2}.

(i) Name the term used to describe an astronomical object that has known luminosity

(ii) Determine the distance of the star from Earth.

You could answer with:

(i) standard candle

(ii) $F=\dfrac{L}{4\pi d^2} \longrightarrow d=\sqrt{\dfrac{4\pi L}{F}}$

$$\mathbf{d} = \sqrt{\frac{4\pi\times4.8\times10^{29}}{2.6\times10^{-9}}} = 4.9\times10^{19}$$

If you encounter a question like this: [Specimen 22, Q(12)(b)]

The Sun has a surface temperature of 5800 K. The wavelength λ_{max} of light for which the maximum rate of emission occurs from the Sun is 500 nm. The scientist observing the star in **(a)** finds that the wavelength for which the maximum rate of emission occurs from the star is 430 nm.

(i) Show that the surface temperature of the star in **(a)** is approximately 6700 K. Explain your reasoning

(ii) Use the information in **(a)** and **(b)(i)** to determine the radius of the star

You could answer with:

(i) $\lambda_{max} \times T = 2.9\times10^{-3}\,m \longrightarrow T=\frac{2.9\times10^{-3}}{\lambda_{max}}$

$T=\frac{2.9\times10^{-3}}{430\times10^{-9}}= 6700$

(ii) $L=4\pi\sigma r^2 T^4 \longrightarrow r=\sqrt{\frac{L}{4\pi\sigma T^4}}$

$r=\sqrt{\frac{4.8\times10^{29}}{4\pi\times5.67\times10^{-8}\times6700^4}} = 1.8\times10^{10}$