Some Math 360 sample questions for review

Incidence, Euclid's and Hilbert's axioms will be provided if needed.

Q 1 Give an example of a finite incidence geometry with at most six points which does not satisfy Playfair's Postulate (that a line has exactly one parallel through each point not on that line)

Q 2 State the definition of a statement being independent of a set of axioms and give an example of a statement which is independant of the incidence axioms.

Q 3 Let ABC be an equilateral triangle. Identify the rigid motion which is the composition of rotation by $2\pi/3$ around C followed by translation from A to B.

Q 4 The set of symmetries of Figure A is the same as that for Figure B- true or false. (where Figures A and B are wallpaper patterns or frieze patterns, for example.)

Q 5 Which of the betweenness axioms hold for the integer lattice geometry, where points are of the form (m, n) where m and n are integers and lines are of the form $p(y - m) = q(x - n)$ for m, n, p and q integers?

Q 6 Find a circle p if possible such that the image of the circle of radius 2 centered at the origin is the line y=12 under inversion through p.

Q 7 Prove that Euclid's parallel postulate implies Playfair's parallel postulate.

Q 8 Prove that if two rigid motions agree on three non-collinear points, then they agree everywhere.

Q 9 Prove that in incidence geometry, if l and m are distinct lines which are not parallel, then they have a unique point in common.

Q 10 Show that if AB is a segment which is perpedicular to a line m, then the composition $\rho_m \circ \tau_{AB} = \rho_n$ where n is a line parallel to m.

Q 11 Give an example of two non-trivial glide reflections whose product is a rotation by $\pi/2$.

Q 12 State the definition of two points being symmetric with respect to a circle p and show that the only points that are symmetric to themselves with respect to p are the points on the circle.

Q 13 Show that $\rho_m \circ \rho_p \circ \rho_l = \rho_l \circ \rho_p \circ \rho_m$ if distinct lines l, m and p have a common perpendicular.

Q 14 Give an example of two rotations in the plane which proves that rotations do not always commute.

Q 15
a) Describe two different models M_1 and M_2 for incidence geometry, both which have exactly four points.
b) Give an example of a statement which is independent of the incidence axioms which is illustrated as being independent by being true in your M_1 and false in your M_2 from part a.

Q 16
a) State the definition of a rigid motion m of the Euclidean plane.

b) The definition of a reflection ρ_l across the line l is that a point P is sent to a point P' if l is the perpendicular bisector of the segment PP'. Prove that a reflection across l is a rigid motion of the Euclidean plane.

Q 17 Let A be the point $(0, 0)$, let B be the point $(0, 2)$ and let C be the point $(2, 2)$. Consider the rigid motion $R_{B,\pi/2} \circ \tau_{AC}$, which is rotation counterclockwise around B by $\pi/2$ following translation by the vector $< 2, 2 >$.
a) What is the image of the point $(0, 0)$ under this composition?
b) Identify the rigid motion $R_{B,\pi/2} \circ \tau_{AC}$.

Q 18
Let the square $ABCD$ have corners at $A = (0, 0)$, $B = (1, 0)$, $C = (1, 1)$, and $D = (0, 1)$. Identify the rigid motion $R_{D,\pi} \circ \gamma_{CD}$ where $R_{D,\pi}$ is rotation by π around D and γ_{CD} is a glide reflection along the segment CD.

Q 19 Sketch an example of a Frieze pattern (a figure with a single axis of discrete translational symmetry, generally a strip-type pattern) with a set of symmetries that contains a glide reflection along a horizontal line and a rotation by π, but does not contain reflectional symmetry across a horizontal line. Indicate centers of rotational symmetry with a $\circ$ symbol.