Some Math 360 sample questions for review

Incidence, Euclid's and Hilbert's axioms will be provided if needed.

Q 1 The set of symmetries of Figure A is the same as that for Figure B- true or false. (where Figures A and B are wallpaper patterns or frieze patterns, for example.)

Q 2 Sketch a pattern which contains some fish with two directions of translational symmetry and rotational symmetry of order 4.

Q 3 Prove that the product of a glide reflection with itself is a translation.

Q 4 Find a circle p if possible such that the image of the circle of radius 2 centered at the origin is the line y=12 under inversion through p.

Q 5 State the definition of two points being symmetric with respect to a circle p and show that the only points that are symmetric to themselves with respect to p are the points on the circle.

Q 6 Give an example of two rotations in the plane which proves that rotations do not always commute.

Q 7 Sketch an example of a Frieze pattern (a figure with a single axis of discrete translational symmetry, generally a strip-type pattern) with a set of symmetries that contains a glide reflection along a horizontal line and a rotation by π, but does not contain reflectional symmetry across a horizontal line. Indicate centers of rotational symmetry with a $\circ$ symbol.

Q 8 Find the hyperbolic length of the Euclidean straight segment from $(1, 3)$ to $(3, 7)$,

Q 9 Find the hyperbolic distance between the points $(1, 3)$ to $(3, 7)$,

Q 10 Let p be the circle of radius 2 centered at $(0, 2)$. What is the image of the point $(0, 1)$ under inversion through p? What is the image of the circle $x^2 + (y + 5)^2 = 36$ under inversion through p?

Q 11 Is there a circle p such that the image of the circle of radius 5 centered at $(0, 2)$ is the line $y = 5$ under inversion through p? If so, find an equation of p. If not, prove that no such p exists.

Q 12 Euclid's Proposition 32 from Book III can be formulated as: If line PAB intersects a circle q in two points A and B, then a line PT with T on q is tangent to q if and only if the angle $\angle ATP$ is equal to the angle $\angle PBT$. Use that form of Proposition 32 to prove: Let P be a point outside a given circle q, and let PT be a tangent to q at T. Let PAB be a segment intersecting q at A and B. Then $PA \cdot PB = PT^2$.

Q 13 Set up an integral to find the hyperbolic length of a parabolic curve from (0,1) to (2,5).