The function to minimize:

```
d = Sqrt[x^2 + 1] + 1/2 Sqrt[(x - 2)^2 + 1]
```

$$\frac{1}{2}\sqrt{1+(-2+x)^2}+\sqrt{1+x^2}$$

Find the derivative:

```
D[d, x]
```

$$\frac{-2+x}{2\sqrt{1+(-2+x)^2}}+\frac{x}{\sqrt{1+x^2}}$$

Set it equal to zero and solve the resulting equation, which gives the intersection point at:

```
Solve[D[d, x] == 0]
```

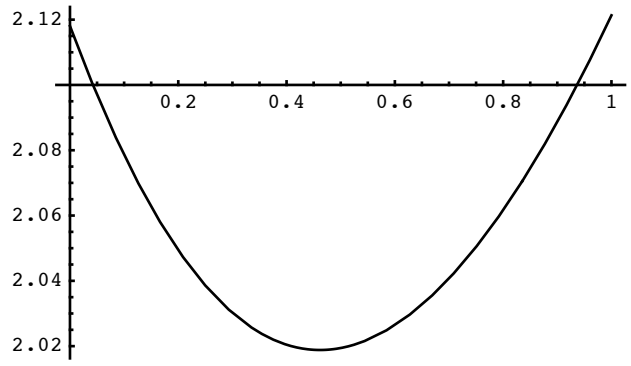
$$\left\{\left\{x \to 1-\frac{1}{2}\sqrt{\frac{2}{3}+\frac{1}{9}\left(5967-216\sqrt{519}\right)^{1/3}+\frac{1}{3}\left(221+8\sqrt{519}\right)^{1/3}}+\right.\right.$$

$$\left.\left.\frac{1}{6}\sqrt{\left(12-\left(5967-216\sqrt{519}\right)^{1/3}-3\left(221+8\sqrt{519}\right)^{1/3}+\right.\right.\right.$$

$$\left.\left.\left.\frac{60}{\sqrt{\frac{2}{3}+\frac{1}{9}\left(5967-216\sqrt{519}\right)^{1/3}+\frac{1}{3}\left(221+8\sqrt{519}\right)^{1/3}}}\right)}\right\}\right\}$$

The distance as a function of the crossing point, wiht the minimum visible at .46 or so:

```
Plot[d, {x, 0, 1}]
```




```
- Graphics -
```

The actual distance, analytically:

```
dist = d /.
```

$$\left\{x \to 1 - \frac{1}{2}\sqrt{\frac{2}{3} + \frac{1}{9}\left(5967 - 216\sqrt{519}\right)^{1/3} + \frac{1}{3}\left(221 + 8\sqrt{519}\right)^{1/3}} + \frac{1}{6}\sqrt{\left(12 - \left(5967 - 216\sqrt{519}\right)^{1/3} - 3\left(221 + 8\sqrt{519}\right)^{1/3} + \frac{60}{\sqrt{\frac{2}{3} + \frac{1}{9}\left(5967 - 216\sqrt{519}\right)^{1/3} + \frac{1}{3}\left(221 + 8\sqrt{519}\right)^{1/3}}}\right)}\right\}$$

$$\frac{1}{2}\sqrt{\left(1 + \left(-1 - \frac{1}{2}\sqrt{\frac{2}{3} + \frac{1}{9}\left(5967 - 216\sqrt{519}\right)^{1/3} + \frac{1}{3}\left(221 + 8\sqrt{519}\right)^{1/3}} + \frac{1}{6}\sqrt{\left(12 - \left(5967 - 216\sqrt{519}\right)^{1/3} - 3\left(221 + 8\sqrt{519}\right)^{1/3} + \frac{60}{\sqrt{\frac{2}{3} + \frac{1}{9}\left(5967 - 216\sqrt{519}\right)^{1/3} + \frac{1}{3}\left(221 + 8\sqrt{519}\right)^{1/3}}}\right)}\right)^2\right) + \sqrt{\left(1 + \left(1 - \frac{1}{2}\sqrt{\frac{2}{3} + \frac{1}{9}\left(5967 - 216\sqrt{519}\right)^{1/3} + \frac{1}{3}\left(221 + 8\sqrt{519}\right)^{1/3}} + \frac{1}{6}\sqrt{\left(12 - \left(5967 - 216\sqrt{519}\right)^{1/3} - 3\left(221 + 8\sqrt{519}\right)^{1/3} + \frac{60}{\sqrt{\frac{2}{3} + \frac{1}{9}\left(5967 - 216\sqrt{519}\right)^{1/3} + \frac{1}{3}\left(221 + 8\sqrt{519}\right)^{1/3}}}\right)}\right)^2\right)}$$

The distance, numerically:

```
N[dist]
```

```
2.01882
```

The crossing point, numerically:

$$N\left[1-\frac{1}{2}\sqrt{\frac{2}{3}+\frac{1}{9}\left(5967-216\sqrt{519}\right)^{1/3}+\frac{1}{3}\left(221+8\sqrt{519}\right)^{1/3}}+\frac{1}{6}\sqrt{\left(12-\left(5967-216\sqrt{519}\right)^{1/3}-3\left(221+8\sqrt{519}\right)^{1/3}+\frac{60}{\sqrt{\frac{2}{3}+\frac{1}{9}\left(5967-216\sqrt{519}\right)^{1/3}+\frac{1}{3}\left(221+8\sqrt{519}\right)^{1/3}}}\right)}\right]$$

```
0.461736
```

Considering the square doesn't make the analysis any simpler:

```
d2 = (Sqrt[x^2 + 1] + 1/2 Sqrt[(x - 2)^2 + 1])^2
```

$$\left(\frac{1}{2}\sqrt{1+(-2+x)^2}+\sqrt{1+x^2}\right)^2$$

$$\left(\frac{1}{2}\sqrt{1+(-2+x)^2}+\sqrt{1+x^2}\right)^2$$

```
D[d2, x]
```

$$2\left(\frac{-2+x}{2\sqrt{1+(-2+x)^2}}+\frac{x}{\sqrt{1+x^2}}\right)\left(\frac{1}{2}\sqrt{1+(-2+x)^2}+\sqrt{1+x^2}\right)$$

```
Solve[D[d2, x] == 0]
```

$$\left\{\left\{x\to 1-\frac{1}{2}\sqrt{\frac{2}{3}+\frac{1}{9}\left(5967-216\sqrt{519}\right)^{1/3}+\frac{1}{3}\left(221+8\sqrt{519}\right)^{1/3}}+\frac{1}{6}\sqrt{\left(12-\left(5967-216\sqrt{519}\right)^{1/3}-3\left(221+8\sqrt{519}\right)^{1/3}+\frac{60}{\sqrt{\frac{2}{3}+\frac{1}{9}\left(5967-216\sqrt{519}\right)^{1/3}+\frac{1}{3}\left(221+8\sqrt{519}\right)^{1/3}}}\right)}\right\}\right\}$$