Some Math 347 sample questions for review

Know the definitions of the terms used and the proofs of the main theorems- see various review materials on Blackboard for some guidance there.

Q 1 State Lagrange's theorem and use it to prove that every group of prime order is cyclic.

Q 2 Find a group isomorphism between the real numbers $\mathbb{R}$ under addition and the positive real numbers $\mathbb{R}^+$ under multiplication or show that no such isomorphism exists.

Q 3 Use the definition of normality to show that if H is a normal subgroup of G, then $ghg^{-1} \in H$ for all $g \in G, h \in H$.

Q 4 Draw an example of a Frieze pattern (plane figure with a symmetry group containing parallel translations) which contains vertical reflections but no horizontal reflection.

Q 5 Prove the following or give a counterexample to show that it is false:
there is no injective automorphism from the cyclic group of order 4 to the non-zero complex numbers $\mathbb{C}^*$ under multiplication.

Q 6 For a, b and c non-zero integers with a and b relatively prime, prove that if $a|bc$ then $a|c$.

Q 7 Each statement is FALSE. Give an example to show that it is false.

- Every group of order 24 is abelian.
- The product of a two-cycle and a three-cycle in S_6 is always a five-cycle.
- If ϕ is a homomorphism from G to H, then for every element g in G, the order of g is the same as the order of $\phi(g)$.
- Every abelian group is a cyclic group.
- In any group G, the product of two elements of order 2 is either order 2 or the identity.
- In the alternating group A_6, there is no element of order 4.
- In the alternating group A_4, there is no element of order 2.
- In the symmetric group S_{10}, there is no element of order 21.
- In the symmetric group S_5, the product of two odd permutations is an even permutation.
- In a finite abelian group, the product of two elements has order which is the product of the orders of the two elements.

Q 8 Prove the following or give a counterexample to show that it is false:
every monomorphism (injection homomorphism) from $\mathbb{Z}$ to $\mathbb{Z}$ is an automorphism.

Q 9 Find an embedding of the cyclic group of order 6 into a group of permutations.

Q 10 Draw a figure with a symmetry group which is isomorphic to the dihedral group D_4 which contains a letter from your first name.

Q 11 List the left cosets of the alternating group A_4 as a subgroup of S_4.

Q 12 Let G be the cyclic group of order 12 generated by a. List all subgroups of G and indicate the order and index of each subgroup. For all nontrivial subgroups of order 5 or fewer, list all generators of each subgroup.

Q 13 Prove the following or give a counterexample to show that it is false:
If ϕ is a homomorphism from G to H, then the order of $\phi(g)$ must divide the order of g for all $g \in G$.

Q 14 Prove the following or give a counterexample to show that it is false:
If $\phi : G \to H$ is a homomorphism of groups, then the kernel of ϕ is a normal subgroup of G.

Q 15 Recall that the center of a group $Z(G)$ is the set of $a \in G$ such that $ax = xa$ for all $x \in G$.
Prove the following or give a counterexample to show that it is false:
the center of a group is a subgroup of G.

Q 16 Give an example of non-identity automorphisms of the integers $\mathbb{Z}$, the rationals $\mathbb{Q}$, and the real numbers $\mathbb{R}$.