Some Math 360 sample questions for review

Incidence, Euclid's and Hilbert's axioms will be provided if needed.

Q 1 Give an example of a finite incidence geometry with at most six points which does not satisfy Playfair's Postulate (that a line has exactly one parallel through each point not on that line)

Q 2 State the definition of a statement being independent of a set of axioms and give an example of a statement which is independent of the incidence axioms.

Q 3 Let ABC be an equilateral triangle. Identify the rigid motion which is the composition of rotation by $2\pi/3$ around C followed by translation from A to B.

Q 4 Which of the order axioms hold for the integer lattice geometry, where points are of the form (m, n) where m and n are integers and lines are of the form $p(y-m) = q(x-n)$ for m, n, p and q integers? What about if we allow rational points (p', q') where p' and q' are rational, with the same set of lines?

Q 5 Prove that Euclid's parallel postulate implies Playfair's parallel postulate.

Q 6 Prove that if two rigid motions agree on three non-collinear points, then they agree everywhere.

Q 7 Prove that in incidence geometry, if l and m are distinct lines which are not parallel, then they have a unique point in common.

Q 8 Show that if AB is a segment which is perpendicular to a line m, then the composition $\rho_m \circ \tau_{AB} = \rho_n$ where n is a line parallel to m.

Q 9 Give an example of two non-trivial glide reflections whose product is a rotation by $\pi/2$.

Q 10 Show that $\rho_m \circ \rho_p \circ \rho_l = \rho_l \circ \rho_p \circ \rho_m$ if distinct lines l, m and p have a common perpendicular.

Q 11 Give an example of two rotations in the plane which proves that rotations do not always commute.

Q 12 Give examples of two models of incidence geometry both with 4 points that show that the statement "Two lines always intersect" is independent of the incidence axioms.

Q 13 Give examples of two models of incidence geometry both with 4 points that show that the statement "Every line contains the same number of points" is independent of the incidence axioms.

Q 14 In Euclidean geometry, show that the angle bisectors of a triangle intersect.

Q 15 In neutral geometry, show that Playfair's postulate implies that the sum of the interior angles of a triangle is always π. That is, assume that it is true that for every line l and every point P not on l, there is a unique line m passing through P which does not intersect l and from that prove that triangles' interior angles add up to π.
You may use Euclid's Proposition 27 if needed:
Prop 27: If a line falling on two lines makes equal alternate interior angles then the two lines are parallel.