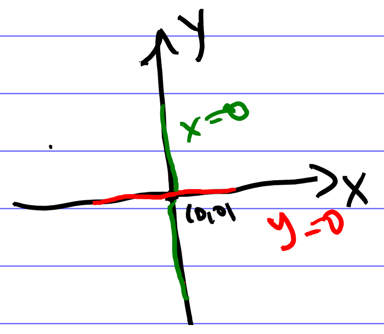
Limites

(1) Mostre que $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2-y^2}{x^2+y^2}$ não existe



p/ $y=0$

$$\lim_{x\to0} \frac{x^2-0^2}{x^2+0^2} = \lim_{x\to0} \frac{x^2}{x^2} = 1$$

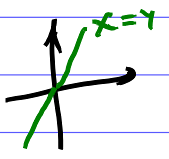
p/ $x=0$

$$\lim_{y\to0} \frac{0^2-y^2}{0^2+y^2} = \lim_{y\to0} \frac{-y^2}{y^2} = -1$$

Como encontrei dois caminhos que levam a valores distintos, então que

$$\lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{x^2+y^2} \nexists$$

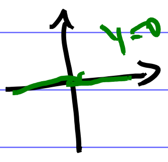
(2) Se $f(x,y) = \dfrac{xy}{x^2+y^2}$, calcule $\lim\limits_{(x,y)\to(0,0)} f(x,y)$



Caminho $x=y$

$$\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2} \qquad \lim_{x\to0} \frac{x^2}{x^2+x^2} = \lim_{x\to0} \frac{x^2}{2x^2} = \frac{1}{2}$$

Caminho $y=0$



$$\lim_{x\to0} \frac{0}{x^2+0} = 0$$

Como encontrei dois caminhos que levam a valores distintos, então que

$$\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2} \nexists$$

$\frac{0}{0}$ é indeterminado?

$$\lim_{x\to0} \frac{x}{x} = 1$$

$$\lim_{x\to0} \frac{x}{2x} = \frac{1}{2}$$

$$\lim_{x\to0} \frac{x}{3x} = \frac{1}{3}$$

$$\lim_{x\to0} \frac{4x}{x} = 4$$

$$\lim_{(x,y)\to(0,0)} \frac{x^2+y^2}{x+2} = 0$$

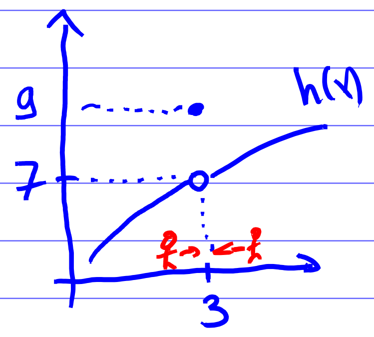
$$\lim_{x\to0} \frac{2\cancel{x}}{\cancel{x}} = 2$$

$$f(x) = \frac{x}{x} \quad x \neq 0$$

$$\neq \quad f(x) = 1 \quad x \in \mathbb{R}$$

$$g(x) = \frac{(x^2-2)}{(x+2)} \qquad x \neq -2$$

$$g(x) = (x-2)$$



$$\lim_{x\to3} h(x) = 7$$

$$h(3) = 9$$

Como $\lim\limits_{x\to3} h(x) \neq h(3)$ a função não é contínua em $x=3$.

H $\longrightarrow$ $h=5$

Umidade relativa (%)

T \ H	40	45	50	55	60	65	70	75	80
26	28	28	29	31	31	32	33	34	35
28	31	32	33	34	35	36	37	38	39
30	34	35	36	37	38	40	41	42	43
32	37	38	39	41	42	43	45	46	47
34	41	42	43	45	47	48	49	51	52
36	43	45	47	48	50	51	53	54	56

Temperatura real (°C) T

$$\frac{df(x)}{dx} = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$\frac{\partial S(H,T)}{\partial H} = \lim_{h\to 0} \frac{S(H+h,T)-S(H,T)}{h}$$

$$\frac{\partial S(H,T)}{\partial T} = \lim_{h\to 0} \frac{S(H,T+h)-S(H,T)}{h}$$

Aproximação

$$\frac{\partial S(H,T)}{\partial H} \approx \frac{S(H+h,T)-S(H,T)}{h}$$

$$\left.\frac{\partial S}{\partial H}\right|_{(60,30)} \approx \frac{S(60+5,30)-S(60,30)}{5}$$

$$\left.\frac{\partial S}{\partial H}\right|_{(60,30)} \approx \frac{40-38}{5} = \frac{2}{5}$$

Outra aproximação

Faça a conta c/ $h=5$ e $h=-5$

e faça a média.

p/ $h=-5$

$$\left.\frac{\partial S}{\partial H}\right|_{(60,30)} \approx \frac{S(60-5,30)-S(60,30)}{(-5)}$$

$$\approx \frac{S(55,30)-S(60,30)}{-5}$$

$$\approx \frac{37-38}{-5} = \frac{-1}{-5} = \frac{1}{5}$$

Fazendo a média p/ $h=5$ e $h=-5$

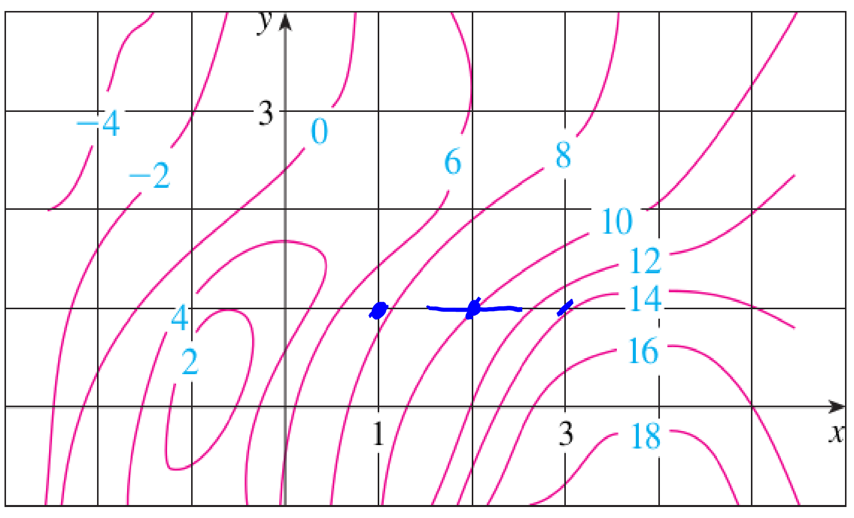
$$\left.\frac{\partial S}{\partial H}\right|_{(60,30)} \approx \frac{\left(\frac{1}{5}+\frac{2}{5}\right)}{2} = \frac{1}{10}+\frac{2}{10} = \frac{3}{10}$$

10. Um mapa de contorno de uma função f é apresentado. Utilize-o para estimar $f_x(2, 1)$ e $f_y(2, 1)$.

$f_x(2,1)$

$\left.\frac{\partial f}{\partial x}\right|_{(2,1)}$

$\frac{\partial f(2,1)}{\partial x}$



$$\left.\frac{\partial f}{\partial x}\right|_{(2,1)} \approx \frac{f(2+h,1)-f(2,1)}{h}$$

p/ $h=1$ $$\left.\frac{\partial f}{\partial x}\right|_{(2,1)} \approx \frac{f(2+1,1)-f(2,1)}{1}$$

$$\approx \frac{14-10}{1}$$

$$\approx 4$$

p/ $h=-1$ $$\left.\frac{\partial f}{\partial x}\right|_{(2,1)} \approx \frac{f(2-1,1)-f(2,1)}{(-1)}$$

$$\left.\frac{\partial f}{\partial x}\right|_{(2,1)} \approx \frac{f(1,1)-f(2,1)}{(-1)}$$

$$\approx \frac{7-10}{-1}$$

$$\approx \frac{-3}{-1} = 3$$

Média

$$\frac{3+4}{2} = 3,5$$

$$\left.\frac{\partial f}{\partial x}\right|_{(2,1)} \approx 3,5$$

Notações para as Derivadas Parciais Se $z = f(x, y)$, escrevemos

$$f_x(x, y) = f_x = \frac{\partial f}{\partial x} = \frac{\partial}{\partial x} f(x, y) = \frac{\partial z}{\partial x} = f_1 = D_1 f = D_x f$$

$$f_y(x, y) = f_y = \frac{\partial f}{\partial y} = \frac{\partial}{\partial y} f(x, y) = \frac{\partial z}{\partial y} = f_2 = D_2 f = D_y f$$

$$\frac{\partial f}{\partial x} = \frac{\partial f(x,y)}{\partial x} = f_x$$

Regra para Determinar as Derivadas Parciais de $z = f(x, y)$

1. Para determinar f_x, trate y como uma constante e derive $f(x, y)$ com relação a x.
2. Para determinar f_y, trate x como uma constante e derive $f(x, y)$ com relação a y.

15–40 Determine as derivadas parciais de primeira ordem da função.

15. $f(x, y) = y^5 - 3xy$ **16.** $f(x, y) = x^4y^3 + 8x^2y$

17. $f(x, t) = e^{-t}\cos \pi x$ **18.** $f(x, t) = \sqrt{x} \ln t$

29. $F(x, y) = \int_y^x \cos(e^t)\, dt$ **30.** $F(\alpha, \beta) = \int_\alpha^\beta \sqrt{t^3 + 1}\, dt$

$$f(x,y) = x^2 + y$$

$$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}(x^2+y) = \frac{\partial x^2}{\partial x} + \frac{\partial y}{\partial x}$$
$$= 2x + 0$$
$$= 2x$$

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}(x^2+y) = \frac{\partial x^2}{\partial y} + \frac{\partial y}{\partial y}$$
$$= 0 + 1$$
$$= 1$$

$$\frac{\partial f}{\partial x} = 2x$$

$$\left.\frac{\partial f}{\partial x}\right|_{(1,3)} = 2 \quad \left.\frac{\partial f}{\partial y}\right|_{(1,3)} = 1$$

$$g(x,y) = x + y$$

$$\frac{\partial g}{\partial x} = 1 \qquad \frac{\partial g}{\partial y} = 1$$

$$h(x,y) = xy$$

$$\frac{\partial h}{\partial x} = \frac{\partial}{\partial x}(xy) = y\frac{\partial x}{\partial x} = y \qquad \frac{\partial h}{\partial y} = \frac{\partial (xy)}{\partial y} = x\frac{\partial y}{\partial y} = x$$

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$$f(x,y) = y^5 - 3xy$$

$$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}(y^5 - 3xy) = \frac{\partial}{\partial x} y^5 - \frac{\partial}{\partial x}(3xy)$$
$$= \frac{\partial y^5}{\partial x} - 3y\frac{\partial x}{\partial x}$$
$$= 0 - 3y = -3y$$

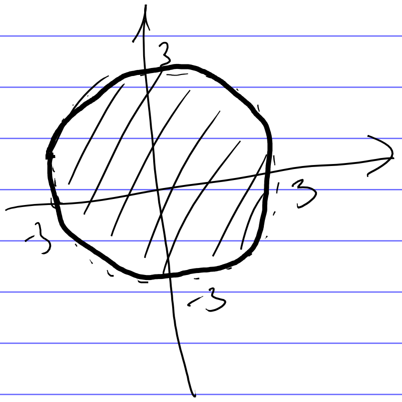
$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}(y^5 - 3xy) = \frac{\partial y^5}{\partial y} - \frac{\partial}{\partial y}(3xy)$$
$$= 5y^4 - 3x$$

A pedido de Alonza.

$$F(\alpha,\beta) = \int_\alpha^\beta \sqrt{t^3+1}\, dt$$

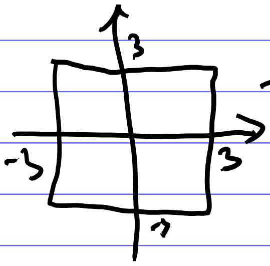
$$\frac{\partial F}{\partial \alpha} = \frac{\partial}{\partial \alpha}\int_\alpha^\beta \sqrt{t^3+1}\, dt$$
$$= -\frac{\partial}{\partial \alpha}\int_\beta^\alpha \sqrt{t^3+1}\, dt$$
$$= -\sqrt{\alpha^3+1}$$

$$\frac{\partial F}{\partial \beta} = \frac{\partial}{\partial \beta}\int_\alpha^\beta \sqrt{t^3+1}\, dt$$
$$= \sqrt{\beta^3+1}$$

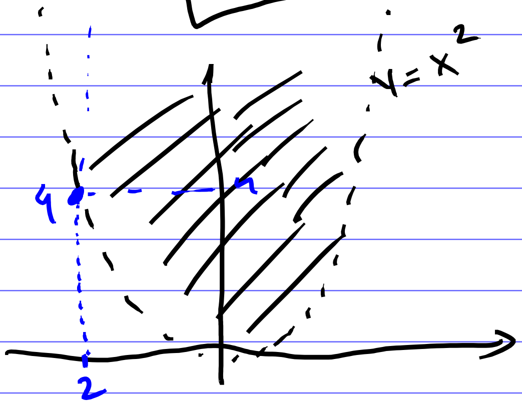


$x^2 + y^2 \leq 3^2$

(x, y)



$\Rightarrow$ $\boxed{\begin{array}{c} -3 \leq x \leq 3 \\ -3 \leq y \leq 3 \end{array}}$



$y > x^2$