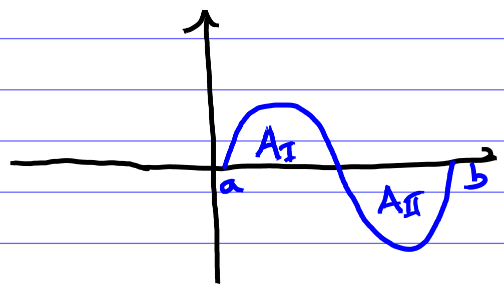
A integral de uma função pode ser entendida como sendo a $\boxed{\text{área líquida}}$ delimitada por essa função.

Anton.

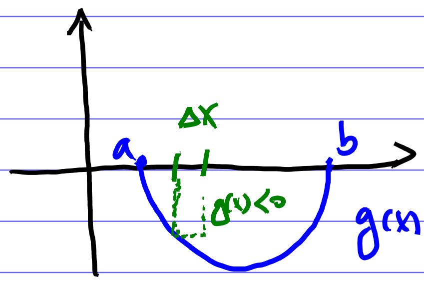
$f(x) \in \mathbb{R}$



$\boxed{\text{Área sempre é positiva em geometria.}}$

$$\int_a^b f(x)\,dx = A_I - A_{II}$$

Pq área abaixo do eixo x é negativa?



$$\int_a^b g(x)\,dx = \lim_{n\to+\infty} \sum_{i=1}^{n} \underbrace{g(x_i)}_{\text{Altura do retângulo}}\ \underbrace{\Delta X}_{\text{base do retângulo}}$$

Nesse caso são negativos — Nesse são positivos

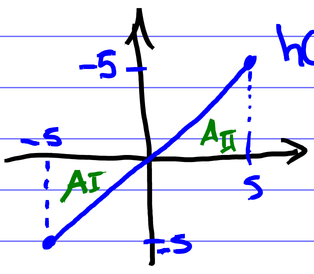
$-$ $\quad$ $+$

$= -$

Exemplo ①

$\Delta X = X_f - X_i$

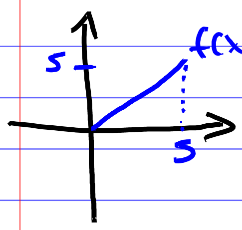
$\Delta ☺ = ☺_f - ☺_i$



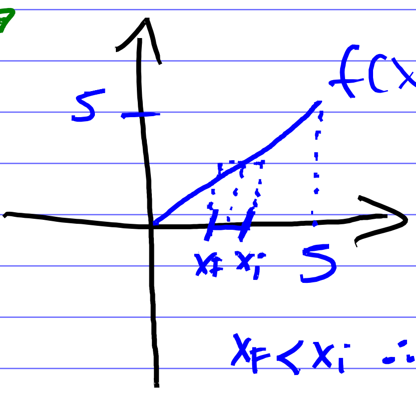
$A_I = \frac{(5)(5)}{2} = \frac{25}{2}$ e $A_{II} = \frac{(5)(5)}{2} = \frac{25}{2}$

$$\int_{-5}^{5} h(x)\,dx = -A_I + A_{II} = -\frac{25}{2} + \frac{25}{2} = 0$$

Exemplo ②



$$\int_5^0 f(x)\,dx = \lim_{n\to+\infty} \sum_{i=1}^{n} f(x_i)\,\Delta X$$



$x_f < x_i \therefore \Delta X < 0$

ΔX é negativo

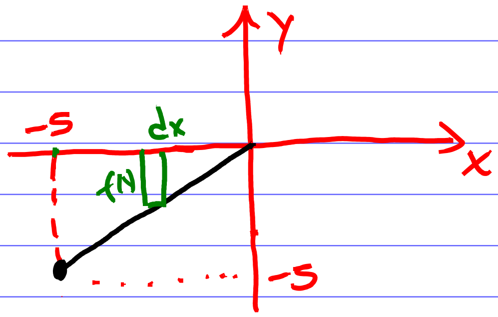
$\left\{ \begin{array}{l} \text{A base é negativa} \\ \text{A altura é positiva} \end{array} \right. \Rightarrow$ sempre teremos área negativa

$$\int_5^0 f(x)\,dx = -\frac{25}{2}$$

Outra propriedade:

$$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$$

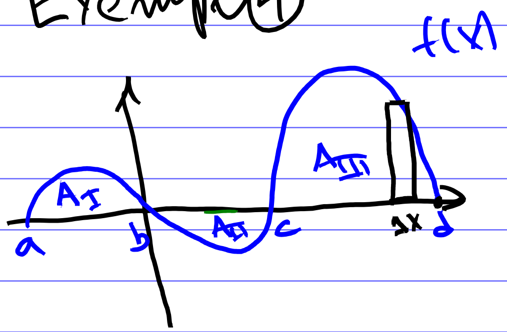
Exemplo ③



A) $\int_0^{-5} \underbrace{f(x)}_{\text{Altura}\ -}\underbrace{dx}_{\text{base}\ -} = \underbrace{\frac{25}{2}}_{\text{Área}\ +}$

B) $\int_{-5}^{0} \underbrace{f(x)}_{\text{Altura}\ -}\underbrace{dx}_{\text{base}\ +} = \underbrace{\frac{-25}{2}}_{\text{Área}\ -}$

Exemplo ④



$\int_a^b f(x)dx = A_I$

$\int_b^c f(x)dx = -A_{II}$

$\int_c^d f(x)dx = A_{III}$

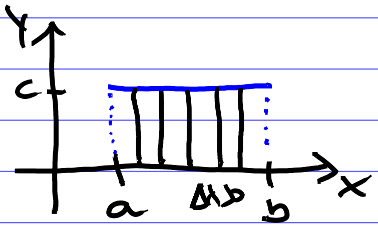
$\int_b^d f(x)dx = -A_{II} + A_{III}$

> 0 Pois $A_{III} > A_{II}$

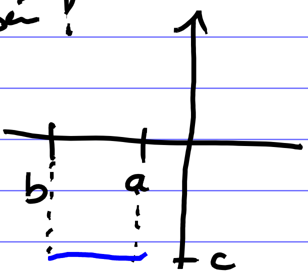
$\int_d^c f(x)dx = -A_{III}$

Outra Propriedade:

$$\int_a^b c\,dx = c(b-a)$$

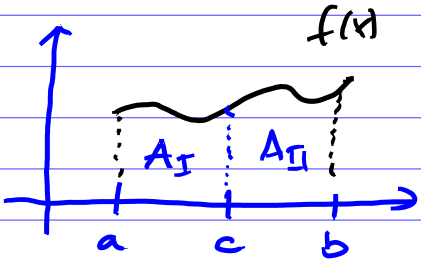


Vale também!

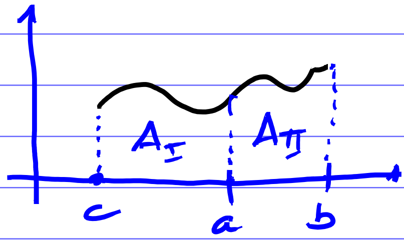


Outra propriedade:

$$\int_{\boxed{a}}^{b} f(x)\,dx = \int_{\boxed{a}}^{c} f(x)\,dx + \int_c^{b} f(x)\,dx$$



Funciona em situações menos intuitivas também!



$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$

$$= -A_I + A_I + A_{II}$$

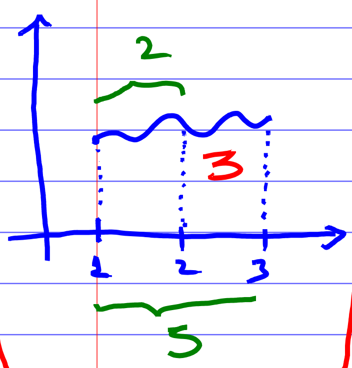
$$= A_{II}$$

Exemplo:

$$\int_1^3 f(x)\,dx = 5 \qquad \int_1^2 f(x)\,dx = 2$$

$$\int_2^3 f(x)\,dx = ?$$

Intuitivamente



Com uso de propriedade

$$\int_2^3 f(x)\,dx = \int_2^1 f(x)\,dx + \int_1^3 f(x)\,dx$$

$$\int_2^3 f(x)\,dx = -2 + 5$$

$$= 3$$

y = variável artificial

$$\int_a^b f(y)\,dy = \int_a^b f(t)\,dt = \int_a^b f(u)\,du$$

$$\int_a^b \boxed{f(u)}\,dx$$

Constante diante de x.

$$= f(u)\int_a^b dx$$

Pois:

$$= \int_a^b c\, f(x)\,dx =$$

$$= c\int_a^b f(x)\,dx$$