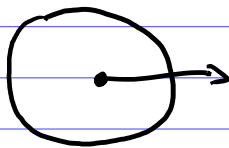
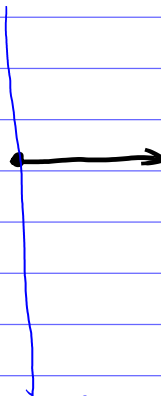


$$\pi = 3$$

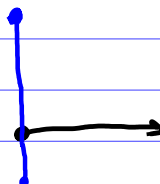


$$\theta = \pi/2$$

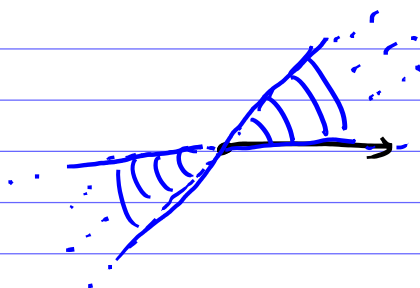


$$\theta = \pi/2 \quad 1 < \pi < 3$$

Segmento de reta.



$$0 \leq \theta \leq \frac{\pi}{4}$$



$$0 \leq \theta \leq \frac{\pi}{4}, \quad 0 < \pi < 2$$

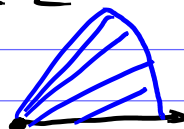


Gráfico de f(x)

$$f(x) = x^2$$

x	f(x)
101	
102	
103	

Não gasta
em calcular f(x)

Fornece melhor ideia
Sobre o gráfico.

x	f(x)
-1	
0	
1	

Conversões

$$r = 2 \cos \theta$$

$$r^2 = 2r \cos \theta$$

$$x^2 + y^2 = 2x \quad \text{Vamos ajustar para a forma}$$

$$(x - x_0)^2 + (y - y_0)^2 = r^2$$

Eq. de uma circunferência
Centro em (x_0, y_0)

$$x^2 + y^2 = 2x$$

$$x^2 - 2x + y^2 = 0$$

$$x^2 - 2x + 1 + y^2 = 1$$

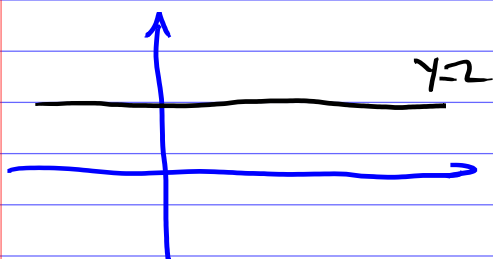
$$(x - 1)^2 + y^2 = 1$$

Circunferência de raio 1, centro em $(1, 0)$.

Exercício

1. Converta a equação a seguir
p/ coord. polares.

$$y = 2$$

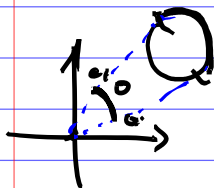


$$r \sin \theta = 2$$

$$r = \frac{2}{\sin \theta}$$

$$0 < \theta < \pi$$

Investiguen!



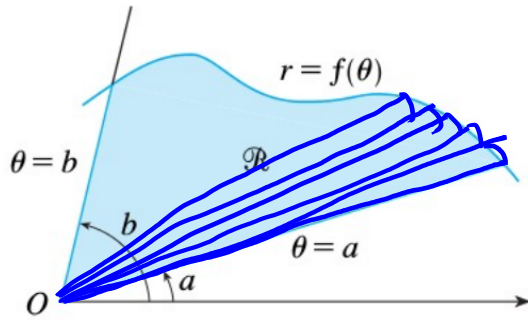
Qual é a eq. de uma circunferência
distante de origem.

$$(x + 10)^2 + (y + 5)^2 = 2^2$$

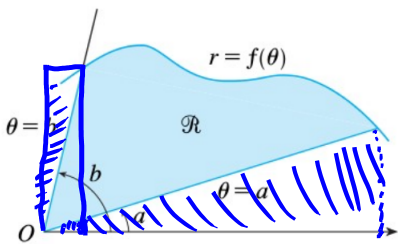
$$(r \cos \theta + 10)^2 + (r \sin \theta + 5)^2 = 2^2$$

Extraia $r(\theta)$ daí!

Área em coordenadas polares



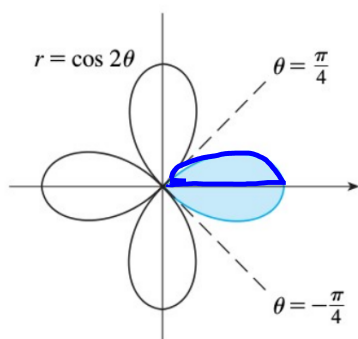
Em coord. cartesianas



Em coord. polares

$$A = \int_a^b \frac{1}{2} f(\theta)^2 d\theta$$

1. Calcule a área delimitada por um laço da rosácea de quatro pétalas $r = \cos(2\theta)$



$$A = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1}{2} r^2 d\theta = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1}{2} \cos^2(2\theta) d\theta$$

$$= \frac{1}{2} \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \left(\frac{1}{2} + \frac{\cos(4\theta)}{2} \right) d\theta$$

$$\boxed{\cos^2(u) = \frac{1}{2} + \frac{\cos(2u)}{2}}$$

$$= \frac{1}{2} \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1}{2} d\theta + \frac{1}{4} \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \cos(4\theta) d\theta$$

$$= \frac{1}{4} \left[\frac{\pi}{4} - \left(-\frac{\pi}{4} \right) \right] + \frac{1}{4} \frac{\sin(4\theta)}{4} \bigg|_{-\frac{\pi}{4}}^{\frac{\pi}{4}}$$

$$= \frac{\pi}{8} + \frac{1}{16} (\sin(\pi) - \sin(-\pi))$$

$$= \frac{\pi}{8} \text{ u.a.}$$

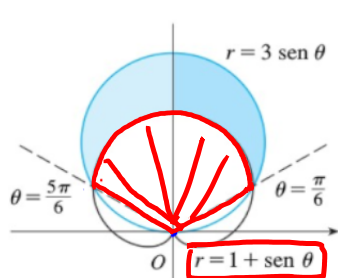
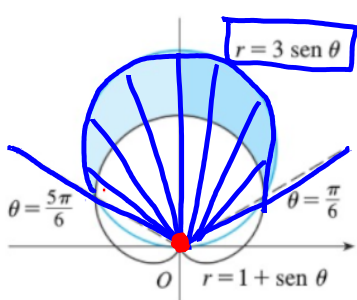
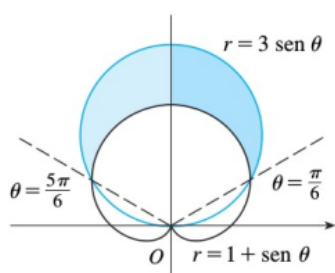
Também poderia ter feito do seguinte modo:

$$A = 2 \int_0^{\frac{\pi}{4}} \frac{1}{2} r^2 d\theta$$

Por simetria.

2. Calcule a área da região que está dentro do círculo $r=3\sin(\theta)$ e fora da cardióide

$$r=1+\sin(\theta)$$



$$A = \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \frac{1}{2} (3\sin\theta)^2 d\theta - \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \frac{1}{2} (1+\sin\theta)^2 d\theta$$

$$A = \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \left[\frac{1}{2} \cdot 9 \sin^2\theta - \frac{1}{2} (1+\sin\theta)^2 \right] d\theta$$

$$\int_a^b A dx - \int_a^b B dx = \int_a^b (A-B) dx$$

$$A = \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \left[\frac{1}{2} \cdot 9 \sin^2\theta - \frac{1}{2} (1+2\sin\theta+\sin^2\theta) \right] d\theta$$

$$A = \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \left[\frac{9}{2} \sin^2\theta - \frac{1}{2} - \sin\theta - \frac{1}{2} \sin^2\theta \right] d\theta$$

$$A = \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \left[4 \sin^2\theta - \frac{1}{2} - \sin\theta \right] d\theta$$

$$A = \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \left[4 \left\{ \frac{1}{2} - \frac{\cos(2\theta)}{2} \right\} - \frac{1}{2} - \sin\theta \right] d\theta$$

$$A = \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \left[\frac{3}{2} - 2\cos(2\theta) - \sin\theta \right] d\theta$$

$$A = \left[\frac{3}{2}\theta - \sin(2\theta) + \cos\theta \right] \Big|_{\frac{\pi}{6}}^{\frac{5\pi}{6}}$$

$$A = \left[\frac{15\pi}{12} - \sin\left(\frac{5\pi}{3}\right) + \cos\left(\frac{5\pi}{6}\right) \right] - \left[\frac{3\pi}{12} - \sin\left(\frac{\pi}{3}\right) + \cos\left(\frac{\pi}{6}\right) \right]$$

$$A = \left[\frac{15\pi}{12} + \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} \right] - \left[\frac{3\pi}{12} - \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} \right]$$

$$A = \frac{15\pi}{12} - \frac{3\pi}{12} = \frac{12\pi}{12} = \pi$$

$$\boxed{A = \pi \text{ m.a.}}$$

WolframAlpha computational intelligence.

int(1/2 (3*sin(theta))^2 - 1/2 (1+sin(theta))^2, theta=pi/6 .. 5*Pi/6)

NATURAL LANGUAGE

MATH INPUT

EXTENDED KEYBOARD

EXAMPLES

UPLOAD

RANDOM

Definite integral

More digits

Step-by-step solution

$$\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \left(\frac{1}{2} (3 \sin(\theta))^2 - \frac{1}{2} (1 + \sin(\theta))^2 \right) d\theta = \pi \approx 3.1416$$

Visual representation of the integral

