17–18 Encontre uma equação polar para a curva representada pela equação cartesiana dada.

17. $x + y = 2$

18. $x^2 + y^2 = 2$

$X + Y = 2$

$X = r\cos\theta$

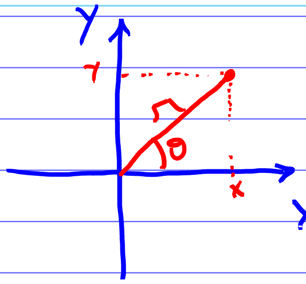
$Y = r\,\text{sen}\,\theta$

$x + Y = 2$

$r\cos\theta + r\,\text{sen}\,\theta = 2$

$r(\cos\theta + \text{sen}\,\theta) = 2$

$$\boxed{r = \frac{2}{\cos\theta + \text{sen}\,\theta}}$$



$r^2 = x^2 + y^2$

então

$x^2 + y^2 = 2$

$r^2 = 2$

$r = -\sqrt{2}$ ou $r = \sqrt{2}$

Em coord. polar essas 2 eq. são equivalent

Há outro modo ?

$X = r\cos\theta$

$Y = r\,\text{sen}\,\theta$

$x^2 + y^2 = 2$

$r^2\cos^2\theta + r^2\text{sen}^2\theta = 2$

$r^2(\cos^2\theta + \text{sen}^2\theta) = 2$

$r^2 = 2$

$r = \sqrt{2}$ ou $r = -\sqrt{2}$

1–4 Encontre a área da região que é delimitada pelas curvas dadas e está no setor especificado.

1. $r = \theta^2, \quad 0 \leqslant \theta \leqslant \pi/4$

$$A = \int_{\theta_i}^{\theta_f} \frac{1}{2} r^2 \, d\theta = \int_0^{\frac{\pi}{4}} \frac{1}{2} \theta^4 \, d\theta$$

$$= \frac{1}{2} \int_0^{\frac{\pi}{4}} \theta^4 \, d\theta = \frac{1}{2} \frac{\theta^5}{5} \Big|_0^{\frac{\pi}{4}}$$

$$= \frac{1}{10} \cdot \left[\left(\frac{\pi}{4}\right)^5 - 0^5 \right]$$

$$= \frac{\pi^5}{10 \cdot 4^5} \text{ u.a.}$$

45–48 Calcule o comprimento exato da curva polar.

45. $r = 2\cos\theta, \quad 0 \leqslant \theta \leqslant \pi$ **46.** $r = 5^\theta, \quad 0 \leqslant \theta \leqslant 2\pi$

45

$$L = \int_{\theta_i}^{\theta_f} \sqrt{r^2 + r'^2} \, d\theta$$

$r = 2\cos\theta$
$r^2 = 4\cos^2\theta$

$\begin{cases} r' = -2\operatorname{sen}\theta \\ (r')^2 = 4\operatorname{sen}^2\theta \end{cases}$

$$L = \int_0^{\pi} \sqrt{4\cos^2\theta + 4\operatorname{sen}^2\theta} \, d\theta$$

$$L = \int_0^{\pi} \sqrt{4} \, d\theta = \int_0^{\pi} 2 \, d\theta$$

$$L = \int_0^{\pi} 2 \, d\theta \xrightarrow{\text{Modo 1}} L = 2(\pi - 0)$$

Uma propriedade da Integral Definida

$L = 2\pi$ u.c.

Modo 2 → TFC

$$L = \int_0^{\pi} 2 \, d\theta = 2\theta \Big|_0^{\pi}$$

$$= (2\pi - 0)$$

$$= 2\pi \text{ u.c.}$$

46

$r = 5^\theta \quad 0 \leq \theta \leq 2\pi$

$$L = \int_0^{2\pi} \sqrt{r^2 + r'^2} \, d\theta$$

$r = 5^\theta$
$r^2 = 5^{2\theta}$

$\begin{cases} r' = 5^\theta \cdot \ln 5 \\ r'^2 = 5^{2\theta} (\ln 5)^2 \end{cases}$

$$L = \int_0^{2\pi} \sqrt{5^{2\theta} + 5^{2\theta} (\ln 5)^2} \, d\theta$$

$$L = \int_0^{2\pi} \sqrt{5^{2\theta} \left(1 + (\ln 5)^2\right)} \, d\theta$$

$$L = \int_0^{2\pi} 5^\theta \sqrt{1 + (\ln 5)^2} \, d\theta$$

$$L = \sqrt{1 + (\ln 5)^2} \int_0^{2\pi} 5^\theta \, d\theta$$

$$L = \sqrt{1 + (\ln 5)^2} \; \frac{5^\theta}{\ln 5} \Big|_0^{2\pi}$$

$$L = \sqrt{1 + (\ln 5)^2} \left[\frac{5^{2\pi}}{\ln 5} - \frac{5^0}{\ln 5} \right]$$

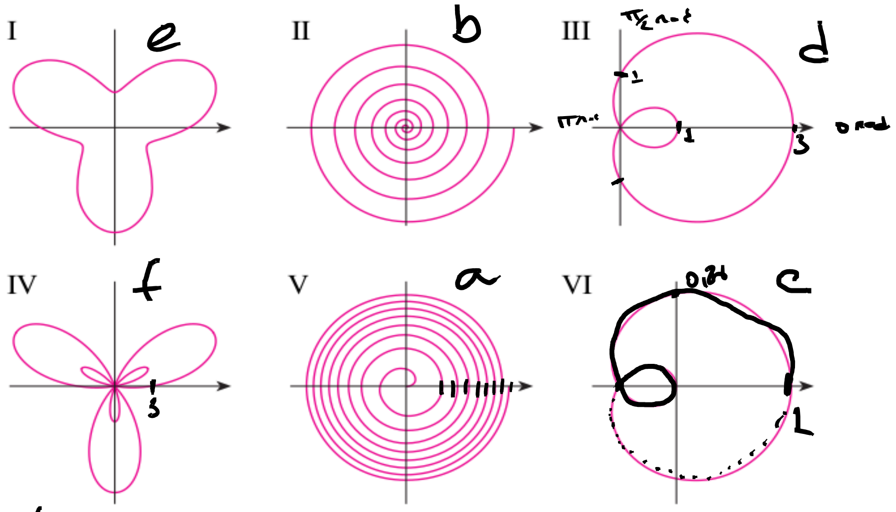
$$\boxed{L = \frac{\sqrt{1 + (\ln 5)^2}}{\ln 5} \left[5^{2\pi} - 1 \right] \text{ u.c.}}$$

54. Associe as curvas polares com seus respectivos gráficos I–VI. Dê razões para suas escolhas. (Não use uma ferramenta gráfica.)

(a) $r = \sqrt{\theta}$, $0 \leqslant \theta \leqslant 16\pi$ (b) $r = \theta^2$, $0 \leqslant \theta \leqslant 16\pi$

(c) $r = \cos(\theta/3)$ (d) $r = 1 + 2\cos\theta$

(e) $r = 2 + \operatorname{sen} 3\theta$ (f) $r = 1 + 2\operatorname{sen} 3\theta$



θ	$r = 1 + 2\cos\theta$
0	3
$\frac{\pi}{2}$	1
π	-1
$\frac{3\pi}{2}$	1
2π	3

$\frac{\theta}{3} = \frac{\pi}{2}$

$\theta = \frac{3\pi}{2}$

θ	$r = \cos\left(\frac{\theta}{3}\right)$
0	1
$\frac{\pi}{2}$	$\frac{\sqrt{3}}{2}$
π	$\frac{1}{2}$
$\frac{3\pi}{2}$	0
2π	

21–26 Encontre uma equação polar para a curva representada pela equação cartesiana dada.

21. $y = 2$ **22.** $y = x$

23. $y = 1 + 3x$ **24.** $4y^2 = x$

25. $x^2 + y^2 = 2cx$ **26.** $xy = 4$

21 $r \operatorname{sen}\theta = 2 \quad \therefore \; r = \dfrac{2}{\operatorname{sen}\theta}$

22 $y = x \Rightarrow r \operatorname{sen}\theta = r\cos\theta$

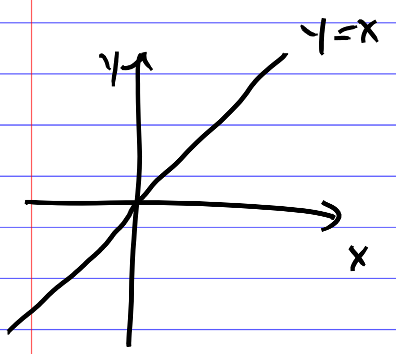
$r\operatorname{sen}\theta - r\cos\theta = 0$

$r(\operatorname{sen}\theta - \cos\theta) = 0$

$r = 0 \;\Big\{\; \operatorname{sen}\theta - \cos\theta = 0$

$\operatorname{sen}\theta = \cos\theta$

$\boxed{\theta = \frac{\pi}{4}}$



inclue todos os pontos, inclusive $r=0$.

23 $y = 1 + 3x$

$r\operatorname{sen}\theta = 1 + 3r\cos\theta$

$r\operatorname{sen}\theta - 3r\cos\theta = 1$

$r(\operatorname{sen}\theta - 3\cos\theta) = 1$

$\boxed{r = \dfrac{1}{\operatorname{sen}\theta - 3\cos\theta}}$

24 $4y^2 = x$

$4(r\operatorname{sen}\theta)^2 = r\cos\theta$

$4r^2\operatorname{sen}^2\theta = r\cos\theta$

$4r^2\operatorname{sen}^2\theta - r\cos\theta = 0$

$r(4r\operatorname{sen}^2\theta - \cos\theta) = 0$

$r = 0 \;\Big\{\; 4r\operatorname{sen}^2\theta - \cos\theta = 0$

$4r\operatorname{sen}^2\theta = \cos\theta$

$\boxed{r = \dfrac{1}{4}\dfrac{\cos\theta}{\operatorname{sen}^2\theta}}$

25 $x^2 + y^2 = 2cx$

$r^2 = 2cr\cos\theta$

$r^2 - 2cr\cos\theta = 0$

$r(r - 2c\cos\theta) = 0$

$r = 0 \;\Big\{\; \boxed{r = 2c\cos\theta}$

26 $xy = 4$

$r\cos\theta \, r\operatorname{sen}\theta = 4$

$r^2\cos\theta\operatorname{sen}\theta = 4$

$r^2 = \dfrac{4}{\cos\theta\operatorname{sen}\theta} \;\Big\{\; r = \sqrt{\dfrac{4}{\cos\theta\operatorname{sen}\theta}}$

ou

$r = -\sqrt{\dfrac{4}{\cos\theta\operatorname{sen}\theta}}$

Formaram o mesmo gráfico.