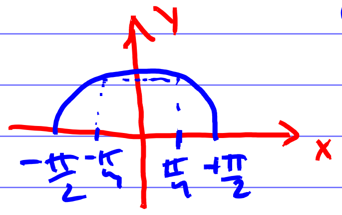
Simetria

$$\int_{-a}^{a} f(x)\,dx = \begin{cases} 0, & \text{se } f(x) = -f(-x) \ \text{ímpar} \\ 2\int_{0}^{a} f(x)\,dx, & \text{se } f(x) = f(-x) \ \text{par} \end{cases}$$

① $\int_{-\pi/2}^{\pi/2} \cos(x)\,dx$

Cos(x) é par ou ímpar?

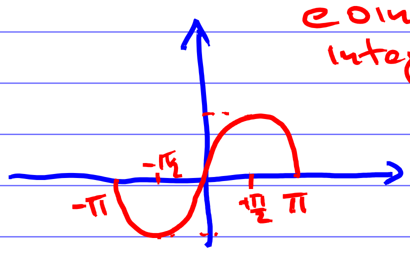


Cos(x) é par.

$$\int_{-\pi/2}^{\pi/2} \cos(x)\,dx = 2\int_{0}^{\pi/2} \cos(x)\,dx$$

② $\int_{-\pi}^{\pi} \operatorname{sen} x\,dx = 0$

Pois sen x é ímpar e o intervalo de integração é simétrico.



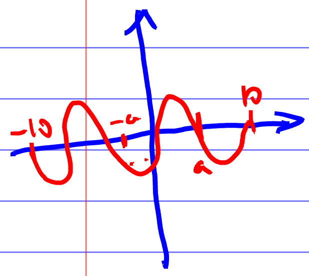
③ $\int_{-3}^{3} x^3\,dx = 0$

Pois x^3 é ímpar e o intervalo de integração é simétrico.

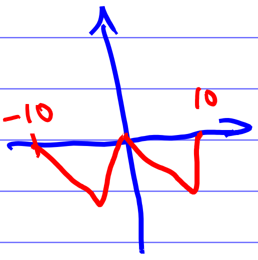
④ $\int_{-2}^{2} x^2\,dx = 2\int_{0}^{2} x^2\,dx$

x^2 é par.

A



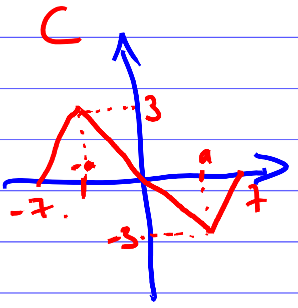
B



Par

ímpar

C



$f(x) = -f(-x)$

$f(a) = -f(-a)$

Técnicas de integração

$$\int_0^{\pi} \operatorname{sen} x \sqrt{\cos x^2}\, dx$$

Equação: $\sqrt{9} = \pm 3$

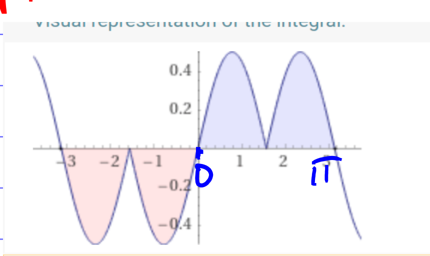
Função: $\sqrt{9} = 3$

$f(x) = \sqrt{x}$

$f(9) = \sqrt{9} = 3$

$\sqrt{x^2} = |x|$

$\sqrt{x^2} \neq x$



$$\int_0^{\pi} \underline{\operatorname{sen} x} \sqrt{\cos x^2}\, \underline{dx}$$

$u = \cos x \quad \therefore \quad \dfrac{du}{dx} = -\operatorname{sen} x$

<u>Limites de integração</u> $\quad \therefore \quad du = -\operatorname{sen} x\, dx$

$(-du) = \operatorname{sen} x\, dx$

Quando $x = 0$, $u = \cos 0 = 1$

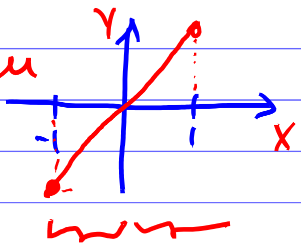
" $x = \pi$, $u = \cos \pi = -1$

$$\int_0^{\pi} \operatorname{sen} x \sqrt{\cos x^2}\, dx = \int_1^{-1} \sqrt{u^2}\,(-du)$$

$$= \int_{-1}^{1} \sqrt{u^2}\, du = \int_{-1}^{1} |u|\, du$$

Como se calcula a integral de uma função modular?

$f(u) = u$



$|u| = \begin{cases} -u, & u < 0 \\ u, & u \geq 0 \end{cases}$

$$\int_{-1}^{1} |u|\, du = \int_{-1}^{0} (-u)\, du + \int_0^{1} (u)\, du$$

$$= \int_0^{-1} u\, du + \int_0^{1} u\, du$$

$$= \frac{u^2}{2}\Big|_0^{-1} + \frac{u^2}{2}\Big|_0^{1}$$

$$= \left[\frac{(-1)^2}{2} - \frac{0}{2}\right] + \left[\frac{1^2}{2} - \frac{0}{2}\right]$$

$$= \frac{1}{2} + \frac{1}{2} = 1$$

Cássio observou que $|u|$ é par e o intervalo é simétrico:

$$\int_{-1}^{1} |u|\, du = 2\int_0^{1} |u|\, du = 2\int_0^{1} u\, du$$

$$= 2\left(\frac{u^2}{2}\right)\Big|_0^{1}$$

$$= 2\left(\frac{1}{2} - 0\right)$$

$$= 1$$

Integração por partes

$$\int_0^{\pi} x \cos x \, dx$$

$$\int_a^b u \, dv = uv \Big|_a^b - \int_a^b v \, du$$

$\boxed{u = x}$ ∴ $\dfrac{du}{dx} = 1$ ∴ $\boxed{du = dx}$

$\boxed{dv = \cos x \, dx}$ ∴ $v = \int dv = \int \cos x \, dx$

$\boxed{v = \operatorname{sen} x}$

$$\int_0^{\pi} x \cos x \, dx = x \operatorname{sen} x \Big|_0^{\pi} - \int_0^{\pi} \operatorname{sen} x \, dx$$

$$= [\pi \operatorname{sen} \pi - 0 \operatorname{sen} 0] - [-\cos x] \Big|_0^{\pi}$$

$$= [0] - [(-\cos \pi) - (-\cos 0)]$$

$$= -[1 - (-1)]$$

$$= -[2] = -2$$

WolframAlpha computational intelligence.

int(x*cos(x),x=0..pi)

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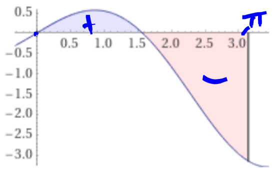
Definite integral:

$$\int_0^{\pi} x \cos(x) \, dx = -2$$

Visual representation of the integral:



Uso de relações trigonométricas

NÃO continue c/ Subst. trigonométrica.

$$\int_0^{\pi/2} \cos^2 x \, dx =$$

Eu sei que

$$\begin{cases} \cos^2 x = \frac{1}{2} + \frac{\cos(2x)}{2} \\ \operatorname{sen}^2 x = \frac{1}{2} - \frac{\cos(2x)}{2} \end{cases}$$

Decorem! Vão usar muito.

$$\int_0^{\pi/2} \cos^2 x \, dx = \int_0^{\pi/2} \left(\frac{1}{2} + \frac{\cos(2x)}{2} \right) dx$$

$$= \int_0^{\pi/2} \frac{1}{2} dx + \int_0^{\pi/2} \frac{\cos(2x)}{2} dx$$

Pode ser feito por Substituição simples.

$$= \frac{1}{2}\left(\frac{\pi}{2} - 0\right) + \left[\frac{\operatorname{sen}(2x)}{4} \right]_0^{\pi/2}$$

Eu fiz me perguntando: "Quem eu derivo para encontrar $\frac{\cos(2x)}{2}$?"

$$= \frac{\pi}{4} + \left[\frac{\operatorname{sen}(\pi)}{4} - \underbrace{\frac{\operatorname{sen} 0}{4}}_{0} \right]$$

$$= \frac{\pi}{4}$$

Definite integral:

$$\int_0^{\frac{\pi}{2}} \cos^2(x)\,dx = \frac{\pi}{4} \approx 0.78540$$

Visual representation of the integral: