Notação Sigma

$$\sum_{i=1}^{5} i = 1+2+3+4+5 = 15$$

$$\sum_{i=3}^{5} i^2 = 3^2 + 4^2 + 5^2 = 9+16+25$$
$$= 50$$

$$\sum_{i=3}^{5} 3 = 3+3+3 = 9$$

$$\sum_{i=a}^{b} K = K(b-a+1)$$

$\underline{3}$ $\underline{4}$ $\underline{5}$

São três elementos, logo $(5-3)+1$

$$\sum_{i=1}^{3} (i+i^2) = (1+1^2) + (2+2^2) + (3+3^2)$$
$$= 2 + 6 + 12$$
$$= 20$$

$$\sum_{i=1}^{3} (i+i^2) = \sum_{i=1}^{3} i + \sum_{i=1}^{3} i^2$$

Na computação:
1 laço
1-for

Na computação:
2 laços
2-for's.

$$\sum_{i=1}^{3} (i-i^2) = \sum_{i=1}^{3} i - \sum_{i=1}^{3} i^2$$

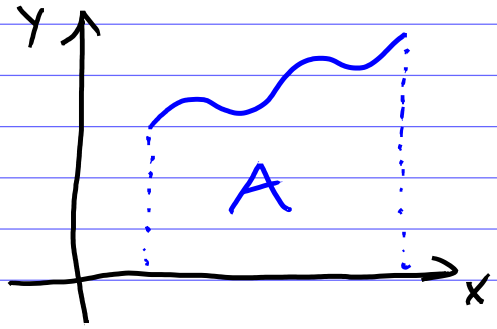
$$\sum_{i=1}^{3} K a_i = K \sum_{i=1}^{3} a_i$$

$$\sum_{i=1}^{3} K = K \sum_{i=1}^{3} 1$$

Área sob a curva

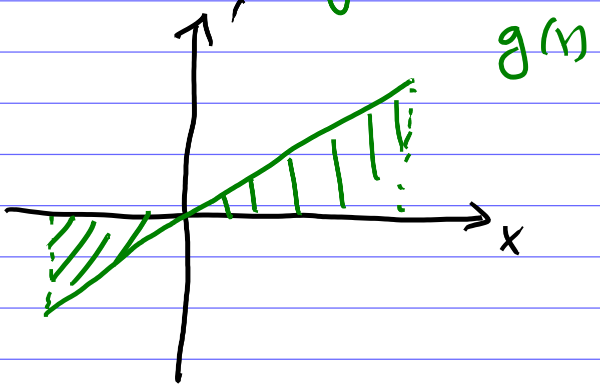
Sob = embaixo

$f(x)$



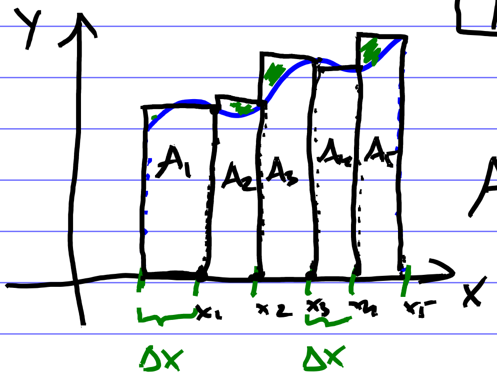
É preciso $f(x) \geq 0$.

Área entre a curva $g(x)$ e o eixo x.



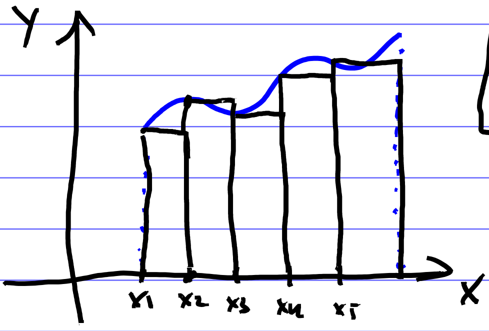
Sob = embaixo

PONTO MAIS À DIREITA



$$A \approx \sum_{i=1}^{5} A_i$$

$$A \approx \sum_{i=1}^{5} f(x_i)\,\Delta x$$

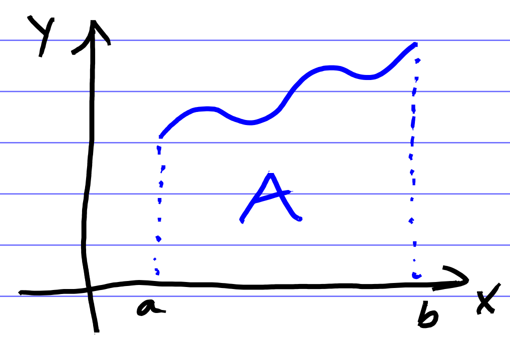


PONTO MAIS À ESQUERDA

$$A \approx \sum_{i=1}^{5} A_i$$

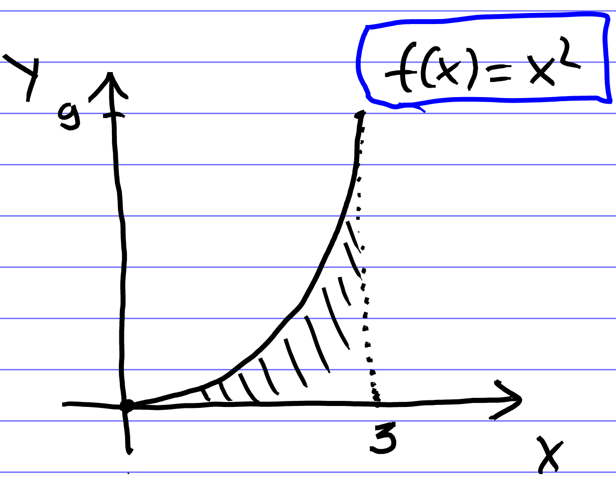
$$A \approx \sum_{i=1}^{5} f(x_i)\,\Delta x$$

E o valor preciso?



$$A = \lim_{n\to+\infty} \sum_{i=1}^{n} f(x_i)\Delta x \qquad \Delta x \to 0$$

$$= \boxed{\int_a^b f(x)\,dx}$$



$$A = \lim_{n\to+\infty} \sum_{i=1}^{n} f(x_i)\Delta x$$

Dividindo em n partes

0 ... 3

$$\Delta x = \frac{3}{n}$$

Escolhemos o ponto mais à direita

$x_1 \quad x_2 \quad x_3 \quad x_4 \quad \dots \quad x_n$

$0 \quad \frac{3}{n} \quad 2\left(\frac{3}{n}\right) \quad 3\left(\frac{3}{n}\right)$

$$x_i = i\left(\frac{3}{n}\right)$$

$$A = \lim_{n\to+\infty} \sum_{i=1}^{n} f(x_i)\Delta x$$

$$f(x) = x^2$$

$$f(x_i) = (x_i)^2$$

$$f(x_i) = \left(i\frac{3}{n}\right)^2$$

$$f(x_i) = \frac{i^2 9}{n^2}$$

$$A = \lim_{n\to+\infty} \sum_{i=1}^{n} \frac{9}{n^2} i^2 \cdot \frac{3}{n}$$

$$A = \lim_{n\to+\infty} \sum_{i=1}^{n} \frac{27 i^2}{n^3} = \lim_{n\to+\infty} \frac{27}{n^3} \sum_{i=1}^{n} i^2$$

$$A = 27 \lim_{n\to+\infty} \frac{1}{n^3} \sum_{i=1}^{n} i^2$$

$$A = 27 \lim_{n\to+\infty} \frac{1}{n^3} \frac{n(n+1)(2n+1)}{6}$$

$$A = 27 \lim_{n\to+\infty} \frac{1}{n^3} \frac{(n^2+n)(2n+1)}{6}$$

Alguns somatórios de funções polinomiais

1. $\sum_{i=m}^{n} 1 = n + 1 - m$
2. $\sum_{i=1}^{n} i = 1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$ (Soma de uma progressão aritmética)
3. $\boxed{\sum_{i=1}^{n} i^2 = 1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}}$ (Número piramidal quadrado)
4. $\sum_{i=1}^{n} i^3 = 1^3 + 2^3 + 3^3 + \dots + n^3 = \left(\frac{n(n+1)}{2}\right)^2$ [3]
5. $\sum_{i=1}^{n} i^4 = 1^4 + 2^4 + 3^4 + \dots + n^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$ [3]
6. $\sum_{i=1}^{n} i^5 = 1^5 + 2^5 + 3^5 + \dots + n^5 = \frac{n^2(n+1)^2(2n^2+2n-1)}{12}$ [3]
7. $\sum_{i=1}^{n} i^6 = 1^6 + 2^6 + 3^6 + \dots + n^6 = \frac{n(n+1)(2n+1)(3n^4+6n^3-3n+1)}{42}$ [3]
8. $\sum_{i=0}^{n} i^p = \frac{(n+1)^{p+1}}{p+1} + \sum_{k=1}^{p} \frac{B_k}{p-k+1}\binom{p}{k}(n+1)^{p-k+1}$

$$A = 27 \lim_{n\to+\infty} \frac{1}{n^3} \frac{(n^2+n)(2n+1)}{6}$$

$$A = 27 \lim_{n\to+\infty} \frac{1}{n^3} \frac{2n^3 + n^2 + 2n^2 + n}{6}$$

$$A = \frac{27}{6} \lim_{n\to+\infty} \left(2 + \frac{3}{n} + \frac{1}{n^2}\right)$$

$$A = \frac{27}{6}(2)$$

$$\boxed{A = \frac{27}{3} = 9 \text{ u.a.}}$$