$$\sum_{i=1}^{3} i^2 = 1^2 + 2^2 + 3^2 = 14$$

$$\sum_{i=2}^{4} i^2 = 2^2 + 3^2 + 4^2$$

$$\sum_{i=1}^{3} (i + i^2) = (1+1^2) + (2+2^2) + (3+3^2)$$

$$\sum_{i=1}^{5} 2 = 2 + 2 + 2 + 2 + 2 = 10$$

$$\sum_{i=a}^{b} K = K(b-a+1)$$

$$\sum_{i=1}^{3} (i + i^2) = \sum_{i=1}^{3} i + \sum_{i=1}^{3} i^2$$

$$\sum_{i=1}^{3} (i - i^2) = \sum_{i=1}^{3} i - \sum_{i=1}^{3} i^2$$

$$\sum_{i=1}^{2} (i\, i^2) \neq \sum_{i=1}^{2} i \cdot \sum_{i=1}^{2} i^2$$

$$1^3 + 2^3 \neq (1+2)(1^2 + 2^2)$$

$$9 \neq (3)(1+4)$$

$$9 \neq 15$$

Tomar cuidado. Tente entender a notação empregada pelo autor do livro.

$$\sum_{i=1}^{2} i \cdot \sum_{i=1}^{2} i^2 = \left(\sum_{i=1}^{2} i\right)\left(\sum_{i=1}^{2} i^2\right)$$

$$\sum_{i=1}^{2} i \cdot \sum_{i=1}^{2} i^2 = \sum_{i=1}^{2} \left(i \cdot \sum_{i=1}^{2} i^2\right)$$

$$\boxed{\sum_{j=1}^{3} \sum_{i=1}^{2} (j^3 i^2) = \sum_{j=1}^{3} \left(j^3 \sum_{i=1}^{2} i^2\right)}$$

$$\sum_{i=1}^{3} 3i = 3\sum_{i=1}^{3} i$$

$$\sum_{i=1}^{3} 3 = 3\sum_{i=1}^{3} 1$$

$$\sum_{i=1}^{5} i = A$$

$$\sum_{i=1}^{5} i^2 = B$$

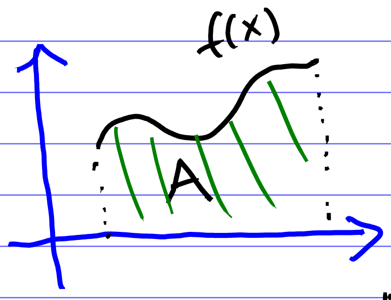
$$\sum_{i=1}^{5} (i+i^2) = \sum_{i=1}^{5} i + \sum_{i=1}^{5} i^2$$

$$= A + B$$

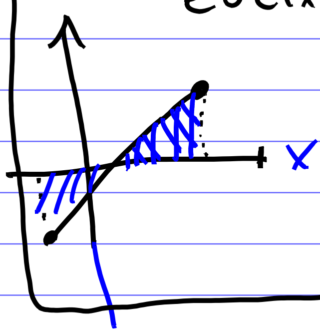
Área sob a curva

$f(x) \geq 0$

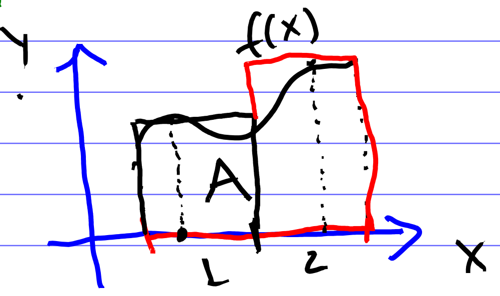
Sob = embaixo



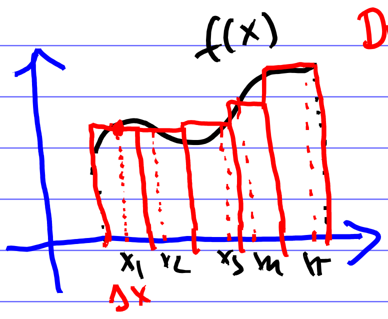
Área entre a curva e o eixo X



Para calcular A iniciamos c/ estimativas.

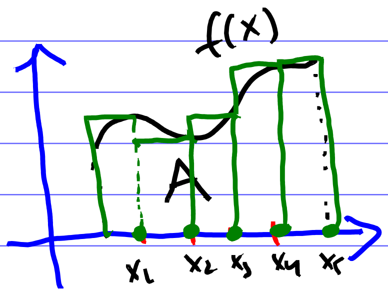


$A \approx \sum_{i=1}^{5} A_i$



Dividi em 5 partes e usei pontos esquerdos para determinar a altura dos retângulos.

$A \approx \sum_{i=1}^{5} A_i$



Dividi em 5 partes e usei os pontos mais a direita p/ determinar a altura dos retângulos.

Definimos a A como sendo

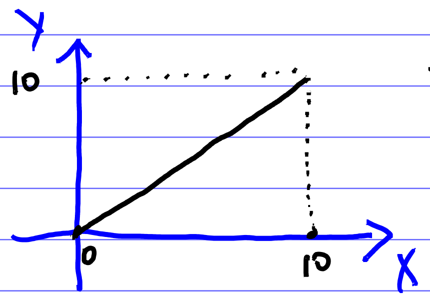
$$A = \lim_{n \to +\infty} \sum_{i=1}^{n} A_i$$

$$\boxed{A = \lim_{n \to +\infty} \sum_{i=1}^{n} f(x_i)\,\Delta x}$$

$$= \int_a^b f(x)\,dx$$

Essa expressão é apenas a representação da anterior.

Qual é a área sob a curva ?

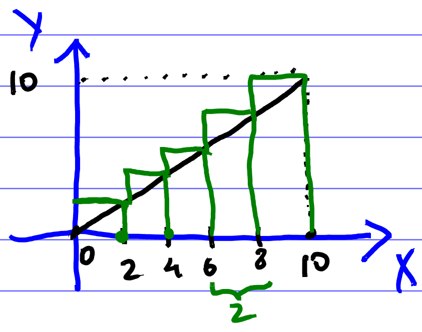


$f(x) = x$

Já sabemos que é

$A = \frac{10(10)}{2} = 50$

Estimativa



$\Delta X = 2$

Usaremos o ponto mais à direita

$$A \approx \sum_{i=1}^{5} A_i$$

$$A \approx \sum_{i=1}^{5} f(x_i)\,\Delta X$$

$$A \approx f(x_1)\Delta X + f(x_2)\Delta X + f(x_3)\Delta X + f(x_4)\Delta X + f(x_5)\Delta X$$

$$A \approx 2\Delta X + 4\Delta X + 6\Delta X + 8\Delta X + 10\Delta X$$

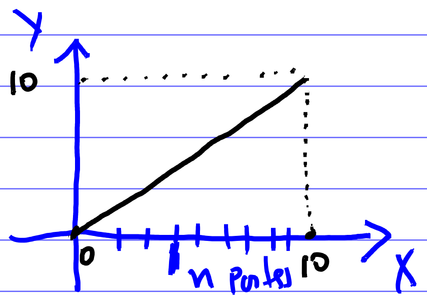
$$A \approx \Delta X(2+4+6+8+10)$$

$$A \approx 2(2+4+6+8+10)$$

$$A \approx 2(30)$$

$$A \approx 60$$

E o valor preciso?



$\Delta X = \frac{10}{n}$

Usaremos o ponto mais à direita

$X_1 = \frac{10}{n}$
$X_2 = 2\left(\frac{10}{n}\right)$
$X_3 = 3\left(\frac{10}{n}\right)$
$\vdots$
$X_i = i\left(\frac{10}{n}\right)$

$$A = \lim_{n \to +\infty} \sum_{i=1}^{n} f(x_i)\,\Delta X$$

$$= \lim_{n \to +\infty} \sum_{i=1}^{n} f\left(i\frac{10}{n}\right)\frac{10}{n}$$

Lembre-se $f(x) = x$

$$= \lim_{n \to +\infty} \sum_{i=1}^{n} i\,\frac{10}{n}\,\frac{10}{n}$$

$$= \lim_{n \to +\infty} \sum_{i=1}^{n} i\,\frac{100}{n^2}$$

$$= \lim_{n \to +\infty} \frac{100}{n^2} \sum_{i=1}^{n} i$$

$$= \lim_{n \to +\infty} \frac{100}{n^2}\,\frac{n(n+1)}{2}$$

Alguns somatórios de funções polinomiais

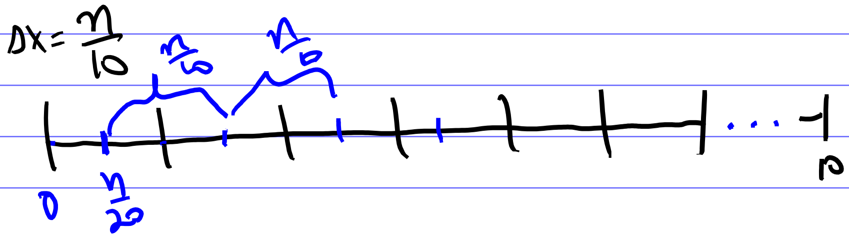
1. $\sum_{i=m}^{n} 1 = n + 1 - m$
2. $\boxed{\sum_{i=1}^{n} i = 1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}}$ (Soma de uma progressão aritmética)
3. $\sum_{i=1}^{n} i^2 = 1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$ (Número piramidal quadrado)
4. $\sum_{i=1}^{n} i^3 = 1^3 + 2^3 + 3^3 + \cdots + n^3 = \left(\frac{n(n+1)}{2}\right)^2$ [3]
5. $\sum_{i=1}^{n} i^4 = 1^4 + 2^4 + 3^4 + \cdots + n^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$ [3]
6. $\sum_{i=1}^{n} i^5 = 1^5 + 2^5 + 3^5 + \cdots + n^5 = \frac{n^2(n+1)^2(2n^2+2n-1)}{12}$ [3]
7. $\sum_{i=1}^{n} i^6 = 1^6 + 2^6 + 3^6 + \cdots + n^6 = \frac{n(n+1)(2n+1)(3n^4+6n^3-3n+1)}{42}$ [3]
8. $\sum_{i=0}^{n} i^p = \frac{(n+1)^{p+1}}{p+1} + \sum_{k=1}^{p} \frac{B_k}{p-k+1}\binom{p}{k}(n+1)^{p-k+1}$

$$= \lim_{n \to +\infty} \frac{100}{n^2}\,\frac{n(n+1)}{2}$$

$$= \lim_{n \to +\infty} \frac{100}{2}\,\frac{n^2+n}{n^2}$$

$$= 50 \lim_{n \to +\infty}\left(1 + \frac{1}{n}\right)$$

$$= 50 \cdot 1 = 50$$



$$x_1 = \frac{\frac{\pi}{10}}{2} = \frac{\pi}{20} \qquad x_2 = \frac{\pi}{20} + \frac{\pi}{10}$$

$$x_3 = \frac{\pi}{20} + \frac{\pi}{10} + \frac{\pi}{10}$$

$$\boxed{x_i = \frac{\pi}{20} + (i-1)\frac{\pi}{10}}$$