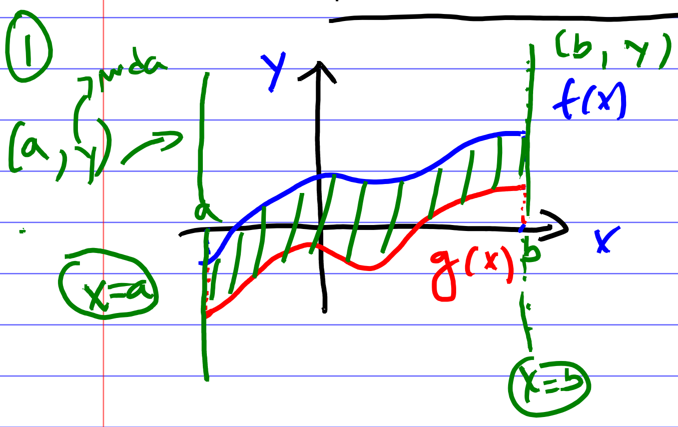
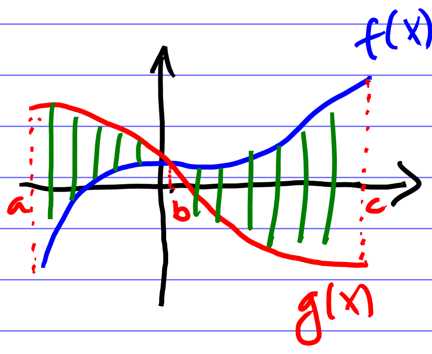
Área entre curvas

(1)



$$\int_a^b (f(x) - g(x))\,dx$$

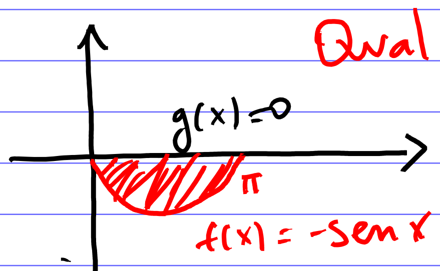
(2)



$$A = \int_a^b (g(x) - f(x))\,dx + \int_b^c (f(x) - g(x))\,dx$$

(3)

Qual é o valor da área hachurada?



$$\int_0^{\pi} (g(x) - f(x))\,dx$$

$$\int_0^{\pi} f(x)\,dx = -2$$

$$= \int_0^{\pi} (0 - f(x))\,dx$$

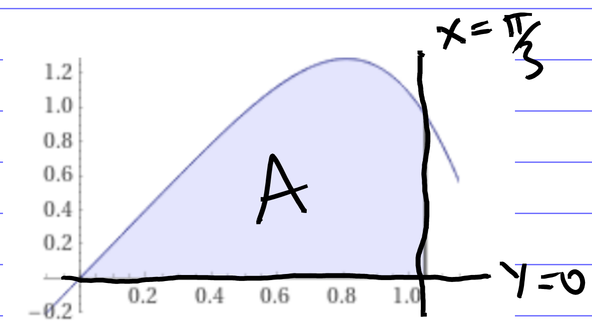
$$= \int_0^{\pi} -(-\operatorname{sen} x)\,dx$$

$$= \int_0^{\pi} \operatorname{sen} x\,dx$$

$$= [-\cos x]\Big|_0^{\pi} = [-\cos \pi] - [-\cos 0]$$

$$= 1 - [-1] = 2$$

(4) $f(x) = 2x\cos(x^2)$



Qual é o valor da área delimitada por $f(x) = 2x\cos(x^2)$, $y=0$ e $x = \frac{\pi}{3}$

Solução

$$A = \int_0^{\pi/3} (2x\cos(x^2))\,dx$$

$$u = x^2$$

$$\frac{du}{dx} = 2x \therefore du = 2x\,dx$$

Mudança nos limites de integração:

Quando $x=0$, $u=0$

" $x = \frac{\pi}{3}$, $u = \left(\frac{\pi}{3}\right)^2 = \frac{\pi^2}{9}$

$$A = \int_0^{\pi^2/9} \cos(u)\,du$$

$$A = \operatorname{sen} u \Big|_0^{\pi^2/9} = \operatorname{sen}\left(\frac{\pi^2}{9}\right) \approx 0.89$$

Se fôssemos fazer via de função. $\int_0^{\pi/3} \cos(u)\,du$

This question already has answers here:

Proving $\sum_{k=0}^{n} \cos(kx) = \frac{1}{2} + \frac{\sin(\frac{2n+1}{2}x)}{2\sin(x/2)}$ (7 answers)

Closed 7 years ago.

Substituição

a) $\int_0^5 2x\sqrt{x^2}\,dx$

$u = x^2 \therefore \frac{du}{dx} = 2x \therefore du = 2x\,dx$

Quando $x=0$, $u=0$
" $x=5$, $u=25$

$= \int_0^{25} \sqrt{u}\,du = \int_0^{25} u^{1/2}\,du$

$= \left.\frac{u^{1/2+1}}{1/2+1}\right|_0^{25} =$

$= \left.\frac{u^{3/2}}{3/2}\right|_0^{25} = \frac{2}{3}u^{3/2}\Big|_0^{25} = \frac{2}{3}\left((25)^{3/2} - 0^{3/2}\right) = \frac{2}{3}\left(25\sqrt{25}\right)$

$= \frac{250}{3}$

$u^{3/2} = u^1 u^{1/2} = u\sqrt{u}$

$7^{3/2} = 7\sqrt{7}$

b) $\int_0^5 x\sqrt{x^2}\,dx$

$u = x^2 \therefore \frac{du}{dx} = 2x$

$\therefore du = 2x\,dx$

$\frac{du}{2} = x\,dx$

$= \int_0^{25} \sqrt{u}\,\frac{du}{2}$

$= \frac{1}{2}\int_0^{25} \sqrt{u}\,du = \frac{1}{2}\left(\frac{250}{3}\right)$

Calculei anteriormente.

c) $\int_0^{\pi} \underbrace{\cos x \operatorname{sen} x}_{u}\,\underbrace{dx}_{du}$

$u = \operatorname{sen} x \quad \therefore \frac{du}{dx} = \cos x$

$\therefore du = \cos x\, dx$

Quando $x = 0$, $u = \operatorname{sen} 0 = 0$

" $x = \pi$, $u = \operatorname{sen} \pi = 0$

$$\int_0^0 u\,du = 0$$

d) $\int_0^{\pi/2} \cos^2 x \operatorname{sen} x\, dx$

$u = \cos x \quad \therefore \frac{du}{dx} = -\operatorname{sen} x$

$\therefore (-du) = \operatorname{sen} x\, dx$

$$-\int_1^0 u^2\,du$$

Quando $x = 0$, $u = \cos 0 = 1$

" $x = \frac{\pi}{2}$, $u = \cos \frac{\pi}{2} = 0$

$$= \int_0^1 u^2 du = \frac{u^3}{3}\Big|_0^1 = \frac{1^3}{3} - \frac{0^3}{3} = \frac{1}{3}$$

WolframAlpha computational intelligence.

int(cos^2(x)*sin(x) , x = 0 .. Pi /2)

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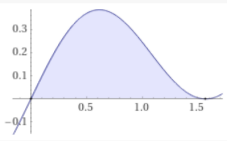
Definite integral: More digits Step-by-step solution

$$\int_0^{\frac{\pi}{2}} \cos^2(x) \sin(x)\, dx = \frac{1}{3} \approx 0.33333$$

Visual representation of the integral:



Indefinite integral: Step-by-step solution

↗ + Fácil.

int(x^2 , x = 0 ..1)

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Definite integral:

$$\int_0^1 x^2\, dx = \frac{1}{3} \approx 0.33333$$

Visual representation of the integral: