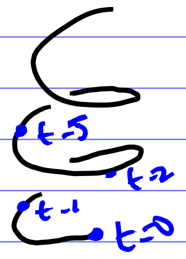
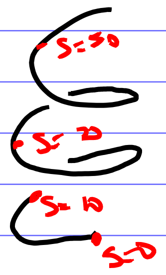
Reparametrização de funções vetoriais

$$\vec{r}(t) = \langle \cos t, \operatorname{sen} t, t \rangle$$



$$\vec{r}(\alpha) = \langle \cos\alpha, \operatorname{sen}\alpha, \alpha \rangle$$

Reparametrização sem alteração no significado.
Somente a letra foi trocada.



$\vec{r}(s)$

É função de comprimento de arco.

Função comprimento de arco.

$$S = \int_0^t |\vec{r}\,'(\tau)|\, d\tau$$

$$\vec{r}(t) = \langle \cos t, \operatorname{sen} t, t \rangle$$

$$\vec{r}(\tau) = \langle \cos\tau, \operatorname{sen}\tau, \tau \rangle$$

$$\vec{r}\,'(\tau) = \langle -\operatorname{sen}\tau, \cos\tau, 1 \rangle$$

$$|\vec{r}\,'(\tau)| = \sqrt{(\operatorname{sen}\tau)^2 + (\cos\tau)^2 + 1}$$

$$= \sqrt{1+1} = \sqrt{2}$$

$$S = \int_0^t \sqrt{2}\, d\tau$$

$$S = \sqrt{2}(t-0)$$

$$S = \sqrt{2}\,t$$

$$t = \frac{s}{\sqrt{2}}$$

$$\vec{r}(t) = \langle \cos t, \operatorname{sen} t, t \rangle$$

$$\vec{r}(s) = \left\langle \cos\left(\frac{s}{\sqrt{2}}\right), \operatorname{sen}\left(\frac{s}{\sqrt{2}}\right), \frac{s}{\sqrt{2}} \right\rangle$$

1–6 Determine o comprimento da curva dada.

4. $\mathbf{r}(t) = \cos t\,\mathbf{i} + \operatorname{sen} t\,\mathbf{j} + \ln \cos t\,\mathbf{k}, \quad 0 \leqslant t \leqslant \pi/4$

$$L = \int_a^b |\vec{r}\,'(t)|\,dt$$

$$\vec{r}\,'(t) = \langle -\operatorname{sen} t, \cos t, \frac{1}{\cos t} \cdot (-\operatorname{sen} t) \rangle$$

$$\vec{r}\,'(t) = \langle -\operatorname{sen} t, \cos t, -\frac{\operatorname{sen} t}{\cos t} \rangle$$

$$\vec{r}\,'(t) = \langle -\operatorname{sen} t, \cos t, -\operatorname{tg}(t) \rangle$$

$$L = \int_0^{\pi/4} \sqrt{(-\operatorname{sen} t)^2 + (\cos t)^2 + (\operatorname{tg} t)^2}\,dt$$

$$L = \int_0^{\pi/4} \sqrt{1 + (\operatorname{tg} t)^2}\,dt$$

$$L = \int_0^{\pi/4} \sqrt{\sec^2 t}\,dt$$

$$\boxed{\sec t = \frac{1}{\cos t} \qquad \sec t > 0 \quad 0 \leq t \leq \frac{\pi}{4}}$$

$$L = \int_0^{\pi/4} \sec t\,dt$$

Funções Trigonométricas

Ver artigo principal: *Lista de integrais de funções trigonométricas*

- $\int \cos x\,dx = \operatorname{sen} x + C$
- $\int \operatorname{sen} x\,dx = -\cos x + C$
- $\int \operatorname{tg} x\,dx = -\ln|\cos x| + C$[12]
- $\int \operatorname{cossec} x\,dx = \ln|\operatorname{cossec} x - \operatorname{cotg} x| + C$
- $\boxed{\int \sec x\,dx = \ln|\sec x + \operatorname{tg} x| + C}$
- $\int \operatorname{cotg} x\,dx = \ln|\operatorname{sen} x| + C$
- $\int \sec x \operatorname{tg} x\,dx = \sec x + C$
- $\int \operatorname{cossec} x \operatorname{cotg} x\,dx = -\operatorname{cossec} x + C$
- $\int \sec^2 x\,dx = \operatorname{tg} x + C$
- $\int \operatorname{cossec}^2 x\,dx = -\operatorname{cotg} x + C$
- $\int \operatorname{sen}^2 x\,dx = \frac{1}{2}(x - \operatorname{sen} x \cos x) + C$
- $\int \cos^2 x\,dx = \frac{1}{2}(x + \operatorname{sen} x \cos x) + C$

$$L = \ln|\sec t + \operatorname{tg} t|\,\Big|_0^{\pi/4}$$

$$L = \ln\left|\sec\frac{\pi}{4} + \operatorname{tg}\frac{\pi}{4}\right| - \ln|\sec 0 + \operatorname{tg} 0|$$

$$L = \ln\left|\frac{2}{\sqrt{2}} + 1\right| - \ln|1 + 0|$$

$$L = \ln\left(\frac{2}{\sqrt{2}} + 1\right)$$

$$L = \ln\left(\frac{2+\sqrt{2}}{\sqrt{2}}\right) = \ln\left(\frac{2\sqrt{2}+2}{2}\right) = \ln(\sqrt{2}+1)$$

$$L = \ln(\sqrt{2}+1) \text{ u.c.}$$

53. Se $\mathbf{r}(t) \neq \mathbf{0}$, mostre que $\dfrac{d}{dt}\,|\,\mathbf{r}(t)\,| = \dfrac{1}{|\,\mathbf{r}(t)\,|}\,\mathbf{r}(t) \cdot \mathbf{r}'(t)$.

$$|\vec{A}| = \sqrt{\vec{A} \circ \vec{A}}$$

$$\frac{d}{dt}|\vec{r}| = \frac{d}{dt}\sqrt{\vec{r} \circ \vec{r}}$$

$$= \frac{d}{dt}(\bar{r} \circ \bar{r})^{1/2}$$

$$= \frac{1}{2}(\bar{r} \circ \bar{r})^{-\frac{1}{2}} \cdot \frac{d}{dt}(\bar{r} \circ \bar{r})$$

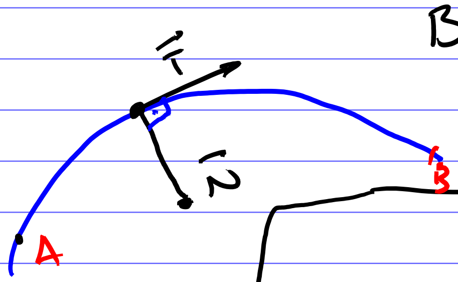
$$= \frac{1}{2} \cdot \frac{1}{\sqrt{\bar{r} \circ \bar{r}}} \cdot (\bar{r}' \circ \bar{r} + \bar{r} \circ \bar{r}')$$

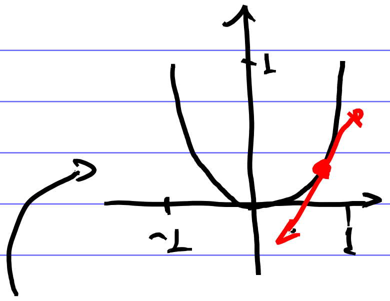
$$= \frac{1}{2\sqrt{\bar{r} \circ \bar{r}}}(2\bar{r}' \circ \bar{r})$$

$$= \frac{\bar{r}' \circ \bar{r}}{\sqrt{\bar{r} \circ \bar{r}'}} = \frac{\bar{r}' \circ \bar{r}}{|\bar{r}|}$$

$$\boxed{\frac{d}{dt}|\vec{r}| = \frac{\bar{r} \circ \bar{r}'}{|\bar{r}|}}$$

$\vec{B} = \vec{T} \times \vec{N}$





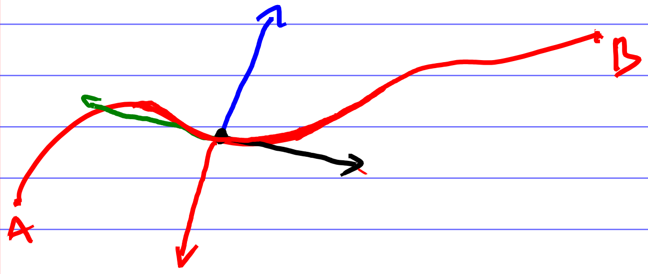
$\vec{r}(t) = \langle \cos t, \cos^2 t \rangle$

$x = \cos t$

$y = \cos^2 t$

$\boxed{y = x^2}$

Exercício



$\vec{T} = \dfrac{\vec{v}}{|\vec{v}|}$

$\vec{v} = \vec{r}\,'$

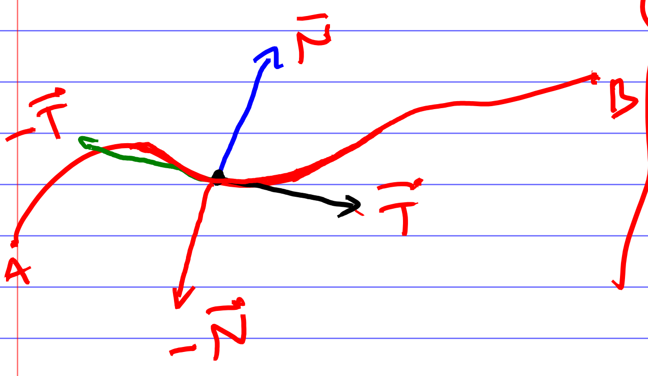
$\boxed{\vec{T} = \dfrac{\vec{r}\,'}{|\vec{r}\,'|}}$

Se a partícula sai de A p/ B:

$\vec{T}$? Preto $\quad \vec{N}$? Azul

Se a partícula sai de B p/ A:

$\vec{T}$? Verde $\quad \vec{N}$? Azul



$$\vec{T} = \frac{\vec{r}'}{|\vec{r}'|} \qquad \vec{N} = \frac{\vec{T}'}{|\vec{T}'|}$$

$$\vec{r} = \langle c, c \rangle$$

$$\frac{d\vec{r}}{dt} = \langle 0, 0 \rangle$$

$$\vec{r} = \langle \cos t, \operatorname{sen} t \rangle$$

$$|\vec{r}| = \sqrt{\cos^2 t + \operatorname{sen}^2 t} = \sqrt{1} = 1$$

$$\frac{d\vec{r}}{dt} = \langle -\operatorname{sen} t, \cos t \rangle$$

$$\vec{r} \circ \frac{d\vec{r}}{dt} = 0$$

São perpendiculares

$$\langle \cos t, \operatorname{sen} t \rangle \circ \langle -\operatorname{sen} t, \cos t \rangle = -\cos t \operatorname{sen} t + \operatorname{sen} t \cos t$$

$$= -\cos t \operatorname{sen} t + \cos t \operatorname{sen} t = 0$$

Determine os vetores normal e binormal da hélice circular

$$\mathbf{r}(t) = \cos t\,\mathbf{i} + \operatorname{sen} t\,\mathbf{j} + t\,\mathbf{k}$$

$$\vec{T} = \frac{\vec{r}'}{|\vec{r}'|} \qquad \vec{r}' = \langle -\operatorname{sen} t, \cos t, 1 \rangle$$

$$|\vec{r}'| = \sqrt{(-\operatorname{sen} t)^2 + (\cos t)^2 + 1^2}$$

$$= \sqrt{1+1}$$

$$= \sqrt{2}$$

$$\boxed{\vec{T} = \left\langle \frac{-\operatorname{sen} t}{\sqrt{2}}, \frac{\cos t}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle}$$

$$\vec{N} = \frac{\vec{T}'}{|\vec{T}'|}$$

$$\vec{T}' = \left\langle \frac{-\cos t}{\sqrt{2}}, \frac{-\operatorname{sen} t}{\sqrt{2}}, 0 \right\rangle$$

$$\vec{T}' = \frac{1}{\sqrt{2}} \langle -\cos t, -\operatorname{sen} t, 0 \rangle$$

$$|\vec{T}'| = \frac{1}{\sqrt{2}} \sqrt{(-\cos t)^2 + (-\operatorname{sen} t)^2 + 0^2}$$

$$|\vec{T}'| = \frac{1}{\sqrt{2}}$$

$$\vec{N} = \frac{\vec{T}'}{|\vec{T}'|} = \frac{\cancel{\frac{1}{\sqrt{2}}} \langle -\cos t, -\operatorname{sen} t, 0 \rangle}{\cancel{\frac{1}{\sqrt{2}}}}$$

$$\boxed{\vec{N} = \langle -\cos t, -\operatorname{sen} t, 0 \rangle}$$

$$\vec{B} = \vec{T} \times \vec{N}$$