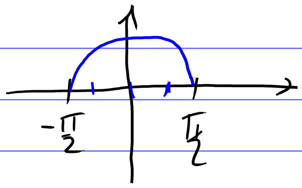
Simetrias

Função Par

$f(x) = f(-x)$

$$\int_{-\pi/2}^{\pi/2} \cos x \, dx$$

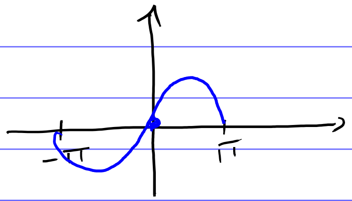


$\cos(\frac{\pi}{4}) = \cos(-\frac{\pi}{4})$

$$= 2\int_{0}^{\pi/2} \cos x \, dx$$

Outra situação

$$\int_{-\pi}^{\pi} \operatorname{sen}(x)\, dx = 0$$

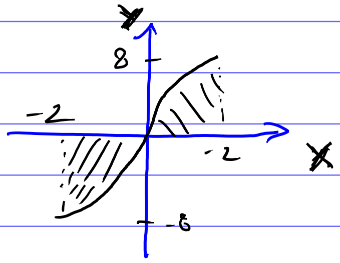


$\int_{-\pi}^{\pi} g(x)\, dx = 0$, se $g(x)$ é ímpar

Outra situação

$$\int_{-2}^{2} x^3 \, dx = 0$$

$$\boxed{\begin{array}{l}\text{ímpar} \\ f(x) = -f(-x)\end{array}}$$



Outra situação

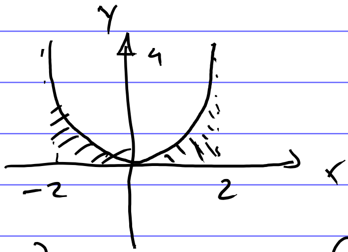
$y = x^2$

É Par

$f(x) = f(-x)$

$x^2 = (-x)^2$

$x^2 = x^2$



$$\int_{-2}^{2} x^2 \, dx = 2\int_{0}^{2} x^2 \, dx$$

Tecnicas de integração

$$\int_0^{\pi} \operatorname{sen} x \sqrt{\cos^2 x}\, dx$$

$u = \cos x \quad \therefore \quad \frac{du}{dx} = -\operatorname{sen} x$

$\therefore \quad du = -\operatorname{sen} x\, dx$

substituição dos limites de integração $\quad (-du) = \operatorname{sen} x\, dx$

$$\boxed{\begin{array}{l} \text{Quando } x = 0,\ u = \cos 0 = 1 \\ \text{"} \quad x = \pi,\ u = \cos \pi = -1 \end{array}}$$

$$\int_1^{-1} \sqrt{u^2}\, (-du)$$

Atenção

$\sqrt{x^2} = \pm x$

Numa equação

Exemplo:

$y = \sqrt{9}$

$y = 3$ ou $y = -3$

$\sqrt{x^2} = |x|$

Numa função

$f(x) = \sqrt{x^2}$

$f(3) = \sqrt{9} = 3$

$g(x) = \sqrt{x}$

$g(9) = \sqrt{9} = 3$

$h(x) = -\sqrt{x}$

$\sqrt{(-3)^2} = +3$

$\sqrt{x^2} = |x|$

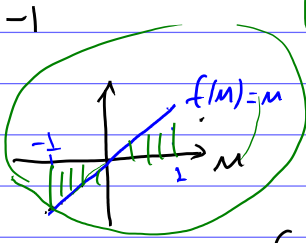
$\sqrt{g^2(x)} = |g(x)|$

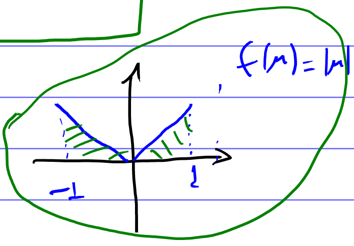
Continuando...

$$\int_1^{-1} \sqrt{u^2}\, (-du)$$

$$= \int_{-1}^{1} \sqrt{u^2}\, du = \boxed{\int_{-1}^{1} |u|\, du}$$

$$|u| = \begin{cases} -u, & u < 0 \\ u, & u > 0 \end{cases}$$





$$= \int_{-1}^{0} (-u)\, du + \int_0^1 u\, du$$

$$= \int_0^{-1} u\, du + \int_0^1 u\, du$$

$$= \frac{u^2}{2}\Big|_0^{-1} + \frac{u^2}{2}\Big|_0^1$$

$$= \left[\frac{(-1)^2}{2} - 0\right] + \left[\frac{1^2}{2} - 0\right]$$

$$= \left[\frac{1}{2}\right] + \left[\frac{1}{2}\right] = 1$$

Exemplo 2: $\displaystyle\int_0^{2\pi}\sqrt{\cos^2 x}\,dx$

ERRADO. NÃO É EQUIVALENTE

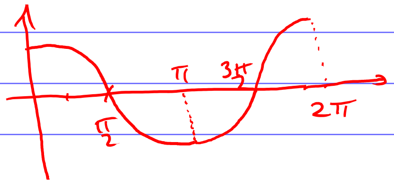
$$\int_0^{2\pi}\cos x\,dx = \operatorname{sen} x\Big|_0^{2\pi}$$

$$= \operatorname{sen} 2\pi - \operatorname{sen} 0$$

$$= 0$$

$$\int_0^{2\pi}\sqrt{\cos^2 x}\,dx = \int_0^{2\pi}|\cos x|\,dx$$

1º Fazer o gráfico de cos x



$$\int_0^{2\pi}|\cos x|\,dx = \int_0^{\pi/2}\cos x\,dx + \int_{\pi/2}^{\frac{3\pi}{2}}[-\cos x]\,dx + \int_{\frac{3\pi}{2}}^{2\pi}\cos x\,dx$$

Continuem por favor ...

Espera-se 4.

Integração por partes

$$\int_0^{\pi} x \boxed{\cos x \, dx}$$

$$\int_a^b u \, dv = uv\Big|_a^b - \int_a^b v \, du$$

vou usar aqui

$$u = x \quad \therefore \quad \frac{du}{dx} = 1 \quad \therefore \quad du = dx$$

$$\boxed{dv = \cos x \, dx} \quad \therefore \quad v = \int dv = \int \cos x \, dx$$

$$\boxed{v = \operatorname{sen} x}$$

$$\int_0^{\pi} x \cos x \, dx = x \operatorname{sen} x \Big|_0^{\pi} - \int_0^{\pi} \operatorname{sen} x \, dx$$

$$= [\pi \operatorname{sen} \pi - 0 \operatorname{sen} 0] - [-\cos x]\Big|_0^{\pi}$$

$$= 0 - ([\cos \pi] - [-\cos 0])$$

$$= -(1 - [-1]) = -(2)$$

$$= -2$$

WolframAlpha computational intelligence

int(x*cos(x),x=0..pi)

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Definite integral:

$$\int_0^{\pi} x \cos(x) \, dx = -2$$

Visual representation of the integral: