$\theta = \frac{\pi}{2}$


$f(x) = x^2$

| x | f(x) |
|---|---|
| -3 | |
| -2 | |
| -1 | |
| 0 | |
| 1 | |
| 2 | |
| 3 | |

| x | f(x) |
|---|---|
| 100 | |
| 101 | |
| 102 | |
| 103 | |

Pontos escolhidos não representam o gráfico muito bem!

## Conversão

$r = 2\cos\theta$

$r^2 = 2r\cos\theta$

$x^2 + y^2 = 2x$

$x^2 - 2x + y^2 = 0$

← completei o quadrado.

$x^2 - 2x + 1 + y^2 = 1$

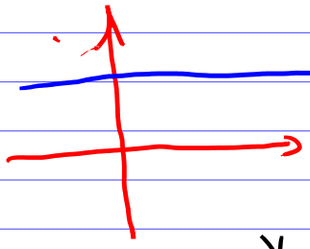
$\boxed{(x-1)^2 + y^2 = 1}$

Eq de um círculo centrado em $(1,0)$

Eq de um círculo centrado em $(x_0, y_0)$

$$\boxed{(x-x_0)^2 + (y-y_0)^2 = r^2}$$

Exercício:




Qual é a expressão de reta $y = 1$ em coord. Polares?

$y = 1$

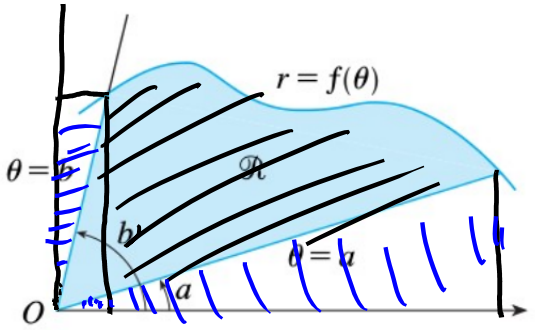
$r\sin\theta = 1$

$$\boxed{r = \frac{1}{\sin\theta}}$$

$0 < r < \pi$

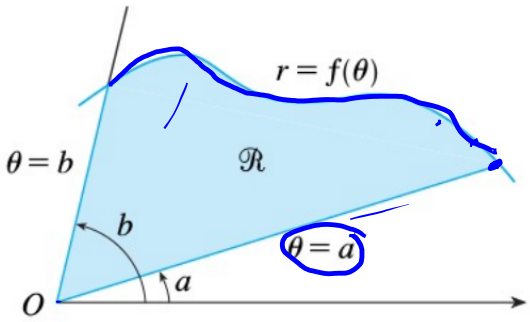
# CÃ¡lculo de Ã¡rea em coodenadas polares

Se fosse em coordenadas cartesianas:



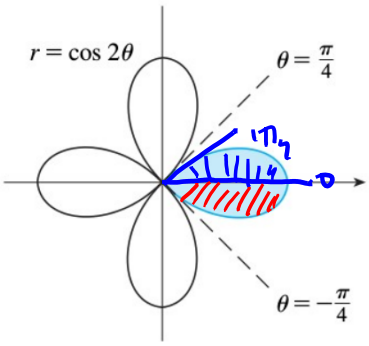

Em coord. polares:

$$A = \int_{\theta_i}^{\theta_f} \frac{1}{2} r^2 \, d\theta$$




$$A = \int_{a}^{b} \frac{1}{2} f(\theta)^2 \, d\theta$$

1. Calcule a área delimitada por um laço da rosácea de quatro pétalas $r=\cos(2\theta)$




$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1}{2}(\cos(2\theta))^2\,d\theta = 2\int_{0}^{\frac{\pi}{4}} \frac{1}{2}(\cos(2\theta))^2\,d\theta$$

OU (Por simetria)

$$= \int_{0}^{\frac{\pi}{4}} (\cos(2\theta))^2\,d\theta = \int_{0}^{\pi/4} \left[\frac{1}{2} + \frac{\cos(4\theta)}{2}\right] d\theta$$

Pois $\cos^2(u) = \frac{1}{2} + \frac{\cos(2u)}{2}$

$$= \int_{0}^{\pi/4} \left[\frac{1}{2}\right] d\theta + \int_{0}^{\pi/4} \left[\frac{\cos(4\theta)}{2}\right] d\theta$$

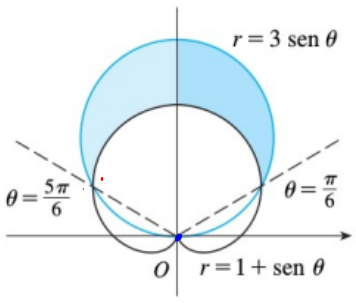
$$= \frac{1}{2}\left[\frac{\pi}{4} - 0\right] + \left[\frac{\operatorname{Sen}(4\theta)}{8}\right]\Bigg|_{0}^{\pi/4}$$

$$= \frac{\pi}{8} + \left[\frac{\operatorname{Sen}(\pi)}{8} - \frac{\operatorname{Sen}(0)}{8}\right]$$

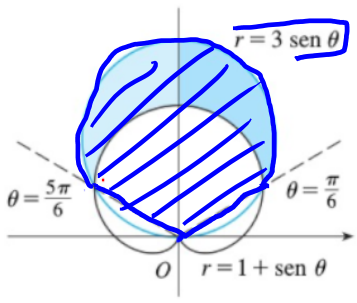
$$= \frac{\pi}{8} \text{ u.a.}$$

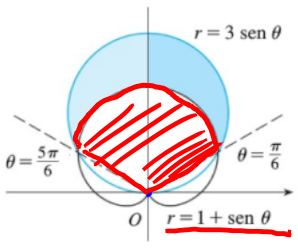
2. Calcule a área da região que está dentro do círculo $r=3\sin(\theta)$ e fora da cardióide

$r=1+\sin(\theta)$




$$A=\int_{\pi/6}^{\frac{5}{6}\pi}\frac{1}{2}(3\operatorname{sen}\theta)^2\,d\theta-\int_{\pi/6}^{\frac{5}{6}\pi}\frac{1}{2}(1+\operatorname{sen}\theta)^2\,d\theta$$




$$A=\int_{\pi/6}^{\frac{5}{6}\pi}\frac{1}{2}(3\operatorname{sen}\theta)^2\,d\theta-\int_{\pi/6}^{\frac{5}{6}\pi}\frac{1}{2}(1+\operatorname{sen}\theta)^2\,d\theta$$

$$A=\int_{\pi/6}^{\frac{5}{6}\pi}\left(\frac{9}{2}\operatorname{sen}^2\theta-\frac{1}{2}(1+2\operatorname{sen}\theta+\operatorname{sen}^2\theta)\right)d\theta$$

$$A=\int_{\pi/6}^{\frac{5}{6}\pi}\left(\frac{9}{2}\cdot\operatorname{sen}^2\theta-\frac{1}{2}-\operatorname{sen}\theta-\frac{1}{2}\operatorname{sen}^2\theta\right)d\theta$$

$$A=\int_{\pi/6}^{\frac{5}{6}\pi}\left(4\operatorname{sen}^2\theta-\operatorname{sen}\theta-\frac{1}{2}\right)d\theta$$

$$A=\int_{\pi/6}^{\frac{5}{6}\pi}\left[4\left(\frac{1}{2}-\frac{\cos(2\theta)}{2}\right)-\operatorname{sen}\theta-\frac{1}{2}\right]d\theta$$

$$A=\int_{\pi/6}^{\frac{5}{6}\pi}\left[2-2\cos(2\theta)-\operatorname{sen}\theta-\frac{1}{2}\right]d\theta$$

$$A=\int_{\pi/6}^{\frac{5}{6}\pi}\left[\frac{3}{2}-2\cos(2\theta)-\operatorname{sen}\theta\right]d\theta$$

$$=\frac{3}{2}\left(\frac{5}{6}\pi-\frac{\pi}{6}\right)-2\frac{\operatorname{Sen}(2\theta)}{2}\Big|_{\pi/6}^{\frac{5}{6}\pi}-\cos\theta\Big|_{\pi/6}^{\frac{5}{6}\pi}$$

$$=\frac{3}{2}\left(\frac{4}{6}\pi\right)-\left[\operatorname{sen}\left(\frac{5}{3}\pi\right)-\operatorname{sen}\left(\frac{\pi}{3}\right)\right]-\left[\cos\frac{5\pi}{6}-\cos\frac{\pi}{6}\right]$$

$$=\pi-\left[\frac{1}{2}-\frac{\sqrt{3}}{2}\right]-\left[-\frac{\sqrt{3}}{2}+\frac{\sqrt{3}}{2}\right)$$

$$=\pi-\frac{1}{2}+\frac{\sqrt{3}}{2}$$

Verificar a conta