15–40 Determine as derivadas parciais de primeira ordem da função.

15. $f(x, y) = y^5 - 3xy$ **16.** $f(x, y) = x^4y^3 + 8x^2y$

33. $w = \ln(x + 2y + 3z)$ **18.** $f(x, t) = \sqrt{x}\ln t$

39. $u = \sqrt{x_1^2 + x_2^2 + \cdots + x_n^2}$

15. $f(x,y)=y^5-3xy$

$$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}(y^5-3xy) = \frac{\partial}{\partial x}y^5 - \frac{\partial}{\partial x}(3xy) = 0 - 3y = -3y$$

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}(y^5-3xy) = \frac{\partial}{\partial y}y^5 - \frac{\partial}{\partial y}(3xy) = 5y^4 - 3x$$

16. $f(x,y)=x^4y^3+8x^2y$

$$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}\left(x^4y^3+8x^2y\right) = \frac{\partial}{\partial x}(x^4y^3) + \frac{\partial}{\partial x}(8x^2y) = y^3\frac{\partial}{\partial x}x^4 + 8y\frac{\partial}{\partial x}x^2 = y^3\,4x^3 + 8y\,2x$$

$$\frac{\partial f}{\partial x} = 4x^3y^3 + 16xy$$

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}\left(x^4y^3+8x^2y\right) = \frac{\partial}{\partial y}(x^4y^3) + \frac{\partial}{\partial y}(8x^2y) = x^4\frac{\partial}{\partial y}y^3 + 8x^2\frac{\partial y}{\partial y} = x^4\,3y^2 + 8x^2 = 3x^4y^2 + 8x^2$$

Outro Exemplo

$f(x,y,z,w)=x$

$$\frac{\partial f}{\partial x} = 1 \left\{ \frac{\partial f}{\partial y} = 0 \right\} \frac{\partial f}{\partial z} = 0 \left\{ \frac{\partial f}{\partial w} = 0 \right.$$

$W = \ln(x+2y+3z)$

$$\frac{\partial W}{\partial x} = \frac{1}{x+2y+3z}\cdot\frac{\partial}{\partial x}(x+2y+3z)$$

$$\boxed{\frac{\partial W}{\partial x} = \frac{1}{x+2y+3z}}$$

$$\frac{\partial W}{\partial y} = \frac{1}{x+2y+3z}\cdot\frac{\partial}{\partial y}(x+2y+3z)$$

$$\boxed{\frac{\partial W}{\partial y} = \frac{2}{x+2y+3z}}$$

$$\frac{\partial W}{\partial z} = \frac{1}{x+2y+3z}\cdot\frac{\partial}{\partial z}(x+2y+3z)$$

$$\boxed{\frac{\partial W}{\partial z} = \frac{3}{x+2y+3z}}$$

$f(x,t)=\sqrt{x}\ln t$

$$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}(\sqrt{x}\ln t) = \ln t\frac{\partial}{\partial x}\sqrt{x} = \ln t\frac{\partial}{\partial x}(x)^{1/2} = \ln t\,\frac{1}{2}(x)^{-1/2} = \ln t\,\frac{1}{2\sqrt{x}}$$

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial t}(\sqrt{x}\ln t) = \sqrt{x}\frac{\partial}{\partial t}\ln t = \sqrt{x}\,\frac{1}{t}$$

$$\frac{\partial f}{\partial t} = \frac{\sqrt{x}}{t}$$

39. $U = \sqrt{x_1^2+x_2^2+\cdots+x_n^2}$

$$\frac{\partial U}{\partial x_1} = \frac{\partial}{\partial x_1}(x_1^2+x_2^2+\cdots+x_n^2)^{1/2}$$

$$= \frac{1}{2}(x_1^2+x_2^2+\cdots+x_n^2)^{-\frac{1}{2}}\cdot\frac{\partial}{\partial x_1}(x_1^2+x_2^2+\cdots+x_n^2)$$

$$= \frac{1}{2}(x_1^2+x_2^2+\cdots+x_n^2)^{-\frac{1}{2}}\cdot 2x_1$$

$$\frac{\partial U}{\partial x_1} = \frac{x_1}{\sqrt{x_1^2+x_2^2+\cdots+x_n^2}}$$

$$\frac{\partial U}{\partial x_2} = \frac{x_2}{\sqrt{x_1^2+x_2^2+\cdots+x_n^2}}$$

$$\frac{\partial U}{\partial x_3} = \frac{x_3}{\sqrt{x_1^2+x_2^2+\cdots+x_n^2}}$$

$\vdots$

$$\boxed{\frac{\partial U}{\partial x_i} = \frac{x_i}{\sqrt{x_1^2+x_2^2+\cdots+x_n^2}} \quad i\in[1,n]}$$

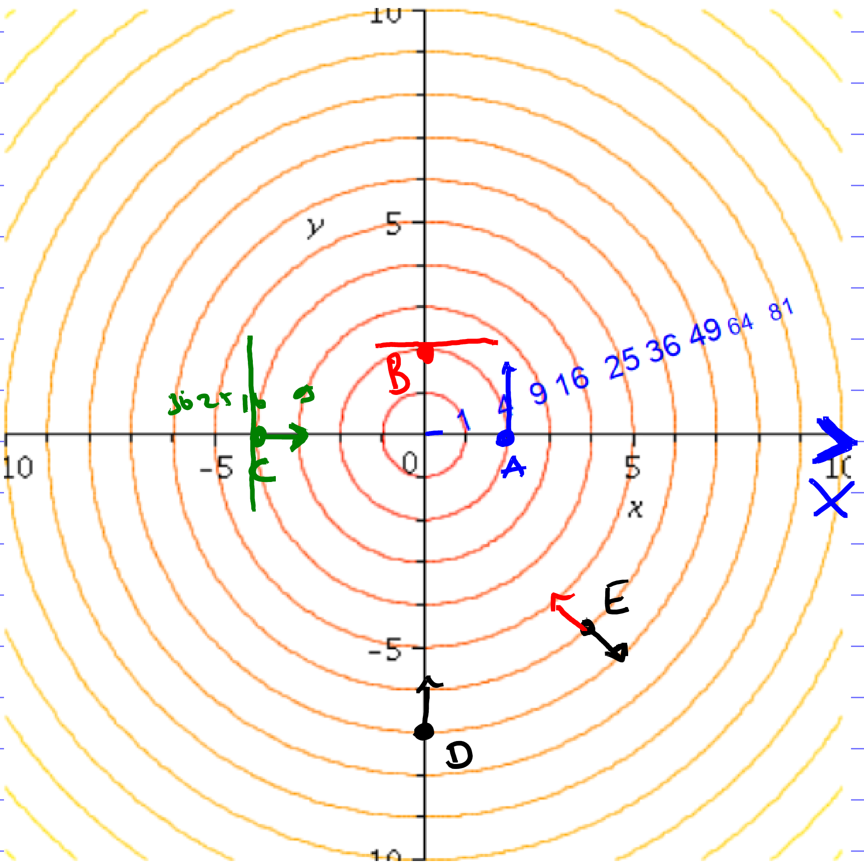
$\vdots$

$f(x,y)=x^2+y^2$

intuitively

$\delta \to 0$

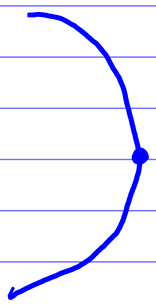
$\delta \neq 0$



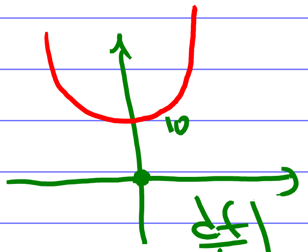
$$\left.\frac{\partial f}{\partial x}\right|_A > 0 \quad \left.\frac{\partial f}{\partial y}\right|_B > 0 \quad \left.\frac{\partial f}{\partial x}\right|_C < 0 \quad \left.\frac{\partial f}{\partial y}\right|_D < 0$$

$$\left.\frac{\partial f}{\partial y}\right|_A = 0 \quad \left.\frac{\partial f}{\partial x}\right|_B = 0 \quad \left.\frac{\partial f}{\partial y}\right|_C = 0 \quad \left.\frac{\partial f}{\partial x}\right|_D = 0$$

$$\frac{\partial f}{\partial y} = \lim_{h \to 0} \frac{f(x,y+h) - f(x,y)}{h}$$



$f(x) = x^2 + 10$



$$\left.\frac{df}{dx}\right|_{x=0} = 0$$

$$\left.\frac{d}{dx}(x^2+10)\right|_{x=0} = \left.2x\right|_{x=0} = 0$$

$$\left.\frac{\partial f}{\partial y}\right|_{(1,2)}$$

$x=1$

$f(1,y)$

$f(x,y)=x^2+y^2$

$f(1,y)=1^2+y^2$

$f(1,y)=1+y^2$

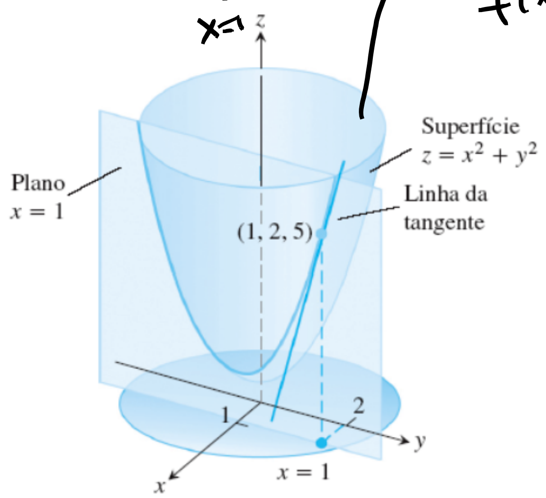
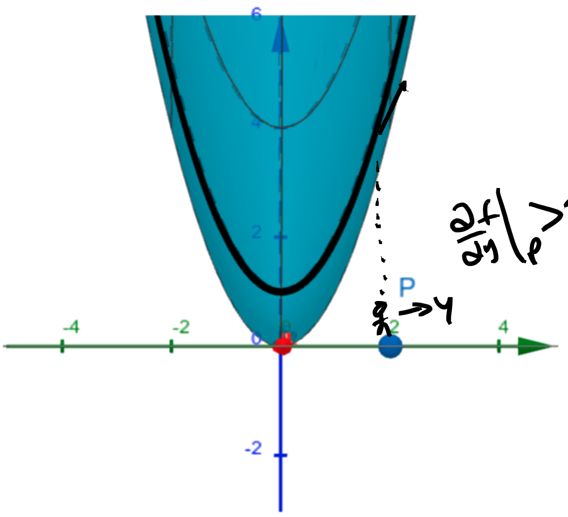
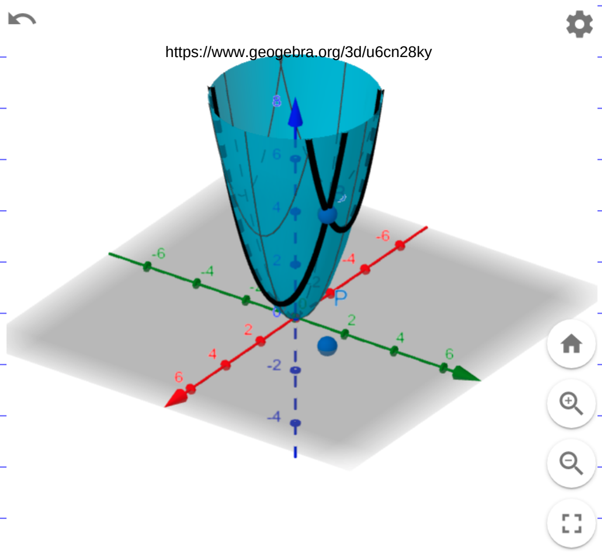


FIGURA 14.16 A tangente à curva de interseção do plano $x = 1$ e da superfície $z = x^2 + y^2$ no ponto (1, 2, 5) (Exemplo 5).



$$\left.\frac{\partial f}{\partial y}\right|_{P} > 0$$



https://www.geogebra.org/3d/u6cn28ky