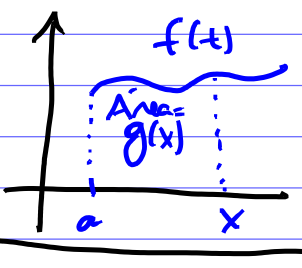
TFC 1

$$g(x)=\int_a^x f(t)\,dt$$



$$\frac{d}{dx}g(x)=\boxed{\frac{d}{dx}\int_a^x f(t)\,dt = f(x)}$$

A integração e a derivação são processos inversos.

NÃO CONFUNDIR COM ANTIDERIVAÇÃO

$$\int x\,dx = \frac{x^2}{2}+c$$

Exemplo:

① $$\frac{d}{dx}\boxed{\int_5^x t^2\,dt} = x^2$$

O t é variável artificial.

$$\int_3^5 t^2\,dt = \int_3^5 y^2\,dy = \int_3^5 a^2\,da$$

② $$\frac{d}{dx}\int_{-3}^x (t^2+t+1)\,dt = x^2+x+1$$

③ $$\frac{d}{dx}\int_{-2}^x \frac{t^2}{1+t^4}\,dt = \frac{x^2}{1+x^4}$$

④ $$\frac{d}{dx}\int_x^{-2}\frac{t^2}{1+t^4}\,dt = -\frac{d}{dx}\int_{-2}^x \frac{t^2}{1+t^4}\,dt$$
$$= -\left(\frac{x^2}{1+x^4}\right)$$

⑤ $$\frac{d}{dx}\int_{-2}^{x^2}\frac{t^2}{1+t^4}\,dt = \frac{d}{du}\int_{-2}^{u}\frac{t^2}{1+t^4}\,dt\cdot\frac{du}{dx}$$

$$= \frac{u^2}{1+u^4}\,2x = \frac{2x\,x^4}{1+(x^2)^4} = \frac{2x^5}{1+x^8}$$

Notação de LEIBNIZ	Notação de LAGRANGE	Notação Newton
$\frac{df}{dx}$	f'	$\dot{x}$
	f''	$\ddot{x}$
$\frac{d^2f}{dx^2}$		

⑥ $$\frac{d}{dx}\int_3^{\cos x}\frac{1}{1+t^5}\,dt = \frac{d}{du}\int_3^{u}\frac{1}{1+t^5}\,dt\cdot\frac{du}{dx}$$

TFC1

$$= \frac{1}{1+u^5}\cdot(-\operatorname{sen}x)$$

$$= \frac{-\operatorname{sen}x}{1+\cos^5 x}$$

⑦ Tentem!

$$\frac{d}{dx}\int_{-\operatorname{sen}x}^{7}\left(\frac{t^2}{1+t^5}\right)dt = -\frac{d}{dx}\int_{7}^{-\operatorname{sen}x}\left(\frac{t^2}{1+t^5}\right)dt$$

$$= -\boxed{\frac{d}{du}\int_{7}^{u}\left(\frac{t^2}{1+t^5}\right)dt}\cdot\frac{du}{dx}$$

TFC

$$= -\frac{u^2}{1+u^5}\cdot(-\cos x)$$

Atenção $u=-\operatorname{sen}x$

$$= +\frac{(-\operatorname{sen}x)^2\cos x}{1+(-\operatorname{sen}x)^5}$$

$$= \frac{\operatorname{sen}^2 x\,\cos x}{1+(-1)^5(\operatorname{sen}^5 x)}$$

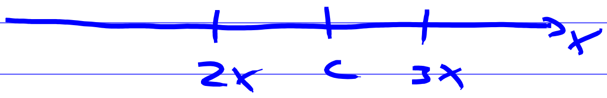
$$= \frac{\operatorname{sen}^2 x\,\cos x}{1-\operatorname{sen}^5 x}$$

Derive $g(x)$:

55. $g(x) = \displaystyle\int_{2x}^{3x} \frac{u^2 - 1}{u^2 + 1}\, du$

$$\frac{d}{dx}\int_{2x}^{3x} \frac{u^2-1}{u^2+1}\,du$$

$$\boxed{\begin{array}{l}\text{Vamos usar esta propriedade}\\ \displaystyle\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx\end{array}}$$



Mesmo que c não estivesse entre 2x e 3x funcionaria.

$$\frac{d}{dx}\left(\int_{2x}^{c} \frac{u^2-1}{u^2+1}\,du + \int_{c}^{3x} \frac{u^2-1}{u^2+1}\,du\right) =$$

$$= \frac{d}{dx}\left(\int_{2x}^{c} \frac{u^2-1}{u^2+1}\,du\right) + \frac{d}{dx}\left(\int_{c}^{3x} \frac{u^2-1}{u^2+1}\,du\right)$$

$$= -\frac{d}{dx}\left(\int_{c}^{2x} \frac{u^2-1}{u^2+1}\,du\right) + \frac{d}{dx}\left(\int_{c}^{3x} \frac{u^2-1}{u^2+1}\,du\right)$$

$$= -\underbrace{\boxed{\frac{d}{dw}\left(\int_{c}^{w} \frac{u^2-1}{u^2+1}\,du\right)}}_{TFC1}\cdot\frac{dw}{dx} + \underbrace{\boxed{\frac{d}{dv}\int_{c}^{v} \frac{u^2-1}{u^2+1}\,du}}_{TFC1}\cdot\frac{dv}{dx}$$

$$= -\frac{w^2-1}{w^2+1}(2) + \frac{v^2-1}{v^2+1}(3)$$

$$= (-2)\frac{(2x)^2-1}{(2x)^2+1} + (3)\frac{(3x)^2-1}{(3x)^2+1} \Leftarrow \text{Já é resposta.}$$

$$= (-2)\frac{(4x^2-1)}{4x^2+1} + 3\frac{(9x^2-1)}{9x^2+1}$$

$$= \frac{-8x^2+2}{4x^2+1} + \frac{27x^2-3}{9x^2+1}$$

TFC 2

$$\int_a^b f(t)\,dt = F(b) - F(a)$$

Exemplo:

Antiderivada $\int x\,dx = \frac{x^2}{2} + c$

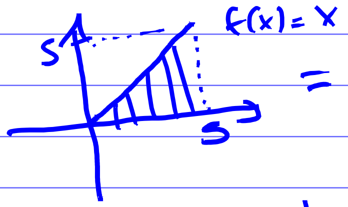
Integral $\int_a^b x\,dx = F(b) - F(a)$

antiderivada

Exemplo (1)

$$\int_0^5 x\,dx$$

1ª Maneira (via gráfico ou via geométrica)



$$= \frac{25}{2}$$

2ª Maneira (via definição)

$$\int_0^5 x\,dx = \lim_{n\to+\infty}\sum_{i=1}^{n} f(x_i)\,\Delta x$$

3ª Maneira (TFC2)

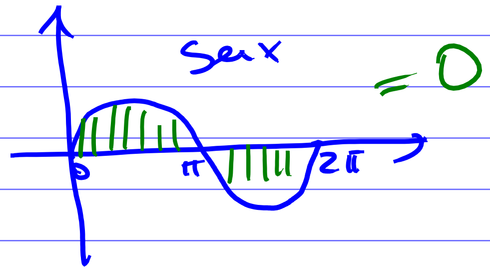
$$\int_0^5 x\,dx = \frac{x^2}{2}\Big|_0^5 = \frac{25}{2} - \frac{0}{2}$$

$$= \frac{25}{2}$$

Exemplo 2

$$\int_0^{2\pi} \operatorname{sen} x \, dx =$$

1º Graficamente



$= 0$

2º Usando o TFC2

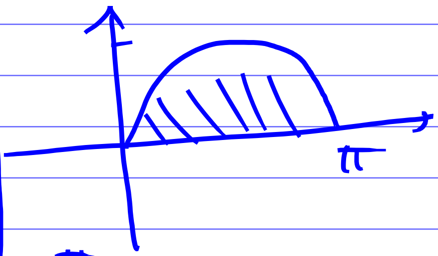
$$\int_0^{2\pi} \operatorname{sen} x \, dx = [-\cos x]\Big|_0^{2\pi}$$

$$= [-\cos(2\pi)] - [-\cos 0]$$

$$= [-1] - [-1]$$

$$= -1 + 1 = 0$$

Exemplo 3



$$\int_0^{\pi} \operatorname{sen} x \, dx = [-\cos x]\Big|_0^{\pi}$$

$$= [-\cos \pi] - [-\cos 0]$$

$$= [-(-1)] - [-1]$$

$$= 1 - (-1)$$

$$= 1 + 1$$

$$= 2$$