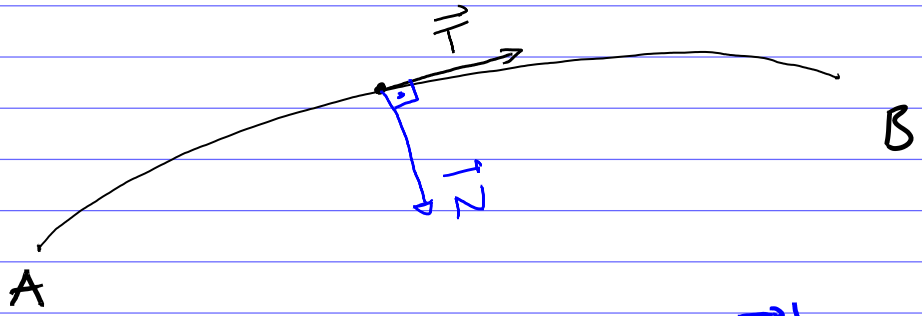
Vetor normal unitário



$$\vec{T} = \frac{\vec{r}'}{|\vec{r}'|} \qquad \vec{N} = \frac{\vec{T}'}{|\vec{T}'|}$$

A derivada de uma função vetorial de módulo constante sempre é perpendicular a ela.

Função constante

$$\boxed{\begin{aligned} \vec{r}(t) &= \langle c, c \rangle \\ \frac{d\vec{r}}{dt} &= \langle 0, 0 \rangle \end{aligned}}$$

Função com módulo constante

$\vec{r}(t) = \langle \cos t, \operatorname{sen} t \rangle$

$|\vec{r}| = \sqrt{\cos^2 t + \operatorname{sen}^2 t}$

$|\vec{r}| = 1$

$\dfrac{d\vec{r}}{dt} = \langle -\operatorname{sen} t, \cos t \rangle$

$\underbrace{\dfrac{d\vec{r}}{dt} \cdot \vec{r}}_{\text{São perpendiculares.}} = 0$

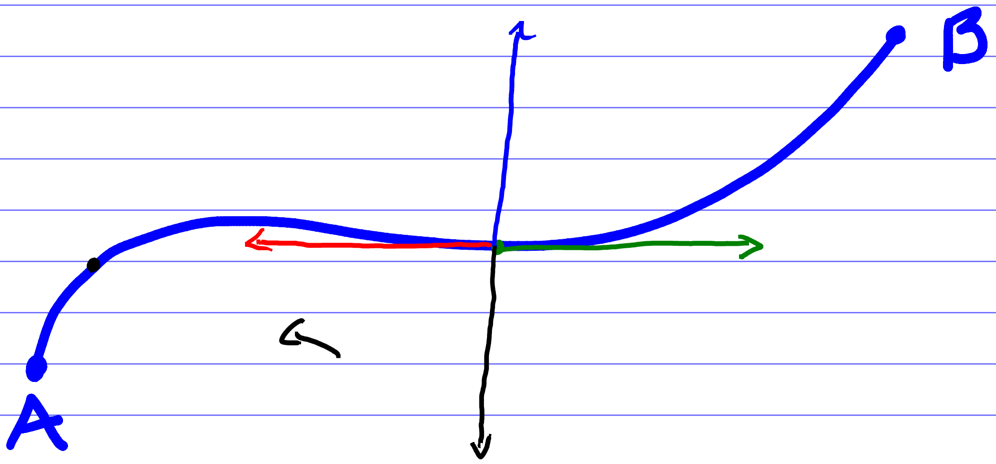
$\langle -\operatorname{sen} t, \cos t \rangle \cdot \langle \cos t, \operatorname{sen} t \rangle =$

$= -\operatorname{sen} t \cos t + \cos t \operatorname{sen} t$

$= -\operatorname{sen} t \cos t + \operatorname{sen} t \cos t$

$= 0$

Exercício



Para uma partícula que vai de A a B :

$\vec{T}$? $\vec{N}$?

Verde   Azul.

Para uma partícula que vai de B a A :

$\vec{T}$? $\vec{N}$?

Vermelho   Azul.

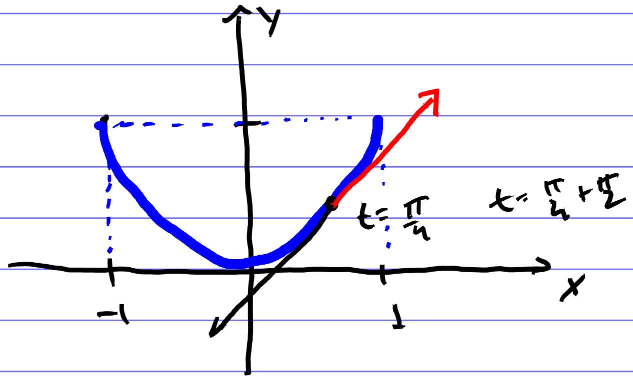
—//—

Analisando uma intersecção

$$\boxed{\vec{r}(t) = \langle \cos t, \cos^2 t \rangle}$$

$x = \cos t$

$y = \cos^2 t \therefore y = x^2$



Determine os vetores normal e binormal da hélice circular

$$\mathbf{r}(t) = \cos t\,\mathbf{i} + \text{sen}\,t\,\mathbf{j} + t\,\mathbf{k}$$

$$\vec{B} = \vec{T} \times \vec{N}$$

$$\vec{T} = \frac{\vec{r}\,'}{|\vec{r}\,'|} = \frac{\langle -\text{sen}\,t, \cos t, 1 \rangle}{\sqrt{\underbrace{(-\text{sen}\,t)^2 + (\cos t)^2}_{\text{I.F. trigonométrica}} + 1}}$$

$$\vec{T} = \frac{\langle -\text{sen}\,t, \cos t, 1 \rangle}{\sqrt{2}}$$

$$\vec{T} = \frac{1}{\sqrt{2}} \langle -\text{sen}\,t, \cos t, 1 \rangle$$

$$\vec{N} = \frac{\vec{T}\,'}{|\vec{T}\,'|} = \frac{\frac{1}{\sqrt{2}} \langle -\cos t, -\text{sen}\,t, 0 \rangle}{\frac{1}{\sqrt{2}} \sqrt{(-\cos t)^2 + (-\text{sen}\,t)^2 + 0^2}}$$

$$\vec{N} = \frac{\vec{T}\,'}{|\vec{T}\,'|} = \frac{\cancel{\frac{1}{\sqrt{2}}} \langle -\cos t, -\text{sen}\,t, 0 \rangle}{\cancel{\frac{1}{\sqrt{2}}}}$$

$$\vec{N} = \langle -\cos t, -\text{sen}\,t, 0 \rangle$$

$$\vec{B} = \vec{T} \times \vec{N}$$

1–6 Determine o comprimento da curva dada.

4. $\mathbf{r}(t) = \cos t\,\mathbf{i} + \operatorname{sen} t\,\mathbf{j} + \boxed{\ln} \cos t\,\mathbf{k}, \quad 0 \leqslant t \leqslant \pi/4$

$$L = \int_a^b |\vec{r}\,'(t)|\,dt$$ → p/ curva expressa como uma função vetorial $\mathbb{R} \to \mathbb{R}^3$

Atenção

$$\int_a^b \sqrt{1 + f'(x)^2}\,dx$$ → p/ curva expressa como uma função de $\mathbb{R} \to \mathbb{R}$

$$\vec{r}\,' = -\operatorname{sen} t\,\hat{i} + \cos t\,\hat{j} + \frac{1}{\cos t} \cdot (-\operatorname{sen} t)\,\hat{k}$$

$$\vec{r}\,' = \langle -\operatorname{sen} t, \cos t, -\operatorname{tg} t \rangle$$

$$|\vec{r}\,'| = \sqrt{(-\operatorname{sen} t)^2 + (\cos t)^2 + (-\operatorname{tg} t)^2}$$

$$|\vec{r}\,'| = \sqrt{1 + \operatorname{tg} t^2}$$

$$|\vec{r}\,'| = \sqrt{\sec^2 t}$$

$$L = \int_0^{\pi/4} \sqrt{\sec^2 t}\,dt$$

$$L = \int_0^{\pi/4} \sec t\,dt$$

$\sec t = \dfrac{1}{\cos t}$

$0 \leq t \leq \frac{\pi}{4}$

$\cos t > 0$

Então

$\sec t > 0$

$$L = \ln|\sec t + \operatorname{tg} t|\Big|_0^{\pi/4}$$

$$L = \ln\left|\sec\tfrac{\pi}{4} + \operatorname{tg}\tfrac{\pi}{4}\right| - \ln|\sec 0 + \operatorname{tg} 0|$$

$$L = \ln\left|\frac{2}{\sqrt{2}} + 1\right| - \ln|1 + 0|$$

$$L = \ln\left(\frac{2}{\sqrt{2}} + 1\right)$$

$$L = \ln\left(\frac{2 + \sqrt{2}}{\sqrt{2}}\right)$$

$$L = \ln\left(\frac{2\sqrt{2} + 2}{2}\right)$$

$$L = \ln(\sqrt{2} + 1)$$

$$\sec\frac{\pi}{4} = \frac{1}{\cos\frac{\pi}{4}}$$

Funções Trigonométricas

Ver artigo principal: Lista de integrais de funções trigonométricas

- $\int \cos x\,dx = \operatorname{sen} x + C$
- $\int \operatorname{sen} x\,dx = -\cos x + C$
- $\int \operatorname{tg} x\,dx = -\ln|\cos x| + C$[12]
- $\int \operatorname{cossec} x\,dx = \ln|\operatorname{cossec} x - \operatorname{cotg} x| + C$
- $\boxed{\int \sec x\,dx = \ln|\sec x + \operatorname{tg} x| + C}$
- $\int \operatorname{cotg} x\,dx = \ln|\operatorname{sen} x| + C$
- $\int \sec x \operatorname{tg} x\,dx = \sec x + C$
- $\int \operatorname{cossec} x \operatorname{cotg} x\,dx = -\operatorname{cossec} x + C$
- $\int \sec^2 x\,dx = \operatorname{tg} x + C$
- $\int \operatorname{cossec}^2 x\,dx = -\operatorname{cotg} x + C$
- $\int \operatorname{sen}^2 x\,dx = \frac{1}{2}(x - \operatorname{sen} x \cos x) + C$
- $\int \cos^2 x\,dx = \frac{1}{2}(x + \operatorname{sen} x \cos x) + C$

Reparametrização

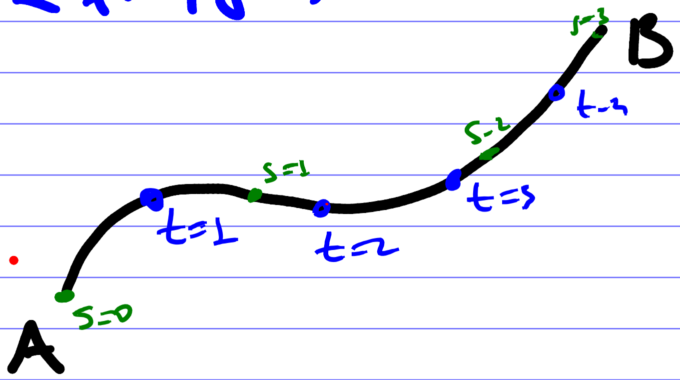
$\vec{r}(t) = \langle \cos t, \operatorname{sen} t \rangle$

$\vec{r}(\alpha) = \langle \cos \alpha, \operatorname{sen} \alpha \rangle$

Reparametrização SEM mudança de significado.

Vamos fazer uma Reparametrização COM mudança de significado.

$\vec{r}(t) = \langle f(t), g(t) \rangle$



$\vec{r}(s) = \langle h(s), l(s) \rangle$

Função comprimento de arco.

$$s = \int_0^t |\vec{r}\,'(\tau)|\, d\tau$$

$\vec{r}(t) = \langle \cos t, \operatorname{sen} t, t \rangle$

$$\boxed{\vec{r}(\tau) = \langle \cos \tau, \operatorname{sen} \tau, \tau \rangle}$$

$\vec{r}\,' = \langle -\operatorname{sen} \tau, \cos \tau, 1 \rangle$

$|\vec{r}\,'| = \sqrt{(\operatorname{sen} \tau)^2 + (\cos \tau)^2 + 1^2} = \sqrt{2}$

Função $\Rightarrow$

$$s = \int_0^t \sqrt{2}\, d\tau$$

$s = \sqrt{2}\, t$

$t = \dfrac{s}{\sqrt{2}}$

$\vec{r}(t) = \langle \cos t, \operatorname{sen} t, t \rangle$

$$\boxed{\vec{r}(s) = \left\langle \cos\left(\frac{s}{\sqrt{2}}\right), \operatorname{sen}\left(\frac{s}{\sqrt{2}}\right), \frac{s}{\sqrt{2}} \right\rangle}$$

$\vec{T}(s) = \dfrac{d\vec{r}}{ds}$ $\left\{ \vec{T}(t) = \dfrac{\frac{d\vec{r}}{dt}}{\left|\frac{d\vec{r}}{dt}\right|} \right.$