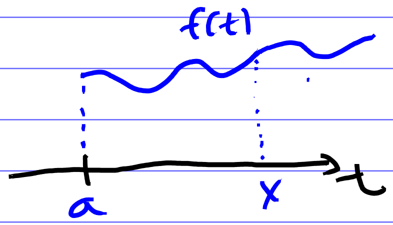
Primeira Parte do TFC

$$g(x) = \int_a^x f(t)\,dt$$



$\boxed{\begin{array}{c}\text{Variável arbitrária} \\ \int_a^b f(x)\,dx = \int_a^b f(t)\,dt = \int_a^b f(w)\,dw\end{array}}$

$\boxed{\begin{array}{c}\text{Antiderivada} \\ \int f(x)\,dx = g(x)\end{array}} \neq \boxed{\begin{array}{c}\text{A derivação é o processo inverso da integração.} \\ \frac{dg(x)}{dx} = f(x)\end{array}}$

Exemplo 1

a) $$\frac{d}{dx}\int_3^x \cos t\,dt = \cos x$$

b) $$\frac{d}{dx}\int_x^3 \cos t\,dt = \frac{d}{dx}\left(-\int_3^x \cos t\,dt\right)$$

$$= -\frac{d}{dx}\left(\int_3^x \cos t\,dt\right) = -\cos x$$

c) $$\frac{d}{dx}\int_3^x \frac{1}{1+t^3}\,dt = \frac{1}{1+x^3}$$

d) $$\frac{d}{dx}\int_3^{x^2} \frac{1}{1+t^3}\,dt = \frac{d}{du}\int_3^{u} \frac{1}{1+t^3}\,dt \cdot \frac{du}{dx}$$

$$= \frac{1}{1+u^3}\cdot 2x$$

$$= \frac{2x}{1+x^6}$$

Notação de Leibniz

$$\frac{df}{dx} = \frac{df}{du}\cdot\frac{du}{dx}$$

$$\frac{dg}{dx} = \frac{dg}{du}\cdot\frac{du}{dv}\cdot\frac{dv}{dw}\cdot\frac{dw}{dx}$$

Notação de Lagrange

$$f' = f'(u)\,u'$$

Notação de Newton

$\dot{x}$ $\ddot{x}$

Refuerzo

$$\frac{d}{dx}\left(\int_3^{x^2} \frac{1}{1+t^3}\,dt\right) =$$

$$= \frac{d}{du}\left(\int_3^{u} \frac{1}{1+t^3}\,dt\right) \cdot \frac{du}{dx}$$

$$= \frac{1}{1+u^3} \cdot 2x = \frac{2x}{1+(x^2)^3}$$

$$= \frac{2x}{1+x^6}$$

e) $$\frac{d}{dx}\int_{-3}^{\cos(x)} \frac{1}{3+t^5}\,dt =$$

$$= \frac{d}{du}\int_{-3}^{u} \frac{1}{3+t^5}\,dt \cdot \frac{du}{dx}$$

$$= \frac{1}{3+u^5} \cdot (-\operatorname{sen} x)$$

$$= \frac{-\operatorname{sen} x}{3+(\cos x)^5} = \frac{-\operatorname{sen} x}{3+\cos^5 x}$$

$$\boxed{\text{Calcule } \frac{dg(x)}{dx}: \quad \textbf{55. } g(x) = \int_{2x}^{3x} \frac{u^2-1}{u^2+1}\,du}$$

$$\frac{d}{dx}\int_{2x}^{3x} \frac{u^2-1}{u^2+1}\,du$$

$$= \frac{d}{dx}\left(\int_{2x}^{a} \frac{u^2-1}{u^2+1}\,du + \int_{a}^{3x} \frac{u^2-1}{u^2+1}\,du\right)$$

Usemos essa Propriedade

$$\boxed{\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx}$$

$$= \frac{d}{dx}\left(\int_{2x}^{a} \frac{u^2-1}{u^2+1}\,du\right) + \frac{d}{dx}\left(\int_{a}^{3x} \frac{u^2-1}{u^2+1}\,du\right)$$

$$= \left(-\frac{d}{dx}\int_{a}^{2x} \frac{u^2-1}{u^2+1}\,du\right) + \left(\frac{d}{dx}\int_{a}^{3x} \frac{u^2-1}{u^2+1}\,du\right)$$

$$= \left(-\frac{d}{dw}\int_{a}^{w} \frac{u^2-1}{u^2+1}\,du \cdot \frac{dw}{dx}\right) + \left(\frac{d}{dv}\int_{a}^{v} \frac{u^2-1}{u^2+1}\,du \cdot \frac{dv}{dx}\right)$$

$$w = 2x \qquad\qquad v = 3x$$

$$= \left(-\frac{w^2-1}{w^2+1} \cdot 2\right) + \left(\frac{v^2-1}{v^2+1} \cdot 3\right)$$

$$= -2\left(\frac{(2x)^2-1}{(2x)^2+1}\right) + 3\left(\frac{(3x)^2-1}{(3x)^2+1}\right) \Leftarrow \text{Resposta}$$

$$= -2\left(\frac{4x^2-1}{4x^2+1}\right) + 3\left(\frac{9x^2-1}{9x^2+1}\right) \Leftarrow$$

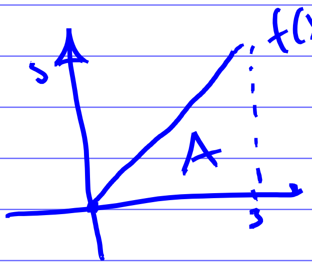
$$= \left(\frac{-8x^2+2}{4x^2+1}\right) + \left(\frac{27x^2-3}{9x^2+1}\right) \Leftarrow$$

TFC2

$$\int_a^b f(t)\,dt = F(b) - F(a)$$

Usando TFC2
$$\int_0^3 x\,dx = \frac{x^2}{2}\Big|_0^3 = \frac{3^2}{2} - \frac{0^2}{2} = \frac{9}{2}$$

Via Geometria

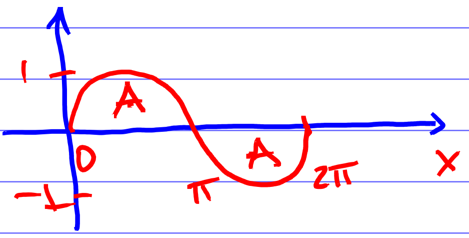


$A = \frac{9}{2}$

Via definição
$$\int_0^3 x\,dx = \lim_{n\to+\infty} \sum_{i=1}^{n} f(x_i)\,\Delta x$$

Exemplo:
$$\int_0^{2\pi} \operatorname{sen}(x)\,dx$$

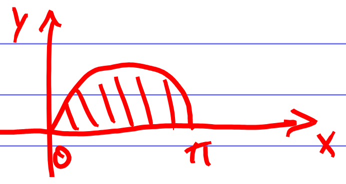
1ª via geometria



$$\int_0^{2\pi} \operatorname{sen}(x)\,dx = A - A = 0$$

2º via TFC
$$\int_0^{2\pi} \operatorname{sen} x\,dx = \left[-\cos x\right]\Big|_0^{2\pi}$$
$$= [-\cos(2\pi)] - [-\cos 0]$$
$$= [-1] - [-1]$$
$$= -1 + 1 = 0$$

Exemplo:



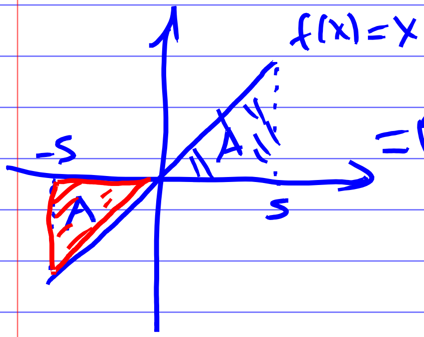
$$\int_0^{\pi} \operatorname{sen}(x)\,dx = \left[-\cos x\right]\Big|_0^{\pi}$$
$$= [-\cos\pi] - [-\cos 0]$$
$$= [-(-1)] - [-1]$$
$$= 1 + 1 = 2$$

38. (5.2) da Lista

$$\int_{-5}^{5}\left(x-\sqrt{25-x^2}\right)dx$$

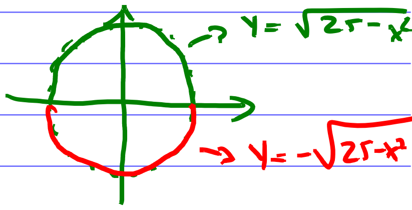
Fazer graficamente.

$$=\int_{-5}^{5}x\,dx-\int_{-5}^{5}\sqrt{25-x^2}\,dx$$



$=0$

$$x^2+y^2=25$$



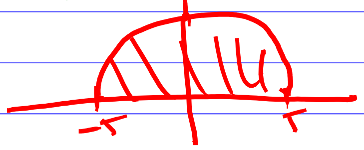
$$y^2=25-x^2$$

$$y=\sqrt{25-x^2}$$

ou

$$y=-\sqrt{25-x^2}$$

$$\sqrt{25-x^2}$$



$$A=\pi r^2$$

$$A=\frac{25\pi}{2}$$

$$\int_{-5}^{5}\left(x-\sqrt{25-x^2}\right)dx=0-\frac{25\pi}{2}=-\frac{25\pi}{2}$$