1

1) Qual é o comprimento de arco da curva dada por $r = 1 + \cos(\theta),\quad 0 \leq \theta \leq \frac{\pi}{3}$?

$$L = \int_{\theta_i}^{\theta_f} \sqrt{r^2 + (r')^2}\, d\theta$$

$$r^2 = (1+\cos\theta)^2 = 1 + 2\cos\theta + \cos^2\theta$$

$$r' = -\operatorname{sen}\theta \quad \therefore \quad (r')^2 = \operatorname{sen}^2\theta$$

$$r^2 + (r')^2 = 1 + 2\cos\theta + \overbrace{\cos^2\theta + \operatorname{sen}^2\theta}^{=1}$$

$$= 2 + 2\cos\theta$$

$$L = \int_0^{\frac{\pi}{3}} \sqrt{2 + 2\cos\theta}\, d\theta$$

$$\cos^2\theta = \frac{1}{2} + \frac{1}{2}\cos(2\theta)$$

$$4\cos^2\theta = 2 + 2\cos(2\theta)$$

$$4\cos^2\left(\frac{\theta}{2}\right) = 2 + 2\cos(\theta)$$

$$L = \int_0^{\pi/3} \sqrt{4\cos^2\left(\frac{\theta}{2}\right)}\, d\theta$$

$$L = \int_0^{\pi/3} 2\cos\left(\frac{\theta}{2}\right) d\theta$$

Pois no intervalo $\cos\left(\frac{\theta}{2}\right) \geq 0$

Substituição simples:

$$u = \frac{\theta}{2} \quad \therefore \quad \frac{du}{d\theta} = \frac{1}{2} \quad \therefore \quad du = \frac{1}{2} d\theta \quad \therefore \quad 2du = d\theta$$

Quando $\theta = 0$, $u = 0$

" $\theta = \frac{\pi}{3}$, $u = \frac{\pi}{6}$

$$L = \int_0^{\pi/6} 2\cos(u)\, 2\, du$$

$$L = 4\int_0^{\pi/6} \cos(u)\, du$$

$$L = 4\ \operatorname{sen} u \Big|_0^{\pi/6}$$

$$L = 4\left(\operatorname{sen}\frac{\pi}{6} - \operatorname{sen} 0\right)$$

$\boxed{L = 2}$ u.c.

2

1) Qual é o valor de k que faz com que a curva dada por $\vec{r}(t) = < 3 \cdot \cos(t),\ 3 \cdot \sin(t),\ k \cdot t >$, com $0 \leq t \leq \pi$ possua comprimento igual a $\sqrt{13}\,\pi$?

$$L = \int_{t_i}^{t_f} |\vec{r}\,'(t)|\, dt$$

$$\vec{r}\,' = < -3\,\text{sen}\,t,\ 3\cos t,\ k >$$

$$|\vec{r}\,'| = \sqrt{9\,\text{sen}^2 t + 9\cos^2 t + k^2}$$

$$|\vec{r}\,'| = \sqrt{9 + k^2}$$

$$L = \int_0^{\pi} \sqrt{9 + k^2}\, dt$$

$$L = \sqrt{9 + k^2} \int_0^{\pi} dt$$

$$L = \sqrt{9 + k^2}\,\pi$$

Queremos

$$\sqrt{13}\,\pi = \sqrt{9 + k^2}\,\pi$$

$$\sqrt{13} = \sqrt{9 + k^2}$$

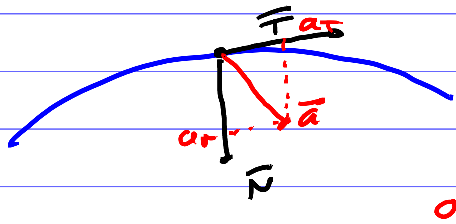
$$13 = 9 + k^2$$

$$4 = k^2 \qquad k = -2 \text{ ou } \boxed{k = 2}$$

No gabarito só existiam números positivos

3

1) Qual é o **módulo da componente tangencial** (em $\frac{m}{s^2}$) da aceleração de uma partícula cuja velocidade escalar no instante em questão é 2 m/s, cuja aceleração escalar é $10\ \frac{m}{s^2}$ e cuja curvatura do ponto onde se encontra é $2\ m^{-1}$?



$$a^2 = a_T^2 + a_N^2$$

$$a^2 - a_N^2 = a_T^2$$

$$a_T^2 = a^2 - a_N^2$$

$$a_T = \sqrt{a^2 - a_N^2}$$

Como quero. $|a_T| = +\sqrt{a^2 - a_N^2}$

Atenção

$a_N = kv^2$

$$|a_T| = +\sqrt{a^2 - a_N^2}$$

$$|a_T| = \sqrt{10^2 - (kv^2)^2}$$

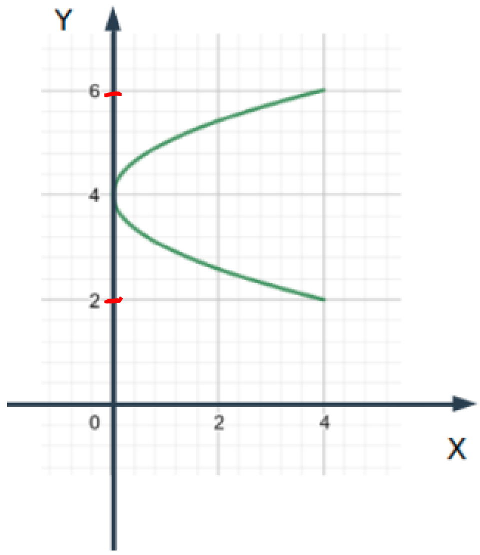
$$|a_T| = \sqrt{10^2 - (2 \cdot 2^2)^2}$$

$$|a_T| = \sqrt{10^2 - (8)^2}$$

$$|a_T| = \sqrt{100 - 64} = \sqrt{36} = 6$$

4

1) Qual é a função vetorial cujo gráfico está ilustrado na figura a seguir?



$x = (y-4)^2$

$2 \leq y \leq 6$

$x = (\underbrace{y-4}_{t})^2$

$x = t^2$

$y - 4 = t \therefore y = t + 4$

$\vec{r}(t) = \langle t^2, t+4 \rangle$

Se o y vai de 2 a 6

o t que é igual y−4

varia de −2 a 2

$-2 \leq t \leq 2$.

(5)

1) Em qual ponto da curva $\vec{r}(t) = \langle \cos(t), \sin(t), t^2 \rangle$ a reta tangente é paralela ao vetor $\langle -3, 0, 3\pi \rangle$?

$$\vec{r}\,'(t) = \langle -\text{sen}\,t, \cos t, 2t \rangle$$

Quando $\vec{r}\,'(t)$ é paralelo a $\langle -3, 0, 3\pi \rangle$?

$$\vec{r}\,'(t) \times \langle -3, 0, 3\pi \rangle = \langle 0, 0, 0 \rangle$$

$$\begin{array}{ccccc} \hat{i} & \hat{j} & \hat{k} & \hat{i} & \hat{j} \\ -\text{sen}\,t & \cos t & 2t & -\text{sen}\,t & \cos t \\ -3 & 0 & 3\pi & -3 & 0 \end{array}$$

$$= 3\cos t\,\hat{k} + 3\pi\,\text{sen}\,t\,\hat{j} + 3\pi \cos t\,\hat{i} - 6t\,\hat{j}$$

$$= \langle 3\pi \cos t, 3\pi\,\text{sen}\,t - 6t, 3\cos t \rangle = \langle 0, 0, 0 \rangle$$

P/ 1ª e 3ª componentes $t = \frac{\pi}{2}, \frac{3\pi}{2}, \dots$

$$3\pi\,\text{sen}\,t - 6t = 0$$

testando p/ $t = \frac{\pi}{2}$ $\Rightarrow$ $3\pi\,\text{sen}\left(\frac{\pi}{2}\right) - 6\left(\frac{\pi}{2}\right) \stackrel{?}{=} 0$

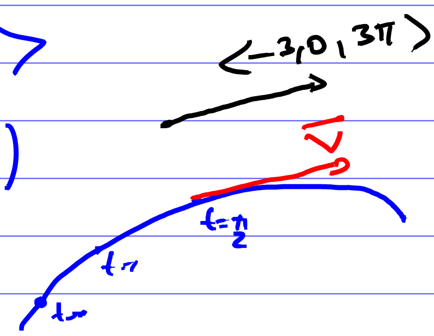
$3\pi - 3\pi \stackrel{?}{=} 0$ SIM

o t procurado é $t = \frac{\pi}{2}$.

$$\vec{r}(t) = \langle \cos t, \text{sen}\,t, t^2 \rangle$$

$$\vec{r}\left(\frac{\pi}{2}\right) = \left\langle 0, 1, \frac{\pi^2}{4} \right\rangle$$

Esse é o ponto: $\left(0, 1, \frac{\pi^2}{4}\right)$



1) Qual é o vetor binormal no ponto $t = \pi$ para a trajetória descrita por $\vec{r}(t) = < \cos(t), t, \sin(t) >$?

a) $< 0, \frac{-\sqrt{2}}{2}, \frac{-\sqrt{2}}{2} >$
b) $< 0, \frac{-\sqrt{2}}{2}, \frac{\sqrt{2}}{2} >$
c) $< \frac{-\sqrt{2}}{2}, \frac{-\sqrt{2}}{2}, 0 >$
d) $< \frac{-1}{2}, \frac{-\sqrt{2}}{2}, \frac{1}{2} >$
e) $< \frac{\sqrt{2}}{2}, \frac{-\sqrt{2}}{2}, 0 >$
f) $< 1, 0, 0 >$
g) $< 0, 1, 0 >$
h) $< 0, 0, 1 >$

$$\vec{B} = \vec{T} \times \vec{N}$$

$$\vec{B}(\pi) = \vec{T}(\pi) \times \vec{N}(\pi)$$

$$\vec{T} = \frac{\vec{r}\,'}{|\vec{r}\,'|} = \frac{<-\text{sen}\,t, 1, \cos t>}{\sqrt{\text{sen}^2 t + 1 + \cos^2 t}} = \frac{<-\text{sen}\,t, 1, \cos t>}{\sqrt{2}}$$

$$\vec{T} = \frac{1}{\sqrt{2}} <-\text{sen}\,t, 1, \cos t>$$

$$\vec{N} = \frac{\vec{T}'}{|\vec{T}'|} = \frac{\frac{1}{\sqrt{2}} <-\cos t, 0, -\text{sen}\,t>}{\frac{1}{\sqrt{2}} \sqrt{\cos^2 t + \text{sen}^2 t}}$$

$$\vec{N} = <-\cos t, 0, -\text{sen}\,t>$$

$$\vec{T}(\pi) = \frac{1}{\sqrt{2}} <-\text{sen}\,\pi, 1, \cos \pi> = \frac{1}{\sqrt{2}} <0, 1, -1>$$

$$\vec{N}(\pi) = <-\cos \pi, 0, -\text{sen}\,\pi> = <1, 0, 0>$$

$$\vec{B}(\pi) = \vec{T}(\pi) \times \vec{N}(\pi)$$

$$\vec{B}(\pi) = \frac{1}{\sqrt{2}} <0, 1, -1> \times <1, 0, 0>$$

$$\vec{B}(\pi) = <0, \frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}> \times <1, 0, 0>$$

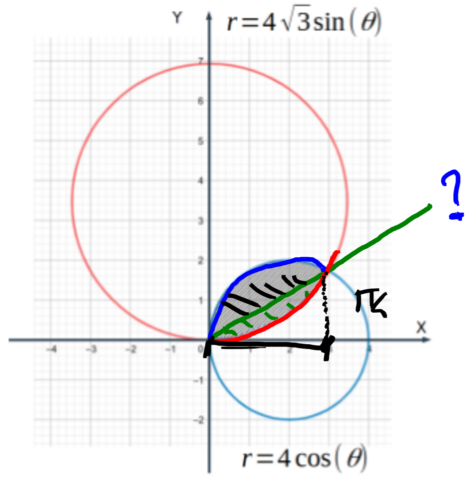
$$= \begin{array}{ccccc} \hat{i} & \hat{j} & \hat{k} & \hat{i} & \hat{j} \\ 0 & \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} \\ 1 & 0 & 0 & 1 & 0 \end{array}$$

$$-\frac{\sqrt{2}}{2}\hat{k} \qquad -\frac{\sqrt{2}}{2}\hat{j}$$

$$= <0, -\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}>$$

7

1) Qual é a expressão que fornece corretamente a área de interseção entre os dois círculos da figura? (A área hachurada).



$$4\cos\theta = 4\sqrt{3}\,\mathrm{sen}\,\theta$$

$$\cos\theta = \sqrt{3}\,\mathrm{sen}\,\theta$$

$$\frac{1}{\sqrt{3}} = \mathrm{tg}\,\theta$$

Fui na tabela em

$$\theta = \mathrm{arctg}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$$

$$A = \int_{\theta_i}^{\theta_f} \frac{1}{2} r^2\, d\theta = \boxed{\int_0^{\pi/6} \frac{1}{2}\left(4\sqrt{3}\,\mathrm{sen}\,\theta\right)^2 d\theta + \int_{\pi/6}^{\pi/2} \frac{1}{2}\left(4\cos\theta\right)^2 d\theta}$$

$\int_0^{\frac{\pi}{6}} \frac{1}{2}\left(\underline{4\sqrt{3}\sin(\theta)}\right)^2 d\theta + \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{1}{2}\left(\underline{4\cos(\theta)}\right)^2 d\theta$

$\int_0^{\frac{\pi}{6}} \frac{1}{2}\left(4\cos(\theta)\right)^2 d\theta + \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{1}{2}\left(4\sqrt{3}\sin(\theta)\right)^2 d\theta$

$\int_0^{\frac{\pi}{4}} \frac{1}{2}\left(4\sqrt{3}\sin(\theta)\right)^2 d\theta + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{1}{2}\left(4\cos(\theta)\right)^2 d\theta$

$\int_0^{\frac{\pi}{4}} \frac{1}{2}\left(4\cos(\theta)\right)^2 d\theta + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{1}{2}\left(4\sqrt{3}\sin(\theta)\right)^2 d\theta$

$\int_0^{2\pi} \frac{1}{2}\left(4\sqrt{3}\sin(\theta)\right)^2 d\theta - \int_0^{2\pi} \frac{1}{2}\left(4\cos(\theta)\right)^2 d\theta$

$\int_0^{2\pi} \frac{1}{2}\left(4\cos(\theta)\right)^2 d\theta - \int_0^{2\pi} \frac{1}{2}\left(4\sqrt{3}\sin(\theta)\right)^2 d\theta$

$\int_0^{\frac{\pi}{2}} \frac{1}{2}\left(4\cos(\theta)\right)^2 d\theta - \int_0^{\frac{\pi}{2}} \frac{1}{2}\left(4\sqrt{3}\sin(\theta)\right)^2 d\theta$

$\int_0^{\frac{\pi}{2}} \frac{1}{2}\left(4\sqrt{3}\sin(\theta)\right)^2 d\theta - \int_0^{\frac{\pi}{2}} \frac{1}{2}\left(4\cos(\theta)\right)^2 d\theta$

8

1) Qual das seguintes funções vetoriais é perpendicular à própria derivada?

a) $< \cos(t^2), \sin(t^2), 1 >$
b) $< \cos(t^2), \sin(t^2), t >$
c) $< \cos(t), \sin(t), t >$
d) $< 5, 3, t >$
e) $< \cos(t), t, t^2 >$
f) $< \cos(t), \sin(t), e^t >$

$\sqrt{\cos^2(t^2) + \sin^2(t^2) + 1}$

$\to \sqrt{2}$

$\sqrt{5^2 + 3^2 + t^2}$

Como a letra c) tem módulo constante, então ela é a função procurada.

$$|\vec{B}| = c$$

$$\vec{B} \circ \vec{B} = c^2$$

$$\frac{d}{dt}(\vec{B} \circ \vec{B}) = 0$$

$$\vec{B}' \circ \vec{B} + \vec{B} \circ \vec{B}' = 0$$

$$2\vec{B} \circ \vec{B}' = 0$$

$$\vec{B} \circ \vec{B}' = 0$$