15–40 Determine as derivadas parciais de primeira ordem da função.

15. $f(x, y) = y^5 - 3xy$ **16.** $f(x, y) = x^4y^3 + 8x^2y$

$\boxed{\textbf{33.}\ w = \ln(x + 2y + 3z)}$ **18.** $f(x, t) = \sqrt{x}\ln t$

$\boxed{\textbf{39.}\ u = \sqrt{x_1^2 + x_2^2 + \cdots + x_n^2}}$

15. $f(x,y) = y^5 - 3xy$

$$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}(y^5 - 3xy) = \frac{\partial y^5}{\partial x} - \frac{\partial}{\partial x}(3xy)$$
$$= 0 - 3y$$
$$\frac{\partial f}{\partial x} = -3y$$

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}(y^5 - 3xy)$$
$$= \frac{\partial y^5}{\partial y} - \frac{\partial}{\partial y}(3xy)$$
$$\frac{\partial f}{\partial y} = 5y^4 - 3x$$

$$\boxed{\frac{d\,10y}{dy} = 10}$$

16 $f(x,y) = x^4y^3 + 8x^2y$

$$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}(x^4y^3 + 8x^2y)$$
$$= \frac{\partial}{\partial x}(x^4y^3) + \frac{\partial}{\partial x}(8x^2y)$$
$$\frac{\partial f}{\partial x} = 4x^3y^3 + 16xy$$

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}(x^4y^3 + 8x^2y)$$
$$= \frac{\partial}{\partial y}(x^4y^3) + \frac{\partial}{\partial y}(8x^2y)$$
$$= x^4\frac{\partial}{\partial y}y^3 + 8x^2\frac{\partial}{\partial y}y$$
$$= x^4\,3\,y^2 + 8x^2$$
$$= 3x^4y^2 + 8x^2$$

33. $W = \ln(x + 2y + 3z)$

Como não há contexto, $W = W(x,y,z)$

> $f(x,y,z) = x$
> f é uma função de 3 variáveis.
>
> $g = x$
> Eu tenho que considerar o contexto se não houver contexto $g = g(x)$.

Derivada de dentro

$$\frac{\partial W}{\partial x} = \frac{1}{x + 2y + 3z} \cdot \frac{\partial}{\partial x}(x + 2y + 3z)$$

Derivada de ln

$$\boxed{\frac{\partial W}{\partial x} = \frac{1}{x + 2y + 3z}}$$

$$\frac{\partial W}{\partial y} = \frac{1}{x + 2y + 3z} \cdot \frac{\partial}{\partial y}(x + 2y + 3z)$$
$$\boxed{\frac{\partial W}{\partial y} = \frac{2}{x + 2y + 3z}}$$

$$\frac{\partial W}{\partial z} = \frac{1}{x + 2y + 3z} \cdot \frac{\partial}{\partial z}(x + 2y + 3z)$$
$$\boxed{\frac{\partial W}{\partial z} = \frac{3}{x + 2y + 3z}}$$

18. $f(x,t) = \sqrt{x}\ln t$

$$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}(\sqrt{x}\ln t) = \ln t\frac{\partial}{\partial x}\sqrt{x}$$
$$= \ln t\frac{\partial}{\partial x}(x^{1/2})$$
$$= \ln t\,\frac{1}{2}x^{-1/2}$$
$$= \ln t\,\frac{1}{2\sqrt{x}}$$
$$\boxed{\frac{\partial f}{\partial x} = \frac{1}{2}\frac{\ln t}{\sqrt{x}}}$$

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial t}(\sqrt{x}\ln t) = \sqrt{x}\frac{\partial}{\partial t}(\ln t)$$
$$= \sqrt{x}\,\frac{1}{t}$$
$$\boxed{\frac{\partial f}{\partial t} = \frac{\sqrt{x}}{t}}$$

39 $U = \sqrt{x_1^2 + x_2^2 + \cdots + x_n^2}$

$U = U(x_1, x_2, x_3, \ldots, x_n)$

Lê-se: "U é função de $x_1, x_2, \ldots, x_n$"

$$\frac{\partial U}{\partial x_1} = \frac{\partial}{\partial x_1}(x_1^2 + x_2^2 + \cdots + x_n^2)^{1/2}$$
$$= \frac{1}{2}(x_1^2 + x_2^2 + \cdots + x_n^2)^{\frac{1}{2}-1} \cdot \frac{\partial}{\partial x_1}(x_1^2 + x_2^2 + \cdots + x_n^2)$$
$$= \frac{1}{2}\frac{2x_1}{\sqrt{x_1^2 + x_2^2 + \cdots + x_n^2}}$$

$$\frac{\partial U}{\partial x_1} = \frac{x_1}{\sqrt{x_1^2 + x_2^2 + \cdots + x_n^2}}$$
$$\frac{\partial U}{\partial x_2} = \frac{x_2}{\sqrt{x_1^2 + x_2^2 + \cdots + x_n^2}}$$
$$\frac{\partial U}{\partial x_3} = \frac{x_3}{\sqrt{x_1^2 + x_2^2 + \cdots + x_n^2}}$$
$$\vdots$$
$$\frac{\partial U}{\partial x_i} = \frac{x_i}{\sqrt{x_1^2 + x_2^2 + \cdots + x_n^2}}$$

A derivada Ã© funÃ§Ã£o das mesmas variÃ¡veis da funÃ§Ã£o original.

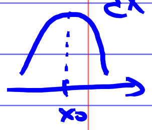
$$\frac{\partial U}{\partial x_i} = \frac{x_i}{U}$$

Vai aparecer em alguma fun. contexto específico

NÃO SE PREOCUPAR AGORA.

Significado da derivada

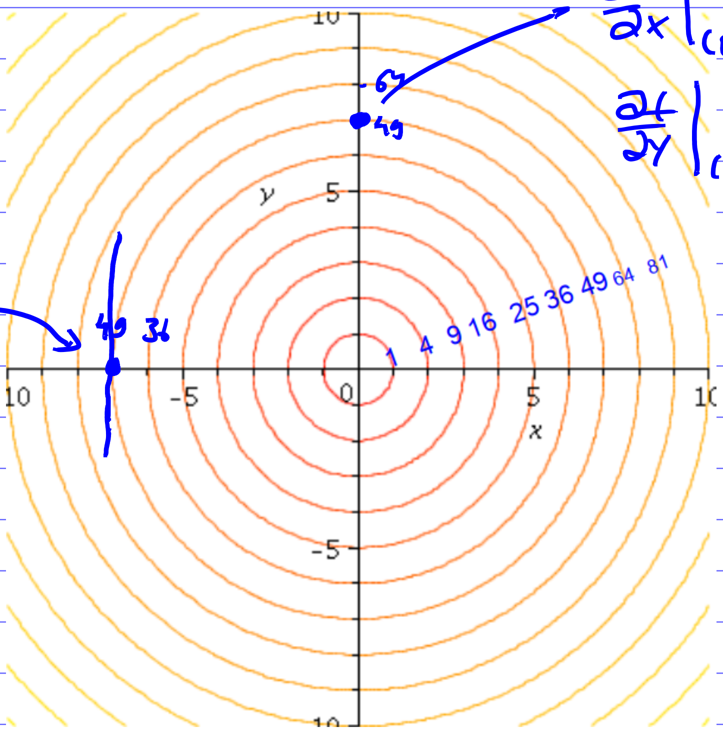
$$\frac{df}{dx}\Big|_{x_0}=0$$



$$\frac{\partial f}{\partial x}\Big|_{(0,7)}=0$$

$$\frac{\partial f}{\partial y}\Big|_{(0,7)}>0$$

$$\frac{\partial f}{\partial x}\Big|_{(-7,0)}$$



$$\frac{\partial f}{\partial x}\Big|_{(-7,0)}<0$$

$$\frac{\partial f}{\partial y}\Big|_{(-7,0)}=0$$

$$\frac{\partial f}{\partial x}\Big|_{(4,0)} \approx \frac{f(4+1,0)-f(4,0)}{1}$$

$$\approx \frac{25-16}{1}$$

$$\approx 9$$

$$\frac{\partial f}{\partial y}\Big|_{(4,0)}=0 \quad \left\{ \begin{array}{l} \text{É a estimativa} \\ \frac{\partial f}{\partial y}\Big|_{(4,0)} \approx \frac{f(4,0+1)-f(4,0)}{1} \end{array} \right.$$

$$\frac{\partial f}{\partial y}\Big|_{(4,0)} \approx \frac{18-16}{1}=2$$

$$\frac{\partial f}{\partial y}\Big|_{(4,0)} \approx 2$$

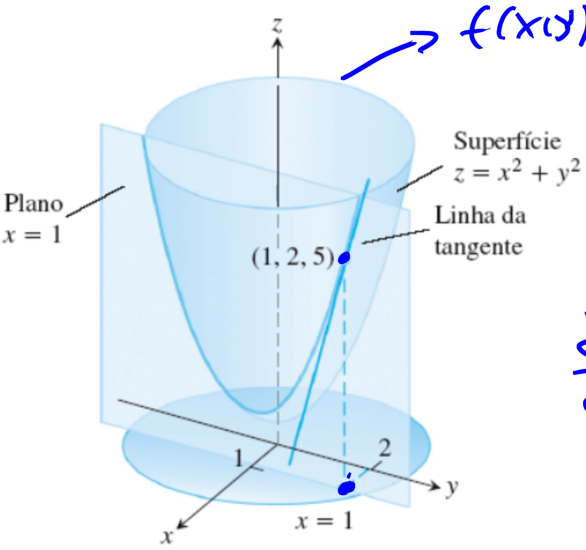
Fazendo a conta exata

$$f(x,y)=x^2+y^2$$

$$\frac{\partial f}{\partial x}=2x \qquad \frac{\partial f}{\partial y}=2y$$

$$\frac{\partial f}{\partial x}\Big|_{(4,0)}=8 \quad \left\{ \quad \frac{\partial f}{\partial y}\Big|_{(4,0)}=0 \right.$$

$f(x,y) = x^2 + y^2$

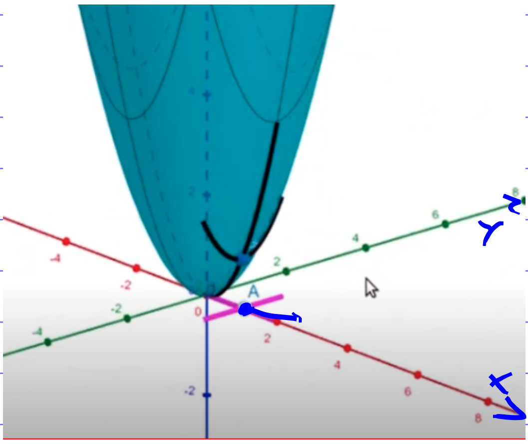


$\left.\dfrac{\partial f}{\partial y}\right|_{(1,2)}$

4

x constante

FIGURA 14.16 A tangente à curva de interseção do plano $x = 1$ e da superfície $z = x^2 + y^2$ no ponto $(1, 2, 5)$ (Exemplo 5).



$\left.\dfrac{\partial f}{\partial x}\right|_{A} > 0$

$\left.\dfrac{\partial f}{\partial y}\right|_{A} = 0$