Regra da cadeia

$f = f(x) \qquad x = x(t)$

$$\frac{df}{dt} = \frac{df}{dx} \cdot \frac{dx}{dt}$$

$f = \cos(x)$ e $x = t^2$

$$\frac{df}{dt} = -\operatorname{sen} x \cdot 2t$$

$$= -\operatorname{sen}(t^2)\, 2t$$

f pode ser função de várias variáveis

$f = f(x, y, z) \qquad x = x(t)$
$y = y(t)$
$z = z(t)$

$f(x, y, z) = x^2 y z \qquad x = t^2$
$y = t$
$z = e^t$

$$f(t) = t^4 t e^t = t^5 e^t$$

Diretamente

$$\frac{df}{dt} = \frac{d}{dt}(t^5 e^t) = 5t^4 e^t + t^5 e^t$$

Via regra da cadeia

$$\frac{df}{dt} = \frac{\partial f}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial f}{\partial y} \frac{dy}{dt} + \frac{\partial f}{\partial z} \cdot \frac{dz}{dt}$$

$x = t^2$	$y = t$	$z = e^t$
$f(x, y, z) = x^2 y z$		

$$\frac{df}{dt} = 2xyz \cdot 2t + x^2 z \cdot 1 + x^2 y\, e^t$$

$$= 2t^2 t e^t 2t + t^4 e^t + t^4 t e^t$$

$$= 4t^4 e^t + t^4 e^t + t^5 e^t$$
$$= 5t^4 e^t + t^5 e^t$$

Regra da Cadeia

1. Sabendo que $h=h(m,n)$ e que $m=m(u)$ e $n=n(u)$, escreva a expressão para o cálculo de

$$\frac{dh}{du}$$

$$\frac{dh}{du} = \frac{\partial h}{\partial m}\frac{dm}{du} + \frac{\partial h}{\partial n}\frac{dn}{du}$$

Atenção

$$f(x,y) = xy$$

$$\boxed{\frac{\partial f}{\partial x} = y}$$

$$\frac{df}{dx} = \frac{\partial f}{\partial x}\cdot\frac{dx}{dx} + \frac{\partial f}{\partial y}\cdot\frac{dy}{dx}$$

$$\boxed{\frac{df}{dx} = y + x\frac{dy}{dx}}$$

Expresse $\dfrac{\partial w}{\partial r}$ e $\dfrac{\partial w}{\partial s}$ em termos de *r* e *s*.

$$w = x + 2y + z^2$$

$$x(r,s) = \frac{r}{s}$$

$$y(r,s) = r^2 + \ln s$$

$$z(r,s) = 2r$$

$$W = W(x, y, z)$$

$$x = x(r, s)$$

$$y = y(r, s)$$

$$z = z(r, s)$$

$$\frac{\partial w}{\partial r} = \frac{\partial w}{\partial x} \cdot \frac{\partial x}{\partial r} + \frac{\partial w}{\partial y} \frac{\partial y}{\partial r} + \frac{\partial w}{\partial z} \frac{\partial z}{\partial r}$$

$$= 1 \cdot \frac{1}{s} + 2 \cdot 2r + 2z\, 2$$

$$= \frac{1}{s} + 4r + 4(2r)$$

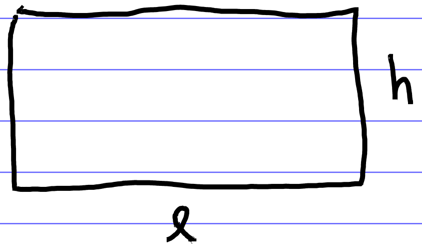
$$= \frac{1}{s} + 4r + 8r = \frac{1}{s} + 12r$$

$$\frac{\partial w}{\partial s} = \frac{\partial w}{\partial x} \cdot \frac{\partial x}{\partial s} + \frac{\partial w}{\partial y} \frac{\partial y}{\partial s} + \frac{\partial w}{\partial z} \frac{\partial z}{\partial s}$$

Vocês tentarão fazer esta.

Regra da Cadeia

Os lados de um retângulo imaginário variam com o tempo. A largura varia a uma taxa de 4 *t* m/s e altura varia a uma taxa de 5 t^2 m/s. A que taxa varia a área do retângulo no instante *t*=2, quando sua altura mede 2 m e sua largura mede 3 m ?



$$A = A(\ell, h) = \ell h$$

$$\frac{d\ell}{dt} = 4t \ \frac{m}{s} \qquad \frac{dh}{dt} = 5t^2 \ \frac{m}{s}$$

$$\frac{dA}{dt} = \frac{\partial A}{\partial \ell} \cdot \frac{d\ell}{dt} + \frac{\partial A}{\partial h} \frac{dh}{dt}$$

$$\frac{dA}{dt} = h \cdot 4t + \ell \, 5t^2$$

$$\left.\frac{dA}{dt}\right|_{t=2} = h \cdot 4t + \ell \, 5t^2$$

$$= 2 \cdot 4(2) + 3 \cdot 5(2)^2$$

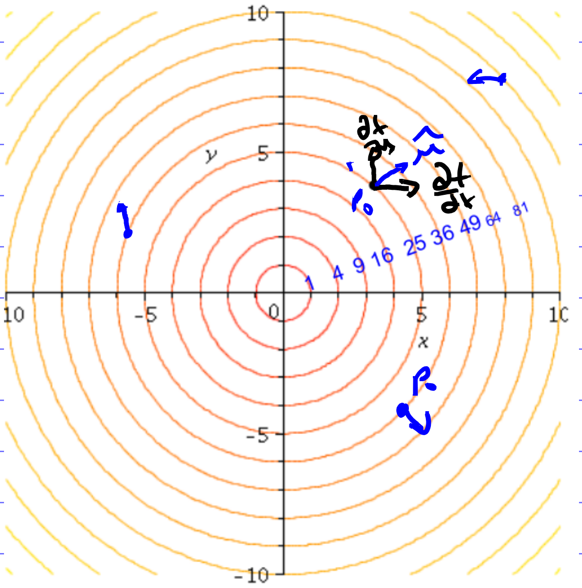
$$= 16 + 60$$

$$= 76 \ m^2/s$$

Derivada Direcional

$f = f(x,y)$

$$\frac{\partial f}{\partial x} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x, y) - f(x,y)}{\Delta x}$$



$|\hat{u}| = 1$ versor.

$$\left.\frac{df}{ds}\right|_{P_0,\hat{u}} = \lim_{s \to 0} \frac{f(x+su_1, y+su_2) - f(x,y)}{s}$$

$\hat{u} = \langle u_1, u_2 \rangle$

$$\left.\frac{df}{ds}\right|_{P_0,\hat{u}} = \left. Df \right|_{P_0,\hat{u}}$$

$$= \left.\left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right\rangle\right|_{P_0} \circ \hat{u}$$

Gradiente da função f.

$$= \left.\vec{\nabla} f\right|_{P_0} \circ \hat{u}$$

Exercício 1: Encontre a derivada da função em P_0 na direção de $\boldsymbol{A}$.

a) $f(x,y)=2xy-3y^2 \quad P_0(5,5), \quad \vec{A}=4\hat{i}+3\hat{j}$

b) $f(x,y,z)=xy+yz+zx$

$P_0(1,-1,2), \quad \vec{A}=3\hat{i}+6\hat{j}-2\hat{k}$

a) $\vec{\nabla} f \circ \hat{u}$

$$\hat{u} = \frac{\vec{A}}{|\vec{A}|} = \frac{\langle 4,3\rangle}{\sqrt{16+9}} = \langle \tfrac{4}{5}, \tfrac{3}{5}\rangle$$

$$\vec{\nabla} f = \left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right\rangle$$

$$\vec{\nabla} f = \langle 2y, 2x-6y \rangle$$

$$\vec{\nabla} f\Big|_{(5,5)} = \langle 10, 10-30\rangle$$
$$= \langle 10, -20\rangle$$

$$\left(\frac{df}{ds}\right)_{P_0,\hat{u}} = \vec{\nabla} f\Big|_{(5,5)} \circ \hat{u}$$

$$= \langle 10,-20\rangle \circ \langle \tfrac{4}{5}, \tfrac{3}{5}\rangle$$

$$= \frac{40}{5} - \frac{60}{5}$$

$$= 8 - 12$$
$$= -4$$