1. Calcule $\frac{\partial z}{\partial x}$ para $z + y = x^2$ ← Função implícita de 2 variáveis.

1º Passo: Explicitar

$$z = x^2 - y$$

$f(x,y) = x^2 - y$ ← Função explícita.

2º Passo: calcular

$$\frac{\partial z}{\partial x} = \frac{\partial}{\partial x}(x^2 - y) = 2x$$

"E em relação a y?"

$$z + y = x^2$$

$$z = x^2 - y$$

$$\frac{\partial z}{\partial y} = \frac{\partial}{\partial y}(x^2 - y) = \frac{\partial}{\partial y}(x^2) - \frac{\partial y}{\partial y}$$

$$= 0 - 1 = -1$$

2. Calcule $\frac{\partial z}{\partial x}$, $z^5 + z\cos(xy) + z^2 x = 0$

Como não consigo/quero explicitar eu calculo a derivada implícita

$$\frac{\partial}{\partial x}\left(z^5 + z\cos(xy) + z^2 x\right) = \frac{\partial 0}{\partial x}$$

Vou tratar o z que aparece na expressão como uma função de x.

$\frac{\partial z}{\partial x}$ → Qualquer letra que não seja z ou x seja tratada como constante.

$$\frac{\partial}{\partial x}(z^5) + \frac{\partial}{\partial x}(z\cos(xy)) + \frac{\partial}{\partial x}(z^2 x) = 0$$

$$\underbrace{5z^4\frac{\partial z}{\partial x}}_{\text{Regra da cadeia}} + \underbrace{\frac{\partial z}{\partial x}\cdot\cos(xy) + z\frac{\partial}{\partial x}\cos(xy)}_{\text{Regra do produto}} + \underbrace{\frac{\partial z^2}{\partial x}x + z^2\frac{\partial x}{\partial x}}_{\text{Regra do produto}} = 0$$

$$5z^4\frac{\partial z}{\partial x} + \frac{\partial z}{\partial x}\cos(xy) + z[-\text{sen}(xy)\cdot y] + 2z\frac{\partial z}{\partial x}\cdot x + z^2 = 0$$

$$\frac{\partial z}{\partial x}\left[5z^4 + \cos(xy) + 2xz\right] = yz\,\text{sen}(xy) - z^2$$

$$\frac{\partial z}{\partial x} = \frac{yz\,\text{sen}(xy) - z^2}{5z^4 + \cos(xy) + 2xz}$$

Exemplo mais simples

$$z\cos(xz) = y$$

$$\frac{\partial z}{\partial x} = ?$$

$$\frac{\partial}{\partial x}\left(z\cos(xz)\right) = \frac{\partial y}{\partial x}$$

$$\frac{\partial z}{\partial x}\cos(xz) + z\frac{\partial}{\partial x}\left(\cos(xz)\right) = 0$$

$$\cos(xz)\frac{\partial z}{\partial x} - z\,\text{sen}(xz)\cdot\frac{\partial}{\partial x}(xz) = 0$$

$$\cos(xz)\frac{\partial z}{\partial x} - z\,\text{sen}(xz)\cdot\frac{\partial x}{\partial x}z + x\frac{\partial z}{\partial x} = 0$$

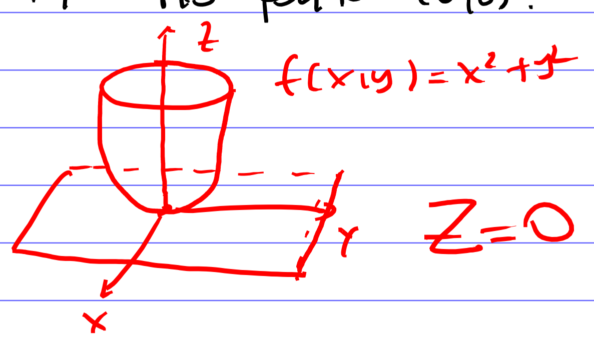
$$\cos(xz)\frac{\partial z}{\partial x} - z\,\text{sen}(xz)\cdot z + x\frac{\partial z}{\partial x} = 0$$

$$\frac{\partial z}{\partial x}\left(\cos(xz) + x\right) = z^2\,\text{sen}(xz)$$

$$\boxed{\frac{\partial z}{\partial x} = \frac{z^2\,\text{sen}(xz)}{\cos(xz) + x}}$$

Plano Tangente

3. Calcule o plano tangente à superfície $f(x,y) = x^2+y^2$ no ponto $(0,0)$.



$(x,y,0)$

$f(x,y)$ é a superfície

$$Z = Z_0 + \frac{\partial f}{\partial x}\Big|_{(x_0,y_0)}(x-x_0) + \frac{\partial f}{\partial y}\Big|_{(x_0,y_0)}(y-y_0)$$

É a eq. do plano tangente à superfície no ponto (x_0,y_0)

Resolvendo:

$f(x,y) = x^2+y^2$

$\frac{\partial f}{\partial x} = 2x \quad \Big\} \quad \frac{\partial f}{\partial y} = 2y$

$\frac{\partial f}{\partial x}\Big|_{(0,0)} = 0 \quad \Big\} \quad \frac{\partial f}{\partial y}\Big|_{(0,0)} = 0$

$Z_0 = ?$ se $x_0 = 0$ e $y_0 = 0$ $\quad Z_0 = x_0^2 + y_0^2$

$Z_0 = 0^2 + 0^2 = 0$

$$Z = Z_0 + \frac{\partial f}{\partial x}\Big|_{(x_0,y_0)}(x-x_0) + \frac{\partial f}{\partial y}\Big|_{(x_0,y_0)}(y-y_0)$$

$$Z = 0 + 0(x-0) + 0(y-0)$$

$$\boxed{Z=0}$$

4. Determine uma equação do plano tangente à superfície do ponto especificado

$Z = xe^{xy} \quad (2,0,2)$

(x_0, y_0, z_0)

$f(x,y) = xe^{xy}$

$$\frac{\partial f}{\partial x}\Big|_{(2,0)} = \frac{\partial}{\partial x}(xe^{xy})\Big|_{(2,0)} = \left[\frac{\partial}{\partial x}x(e^{xy}) + x\frac{\partial}{\partial x}(e^{xy})\right]\Big|_{(2,0)}$$

$$= \left[e^{xy} + xye^{xy}\right]\Big|_{(2,0)}$$

$$= \left[e^0 + 2 \cdot 0\, e^0\right]$$

$$\frac{\partial f}{\partial x}\Big|_{(2,0)} = 1$$

$$\frac{\partial f}{\partial y}\Big|_{(2,0)} = \frac{\partial}{\partial y}(xe^{xy})\Big|_{(2,0)} = x\frac{\partial}{\partial y}(e^{xy})\Big|_{(2,0)}$$

$$= x\, e^{xy} \cdot x\Big|_{(2,0)} = 4e^0 = 4$$

$$Z = Z_0 + \frac{\partial f}{\partial x}\Big|_{(2,0)}(x-x_0) + \frac{\partial f}{\partial y}\Big|_{(2,0)}(y-y_0)$$

$$Z = 2 + 1(x-2) + 4(y-0)$$

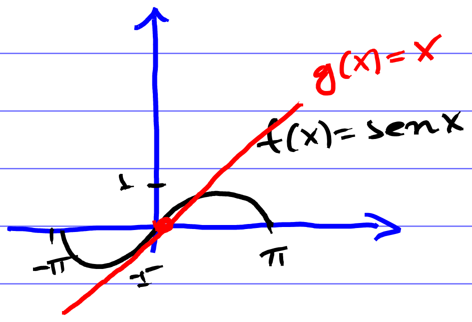
$$Z = 2 + (x-2) + 4y$$

$$Z = 2 + x - 2 + 4y$$

$$\boxed{Z = x+4y}$$

https://www.geogebra.org/3d/z5thncxs

Aproximação Linear

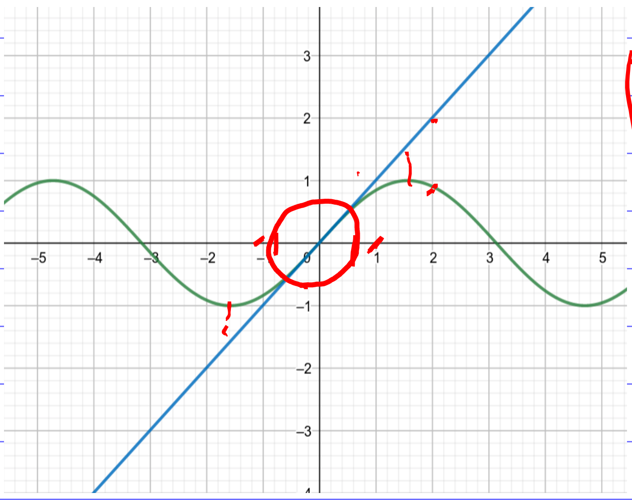


$\operatorname{Sen} x \approx x$

sin(0,02)

0,019998667

Em torno de x=0



$$\boxed{Y = Y_0 + \frac{df}{dx}\Big|_{x_0}(x-x_0)}$$

5. Qual é a aproximação linear em (0,0) da função $e^x\cos(xy)$

$$Z = z_0 + \frac{\partial f}{\partial x}\Big|_{(x_0,y_0)}(x-x_0) + \frac{\partial f}{\partial y}\Big|_{(x_0,y_0)}(y-y_0)$$

$$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}\left(e^x\cos(xy)\right) = \frac{\partial e^x}{\partial x}\cos(xy) + e^x\frac{\partial}{\partial x}\cos(xy)$$

$$= e^x\cos(xy) - e^x\operatorname{sen}(xy)\,y$$

$$\frac{\partial f}{\partial x} = e^x\cos(xy) - y\,e^x\operatorname{sen}(xy)$$

$$\frac{\partial f}{\partial x}\Big|_{(0,0)} = e^0\cos(0) - 0\,e^0\operatorname{sen}(0)$$

$$= 1$$

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}\left(e^x\cos(xy)\right) = e^x\frac{\partial}{\partial y}\cos(xy)$$

$$= -e^x\operatorname{sen}(xy)\cdot x$$

$$\frac{\partial f}{\partial y}\Big|_{(0,0)} = -e^0\operatorname{sen}(0)\cdot 0$$

$$= 0$$

$$Z = z_0 + \frac{\partial f}{\partial x}\Big|_{(x_0,y_0)}(x-x_0) + \frac{\partial f}{\partial y}\Big|_{(x_0,y_0)}(y-y_0)$$

$$z_0 = e^{x_0}\cos(x_0y_0) \;\therefore\; z_0 = e^0\cos(0)$$

$$\therefore\; z_0 = 1$$

$$Z = 1 + 1(x-0) + 0(y-0)$$

$Z = 1 + x$ eq. do plano tangente

$f(x,y) = e^x\cos(xy) \approx 1 + x$

em torno de (0,0)

$f(0.01, 0.2)$

$f(-0.1, -0.01)$

$f(0,03, 0.02)$