17–18 Encontre uma equação polar para a curva representada pela equação cartesiana dada.

17. $x + y = 2$ **18.** $x^2 + y^2 = 2$

$x = r\cos\theta$

$y = r\,\text{sen}\,\theta$

$x + y = 2$

$r\cos\theta + r\,\text{sen}\,\theta = 2$

$r(\cos\theta + \text{sen}\,\theta) = 2$

$$\boxed{r = \frac{2}{\cos\theta + \text{sen}\,\theta}}$$

$r^2 = 2$

$\boxed{r = \sqrt{2}}$ ou

$\boxed{r = -\sqrt{2}}$

↑

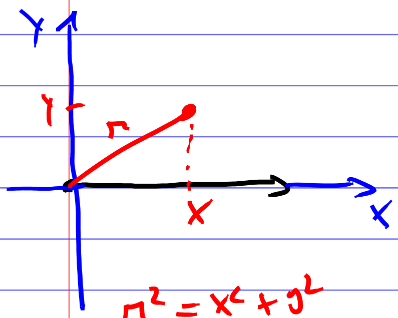
Isso não significa raio negativo.

Essas duas expressões são equivalentes em coord. polares.

Outro modo

$x = r\cos\theta$

$y = r\,\text{sen}\,\theta$



$r^2 = x^2 + y^2$

$x^2 + y^2 = 2$

$(r\cos\theta)^2 + (r\,\text{sen}\,\theta)^2 = 2$

$r^2\cos^2\theta + r^2\,\text{sen}^2\theta = 2$

$r^2(\cos^2\theta + \text{sen}^2\theta) = 2$

$r^2 = 2$

1–4 Encontre a área da região que é delimitada pelas curvas dadas e está no setor especificado.

1. $r = \theta^2, \quad 0 \leqslant \theta \leqslant \pi/4$

$$A = \int_0^{\pi/4} \frac{1}{2} r^2 \, d\theta$$

$$A = \int_0^{\pi/4} \frac{1}{2} \theta^4 \, d\theta$$

$$A = \frac{1}{2} \frac{\theta^5}{5} \Big|_0^{\pi/4} = \frac{1}{10} \left[\left(\frac{\pi}{4}\right)^5 - 0^5 \right]$$

$$A = \frac{1}{10} \cdot \frac{\pi^5}{4^5} \quad u.a$$

45–48 Calcule o comprimento exato da curva polar.

45. $r = 2\cos\theta, \quad 0 \leqslant \theta \leqslant \pi$ **46.** $r = 5^{\theta}, \quad 0 \leqslant \theta \leqslant 2\pi$

$$L = \int_{\theta_i}^{\theta_f} \sqrt{r^2 + r'^2}\, d\theta$$

45. $r = 2\cos\theta \therefore r' = -2\,\text{sen}\,\theta$

$r^2 = 4\cos^2\theta$ e $r'^2 = 4\,\text{sen}^2\theta$

$$L = \int_0^{\pi} \sqrt{4\cos^2\theta + 4\,\text{sen}^2\theta}\, d\theta$$

$$L = \int_0^{\pi} \sqrt{4}\, d\theta$$

(Uma Propriedade da Integral Definida)

$$L = \int_0^{\pi} 2\, d\theta$$

Modo 1 → $2(\pi - 0) = 2\pi$

Modo 2 (TFC) → $2\theta \Big|_0^{\pi} = 2\pi - 0 = 2\pi$

46. $r = 5^{\theta} \qquad 0 \leq \theta \leq 2\pi$

$$L = \int_0^{2\pi} \sqrt{r^2 + r'^2}\, d\theta$$

$r = 5^{\theta} \therefore r^2 = 5^{2\theta}$

$r' = 5^{\theta} \cdot \ln 5 \therefore r'^2 = 5^{2\theta}(\ln 5)^2$

$$L = \int_0^{2\pi} \sqrt{5^{2\theta} + 5^{2\theta}(\ln 5)^2}\, d\theta$$

$$L = \int_0^{2\pi} \sqrt{5^{2\theta}(1 + [\ln 5]^2)}\, d\theta$$

$$L = \int_0^{2\pi} 5^{\theta} \sqrt{1 + [\ln 5]^2}\, d\theta$$

$$L = \sqrt{1 + [\ln 5]^2} \int_0^{2\pi} 5^{\theta}\, d\theta$$

$$L = \sqrt{1 + [\ln 5]^2}\, \frac{5^{\theta}}{\ln 5} \Big|_0^{2\pi}$$

$$L = \sqrt{1 + [\ln 5]^2} \left(\frac{5^{2\pi}}{\ln 5} - \frac{5^{0}}{\ln 5} \right)$$

$$L = \frac{\sqrt{1 + [\ln 5]^2}\,(5^{2\pi} - 1)}{\ln 5} \quad \text{u.c.}$$

54. Associe as curvas polares com seus respectivos gráficos I–VI. Dê razões para suas escolhas. (Não use uma ferramenta gráfica.)

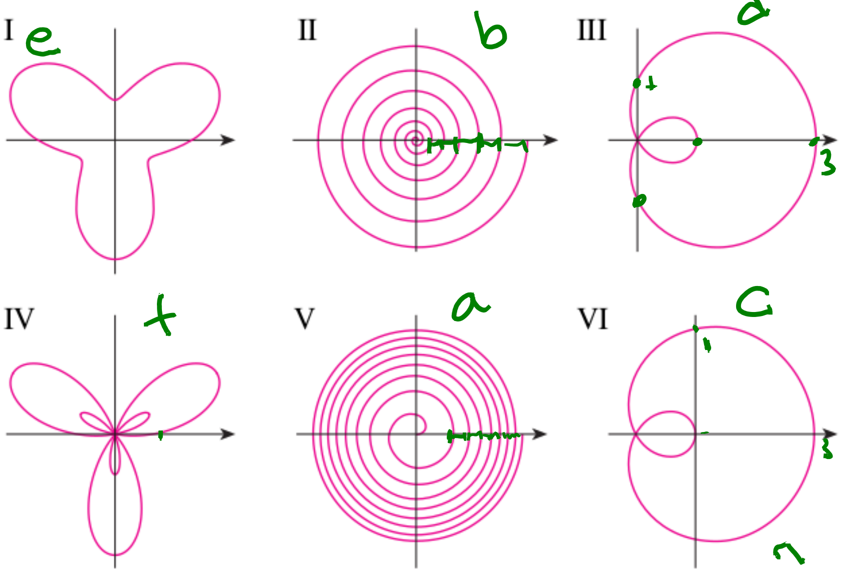
(a) $r = \sqrt{\theta}, \quad 0 \leqslant \theta \leqslant 16\pi$ (b) $r = \theta^2, \quad 0 \leqslant \theta \leqslant 16\pi$

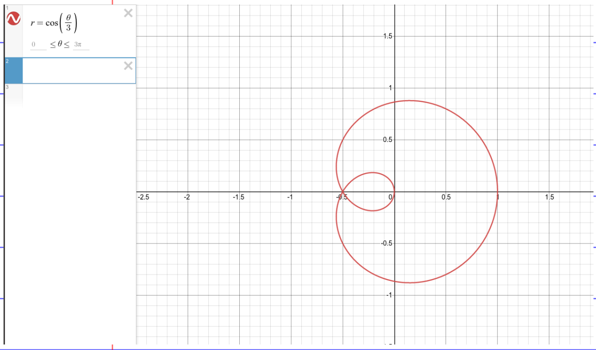
(c) $r = \cos(\theta/3)$ (d) $r = 1 + 2\cos\theta$

(e) $r = 2 + \text{sen } 3\theta$ (f) $r = 1 + 2 \text{ sen } 3\theta$

$\frac{\theta}{3} = \frac{\pi}{2}$

$\theta = 3\frac{\pi}{2}$





$r = 1 + 2\cos\theta$

θ	$1+2\cos\theta$
0	3
$\frac{\pi}{2}$	1
π	-1
$\frac{3\pi}{2}$	1
2π	3

d

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