Regra da Cadeia

$$f = f(x) \qquad x = x(t)$$

$$f = f(x(t))$$

$$\frac{df}{dt} = \frac{df}{dx} \cdot \frac{dx}{dt}$$

'De fora' 'De dentro'

$$f = f(x, y, z)$$

$$x = x(t) \qquad y = y(t) \quad e \quad z = z(t)$$

$$f(x(t), y(t), z(t)) = f(t)$$

$$\frac{df}{dt} = \frac{\partial f}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt} + \frac{\partial f}{\partial z}\frac{dz}{dt}$$

Exercíco

$$f(x, y, z) = x^2 y^2 z$$

$$x(t) = t^3$$

$$y(t) = t$$

$$\frac{df}{dt} = ? \qquad z(t) = e^t$$

Diretamente

$$f(x, y, z) = x^2 y^2 z$$

$$f = t^6 t^2 e^t$$

$$f(t) = t^8 e^t$$

$$\frac{df}{dt} = \frac{d}{dt}(t^8 e^t) = 8t^7 e^t + t^8 e^t$$

$$\boxed{\frac{df}{dt} = e^t(8t^7 + t^8)}$$

Regra da Cadeia

$$\frac{df}{dt} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt} + \frac{\partial f}{\partial z}\frac{dz}{dt}$$

$$\frac{df}{dt} = 2xy^2 z \cdot 3t^2 + x^2 2yz\, 1 + x^2 y^2 e^t$$

$$\frac{df}{dt} = 2t^3 t^2 e^t 3t^2 + t^6 2t e^t + t^6 t^2 e^t$$

$$\frac{df}{dt} = e^t(6t^7 + 2t^7 + t^8)$$

$x(t) = t^3$
$y(t) = t$
$z(t) = e^t$

$$= e^t(8t^7 + t^8)$$

Regra da Cadeia

1. Sabendo que $h=h(m,n)$ e que $m=m(u)$ e $n=n(u)$, escreva a expressão para o cálculo de

$$\frac{dh}{du}$$

$$\frac{dh}{du} = \frac{\partial h}{\partial m}\frac{dm}{du} + \frac{\partial h}{\partial n}\frac{dn}{du}$$

Expresse $\frac{\partial w}{\partial r}$ e $\frac{\partial w}{\partial s}$ em termos de *r* e *s*.

$w = x + 2y + z^2$

$x(r,s) = \frac{r}{s}$

$y(r,s) = r^2 + \ln s$

$z(r,s) = 2r$

$W = W(x, y, z)$

$x = x(r, s)$

$y = y(r, s)$

$z = z(r, s)$

$$\frac{\partial w}{\partial r} = \frac{\partial w}{\partial x} \cdot \frac{\partial x}{\partial r} + \frac{\partial w}{\partial y} \cdot \frac{\partial y}{\partial r} + \frac{\partial w}{\partial z} \cdot \frac{\partial z}{\partial r}$$

$$= 1 \cdot \frac{1}{s} + 2 \, 2r + 2z \cdot 2$$

$$\frac{\partial w}{\partial r} = \frac{1}{s} + 4r + 4z$$

$$= \frac{1}{s} + 4r + 4(2r)$$

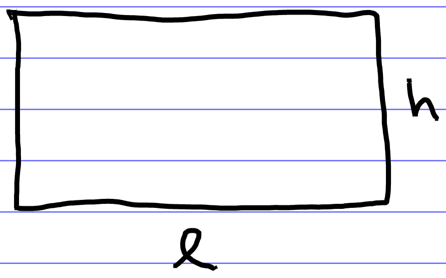
$$\boxed{\frac{\partial w}{\partial r} = \frac{1}{s} + 12r}$$

$$\frac{\partial w}{\partial s} = \frac{\partial w}{\partial x} \cdot \frac{\partial x}{\partial s} + \frac{\partial w}{\partial y} \cdot \frac{\partial y}{\partial s} + \frac{\partial w}{\partial z} \cdot \frac{\partial z}{\partial s}$$

Vocês tentarão fazer em casa.

Regra da Cadeia

Os lados de um retângulo imaginário variam com o tempo. A largura varia a uma taxa de 4 t m/s e altura varia a uma taxa de 5 t^2 m/s. A que taxa varia a área do retângulo no instante t=2, quando sua altura mede 2 m e sua largura mede 3 m ?



$$A(l,h) = lh$$

$$\frac{dl}{dt} = 4t \qquad \frac{dh}{dt} = 5t^2$$

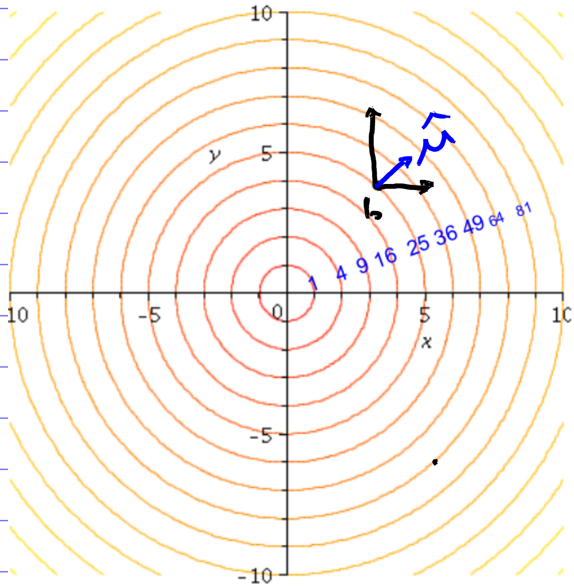
$$\frac{dA}{dt} = \frac{\partial A}{\partial l}\frac{dl}{dt} + \frac{\partial A}{\partial h}\frac{dh}{dt}$$

$$\frac{dA}{dt} = h \cdot 4t + l \, 5t^2$$

$$\left.\frac{dA}{dt}\right|_{t=2} = 2 \cdot 4 \cdot 2 + 3 \cdot 5 \cdot 2^2 = 16 + 60$$

$$= 76 \, \frac{m^2}{s}$$

Derivada direcional



$$\left.\frac{\partial f}{\partial y}\right|_{P_0} > 0$$

$$\left.\frac{\partial f}{\partial x}\right|_{P_0} > 0$$

$\hat{u}$ = versor . vetor de módulo 1 usado p/ indicar direção.

$$\left(\frac{df}{ds}\right)_{P_0,\hat{u}} = \lim_{s\to 0} \frac{f(x+su_1, y+su_2) - f(x,y)}{s}$$

$$\hat{u} = \langle u_1, u_2 \rangle$$

$$\left(\frac{df}{ds}\right)_{P_0,\hat{u}} = \underbrace{\left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right\rangle}_{\text{Vetor Gradiente}} \circ \hat{u}$$

$$\left.\left(\frac{df}{ds}\right)\right|_{P_0,\hat{u}} = \left.\vec{\nabla} f\right|_{P_0} \circ \hat{u}$$

Exercício 1: Encontre a derivada da função em P_0 na direção de ***A***.

a) $f(x,y)=2xy-3y^2 \quad P_0(5,5), \quad \vec{A}=4\hat{\imath}+3\hat{\jmath}$

b) $f(x,y,z)=xy+yz+zx$

$P_0(1,-1,2), \quad \vec{A}=3\hat{\imath}+6\hat{\jmath}-2\hat{k}$

a) $\vec{\nabla} f = \left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right\rangle$

$= \langle 2y, 2x-6y \rangle$

$\vec{\nabla} f \Big|_{(5,5)} = \langle 10, 10-30 \rangle$

$= \langle 10, -20 \rangle$

$\hat{u} = \frac{\langle 4,3 \rangle}{\sqrt{4^2+3^2}} = \frac{\langle 4,3 \rangle}{\sqrt{25}} = \left\langle \frac{4}{5}, \frac{3}{5} \right\rangle$

$\left(\frac{df}{ds}\right)\Big|_{P_0,\hat{u}} = \langle 10,-20 \rangle \circ \left\langle \frac{4}{5}, \frac{3}{5} \right\rangle$

$= \frac{40}{5} - \frac{60}{5} = 8 - 12 = -4$

(2) $f(x,y,z) = xy + yz + zx$

$P_0(1,-1,2) \quad \vec{A} = \langle 3, 6, -2 \rangle$

$\hat{u} = \frac{\langle 3,6,-2 \rangle}{\sqrt{9+36+4}} = \left\langle \frac{3}{7}, \frac{6}{7}, \frac{-2}{7} \right\rangle$

$\vec{\nabla} f = \left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right\rangle = \langle y+z, x+z, y+x \rangle$

$\vec{\nabla} f \Big|_{(1,-1,2)} = \langle -1+2, 1+2, -1+2 \rangle$

$= \langle 1, 3, 1 \rangle$

$\left(\frac{df}{ds}\right)\Big|_{P_0,\hat{u}} = \vec{\nabla} f \Big|_{P_0} \circ \hat{u}$

$= \langle 1,3,1 \rangle \circ \left\langle \frac{3}{7}, \frac{6}{7}, \frac{-2}{7} \right\rangle$

$= \frac{3}{7} + \frac{18}{7} - \frac{2}{7} = \frac{19}{7}$

$= \frac{19}{7}$