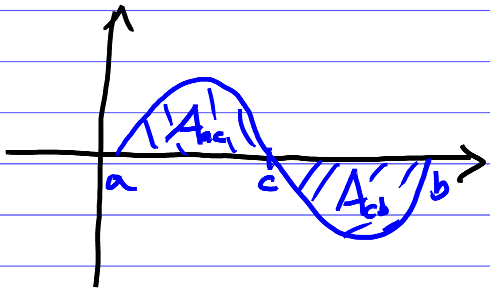
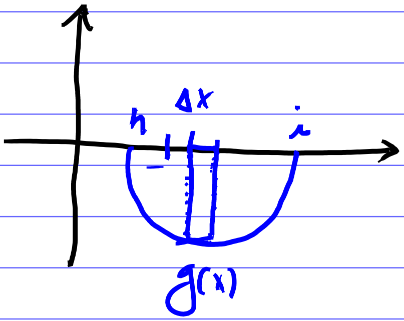
Área Líquida

Considere $f(x) \in \mathbb{R}$



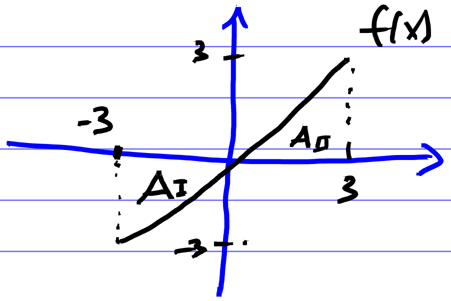
$$\int_a^b f(x)\,dx = ?$$

$$\int_a^b f(x)\,dx = A_{ac} - A_{cb}$$



$$\int_h^i g(x)\,dx = \lim_{n \to \infty} \sum_{j=1}^{n} \underbrace{g(x_j)\Delta x}$$

Área do retângulo cuja altura será negativa.

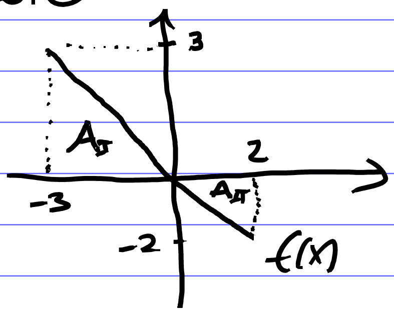


$$\int_{-3}^{3} f(x)\,dx$$

$A_I = \frac{(3)(3)}{2} = \frac{9}{2}$ $\qquad$ $A_{II} = \frac{(3)(3)}{2} = \frac{9}{2}$

$$\int_{-3}^{3} f(x)\,dx = -A_I + A_{II} = -\frac{9}{2} + \frac{9}{2} = 0$$

Exemplo: 1

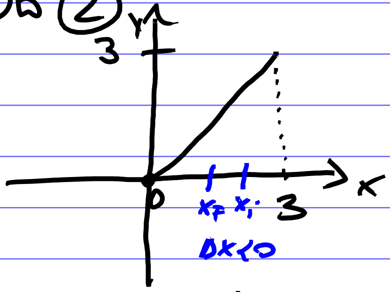


$$\int_{-3}^{2} f(x)\,dx \overset{?}{=} A_I - A_{II}$$

$$= \frac{9}{2} - 2$$

$$= \frac{9-4}{2} = \frac{5}{2} = 2.5 \text{ u.a.}$$

Exemplo 2



$$\int_{3}^{0} f(x)\,dx$$

$$\int_{a}^{b} f(x)\,dx = -\int_{b}^{a} f(x)\,dx$$

$$\int_{3}^{0} f(x)\,dx = -\int_{0}^{3} f(x)\,dx$$

$$= -4.5$$

Diretamente

$$\int_{3}^{0} f(x)\,dx = \lim_{n\to+\infty} \sum_{i=1}^{n} f(x_i)\,\Delta x$$

Note que nesse caso

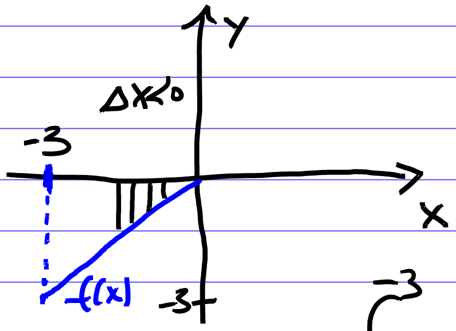
$\Delta x < 0$

é negativo.

$\Delta x = x_F - x_i$

$\Delta 3 = 3_F - 3_i$

Exemplo ③



$$\int_0^{-3} f(x)\,dx = 4.5$$

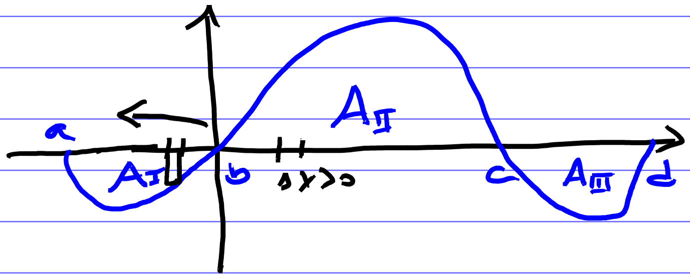
$$\boxed{\int_0^{-3} \underbrace{f(x)}\,\underbrace{dx}} = \lim_{n\to+\infty} \sum_{i=1}^{n} \underbrace{f(x_i)}_{<0}\,\underbrace{\Delta x}_{<0}$$

O produto é positivo

Altura   Base

do retângulo

Exemplo ④



$$\int_a^b f(x)\,dx = -A_I \qquad \int_b^a f(x)\,dx = +A_I$$

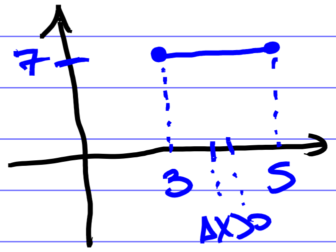
$$\int_b^c f(x)\,dx = A_{II} > 0 \qquad \int_c^b f(x)\,dx = -A_{II}$$

$$\int_b^d f(x)\,dx = A_{II} - A_{III} > 0$$

Outra propriedade:

$$\int_a^b c\,dx = c(b-a)$$

Exemplo



$$\int_3^5 f(x)\,dx$$

$$= \int_3^5 7\,dx = 7(5-3)$$

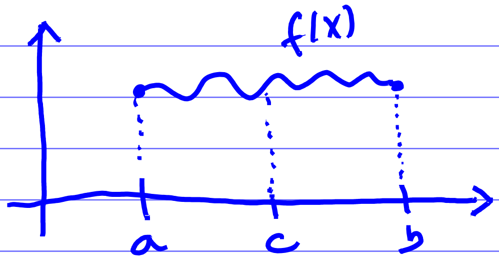
$$= 7(2)$$

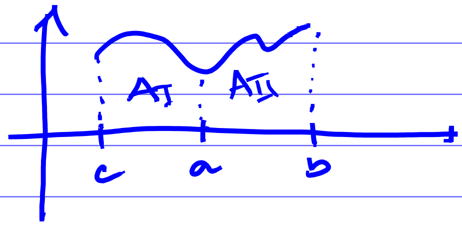
$$= 14$$

$$\int_5^3 7\,dx = 7(-2) = -14$$

Outra propriedade:

$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$





$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$

$$= -A_I + A_I + A_{II}$$

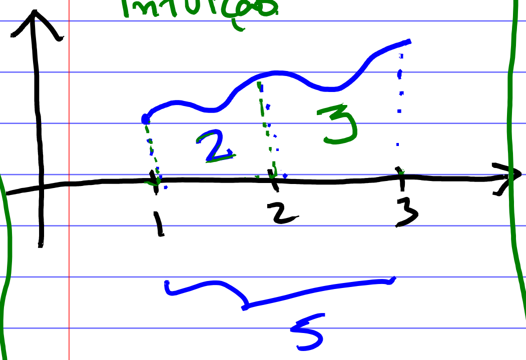
$$= A_{II}$$

Exemplo:

$$\int_1^3 f(x)\,dx = 5 \qquad \int_1^2 f(x)\,dx = 2$$

$$\int_2^3 f(x)\,dx = ?$$

Intuição



Atenção: eu "chutei" uma função para usar a intuição.

Usando a Propriedade

$$\int_2^3 f(x)\,dx = \int_2^1 f(x)\,dx + \int_1^3 f(x)\,dx$$

$$= -2 + 5$$

$$= 3$$

$$\boxed{\int_2^3 f(x)\,dx = 3}$$